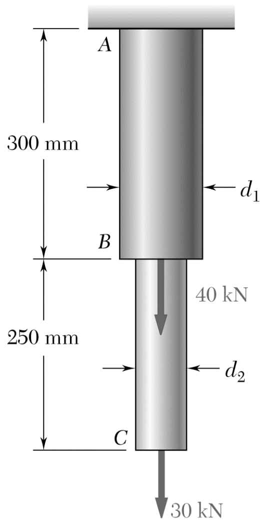
CHAPTER 1

PROBLEM 1.1



Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod *AB* and 150 MPa in rod *BC*, determine the smallest allowable values of d_1 and d_2.

SOLUTION

(*a*) Rod *AB*

$$P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{N}$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{\frac{\pi}{4} d_1^2} = \frac{4P}{\pi d_1^2}$$

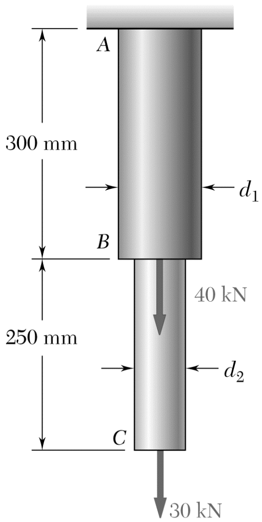
$$d_1 = \sqrt{\frac{4P}{\pi \sigma_{AB}}} = \sqrt{\frac{(4)(70 \times 10^3)}{\pi(175 \times 10^6)}} = 22.6 \times 10^{-3} \text{m} \qquad d_1 = 22.6 \text{ mm} \blacktriangleleft$$

(*b*) Rod *BC*

$$P = 30 \text{ kN} = 30 \times 10^3 \text{N}$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{P}{\frac{\pi}{4} d_2^2} = \frac{4P}{\pi d_2^2}$$

$$d_2 = \sqrt{\frac{4P}{\pi \sigma_{BC}}} = \sqrt{\frac{(4)(30 \times 10^3)}{\pi(150 \times 10^6)}} = 15.96 \times 10^{-3} \text{m} \qquad d_2 = 15.96 \text{ mm} \blacktriangleleft$$



PROBLEM 1.2

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 50$ mm and $d_2 = 30$ mm, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

(*a*) Rod *AB*

$$P = 40 + 30 = 70 \text{ kN} = 70\times 10^3 \text{N}$$

$$A = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(50)^2 = 1.9635\times 10^3 \text{mm}^2 = 1.9635\times 10^{-3}\text{m}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{70\times 10^3}{1.9635\times 10^{-3}} = 35.7\times 10^6 \text{Pa}$$

$\sigma_{AB} = 35.7$ MPa ◀

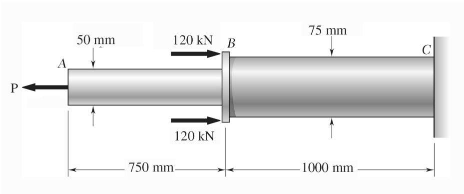
(*b*) Rod *BC*

$$P = 30 \text{ kN} = 30\times 10^3 \text{N}$$

$$A = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86\times 10^{-6}\text{m}^2$$

$$\sigma_{BC} = \frac{P}{A} = \frac{30\times 10^3}{706.86\times 10^{-6}} = 42.4\times 10^6 \text{Pa}$$

$\sigma_{BC} = 42.4$ MPa ◀



PROBLEM 1.3

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(50)^2 = 1963.5 \text{ mm}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{1963.5}$$

$$A_{BC} = \frac{\pi}{4}(75)^2 = 4417.9 \text{ mm}^2$$

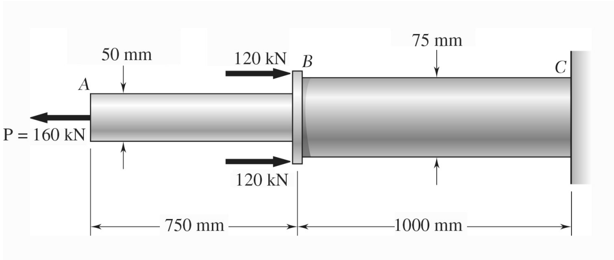
$$\sigma_{BC} = \frac{2(120) - P}{A_{BC}}$$

$$= \frac{240 - P}{4417.9}$$

Equating σ_{AB} to $2\sigma_{BC}$

$$\frac{P}{1963.5} = \frac{2(240 - P)}{4417.9}$$

$P = 112.9$ kN ◀



PROBLEM 1.4

In Prob. 1.3, knowing that $P = 160$ kN, determine the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

PROBLEM 1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.

SOLUTION

(*a*) Rod *AB*

$P = 160$ kN (tension) $$A_{AB} = \frac{\pi d_{AB}^2}{4} = \frac{\pi}{4}(50)^2 = 1963.5 \text{ mm}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{160 \times 10^3 \text{ N}}{1963.5 \times 10^{-6} \text{ m}^2}$$

$\sigma_{AB} = 81.5$ MPa ◀

(*b*) Rod *BC*

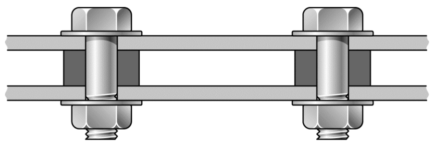
$F = 160 - 2(120) = -80$ kN i.e., 80 kN compression.

$$A_{BC} = \frac{\pi d_{BC}^2}{4} = \frac{\pi}{4}(75)^2 = 4417.9 \text{ mm}^2$$

$$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{-80 \times 10^3 \text{ N}}{4417.9 \times 10^{-6} \text{ m}^2}$$

$\sigma_{BC} = -18.11$ MPa ◀

PROBLEM 1.5



Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

SOLUTION

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time, the spacer pushes that plate upward with a compressive force P_s in order to maintain equilibrium.

$$P_b = P_s$$

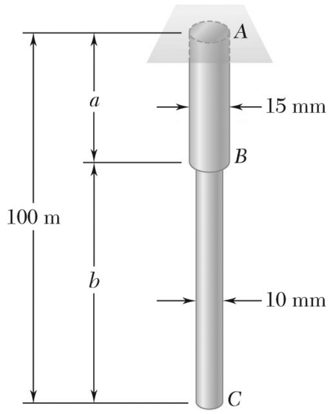
For the bolt, $\sigma_b = \frac{F_b}{A_b} = \frac{4P_b}{\pi d_b^2}$ or $P_b = \frac{\pi}{4}\sigma_b d_b^2$

For the spacer, $\sigma_s = \frac{P_s}{A_s} = \frac{4P_s}{\pi(d_s^2 - d_b^2)}$ or $P_s = \frac{\pi}{4}\sigma_s (d_s^2 - d_b^2)$

Equating P_b and P_s,

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s (d_s^2 - d_b^2)$$

$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)} d_b = \sqrt{\left(1 + \frac{200}{130}\right)}(16) \qquad d_s = 25.2 \text{ mm} \blacktriangleleft$$



PROBLEM 1.6

Two brass rods AB and BC, each of uniform diameter, will be brazed together at B to form a nonuniform rod of total length 100 m, which will be suspended from a support at A as shown. Knowing that the density of brass is 8470 kg/m^3, determine (*a*) the length of rod AB for which the maximum normal stress in ABC is minimum, (*b*) the corresponding value of the maximum normal stress.

SOLUTION

Areas: $$A_{AB} = \frac{\pi}{4}(15\text{ mm})^2 = 176.71\text{ mm}^2 = 176.71\times 10^{-6}\text{m}^2$$

$$A_{BC} = \frac{\pi}{4}(10\text{ mm})^2 = 78.54\text{ mm}^2 = 78.54\times 10^{-6}\text{m}^2$$

From geometry, $b = 100 - a$

Weights: $$W_{AB} = \rho g\, A_{AB}\ell_{AB} = (8470)(9.81)(176.71\times 10^{-6})a = 14.683a$$

$$W_{BC} = \rho g\, A_{BC}\ell_{BC} = (8470)(9.81)(78.54\times 10^{-6})(100 - a) = 652.59 - 6.526a$$

Normal stresses:

At A, $$P_A = W_{AB} + W_{BC} = 652.59 + 8.157a \qquad (1)$$

$$\sigma_A = \frac{P_A}{A_{AB}} = 3.6930\times 10^6 + 46.160\times 10^3 a$$

At B, $$P_B = W_{BC} = 652.59 - 6.526a \qquad (2)$$

$$\sigma_B = \frac{P_B}{A_{BC}} = 8.3090\times 10^6 - 83.090\times 10^3 a$$

(*a*) Length of rod AB. The maximum stress in ABC is minimum when $\sigma_A = \sigma_B$ or

$$4.6160\times 10^6 - 129.25\times 10^3 a = 0$$

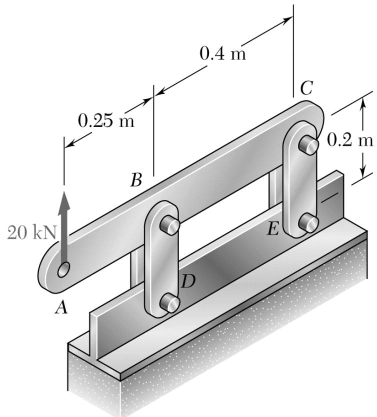
$$a = 35.71\text{ m} \qquad \ell_{AB} = a = 35.7\text{ m} \blacktriangleleft$$

(*b*) Maximum normal stress.

$$\sigma_A = 3.6930\times 10^6 + (46.160\times 10^3)(35.71)$$

$$\sigma_B = 8.3090\times 10^6 - (83.090\times 10^3)(35.71)$$

$$\sigma_A = \sigma_B = 5.34\times 10^6\text{ Pa} \qquad \sigma = 5.34\text{ MPa} \blacktriangleleft$$

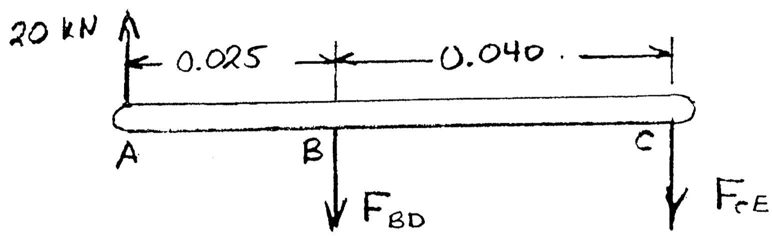


PROBLEM 1.7

Each of the four vertical links has an 8×36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$$\Sigma M_C = 0: \quad (0.040) F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{N} \qquad \text{Link } BD \text{ is in tension.}$$

$$\Sigma M_B = 0: \quad -(0.040) F_{CE} - (0.025)(20 \times 10^3) = 0$$

$$F_{CE} = -12.5 \times 10^3 \text{N} \qquad \text{Link } CE \text{ is in compression.}$$

Net area of one link for tension $= (0.008)(0.036 - 0.016) = 160 \times 10^{-6} \text{m}^2$.

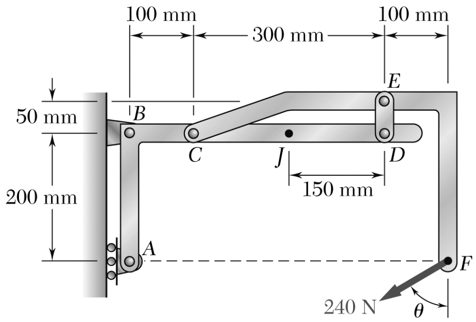
For two parallel links, $A_{\text{net}} = 320 \times 10^{-6} \text{m}^2$

(*a*) $\sigma_{BD} = \dfrac{F_{BD}}{A_{\text{net}}} = \dfrac{32.5 \times 10^3}{320 \times 10^{-6}} = 101.56 \times 10^6$ $\qquad \sigma_{BD} = 101.6 \text{ MPa}$ ◀

Area for one link in compression $= (0.008)(0.036) = 288 \times 10^{-6} \text{m}^2$.

For two parallel links, $A = 576 \times 10^{-6} \text{m}^2$

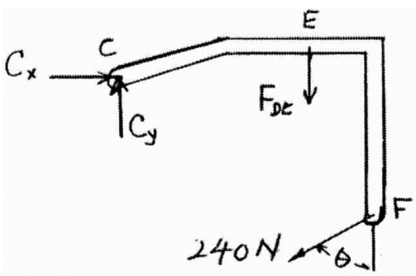
(*b*) $\sigma_{CE} = \dfrac{F_{CE}}{A} = \dfrac{-12.5 \times 10^3}{576 \times 10^{-6}} = -21.70 \times 10^{-6}$ $\qquad \sigma_{CE} = -21.7 \text{ MPa}$ ◀



PROBLEM 1.8

Knowing that link DE is 25 mm wide and 3 mm thick, determine the normal stress in the central portion of that link when (a) $\theta = 0$, (b) $\theta = 90°$.

SOLUTION



Use member CEF as a free body

$$+\circlearrowleft \Sigma M_C = 0: \quad -0.3F_{DE} - (0.2)(240\sin\theta) - (0.4)(240\cos\theta) = 0$$

$$F_{DE} = -160\sin\theta - 320\cos\theta \text{ N}$$

$$A_{DE} = (0.025)(0.003) = 75\times 10^{-6}\text{m}^2$$

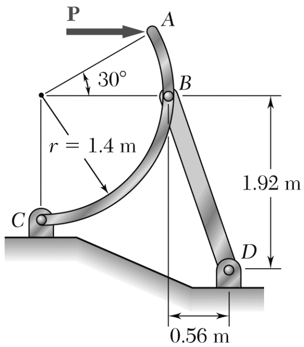
$$\sigma_{DE} = \frac{F_{DE}}{A_{DE}}$$

(a) $\theta = 0$: $F_{DE} = -320$ N

$$\sigma_{DE} = \frac{-320}{75\times 10^{-6}} = -4.27 \text{ MPa} \blacktriangleleft$$

(b) $\theta = 90°$: $F_{DE} = -160$ N

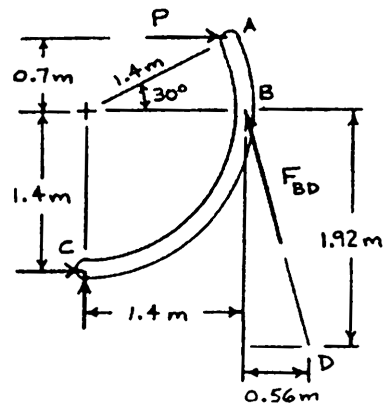
$$\sigma_{DE} = \frac{-160}{75\times 10^{-6}} = -2.13 \text{ MPa} \blacktriangleleft$$



PROBLEM 1.9

Knowing that the central portion of the link *BD* has a uniform cross-sectional area of 800 mm^2, determine the magnitude of the load **P** for which the normal stress in that portion of *BD* is 50 MPa.

SOLUTION



$$\begin{aligned} F_{BD} &= \sigma A \\ &= (50\times 10^6)(800\times 10^{-6}) \\ &= 40\times 10^3 \text{ N} \end{aligned}$$

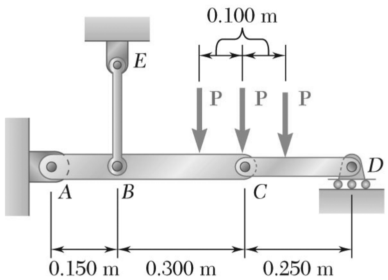
$$\begin{aligned} BD &= \sqrt{(0.56)^2 + (1.92)^2} \\ &= 2.00 \text{ m} \end{aligned}$$

Use Free Body *AC* for statics.

$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{0.56}{2.00}(40\times 10^3)(1.4) + \frac{1.92}{2.00}(40\times 10^3)(1.4) - P(0.7+1.4) = 0$$

$$P = 33.1\times 10^3 \text{ N}$$

$P = 33.1$ kN ◀

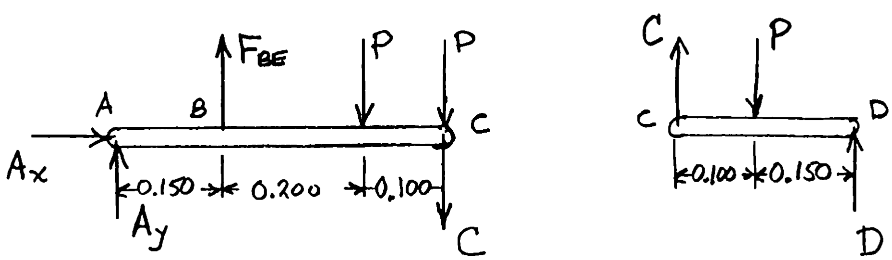


PROBLEM 1.10

Three forces, each of magnitude $P = 4$ kN, are applied to the mechanism shown. Determine the cross-sectional area of the uniform portion of rod BE for which the normal stress in that portion is +100 MPa.

