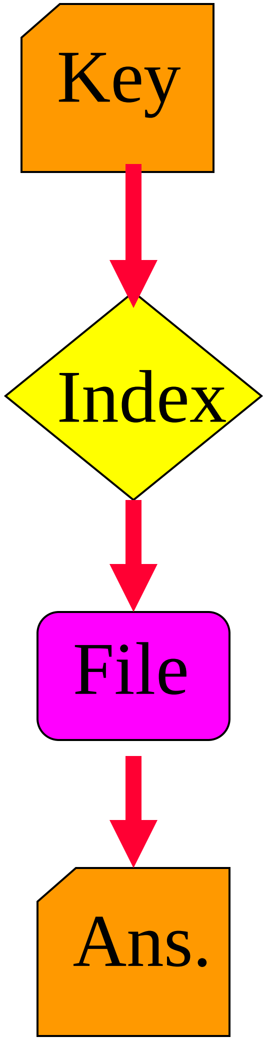
Bloom Filters

- Differential Files
- Simple large database.
  - File of records residing on disk.
  - Single key.
  - Index to records.
- Operations.
  - Retrieve.
  - Update.
    - Insert a new record.
    - Make changes to an existing record.
    - Delete a record.

Naïve Mode Of Operation

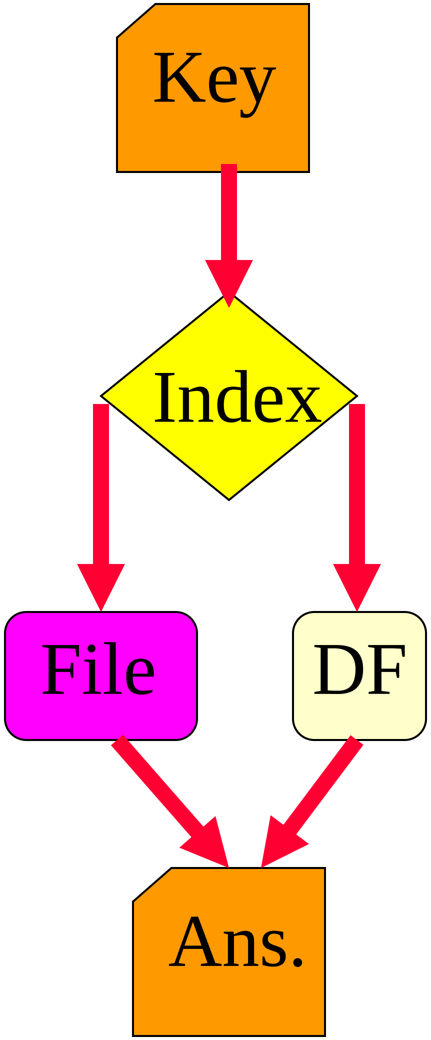


- Problems.
  - Index and File change with time.
  - Sooner or later, system will crash.
  - Recovery =>
    - Copy Master File (MF) from backup.
    - Copy Master Index (MI) from backup.
    - Process all transactions since last backup.
  - Recovery time depends on MF & MI size + #transactions since last backup.

Differential File

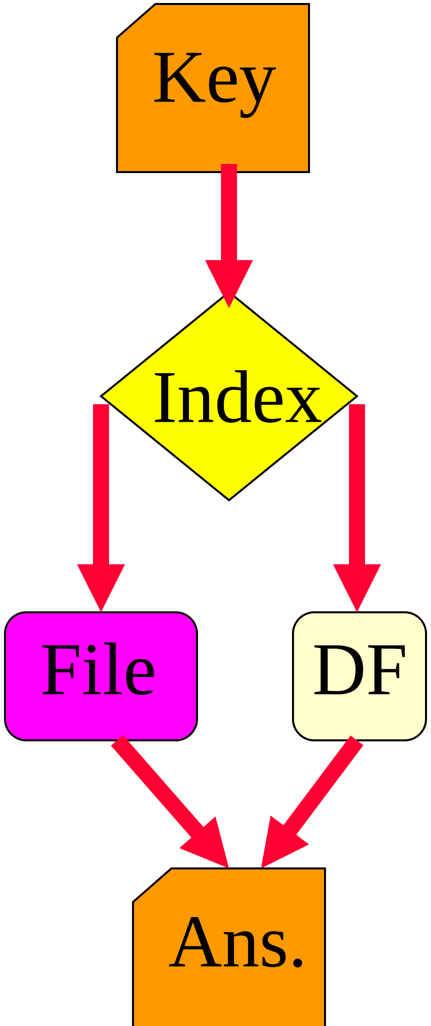
- Make no changes to master file.
- Alter index and write updated record to a new file called differential file.

