Symmetric Min-Max Heaps (SMMH)

- A double-ended priority queue (DEPQ) may be represented using a SMMH.
- Definition and Properties
  - A SMMH is a complete binary tree.
  - Let N be any node of the SMMH. Let *elements*(N) be the elements in the subtree rooted at N but excluding N.
- N must satisfy the following properties
  - The left child of N has the minimum element in *elements*(N).
  - The right child of N (if any) has the maximum element in *elements*(N).

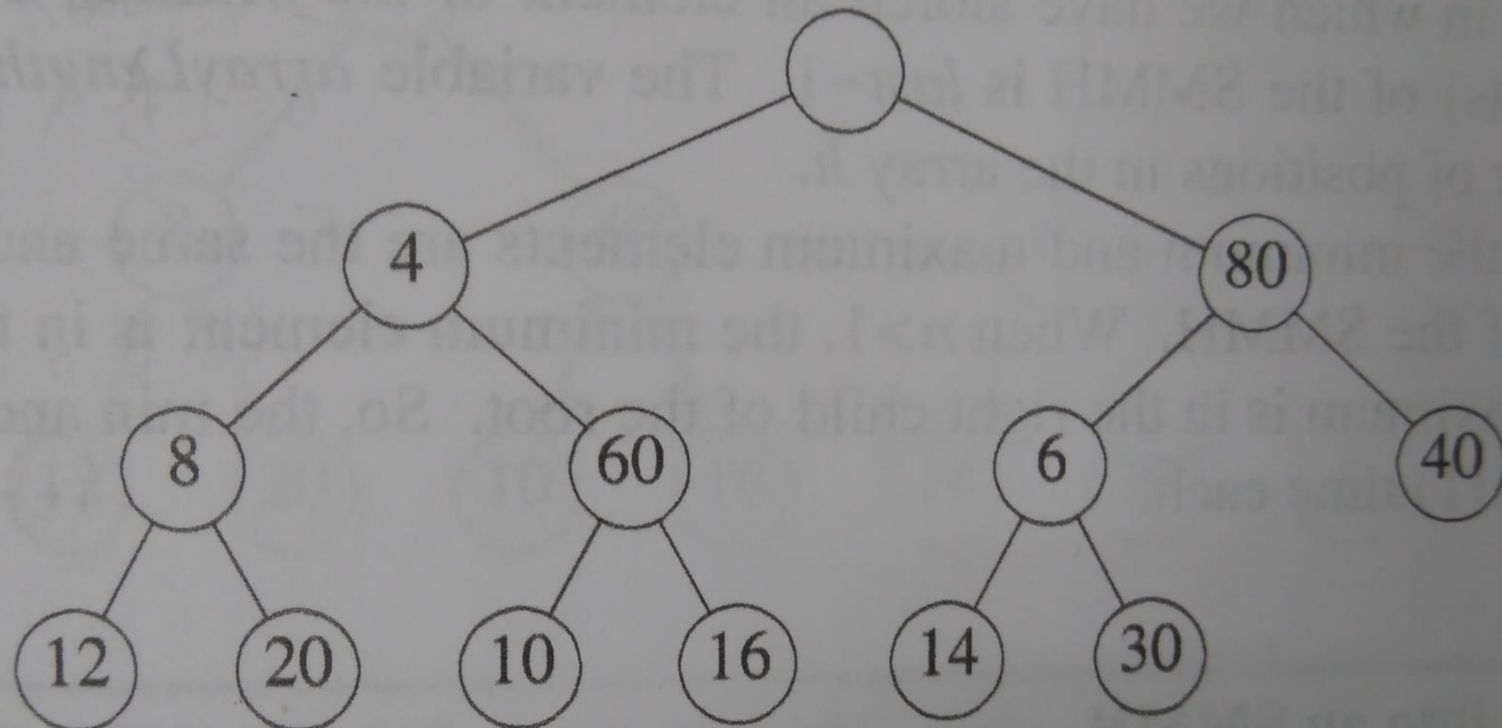

Figure 9.25: A symmetric min-max heap

- For example, elements(80) = {6, 14, 30, 40}.

We can easily infer that the following are true.

- The element in each node is less than or equal to that in its right sibling (if any).
- For every node N that has a grandparent, the element in the left child of the grandparent is less than or equal to that in N.
- For every node N that has a grandparent, the element in the right chid of the grandparent is greater than or equal to that in N.

