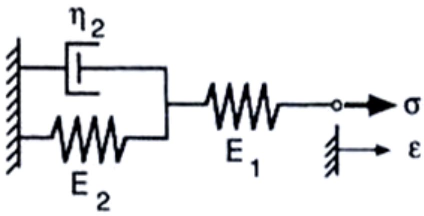
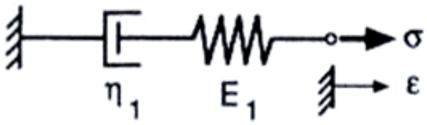
材料機械性質學

Exam. # 2 （05/21/2020）

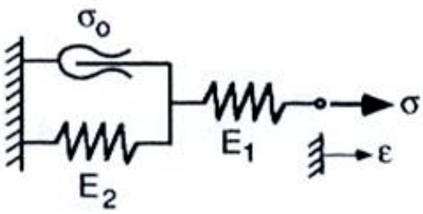
1. (10pts) 請以以下的模式推導潛變時的潛變應變和時間關係。



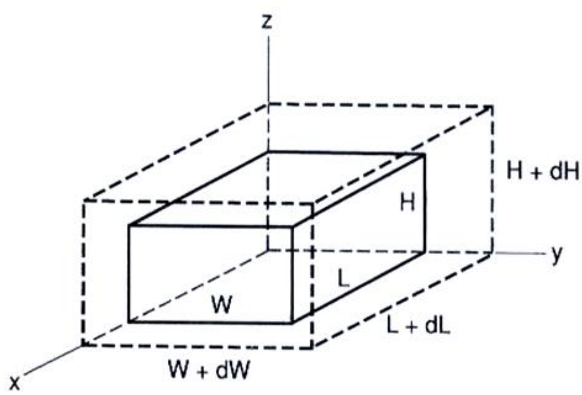
2. (10pts) 請以以下的模式推導鬆弛時的鬆弛應力和時間關係。



3. (10pts) 請根據以下的模式推導塑性變形時的應力和應變關係。

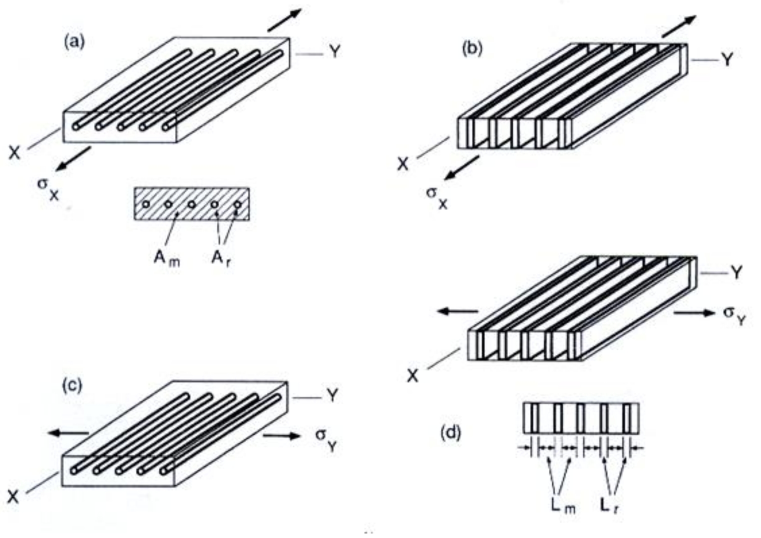


4. (10pts) 請推導靜水應力σ_h、體積應變ε_v及體積模數 B 之間的關係。



5. (10pts) 請以矩陣方式表示(a)非等向性及(b)均質且等向性材料的應變-應力關係。

6. (20pts) 請推導平行與垂直纖維方向複合材料的彈性係數方程式。



7. (20pts) 三度座標點的應力分量為:

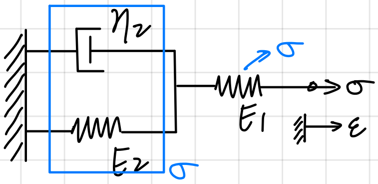
$\sigma_x = 100, \sigma_y = \text{-}60, \sigma_z = 40, \tau_{xy} = 80, \tau_{xz} = \tau_{yz} = 0\ MPa$

(a) 請計算主應力。

(b) 請計算各主應力面方向的單位向量。

8. (10pts) 一鋁合金工程元件的自由面上測得以下的應變為: ε_x = -0.0005、ε_y = 0.0035 和γ_{xy} = 0.003。假設材料沒有發生屈服且蒲松比ν為 0.345，請計算主應變及主剪應變。

1.



