Technical University of Denmark

42002 Modelling and Analysis of Sustainable Energy Systems using Operations Research

First Assignment

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3 Written Assignment

3.A

The mathematical model that satisfy the charging Tesla problem is the following:

$$\text{Max. } Z = 0.025x_1 - 0.015x_2 \tag{1}$$

$$\text{s.t. } x_1 - x_2 = 55 \tag{2}$$

$$x_1 >= 48 \tag{3}$$

$$x_1 <= 100 \tag{4}$$

$$x_2 >= -11 \tag{5}$$

$$x_2 <= 11 \tag{6}$$

$$x_1 >= 0 \tag{7}$$

$$x_2 \in \mathbb{R} \tag{8}$$

where:
(1) maximizes the earned money by subtracting the storage value of 0.025 EUR/kWh multiplied by the storage level x_1 and the charge/discharge value 0.015 EUR/kWh multiplied by the charge/discharge level x_2.
(2) represents the initial storage level and the charge/discharge level. When there's no charge or discharge of the battery, the battery stays at its initial value of 55 kWh.
(3) and (4) constraints the storage level between 48 kWh, which corresponds to the required storage to travel home, and 100 kWh, which corresponds to the maximum battery capacity.
(5) and (6) constraints the charge or discharge level between -11 kW and 11 kW, which corresponds to the minimum and maximum charge/discharge capacity of the power charger.
(7) and (8) constraints storage level to positive values and the charge/discharge level to real numbers.

3.B

Given the mathematical model represented by the set of equations (1)-(8), we can see from Figure 1 that the feasible region is the red line, which is part of constraint (2), inside constraints (3), (4), (5), and (6).

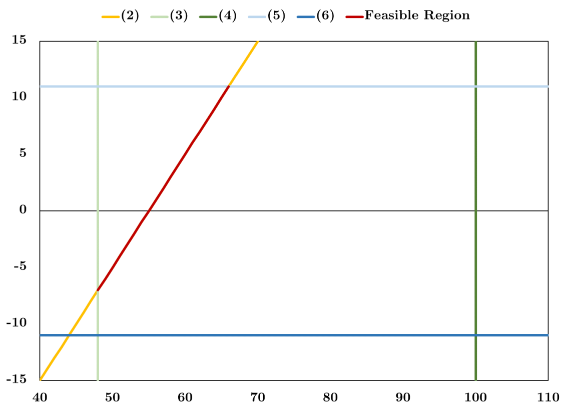

Figure 1: Constraints and Feasible Region

Given that this is a maximization problem, the solution **Z** will try to obtain the maximum value, which can be interpreted as the point where constraint (2) and (5) crosses. Therefore, from Figure 2, the solution is the optimal solution is $Z = 1.485$ EUR at $x_1 = 66$ and $x_2 = 11$.

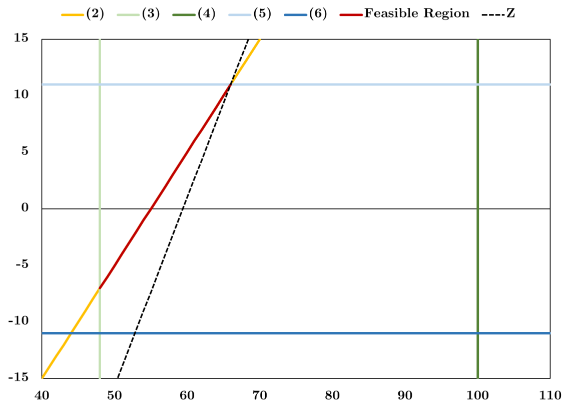

Figure 2: Constraints, Feasible Region and Solution

3.C

If the dual problem has been solved to optimality, we can know from Figure 2, that the dual variables corresponding to equation (2) and (6) will not be zero since they're binding the solution. Therefore, the marginal change in any of these two dual variables (or its corresponding equations) will change the optimal solution. Regarding the power charger, the dual variable corresponding to equation (6) should be investigated, since this dual variable represents the maximum charging capacity of the power charger. If we could change this value, the optimal solution (and the maximum value of the function) would change.

3.D

The mathematical model of the second part of the problem is the following:

$$\text{Max. } H = 12y_1 + 23y_2 \tag{9}$$

$$\text{s.t. } 4y_1 + 7y_2 <= 63 \tag{10}$$

$$y_1 >= 2 \tag{11}$$

$$y_2 >= 3 \tag{12}$$

$$y_1 \rightarrow \text{positive integer} \tag{13}$$

$$y_2 \rightarrow \text{positive integer} \tag{14}$$

where:
(9) represents the objective function of multiplying the visits to each customer and its corresponding satisfaction index to obtain the happiness of our customers.
(10) represents the usage of battery of each customer and is constrained to 63 kWh, since we need to have at least 37 kWh at the last hour of the shift to charge the car and travel home.
(11) and (12) represent the minimum necessary trips to each corresponding client.
(13) and (14) constraints the customer visits to positive integer numbers, since divisibility in this problem can't represent the physical meaning of "a visit".

3.E

Since this problem constraints the variables to positive integers, i.e. a pure integer programming problem, the appropriate method to solve it is using the *branch-and-bound technique*. This is a cleverly structured enumeration procedure where only a tiny fraction of feasible solutions are examined, thus preventing enumerating the whole finite number of feasible solutions.

Following the theory, we begin by initializing our solution $Z^* = -\infty$ and its corresponding solution variables $x^* =$ null. For each iteration, we compute the relaxed LP problem, i.e. we remove the positive integer constraints (13) and (14), and solve. Then, we have three options:

- If the solution is feasible but x values are not integer, we proceed to round-down the objective function value and name it as Z^{UP}. Then, we divide the branch by setting two additional constraint options, one rounding-down the non-integer variable value and another rounding-up the non-integer variable value.
- If the solution is feasible and x values are integer, we proceed to store the solution as x^* and Z^*. Then, we fathom the branch and keep the constraint of the branch.
- If the solution is not feasible, i.e. it violates one of the constraints, we fathom the branch and forget about the additional constraint we used in this branch.

By following the pseudo-code, the solution we get is shown in the following tree-diagram, where the final optimal solution is found at node J with $Z^* = 199$ with $x^* = \{7, 5\}$