Differential File Operation



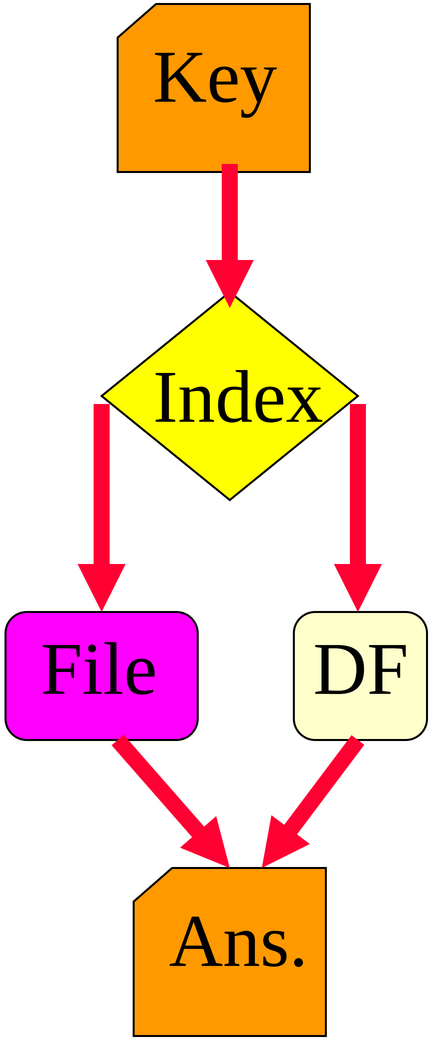
- Advantage.
  - DF is smaller than File and so may be backed up more frequently.
  - Index needs to be backed up whenever DF is. So, index should be no larger than DF.
  - Recovery time is reduced.

Differential File Operation



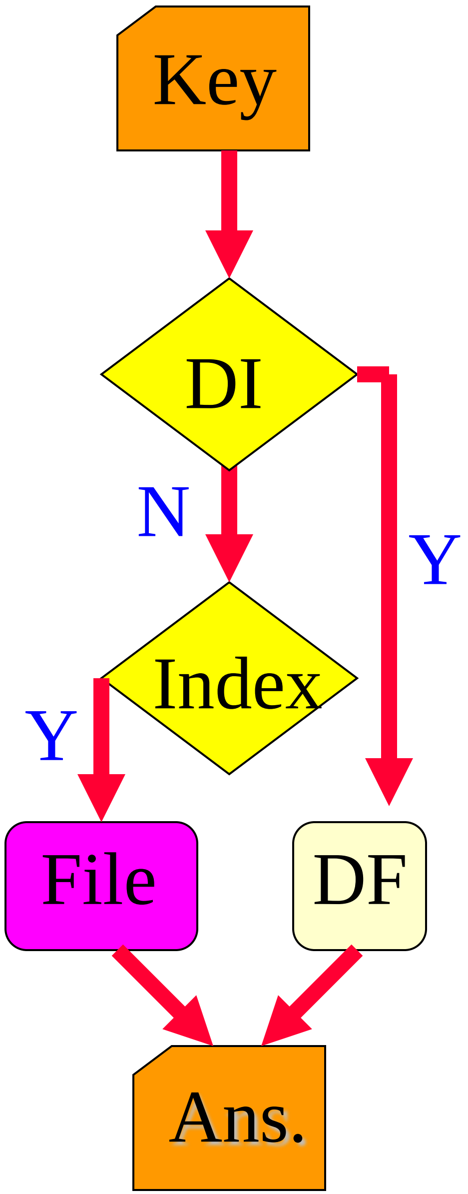
- Disadvantage.
  - Eventually DF becomes large and can no longer be backed up with desired frequency.
  - Must integrate File and DF now.
  - Following integration, DF is empty.

