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# Correlated Parameters in Driving Behavior Models

## Car-Following Example and Implications for Traffic Microsimulation

Jiwon Kim and Hani S. Mahmassani

Behavioral parameters in car following and other models of driving behavior are expected to be correlated. An investigation is conducted into the effect of ignoring correlations in three parameters of car-following models on the resulting movement and properties of a simulated heterogeneous vehicle traffic stream. For each model specification, parameters are calibrated for the entire sample of individual drivers with Next Generation Simulation trajectory data. Factor analysis is performed to understand the pattern of relationships between parameters on the basis of calibrated data. Correlation coefficients have been used to show statistically significant correlation between the parameters. Simulation experiments are performed with vehicle parameter sets generated with and without considering such correlation. First, parameter values are sampled from the empirical mass functions, and simulated results show significant difference in output measures when parameter correlation is captured (versus ignored). Next, parameters are sampled under the assumption that they follow the multivariate normal distribution. Results suggest that the use of parametric distribution with known correlation structure may not sufficiently reduce the error due to ignoring correlation if the underlying assumption does not hold for both marginal and joint distributions.

Microsimulation has long been recognized as an effective approach to the analysis of vehicular traffic flow because individual driving behaviors can be captured and interaction between vehicles is reflected in the resulting collective traffic flow pattern. Accordingly, many microscopic models of driving behavior—including car-following, lane-changing, acceleration and deceleration, gap acceptance, and merging models—have been incorporated in traffic microsimulation tools (1–3). Typically, these models include numerous parameters that must be calibrated against real data and provided as input to the simulation (4–6). Many existing simulators adopt Monte Carlo sampling of parameter values to model heterogeneous drivers by allowing users to specify a marginal distribution for each parameter, represented by the mean and standard deviation (SD) or, in some instances, by empirical mass functions (1, 2). Concerns may arise when a set of

individual parameter values is independently and randomly drawn for each driver from the separate marginal distributions under the (explicit or implicit) assumption that parameters are uncorrelated.

Behavioral parameters in car-following and other models of driving behavior are expected to be correlated, to the extent that these parameters may reflect underlying factors such as risk aversion or personality characteristics. For example, a driver with long reaction time may also exhibit a high tendency to decelerate to avoid collision. Hence the likelihood of particular parameter combinations reflects particular correlation patterns that characterize the driving population. The presence of correlations affects the shape of the joint distribution (i.e., the sampling space for the parameter set). When parameter correlation is not considered, the sampling space enlarges, thereby allowing the simulator to produce unrealistic parameter combinations (which may not exist in real-world traffic) or to generate a biased representation of the driving population. In other words, independent draws of individual model parameters may produce driving behaviors that have no counterpart (or that have a low likelihood of occurring) in the real world. Therefore, ignoring the correlations of input parameters in the random sampling procedure could lead to unrealistic system performance, biased simulation outputs, and erroneous interpretations.

Image-processing techniques used in collecting microscopic trajectory data enable many in-depth empirical studies to explore heterogeneous car-following behaviors beyond model parameter calibration. Ossen and Hoogendoorn find considerable differences in the car-following behaviors of individual drivers by estimating optimal parameter values of different specifications of the Gazis–Herman–Rothery model for each driver (7). Hamdar et al. introduce a cognitive-based stochastic car-following model in which acceleration is determined by evaluating the probability of rear-end collision with candidate acceleration (8). They reveal the existence of considerable interindividual variation and correlation among behavioral parameters. Hamdar presents detailed distributions and a corresponding correlation matrix (9).

The next logical questions to ask are, how can such heterogeneity be generated in microsimulation models? What is the impact of parameter correlation on the resulting traffic stream properties and simulation results—and conversely, what happens when such correlation is ignored? How can such correlation be captured in traffic microsimulation? These questions, which have not been addressed in previous studies, are the focus of this paper. In particular, these questions are addressed in connection with car-following models. Thus, an investigation is conducted into the effect of ignoring correlations in the parameters of car-following models on the resulting movement and properties of a simulated heterogeneous vehicle traffic stream.

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Three car-following model specifications were selected: the Gipps model (10), the Helly model (11), and the intelligent driver model (IDM) (12, 13). For each model specification, the parameters were calibrated for the entire sample of individual drivers using Next Generation Simulation (NGSIM) trajectory data (14). These results form the basis for investigating three main aspects: understanding qualitative relationships among parameters in selected car-following models and identifying correlation structures, comparing simulated car-following behaviors between parameter sets with and without preserving correlations, and evaluating the performance of conventional sampling approaches with parametric distribution assumption. Statistical hypothesis tests are conducted to determine the significance of differences in output measures with respect to sampling methods for correlated input parameters.

## SELECTED MODELS

For calibration and subsequent correlation analysis, three car-following models were selected: Gipps, Helly, and IDM. The model specifications are presented in the following equations.

### Gipps Model

$$v_{a,n}(t+T) = v_n(t) + 2.5 aT \left( 1 - \frac{v_n(t)}{V^*} \right) \sqrt{0.025 + \frac{v_n(t)}{V^*}} \quad (1)$$

$$v_{b,n}(t+T) = dT + \sqrt{d^2 T^2 - d \left[ \frac{2\{x_{n-1}(t) - (l_{n-1} + s^*) - x_n(t)\}}{-v_n(t)T - \frac{v_{n-1}(t)^2}{\alpha d}} \right]} \quad (2)$$

$$v_n(t+T) = \min\{v_{a,n}(t+T), v_{b,n}(t+T)\} \quad (3)$$

### Helly Linear Model

$$\begin{aligned} a_n(t) &= C_1 [v_{n-1}(t-T) - v_n(t-T)] \\ &\quad + C_2 [x_{n-1}(t-T) - x_n(t-T) - D_n(t)] \\ D_n(t) &= (d^* + l_{n-1}) + \gamma v_n(t-T) \end{aligned} \quad (4)$$

### IDM

$$a_n(t) = a \left[ \begin{aligned} &1 - \left( \frac{v_n(t)}{V_0} \right)^4 \\ &- \left[ \frac{s_0 + \left( v_n(t) T_{HW} + \frac{v_n(t)[v_n(t) - v_{n-1}(t)]}{2\sqrt{ab}} \right)^2}{x_{n-1}(t) - l_{n-1}(t) - x_n(t)} \right] \end{aligned} \right] \quad (5)$$

where

$a_n(t)$  = acceleration of follower  $n$  (and leader  $n-1$ ) at time  $t$ ,  
 $v_n(t)$  = speed of follower  $n$  (and leader  $n-1$ ) at time  $t$ ,  
 $x_n(t)$  = location of follower  $n$  (and leader  $n-1$ ) at time  $t$ , and  
 $l_n$  = physical length of vehicle  $n$ .

The parameters to be estimated for each model are listed in Table 1.

## MODEL CALIBRATION

For model calibration, the downhill simplex (gradient-free optimization) method was used to obtain optimal parameters that minimize the objective function in Equation 6, which is the discrepancy between the simulated value and the observed value for the speed and the location (15). For each follower-leader pair with no lane changing in the NGSIM trajectory data, three parameter sets for the Gipps model, the Helly model, and the IDM are estimated separately. To prevent the algorithm from falling into local minima with

TABLE 1 Model Parameters for Estimation

Model	Parameter No.	Parameter	Description
Gipps	1	$T$	Reaction time
	2	$a$	Maximum acceleration
	3	$d$	Maximum desirable deceleration ( $< 0$ )
	4	$V^*$	Desired speed
	5	$s^*$	Minimum net stopped distance from the leader
	6	$\alpha$	Sensitivity factor: $\alpha d = \hat{d}_{n-1}$ , where $\hat{d}_{n-1}$ is the leader's desired deceleration estimated by the follower $n$ . When $\alpha < 1$ , the vehicle underestimates the deceleration of the leader and becomes more aggressive; more careful when $\alpha > 1$ (1)
Helly	1	$T$	Reaction time
	2	$C_1$	Constant for the relative speed
	3	$C_2$	Constant for the spacing
	4	$d^*$	Desired net stopped distance from the leader
	5	$\gamma$	Constant for the speed in the desired following distance ( $D_n$ )
IDM	1	$a$	Maximum acceleration
	2	$b$	Desired deceleration
	3	$V_0$	Desired speed
	4	$s_0$	Minimum net stopped distance from the leader
	5	$T_{HW}$	Desired safety time headway

TABLE 2 Boundary Constraints for Parameters

Gipps Model			Helly Model			IDM		
Model	Variable	Parameter	Model	Variable	Parameter	Model	Variable	Parameter
0.2 s <	$T$	< 3.5 s	0.2 s <	$T$	< 3.5 s	0 m/s <sup>2</sup> <	$a$	< 8 m/s <sup>2</sup>
0 m/s <sup>2</sup> <	$a$	< 8 m/s <sup>2</sup>	0.1 <	$C_1$	< 3.0	0 m/s <sup>2</sup> <	$b$	< 8 m/s <sup>2</sup>
-8 m/s <sup>2</sup> <	$d$	< 0 m/s <sup>2</sup>	0.01 <	$C_2$	< 3.0	50 km/h <	$V_0$	< 150 km/h
50 km/h <	$V^*$	< 150 km/h	0 m <	$d^*$	< 10 m	0 m <	$s_0$	< 10 m
0 m <	$s^*$	< 20 m	0 <	$\gamma$	< 2.0	0 s <	$T_{HW}$	< 10 s
0.5 <	$\alpha$	< 2.0						

unreasonable values, boundary constraints are imposed on the parameters. These values are determined from the published literature, summarized in Table 2 (4, 13, 15).