SOLUTION

Draw free body diagrams of AC and CD.



Free Body CD: $\quad +\circlearrowleft \Sigma M_D = 0: \quad 0.150P - 0.250C = 0$

$$C = 0.6P$$

Free Body AC: $\quad +\circlearrowleft M_A = 0: \quad 0.150F_{BE} - 0.350P - 0.450P - 0.450C = 0$

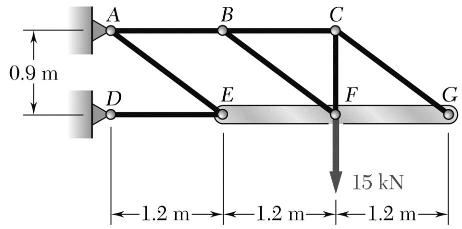
$$F_{BE} = \frac{1.07}{0.150}P = 7.1333\,P = (7.133)(4\text{ kN}) = 28.533\text{ kN}$$

Required area of BE:

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}}$$

$$A_{BE} = \frac{F_{BE}}{\sigma_{BE}} = \frac{28.533 \times 10^3}{100 \times 10^6} = 285.33 \times 10^{-6}\,\text{m}^2$$

$A_{BE} = 285\text{ mm}^2$ ◀



PROBLEM 1.11

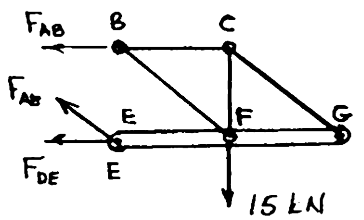
The rigid bar *EFG* is supported by the truss system shown. Knowing that the member *CG* is a solid circular rod of 18 mm diameter, determine the normal stress in *CG*.

SOLUTION

Using portion *EFGCB* as a free body

$$+\uparrow \Sigma F_y = 0: \quad \frac{0.9}{1.5} \quad F_{AB} - 15 = 0$$

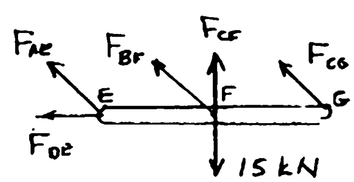
$$F_{AE} = 25 \text{ kN}$$



Using beam *EFG* as a free body

$$+\circlearrowleft M_F = 0: \quad -(1.2)\frac{0.9}{1.2}F_{AE} + (1.2)\left(\frac{0.9}{1.2}F_{CG}\right) = 0$$

$$F_{CG} = F_{AE} = 25 \text{ kN}$$

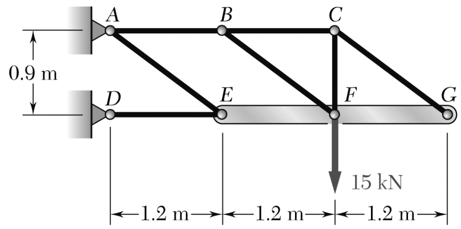


Cross sectional area of member *CG*

$$A_{CG} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.018)^2 = 254.4 \times 10^{-6} \text{m}^2$$

Normal stress in *CG*.

$$\sigma_{CG} = \frac{F_{CG}}{A_{CG}} = \frac{25}{254.4 \times 10^{-6}} = 98.3 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 1.12

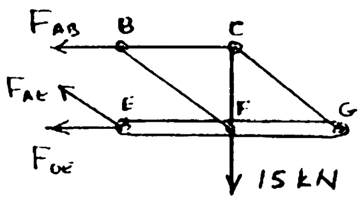
The rigid bar *EFG* is supported by the truss system shown. Determine the cross-sectional area of member *AE* for which the normal stress in the member is 105 MPa.

SOLUTION

Using portion *EFGCB* as a free body

$$+\uparrow \Sigma F_y = 0: \quad \frac{0.9}{1.5} \quad F_{AE} - 15 = 0$$

$$F_{AE} = 25 \text{ kN}$$

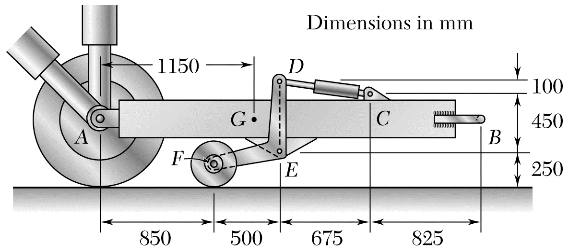


Stress in member *AE* $\qquad \sigma_{AE} = 105 \text{ MPa}$

$$\sigma_{AE} = \frac{F_{AE}}{A_{AE}}$$

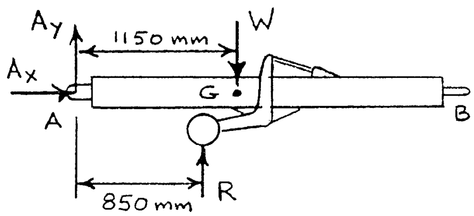
$$A_{AE} = \frac{F_{AC}}{\sigma_{AE}} = \frac{25 \times 10^3}{105 \times 10^{-6}} = 238.1 \times 10^{-6} \text{m}^2 \quad \blacktriangleleft$$

PROBLEM 1.13



An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.

SOLUTION

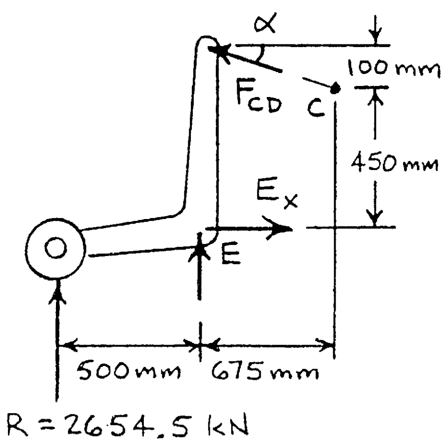


FREE BODY – ENTIRE TOW BAR:

$$W = (200\text{ kg})(9.81\text{ m/s}^2) = 1962.00\text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 850R - 1150(1962.00\text{ N}) = 0$$

$$R = 2654.5\text{ N}$$



FREE BODY – BOTH ARM & WHEEL UNITS:

$$\tan\alpha = \frac{100}{675} \qquad \alpha = 8.4270°$$

$$+\circlearrowleft \Sigma M_E = 0: \quad (F_{CD}\cos\alpha)(550) - R(500) = 0$$

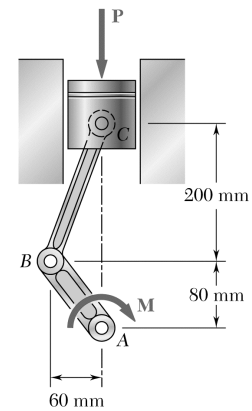
$$F_{CD} = \frac{500}{550\cos 8.4270°}(2654.5\text{ N})$$

$$= 2439.5\text{ N} \quad \text{(comp.)}$$

$$\sigma_{CD} = -\frac{F_{CD}}{A_{CD}} = -\frac{2439.5\text{ N}}{\pi(0.0125\text{ m})^2}$$

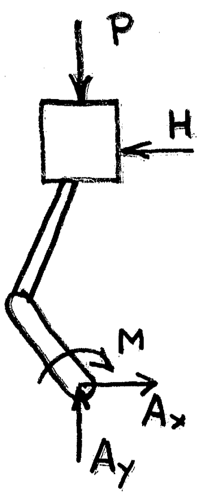
$$= -4.9697\times 10^6\text{Pa} \qquad \sigma_{CD} = -4.97\text{ MPa} \blacktriangleleft$$

PROBLEM 1.14



A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm^2 uniform cross section.

SOLUTION



Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y.

$$+\circlearrowleft \Sigma M_A = 0: \quad (0.280 \text{ m})H - 1500 \text{ N} \cdot \text{m} = 0$$

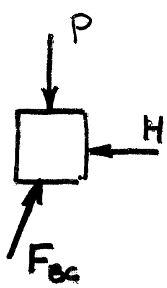
$$H = 5.3571 \times 10^3 \text{N}$$

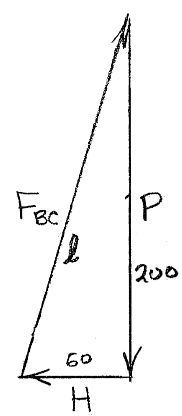
Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BE} by proportions.

$$l = \sqrt{200^2 + 60^2} = 208.81 \text{ mm}$$

$$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86 \times 10^3 \text{ N}$$

(*a*) $P = 17.86$ kN ◀





$$\frac{F_{BC}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{BC} = 18.643 \times 10^3 \text{ N}$$

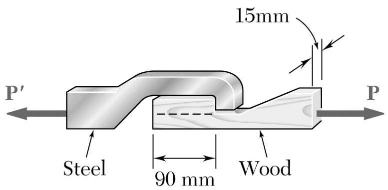
Rod *BC* is a compression member. Its area is

$$450 \text{ mm}^2 = 450 \times 10^{-6} \text{m}^2$$

Stress,

$$\sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-18.643 \times 10^3}{450 \times 10^{-6}} = -41.4 \times 10^6 \text{ Pa}$$

(*b*) $\sigma_{BC} = -41.4$ MPa ◀



PROBLEM 1.15

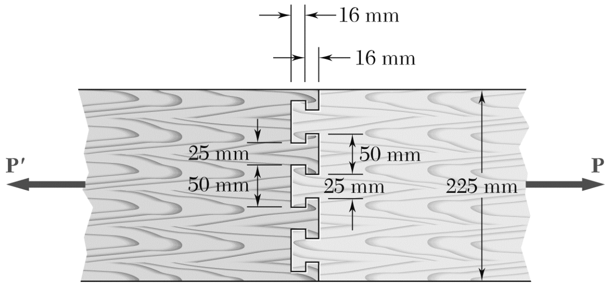
When the force **P** reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

SOLUTION

Area being sheared: $A = 90 \text{ mm} \times 15 \text{ mm} = 1350 \text{ mm}^2 = 1350 \times 10^{-6} \text{m}^2$

Force: $P = 8 \times 10^3 \text{ N}$

Shearing stress: $\tau = \dfrac{P}{A} - \dfrac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^6 \text{ Pa}$ $\qquad \tau = 5.93 \text{ MPa}$ ◀



PROBLEM 1.16

Two wooden planks, each 12 mm thick and 225 mm wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 8 MPa, determine the magnitude P of the axial load which will cause the joint to fail.

SOLUTION

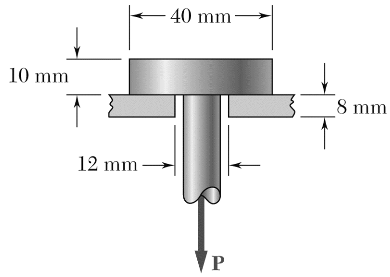
Six areas must be sheared off when the joint fails. Each of these areas has dimensions 16 mm × 12 mm, its area being

$$A = (16)(12) = 192\text{ mm}^2 = 192\times 10^{-6}\text{m}^2$$

At failure the force F carried by each of areas is

$$F = \tau A = (8\times 10^{6})(192\times 10^{-6}) = 1536\text{ N} = 1.536\text{ kN}$$

Since there are six failure areas $\quad P = 6F = (6)(1.536) = 9.22\text{ kN}$ ◀



PROBLEM 1.17

A load P is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load **P** that can be applied to the rod.

SOLUTION

For the steel rod,

$$A_1 = \pi d_1 t_1 = (\pi)(0.012)(0.010)$$

$$= 376.99 \times 10^{-6}\,\text{m}^2$$

$$\tau_1 = \frac{P}{A_1} \longrightarrow P_1 = \tau_1 A_1$$

$$P_1 = (180 \times 10^6)(376.99 \times 10^{-6}) = 67.86 \times 10^3\ \text{N}$$

For the aluminum plate,

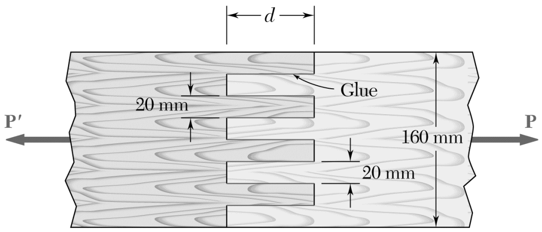
$$A_2 = \pi d_2 t_2 = (\pi)(0.040)(0.008) = 1.00531 \times 10^{-3}\,\text{m}^2$$

$$\tau_2 = \frac{P_2}{A_2} \longrightarrow P_2 = \tau_2 A_2$$

$$P_2 = (70 \times 10^6)(1.0053 \times 10^{-6}) = 70.372 \times 10^3\ \text{N}$$

The limiting value for the load P is the smaller of P_1 and P_2.

$$P = 67.86 \times 10^3\ \text{N} \qquad\qquad P = 67.9\ \text{kN} \blacktriangleleft$$



PROBLEM 1.18

Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6$ kN.

SOLUTION

Seven surfaces carry the total load $P = 7.6 \text{ kN} = 7.6 \times 10^3$.

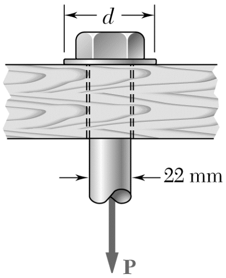
Let $t = 22$ mm.

Each glue area is $A = dt$

$$\tau = \frac{P}{7A} \qquad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} = 1.32404 \times 10^{-3} \text{m}^2$$

$$= 1.32404 \times 10^3 \text{mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} = 60.2 \qquad d = 60.2 \text{ mm} \blacktriangleleft$$



PROBLEM 1.19

The load **P** applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter *d* of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.

SOLUTION

Steel rod: $A = \frac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6}\text{m}^2$

$$\sigma = 35 \times 10^6 \text{Pa}$$

$$P = \sigma A = (35 \times 10^6)(380.13 \times 10^{-6})$$

$$= 13.305 \times 10^3 \text{N}$$

Washer: $\sigma_b = 5 \times 10^6 \text{Pa}$

Required bearing area:

$$A_b = \frac{P}{\sigma_b} = \frac{13.305 \times 10^3}{5 \times 10^6} = 2.6609 \times 10^{-3}\text{m}^2$$

But, $A_b = \frac{\pi}{4}(d^2 - d_i^2)$

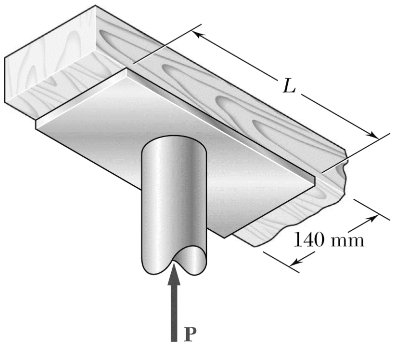
$$d^2 = d_i^2 + \frac{4A_b}{\pi}$$

$$= (0.025)^2 + \frac{(4)(2.6609 \times 10^{-3})}{\pi}$$

$$= 4.013 \times 10^{-3}\text{m}^2$$

$$d = 63.3 \times 10^{-3}\text{m} \qquad d = 63.3 \text{ mm} \blacktriangleleft$$

PROBLEM 1.20



The axial force in the column supporting the timber beam shown is $P = 75$ kN. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 3.0 MPa.

SOLUTION

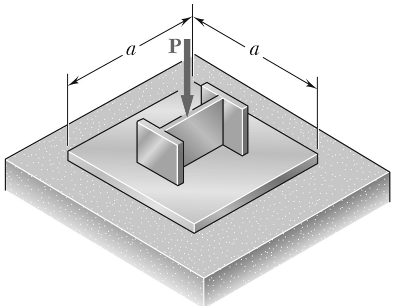
$$\sigma_b = \frac{P}{A} = \frac{P}{LW}$$

Solving for L:

$$L = \frac{P}{\sigma_b W} = \frac{75 \times 10^3}{(3.0 \times 10^6)(0.140)}$$

$$= 178.6 \times 10^{-3}\,\text{m}$$

$L = 178.6$ mm ◀



PROBLEM 1.21

An axial load **P** is supported by a short 200×59 column of cross-sectional area $A = 7650\ \text{mm}^2$ and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 200 MPa and that the bearing stress on the concrete foundation must not exceed 20 MPa determine the side a of the plate that will provide the most economical and safe design.