並聯部分 = $\sigma = E_2\varepsilon_c + \eta_2\varepsilon_c'$ (ε_c = 並聯部分的應變)

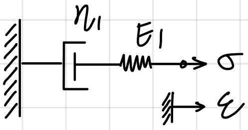
$$\xrightarrow{\text{潛變時}} \sigma' = E_2\varepsilon_c + \eta_2\varepsilon_c' \longrightarrow \varepsilon_c' + \frac{E_2}{\eta_2}\varepsilon_c = \frac{\sigma'}{\eta_2}$$

$$\therefore \varepsilon_c = C_1\exp\left[-\frac{E_2}{\eta_2}t\right] + \frac{\sigma'}{E_2} = \frac{\sigma'}{E_2}\left(1-e^{-E_2t/\eta_2}\right),$$

△ From initial condition: $\because t=0,\ \varepsilon_c=0\ \therefore \varepsilon_c = C_1 + \frac{\sigma'}{E_2} \Rightarrow C_1 = -\frac{\sigma'}{E_2}$

$$\Rightarrow \varepsilon = \varepsilon_e + \varepsilon_c = \underline{\frac{\sigma'}{E_1} + \frac{\sigma'}{E_2}\left(1-e^{-E_2t/\eta_2}\right)}\ \#$$

2.



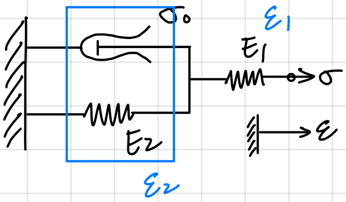
$$\varepsilon' = \varepsilon_e + \varepsilon_c$$

$$\xrightarrow{\text{取微分}} \frac{d\varepsilon'}{dt} = \frac{d\varepsilon_e}{dt} + \frac{d\varepsilon_c}{dt} = \frac{\sigma'}{E_1} + \varepsilon_c' = \frac{\sigma'}{E_1} + \frac{\sigma}{\eta_1} = 0$$

$$\therefore \sigma = C_1\exp\left[-\frac{E_1}{\eta_1}t\right] = \underline{E_1\varepsilon' e^{-E_1t/\eta_1}}\ \#$$

△ From initial condition: $\because t=0,\ \varepsilon=\varepsilon' \Rightarrow \sigma = E_1\varepsilon'\ \therefore C_1 = \varepsilon/E_1$

3.



$$\varepsilon = \varepsilon_1 + \varepsilon_2 = \frac{\sigma}{E_1} + \varepsilon_2$$

① $\varepsilon_2 = 0,\ \varepsilon = \frac{\sigma}{E_1}$ (when $\sigma \leq \sigma_0$)

② $\varepsilon_2 = \frac{\sigma-\sigma_0}{E_2},\ \varepsilon = \frac{\sigma}{E_1} + \frac{\sigma-\sigma_0}{E_2}$ (when $\sigma \geq \sigma_0$)

$$\Rightarrow \varepsilon = \frac{(E_1+E_2)\sigma}{E_1E_2} - \frac{E_1\sigma_0}{E_1E_2}\quad \therefore \frac{d\varepsilon}{d\sigma} = \frac{E_1+E_2}{E_1E_2} \Rightarrow \underline{\frac{d\sigma}{d\varepsilon} = \frac{E_1E_2}{E_1+E_2}}\ \#$$

4.

$$\varepsilon_x = \frac{dL}{L},\quad \varepsilon_y = \frac{dW}{W},\quad \varepsilon_z = \frac{dH}{H}$$

