110-1 ENGINEERING MATHEMATICS HW3

Due Date: 2021/12/16 18:00

Part I: Use Definition 4.1.1 to find $L\{f(t)\}$

1. $f(t) = e^{t+7}$

Part II: Use Theorem 4.1.1 to find $L\{f(t)\}$

2. $f(t) = \mathrm{e}^{\mathrm{t}}\sinh t$

Part III: Find the given inverse transform

3. $L^{-1}\{\frac{(s+1)^3}{s^4}\}$
4. $L^{-1}\{\frac{5}{s^2+49}\}$

Part IV: Find either F(s) or f(t)

5. $L\{e^{3t}(9 - 4\mathrm{t} + 10\sin\frac{t}{2})\}$
6. $L^{-1}\{\frac{2s-1}{s^2(s+1)^3}\}$

Part V: Find the convolution $f * g$ and Find the Laplace transform $f * g$

7. $f(t) = 4t, g(t) = 3t^2$

Part VI: Use the Laplace transform to solve the given integral equation

8. $f(t) = te^t + \int_0^t \tau f(\mathrm{t} - \tau)\mathrm{d}\tau$

Part VII: Use the Laplace transform to solve the given differential equation

9. $y'' + 4y' + 13y = \delta(t - \pi) + \delta(t - 3\pi)$

$$y(0) = 1, y'(0) = 0$$

10.

$$\frac{dx}{dt} = -x + y$$

$$\frac{dy}{dt} = 2x$$

$$x(0) = 0, y(0) = 1$$

Definition 4.1.1 Laplace Transform

If $f(t)$ is defined for $t \geq 0$, then

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$$

is said to be the **Laplace Transform** of f.

Theorem 4.1.1 Transform of Some Basic Functions

(a) $\mathcal{L}\{1\} = \frac{1}{s}$

(b) $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$, $n = 1, 2, 3, \cdots$

(c) $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

(d) $\mathcal{L}\{\sin kt\} = \frac{k}{s^2 + k^2}$

(e) $\mathcal{L}\{\cos kt\} = \frac{s}{s^2 + k^2}$

(f) $\mathcal{L}\{\sin kt\} = \frac{k}{s^2 - k^2}$

(g) $\mathcal{L}\{\cosh kt\} = \frac{s}{s^2 - k^2}$