HW10 CH31 Solutions

31.12. **IDENTIFY:** Calculate the reactance of the inductor and of the capacitor. Calculate the impedance and use that result to calculate the current amplitude.

SET UP: With no capacitor, $Z=\sqrt{R^2+X_L^2}$ and $\tan\phi=\dfrac{X_L}{R}$. $X_L=\omega L$. $I=\dfrac{V}{Z}$. $V_L=IX_L$ and $V_R=IR$. For an inductor, the voltage leads the current.

EXECUTE: **(a)** $X_L=\omega L=(250\text{ rad/s})(0.400\text{ H})=100\,\Omega$. $Z=\sqrt{(200\,\Omega)^2+(100\,\Omega)^2}=224\,\Omega$.

(b) $I=\dfrac{V}{Z}=\dfrac{30.0\text{ V}}{224\,\Omega}=0.134\text{ A}$.

(c) $V_R=IR=(0.134\text{ A})(200\,\Omega)=26.8\text{ V}$. $V_L=IX_L=(0.134\text{ A})(100\,\Omega)=13.4\text{ V}$.

(d) $\tan\phi=\dfrac{X_L}{R}=\dfrac{100\,\Omega}{200\,\Omega}$ and $\phi=+26.6°$. Since ϕ is positive, the source voltage leads the current.

(e) The phasor diagram is sketched in Figure 31.12.

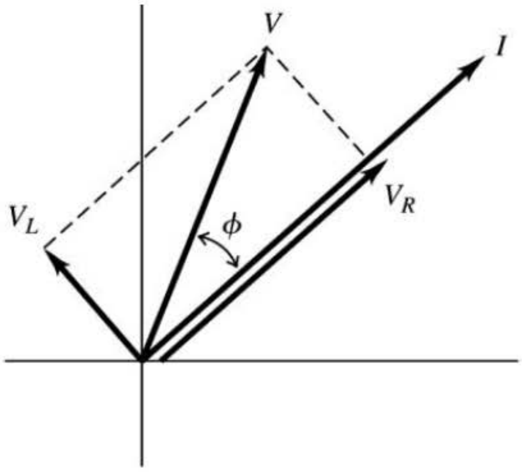


Figure 31.12

31.24. **IDENTIFY** and **SET UP:** $P_{av}=V_{rms}I_{rms}\cos\phi$. $I_{rms}=\dfrac{V_{rms}}{Z}$. $\cos\phi=\dfrac{R}{Z}$.

EXECUTE: $I_{rms}=\dfrac{73.0\text{ V}}{119\,\Omega}=0.613\text{ A}$. $\cos\phi=\dfrac{76.0\,\Omega}{119\,\Omega}=0.639$.

$P_{av}=(73.0\text{ V})(0.613\text{ A})(0.639)=28.6\text{ W}$.

31.38. **IDENTIFY:** We have an *L-R-C* series ac circuit.

SET UP: $\tan\phi=\dfrac{X_L-X_C}{R}$, $P_{av}=\dfrac{1}{2}I^2R$, $P_{av}=\dfrac{1}{2}IV\cos\phi$.

EXECUTE: **(a)** We want X_L. Use $\tan\phi=\dfrac{X_L-X_C}{R}$. $X_L=R\tan\phi+X_C$. Using the numbers gives $X_L=(300\,\Omega)\tan(-53.0°)+500\,\Omega=102\,\Omega$.

(b) We want I. Use $P_{av}=\dfrac{1}{2}I^2R$ and solve for I. $I=\sqrt{2P_{av}/R}$ gives $I=0.730$ A.

(c) We want V. Use $P_{av}=\dfrac{1}{2}IV\cos\phi$ and solve for V. $V=\dfrac{2P_{av}}{I\cos\phi}=364$ V.

31.39. **IDENTIFY:** An *L-R-C* ac circuit operates at resonance. We know *L*, *C*, and *V* and want to find *R*.

SET UP: At resonance, $Z = R$ and $\omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$. $X_C = \dfrac{1}{\omega C}$, $I = V/Z$.