And $\because V = LWH\ \therefore dV = \frac{\partial V}{\partial L}dL + \frac{\partial V}{\partial W}dW + \frac{\partial V}{\partial H}dH \xrightarrow{V=LWH} dV = WHdL + LHdW + LWdH$

$$\longrightarrow \frac{dV}{V} = \frac{dL}{L} + \frac{dW}{W} + \frac{dH}{H} \longrightarrow \varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\begin{pmatrix} \varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y+\sigma_z)] \\ \varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x+\sigma_z)] \\ \varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x+\sigma_y)] \end{pmatrix}$$

$$\longrightarrow \varepsilon_V = \frac{1}{E}\left[\sigma_x+\sigma_y+\sigma_z - 2\nu(\sigma_x+\sigma_y+\sigma_z)\right] = \frac{1-2\nu}{E}(\sigma_x+\sigma_y+\sigma_z),$$

And $\sigma_h = \frac{1}{3}(\sigma_x+\sigma_y+\sigma_z)$, $\frac{\varepsilon_V E}{1-2\nu} = 3\sigma_h \Rightarrow \varepsilon_V = \frac{3(1-2\nu)}{E}\sigma_h\ \therefore \underline{B = \frac{\sigma_h}{\varepsilon_V} = \frac{E}{3(1-2\nu)}}\ \#$

5. (a)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}\ \#$$

(b)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}\ \#$$

6. (1) 平行方向：

$A=A_r+A_m \longrightarrow \sigma_x A=\sigma_r A_r+\sigma_m A_m$, $(\because \sigma=\frac{P}{A}, P=\sigma A)$

$\sigma_x=E_x\varepsilon_x$, $\sigma_r=E_r\varepsilon_r$, $\sigma_m=E_m\varepsilon_m$, and $\varepsilon_x=\varepsilon_r=\varepsilon_m$

$\longrightarrow E_x\cancel{\varepsilon_x}A=E_r\cancel{\varepsilon_r}A_r+E_m\cancel{\varepsilon_m}A_m$, $E_x=\frac{E_rA_r+E_mA_m}{A}=\frac{E_rA_rL+E_mA_mL}{AL}=E_r\left(\frac{A_rL}{AL}\right)+E_m\left(\frac{A_mL}{AL}\right)$, $=E_r\boxed{V_r}+E_m\boxed{V_m}$ #

V_r V_m 加強材體積分數(V_r) 基材体積分數(V_m)

(2) 垂直方向：

$L=L_r+L_m$ and $\varepsilon_y=\frac{\Delta L}{L}$, $\varepsilon_r=\frac{\Delta L_r}{L_r}$, $\varepsilon_m=\frac{\Delta L_m}{L_m}$, $\Delta L=\Delta L_r+\Delta L_m$

And $\sigma_y=\sigma_r=\sigma_m$, $\sigma_y=E_y\varepsilon_y$, $\sigma_r=E_r\varepsilon_r$, $\sigma_m=E_m\varepsilon_m$.

$\longrightarrow \varepsilon_yL=\varepsilon_rL_r+\varepsilon_mL_m$, $\varepsilon_y=\frac{\varepsilon_rL_r+\varepsilon_mL_m}{L}$

$\longrightarrow \frac{\cancel{\sigma_y}}{E_y}=\frac{\cancel{\sigma_r}}{E_r}\times\frac{L_r}{L}+\frac{\cancel{\sigma_m}}{E_m}\times\frac{L_m}{L} \longrightarrow \frac{1}{E_y}=\frac{1}{E_r}\left(\frac{AL_r}{AL}\right)+\frac{1}{E_m}\left(\frac{AL_m}{AL}\right)=\frac{V_r}{E_r}+\frac{V_m}{E_m}=\frac{V_rE_m+V_mE_r}{E_rE_m}$ $\therefore E_y=\frac{E_rE_m}{V_rE_m+V_mE_r}$ #

7. $\sigma_x=100$, $\sigma_y=-60$, $\sigma_z=40$, $\tau_{xy}=80$, $\tau_{xz}=\tau_{yz}=0$ MPa

