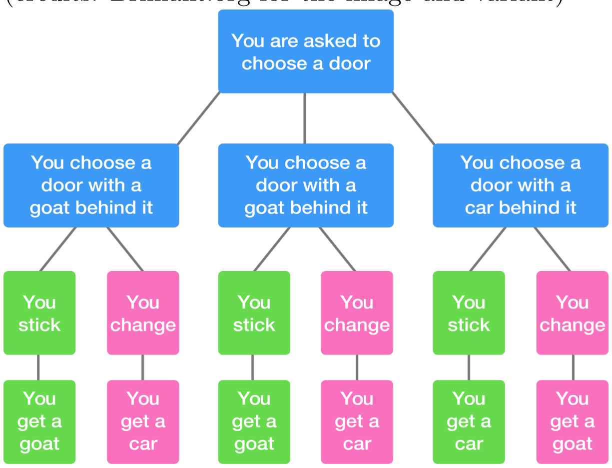
February 27, 2021

2.5 Monty Hall Problem

(credits: Brilliant.org for the image and variant)



We can solve this problem using Bayes' Rule, and it can also help to envision the possible worlds using a tree diagram.

Let W be the event that you chose the car in the initial step.

Let E be the event that the host chooses a goat.

We are concerned with the event that you did or did not choose the car conditioned on the event that the host chooses a goat, i.e. $P(W|E)$. By Bayes' rule,

$$\begin{aligned} P(W|E) &= \frac{P(E|W)P(W)}{P(E)} \\ &= \frac{1 \cdot \frac{1}{3}}{1} \end{aligned}$$

Compared to the complement conditional probability that you did not choose a winning door, $\frac{2}{3}$, this indicates that you should switch.

Crucial to the structure of this problem is your low probability of having chosen the right door (given a uniform prior), and the fact that the host will deterministically reveal doors such that there are only two doors left, and the prize is guaranteed to be behind one of them.

To see this consider a variant where instead of revealing a door behind which lies a goat, the host will randomly choose one of the two doors to reveal. Then $P(E) = \frac{2}{3}$.

$$\begin{aligned} P(E) &= P(E|W)P(W) + P(E|\neg W)P(\neg W) \\ &= 1 \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3} \\ * &= \frac{2}{3} \end{aligned}$$

This gives $P(W|E) = \frac{1}{2}$, indicating that it doesn't matter whether you switch or not.