SOLUTION

For the column $\sigma = \dfrac{P}{A}$ or

$$P = \sigma A = (200\times10^6)(7650\times10^{-6})\ \text{N}$$

For the $a\times a$ plate, $\sigma = 20$ MPa

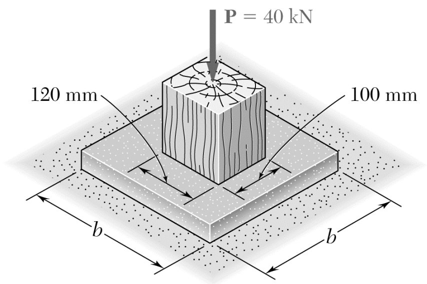
$$= 1530\ \text{kN}$$

$$A = \frac{P}{\sigma} = \frac{1530\times10^3}{20\times10^6} = 0.0765\ \text{m}^2$$

Since the plate is square, $A = a^2$

$$a = \sqrt{A} = \sqrt{0.0765} = 0.2766\ \text{m}$$

$a = 277$ mm ◀



PROBLEM 1.22

A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

SOLUTION

(*a*) Bearing stress on concrete footing.

$$P = 40\,\text{kN} = 40 \times 10^3\,\text{N}$$

$$A = (100)(120) = 12 \times 10^3\,\text{mm}^2 = 12 \times 10^{-3}\,\text{m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.333 \times 10^6\,\text{Pa}$$

3.33 MPa ◀

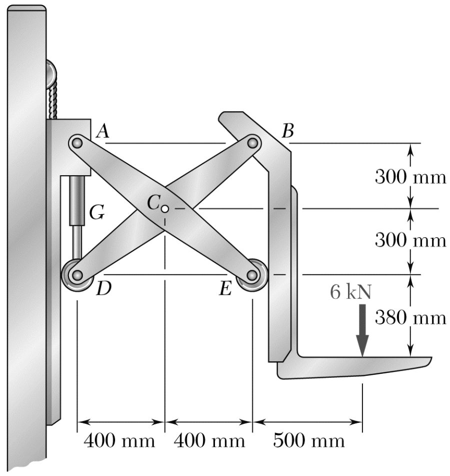
(*b*) Footing area. $P = 40 \times 10^3\,\text{N}$ $\quad \sigma = 145\text{ kPa} = 45 \times 10^3\,\text{Pa}$

$$\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586\text{ m}^2$$

Since the area is square, $A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525\text{ m}$$

$b = 525$ mm ◀

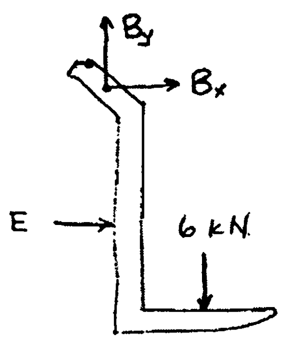


PROBLEM 1.23

Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 6 kN. Knowing that the thickness of member *BD* is 16 mm, determine (*a*) the average shearing stress in the 12-mm-diameter pin at *B*, (*b*) the bearing stress at *B* in member *BD*.

SOLUTION

Use one fork as a free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.6E - (0.5)(6) = 0$$

$$E = 5 \text{ kN} \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E + B_x = 0 \quad B_x = -E$$

$$B_x = 5 \text{ kN} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 6 = 0 \qquad B_y = 6 \text{ kN}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{5^2 + 6^2} = 7.81 \text{ kN}$$

(*a*) Shearing stress in pin at *B*.

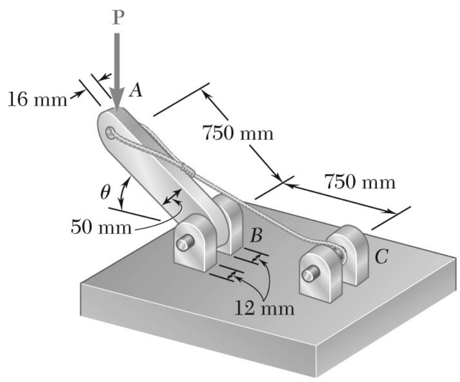
$$A_{\text{pin}} = \frac{\pi}{4} d_{\text{pin}}^2 = \frac{\pi}{4}(0.012)^2 = 113.1 \times 10^{-6} \text{m}^2$$

$$\tau = \frac{B}{A_{\text{pin}}} = \frac{7.81 \times 10^3}{113.1 \times 10^{-6}} = 69 \text{ MPa} \qquad \blacktriangleleft$$

(*b*) Bearing stress at *B*.

$$\sigma = \frac{B}{dt} = \frac{7.8 \times 10^3}{(0.012)(0.016)} = 40.6 \text{ MPa} \qquad \blacktriangleleft$$

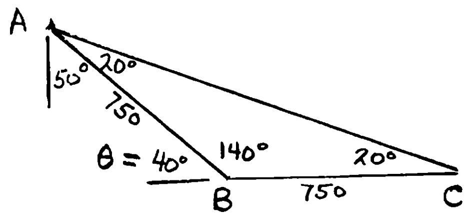
PROBLEM 1.24



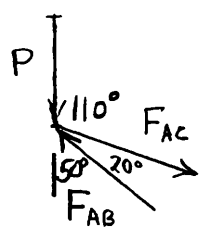
Knowing that $\theta = 40°$ and $P = 9$ kN, determine (*a*) the smallest allowable diameter of the pin at *B* if the average shearing stress in the pin is not to exceed 120 MPa, (*b*) the corresponding average bearing stress in member *AB* at *B*, (*c*) the corresponding average bearing stress in each of the support brackets at *B*.

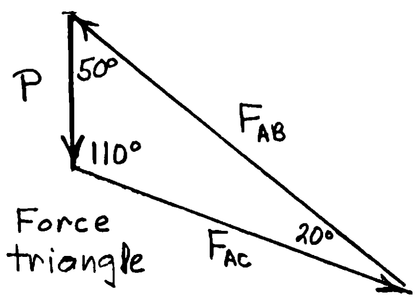
SOLUTION

Geometry: Triangle *ABC* is an isoseles triangle with angles shown here.



Use joint *A* as a free body.





Law of sines applied to force triangle

$$\frac{P}{\sin 20°} = \frac{F_{AB}}{\sin 110°} = \frac{F_{AC}}{\sin 50°}$$

$$F_{AB} = \frac{P \sin 110°}{\sin 20°}$$

$$= \frac{(9)\sin 110°}{\sin 20°} = 24.73 \text{ kN}$$

PROBLEM 1.24 *(Continued)*

(*a*) Allowable pin diameter.

$$\tau = \frac{F_{AB}}{2A_P} = \frac{F_{AB}}{2\frac{\pi}{4}d^2} = \frac{2F_{AB}}{\pi d^2} \text{ where } F_{AB} = 24.73\times 10^3\,\text{N}$$

$$d^2 = \frac{2F_{AB}}{\pi\tau} = \frac{(2)(24.73\times 10^3)}{\pi(120\times 10^6)} = 131.18\times 10^{-6}\,\text{m}^2$$

$$d = 11.45\times 10^{-3}\,\text{m}$$

11.45 mm ◀

(*b*) Bearing stress in *AB* at *A*.

$$A_b = td = (0.016)(11.45\times 10^{-3}) = 183.26\times 10^{-6}\,\text{m}^2$$

$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{24.73\times 10^3}{183.26\times 10^{-6}} = 134.9\times 10^6$$

134.9 MPa ◀

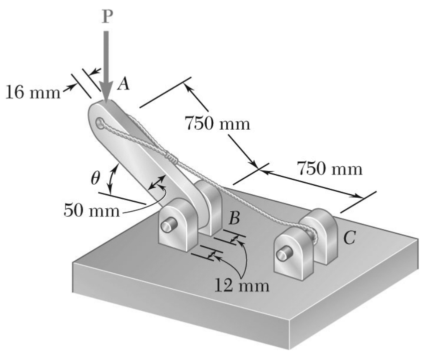
(*c*) Bearing stress in support brackets at *B*.

$$A = td = (0.012)(11.45\times 10^{-3}) = 137.4\times 10^{-6}\,\text{m}^2$$

$$\sigma_b = \frac{\frac{1}{2}F_{AB}}{A} = \frac{(0.5)(24.73\times 10^3)}{137.4\times 10^{-6}} = 90.0\times 10^6$$

90.0 MPa ◀

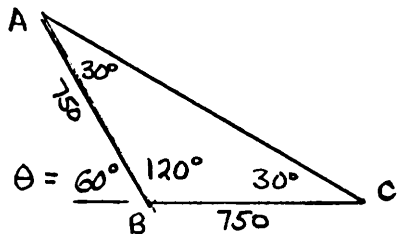
PROBLEM 1.25



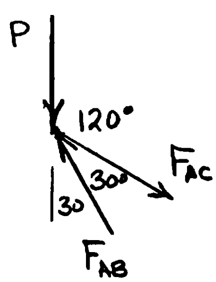
Determine the largest load **P** which may be applied at A when $\theta = 60°$, knowing that the average shearing stress in the 10-mm-diameter pin at B must not exceed 120 MPa and that the average bearing stress in member AB and in the bracket at B must not exceed 90 MPa.

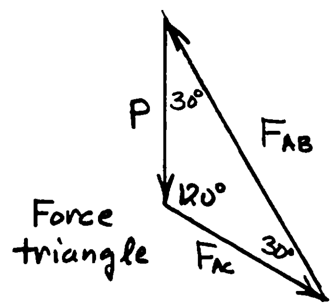
SOLUTION

Geometry: Triangle ABC is an isoseles triangle with angles shown here.



Use joint A as a free body.





Law of sines applied to force triangle

$$\frac{P}{\sin 30°} = \frac{F_{AB}}{\sin 120°} = \frac{F_{AC}}{\sin 30°}$$

$$P = \frac{F_{AB} \sin 30°}{\sin 120°} = 0.57735\, F_{AB}$$

$$P = \frac{F_{AC} \sin 30°}{\sin 30°} = F_{AC}$$

PROBLEM 1.25 *(Continued)*

If shearing stress in pin at B is critical,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\,\text{m}^2$$

$$F_{AB} = 2A\tau = (2)(78.54 \times 10^{-6})(120 \times 10^6) = 18.850 \times 10^3\,\text{N}$$

If bearing stress in member AB at bracket at A is critical,

$$A_b = td = (0.016)(0.010) = 160 \times 10^{-6}\,\text{m}^2$$

$$F_{AB} = A_b\sigma_b = (160 \times 10^{-6})(90 \times 10^6) = 14.40 \times 10^3\,\text{N}$$

If bearing stress in the bracket at B is critical,

$$A_b = 2td = (2)(0.012)(0.010) = 240 \times 10^{-6}\,\text{m}^2$$

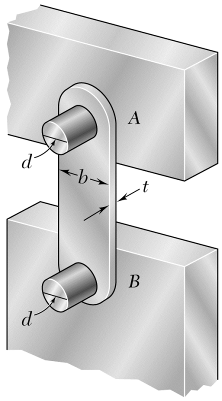
$$F_{AB} = A_b\sigma_b = (240 \times 10^{-6})(90 \times 10^6) = 21.6 \times 10^3\,\text{N}$$

Allowable F_{AB} is the smallest, i.e., $14.40 \times 10^3\,\text{N}$

Then from Statics $\quad P_{\text{allow}} = (0.57735)(14.40 \times 10^3)$

$$= 8.31 \times 10^3\,\text{N}$$

8.31 kN ◀



PROBLEM 1.26

Link AB, of width $b = 50$ mm and thickness $t = 6$ mm, is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is -140 MPa, and that the average shearing stress in each of the two pins is 80 MPa, determine (*a*) the diameter d of the pins, (*b*) the average bearing stress in the link.

SOLUTION

Rod AB is in compression.

$$A = bt \quad \text{where} \quad b = 50 \text{ mm} \quad \text{and} \quad t = 6 \text{ mm}$$

$$A = (0.050)(0.006) = 300 \times 10^{-6} \text{m}^2$$

$$P = -\sigma A = -(-140 \times 10^6)(300 \times 10^{-6})$$

$$= 42 \times 10^3 \text{N}$$

For the pin,

$$A_p = \frac{\pi}{4} d^2 \quad \text{and} \quad \tau = \frac{P}{A_p}$$

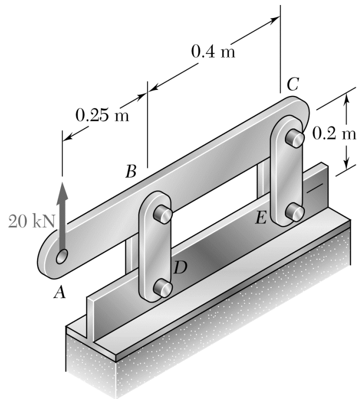
$$A_p = \frac{P}{\tau} = \frac{42 \times 10^3}{80 \times 10^6} = 525 \times 10^{-6} \text{m}^2$$

(*a*) Diameter d

$$d = \sqrt{\frac{4A_p}{\pi}} = \sqrt{\frac{(4)(525 \times 10^{-6})}{\pi}} = 2.585 \times 10^{-3} \text{m} \qquad d = 25.9 \text{ mm} \blacktriangleleft$$

(*b*) Bearing stress

$$\sigma_b = \frac{P}{dt} = \frac{42 \times 10^3}{(25.85 \times 10^{-3})(0.006)} = 271 \times 10^6 \text{ Pa} \qquad \sigma_b = 271 \text{ MPa} \blacktriangleleft$$



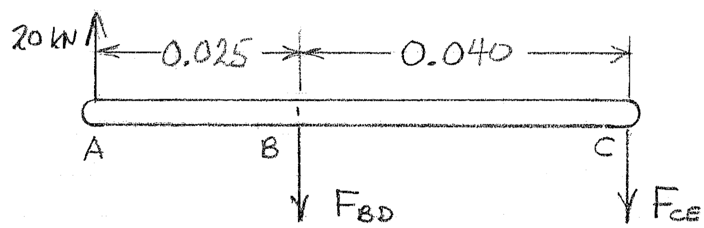
PROBLEM 1.27

For the assembly and loading of Problem 1.7, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10×50-mm uniform rectangular cross section.

PROBLEM 1.7 Each of the four vertical links has an 8×36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(*a*) Shear pin at *B* $\qquad \tau = \dfrac{F_{BD}}{2A}$ for double shear,

where $\qquad A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6}\text{m}^2$

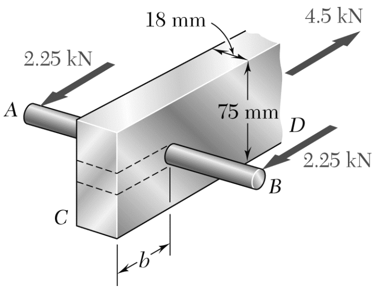
$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \qquad \tau = 80.8 \text{ MPa} \blacktriangleleft$$

(*b*) Bearing: link *BD* $\qquad A = dt = (0.016)(0.008) = 128 \times 10^{-6}\text{m}^2$

$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \qquad \sigma_b = 127.0 \text{ MPa} \blacktriangleleft$$

(*c*) Bearing in *ABC* at *B* $\qquad A = dt = (0.016)(0.010) = 160 \times 10^{-6}\text{m}^2$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \qquad \sigma_b = 203 \text{ MPa} \blacktriangleleft$$



PROBLEM 1.28

A 12-mm-diameter steel rod *AB* is fitted to a round hole near end *C* of the wooden member *CD*. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance *b* for which the average shearing stress is 620 kPa on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

SOLUTION

(*a*) Maximum average normal stress in the wood.

$$A_{\text{net}} = (75 - 12)(18) = 1.134 \times 10^3\,\text{mm}^2 = 1.134 \times 10^{-3}\,\text{m}^2$$

$$P = 4.50\text{ kN} = 4.50 \times 10^3\text{ N}$$

$$\sigma = \frac{P}{A_{\text{net}}} = \frac{4.50 \times 10^3}{1.134 \times 10^{-3}} = 3.97 \times 10^6\text{ Pa}$$

3.97 MPa ◀

(*b*)

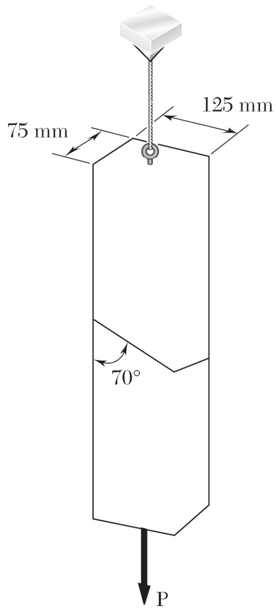
$$\tau = \frac{P}{A} = \frac{P}{2bt} \qquad b = \frac{P}{2t\tau} = \frac{4.50 \times 10^3}{(2)(18 \times 10^{-3})(620 \times 10^3)} = 202 \times 10^{-3}\,\text{m}$$

$b = 202$ mm ◀

(*c*)

$$\sigma_b = \frac{P}{dt} = \frac{4.50 \times 10^3}{(12 \times 10^{-3})(18 \times 10^{-3})} = 20.8 \times 10^6\text{ Pa}$$

20.8 MPa ◀



PROBLEM 1.29

The 6 kN load **P** is supported by two wooden members of 75 × 125 mm uniform rectangular cross section which are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

SOLUTION

$$P = 6\times 10^3 \text{ N} \quad \theta = 90° - 60° = 30°$$

$$A_o = (0.075)(0.125) = 9.375\times 10^{-3} \text{m}^3$$

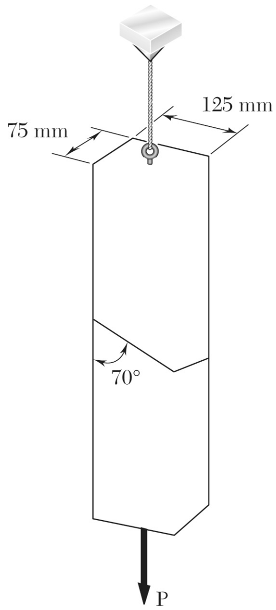
$$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{(6\times 10^3)\cos^2 30°}{9.375\times 10^{-3}} = 480\times 10^3$$

$$\sigma = 480 \text{ kPa} \quad ◀$$

$$\tau = \frac{P}{2A_o}\sin 2\theta = \frac{(6\times 10^3)\sin 60°}{(2)(9.375\times 10^{-3})} = 277\times 10^3$$

$$\tau = 277 \text{ kPa} \quad ◀$$

PROBLEM 1.30



Two wooden members of 75 × 125 mm uniform rectangular cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 500 kPa, determine (*a*) the largest load ***P*** which can be safely supported, (*b*) the corresponding shearing stress in the splice.

