110-1 ENGINEERING MATHEMATICS PRACTICE
(考前練習題)

1. $y'''' = \delta(x-1), y(0)=1, y'(0)=y''(0)=y'''(0)=0$
   Ans: $y(x) = 1 + \frac{1}{6}(x-1)^3 H(x-1)$

2. $xy'' + (x-1)y' + y = 0$, $y(0)=0, y(1)=e^{-1}$
   Ans: $y(x) = x^2 e^{-x}$

3. $y'' + 4xy' - 4y = 3\,\delta(t), y(0)=0, y'(0)=0$
   Ans: $y = 3t$

4. $\frac{dx}{dt} = -15x + 12y, \frac{dy}{dt} = -24x + 19y, x(0)=1, y(0)=-1$
   Ans: $x(t) = 15e^t - 14e^{3t}, y(t) = 20e^t - 21e^{3t}$

5. $f(x) = e^{-x} + 2\int_0^x e^{-3\tau} f(x-\tau)\, d\tau$
   Ans: $f(x) = e^{-x} + 2xe^{-x}$

6. $f'(x) = x + \int_0^x f(x-\tau)\cos\tau\, d\tau, f(0)=4$
   Ans: $f(x) = 4 + \frac{5}{2}x^2 + \frac{1}{24}x^4$

7. $y'(t) + 3y(t) + 2\int_0^t y(\tau)\,d\tau = f(t), y(0)=1, f(t)=1$, when $0 < t < 1$, $f(t)=0, when\ 1 < t < 2$
   Ans: $y(t) = -e^{-t} + 2e^{-2t} + \Sigma_{n=0}^{\infty}\left(e^{-(t-n)} - e^{-2(t-n)}\right)(-1)^n H(t-n)$

8. *Find the general solution in terms of Bessel's function*
   $x^2 y'' + xy' + (\lambda^2 x^2 - v^2)y = 0, 2v \notin N$
   Ans: $y = C_1 J_v(\lambda x) + C_2 J_{-v}(\lambda x)$ (考試須另外將$J_v(\lambda x), J_{-v}(\lambda x)$完整寫出)

9. $\frac{1}{2}x^2 y'' + \frac{1}{2}xy' + \left(2x^4 - \frac{1}{8}\right)y = 0$
   Ans: $y = C_1 J_{\frac{1}{4}}(x^2) + C_2 J_{-\frac{1}{4}}(x^2)$ (考試須另外將$J_{\frac{1}{4}}(x^2), J_{-\frac{1}{4}}(x^2)$完整寫出)

10. 假設 $x = 0$ 為 $x^2 y'' + xp(x)y' + q(x)y = 0$ 之一規則異點 Qn is a bipartite graph.

   其中 $p(x) = a_0 + a_1 x + a_2 x^2 + \cdots$
   $q(x) = b_0 + b_1 x + b_2 x^2 + \cdots$
   請問在 $C_0 \neq 0$ 之指標方程式為何?
   Ans: $r(r-1) + a_0 r + b_0 = 0$

11. $f(t) = \cos t + e^{-2t}\int_0^t f(\tau)e^{2\tau}\, d\tau$
   Ans: $f(t) = -\frac{1}{2}e^{-t} + \frac{3}{2}\cos t + \frac{1}{2}\sin t$

12. Please summarize the whole series solution for the general second order D.E.