EXECUTE: $\omega = \dfrac{1}{\sqrt{LC}} = 626.0 \text{ rad/s}$ $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(626 \text{ rad/s})(5.00\times 10^{-6} \text{ F})} = 319.4\,\Omega$.

$I = \dfrac{V_C}{X_C} = \dfrac{77.0 \text{ V}}{319.4\,\Omega} = 0.2411 \text{ A}$. At resonance $Z = R$, so $I = \dfrac{V}{R}$. $R = \dfrac{V}{I} = \dfrac{57.5 \text{ V}}{0.2411 \text{ A}} = 238\,\Omega$.

31.42. **IDENTIFY:** Use geometry to calculate the self-inductance of the toroidal solenoid. Then find its reactance and use this to find the impedance, and finally the current amplitude, of the circuit.

SET UP: $L = \dfrac{\mu_0 N^2 A}{2\pi r}$, $X_L = 2\pi f L$, $Z = \sqrt{R^2 + X_L^2}$, and $I = V/Z$.

EXECUTE: $L = \dfrac{\mu_0 N^2 A}{2\pi r} = (2\times 10^{-7} \text{ T}\cdot\text{m/A})\dfrac{(3000)^2(0.500\times 10^{-4} \text{ m}^2)}{9.25\times 10^{-2} \text{ m}} = 9.73\times 10^{-4} \text{ H}$.

$X_L = 2\pi f L = (2\pi)(375 \text{ Hz})(9.73\times 10^{-4} \text{ H}) = 2.292\,\Omega$. $Z = \sqrt{R^2 + X_L^2} = 3.618\,\Omega$.

$I = \dfrac{V}{Z} = \dfrac{23.5 \text{ V}}{3.618\,\Omega} = 6.49 \text{ A}$.

31.43. **IDENTIFY and SET UP:** Source voltage lags current so it must be that $X_C > X_L$.

EXECUTE: **(a)** We must add an inductor in series with the circuit. When $X_C = X_L$ the power factor has its maximum value of unity, so calculate the additional *L* needed to raise X_L to equal X_C.

(b) Power factor $\cos\phi$ equals 1 so $\phi = 0$ and $X_C = X_L$. Calculate the present value of $X_C - X_L$ to see how much more X_L is needed: $R = Z\cos\phi = (58.0\,\Omega)(0.720) = 41.8\,\Omega$

$\tan\phi = \dfrac{X_L - X_C}{R}$ so $X_L - X_C = R\tan\phi$.

$\cos\phi = 0.720$ gives $\phi = -43.95°$ (ϕ is negative since the voltage lags the current).

Then $X_L - X_C = R\tan\phi = (41.8\,\Omega)\tan(-43.95°) = -40.26\,\Omega$.

Therefore need to add $40.26\,\Omega$ of X_L.

$X_L = \omega L = 2\pi f L$ and $L = \dfrac{X_L}{2\pi f} = \dfrac{40.26\,\Omega}{2\pi(45.0 \text{ Hz})} = 0.142 \text{ H}$, amount of inductance to add.

31.44. **IDENTIFY:** We are dealing with a transformer as an ac adapter.

SET UP and EXECUTE: **(a)** Voltage = 19.5 V, current = 6.7 A.

(b) Power = 130 W. $IV = (19.5 \text{ V})(6.7 \text{ A}) = 131$ W, so $P = IV$.

(c) Primary: 230 V rms, 200 turns. The full-wave rectifier following the secondary coil maintains a voltage amplitude $V = 19.5$ V. We want the number of turns in the secondary. $V_1 = V_{\text{rms}}\sqrt{2}$. The secondary output should be $V_2 = 19.5$ *V*. $N_2 = \dfrac{V_2}{V_1}N_1 = \dfrac{19.5 \text{ V}}{230\sqrt{2} \text{ V}}(200) = 12$ turns.

(d) We want I_1. $P = IV$ gives 130 W = I_1(230 V), so $I_1 = 0.57$ A.

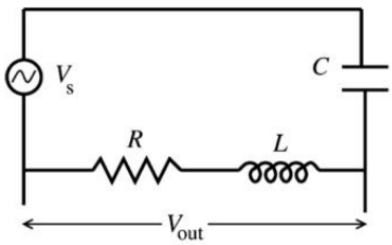
(e) Estimate the size: Outside: 5 cm by 5 cm by 1.5 cm. Inside: 3 cm by 3 cm by 1 cm. One coil: 4 cm. 200 coils: 800 cm = 8.0 m.