SOLUTION

$$A_o = (0.075)(0.125) = 9.375\times 10^{-3}\,\text{m}^2$$

$$\theta = 90° - 60° = 30° \quad \sigma = 500\times 10^3\ \text{Pa}$$

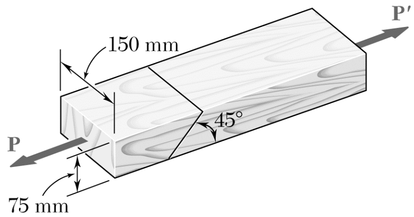
$$\sigma = \frac{P}{A_o}\cos^2\theta$$

$$P = \frac{A_o\sigma}{\cos^2\theta} = \frac{(9.375\times 10^{-3})(500\times 10^3)}{\cos^2 30°} = 6.25\times 10^3\ \text{N}$$

(*a*) $P = 6.25\ \text{kN}$ ◀

$$\tau = \frac{P\sin 2\theta}{2\,A_\theta} = \frac{(6.25\times 10^3)\sin 60°}{(2)(9.375\times 10^{-3})} = 288.68\times 10^3$$

(*b*) $\tau = 289\ \text{kPa}$ ◀



PROBLEM 1.31

Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice.

SOLUTION

$$\theta = 90° - 45° = 45°$$

$$P = 11 \text{ kN} = 11 \times 10^3 \text{N}$$

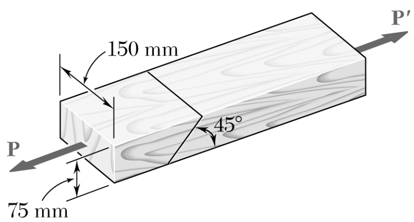
$$A_0 = (150)(75) = 11.25 \times 10^3 \text{mm}^2 = 11.25 \times 10^{-3} \text{m}^2$$

$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(11 \times 10^3)\cos^2 45°}{11.25 \times 10^{-3}} = 489 \times 10^3 \text{Pa}$$

$\sigma = 489$ kPa ◀

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(11 \times 10^3)(\sin 90°)}{(2)(11.25 \times 10^{-3})} = 489 \times 10^3 \text{Pa}$$

$\tau = 489$ kPa ◀



PROBLEM 1.32

Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine (*a*) the largest load **P** that can be safely applied, (*b*) the corresponding tensile stress in the splice.

SOLUTION

$$\theta = 90° - 45° = 45°$$

$$A_0 = (150)(75) = 11.25 \times 10^3 \text{mm}^2 = 11.25 \times 10^{-3} \text{m}^2$$

$$\tau = 620 \text{ kPa} = 620 \times 10^3 \text{Pa}$$

$$\tau = \frac{P \sin 2\theta}{2A_0}$$

(*a*) $$P = \frac{2A_0\tau}{\sin 2\theta} = \frac{(2)(11.25 \times 10^{-3})(620 \times 10^3)}{\sin 90°}$$

$$= 13.95 \times 10^3 \text{N}$$

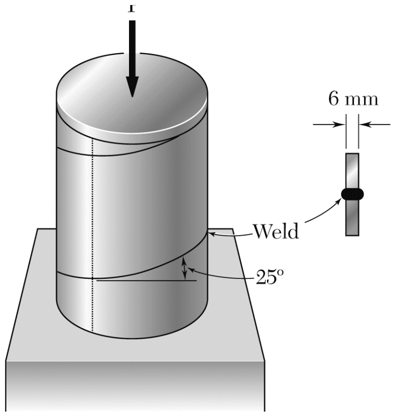
$P = 13.95$ kN ◀

(*b*) $$\sigma = \frac{P \cos^2\theta}{A_0} = \frac{(13.95 \times 10^3)(\cos 45°)^2}{11.25 \times 10^{-3}}$$

$$= 620 \times 10^3 \text{Pa}$$

$\sigma = 620$ kPa ◀

PROBLEM 1.33



A steel pipe of 300 mm outer diameter is fabricated from 6 mm thick plate by welding along a helix which forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in directions respectively normal and tangential to the weld are $\sigma = 50$ MPa and $\tau = 30$ MPa, determine the magnitude **P** of the largest axial force that can be applied to the pipe.

SOLUTION

$$d_o = 0.300 \text{ m} \qquad r_o = \frac{1}{2}d_o = 0.150 \text{ m}$$

$$r_i = r_o - t = 0.150 - 0.006 = 0.144 \text{ m}$$

$$A_0 = \pi\left(r_0^2 - r_i^2\right) = \pi(0.150^2 - 0.144^2)$$

$$= 5.54 \times 10^{-3} \text{m}^2$$

$$\theta = 25°$$

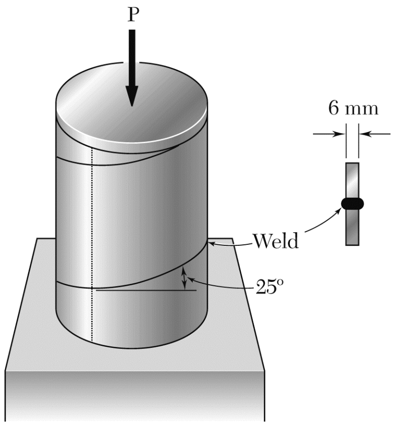
Based on $|\sigma| = 50$ MPa: $\quad \sigma = \dfrac{P}{A_o}\cos^2\theta$

$$P = \frac{A_o \sigma}{\cos^2\theta} = \frac{(5.54 \times 10^{-3})(50 \times 10^6)}{\cos^2 25°} = 337 \times 10^3 \text{ N}$$

Based on $|\tau| = 30$ MPa: $\quad \tau = \dfrac{P}{2A_o}\sin 2\theta$

$$P = \frac{2A_o \tau}{\sin 2\theta} = \frac{(2)(5.54 \times 10^{-3})(30 \times 10^6)}{\sin 50°} = 434 \times 10^3 \text{ N}$$

Smaller value is the allowable value of P $\qquad \therefore P = 337$ kN ◀



PROBLEM 1.34

A steel pipe of 300 mm outer diameter is fabricated from 6 mm-thick plate by welding along a helix which forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that a 250 kN axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

SOLUTION

$$d_o = 0.300 \text{ m} \qquad r_o = \frac{1}{2}d_o = 0.150 \text{ m}$$

$$r_i = r_o - t = 0.150 - 0.006 = 0.144 \text{ m}$$

$$A_o = \pi\left(r_0^2 - r_i^2\right) = \pi(0.150^2 - 0.144^2)$$

$$= 5.54 \times 10^{-3} \text{m}^2$$

$$\theta = 25°$$

$$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{-250 \times 10^3 \cos^2 25°}{5.54 \times 10^{-3}}$$

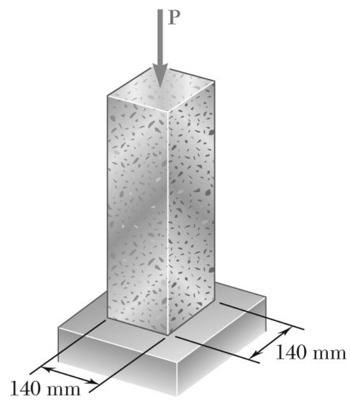
$$= -37.1 \times 10^6$$

$\sigma = -37.1$ MPa ◀

$$\tau = \frac{P}{2A_o}\sin 2\theta = \frac{-250 \times 10^3 \sin 50°}{(2)(5.54 \times 10^{-3})}$$

$$= -17.28 \times 10^6$$

$\tau = 17.28$ MPa ◀



PROBLEM 1.35

A 1060-kN load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.

SOLUTION

$$A_0 = (140 \text{ mm})(140 \text{ mm}) = 19.6 \times 10^3 \text{mm}^2 = 19.6 \times 10^{-3} \text{m}^2$$

$$P = 1060 \times 10^3 \text{N}$$

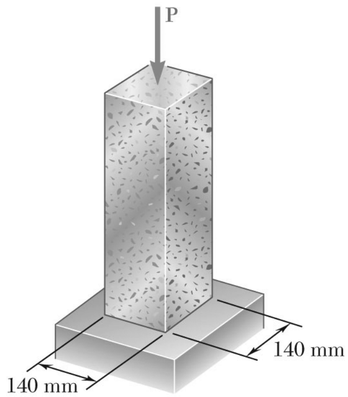
$$\sigma = \frac{P}{A_0}\cos^2\theta = \frac{1060 \times 10^3}{19.6 \times 10^{-3}}\cos^2\theta = 54.082 \times 10^6 \cos^2\theta$$

(*a*) Maximum tensile stress = 0 at $\theta = 90°$.

Maximum compressive stress = 54.1×10^6 at $\theta = 0°$. $|\sigma|_{max} = 54.1 \text{ MPa}$ ◀

(*b*) Maximum shearing stress:

$$\tau_{max} = \frac{P}{2A_0} = \frac{1060 \times 10^3}{(2)(19.6 \times 10^{-3})} = 27.0 \times 10^6 \text{ Pa at } \theta = 45°. \qquad \tau_{max} = 27.0 \text{ MPa} ◀$$



PROBLEM 1.36

A centric load **P** is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 18 MPa, determine (*a*) the magnitude of **P**, (*b*) the orientation of the surface on which the maximum shearing stress occurs, (*c*) the normal stress exerted on that surface, (*d*) the maximum value of the normal stress in the block.

SOLUTION

$$A_0 = (140\text{ mm})(140\text{ mm}) = 19.6\times10^3\text{mm}^2 = 19.6\times10^{-3}\text{m}^2$$

$$\tau_{\max} = 18\text{ MPa} = 18\times10^6\text{ Pa}$$

$$\theta = 45° \text{ for plane of } \tau_{\max}$$

(*a*) Magnitude of P. $\tau_{\max} = \dfrac{|P|}{2A_0}$ so $P = 2A_0\,\tau_{\max}$

$$P = (2)(19.6\times10^{-3})(18\times10^6) = 705.6\times10^3\text{N}$$

$P = 706\text{ kN}$ ◀

(*b*) Orientation. $\sin 2\theta$ is maximum when $2\theta = 90°$

$\theta = 45°$ ◀

(*c*) Normal stress at $\theta = 45°$.

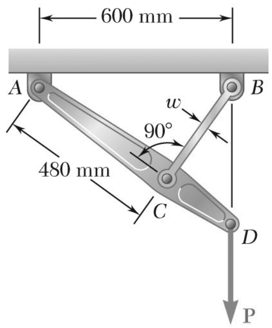
$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(705.8\times10^3)\cos^2 45°}{19.6\times10^{-3}} = 18.00\times10^6\text{ Pa}$$

$\sigma = 18.00\text{ MPa}$ ◀

(*d*) Maximum normal stress: $\sigma_{\max} = \dfrac{P}{A_0}$

$$\sigma_{\max} = \frac{705.8\times10^3}{19.6\times10^{-3}} = 36.0\times10^6\text{ Pa}$$

$\sigma_{\max} = 36.0\text{ MPa}$ (compression) ◀

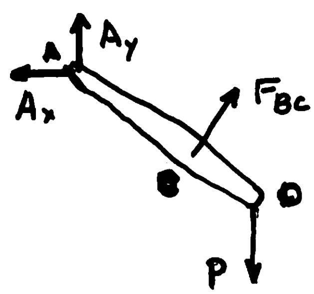


PROBLEM 1.37

Link BC is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480-MPa ultimate strength in tension. What was the safety factor used if the structure shown was designed to support a 16-kN load **P**?

SOLUTION

Use bar ACD as a free body and note that member BC is a two-force member.



$$\Sigma M_A = 0:$$

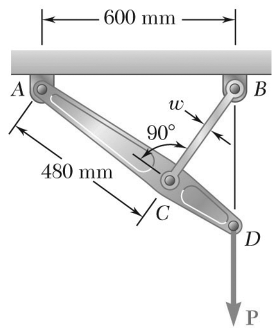
$$(480)F_{BC} - (600)P = 0$$

$$F_{BC} = \frac{600}{480}P = \frac{(600)(16 \times 10^3)}{480} = 20 \times 10^3 \text{N}$$

Ultimate load for member BC: $F_U = \sigma_U A$

$$F_U = (480 \times 10^6)(0.006)(0.025) = 72 \times 10^3 \text{N}$$

Factor of safety: $\text{F.S.} = \dfrac{F_U}{F_{BC}} = \dfrac{72 \times 10^3}{20 \times 10^3}$ F.S. = 3.60 ◀

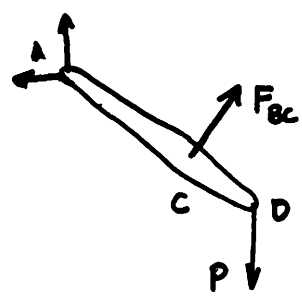


PROBLEM 1.38

Link *BC* is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be its width *w* if the structure shown is being designed to support a 20-kN load **P** with a factor of safety of 3?

SOLUTION

Use bar *ACD* as a free body and note that member *BC* is a two-force member.



$$\Sigma M_A = 0:$$

$$(480)F_{BC} - 600P = 0$$

$$F_{BC} = \frac{600P}{480} = \frac{(600)(20\times10^3)}{480} = 25\times10^3\,\text{N}$$

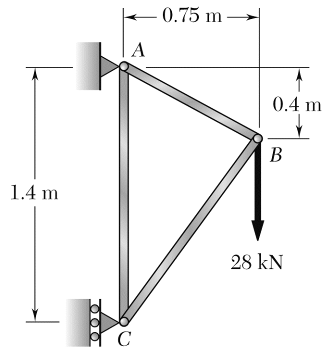
For a factor of safety F.S. = 3, the ultimate load of member *BC* is

$$F_U = (\text{F.S.})(F_{BC}) = (3)(25\times10^3) = 75\times10^3\,\text{N}$$

But $F_U = \sigma_U A \quad \therefore A = \dfrac{F_U}{\sigma_U} = \dfrac{75\times10^3}{450\times10^6} = 166.67\times10^{-6}\,\text{m}^2$

For a rectangular section $\quad A = wt$ or $w = \dfrac{A}{t} = \dfrac{166.67\times10^{-6}}{0.006}$

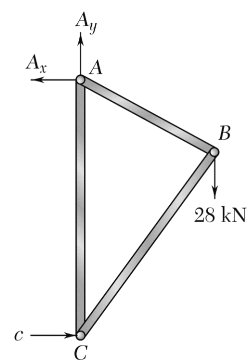
$$w = 27.8\times10^{-3}\,\text{m or } 27.8\text{ mm} \blacktriangleleft$$



PROBLEM 1.39

Members AB and AC of the truss shown consist of bars of square cross section made of the same alloy. It is known that a 20 mm square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If bar AB has a 15 mm square cross section, determine (*a*) the factor of safety for bar AB, (*b*) the dimensions of the cross section of bar AC if it is to have the same factor of safety as bar AB.