Differential File Operation



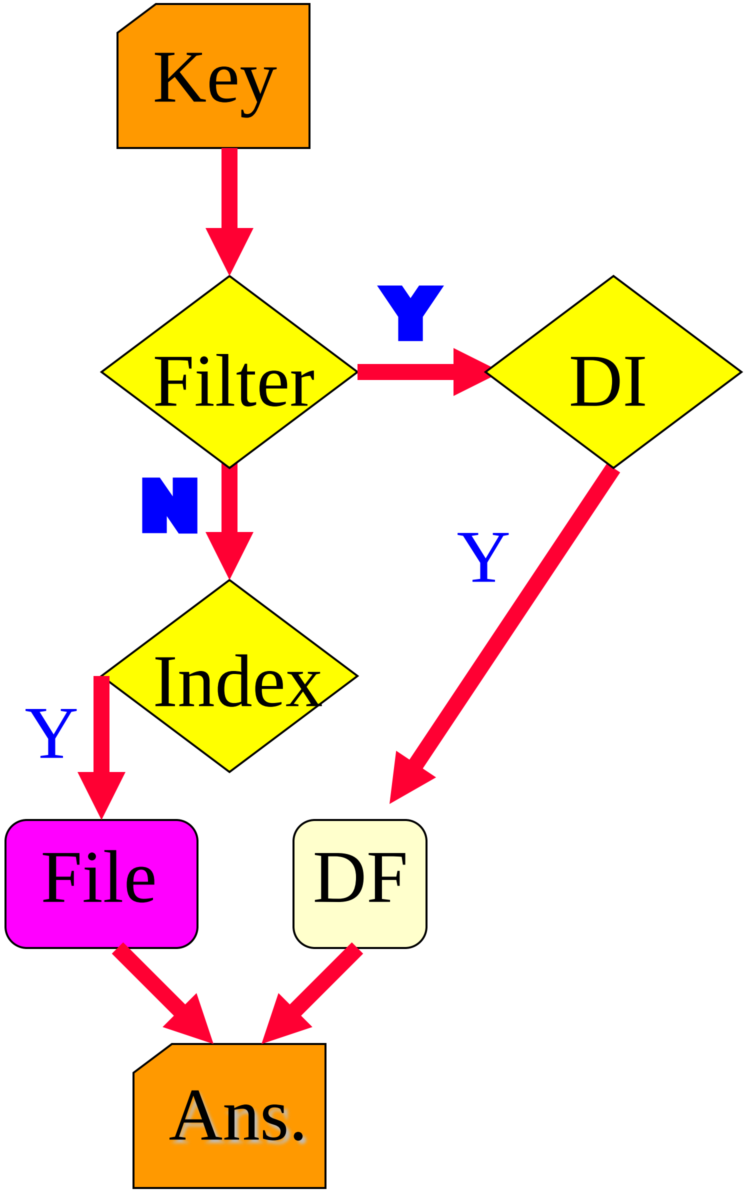
- Large Index.
  - Index cannot be backed up as frequently as desired.
  - Time to recover current state of index & DF is excessive.
  - Use a differential index.
    - Make no changes to Index.
    - DI is an index to all deleted records and updated records in DF.