$$F(v_n^{\text{sim}}, \Delta x_n^{\text{sim}}) = \frac{\sqrt{\frac{1}{T} \sum_{t=1}^T (v_{n,t}^{\text{obs}} - v_{n,t}^{\text{sim}})^2}}{\sqrt{\frac{1}{T} \sum_{t=1}^T (v_{n,t}^{\text{obs}})^2} + \sqrt{\frac{1}{T} \sum_{t=1}^T (v_{n,t}^{\text{sim}})^2}} + \frac{\sqrt{\frac{1}{T} \sum_{t=1}^T (\Delta x_{n,t}^{\text{obs}} - \Delta x_{n,t}^{\text{sim}})^2}}{\sqrt{\frac{1}{T} \sum_{t=1}^T (\Delta x_{n,t}^{\text{obs}})^2} + \sqrt{\frac{1}{T} \sum_{t=1}^T (\Delta x_{n,t}^{\text{sim}})^2}} \quad (6)$$

where

$v_{n,t}^{\text{sim}}$  and  $v_{n,t}^{\text{obs}}$  = simulated and observed speeds of vehicle  $n$  at time  $t$ , respectively;

$\Delta x_{n,t}^{\text{sim}}$  and  $\Delta x_{n,t}^{\text{obs}}$  = simulated and observed distances between the leader ( $n-1$ ) and the follower ( $n$ ) at time  $t$ , respectively; and

$T$  = number of observations in trajectory data.

NGSIM trajectory data were collected from video recorded at 10 frames per second on I-80 eastbound in the San Francisco Bay Area of California. The study area consisted of six freeway lanes approximately 500 m long. In the full data set, 45 min of data are available, segmented into three 15-min periods. Data from each period were calibrated separately to check the consistency and reliability of the optimization process. Calibration results show identical distribution patterns for all three data sets. Results with the objective function value greater than or equal to 0.2 were considered invalid and therefore were discarded. The final descriptive statistics are listed in Table 3.

## PARAMETER CORRELATION

### Factor Analysis

Before the correlation structure of the parameters was investigated, a factor analysis was performed to understand how behavioral parameters are influenced by underlying common factors (16). Based on the eigenvalue of factors, factor loading patterns with the two most influential factors are plotted on rotated axes such that the points fall close to Factor 1 or Factor 2 (Figure 1); these two factors account for nearly half of the total variance.

For the Gipps model (Figure 1a), Factor 1 is primarily a measure of  $\alpha$ , which shows negative relationships with  $d$  and  $T$  and might explain why a long reaction time tends to lead to high deceleration due to an abrupt driving maneuver. For the Helly model (Figure 1b), Factor 1 is strongly related to  $C_1$  and  $\gamma$ , which are associated with speed, whereas Factor 2 is strongly related to  $C_2$  and  $d^*$ , which are associated with spacing. The plot suggests a positive relationship between  $C_1$  and  $C_2$  and a negative relationship between  $\gamma$  and  $d^*$  with respect to both factors. For the IDM (Figure 1c), Factor 1 reveals a positive relationship between  $V_0$  and  $s_0$ . If Equation 5 is simplified as  $a_n(t) = a[1 - f(V_0^{-1}) - g(s_0)]$  using functions of  $V_0^{-1}$  and  $s_0$ ,  $g(s_0)$  decreases when  $f(V_0^{-1})$  increases and vice versa, which leads to the positive relationship between  $V_0$  and  $s_0$ , depending on the prevailing traffic mode (i.e., free flow versus car following) (14).

TABLE 3 Descriptive Statistics of Calibrated Parameters

Parameter	Mean	SD	No. Obs.
Gipps			
$T$	0.78	0.42	484
$a$	2.38	1.44	484
$d$	-3.09	1.40	484
$V^*$	76.08	21.42	484
$s^*$	3.43	2.56	484
$\alpha$	1.08	0.32	484
Helly			
$T$	0.44	0.24	803
$C_1$	0.51	0.24	803
$C_2$	0.13	0.11	803
$d^*$	4.85	2.57	803
$\gamma$	0.86	0.46	803
IDM			
$a$	1.41	1.01	465
$b$	2.23	1.85	465
$V_0$	85.72	26.55	465
$s_0$	2.17	1.15	465
$T_{HW}$	1.27	0.51	465

NOTE: Data Sets I (4:00–4:15 p.m.), II (5:00–5:15 p.m.), and III (5:15–5:30 p.m.). For  $d$  in Gipps model, absolute values are used for subsequent analyses throughout this paper. No. Obs. = number of observations.

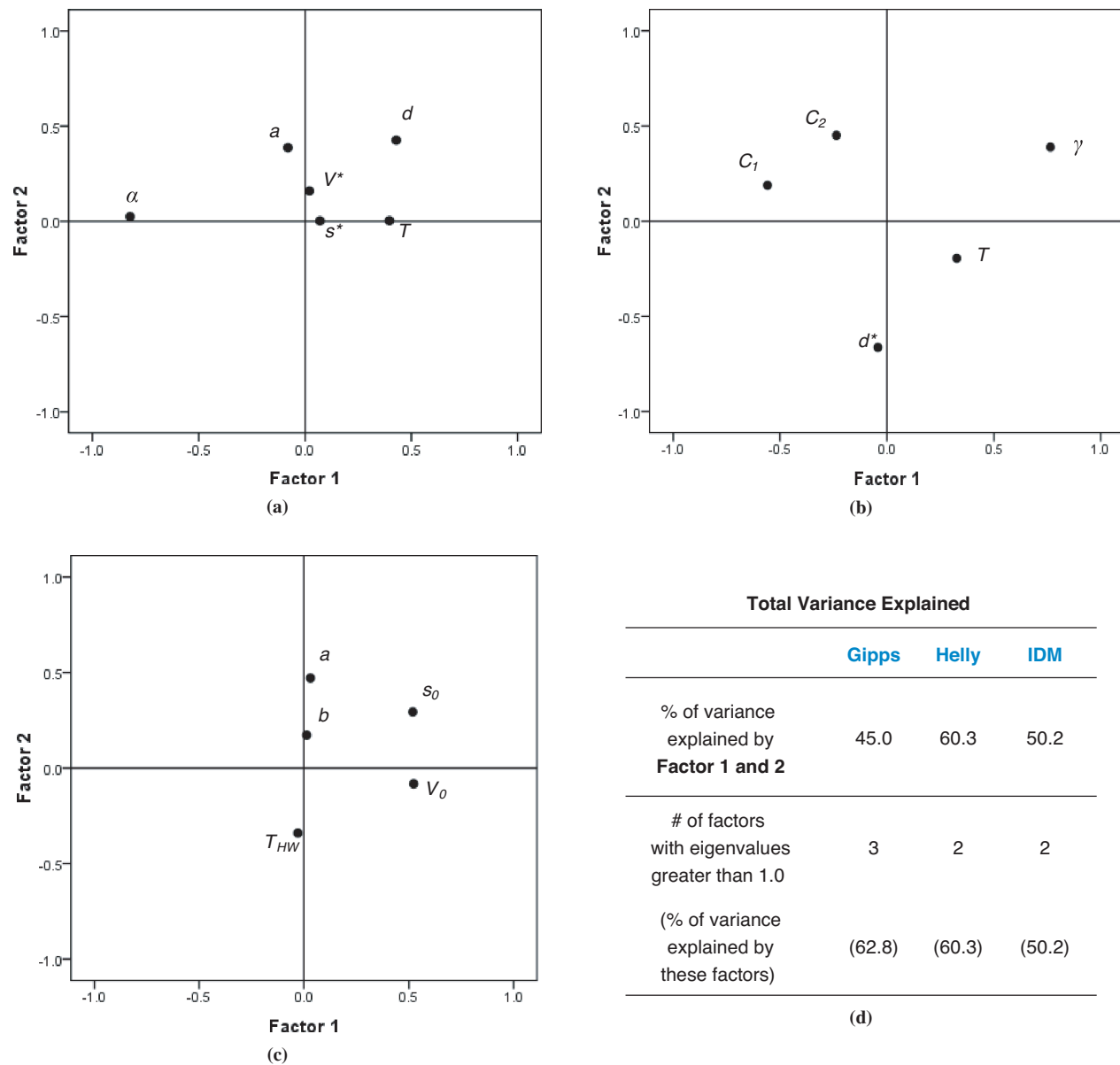


FIGURE 1 Factor plots in rotated factor space for (a) Gipps model, (b) Helly model, and (c) IDM, with (d) total variance explained.

## Correlation Coefficient

Although factor analysis provides useful insight for qualitative interpretations of parameter relationships, correlation coefficients provide direct quantitative measures that can be used in the sampling procedure for simulation. Pearson correlation coefficients for parameters in the three models are presented in Tables 4, 5, and 6, and several parameter pairs show statistically significant correlation. Overall, correlation patterns are consistent with the preceding factor analysis, particularly for pairs with relatively high correlation. Scatter plots for six selected pairs are presented in Figure 2, in which various levels and types of dependencies are observed. Certainly, less-frequent combinations of parameters exist (represented by sparse areas in the plots) whose patterns may not be captured by individual marginal distributions.

The questions addressed in the remainder of the paper are what would happen if the parameter values used in a simulation study were sampled from the sparse areas of the plots (which represent parameter

value combinations not observed in real data) and how to prevent the undesirable consequences of otherwise ignoring these correlations.