SOLUTION



Length of member AB

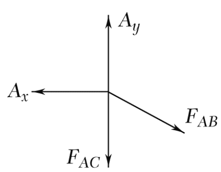
$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

Use entire truss as a free body

$$+\circlearrowleft \Sigma M_C = 0 \quad 1.4\,A_x - (0.75)(28) = 0 \qquad A_x = 15 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0 \qquad A_y - 28 = 0 \qquad A_y = 28 \text{ kN}$$

Use joint A as free body



$$\xrightarrow{+} \Sigma F_y = 0 \qquad \frac{0.75}{0.85} F_{AB} - A_x = 0$$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0 \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar $\qquad A = (0.020)^2 = 400 \times 10^{-6} \text{m}^2 \qquad P_U = 120 \times 10^3 \text{ N}$

For the material

$$\sigma_U = \frac{P_U}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$$

(*a*) For bar AB

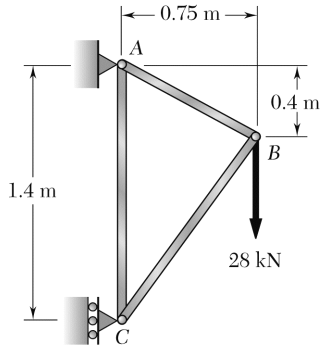
$$\text{F.S.} = \frac{F_U}{F_{AB}} = \frac{\sigma_U A}{F_{AB}} = \frac{(300 \times 10^6)(0.015)^2}{17 \times 10^3} = 3.97 \quad \blacktriangleleft$$

(*b*) For bar AC

$$\text{F.S.} = \frac{F_U}{F_{AC}} = \frac{\sigma_U A}{F_{AC}} = \frac{\sigma_U a^2}{F_{AC}}$$

$$a^2 = \frac{(\text{F.S.})F_{AC}}{\sigma_U} = \frac{(3.97)(20 \times 10^3)}{300 \times 10^6} = 264.7 \times 10^{-6} \text{m}^2$$

$$a = 16.27 \times 10^{-3} \text{m} \quad \text{or} \quad 16.27 \text{ mm} \quad \blacktriangleleft$$



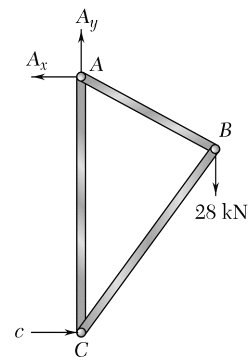
PROBLEM 1.40

Members AB and AC of the truss shown consist of bars of square cross section made of the same alloy. It is known that a 20 mm square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If a factor of safety of 3.2 is to be achieved for bars, determine the required dimensions of the cross section of (*a*) bar AB, (*b*) bar AC.

SOLUTION

Length of member AB

$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

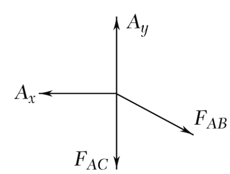


Use entire truss as a free body

$$+\circlearrowleft \Sigma M_C = 0 \quad 1.4\, A_x - (0.75)(28) = 0 \qquad A_x = 15 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0 \qquad A_y - 28 = 0 \qquad A_y = 28 \text{ kN}$$

Use joint A as free body



$$\xrightarrow{+} \Sigma F_x = 0 \qquad \frac{0.75}{0.85} F_{AB} - A_x = 0$$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0 \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar $\quad A = (0.020)^2 = 400 \times 10^{-6} \text{m}^2 \qquad P_U = 120 \times 10^3 \text{ N}$

For the material $\quad \sigma_U = \dfrac{P_U}{A} = \dfrac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(*a*) For member AB $\quad \text{F.S.} = \dfrac{P_U}{F_{AB}} = \dfrac{\sigma_U A}{F_{AB}} = \dfrac{\sigma_U a^2}{F_{AB}}$

$$a^2 = \frac{(\text{F.S.}) F_{AB}}{\sigma_U} = \frac{(3.2)(17 \times 10^3)}{300 \times 10^6} = 181.33 \times 10^{-6} \text{m}^2$$

$$a = 13.47 \times 10^{-3} \text{m} \quad \text{or} \quad 13.47 \text{ mm}$$ ◀

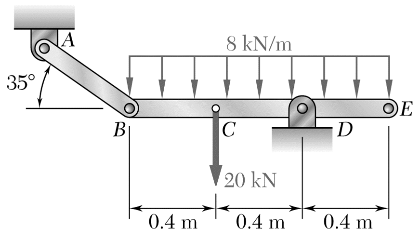
PROBLEM 1.40 *(Continued)*

(*b*) For member AC

$$\text{F.S.} = \frac{P_U}{F_{AC}} = \frac{\sigma_U A}{F_{AC}} = \frac{\sigma_U b^2}{F_{AC}}$$

$$b^2 = \frac{(\text{F.S.})F_{AC}}{\sigma_U} = \frac{(3.2)(20\times 10^3)}{300\times 10^6} = 213.33\times 10^{-6}\,\text{m}^2$$

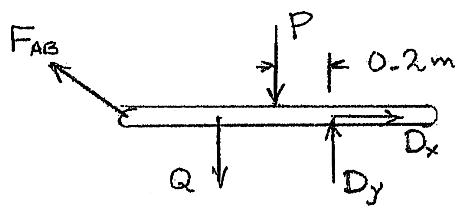
$$b = 14.61\times 10^{-3}\,\text{m} \quad \text{or} \quad 14.61\ \text{mm} \quad \blacktriangleleft$$



PROBLEM 1.41

Link AB is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area for AB for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at A and B.

SOLUTION



$$P = (1.2)(8) = 9.6 \text{ kN}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(0.8)(F_{AB} \sin 35°) + (0.2)(9.6) + (0.4)(20) = 0$$

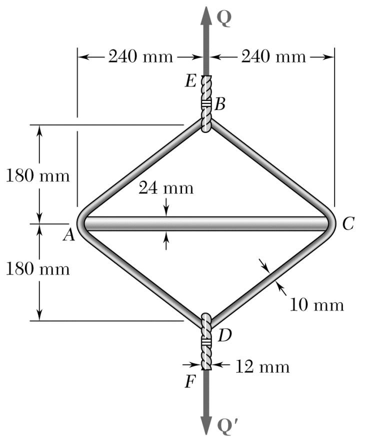
$$F_{AB} = 21.619 \text{ kN} = 21.619 \times 10^3 \text{N}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sigma_{\text{ult}}}{\text{F.S.}}$$

$$A_{AB} = \frac{(\text{F.S.})F_{AB}}{\sigma_{\text{ult}}} = \frac{(3.50)(21.619 \times 10^3)}{450 \times 10^6}$$

$$= 168.1 \times 10^{-6} \text{m}^2$$

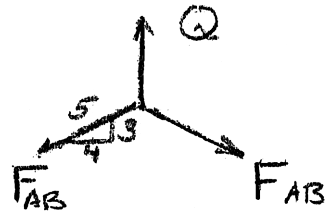
$$A_{AB} = 168.1 \text{ mm}^2 \blacktriangleleft$$



PROBLEM 1.42

A steel loop *ABCD* of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod *AC*. Cables *BE* and *DF*, each of 12-mm diameter, are used to apply the load **Q**. Knowing that the ultimate strength of the steel used for the loop and the cables is 480 MPa and that the ultimate strength of the aluminum used for the rod is 260 MPa, determine the largest load **Q** that can be applied if an overall factor of safety of 3 is desired.

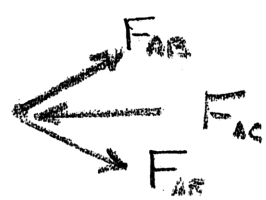
SOLUTION



Using joint *B* as a free body and considering symmetry,

$$2 \cdot \frac{3}{5} F_{AB} - Q = 0 \quad Q = \frac{6}{5} F_{AB}$$

Using joint *A* as a free body and considering symmetry,



$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$

Based on strength of cable *BF*:

$$Q_U = \sigma_U A = \sigma_U \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{ N}$$

Based on strength of steel loop:

$$Q_U = \frac{6}{5} F_{AB,U} = \frac{6}{5} \sigma_U A = \frac{6}{5} \sigma_U \frac{\pi}{4} d^2$$

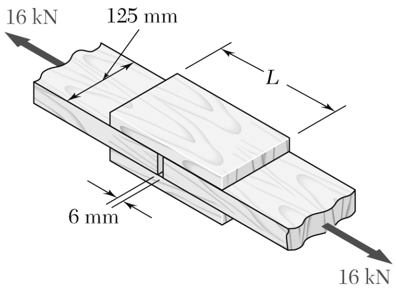
$$= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{ N}$$

Based on strength of rod *AC*:

$$Q_U = \frac{3}{4} F_{AC,U} = \frac{3}{4} \sigma_U A = \frac{3}{4} \sigma_U \frac{\pi}{4} d^2 = \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{ N}$$

Actual ultimate load Q_U is the smallest, $\therefore \quad Q_U = 45.24 \times 10^3$ N

Allowable load: $$Q = \frac{Q_U}{\text{F.S.}} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3 \text{ N}$$ $Q = 15.08$ kN ◀



PROBLEM 1.43

The two wooden members shown, which support a 16-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.5 MPa and the clearance between the members is 6 mm. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.

SOLUTION

There are 4 separate areas of glue.

Each glue area must transmit 8 kN of shear load.

$$P = 8 \text{ kN} = 8 \times 10^3 \text{ N}$$

Required ultimate load.

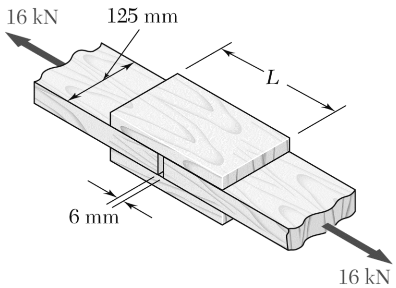
$$P_U = (\text{F.S.})P = (2.75)(8 \times 10^3) = 22 \times 10^3 \text{ N}$$

Required length of each glue area.

$$P_U = \tau_U A = \tau_U \ell w \qquad \ell = \frac{P_U}{\tau_U w} = \frac{22 \times 10^3}{(2.5 \times 10^6)(0.125)} = 70.4 \times 10^{-3} \text{m}$$

Length of splice: $\quad L = 2\ell + C = (2)(70.4 \times 10^{-3}) + 0.006 = 0.1468 \times 10^{-3} \text{m}$

$L = 146.8$ mm ◀



PROBLEM 1.44

For the joint and loading of Problem 1.43, determine the factor of safety, knowing that the length of each splice is $L = 180$ mm.

PROBLEM 1.43 The two wooden members shown, which support a 16-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.5 MPa and the clearance between the members is 6 mm. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.

SOLUTION

There are 4 separate areas of glue.

Each glue area must transmit 8 kN of shear load.

$$P = 8 \text{ kN} = 8 \times 10^3 \text{ N}$$

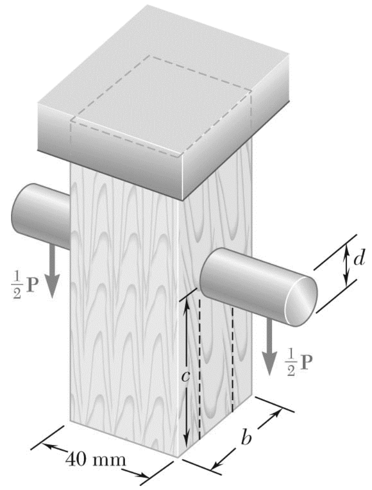
Length of splice. $L = 2\ell + C$ where ℓ = length of glue and C = clearance.

$$\ell = \frac{1}{2}(L - C) = \frac{1}{2}(0.180 - 0.006) = 0.087 \text{ m}$$

Area of glue. $A = \ell w = (0.087)(0.125) = 10.875 \times 10^{-3} \text{m}$

Ultimate load. $P_U = \tau_U A = (2.5 \times 10^6)(10.875 \times 10^{-3}) = 27.1875 \times 10^3 \text{ N}$

Factor of safety. $\text{F.S.} = \dfrac{P_U}{P} = \dfrac{27.1875 \times 10^3}{8 \times 10^3}$ F.S. = 3.40 ◀



PROBLEM 1.45

A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

Based on double shear in pin:

$$P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U$$

$$= \frac{\pi}{4}(2)(0.012)^2(145\times 10^6) = 32.80\times 10^3\,\text{N}$$

Based on tension in wood:

$$P_U = A\sigma_U = w(b-d)\sigma_U$$

$$= (0.040)(0.040-0.012)(60\times 10^6)$$

$$= 67.2\times 10^3\,\text{N}$$

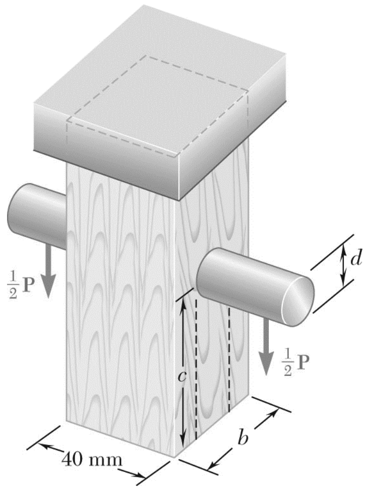
Based on double shear in the wood:

$$P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times 10^6)$$

$$= 33.0\times 10^3\,\text{N}$$

Use smallest $\quad P_U = 32.8\times 10^3\,\text{N}$

Allowable: $\quad P = \dfrac{P_U}{\text{F.S.}} = \dfrac{32.8\times 10^3}{3.2} = 10.25\times 10^3\,\text{N} \qquad$ 10.25 kN ◀



PROBLEM 1.46

For the support of Problem 1.45, knowing that the diameter of the pin is $d = 16$ mm and that the magnitude of the load is $P = 20$ kN, determine (*a*) the factor of safety for the pin, (*b*) the required values of *b* and *c* if the factor of safety for the wooden members is the same as that found in part *a* for the pin.

PROBLEM 1.45 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

$$P = 20 \text{ kN} = 20 \times 10^3 \text{N}$$

(*a*) Pin: $$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.016)^2 = 2.01.06 \times 10^{-6} \text{m}^2$$

Double shear: $$\tau = \frac{P}{2A} \quad \tau_U = \frac{P_U}{2A}$$

$$P_U = 2A\tau_U = (2)(201.16 \times 10^{-6})(145 \times 10^6) = 58.336 \times 10^3 \text{N}$$

$$\text{F.S.} = \frac{P_U}{P} = \frac{58.336 \times 10^3}{20 \times 10^3}$$

F.S. = 2.92 ◀

(*b*) Tension in wood: $P_U = 58.336 \times 10^3 \text{N}$ for same F.S.

$$\sigma_U = \frac{P_U}{A} = \frac{P_U}{w(b-d)} \quad \text{where} \quad w = 40 \text{ mm} = 0.040 \text{ m}$$

$$b = d + \frac{P_U}{w\sigma_U} = 0.016 + \frac{58.336 \times 10^3}{(0.040)(60 \times 10^6)} = 40.3 \times 10^{-3} \text{m}$$

$b = 40.3$ mm ◀

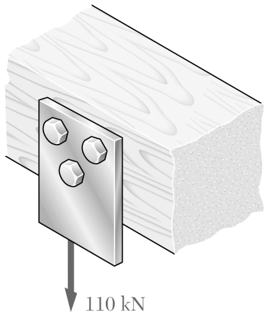
Shear in wood: $P_U = 58.336 \times 10^3 \text{N}$ for same F.S.

Double shear; each area is $A = wc$ $$\tau_U = \frac{P_U}{2A} = \frac{P_U}{2wc}$$

$$c = \frac{P_U}{2w\tau_U} = \frac{58.336 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 97.2 \times 10^{-3} \text{m}$$

$c = 97.2$ mm ◀

PROBLEM 1.47



Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.

SOLUTION

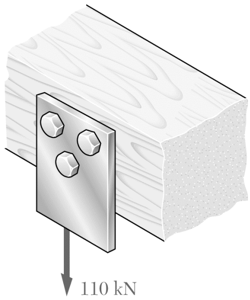
For each bolt, $$P = \frac{110}{3} = 36.667 \text{ kN}$$

Required: $$P_U = (\text{F.S.})P = (3.35)(36.667) = 122.83 \text{ kN}$$

$$\tau_U = \frac{P_U}{A} = \frac{P_U}{\frac{\pi}{4}d^2} = \frac{4P_U}{\pi d^2}$$

$$d = \sqrt{\frac{4P_U}{\pi\tau_U}} = \sqrt{\frac{(4)(122.83\times 10^3)}{\pi(360\times 10^6)}} = 20.8\times 10^{-3}\text{m}$$

$d = 20.8 \text{ mm}$ ◀



PROBLEM 1.48

Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.