(f) We want *B* inside the core. Apply Ampere's law. Use permeability μ instead of μ_0, where $\mu = K_m\mu_0$, with K_m being the relative permeability. $\oint \vec{B}\cdot d\hat{l} = K_m\mu_0 I_{\text{encl}}$. Use a rectangular path with one side of length $l = 3$ cm inside the core enclosing all the loops. $Bl = K\mu_0 N_1 I_{\text{encl}}$.

$B = K\mu_0 N_1 I_{\text{encl}}/l = (5000)\,\mu_0\,(200)(0.57 \text{ A})/(0.030 \text{ m}) = 24$ T.

31.47. **IDENTIFY and SET UP:** Express Z and I in terms of ω, L, C, and R. The voltages across the resistor and the inductor are $90°$ out of phase, so $V_{\text{out}} = \sqrt{V_R^2 + V_L^2}$.

EXECUTE: The circuit is sketched in Figure 31.47.



$$X_L = \omega L, X_C = \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$I = \frac{V_s}{Z} = \frac{V_s}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Figure 31.47

$$V_{\text{out}} = I\sqrt{R^2 + X_L^2} = I\sqrt{R^2 + \omega^2 L^2} = V_s\sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\frac{V_{\text{out}}}{V_s} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$\underline{\omega \text{ small}}$:

As ω gets small, $R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \to \frac{1}{\omega^2 C^2}$, $R^2 + \omega^2 L^2 \to R^2$.

Therefore, $\frac{V_{\text{out}}}{V_s} \to \sqrt{\frac{R^2}{(1/\omega^2 C^2)}} = \omega RC$ as ω becomes small.

$\underline{\omega \text{ large}}$:

As ω gets large, $R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \to R^2 + \omega^2 L^2 \to \omega^2 L^2$, $R^2 + \omega^2 L^2 \to \omega^2 L^2$.

Therefore, $\frac{V_{\text{out}}}{V_s} \to \sqrt{\frac{\omega^2 L^2}{\omega^2 L^2}} = 1$ as ω becomes large.

31.48. **IDENTIFY:** $V = V_C = IX_C$. $I = V/Z$.

SET UP: $X_L = \omega L$, $X_C = \frac{1}{\omega C}$.

EXECUTE: $V_{\text{out}} = V_C = \frac{I}{\omega C} \Rightarrow \frac{V_{\text{out}}}{V_s} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

If ω is large: $\frac{V_{\text{out}}}{V_s} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \approx \frac{1}{\omega C\sqrt{(\omega L)^2}} = \frac{1}{(LC)\omega^2}$.

If ω is small: $\frac{V_{\text{out}}}{V_s} \approx \frac{1}{\omega C\sqrt{(1/\omega C)^2}} = \frac{\omega C}{\omega C} = 1$.

31.51. **IDENTIFY:** We know R, X_C, and ϕ so $\tan\phi = \dfrac{X_L - X_C}{R}$ tells us X_L. Use $P_{av} = I_{rms}^2 R$ to calculate I_{rms}. Then calculate Z and use $V_{rms} = I_{rms}Z$ to calculate V_{rms} for the source.

SET UP: Source voltage lags current so $\phi = -54.0°$. $X_C = 350\ \Omega$, $R = 180\ \Omega$, $P_{av} = 140$ W.

EXECUTE: **(a)** $\tan\phi = \dfrac{X_L - X_C}{R}$.

$X_L = R\tan\phi + X_C = (180\ \Omega)\tan(-54.0°) + 350\ \Omega = -248\ \Omega + 350\ \Omega = 102\ \Omega.$

(b) $P_{av} = V_{rms}I_{rms}\cos\phi = I_{rms}^2 R$ (Exercise 31.22). $I_{rms} = \sqrt{\dfrac{P_{av}}{R}} = \sqrt{\dfrac{140\ \text{W}}{180\ \Omega}} = 0.882$ A.