Inserting into SMMH （a small value）

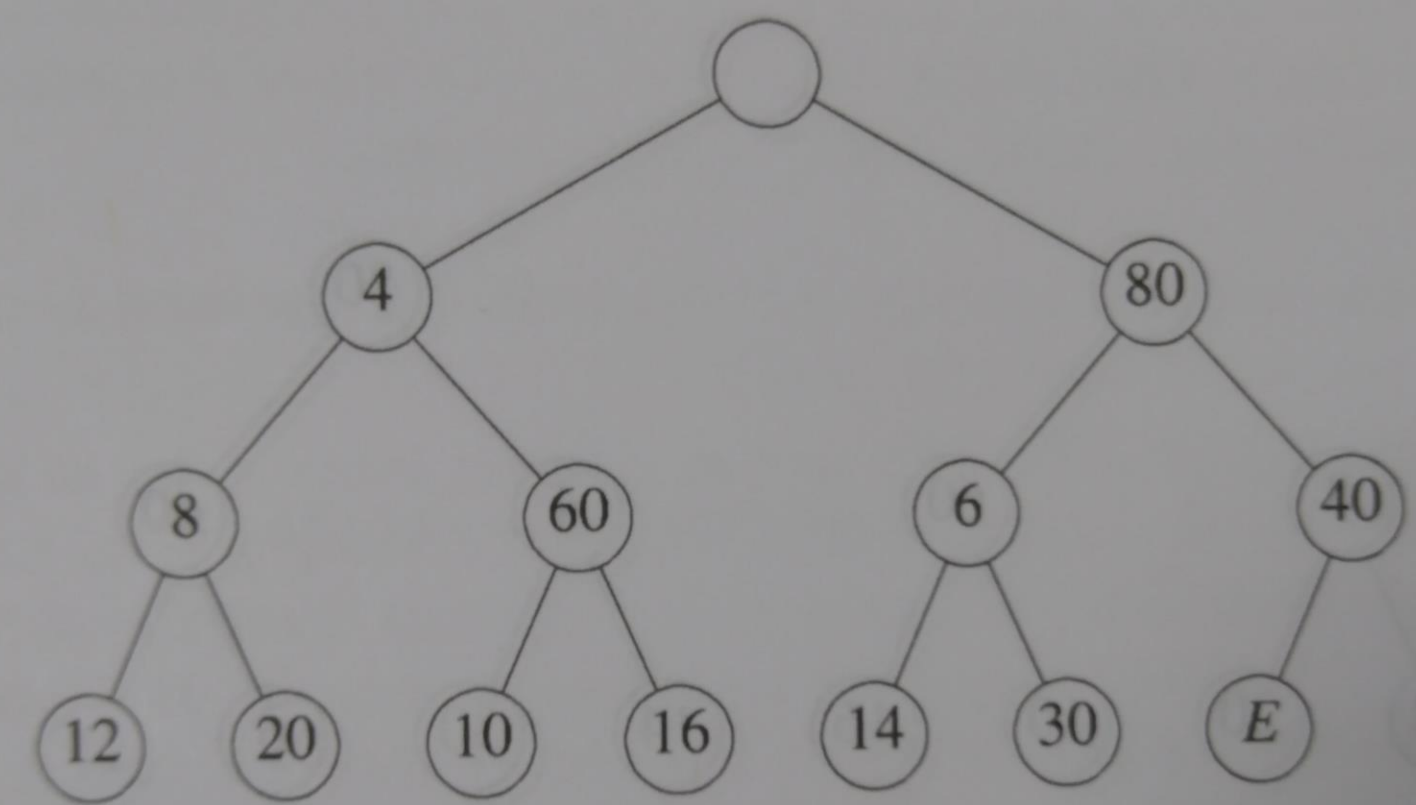

Figure 9.26: The SMMH of Figure 9.25 with a node added

Inserting into SMMH （a small value）

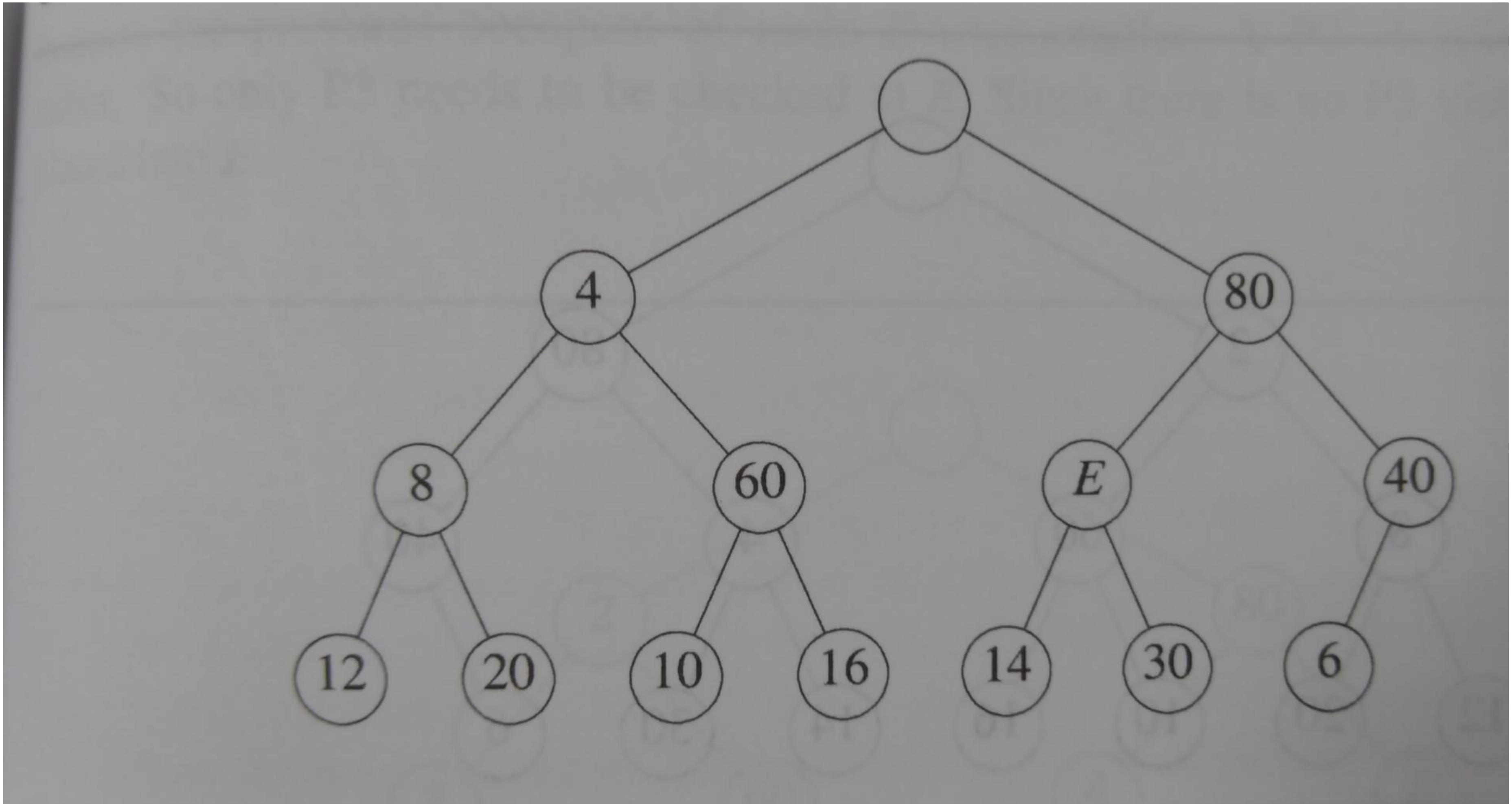

Figure 9.27: The SMMH of Figure 9.26 with 6 moved down

Inserting into SMMH （a small value）

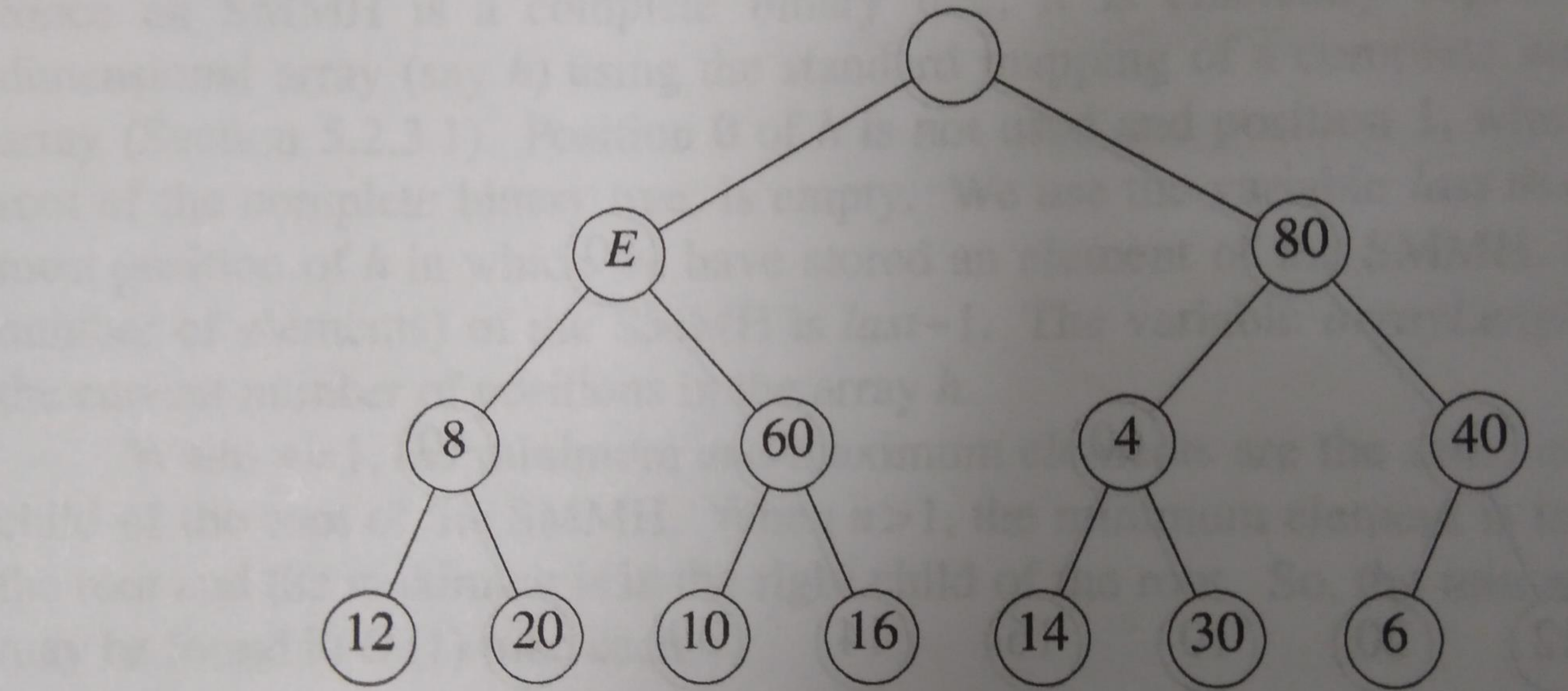

Figure 9.28: The SMMH of Figure 9.27 with 4 moved down

Inserting into SMMH （a small value）

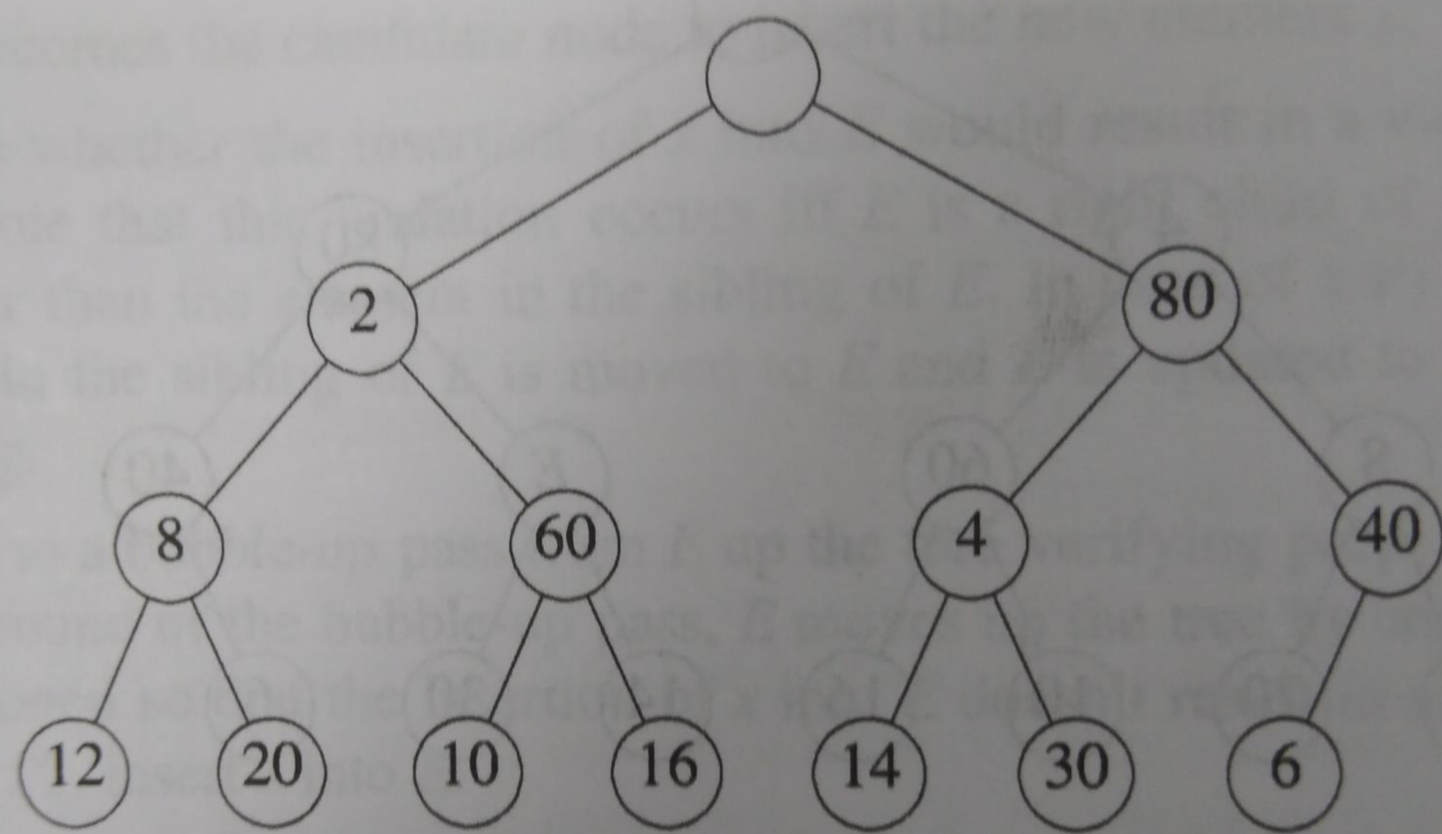

Figure 9.29: The SMMH of Figure 9.28 with 2 inserted

Inserting into SMMH （a large value）