SOLUTION

For each bolt, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(18)^2 = 254.47 \text{ mm}^2 = 254.47 \times 10^{-6} \text{m}^2$

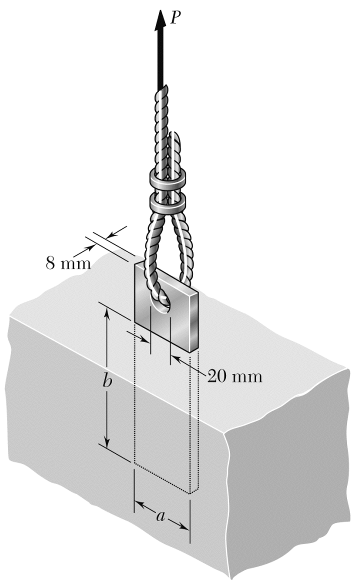
$$P_U = A\tau_U = (254.47 \times 10^{-6})(360 \times 10^6)$$
$$= 91.609 \times 10^3 \text{N}$$

For the three bolts, $P_U = (3)(91.609 \times 10^3) = 274.83 \times 10^3 \text{N}$

Factor of safety:

$$\text{F.S.} = \frac{P_U}{P} = \frac{274.83 \times 10^3}{110 \times 10^3}$$

F.S. = 2.50 ◀



PROBLEM 1.49

A steel plate 8 mm thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is 20 mm the ultimate strength of the steel used is 250 MPa, and the ultimate bonding stress between plate and concrete is 2 MPa. Knowing that a factor of safety of 3.60 is desired when $P = 10$ kN, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)

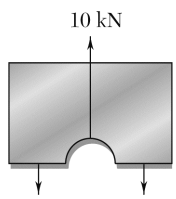
SOLUTION

Based on tension in plate

$$A = (a - d)t$$

$$P_U = \sigma_U A$$

$$\text{F.S.} = \frac{P_U}{P} = \frac{\sigma_U (a - d)t}{P}$$



Solving for *b*

$$a = d + \frac{(\text{F.S.})P}{\sigma_U t} = 20 + \frac{(3.60)(10000)}{(250)(8)}$$

$$a = 38 \text{ mm} \quad \blacktriangleleft$$

Based on shear between plate and concrete slab

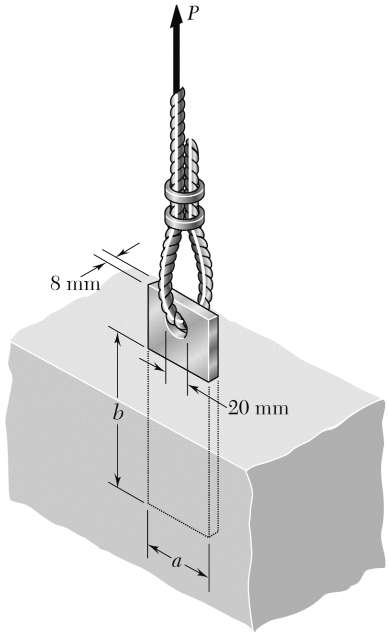
$$A = \text{perimeter} \times \text{depth} = 2(a + t)b \qquad \tau_U = 2 \text{ MPa}$$

$$P_U = \tau_U A = 2\tau_U (a + t)b \qquad \text{F.S.} = \frac{P_U}{P}$$

Solving for *b*

$$b = \frac{(\text{F.S.})P}{2(a + t)\tau_U} = \frac{(3.6)(10000)}{(2)(38 + 8)(2)}$$

$$b = 195.6 \text{ mm} \quad \blacktriangleleft$$

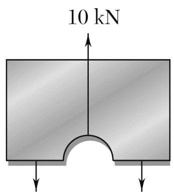


PROBLEM 1.50

A steel plate 8 mm thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is 20 mm, the ultimate strength of the steel used is 250 MPa, and the ultimate bonding stress between plate and concrete is 2 MPa. Determine the factor of safety for the cable anchor of Problem 1.49 when $P = 14$ kN, knowing that $a = 50$ mm and $b = 190$ mm.

SOLUTION

Based on tension in plate

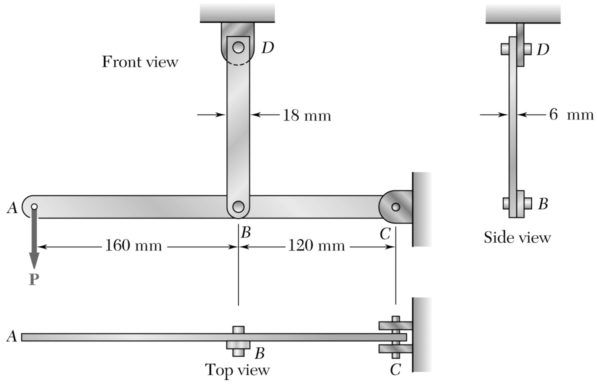


$$A = (a - d)t$$
$$= (50 - 20)(8) = 240 \text{ mm}^2$$
$$P_U = \sigma_U A$$
$$= (250)(240) = 60 \text{ kN}$$
$$\text{F.S.} = \frac{P_U}{P} = \frac{60}{14} = 4.29$$

Based on shear between plate and concrete slab

$$A = \text{perimeter} \times \text{depth} = 2(a + t)b = 2(50 + 8)(190)$$
$$A = 22040 \text{ mm}^2 \qquad \tau_U = 2 \text{ MPa}$$
$$P_U = \tau_U A = (2)(22040) = 44.08 \text{ kN}$$
$$\text{F.S.} = \frac{P_U}{P} = \frac{44.08}{14} = 3.15$$

Actual factor of safety is the smaller value F.S. = 3.15 ◀

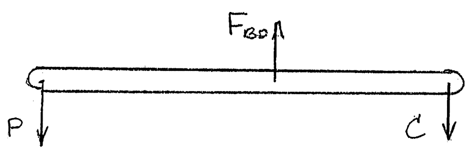


PROBLEM 1.51

In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at A. Note that link BD is not reinforced around the pin holes.

SOLUTION

Use free body ABC.



$$+\circlearrowleft \Sigma M_C = 0: \quad 0.280P - 0.120\, F_{BD} = 0$$

$$P = \frac{3}{7} F_{BD} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.160P - 0.120C = 0$$

$$P = \frac{3}{4} C \qquad (2)$$

Tension on net section of link BD.

$$F_{BD} = \sigma A_{\text{net}} = \frac{\sigma_U}{\text{F.S.}} A_{\text{net}} = \left(\frac{400 \times 10^6}{3}\right)(6 \times 10^{-3})(18 - 10)(10^{-3}) = 6.40 \times 10^3 \text{N}$$

Shear in pins at B and D.

$$F_{BD} = \tau A_{\text{pin}} = \frac{\tau_U}{\text{F.S.}} \frac{\pi}{4} d^2 = \left(\frac{150 \times 10^6}{3}\right)\left(\frac{\pi}{4}\right)(10 \times 10^{-3})^2 = 3.9270 \times 10^3 \text{N}$$

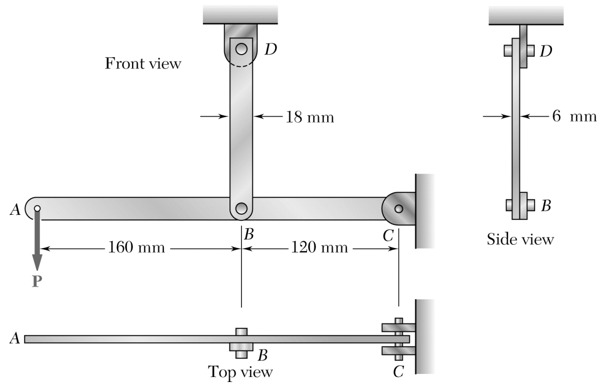
Smaller value of F_{BD} is 3.9270×10^3N.

From (1) $\quad P = \left(\frac{3}{7}\right)(3.9270 \times 10^3) = 1.683 \times 10^3 \text{N}$

Shear in pin at C. $\quad C = 2\tau A_{\text{pin}} = 2\frac{\tau_U}{\text{F.S.}} \frac{\pi}{4} d^2 = (2)\left(\frac{150 \times 10^6}{3}\right)\left(\frac{\pi}{4}\right)(6 \times 10^{-3})^2 = 2.8274 \times 10^3 \text{N}$

From (2) $\quad P = \left(\frac{3}{4}\right)(2.8274 \times 10^3) = 2.12 \times 10^3 \text{N}$

Smaller value of P is allowable value. $\quad P = 1.683 \times 10^3 \text{N} \qquad P = 1.683 \text{ kN}$ ◀



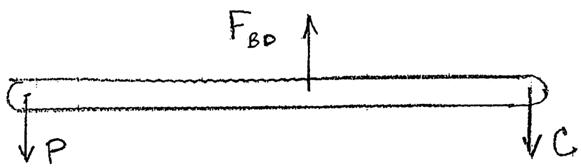
PROBLEM 1.52

Solve Problem 1.51, assuming that the structure has been redesigned to use 12-mm-diameter pins at *B* and *D* and no other change has been made.

PROBLEM 1.51 In the steel structure shown, a 6-mm-diameter pin is used at *C* and 10-mm-diameter pins are used at *B* and *D*. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link *BD*. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at *A*. Note that link *BD* is not reinforced around the pin holes.

SOLUTION

Use free body *ABC*.



$$+\circlearrowleft \Sigma M_C = 0: \quad 0.280P - 0.120\,F_{BD} = 0$$

$$P = \frac{3}{7}F_{BD} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.160P - 0.120C = 0$$

$$P = \frac{3}{4}C \qquad (2)$$

Tension on net section of link *BD*.

$$F_{BD} = \sigma A_{\text{net}} = \frac{\sigma_U}{\text{F.S.}}A_{\text{net}} = \left(\frac{400\times10^6}{3}\right)(6\times10^{-3})(18-12)(10^{-3}) = 4.80\times10^3\,\text{N}$$

Shear in pins at *B* and *D*.

$$F_{BD} = \tau A_{\text{pin}} = \frac{\tau_U}{\text{F.S.}}\frac{\pi}{4}d^2 = \left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(10\times10^{-3})^2 = 5.6549\times10^3\,\text{N}$$

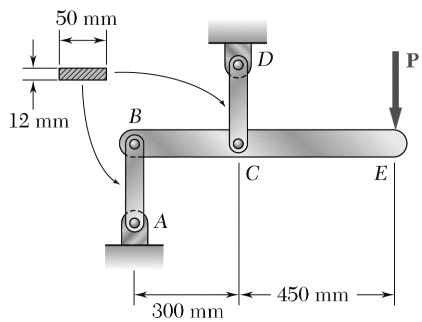
Smaller value of F_{BD} is 4.80×10^3 N.

From (1) $\qquad P = \left(\frac{3}{7}\right)(4.80\times10^3) = 2.06\times10^3\,\text{N}$

Shear in pin at *C*. $\qquad C = 2\tau A_{\text{pin}} = 2\frac{\tau_U}{\text{F.S.}}\frac{\pi}{4}d^2 = (2)\left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(6\times10^{-3})^2 = 2.8274\times10^3\,\text{N}$

From (2) $\qquad P = \left(\frac{3}{4}\right)(2.8274\times10^3) = 2.12\times10^3\,\text{N}$

Smaller value of *P* is allowable value. $\qquad P = 2.06\times10^3\,\text{N}$ $\qquad P = 2.06\text{ kN}$ ◀

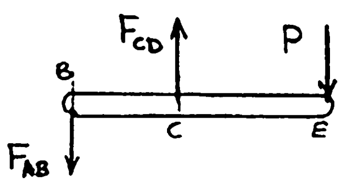


PROBLEM 1.53

Each of the steel links *AB* and *CD* is connected to a support and to member *BCE* by 25-mm-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 210 MPa for the steel used in the pins and that the ultimate normal stress is 490 MPa for the steel used in the links, determine the allowable load **P** if an overall factor of safety of 3.0 is desired. (Note that the links are not reinforced around the pin holes.)

SOLUTION

Use member *BCE* as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.3F_{CD} - 0.75P = 0$$

$$P = \frac{2}{5}F_{CD}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad 0.3F_{AB} - 0.45P = 0$$

$$P = \frac{2}{3}F_{AB}$$

Both links have the same area, pin diameter and material. Therefore, they have the same ultimate load.

Failure by pin in single shear.

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.025)^2 = 490.9 \times 10^{-6}\text{m}^2$$

$$F_U = \tau_U A = (210 \times 10^6)(490.9 \times 10^{-6}) = 103.09 \text{ kN}$$

Failure by tension in link.

$$A = (b - d)t = (0.05 - 0.025)0.012 = 3 \times 10^{-4}\text{m}^2$$

$$F_U = \sigma_U A = (490 \times 10^6)(3 \times 10^{-4}) = 147 \text{ kN}$$

Ultimate load for link and pin is the smaller.

$$F_U = 103.09 \text{ kN}$$

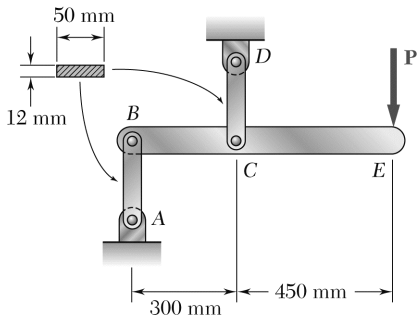
Allowable values of F_{CD} and F_{AB}.

$$F_{\text{all}} = \frac{F_U}{\text{F.S.}} = \frac{103.09}{3.0} = 34.36 \text{ kN}$$

Allowable load for structure is the smaller of $\frac{2}{3}F_{\text{all}}$ and $\frac{2}{5}F_{\text{all}}$.

$$P = \frac{2}{5}(34.36)$$

$P = 13.7$ kN ◀



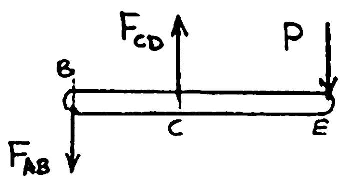
PROBLEM 1.54

An alternative design is being considered to support member *BCE* of Problem 1.53, in which link *CD* will be replaced by two links, each of 6×50-mm cross section, causing the pins at *C* and *D* to be in double shear. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

PROBLEM 1.53 Each of the steel links *AB* and *CD* is connected to a support and to member *BCE* by 25-mm-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 210 MPa for the steel used in the pins and that the ultimate normal stress is 490 MPa for the steel used in the links, determine the allowable load **P** if an overall factor of safety of 3.0 is desired. (Note that the links are not reinforced around the pin holes.)

SOLUTION

Use member *BCE* as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.3F_{CD} - 0.75P = 0$$

$$P = \frac{2}{5}F_{CD}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad 0.3F_{AB} - 0.45P = 0$$

$$P = \frac{2}{3}F_{AB}$$

Area of all pins: $A_{\text{pin}} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.025)^2 = 490.9 \times 10^{-6}\,\text{m}^2$

Net section area of link *AB*: $A_{\text{net}} = (b - d)t_{AB} = (0.05 - 0.025)(0.012) = 3 \times 10^{-4}\,\text{m}^2$

Net section area of the 2 links *CD* is the same.

Failure by pins *A* and *B* in single shear. $(F_{AB})_U = \tau_U A_{\text{pin}}$

$$(F_{AB})_U = (210 \times 10^6)(490.9 \times 10^{-6}) = 103.09 \text{ kN}$$

Failure by tension in link *AB*. $(F_{AB})_U = \sigma_U A_{\text{net}}$

$$(F_{AB})_U = (490 \times 10^6)(3 \times 10^{-4}) = 147 \text{ kN}$$

Ultimate load for link and pins *AB* is the smaller: $(F_{AB})_U = 103.09$ kN

Corresponding ultimate load, $P_U = \frac{2}{3}(F_{AB})_U = 68.73$ kN

Failure by pins *C* and in double shear. $(F_{CD})_U = 2\tau_U A_{\text{pin}}$

$$(F_{CD})_U = (2)(210 \times 10^6)(490.9 \times 10^{-6}) = 206.18 \text{ kN}$$

PROBLEM 1.54 *(Continued)*

Failure by tension in links *CD*. $(F_{CD})_U = \sigma_U A_{\text{net}}$

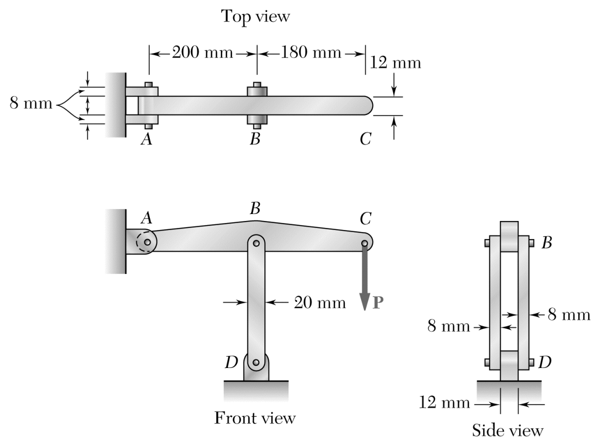
$$(F_{CD})_U = (490 \times 10^6)(3 \times 10^{-4}) = 147 \text{ kN}$$

Ultimate load for links and pins *CD* is the smaller: $(F_{CD})_U = 147 \text{ kN}$

Corresponding ultimate load: $P_U = \frac{2}{5}(F_{CD})_U = 58.8 \text{ kN}$

Actual ultimate load is the smaller. $P_U = 58.8 \text{ kN}$

<u>Allowable load *P*</u>: $P = \frac{P_U}{\text{F.S.}} = \frac{58.8}{3.0}$ $P = 19.6 \text{ kN}$ ◀

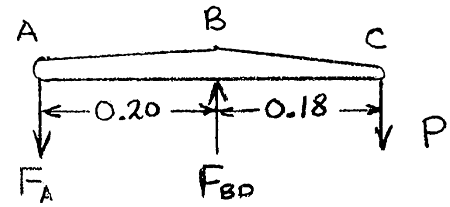


PROBLEM 1.55

In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load $\mathbf{P}$ if an overall factor of safety of 3.0 is desired.