## PARAMETER SAMPLING

### Empirical Data

#### Input Data Preparation

To investigate the impact of parameter correlation on the resulting traffic stream and simulation results, a simple simulation experiment is designed using two vehicle sets: one with correlated parameters (in which the original correlation structure in the calibrated parameters is preserved) and another with uncorrelated parameters (in which correlations between parameters are ignored). These two vehicle sets are generated as described below.

TABLE 4 Pearson Correlation Coefficients: Gipps Model

		Gipps					
Parameter		$T$	$a$	$d$	$V^*$	$s^*$	$\alpha$
$T$	Corr. coeff.	1.000	-0.042	0.173	0.037	-0.090	-0.334
	Sig.		0.357	0.000	0.419	0.048	0.000
$a$	Corr. coeff.	-0.042	1.000	0.130	0.062	-0.025	0.068
	Sig.	0.357		0.004	0.176	0.579	0.134
$d$	Corr. coeff.	0.173	0.130	1.000	0.075	0.063	-0.339
	Sig.	0.000	0.004		0.100	0.166	0.000
$V^*$	Corr. coeff.	0.037	0.062	0.075	1.000	-0.023	-0.003
	Sig.	0.419	0.176	0.100		0.610	0.948
$s^*$	Corr. coeff.	-0.090	-0.025	0.063	-0.023	1.000	-0.097
	Sig.	0.048	0.579	0.166	0.610		0.033
$\alpha$	Corr. coeff.	-0.334	0.068	-0.339	-0.003	-0.097	1.000
	Sig.	0.000	0.134	0.000	0.948	0.033	

NOTE: Correlation (corr.) is significant (sig.) at the 0.01 level (2-tailed). Coeff. = coefficient.

Correlation is significant at the 0.05 level (2-tailed).

Because calibrated parameter sets are available for each vehicle from the NGSIM data, 500 randomly sampled vehicles with the entire set of corresponding parameters were used. That is, correlated parameters represent a subset of the original NGSIM data so the population correlation structure is preserved in correlated parameters. For uncorrelated parameters, each parameter is sampled separately from the respective calibrated parameters such that the marginal distribution of the individual parameter remains the same as the NGSIM data but the joint distribution (correlation structure) is not preserved. This procedure is illustrated in Figure 3.

In the Gipps model, however,  $\alpha$  has an instability issue when the ratio between the follower's and leader's deceleration capabilities is high (i.e.,  $\alpha$  is far from 1), thereby requiring certain caution in choosing  $\alpha$  (1). To exclude the effect of a relationship between two parameters, which is a known source of problematic driving behaviors,  $d$  and  $\alpha$  are tied in simulation outputs with uncorrelated parameter sampling, and consequently, the correlation between the two is preserved as in correlated parameters.

Vehicle movement is simulated on a 1,000-m straight single-lane highway section. Given a virtual initial leader with a fixed trajectory (which is created from actual NGSIM data and used identically for

all simulation runs), 500 vehicles are inserted into the highway according to the Poisson process with an interarrival time of 2.0 s. The process of vehicle generation and simulation is replicated 50 times to obtain the distribution of output results for correlated and uncorrelated parameters, respectively. This procedure is repeated for each model specification.

#### Output Performance and Car-Following Behavior Measures

To examine the effect of input parameter sets with and without correlation on the simulation results, six output measures are defined below. The first two measures reflect overall network performance, and the other four reflect car-following dynamics in terms of spacing.

1. Network exit time (s) is the time until all 500 vehicles exit the network (i.e., the last vehicle's exit time).
2. Total travel time (min) is the sum of the net driving time spent by all vehicles after entering the network.

TABLE 5 Pearson Correlation Coefficients: Helly Model

		Helly				
Parameter		$T$	$C_1$	$C_2$	$d^*$	$\gamma$
$T$	Corr. coeff.	1.000	-0.206	-0.177	0.114	0.179
	Sig.		0.000	0.000	0.001	0.000
$C_1$	Corr. coeff.	-0.206	1.000	0.217	-0.108	-0.359
	Sig.	0.000		0.000	0.002	0.000
$C_2$	Corr. coeff.	-0.177	0.217	1.000	-0.282	0.000
	Sig.	0.000	0.000		0.000	0.995
$d^*$	Corr. coeff.	0.114	-0.108	-0.282	1.000	-0.292
	Sig.	0.001	0.002	0.000		0.000
$\gamma$	Corr. coeff.	0.179	-0.359	0.000	-0.292	1.000
	Sig.	0.000	0.000	0.995	0.000	

NOTE: Correlation is significant at the 0.01 level (2-tailed).

TABLE 6 Pearson Correlation Coefficients: IDM

		IDM				
Parameter		$a$	$b$	$V_0$	$s_0$	$T_{HW}$
$a$	Corr. coeff.	1.000	0.126	-0.021	0.152	-0.142
	Sig.		0.006	0.656	0.001	0.002
$b$	Corr. coeff.	0.126	1.000	0.038	0.009	-0.049
	Sig.	0.006		0.412	0.839	0.288
$V_0$	Corr. coeff.	-0.021	0.038	1.000	0.248	0.039
	Sig.	0.656	0.412		0.000	0.399
$s_0$	Corr. coeff.	0.152	0.009	0.248	1.000	-0.143
	Sig.	0.001	0.839	0.000		0.002
$T_{HW}$	Corr. coeff.	-0.142	-0.049	0.039	-0.143	1.000
	Sig.	0.002	0.288	0.399	0.002	

NOTE: Correlation is significant at the 0.01 level (2-tailed).

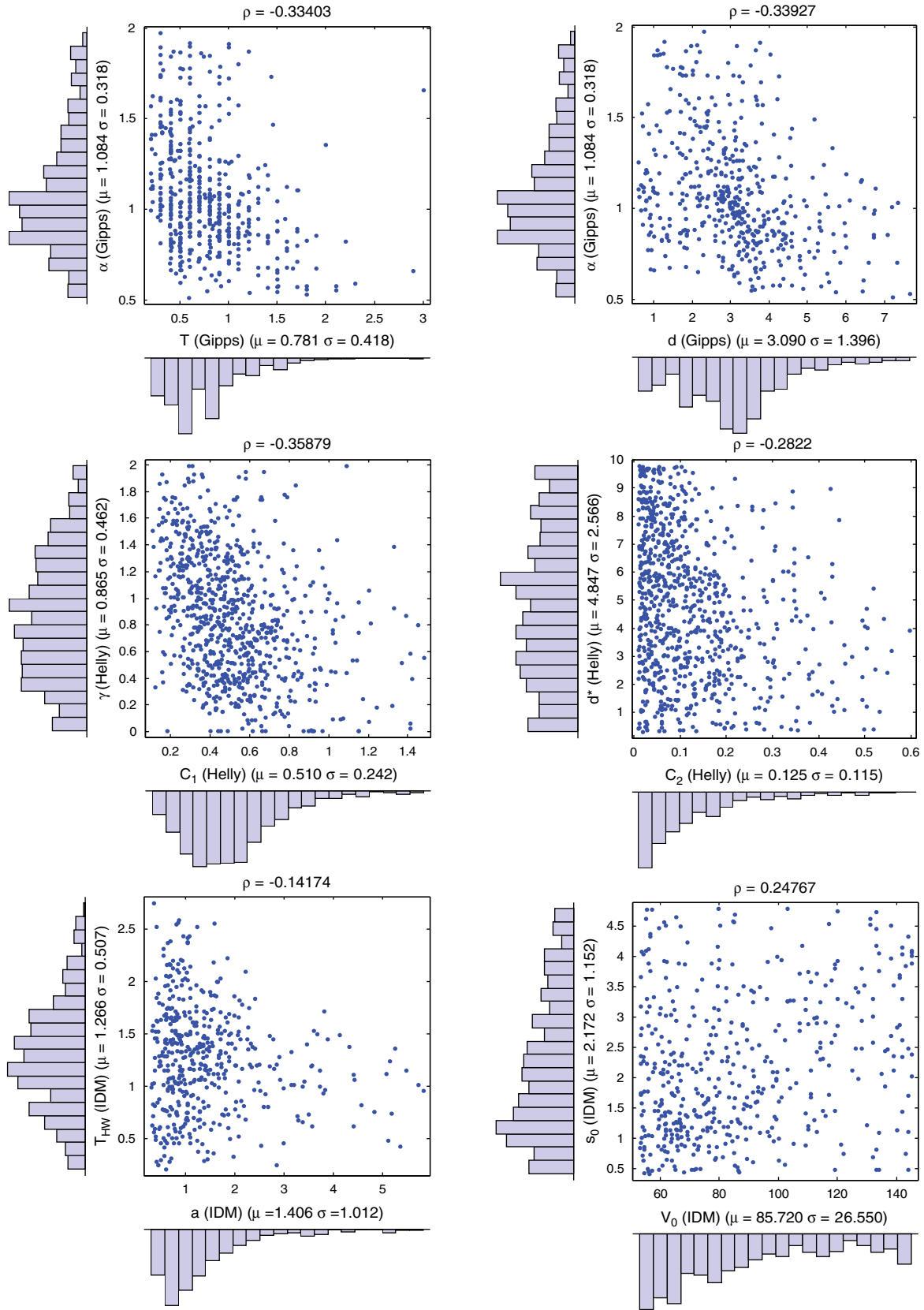


FIGURE 2 Joint distributions of paired parameters.