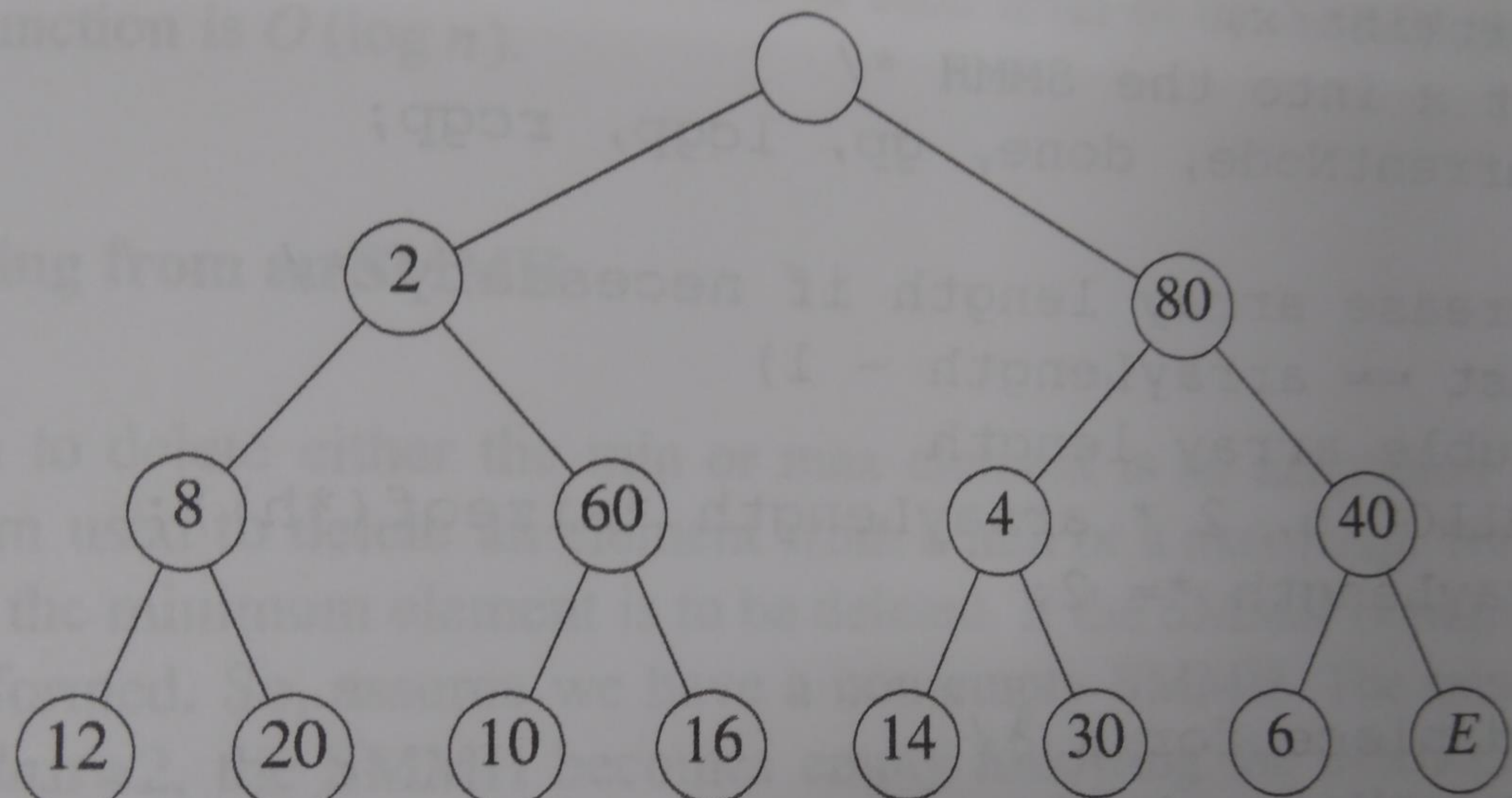

Figure 9.30: The SMMH of Figure 9.29 with a node added

Inserting into SMMH （a large value）

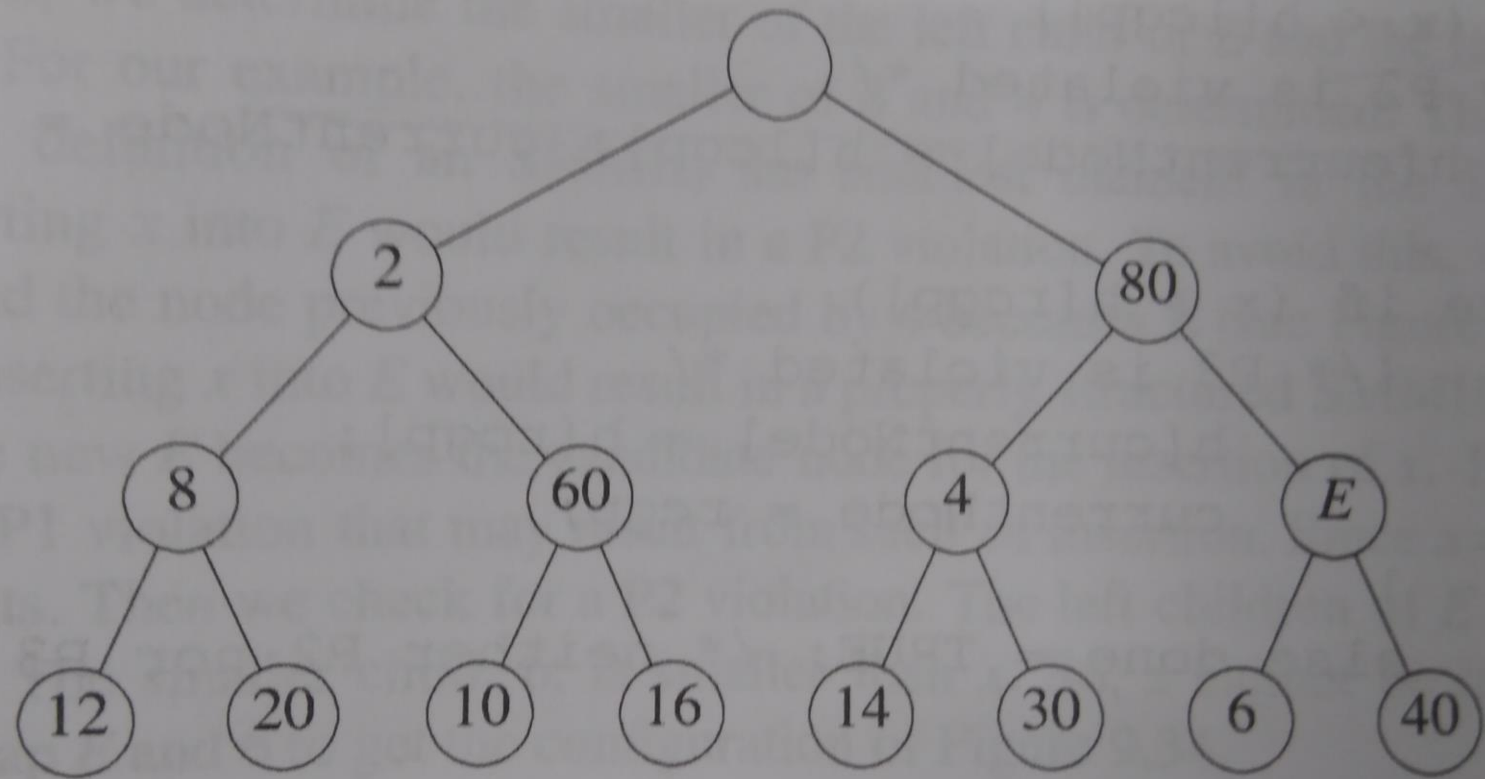

Figure 9.31: The SMMH of Figure 9.30 with 40 moved down

Deletion from SMMH （the smallest value）

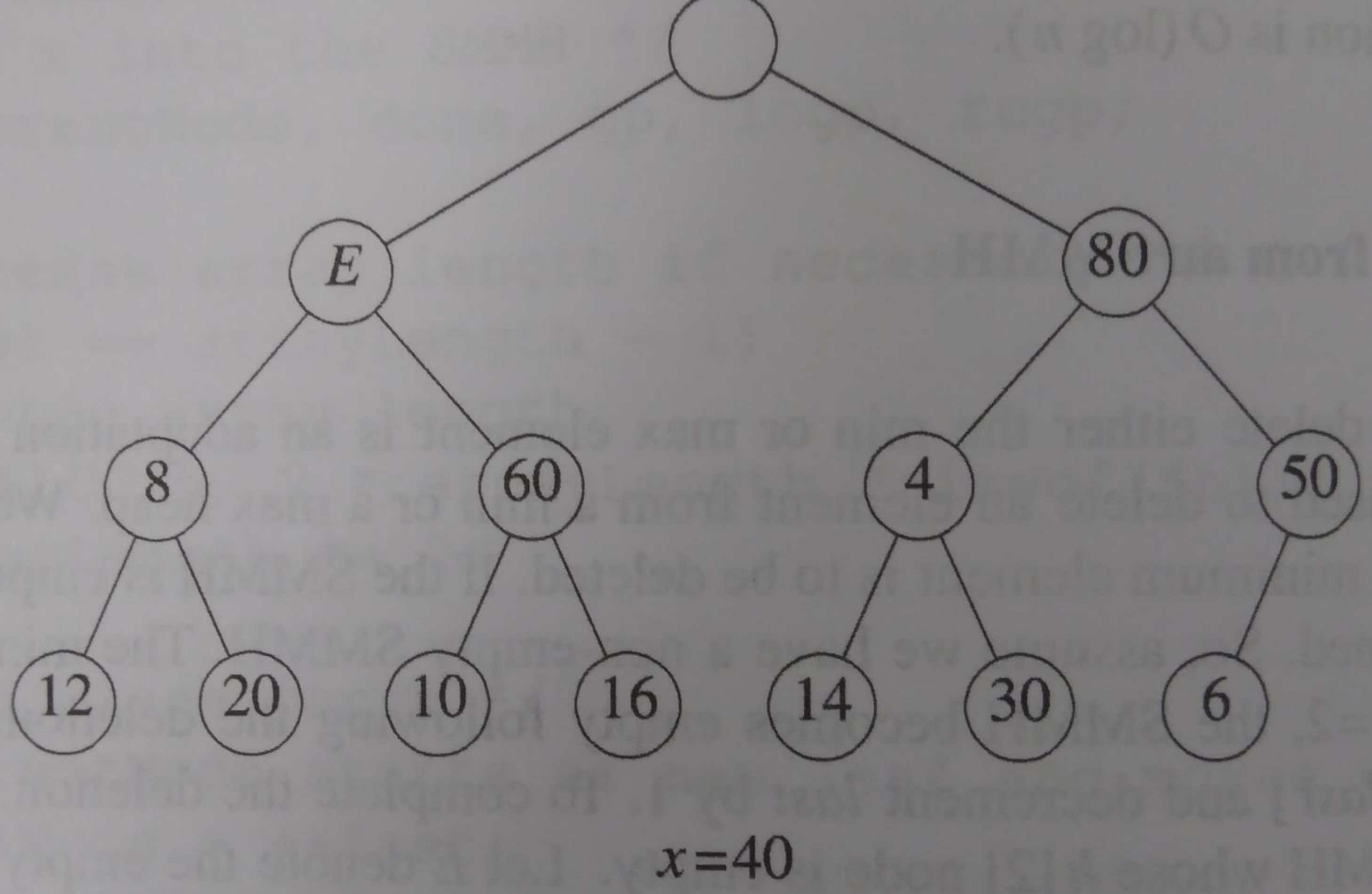

Figure 9.32: The SMMH of Figure 9.31 with 2 deleted

Deletion from SMMH （the smallest value）

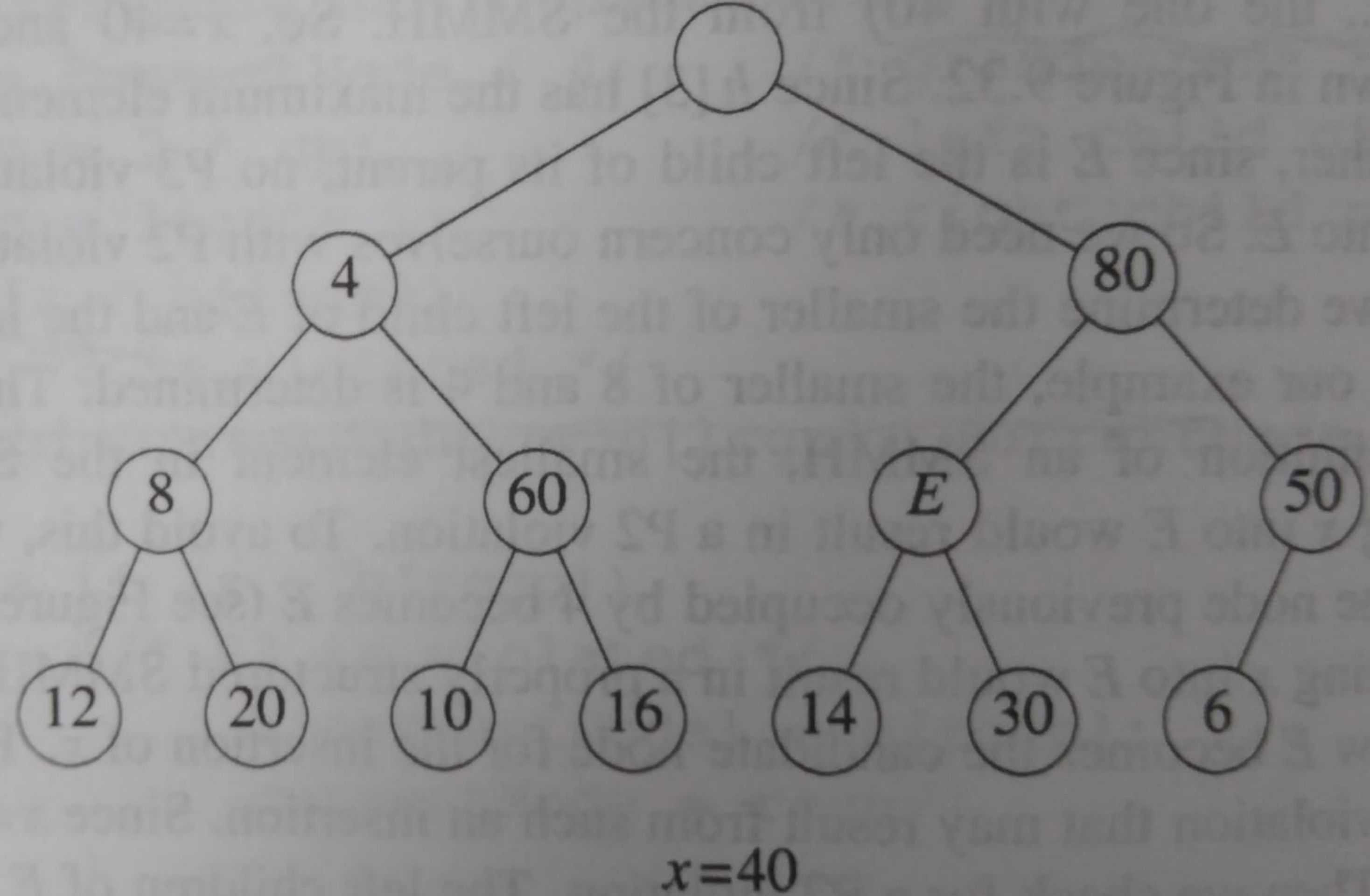

Figure 9.33: The SMMH of Figure 9.32 with *E* and 4 interchanged

Deletion from SMMH （the smallest value）