(c) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(180\ \Omega)^2 + (102\ \Omega - 350\ \Omega)^2} = 306\ \Omega.$

$V_{rms} = I_{rms}Z = (0.882\ \text{A})(306\ \Omega) = 270$ V.

31.54. **IDENTIFY:** At any instant of time the same rules apply to the parallel ac circuit as to the parallel dc circuit: the voltages are the same and the currents add.

SET UP: For a resistor the current and voltage in phase. For an inductor the voltage leads the current by $90°$ and for a capacitor the voltage lags the current by $90°$.

EXECUTE: **(a)** The parallel L-R-C circuit must have equal potential drops over the capacitor, inductor and resistor, so $v_R = v_L = v_C = v$. Also, the sum of currents entering any junction must equal the current leaving the junction. Therefore, the sum of the currents in the branches must equal the current through the source: $i = i_R + i_L + i_C$.

(b) $i_R = \dfrac{v}{R}$ is always in phase with the voltage. $i_L = \dfrac{v}{\omega L}$ lags the voltage by $90°$, and $i_C = v\omega C$ leads the voltage by $90°$. The phasor diagram is sketched in Figure 31.54.

(c) From the diagram, $I^2 = I_R^2 + (I_C - I_L)^2 = \left(\dfrac{V}{R}\right)^2 + \left(V\omega C - \dfrac{V}{\omega L}\right)^2$.

(d) From part (c): $I = V\sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}$. But $I = \dfrac{V}{Z}$, so $\dfrac{1}{Z} = \sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}$.

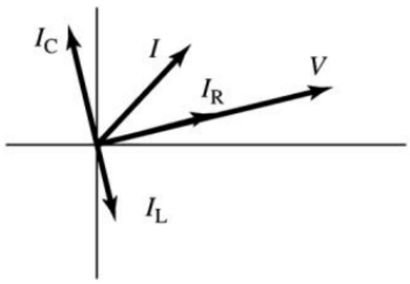


Figure 31.54

31.65. IDENTIFY: We have an *L*-*R* ac circuit.

SET UP: $v_{\text{in}} = V_{\text{in}} \cos \omega t$, $G = (20\text{ dB})\ln(V_{\text{out}}/V_{\text{in}})$, $I = V_{\text{in}}/Z$, $Z = \sqrt{R^2 + (\omega L)^2}$, $\phi = \arctan\left(\dfrac{\omega L}{R}\right)$.

EXECUTE: **(a)** We want the current. $I = \dfrac{V_{\text{in}}}{Z} = \dfrac{V_{\text{in}}}{\sqrt{R^2 + (\omega L)^2}}$.

(b) We want ϕ. $\phi = \arctan\left(\dfrac{\omega L}{R}\right)$.

(c) We want $V_{\text{out}}/V_{\text{in}}$. $\dfrac{V_{\text{out}}}{V_{\text{in}}} = \dfrac{V_L}{V_{\text{in}}} = \dfrac{IX_L}{V_{\text{in}}} = \dfrac{\dfrac{V_{\text{in}}}{\sqrt{R^2 + (\omega L)^2}} \cdot \omega L}{V_{\text{in}}} = \dfrac{\omega L}{\sqrt{R^2 + (\omega L)^2}} = \dfrac{1}{\sqrt{1 + (R/\omega L)^2}}$.

(d) We want *f* so $G = -3.0$ dB. Use $G = (20\text{ dB})\ln(V_{\text{out}}/V_{\text{in}})$. $-3.0\text{ dB} = (20\text{ dB})\ln(V_{\text{out}}/V_{\text{in}})$.

$-3.0/20 = \ln(V_{\text{out}}/V_{\text{in}})$. Write this result in terms of exponents and use the result from (c).

$10^{-3.0/20} = 0.708 = \dfrac{V_{\text{out}}}{V_{\text{in}}} = \dfrac{1}{\sqrt{1 + (R/\omega L)^2}}$. Use $\omega = 2\pi f$ and solve for *f*, giving $f = \dfrac{R}{2.00\pi L}$.

(e) We want *L*. Solve the result in (d) when *f* = 10.0 kHz and *R* = 100 Ω, giving *L* = 1.6 mH.