Vehicles in NGSIM trajectory data

Calibrated parameter set for the Gipps model for each vehicle

	$T$	$a$	$d$	$V^*$	$s^*$	$\alpha$
Vehicle 1	0.8	4.0	4.7	80.4	2.9	1.2
Vehicle 2	0.7	2.3	3.6	104.0	2.3	0.9
Vehicle 3	0.4	3.9	4.3	146.0	4.3	1.1
Vehicle 4	0.5	2.1	3.0	84.9	6.9	1.2
Vehicle 5	1.5	3.3	3.4	54.9	2.5	0.9
Vehicle 6	1.0	7.8	3.3	95.0	1.6	1.0
Vehicle 7	0.5	4.7	1.9	70.8	1.1	1.4
Vehicle 8	1.3	5.6	1.7	80.7	1.9	1.5
Vehicle 9	1.0	3.1	2.9	149.0	3.3	1.1
:	:	:	:	:	:	:
:	:	:	:	:	:	:

Parameter set for one vehicle in CO vehicle set

Parameter set for one vehicle in UC vehicle set

FIGURE 3 Parameter sampling methods for correlated parameters (CO) and uncorrelated parameters (UC). (Note: For each model, 500 vehicles for CO and 500 vehicles for UC are generated.)

3. Mean of average spacing ( $m$ ) is the mean of each vehicle's average spacing, where  $\Delta x_n^{\text{net}}(t)$  is the net distance between the vehicle  $n$  and its leader at time  $t$ ,  $N$  is the number of vehicles, and  $\mu_n^{\text{spacing}}$  denotes the average spacing over time for vehicle  $n$ .

$$\bar{\mu}^{\text{spacing}} = \frac{1}{N} \sum_{n=1}^N (\mu_n^{\text{spacing}}) = \frac{1}{N} \sum_{n=1}^N \left( \frac{1}{T} \sum_{t=1}^T \Delta x_n^{\text{net}}(t) \right)$$

4. SD of average spacing ( $m$ ) is the intervehicle variability.

$$\bar{\sigma}^{\text{spacing}} = \sqrt{\frac{1}{N} \sum_{n=1}^N (\mu_n^{\text{spacing}} - \bar{\mu}^{\text{spacing}})^2}$$

5. Mean of coefficient of variation ( $CV = \sigma/\mu$ ) of spacing is the average intravehicle variation, where  $\sigma_n^{\text{spacing}}$  denotes the SD of the spacing over time for vehicle  $n$  and  $CV_n^{\text{spacing}}$  denotes the ratio of the SD over time to its mean for vehicle  $n$ .

$$\overline{CV}^{\text{spacing}} = \frac{1}{N} \sum_{n=1}^N CV_n^{\text{spacing}} = \frac{1}{N} \sum_{n=1}^N \left( \frac{\sigma_n^{\text{spacing}}}{\mu_n^{\text{spacing}}} \right)$$

6. SD of CV of spacing is the intervehicle variability of average intravehicle variation.

$$SD(CV^{\text{spacing}}) = \sqrt{\frac{1}{N} \sum_{n=1}^N (CV_n^{\text{spacing}} - \overline{CV}^{\text{spacing}})^2}$$

### Simulation Results

To determine whether the output distribution from correlated parameters was significantly different from the output distribution from uncorrelated parameters, the two-sample Kolmogorov–Smirnov (K-S) test was conducted on 100 realizations (50 from correlated parameters and 50 from uncorrelated parameters) of each output measure.

Fisher's exact test was used in the analysis of contingency tables for categorical data (17). For each output measure, 100 realizations were combined and divided into two categories: outputs greater than or equal to the median, and outputs less than the median. Then a contingency table was built with corresponding counts (Table 7). If the range of output values from correlated and uncorrelated parameters are significantly different, the ratio of correlated and uncorrelated parameters in one category will be significantly different from that in the other category. The null hypotheses  $H_0$  of both tests claim identically that outcomes from correlated and uncorrelated parameters are not different. Both tests were conducted for all six output measures for each model.

The K-S test and Fisher's test yield similar results at the 5% significance level (Table 8). For the Gipps model and the IDM, four of six measures reject  $H_0$  in both tests, indicating that the corresponding simulation outputs are significantly different and affected by ignoring correlations in model parameters. For the Helly model,  $H_0$  is rejected for two output measures in the K-S test and three in the Fisher's test.

The results can be better examined from the empirical cumulative distributions for each measure presented in Figure 4, with the K-S test results and  $p$ -values presented at the top of each plot. Clear differences in correlated and uncorrelated parameters are observed in the last three plots for the Gipps model (Figure 4a). Ignoring correlation appears to increase drivers' sensitivity because both inter- and intravehicle vari-

TABLE 7 Example of Contingency Table for Fisher's Exact Test

Output Measure	Category	CO	UC	Total
Network exit time (example)	< median	$a$	$b$	$a + b$
	$\geq$ median	$c$	$d$	$c + d$
	Total	50	50	100

NOTE:  $a, b, c, d$  = frequency counts for each category.

TABLE 8 Significance Test Results

Output Measure	2-Sample K-S Test			Fisher's Exact Test		
	Gipps	Helly	IDM	Gipps	Helly	IDM
Network exit time	Not reject	Reject	Reject	Not reject	Reject	Reject
Total travel time	Reject	Reject	Not reject	Reject	Reject	Not reject
Mean of avg. spacing	Not reject	Not reject	Not reject	Not reject	Not reject	Not reject
SD of avg. spacing	Reject	Not reject	Reject	Reject	Reject	Reject
Mean of CV of spacing	Reject	Not reject	Reject	Reject	Not reject	Reject
SD of CV of spacing	Reject	Not reject	Reject	Reject	Not reject	Reject
Number of $H_0$ rejections	4	2	4	4	3	4

NOTE: Avg. = average.

ation of spacing increase for uncorrelated parameters related to correlated parameters. The IDM shows a similar yet less obvious pattern for the spacing variation (Figure 4b). Such results may lead to a wrong interpretation of unstable driving behavior or suggest false heterogeneity. Moreover, after the network becomes complicated, how such variation would affect the entire traffic flow is unpredictable. For the Helly model, ignoring parameter correlations results in statistically significant differences not in spacing variation, but in the overall travel time, as shown in the first two plots (Figure 4c). However, the Helly model does not produce realistic perturbation propagation in the scenario used for this study. Therefore, most vehicles travel in free-flow mode (as

determined from average spacings larger than in the other two models), and the true effect of correlation on leader–follower interactions is not sufficiently reflected in simulation outputs.

### Parametric Distribution

Results in the previous section confirm the importance of considering correlation in the parameter-sampling procedure. However, empirical joint distributions may not be readily available in practice. In such a case, the use of a parametric model with an appropriate distribution