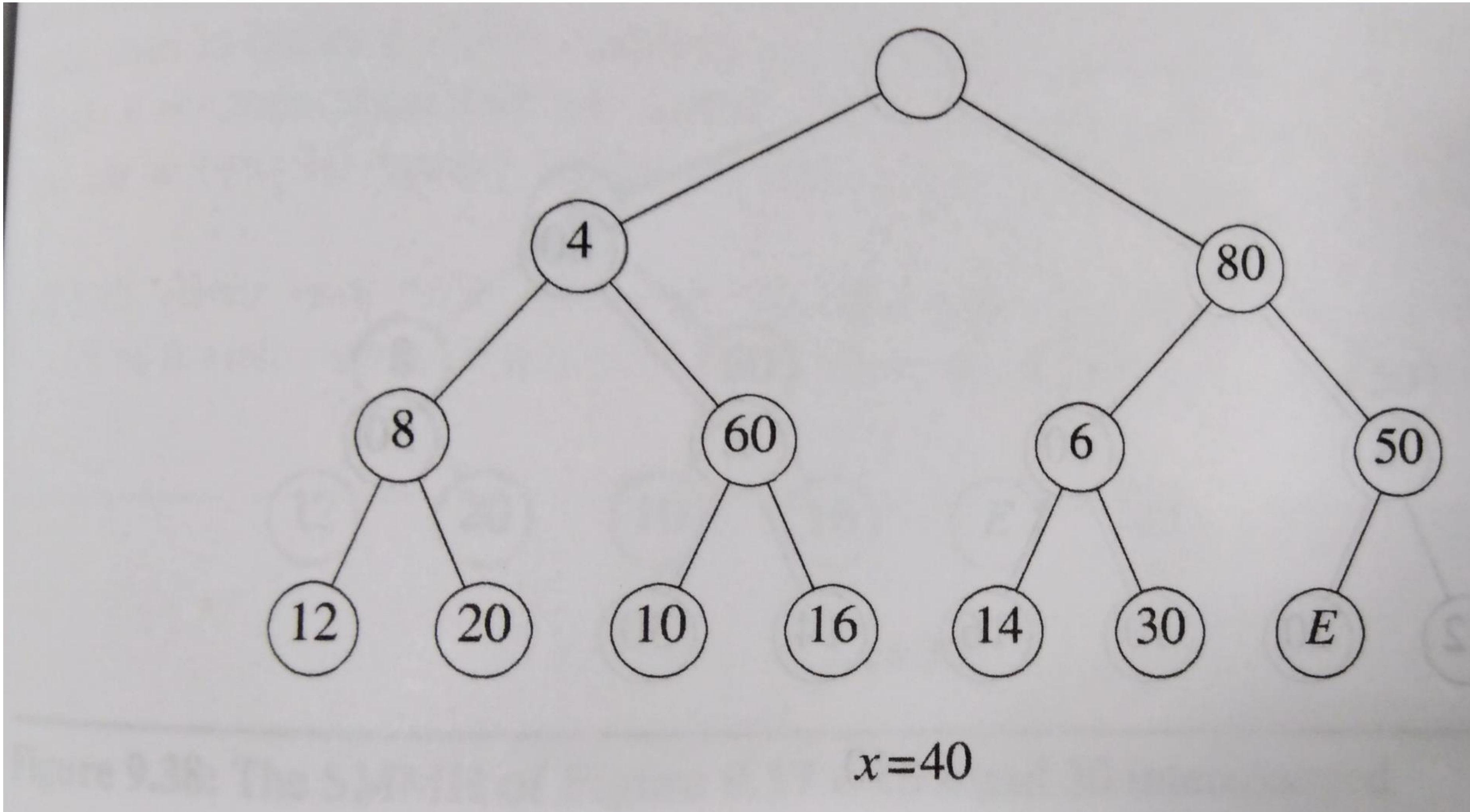

Figure 9.34: The SMMH of Figure 9.33 with *E* and 6 interchanged

Deletion from SMMH （the smallest value）

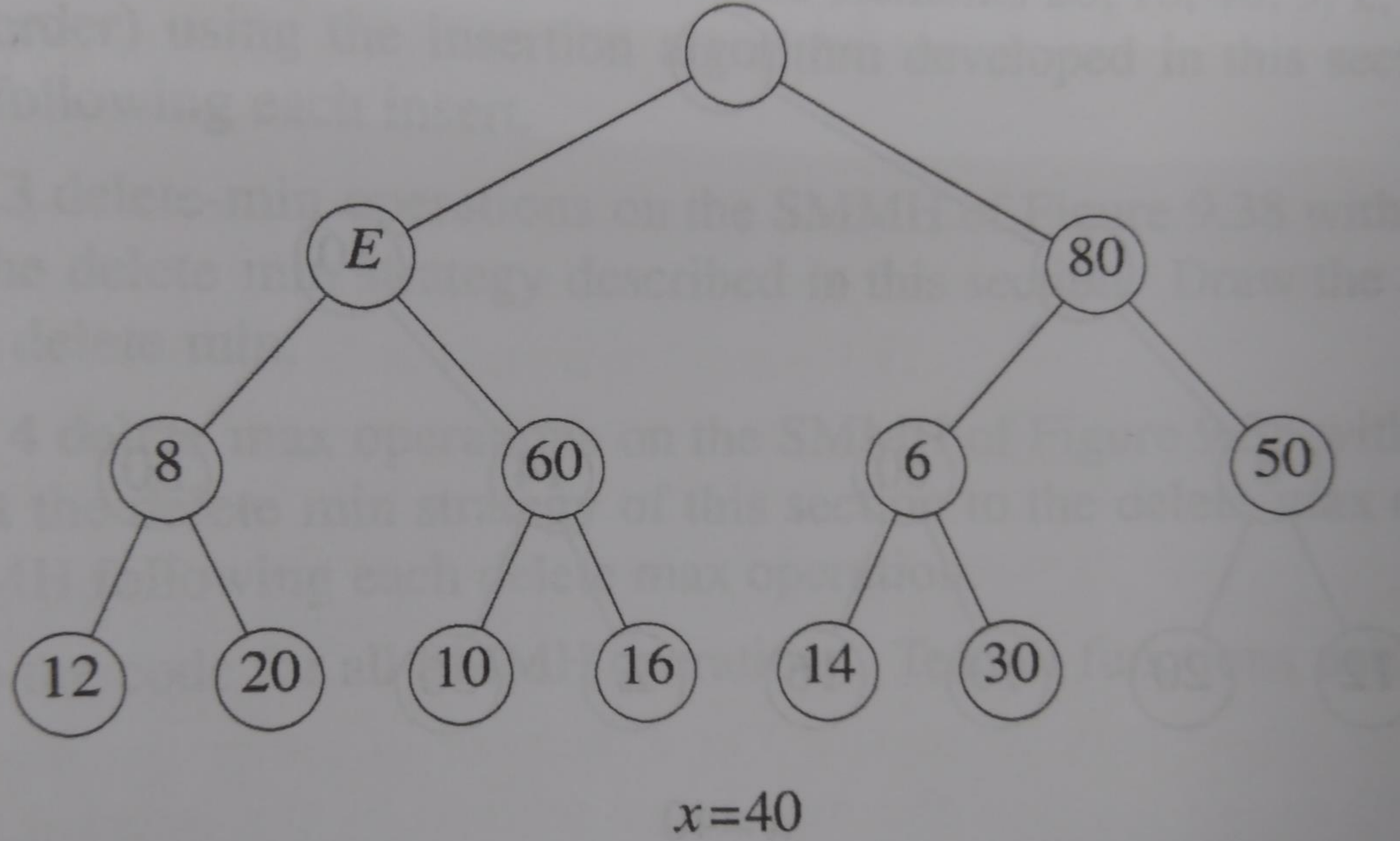

Figure 9.35: First step of another delete min

Deletion from SMMH （the smallest value）

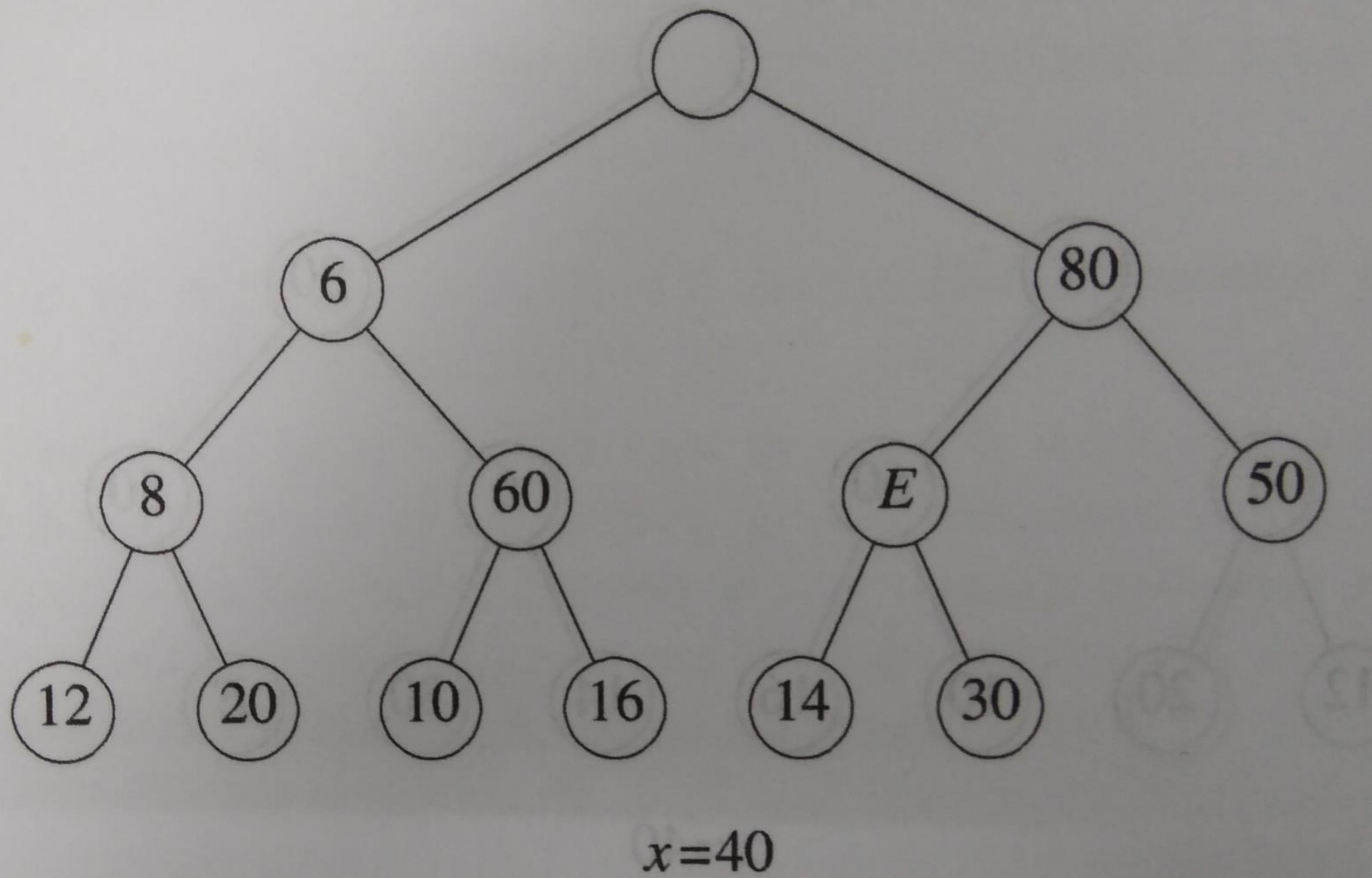

Figure 9.36: The SMMH of Figure 9.35 with *E* and 6 interchanged

Deletion from SMMH （the smallest value）

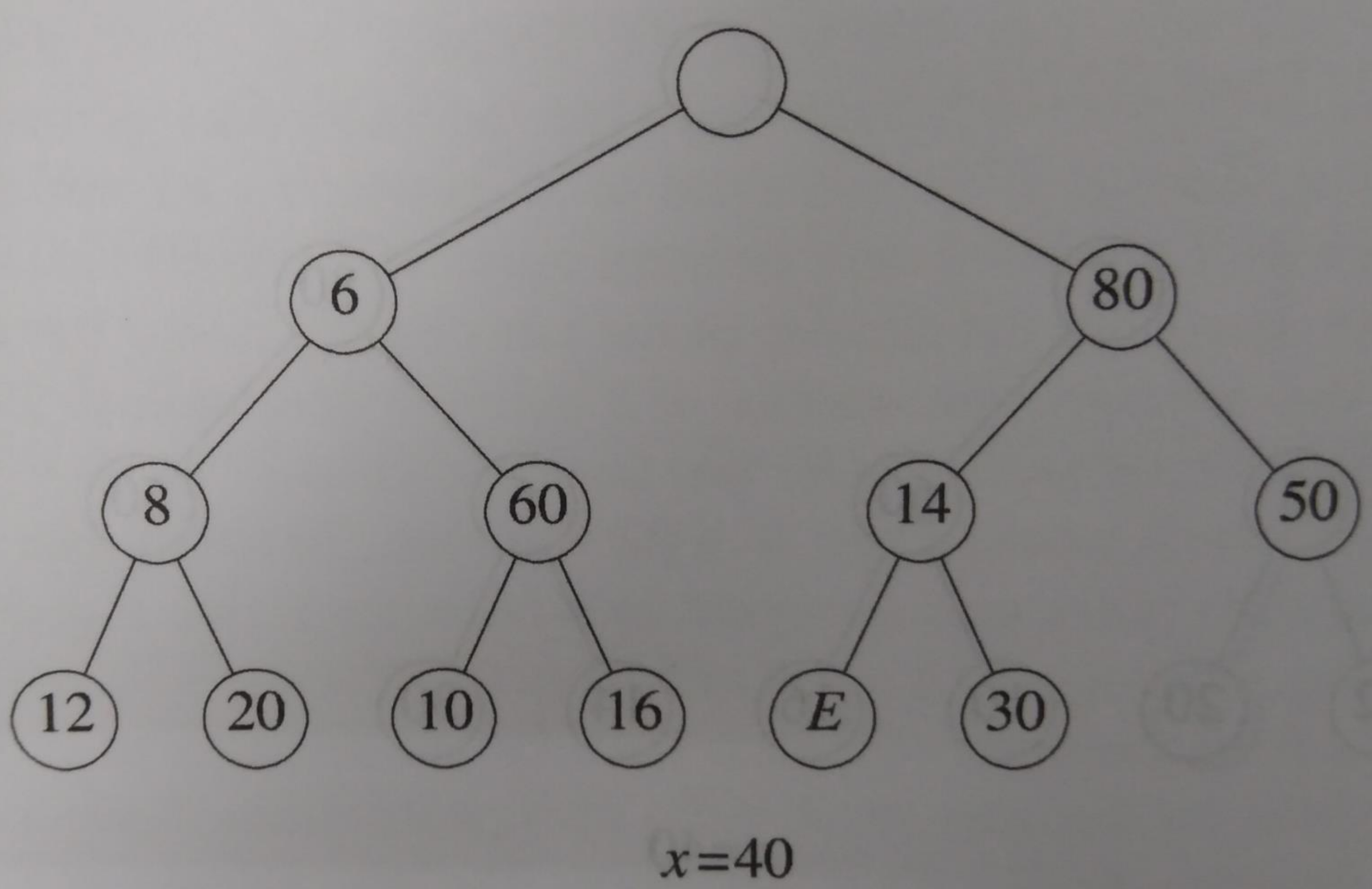

Figure 9.37: The SMMH of Figure 9.36 with *E* and 14 interchanged

Deletion from SMMH （the smallest value）

