First assignment

Modelling and analysis of Sustainable Energy Systems using Operations Research

Handed out on October 10th 2019 at 17:00. The assignment is to be handed in on Inside on October 24th before 23:59.

Parts 1 and 2 are multiple choice questions. You find the different choices on Inside under assignments and "First assignment - multiple choice", which is also where you upload your answers. Note that part 2 requires you to also upload your GAMS code. This should be done in the assignment named "First assignment".

For part 3, you need to upload your answers under "First assignment". The answers can be written in hand or on computer, but please ensure that your drawings and explanations are in a clear and readable state. Furthermore, remember to include explanations to your solutions.

Part 1 - Theoretical Multiple Choice

1. In linear programming, only one of the corner-point feasible solutions can be an optimal solution.

2. What is the standard form of the following problem?

$$\begin{aligned} \text{Min.} \quad & -Z = 2x_1 - 4x_2 && (1)\\ \text{s.t.} \quad & 7x_1 + 2x_2 \geq 21 && (2)\\ & 3x_1 + x_2 - x_1 \geq 17 && (3)\\ & x_1 \geq 0 && (4)\\ & x_2 \geq 0 && (5) \end{aligned}$$

3. Which statement(s) are true regarding duality.

4. Which statement(s) are true.

5. Find the dual problem for the following problem.

$$
\begin{aligned}
\text{Max. } & Z = 3x_1 + 2x_2 + 8x_3 + 9x_4 && (6)\\
\text{s.t. } & 4x_2 - 9x_3 + x_1 \geq 2 && (7)\\
& 6x_1 + 5x_2 + 3x_4 \leq -5 && (8)\\
& x_1 \leq 0 && (9)\\
& x_2 \geq 0 && (10)\\
& x_3 \leq 0 && (11)\\
& x_4 \leq 0 && (12)
\end{aligned}
$$

Part 2 - GAMS Multiple Choice

As a student at DTU, you are trying to maximize your daily happiness (H). Your daily happiness is dependent on three variables: amount you study (s), amount of coffee you drink (c), and the amount you bike each day (b). Additionally, you are affected by your energy level throughout day (e(t)=100-t).

Using GAMS and the information provided, answer the multiple choice questions posted on DTU Inside and **upload your GAMS file** (.gms).

$$
\begin{aligned}
\text{Max. } & H = \sum_{t=1}^{t=24} 4s - c * e(t) + 2b && (13)\\
\text{s.t. } & s + 3c \geq 47 && (14)\\
& \sum_{t=1}^{t=24} -3s + c * e(t) \geq 21 && (15)\\
& 5s + b + c \leq 500 && (16)\\
& b + s \leq 100 && (17)\\
& s \geq 0 && (18)\\
& c \geq 0 && (19)\\
& b \geq 0 && (20)
\end{aligned}
$$

6. Solve the problem in GAMS. What is the value of H, the objective value? Choose the closest value.

7. What are the values of the non-zero shadow prices of the constraints? Choose the closest values.

8. Which of the listed constraints are affecting the optimal solution?

9. Now formulate the problem so that the variable 'c' is an integer. What is the new value of variable 'b'?

10. Omit the changes you made in Question 4. What is the objective value when you add a constraint that makes the value of variable 'b' less than or equal to 12.

Part 3 - Written assignment

You won the lottery and are now able to invest in your biggest dream—a Tesla Model S, P100D. Tesla just found a way to sell back the stored electricity to the grid at the same cost as charging. However, each workday you have a busy travelling salesman schedule, meaning that you have only one hour at work to charge/discharge your car before you drive 200 km to your home, corresponding to a need of 48 kWh. Now you want to figure out how to charge or discharge

Parameter	Unit	Value
Min. storage/your demand	kWh	48
Max. storage	kWh	100
Max. charge to car	kW	11
Max. discharge to grid	kW	11
Initial storage level	kWh	55
Storage value	€/kWh	0.025
Cost of charging/profit of discharging	€/kWh	0.015

Table 1: Data for the Tesla car

your car in the hour before going home in order to earn the most money and still being able to return home in your car. Remember that the storage level also has a value as you might sell the electricity later. Assume that the there are no transmission losses, i.e. charge/discharge has an efficiency of 100%.

Let x_1 denote the storage level, and x_2 denote the charge and discharge of the battery such that if $x_2 > 0$ the car is charging and if $x_2 < 0$ the car is discharging.

A Write the mathematical model for the problem.

B Find the feasible region and solve the problem.

C Imagine that the dual problem has been solved to optimality. Which of the dual variables are not zero? Which of the dual variables would you investigate if you want to see if it makes sense to invest in a power charger with a higher charging/discharging capacity?

Your employer is a bit concerned with how you are going to serve the customers of the company with your new car. You ensure him, that as long as you have enough battery to reach home at the end of the day, you are as flexible as before. This means, that you should at least have 37 kWh on the car before your one hour in the office—or in other words that you have 63 kWh to serve customers.

The company have divided their customers into two groups, with the needed use of battery and their satisfaction index given in table 2. Furthermore, the company has two important customers of type 1 and three of type 2 that you need to visit each day.

Parameter	Unit	Type 1	Type 2
Usage of battery	kWh/visit	4	7
Satisfaction index	happiness/visit	12	23
Necessary trips	#	2	3

Table 2: Data for the customers

Let y_1 denote the number of trips of type 1 customers and y_2 denote the number of trips of type 2 customers. Your task is to find the number of each type of trips you should schedule to maximise customer satisfaction.

D Write the mathematical model for the problem.

E Select the appropriate method for solving the problem and find the optimal solution.