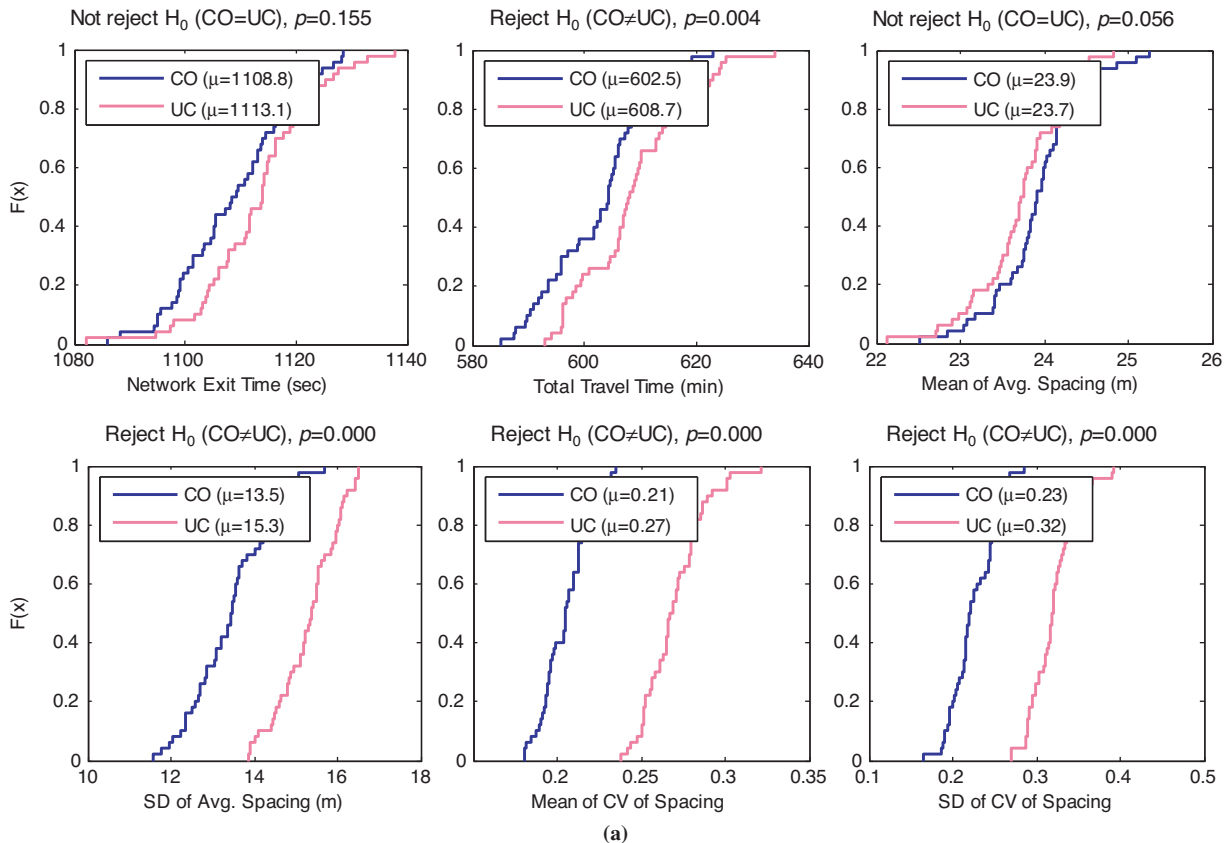


FIGURE 4 Cumulative distribution function (CDF) for six output measures (CO versus UC) for (a) Gipps model.  
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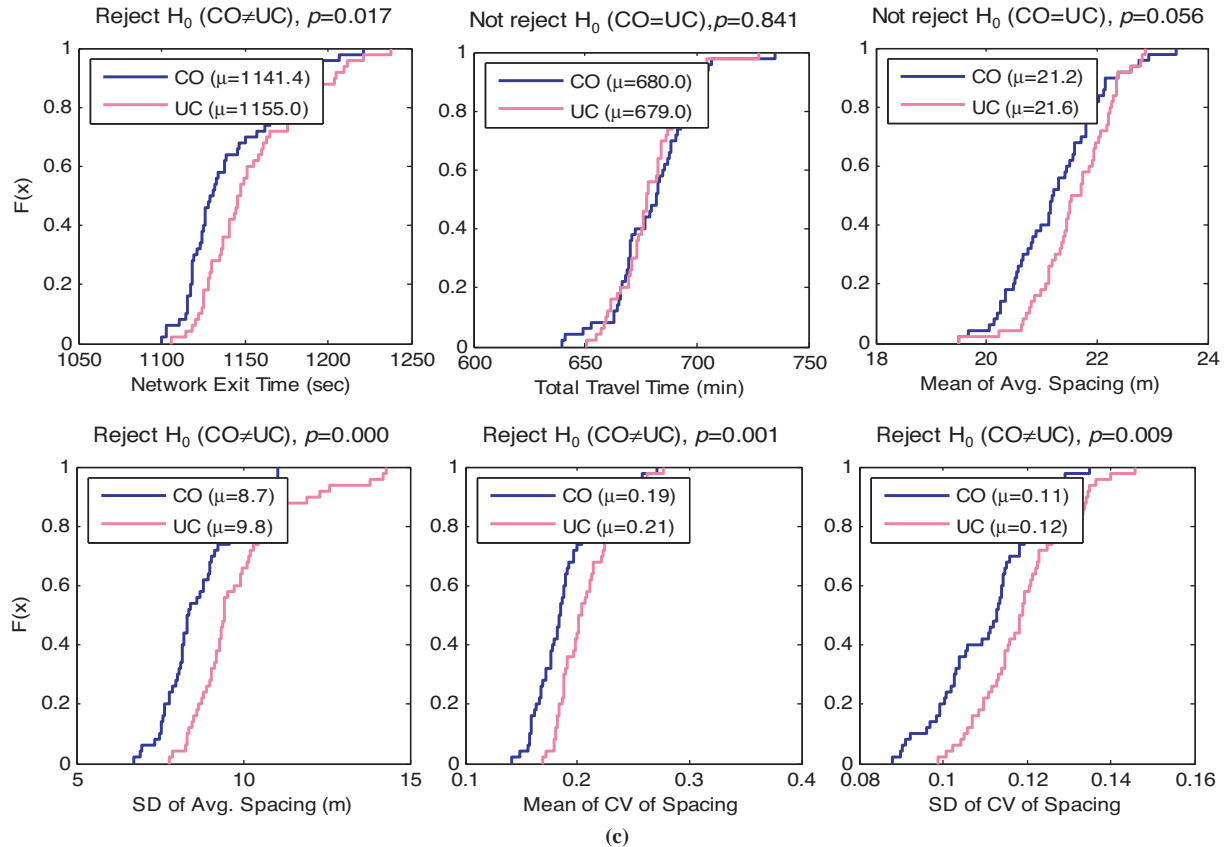
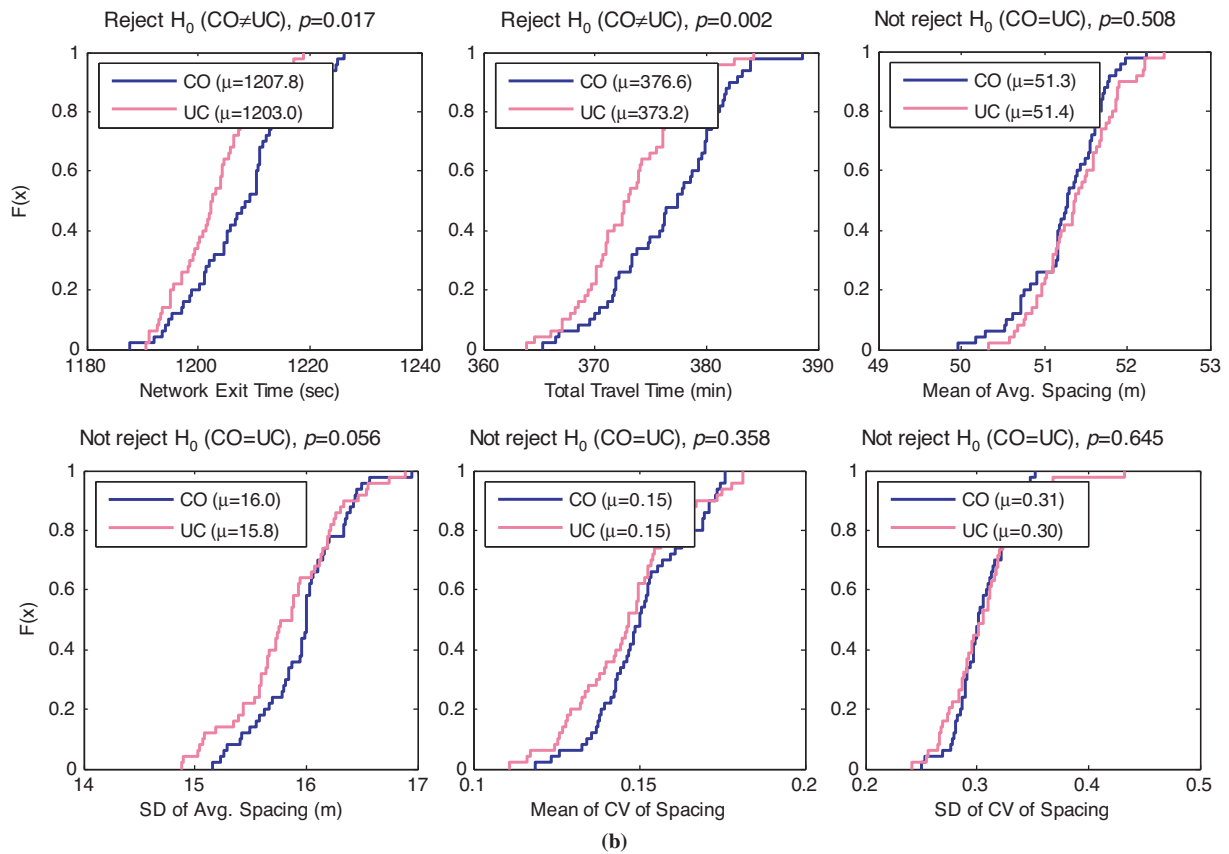


FIGURE 4 (continued) Cumulative distribution function (CDF) for six output measures (CO versus UC) for (b) Helly model, and (c) IDM.

assumption provides an alternative way to sample correlated parameters, as long as correlation coefficients or covariance matrix are known, in addition to the mean and SD of the marginal distributions.

This section focuses on the question of how to capture correlation in conventional microsimulation tools, and how closely the correlation effect is replicated when using a distributional assumption instead of an empirical distribution.

### Multivariate Normal Distribution

When parameters have a multivariate normal (MVN) distribution with mean  $\mu$  and covariance matrix  $\Sigma$ , the correlated parameter set can be drawn from  $N(\mu, \Sigma)$  using the Choleski decomposition approach (18). To use the MVN distribution assumption, every linear combination of each parameter component must follow the nor-

mal distribution. The lognormal distribution also can be used by taking logarithms. Figure 5 presents empirical distributions for all parameters and the corresponding best-fitting normal or lognormal density function. Overall, parameter marginal distributions appear well fitted by the normal or the lognormal distribution.

Assuming an MVN distribution, noting that joint or marginal distributions of some parameters might not be normal, 500 draws are taken from  $N(\mu, \Sigma_{\text{cov}})$ , where  $\mu$  is mean values in Table 3 and  $\Sigma_{\text{cov}}$  is the covariance matrix, which reflects the correlation structure in Tables 4, 5, and 6. This correlated parameter set from a MVN distribution is denoted as  $\text{CO}_{\text{MVN}}$ . By simply replacing  $\Sigma_{\text{cov}}$  with  $\Sigma_{\text{var}}$ —variance matrix with the same diagonal elements as  $\Sigma_{\text{cov}}$  but with zero off-diagonal elements—an uncorrelated parameter set ( $\text{UC}_{\text{MVN}}$ ) also can be generated. The  $\text{UC}_{\text{MVN}}$  case can be seen as the conventional sampling approach, where only the mean and the variance for each parameter are used. The boundary constraints in Table 2 also are applied.