SOLUTION

Statics: Use ABC as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20\, F_A - 0.18\, P = 0 \qquad P = \frac{10}{9} F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20\, F_{BD} - 0.38\, P = 0 \qquad P = \frac{10}{19} F_{BD}$$

Based on double shear in pin A: $A = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4}(0.008)^2 = 50.266 \times 10^{-6}\,\text{m}^2$

$$F_A = \frac{2\tau_U A}{\text{F.S.}} = \frac{(2)(100\times 10^6)(50.266\times 10^{-6})}{3.0} = 3.351\times 10^3\,\text{N}$$

$$P = \frac{10}{9} F_A = 3.72\times 10^3\,\text{N}$$

Based on double shear in pins at B and D: $A = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\,\text{m}^2$

$$F_{BD} = \frac{2\tau_U A}{\text{F.S.}} = \frac{(2)(100\times 10^6)(113.10\times 10^{-6})}{3.0} = 7.54\times 10^3\,\text{N}$$

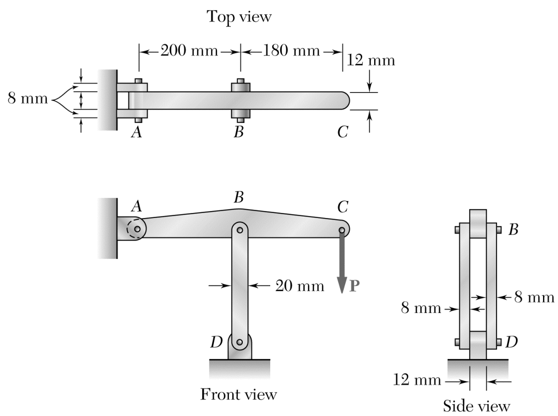
$$P = \frac{10}{19} F_{BD} = 3.97\times 10^3\,\text{N}$$

Based on compression in links BD: For one link, $A = (0.020)(0.008) = 160 \times 10^{-6}\,\text{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{\text{F.S.}} = \frac{(2)(250\times 10^6)(160\times 10^{-6})}{3.0} = 26.7\times 10^3\,\text{N}$$

$$P = \frac{10}{19} F_{BD} = 14.04\times 10^3\,\text{N}$$

Allowable value of P is smallest, $\therefore P = 3.72\times 10^3\,\text{N}$ $\qquad P = 3.72\text{ kN}$ ◀

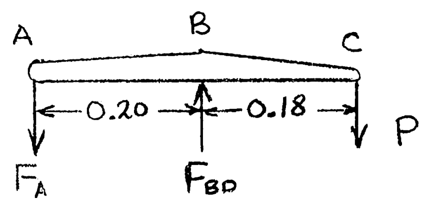


PROBLEM 1.56

In an alternative design for the structure of Problem 1.55, a pin of 10-mm-diameter is to be used at *A*. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

PROBLEM 1.55 In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

SOLUTION



Statics: Use *ABC* as free body.

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20\, F_A - 0.18P = 0 \qquad P = \frac{10}{9}F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20\, F_{BD} - 0.38P = 0 \qquad P = \frac{10}{19}F_{BD}$$

Based on double shear in pin *A*: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\,\text{m}^2$

$$F_A = \frac{2\tau_U A}{\text{F.S.}} = \frac{(2)(100\times 10^6)(78.54\times 10^{-6})}{3.0} = 5.236\times 10^3\,\text{N}$$

$$P = \frac{10}{9}F_A = 5.82\times 10^3\,\text{N}$$

Based on double shear in pins at *B* and *D*: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.012)^2 = 113.10\times 10^{-6}\,\text{m}^2$

$$F_{BD} = \frac{2\tau_U A}{\text{F.S.}} = \frac{(2)(100\times 10^6)(113.10\times 10^{-6})}{3.0} = 7.54\times 10^3\,\text{N}$$

$$P = \frac{10}{19}F_{BD} = 3.97\times 10^3\,\text{N}$$

Based on compression in links *BD*: For one link, $A = (0.020)(0.008) = 160\times 10^{-6}\,\text{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{\text{F.S.}} = \frac{(2)(250\times 10^6)(160\times 10^{-6})}{3.0} = 26.7\times 10^3\,\text{N}$$

$$P = \frac{10}{19}F_{BD} = 14.04\times 10^3\,\text{N}$$

Allowable value of *P* is smallest, $\therefore P = 3.97\times 10^3\,\text{N}$ $\qquad P = 3.97\text{ kN}$ ◀

PROBLEM 1.57

The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weighs 72 kg and each of the window washers is assumed to weigh 88 kg with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the conventional factor of safety for the selected cables?

SOLUTION

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

$$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\phi}$$

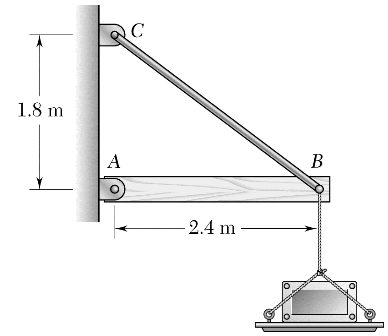
$$= \frac{(1.2)\left(\frac{1}{2} \times 72\right) + (1.5)\left(\frac{3}{4} \times 2 \times 88\right)}{0.85}$$

$$= 283.76 \text{ kg} = 2.78 \text{ kN} \quad \blacktriangleleft$$

Conventional factor of safety

$$P = P_D + P_L = \frac{1}{2} \times 72 + 0.75 \times 2 \times 88 = 168 \text{ kg} = 1.648 \text{ kN}$$

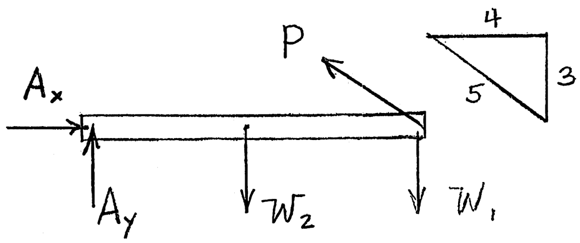
$$\text{F.S.} = \frac{P_U}{P} = \frac{2.784}{1.648} = 1.69 \quad \blacktriangleleft$$



PROBLEM 1.58

A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (*a*) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load that can be safely placed on the platform. (*b*) What is the corresponding conventional factor of safety for rod BC?

SOLUTION



$$\circlearrowleft\!+\ \Sigma M_A = 0: \quad (2.4)\frac{3}{5}P - 2.4W_1 - 1.2W_2$$

$$\therefore \quad P = \frac{5}{3}W_1 + \frac{5}{6}W_2$$

For dead loading, $W_1 = (40)(9.81) = 392.4\text{ N}, \quad W_2 = (50)(9.81) = 490.5\text{ N}$

$$P_D = \left(\frac{5}{3}\right)(392.4) + \left(\frac{5}{6}\right)(490.5) = 1.0628 \times 10^3\text{N}$$

For live loading, $W_1 = mg \quad W_2 = 0 \quad P_L = \frac{5}{3}mg$

From which $m = \frac{3}{5}\frac{P_L}{g}$

Design criterion. $\gamma_D P_D + \gamma_L P_L = \phi P_U$

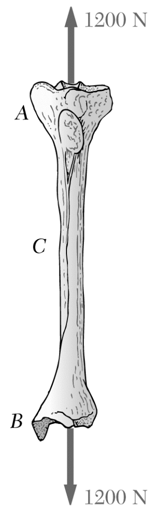
$$P_L = \frac{\phi P_U - \gamma_D P_D}{\gamma_L} = \frac{(0.90)(12 \times 10^3) - (1.25)(1.0628 \times 10^{-3})}{1.6}$$
$$= 5.920 \times 10^3\text{N}$$

(*a*) Allowable load. $m = \frac{3}{5}\frac{5.92 \times 10^3}{9.81}$ $\quad m = 362\text{ kg}$ ◀

Conventional factor of safety.

$$P = P_D + P_L = 1.0628 \times 10^3 + 5.920 \times 10^3 = 6.983 \times 10^3\text{N}$$

(*b*) $\text{F.S.} = \frac{P_U}{P} = \frac{12 \times 10^3}{6.983 \times 10^3}$ $\quad \text{F.S.} = 1.718$ ◀



PROBLEM 1.59

A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.

SOLUTION

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $A = \dfrac{\pi}{4}(d_1^2 - d_2^2)$

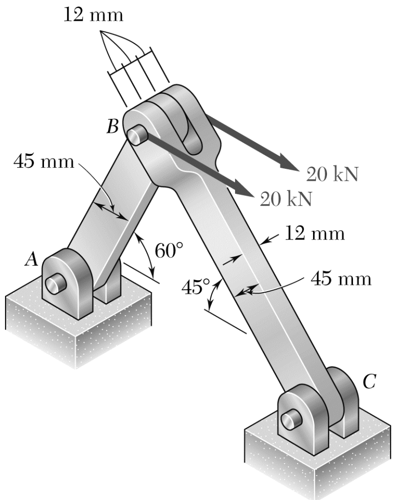
$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25 \times 10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80 \times 10^6)}$$

$$= 222.9 \times 10^{-6}\,\text{m}^2$$

$$d_2 = 14.93 \times 10^{-3}\,\text{m}$$

$d_2 = 14.93$ mm ◀

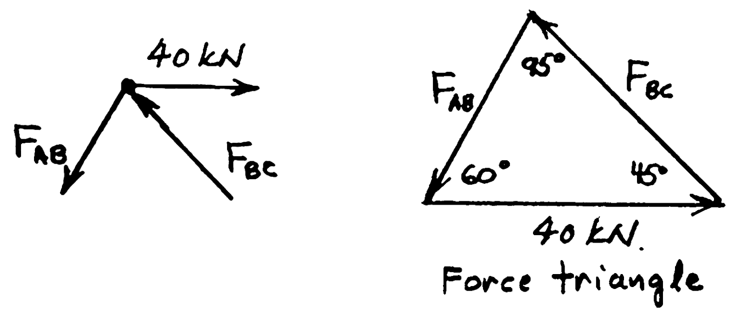


PROBLEM 1.60

Two horizontal 20 kN forces are applied to pin B of the assembly shown. Knowing that a pin of 20 mm diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link AB, (*b*) in link BC.

SOLUTION

Use joint B as free body.



Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{40}{\sin 95^\circ}$$

$$F_{AB} = 28.4 \text{ kN}$$

$$F_{BC} = 34.8 \text{ kN}$$

Link AB is a tension member.

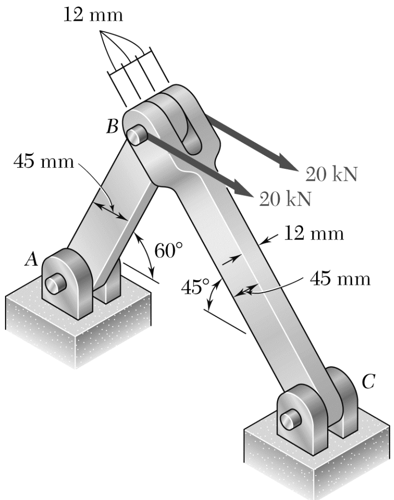
Minimum section at pin. $A_{\text{net}} = (0.045 - 0.02)(0.012) = 300 \times 10^{-6} \text{m}^2$

(*a*) Stress in AB $\quad \sigma_{AB} = \dfrac{F_{AB}}{A_{\text{net}}} = \dfrac{28.4}{300 \times 10^{-6}} = 94.7 \text{ MPa}$ ◀

Link BC is a compression member.

Cross sectional area is $A = (0.045)(0.012) = 540 \times 10^{-6} \text{m}^2$

(*b*) Stress in BC $\quad \sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{34.8}{540 \times 10^{-6}} = -64.4 \text{ MPa}$ ◀



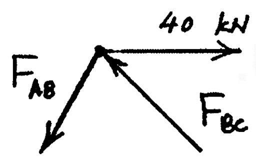
PROBLEM 1.61

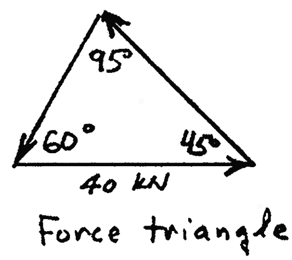
For the assembly and loading of Problem 1.60, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in member *BC*, (*c*) the average bearing stress at *B* in member *BC*.

PROBLEM 1.60 Two horizontal 20-kN forces are applied to pin *B* of the assembly shown. Knowing that a pin of 20-mm diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

SOLUTION

Use joint *B* as free body.





Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{40}{\sin 95^\circ} \qquad F_{BC} = 34.77 \text{ kN}$$

(*a*) Shearing stress in pin at *C*. $\tau = \dfrac{F_{BC}}{2A_P}$

$$A_P = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2$$

$$\tau = \frac{34770}{(2)(314.16 \times 10^{-6})} = 55.338 \times 10^6 \qquad \tau = 55.3 \text{ MPa} \blacktriangleleft$$

(*b*) Bearing stress at *C* in member *BC*. $\sigma_b = \dfrac{F_{BC}}{A}$

$$A = td = (12)(20) = 240 \text{ mm}^2$$

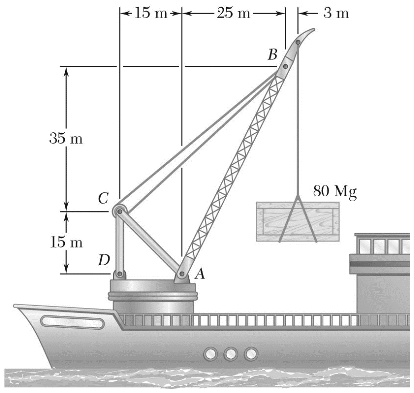
$$\sigma_b = \frac{34770}{240 \times 10^{-6}} = 144.875 \times 10^6 \qquad \sigma_b = 144.9 \text{ MPa} \blacktriangleleft$$

PROBLEM 1.61 *(Continued)*

(*c*) <u>Bearing stress at *B* in member *BC*.</u> $\sigma_b = \dfrac{F_{BC}}{A}$

$$A = 2td = 2(12)(20) = 480 \text{ mm}^2$$

$$\sigma_b = \frac{34770}{480 \times 10^{-6}} = 72.437 \times 10^6 \qquad \sigma_b = 72.4 \text{ MPa} \blacktriangleleft$$



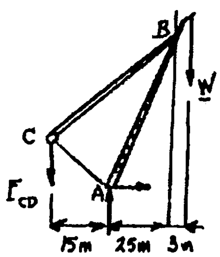
PROBLEM 1.62

In the marine crane shown, link CD is known to have a uniform cross section of 50×150 mm. For the loading shown, determine the normal stress in the central portion of that link.

SOLUTION

Weight of loading: $W = (80 \text{ Mg})(9.81 \text{ m/s}^2) = 784.8 \text{ kN}$

Free Body: Portion ABC

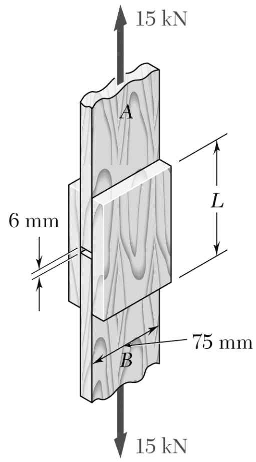


$$+\circlearrowleft \Sigma M_A = 0: \quad F_{CD}(15 \text{ m}) - W(28 \text{ m}) = 0$$

$$F_{CD} = \frac{28}{15}W = \frac{28}{15}(784.8 \text{ kN})$$

$$F_{CD} = +1465 \text{ kN}$$

$$\sigma_{CD} = \frac{F_{CD}}{A} = \frac{+1465 \times 10^3 \text{N}}{(0.050 \text{ m})(0.150 \text{ m})} = +195.3 \times 10^6 \text{ Pa} \qquad \sigma_{CD} = +195.3 \text{ MPa} \blacktriangleleft$$



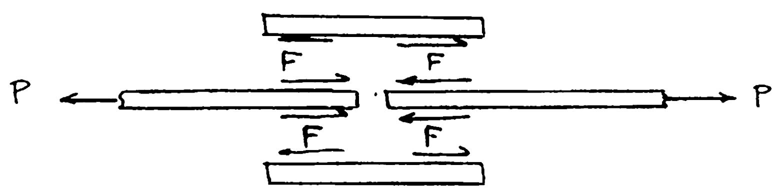
PROBLEM 1.63

The wooden members A and B are to be joined by plywood splice plates that will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be 6 mm, determine the smallest allowable length L if the average shearing stress in the glue is not to exceed 700 kPa.