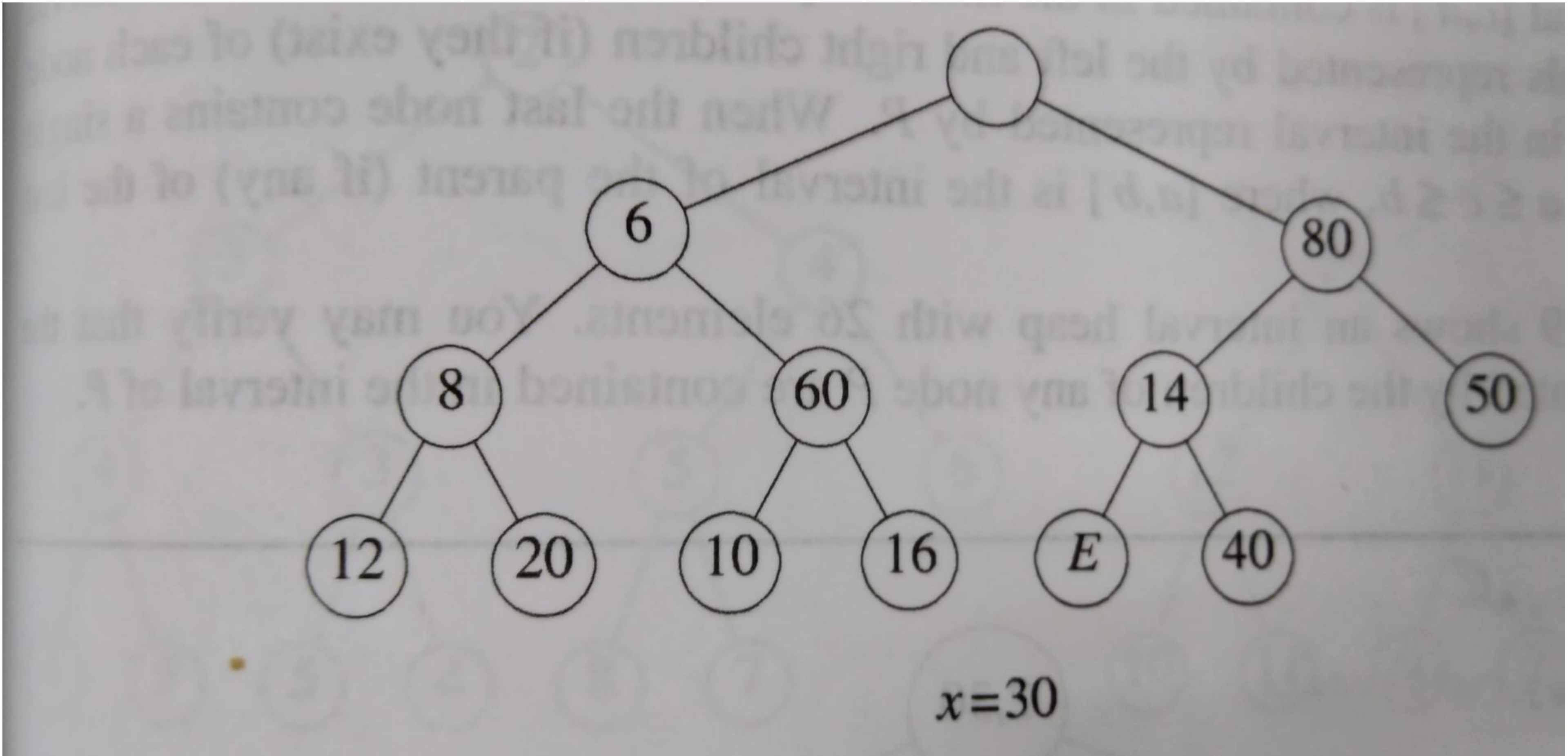

Figure 9.38: The SMMH of Figure 9.37 with x and 30 interchanged

課本沒有給 Deleting the Max Element，但道理是一樣的，所以你可以照著做即可

Interval Heaps

Definition and Properties

- An interval heap is a complete binary tree in which each node, except the last one, must contain two element.
- Let the two elements in a node be a and b. Then, $a \leq b$. We call a the left end of the interval and b the right end.
- In an interval heap, if [a, b] is the interval of any node X and [c, d] is the interval of X's child, then $a \leq c \leq d \leq b$ must hold.