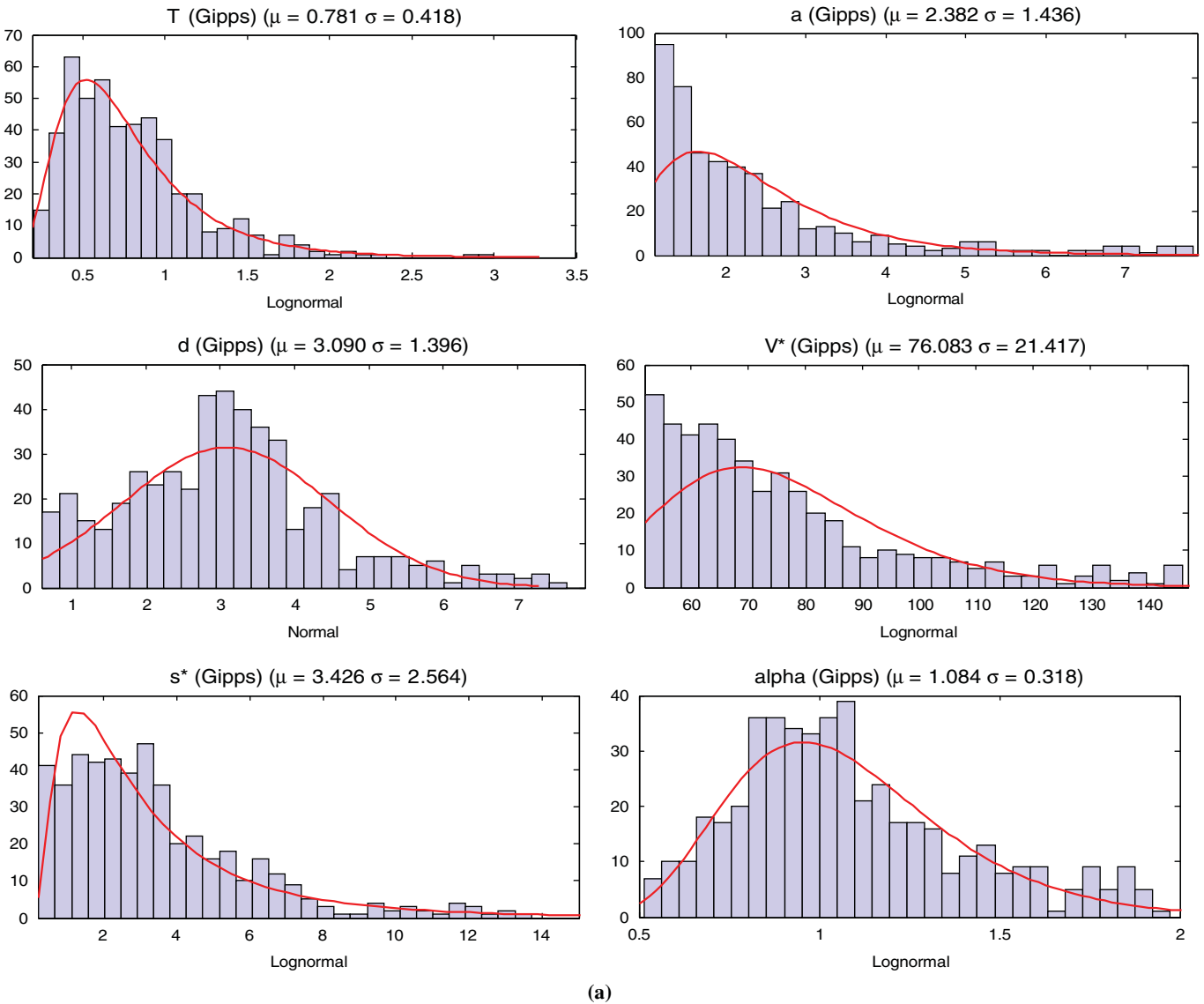


FIGURE 5 Histogram of calibrated parameters with fitted lognormal distribution for (a) Gipps model.

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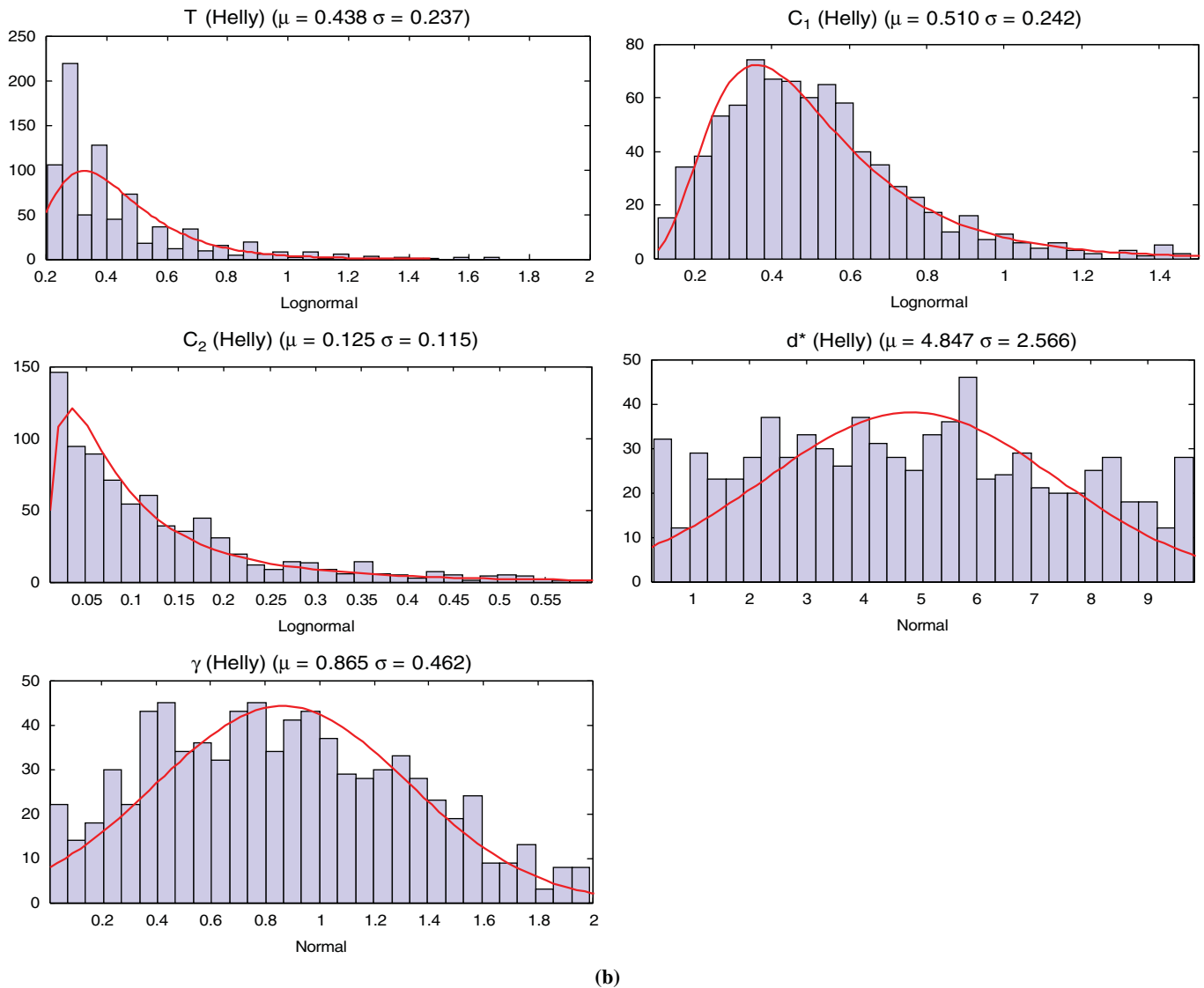
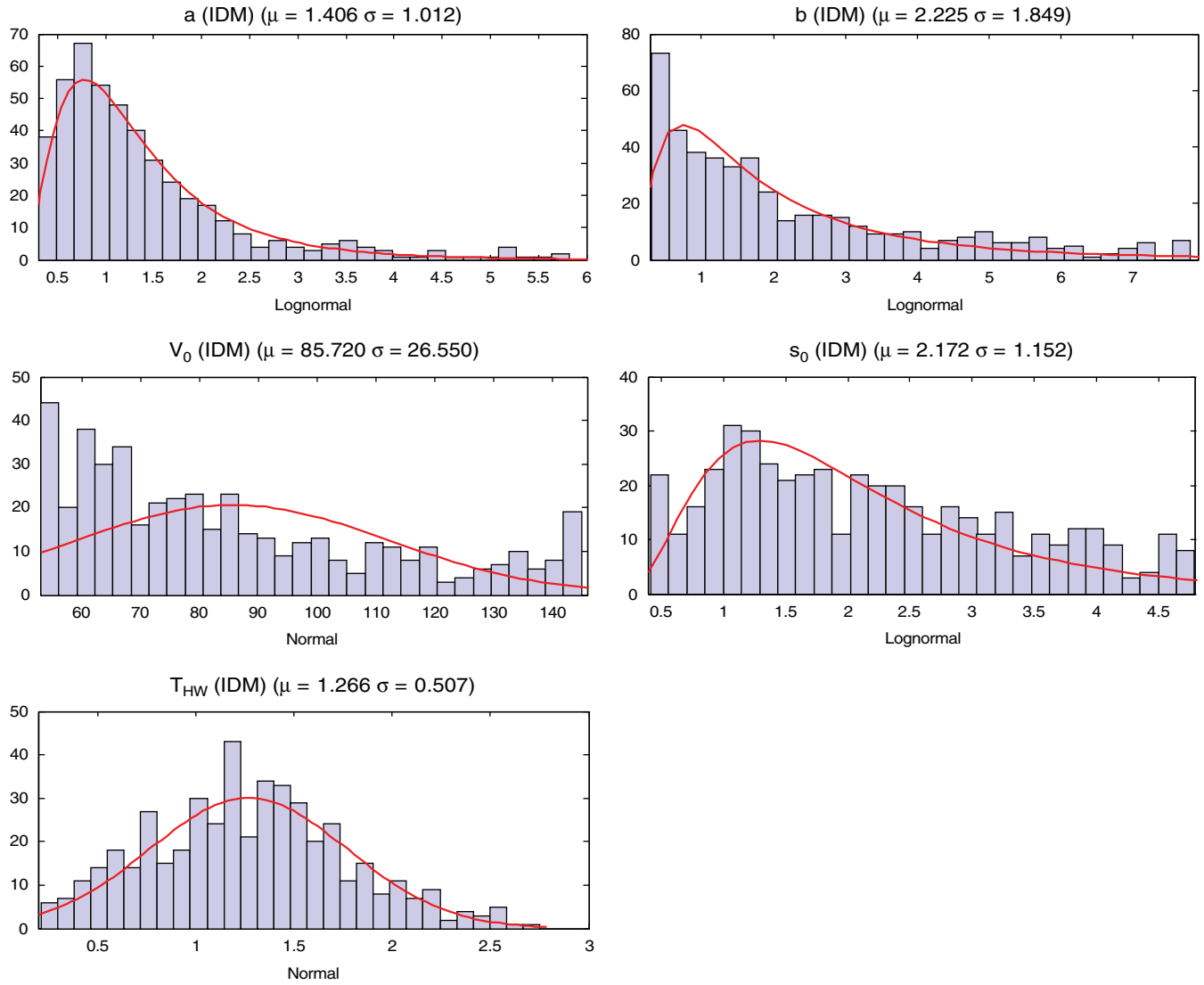