SOLUTION

There are four separate areas that are glued. Each of these areas transmits one half the 15 kN load. Thus

$$F = \frac{1}{2}P = \frac{1}{2}(15) = 7.5 \text{ kN} = 7500 \text{ N}$$



Let ℓ = length of one glued area and w = 75 mm = 0.075 m be its width.

For each glued area, $\quad A = \ell w$

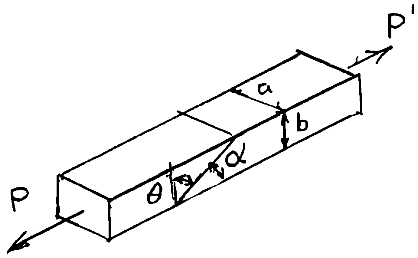
Average shearing stress: $\quad \tau = \dfrac{F}{A} = \dfrac{F}{\ell w}$

The allowable shearing stress is $\quad \tau = 700 \times 10^3$ Pa

Solving for ℓ, $\quad \ell = \dfrac{F}{\tau w} - \dfrac{7500}{(700 \times 10^3)(0.025)} = 0.14286 \text{ m} = 142.85 \text{ mm}$

Total length L: $\quad L = \ell + (\text{gap}) + \ell = 142.85 + 6 + 142.85$

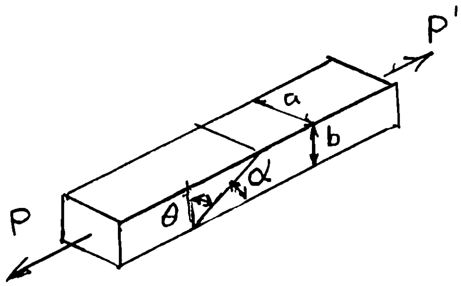
$L = 292$ mm ◀



PROBLEM 1.64

Two wooden members of uniform rectangular cross section of sides $a = 100$ mm and $b = 60$ mm are joined by a simple glued joint as shown. Knowing that the ultimate stresses for the joint are $\sigma_U = 1.26$ MPa in tension and $\tau_U = 1.50$ MPa in shear, and that $P = 6$ kN, determine the factor of safety for the joint when (*a*) $\alpha = 20°$, (*b*) $\alpha = 35°$, (*c*) $\alpha = 45°$. For each of these values of α, also determine whether the joint will fail in tension or in shear if P is increased until rupture occurs.

SOLUTION



Let $\theta = 90° - \alpha$ as shown.

From the text book:

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta$$

or

$$\sigma = \frac{P}{A_0}\sin^2\alpha \tag{1}$$

$$\tau = \frac{P}{A_0}\sin\alpha\cos\alpha \tag{2}$$

$$A_0 = ab = (100\text{ mm})(60\text{ mm}) = 6000\text{ mm}^2 = 6\times10^{-3}\text{m}^2$$

$$\sigma_U = 1.26\times10^6\text{ Pa} \qquad \tau_U = 1.50\times10^6\text{ Pa}$$

Ultimate load based on tension across the joint:

$$(P_U)_\sigma = \frac{\sigma_U A_0}{\sin^2\alpha} = \frac{(1.26\times10^6)(6\times10^{-3})}{\sin^2\alpha} = \frac{7560}{\sin^2\alpha} = \frac{7.56}{\sin^2\alpha}\text{kN}$$

Ultimate load based on shear across the joint:

$$(P_U)_\tau = \frac{\tau_U A_0}{\sin\alpha\cos\alpha} = \frac{(1.50\times10^6)(6\times10^{-3})}{\sin\alpha\cos\alpha} = \frac{9000}{\sin\alpha\cos\alpha} = \frac{9.00}{\sin\alpha\cos\alpha}\text{kN}$$

PROBLEM 1.64 *(Continued)*

(*a*) $\alpha = 20°$: $(P_U)_\sigma = \dfrac{7.56}{\sin^2 20°} = 64.63 \text{ kN}$

$$= (P_U)_\tau = \frac{9.00}{\sin 20° \cos 20°} = 28.00 \text{ kN}$$

The smaller value governs. The joint will fail in shear and $P_U = 28.00$ kN.

$$\text{F.S.} = \frac{P_U}{P} = \frac{28.00}{6} \qquad \text{F.S.} = 4.67 \blacktriangleleft$$

(*b*) $\alpha = 35°$: $(P_U)_\sigma = \dfrac{7.56}{\sin^2 35°} = 22.98 \text{ kN}$

$$(P_U)_\tau = \frac{9.00}{\sin 35° \cos 35°} = 19.155 \text{ kN}$$

The joint will fail in shear and $P_U = 19.155$ kN.

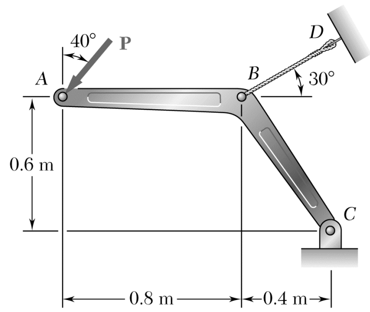
$$\text{F.S.} = \frac{P_U}{P} = \frac{19.155}{6} \qquad \text{F.S.} = 3.19 \blacktriangleleft$$

(*c*) $\alpha = 45°$: $(P_U)_\sigma = \dfrac{7.56}{\sin^2 45°} = 15.12 \text{ kN}$

$$(P_U)_\tau = \frac{9.00}{\sin 45° \cos 45°} = 18.00 \text{ kN}$$

The joint will fail in tension and $P_U = 15.12$ kN.

$$\text{F.S.} = \frac{P_U}{P} = \frac{15.12}{6} \qquad \text{F.S.} = 2.52 \blacktriangleleft$$



PROBLEM 1.65

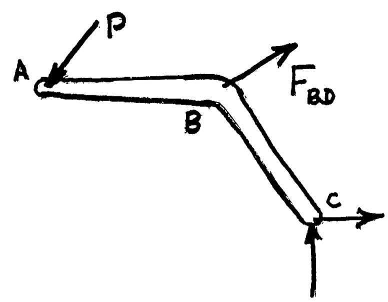
Member ABC, which is supported by a pin and bracket at C and a cable BD, was designed to support the 16-kN load **P** as shown. Knowing that the ultimate load for cable BD is 100 kN, determine the factor of safety with respect to cable failure.

SOLUTION

Use member ABC as a free body, and note that member BD is a two-force member.

$$\circlearrowleft + \; \Sigma M_c = 0: \quad (P\cos 40°)(1.2) + (P\sin 40°)(0.6) - (F_{BD}\cos 30°)(0.6) - (F_{BD}\sin 30°)(0.4) = 0$$

$$1.30493P - 0.71962F_{BD} = 0$$

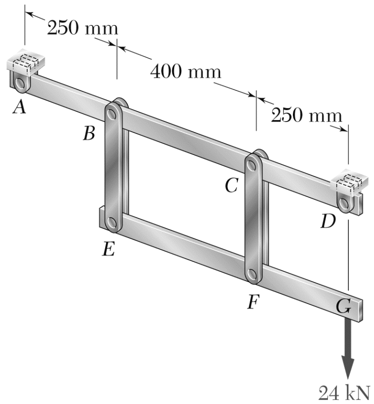


$$F_{BD} = 1.81335\ P = (1.81335)(16 \times 10^3) = 29.014 \times 10^3 \text{ N}$$

$$F_U = 100 \times 10^3 \text{ N}$$

$$\text{F.S.} = \frac{F_U}{F_{BD}} = \frac{100 \times 10^3}{29.014 \times 10^3}$$

F.S. = 3.45 ◀

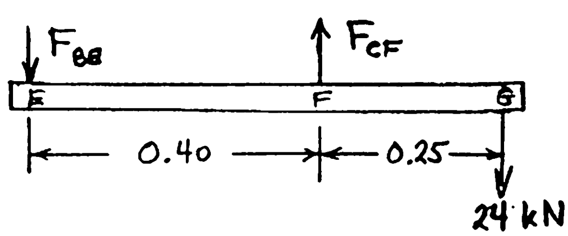


PROBLEM 1.66

Each of the two vertical links CF connecting the two horizontal members AD and EG has a 10×40-mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at C and F has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links CF and the pins connecting them to the horizontal members.

SOLUTION

Use member EFG as free body.



$$+\circlearrowleft \Sigma M_E = 0 \quad 0.40F_{CF} - (0.65)(24 \times 10^3) = 0$$

$$F_{CF} = 39 \times 10^3 \text{ N}$$

Based on tension in links CF

$$A = (b-d)t = (0.040 - 0.02)(0.010) = 200 \times 10^{-6}\text{m}^2 \text{ (one link)}$$

$$F_U = 2\sigma_U A = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160.0 \times 10^3 \text{ N}$$

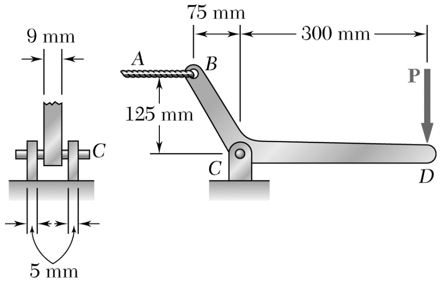
Based on double shear in pins

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\text{m}^2$$

$$F_U = 2\tau_U A = (2)(150 \times 10^6)(314.16 \times 10^{-6}) = 94.248 \times 10^3 \text{ N}$$

Actual F_U is smaller value, i.e.: $F_U = 94.248 \times 10^3$ N

Factor of safety $\quad \text{F.S.} = \dfrac{F_U}{F_{CF}} = \dfrac{94.248 \times 10^3}{39 \times 10^3} = 2.42$ ◀

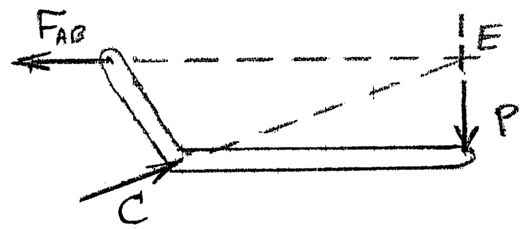


PROBLEM 1.67

Knowing that a force **P** of magnitude 750 N is applied to the pedal shown, determine (*a*) the diameter of the pin at *C* for which the average shearing stress in the pin is 40 MPa, (*b*) the corresponding bearing stress in the pedal at *C*, (*c*) the corresponding bearing stress in each support bracket at *C*.

SOLUTION

Draw free body diagram of *BCD*. Since *BCD* is a 3-force member, the reaction at *C* is directed toward Point *E*, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325 \text{ mm}$

$$+\uparrow \Sigma F_y = 0: \quad \frac{125}{325} C - P = 0 \quad C = 2.6P = (2.6)(750) = 1950 \text{ N}$$

(*a*) $$\tau_{\text{pin}} = \frac{\frac{1}{2}C}{A_{\text{pin}}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} \quad d = \sqrt{\frac{2C}{\pi\tau_{\text{pin}}}} = \sqrt{\frac{(2)(1950)}{\pi(40 \times 10^6)}} = 5.57 \times 10^{-3} \text{m}$$

$d = 5.57$ mm ◀

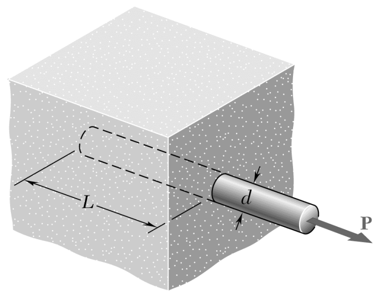
(*b*) $$\sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1950}{(5.57 \times 10^{-3})(9 \times 10^{-3})} = 38.9 \times 10^6 \text{ Pa}$$

$\sigma_b = 38.9$ MPa ◀

(*c*) $$\sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1950}{(2)(5.57 \times 10^{-3})(5 \times 10^{-3})} = 35.0 \times 10^6 \text{ Pa}$$

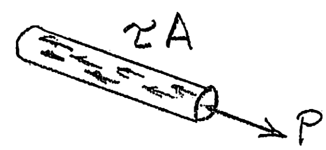
$\sigma_b = 35.0$ MPa ◀

PROBLEM 1.68



A force **P** is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)

SOLUTION



For shear, $A = \pi dL$

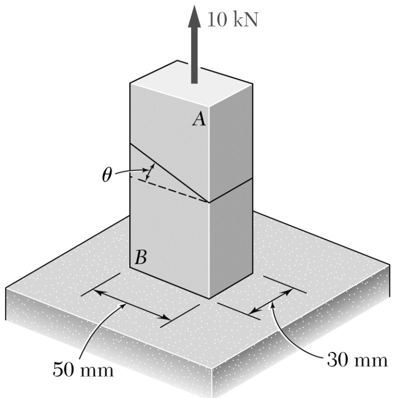
$$P = \tau_{all} A = \tau_{all} \pi dL$$

For tension, $A = \frac{\pi}{4} d^2$

$$P = \sigma_{all} A = \sigma_{all} \left(\frac{\pi}{4} d^2 \right)$$

Equating, $\tau_{all} \pi dL = \sigma_{all} \frac{\pi}{4} d^2$

Solving for L, $L_{min} = \sigma_{all} d / 4\tau_{all}$ ◀



PROBLEM 1.69

The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine the range of values of θ for which the factor of safety of the members is at least 3.0.

SOLUTION

$$A_o = (0.05)(0.03) = 0.0015 \text{ m}^2$$

$$P = 10 \text{ kN}$$

$$P_U = (F.S.)P = 30 \text{ kN}$$

Based on tensile stress

$$\sigma_U = \frac{P_U}{A_o}\cos^2\theta$$

$$\cos^2\theta = \frac{\sigma_U A_o}{P_U} = \frac{(17\times 10^6)(0.0015)}{30\times 10^3} = 0.85$$

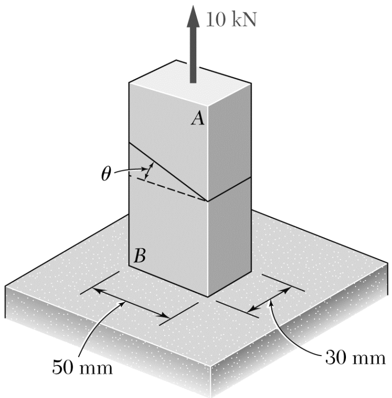
$$\cos\theta = 0.922 \qquad \theta = 22.8° \qquad \theta \geq 22.8°$$

Based on shearing stress $\qquad \tau_U = \dfrac{P_U}{A_o}\sin\theta\cos\theta = \dfrac{P_U}{2A_o}\sin 2\theta$

$$\sin 2\theta = \frac{2A_o\tau_U}{P_U} = \frac{(2)(0.0015)(9\times 10^6)}{30\times 10^3} = 0.9$$

$$2\theta = 64.16° \qquad \theta = 32.1° \qquad \theta \leq 32.1°$$

Hence, $\qquad 22.8° \leq \theta \leq 32.1°$ ◀



PROBLEM 1.70

The two portions of member *AB* are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine (*a*) the value of θ for which the factor of safety of the member is maximum, (*b*) the corresponding value of the factor of safety. (*Hint:* Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

SOLUTION

$$A_o = (0.05)(0.03) = 0.0015 \text{ m}^2$$

At the optimum angle $\qquad (\text{F.S.})_\sigma = (\text{F.S.})_\tau$

Normal stress: $\quad \sigma = \dfrac{P}{A_o}\cos^2\theta \quad \therefore \quad P_{U,\sigma} = \dfrac{\sigma_U A_o}{\cos^2\theta}$

$$(\text{F.S.})_\sigma = \frac{P_{U,\sigma}}{P} = \frac{\sigma_U A_o}{P\cos^2\theta}$$

Shearing stress: $\tau = \dfrac{P}{A_o}\sin\theta\cos\theta \quad \therefore \quad P_{U,\tau} = \dfrac{\tau_U A_o}{\sin\theta\cos\theta}$

$$(\text{F.S.})_\tau = \frac{P_{U,\tau}}{P} = \frac{\tau_U A_o}{P\sin\theta\cos\theta}$$

Equating: $\dfrac{\sigma_U A_o}{P\cos^2\theta} = \dfrac{\tau_U A_o}{P\sin\theta\cos\theta}$;

Solving: $\quad \dfrac{\sin\theta}{\cos\theta} = \tan\theta = \dfrac{\tau_U}{\sigma_U} = \dfrac{9}{17} = 0.529$ $\qquad$ (*a*) $\quad \theta_{\text{opt}} = 27.9° ◀$

(*b*) $\quad P_U = \dfrac{\sigma_U A_o}{\cos^2\theta} = \dfrac{(17\times10^6)(0.0015)}{\cos^2 27.9°} = 32.65 \text{ kN}$

$$\text{F.S.} = \frac{P_U}{P} = \frac{32.64}{10} = 3.26 ◀$$