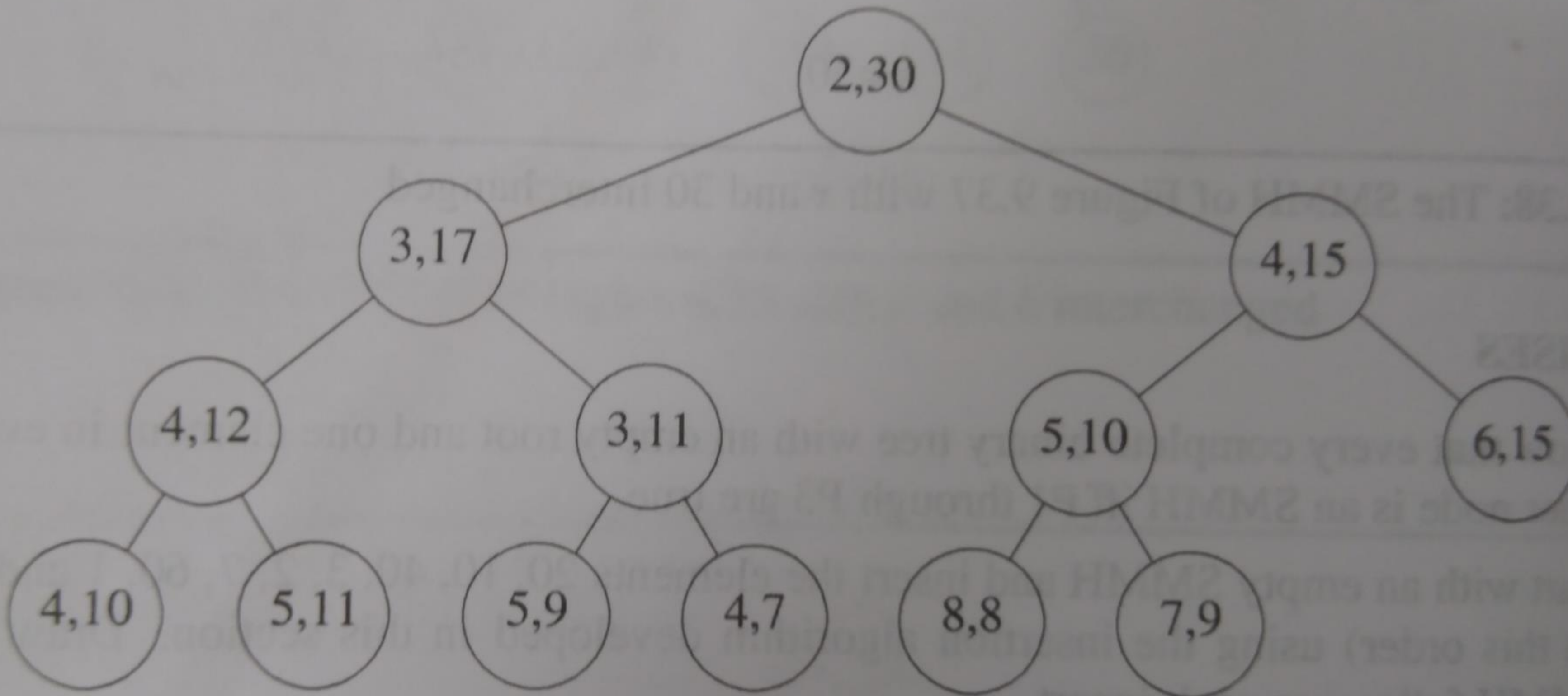

Figure 9.39: An interval heap

We can easily infer the following facts.

- The left values of the node intervals define a min heap, and the right values define a max heap. Fig. 9.40 shows the min and max heaps defined by the interval heap of Fig. 9.39.
- In case the total number of elements (i.e., values) is odd, the last node has a single element which may be regarded as a member of either the min or max heap.
- When the root has two elements (i.e., values), the left value of the root is the minimum in the interval heap and the right value is the maximum. When the root has only one element, the interval heap contains just one element. It is both the minimum and the maximum.

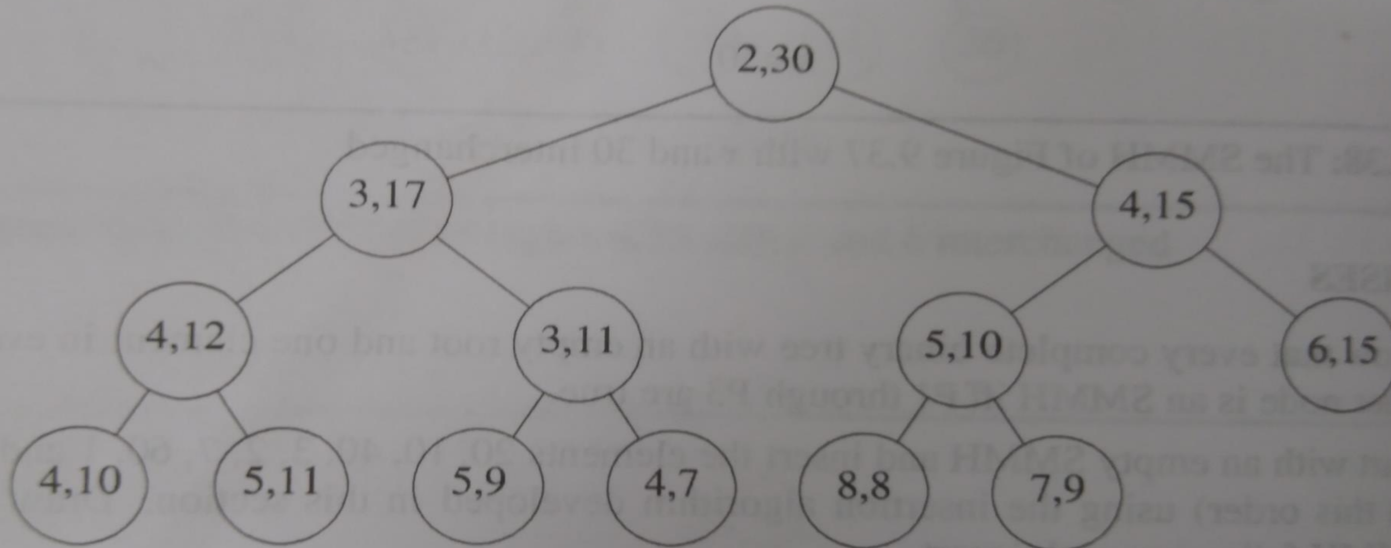

Figure 9.39: An interval heap

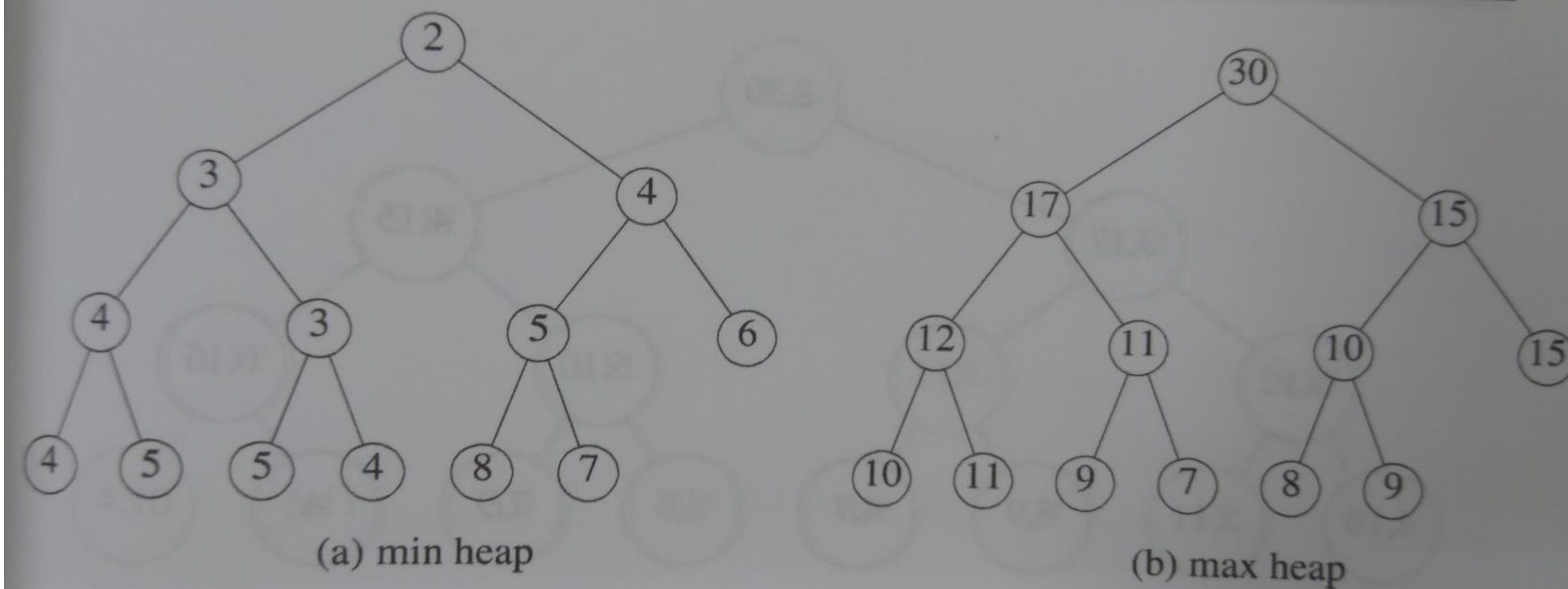

Figure 9.40: Min and max heaps embedded in Figure 9.39

Inserting into Interval Heap (a small value)