(1) 主應力：

$$\boxed{\begin{vmatrix} \sigma_x-\sigma & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y-\sigma & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z-\sigma \end{vmatrix}}=\begin{vmatrix} 100-\sigma & 80 & 0 \\ 80 & -60-\sigma & 0 \\ 0 & 0 & 40-\sigma \end{vmatrix}=0$$

$\longrightarrow (40-\sigma)\left[(100-\sigma)(-60-\sigma)-6400\right]=0$

$\longrightarrow (40-\sigma)(\sigma^2-40\sigma-12400)=0$ $\therefore \sigma_1=133.14$ MPa, $\sigma_2=-93.14$ MPa, $\sigma_3=40$ MPa #

(2) 主應力面方向的單位向量：

a. when $\sigma_i=\sigma_1=133.14$ MPa：

$$\begin{cases} (100-133.14)l_1+80m_1+0=0 \\ 80l_1+(-60-133.14)m_1+0=0 \\ 0+0+(40-133.14)n_1=0 \end{cases} \Rightarrow n_1=0,$$

$l_1=2.414m_1$

$\because (l,m,n)$為單位向量

又 $l_1^2+m_1^2+n_1^2=1 \rightarrow (2.414m_1)^2+m_1^2+0=1$ $\therefore m_1=0.383$, $l_1=2.414m_1=0.924$

$\Rightarrow l_1=0.924$, $m_1=0.383$, $n_1=0$ #

b. when $\sigma_i=\sigma_2=-93.14$ MPa：

$$\begin{cases} (100+93.14)l_2+80m_2=0 \\ 80l_2+(-60+93.14)m_2=0 \\ (40+93.14)n_2=0 \end{cases} \Rightarrow n_2=0$$

and $l_2=-0.414m_2$

$\Rightarrow l_2=-0.383$, $m_2=0.924$, $n_2=0$ #

又 $l_2^2+m_2^2+n_2^2=1 \rightarrow (-0.414m_2)^2+m_2^2=1$ $\therefore m_2=0.924$, $l_2=-0.414m_2=-0.383$

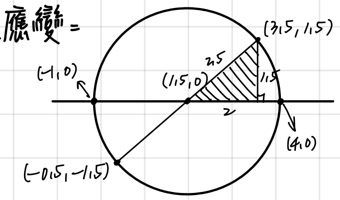
c when $\sigma_i = \sigma_3 = 40MPa$ =

$$\begin{cases} (100-40)l_3 + 80m_3 = 0 \longrightarrow l_3 = -\frac{80}{60}m_3 = -\frac{4}{3}m_3 \\ 80l_3 + (-60-40)m_3 = 0 \longrightarrow l_3 = \frac{100}{80}m_3 = \frac{5}{4}m_3 \Rightarrow l_3 = m_3 = 0, \\ (40-40)n_3 = 0 \end{cases}$$

又 $l_3^2 + m_3^2 + n_3^2 = 1 \rightarrow n_3^2 = 1$ $\therefore n_3 = 1$ $\Rightarrow \underline{l_3 = 0,\ m_3 = 0,\ n_3 = 1}$ #

8. $\varepsilon_x = -0.0005$, $\varepsilon_y = 0.0035$, $\gamma_{xy} = 0.003$, $\nu = 0.345$

(1) 主應變 =



$r = 2.5 \times 10^{-3}$ and $\boxed{\varepsilon_3 = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)} = \frac{-0.345}{1-0.345}(-0.0005 + 0.0035) = -1.58 \times 10^{-3}$

$\Rightarrow \underline{\varepsilon_1 = 4 \times 10^{-3},\ \varepsilon_2 = -1 \times 10^{-3},\ \varepsilon_3 = -1.58 \times 10^{-3}}$ #

(2) 主剪應變 = $\gamma_1 = |\varepsilon_2 - \varepsilon_3| = \underline{0.58 \times 10^{-3}}$ #

$\gamma_2 = |\varepsilon_1 - \varepsilon_3| = \underline{5.58 \times 10^{-3}}$ #

$\gamma_3 = |\varepsilon_1 - \varepsilon_2| = \underline{5 \times 10^{-3}}$ #