FIGURE 5 (continued) Histogram of calibrated parameters with fitted lognormal distribution for (b) Helly model.  
(continued)



(c)

FIGURE 5 (continued) Histogram of calibrated parameters with fitted lognormal distribution for (c) IDM.

### Simulation Results

The same scenario as in the previous section is used for the simulation experiments, whereby 500 vehicles for each  $CO_{MVN}$  and  $UC_{MVN}$  are simulated 50 times, respectively, and the resulting outputs are measured. The four cases are

1. CO, a correlated parameter set from the empirical joint distribution;
2. UC, an uncorrelated parameter set from the empirical marginal distributions;

3.  $CO_{MVN}$ , a correlated parameter set from the MVN distribution with correlation; and
4.  $UC_{MVN}$ , an uncorrelated parameter set from the MVN distribution without correlation.

The correlation coefficients of the sampled parameters (averaged over 50 simulation runs) for the four cases with the original correlation coefficients obtained from the calibrated data presented in Tables 4, 5, and 6 (denoted as NGSIM) are compared in Figure 6. The  $x$ -axis represents all pairs of parameters;  $R_{ij}$  denotes the correlation coefficient between parameters  $i$  and  $j$ , where  $i$  and  $j$  represent the order of

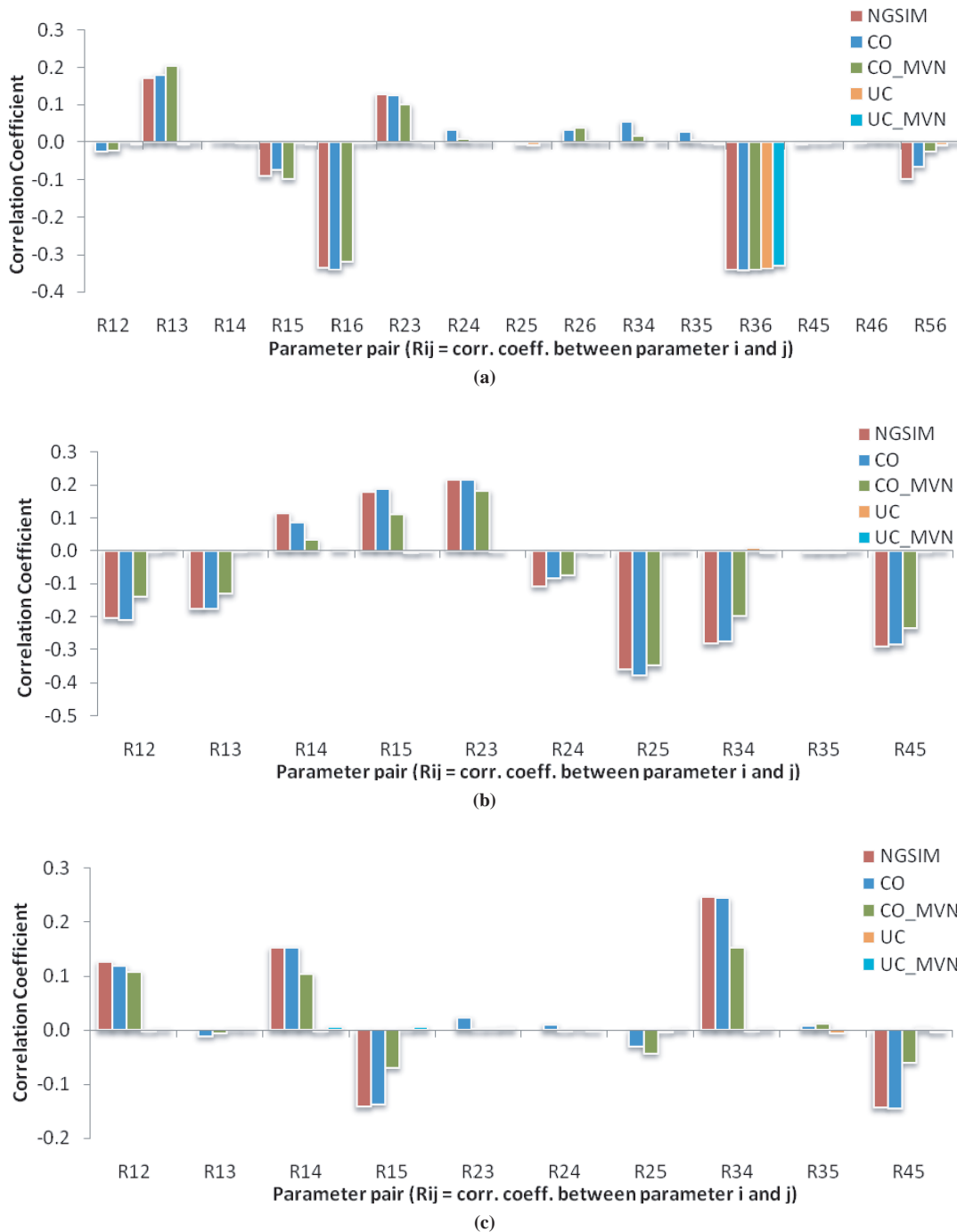


FIGURE 6 Comparison of parameter correlation coefficients for (a) Gipps model, (b) Helly model, and (c) IDM.

parameters in Table 1. As expected, CO and  $CO_{MVN}$  preserve the correlation coefficients close to those of NGSIM, whereas UC and  $UC_{MVN}$  exhibit nearly no correlation between parameters. However, for  $R_{36}$  (correlation between  $d$  and  $\alpha$ ) in the Gipps model, correlation is maintained for all cases to avoid the known instability issue mentioned in the sampling procedure for uncorrelated parameters. For some parameters in  $CO_{MVN}$ , the correlation structure is insufficiently replicated, indicating the limitation of the MVN assumption.

With the output measures for all four cases, the two-sample K-S test was performed again to determine whether output measures from each case were significantly different. In addition to the previous results for CO versus UC, CO versus  $CO_{MVN}$  and CO versus  $UC_{MVN}$  were tested; results are presented in Figure 7. For the Gipps model, sampling from the MVN distribution with known correlation does not replicate the output performance under sampling from the empirical distribution (i.e.,  $CO \neq CO_{MVN}$ ) for all six measures (Figure 7a). However,  $CO_{MVN}$  shows less bias than the conventional approach represented by  $UC_{MVN}$ , which is farthest away from CO in the last three plots. For the Helly model, it is hard to see the statistically significant effect of sampling methods in the test results (Figure 7b). However, there is a slight distinction between outputs from correlated parameters ( $CO_{MVN}$  and CO) and outputs from uncorrelated parameters ( $UC_{MVN}$  and UC), especially in the first two plots, providing evidence of correlation effects. The IDM produces a pattern similar to

that of the Gipps model; CO is not equal to  $CO_{MVN}$  for five of six output measures, but  $CO_{MVN}$  is closer to CO than  $UC_{MVN}$  is for all six measures (Figure 7c).

## CONCLUSIONS

This study presents a detailed analysis of the existence and extent of correlation between parameters in car-following models and its impact on microsimulation results. Three selected models (Gipps, Helly, and IDM) were calibrated and the correlation structures identified. Factor analysis was conducted to examine underlying common factors that characterize interrelation among parameters. Qualitative interpretations from factor analysis agree with the results from the computed correlation coefficients between parameters.