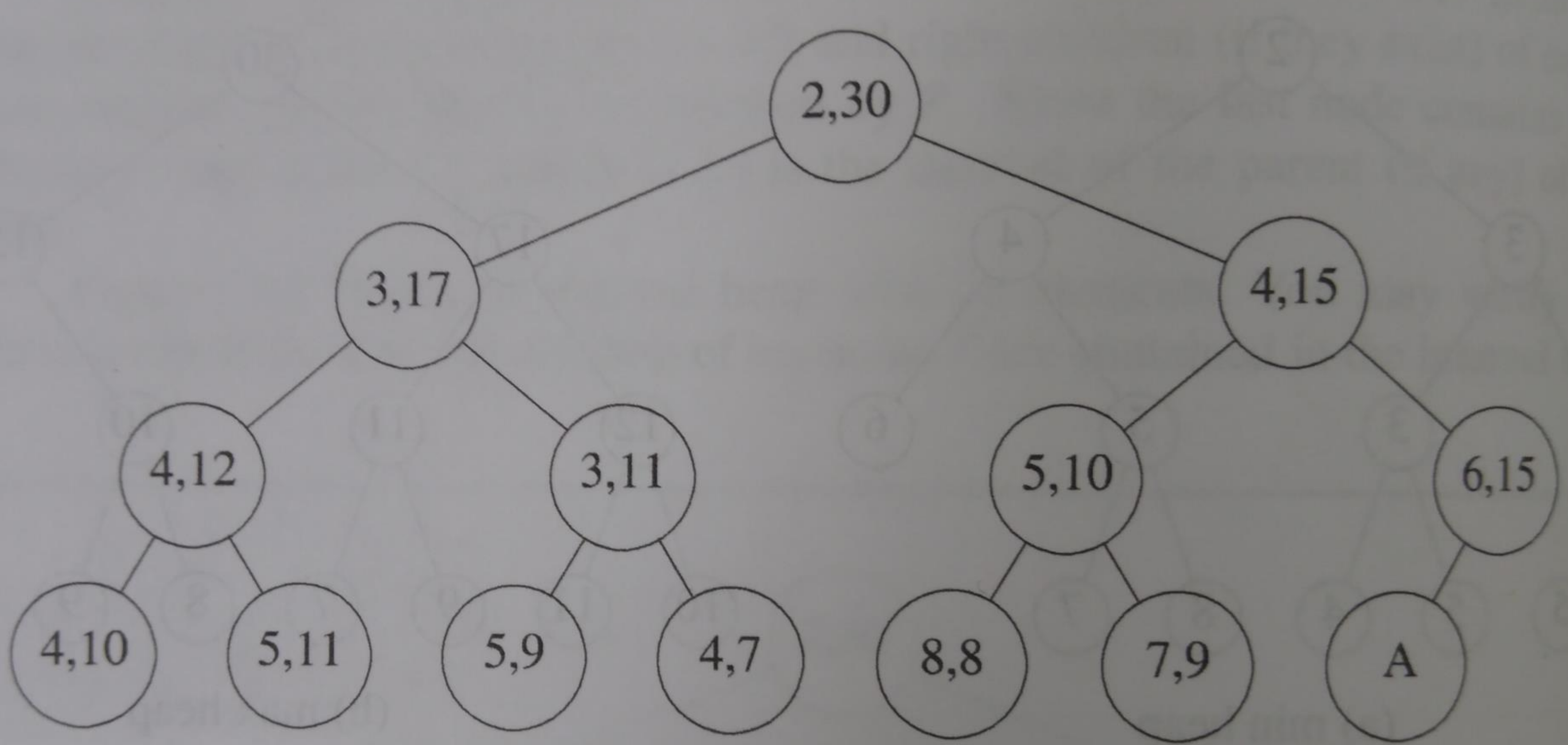

Figure 9.41: Interval heap of Figure 9.39 after one node is added

Inserting into Interval Heap (a small value)

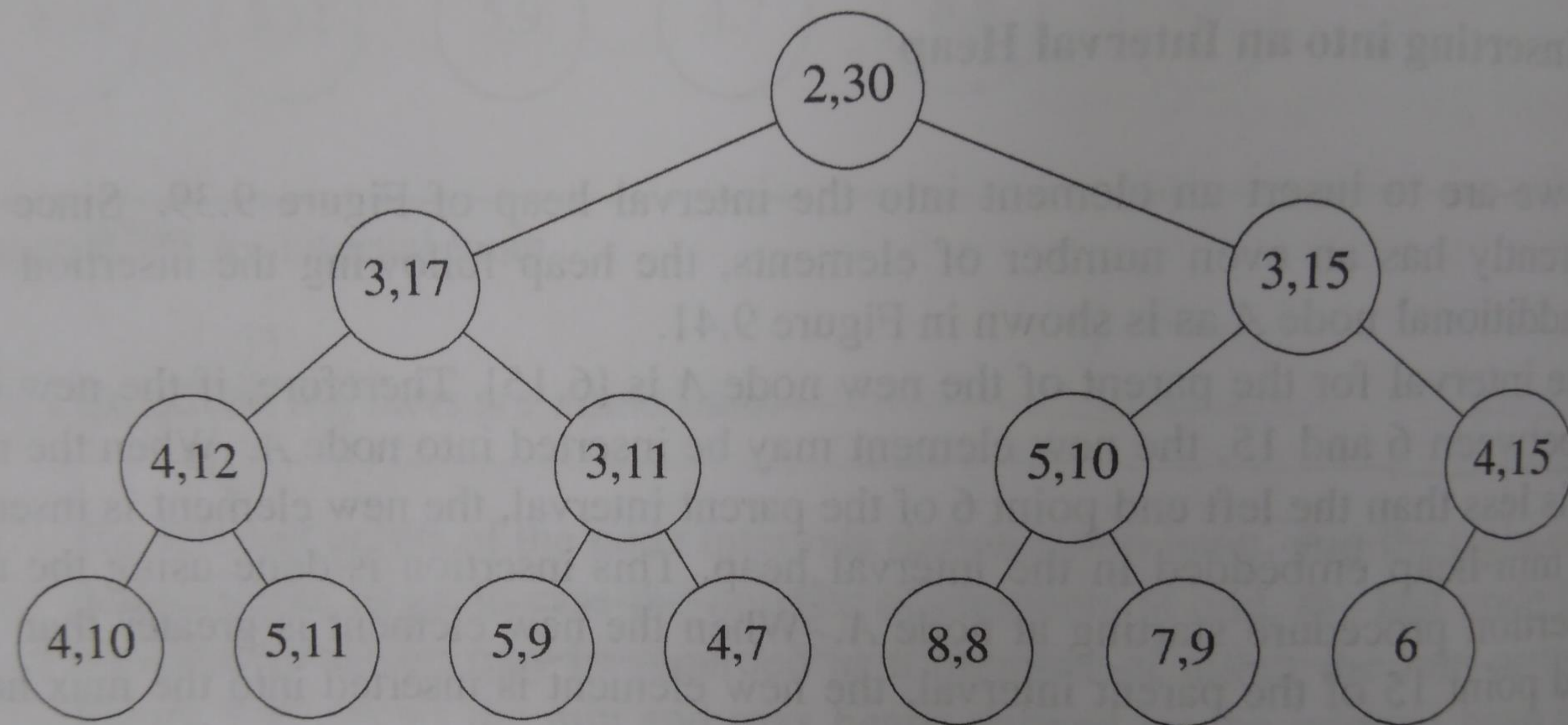

Figure 9.42: The interval heap of Figure 9.39 with 3 inserted

Inserting into Interval Heap (a large value)

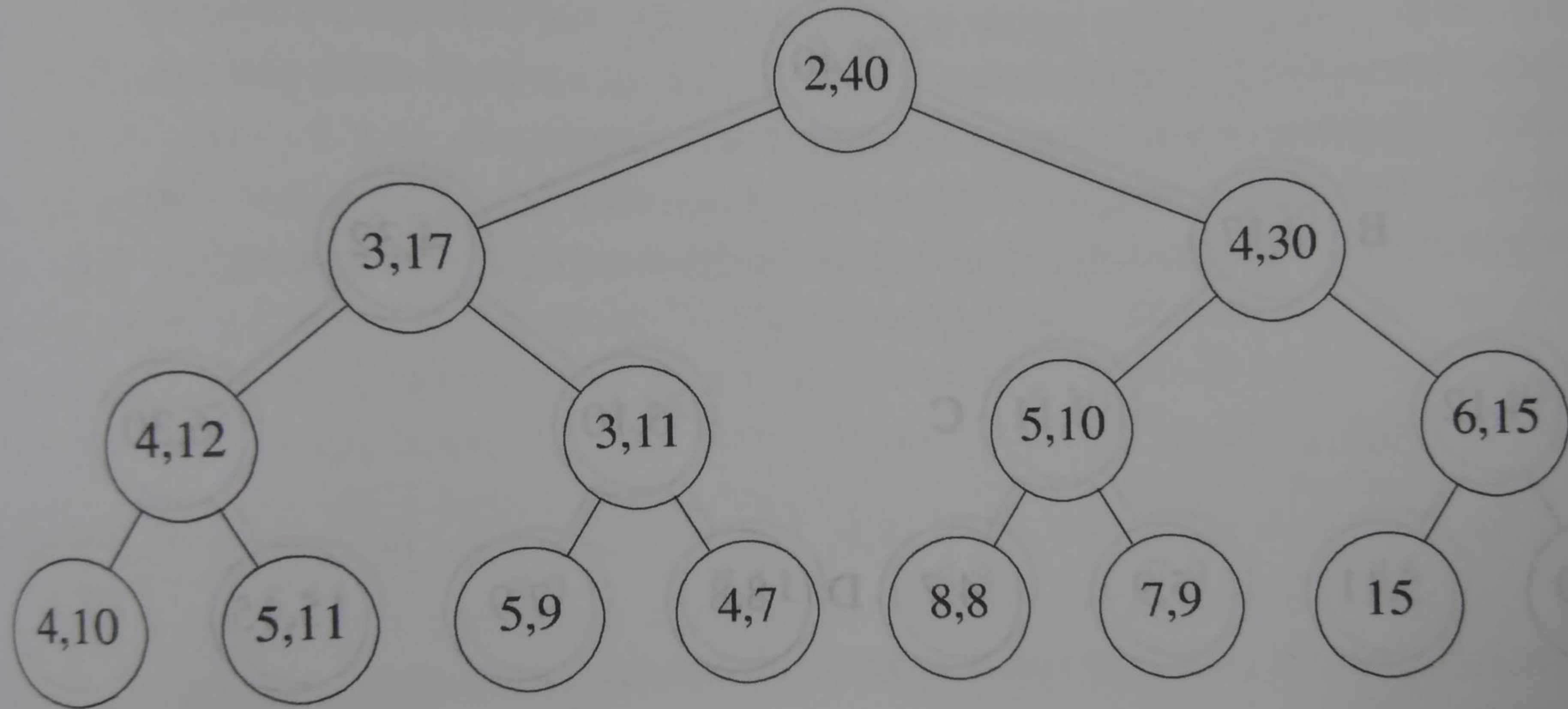

Figure 9.43: The interval heap of Figure 9.39 with 40 inserted

Inserting into Interval Heap (a large value)