Differential File & Index Operation



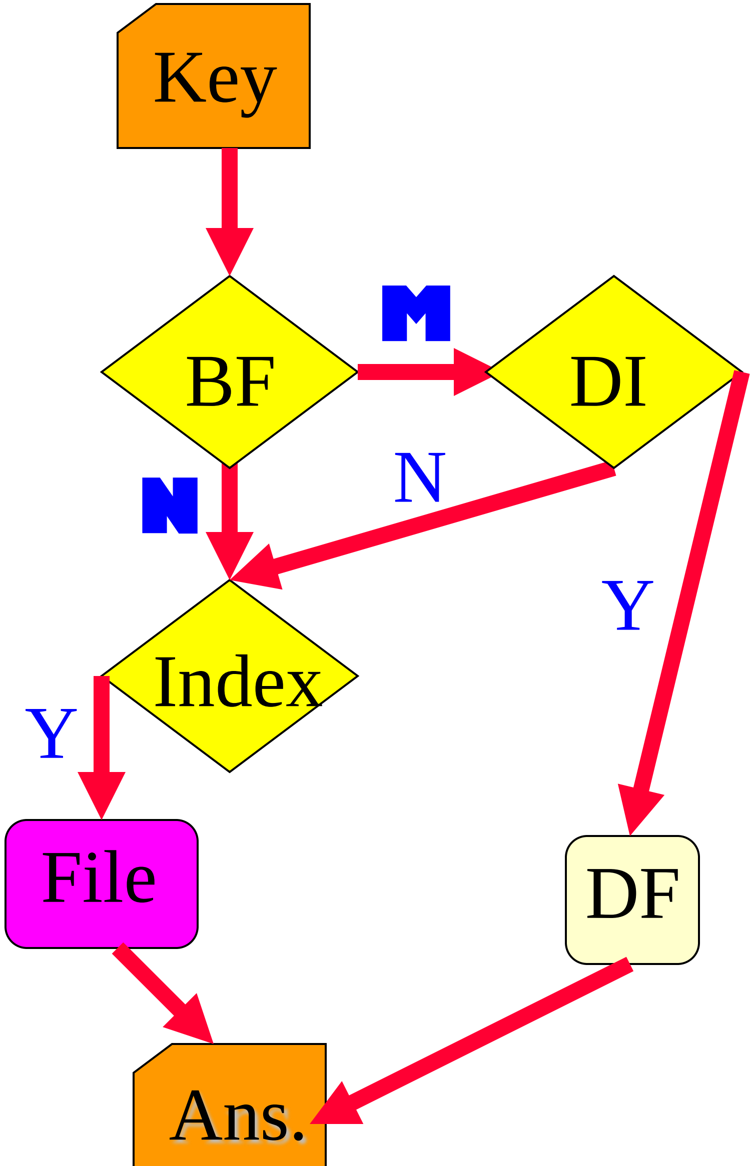
- Performance hit.
  - Most queries search both DI and Index.
  - Increase in # of disk accesses/query.
- Use a filter to decide whether or not DI should be searched.

Ideal Filter



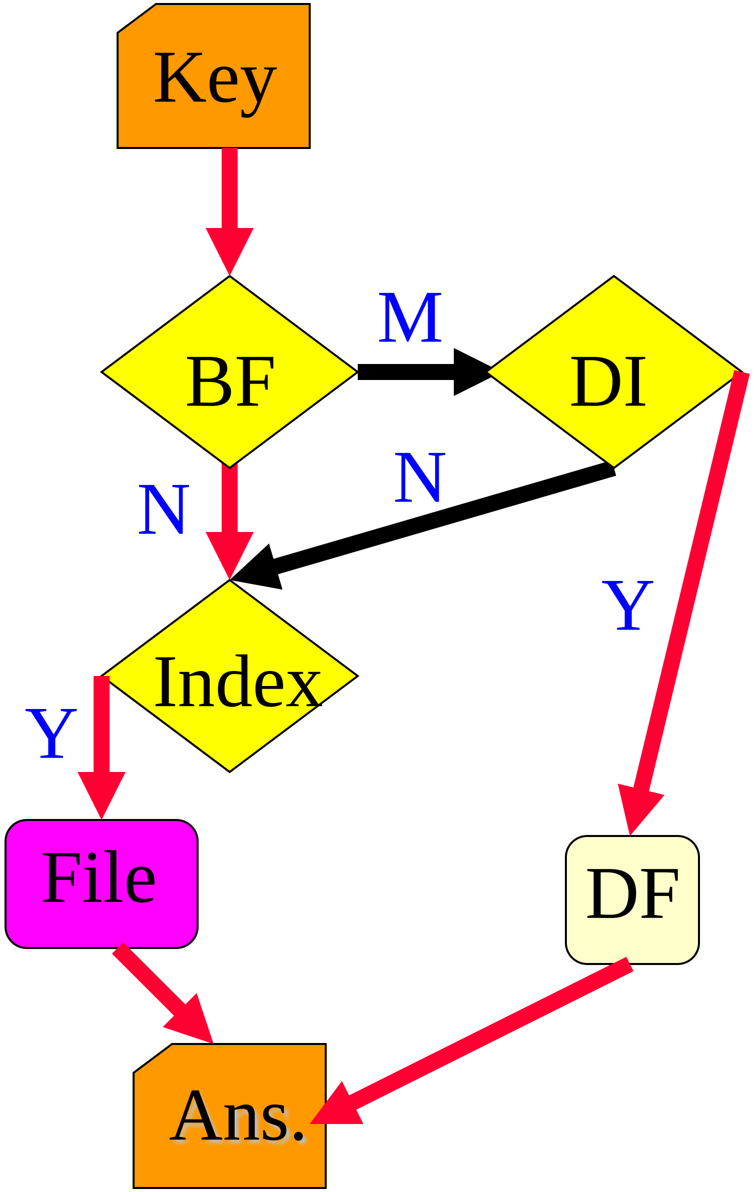
- **Y** => this key is in the DI.
- **N** => this key is not in the DI.
- Functionality of ideal filter is same as that of DI.
- So, a filter that eliminates performance hit of DI doesn't exist.