Two simulation experiments were conducted. First, based on empirical distributions of calibrated parameters, parameter sets were sampled with and without correlation, and car-following movements were simulated for each model. Six output measures were defined and collected during the simulation to compare performance. For the Gipps model and IDM, statistically significant differences between CO and UC parameter sets are observed for four output measures, three of which are related to inter- and intraindividual variations of the spacing. Depending on the hypothesis test type, two to three measures show significant differences for the Helly model.

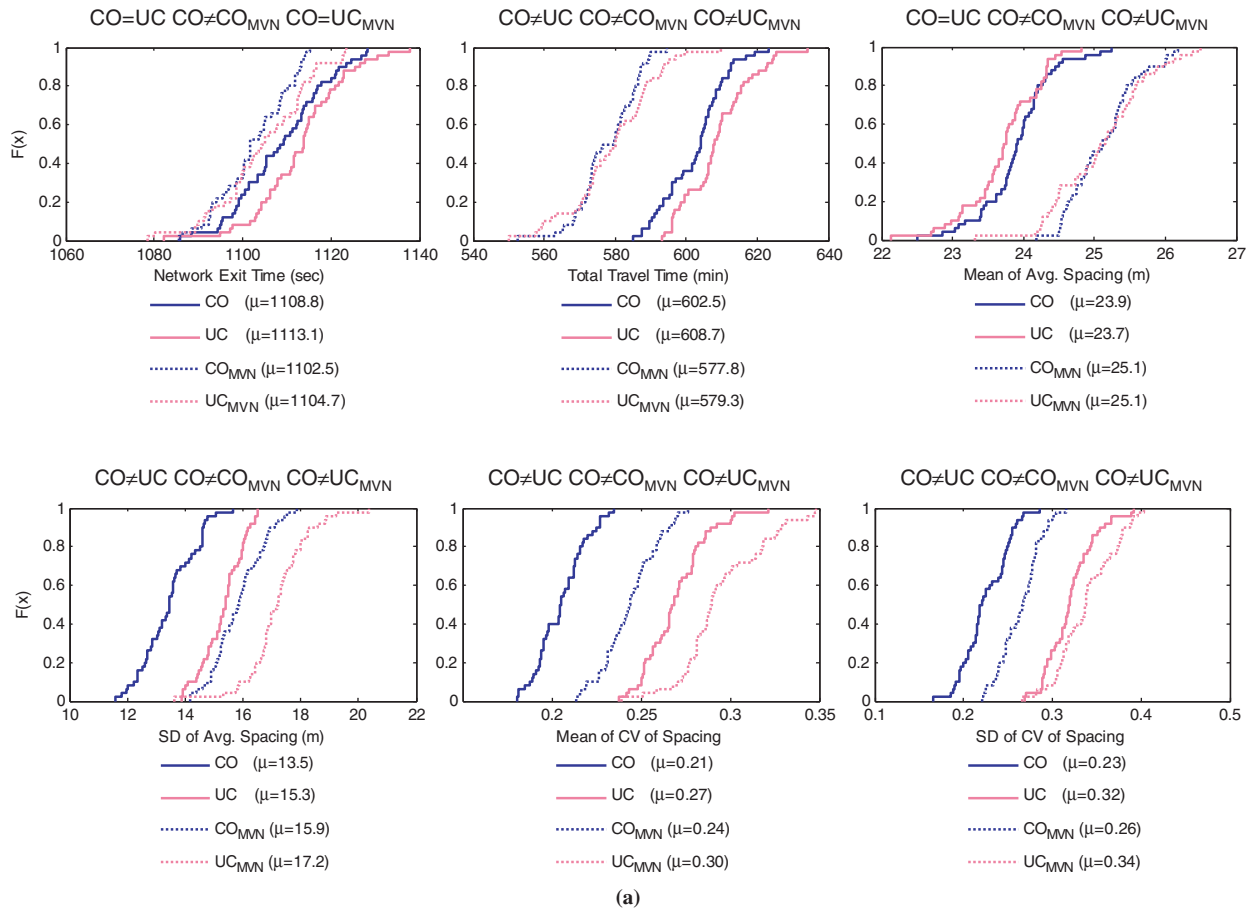


FIGURE 7 CDF for six output measures for all four cases (CO, UC,  $CO_{MVN}$ , and  $UC_{MVN}$ ) for (a) Gipps model.  
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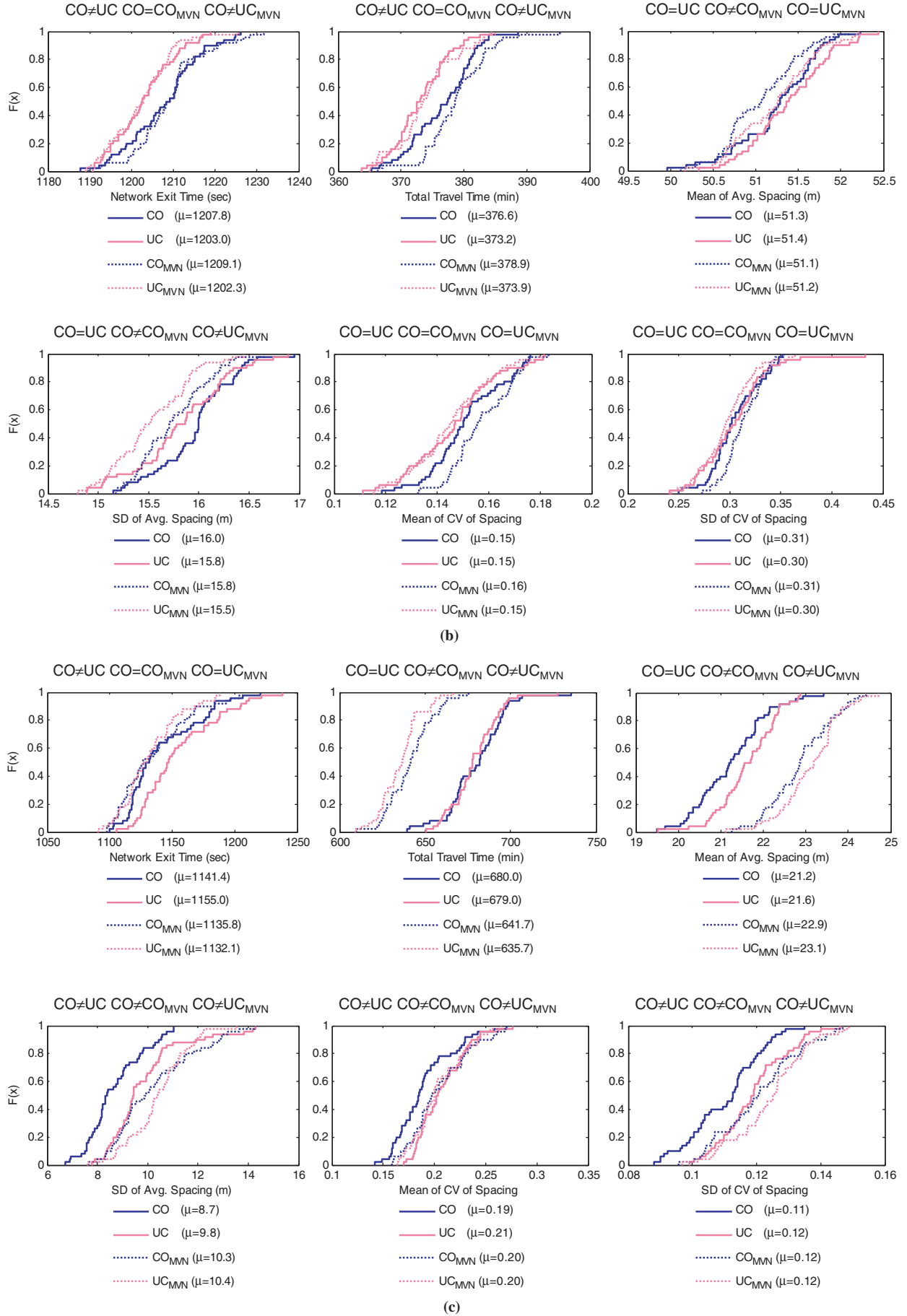


FIGURE 7 (continued) CDF for six output measures for all four cases (CO, UC, CO<sub>MVN</sub>, and UC<sub>MVN</sub>) for (b) Helly model, and (c) IDM ( $A \neq B$  if  $H_0$  is rejected at the 5% level for two samples from  $A$  and  $B$ ;  $A = B$  otherwise).

Next, assuming that parameters have the MVN distribution, both CO and UC parameter sets were generated using the estimated covariance matrix from the calibrated data. The aim was to explore a practical way to capture correlation in microsimulation when empirical data are not available. For the Gipps model and the IDM, even though correlations between parameters were replicated well, parameters drawn from the MVN distribution did not provide the same results (statistically) as those with the correlated empirical distribution. One reason for this disagreement may be the poor fit of the normal marginal distributions to some parameters. Another reason is the possible existence of nonlinear correlations between parameters. Because the Pearson correlation coefficient detects only a linear relationship, any nonlinear correlation cannot be replicated through the correlation structure used in MVN-based sampling, thereby generating parameters inconsistent with the original calibrated parameters. Finally, the Helly model shows limited effects of parameter correlation because insufficient leader–follower interactions were captured in the present car-following simulation experiment.

To summarize, when correlation is present between parameters in car-following models, parameters simply drawn from uncorrelated marginal distributions could yield unreliable results in simulation and consequently inaccurate interpretation. However, the use of parametric distributions with an estimated correlation structure may not necessarily reduce the error due to ignoring correlation if the underlying distributional assumption does not sufficiently hold for both marginal and joint distributions.

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