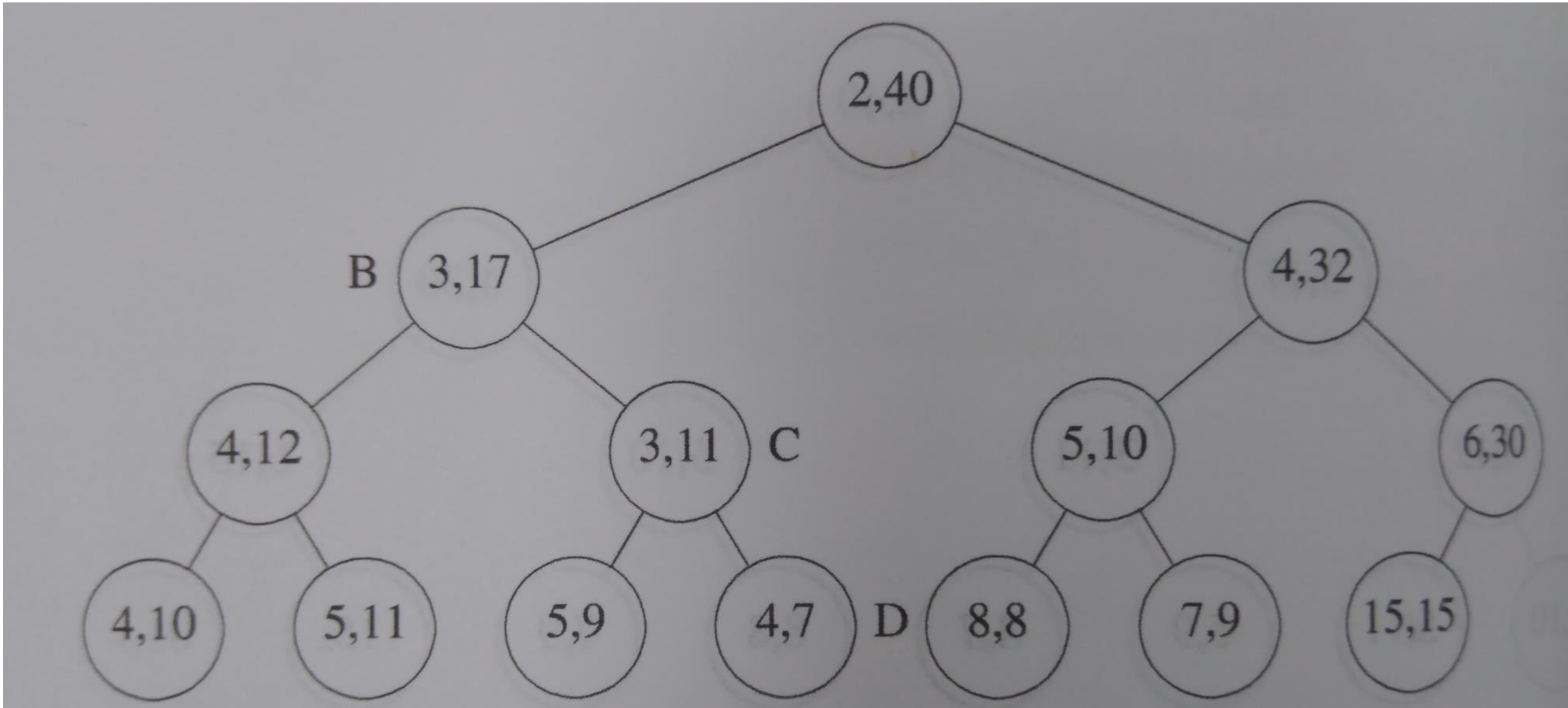

Figure 9.44: The interval heap of Figure 9.43 with 32 inserted

Deleting the Min Element

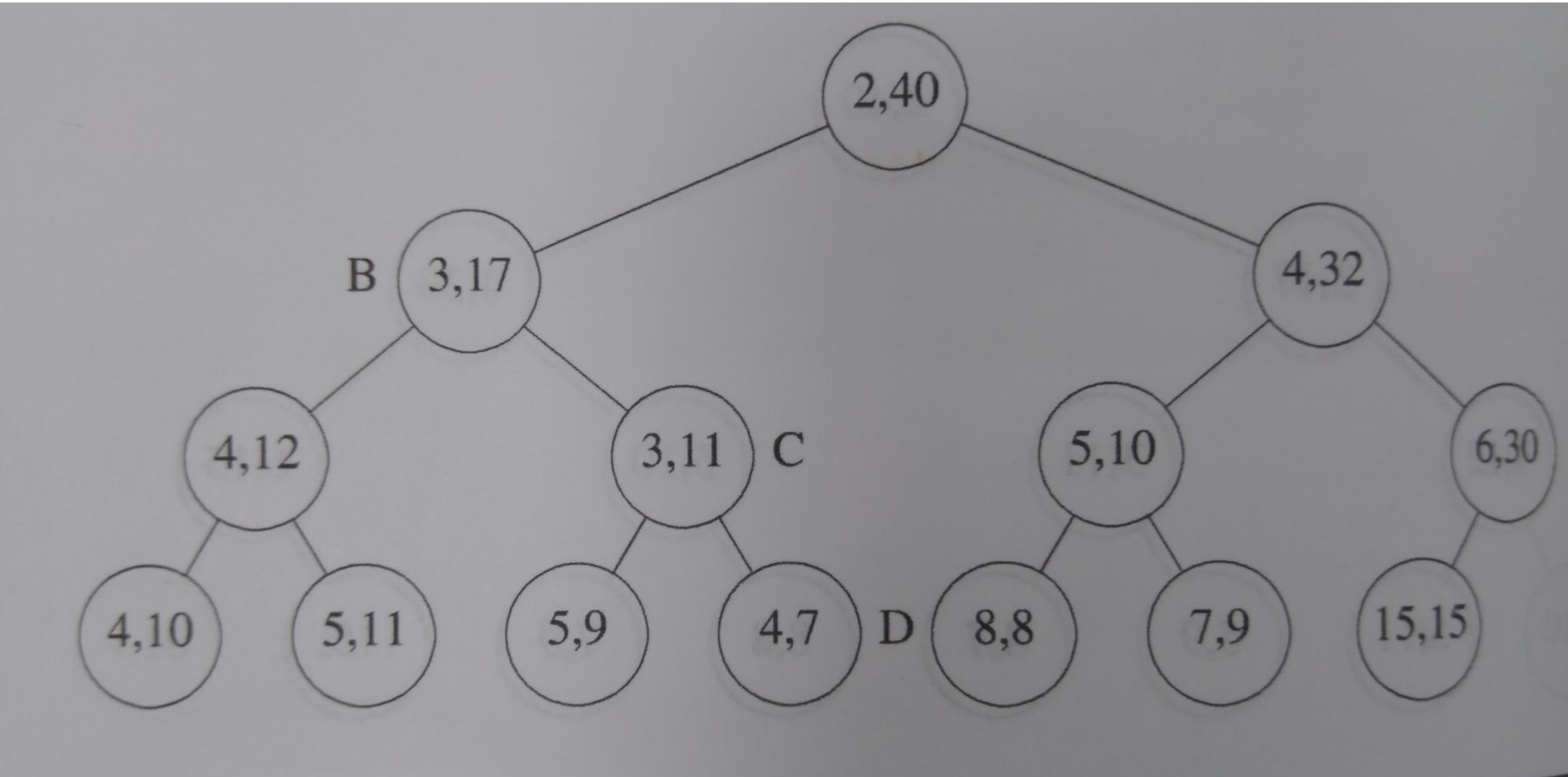

Figure 9.44: The interval heap of Figure 9.43 with 32 inserted

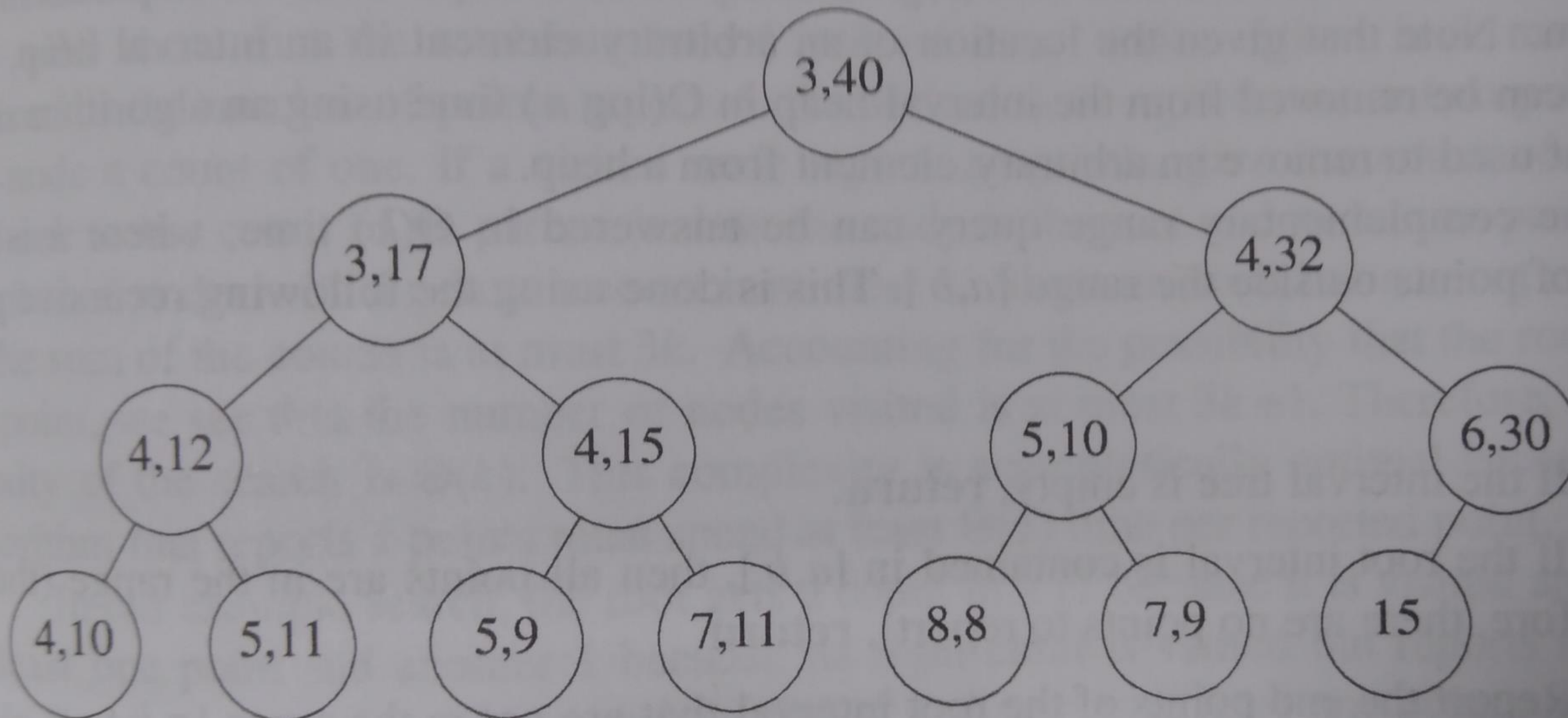

Figure 9.45: The interval heap of Figure 9.44 with minimum element removed

課本沒有給 Deleting the Max Element，但道理是一樣的，所以你可以照著做即可