Bloom Filter (BF)



- N => this key is not in the DI.
- M (maybe) => this key may be in the DI.
- Filter error.
  - BF says Maybe.
  - DI says No.

Bloom Filter (BF)



- Filter error.
  - BF says Maybe.
  - DI says No.

- BF resides in memory.
- Performance hit paid only when there is a filter error.

Bloom Filter Design

- Use m bits of memory for the BF.
- Larger m => fewer filter errors.
- When DI empty, all m bits $= 0$.
- Use $h > 0$ hash functions: $f_1()$, $f_2()$, …, $f_h()$.
- When key k inserted into DI, set bits $f_1(k)$, $f_2(k)$, …, and $f_h(k)$ to 1.
- $f_1(k)$, $f_2(k)$, …, $f_h(k)$ is the signature of key k.

Example

0	1	0	0	1	0	1	0	1	0	0
0	1	2	3	4	5	6	7	8	9	

- $m = 11$ (normally, m would be much much larger).
- $h = 2$ (2 hash functions).
- $f_1(k) = k \bmod m$.
- $f_2(k) = (2k) \bmod m$.
- $k = 15$.
- $k = 17$.

Example

0	1	0	0	1	0	1	0	1	0	0
0	1	2	3	4	5	6	7	8	9	

- DI has $k = 15$ and $k = 17$.
- Search for k.
  - $f_1(k) = 0$ or $f_2(k) = 0$ => k not in DI.
  - $f_1(k) = 1$ and $f_2(k) = 1$ => k may be in DI.
- $k = 6$ => filter error.

Bloom Filter Design

- Choose m (filter size in bits).
  - Use as much memory as is available.
- Pick h (number of hash functions).
  - h too small => probability of different keys having same signature is high.
  - h too large => filter becomes filled with ones too soon.
- Select the h hash functions.
  - Hash functions should be relatively independent.

Optimal Choice Of h

- Probability of a filter error depends on:
  - Filter size … m.
  - # of hash functions … h.
  - # of updates before filter is reset to 0 … u.
    - Insert
    - Delete
    - Change
- Assume that m and u are constant.
- # of master file records = n >> u.

Probability Of Filter Error

- p(u) = probability of a filter error after u updates

  = A * B
- A = p(request for an unmodified record after u updates)
- B = p(filter bits are all 1 for this request for an unmodified record)

$A = p(\text{request for unmodified record})$

- $p($update j is for record $i) = 1/n$.
- $p($record i not modified by update $j) = 1 - 1/n$.
- $p($record i not modified by any of the u updates$)$

  $= (1 - 1/n)^u$

  $= A$.

