110-1 ENGINEERING MATHEMATICS HW3_SOL

1.

$$\mathscr{L}\{f(t)\} = \int_0^\infty e^{t+7}e^{-st}\,dt = e^7\int_0^\infty e^{(1-s)t}\,dt$$

$$= \frac{e^7}{1-s}e^{(1-s)t}\Big|_0^\infty = 0 - \frac{e^7}{1-s} = \frac{e^7}{s-1}$$

2.

$$\mathscr{L}\{e^t\sinh t\} = \mathscr{L}\left\{e^t\,\frac{e^t - e^{-t}}{2}\right\} = \mathscr{L}\left\{\frac{1}{2}e^{2t} - \frac{1}{2}\right\} = \frac{1}{2(s-2)} - \frac{1}{2s}$$

3.

$$\mathscr{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s} + 3\cdot\frac{1}{s^2} + \frac{3}{2}\cdot\frac{2}{s^3} + \frac{1}{6}\cdot\frac{3!}{s^4}\right\} = 1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3$$

4.

$$\mathscr{L}^{-1}\left\{\frac{5}{s^2+49}\right\} = \mathscr{L}^{-1}\left\{\frac{5}{7}\cdot\frac{7}{s^2+49}\right\} = \frac{5}{7}\sin 7t$$

5.

$$\mathscr{L}\left\{e^{3t}\left(9 - 4t + 10\sin\frac{t}{2}\right)\right\} = \mathscr{L}\left\{9e^{3t} - 4te^{3t} + 10e^{3t}\sin\frac{t}{2}\right\}$$

$$= \frac{9}{s-3} - \frac{4}{(s-3)^2} + \frac{5}{(s-3)^2 + 1/4}$$

6.

$$\mathscr{L}^{-1}\left\{\frac{2s-1}{s^2(s+1)^3}\right\} = \mathscr{L}^{-1}\left\{\frac{5}{s}-\frac{1}{s^2}-\frac{5}{s+1}-\frac{4}{(s+1)^2}-\frac{3}{2}\frac{2}{(s+1)^3}\right\}$$
$$= 5-t-5e^{-t}-4te^{-t}-\frac{3}{2}t^2e^{-t}$$

7.

Identify $f(\tau)=4\tau$ and $g(t-\tau)=3\,(t-\tau)^2$. Therefore,

$$f*g=\int_0^t (4\tau)\,3\,(t-\tau)^2\;d\tau = 12\int_0^t \left(t^2\tau-2t\tau^2+\tau^3\right)\,d\tau$$
$$=12\left(t^2\cdot\frac{1}{2}t^2-2t\cdot\frac{1}{3}t^3+\frac{1}{4}t^4\right)=t^4$$

Then

$$\mathscr{L}\left\{f*g\right\}=\mathscr{L}\left\{t^4\right\}=\frac{4!}{s^5}=\frac{24}{s^5}$$

8.

The Laplace transform of the given equation is

$$\mathscr{L}\{f\}=\mathscr{L}\left\{te^t\right\}+\mathscr{L}\{t\}\mathscr{L}\{f\}.$$

Solving for $\mathscr{L}\{f\}$ we obtain

$$\mathscr{L}\{f\}=\frac{s^2}{(s-1)^3(s+1)}=\frac{1}{8}\frac{1}{s-1}+\frac{3}{4}\frac{1}{(s-1)^2}+\frac{1}{4}\frac{2}{(s-1)^3}-\frac{1}{8}\frac{1}{s+1}$$

Thus

$$f(t)=\frac{1}{8}e^t+\frac{3}{4}te^t+\frac{1}{4}t^2e^t-\frac{1}{8}e^{-t}$$

9.

The Laplace transform of the differential equation yields

$$\begin{aligned}\mathscr{L}\{y\} &= \frac{4+s}{s^2+4s+13} + \frac{e^{-\pi s}+e^{-3\pi s}}{s^2+4s+13} \\ &= \frac{2}{3}\frac{3}{(s+2)^2+3^2} + \frac{s+2}{(s+2)^2+3^2} + \frac{1}{3}\frac{3}{(s+2)^2+3^2}\left(e^{-\pi s}+e^{-3\pi s}\right)\end{aligned}$$

so that

$$\begin{aligned}y = {}& \frac{2}{3}e^{-2t}\sin 3t + e^{-2t}\cos 3t + \frac{1}{3}e^{-2(t-\pi)}\sin 3(t-\pi)\mathscr{U}(t-\pi) \\ &+ \frac{1}{3}e^{-2(t-3\pi)}\sin 3(t-3\pi)\mathscr{U}(t-3\pi).\end{aligned}$$

10.

Taking the Laplace transform of the system gives

$$\begin{aligned}s\mathscr{L}\{x\} &= -\mathscr{L}\{x\} + \mathscr{L}\{y\} \\ s\mathscr{L}\{y\} - 1 &= 2\mathscr{L}\{x\}\end{aligned}$$

so that

$$\mathscr{L}\{x\} = \frac{1}{(s-1)(s+2)} = \frac{1}{3}\frac{1}{s-1} - \frac{1}{3}\frac{1}{s+2}$$

and

$$\mathscr{L}\{y\} = \frac{1}{s} + \frac{2}{s(s-1)(s+2)} = \frac{2}{3}\frac{1}{s-1} + \frac{1}{3}\frac{1}{s+2}.$$

Then

$$x = \frac{1}{3}e^{t} - \frac{1}{3}e^{-2t} \qquad \text{and} \qquad y = \frac{2}{3}e^{t} + \frac{1}{3}e^{-2t}.$$