B = p(filter bits are all 1 for this request)

- Consider an update with key K.
- $p(f_j(K) \mathrel{!=} i) = 1 - 1/m$.
- $p(f_j(K) \mathrel{!=} i$ for all $j) = (1 - 1/m)^h$.
- p(bit $i = 0$ after one update) $= (1 - 1/m)^h$.
- p(bit $i = 0$ after u updates) $= (1 - 1/m)^{uh}$.
- p(bit $i = 1$ after u updates) $= 1 - (1 - 1/m)^{uh}$.
- p(signature of K is 1 after u updates)

  $= [1 - (1 - 1/m)^{uh}]^h$

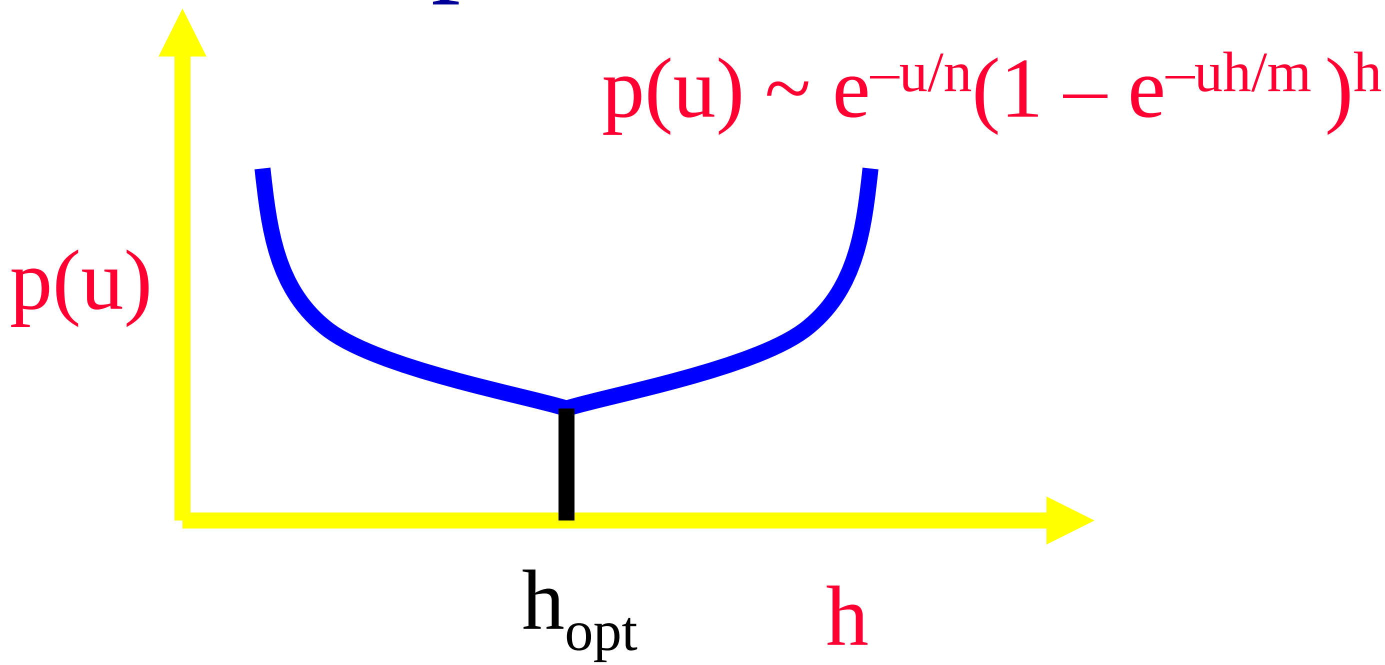
  $= B$.

Probability Of Filter Error

- $p(u) = A * B$

  $= (1 - 1/n)^{u} * [1 - (1 - 1/m)^{uh}]^{h}$
- $(1 - 1/x)^{q} \sim e^{-q/x}$ when x is large.
- $p(u) \sim e^{-u/n}(1 - e^{-uh/m})^{h}$
- $d\,p(u)/dh = 0 \Rightarrow h = (\ln 2)m/u \sim 0.693m/u$.

Optimal h



- $h \sim 0.693m/u$.
- $m = 10^6$, $u = 10^6/2$
  - $h \sim 1.386$
  - Use $h = 1$ or $h = 2$.
- $m = 2*10^6$, $u = 10^6/2$
  - $h \sim 2.772$
  - Use $h = 2$ or $h = 3$.