29.11. **IDENTIFY:** We are dealing with an induced emf due to changing magnetic flux.

SET UP: $\mathcal{E} = -\frac{d\Phi_B}{dt}$, $\mathcal{E} = RI$, $B = B_0 e^{-t/\tau}$. The target variable is the current.

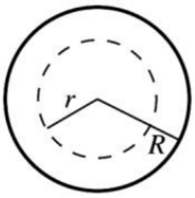
EXECUTE: **(a)** $\mathcal{E} = \frac{dBA}{dt} = \frac{dB_0 e^{-t/\tau} A}{dt} = -\frac{B_0 A}{\tau} e^{-t/\tau}$. $|I| = \frac{\mathcal{E}}{R} = \frac{B_0 A}{R\tau} e^{-t/\tau}$. I is a maximum when $t = 0$.

Using $A = \pi r^2$ and the given values, we get $I_{max} = 12.6$ mA.

(b) At $t = 1.50$ s, we use the result from (a) for the emf with $t = 1.50$ s and $I_{max} = 12.6$ mA. This gives $I = 0.626$ mA $= 626\ \mu$A.

29.36. **IDENTIFY:** Use $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ to calculate the induced electric field E at a distance r from the center of the solenoid. Away from the ends of the solenoid, $B = \mu_0 nI$ inside and $B = 0$ outside.

SET UP: The end view of the solenoid is sketched in Figure 29.36.



Let R be the radius of the solenoid.

Figure 29.36

Apply $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ to an integration path that is a circle of radius r, where $r < R$. We need to calculate just the magnitude of E so we can take absolute values.

EXECUTE: **(a)** $\oint |\vec{E} \cdot d\vec{l}| = E(2\pi r)$.

$\Phi_B = B\pi r^2$, $\left| -\frac{d\Phi_B}{dt} \right| = \pi r^2 \left| \frac{dB}{dt} \right|$.

$\oint |\mathrm{r}\vec{E} \cdot d\vec{l}| = \left| -\frac{d\Phi_B}{dt} \right|$ implies $E(2\pi r) = \pi r^2 \left| \frac{dB}{dt} \right|$.

$E = \frac{1}{2} r \left| \frac{dB}{dt} \right|$.

$B = \mu_0 nI$, so $\frac{dB}{dt} = \mu_0 n \frac{dI}{dt}$.

Thus $E = \frac{1}{2} r \mu_0 n \frac{dI}{dt} = \frac{1}{2}(0.00500\text{ m})(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(900\text{ m}^{-1})(36.0\text{ A/s}) = 1.02 \times 10^{-4}$ V/m.

(b) $r = 0.0100$ cm is still inside the solenoid so the expression in part (a) applies.

$E = \frac{1}{2} r \mu_0 n \frac{dI}{dt} = \frac{1}{2}(0.0100\text{ m})(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(900\text{ m}^{-1})(36.0\text{ A/s}) = 2.04 \times 10^{-4}$ V/m.

29.40. IDENTIFY and SET UP: Use $i_C = q/t$ to calculate the charge q that the current has carried to the plates in time t. The equations $V = Ed$ and $E = \dfrac{\sigma}{\epsilon_0}$ relate q to the electric field E and the potential difference between the plates. The displacement current density is $j_D = \mathcal{E}\dfrac{dE}{dt}$.

EXECUTE: (a) $i_C = 1.80 \times 10^{-3}$ A.

$q = 0$ at $t = 0$.

The amount of charge brought to the plates by the charging current in time t is

$q = i_C t = (1.80 \times 10^{-3}\ \text{A})(0.500 \times 10^{-6}\ \text{s}) = 9.00 \times 10^{-10}$ C.

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} = \frac{9.00 \times 10^{-10}\ \text{C}}{(8.854 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00 \times 10^{-4}\ \text{m}^2)} = 2.03 \times 10^5\ \text{V/m}.$$

$V = Ed = (2.03 \times 10^5\ \text{V/m})(2.00 \times 10^{-3}\text{m}) = 406$ V.

(b) $E = q/\epsilon_0 A$.

$$\frac{dE}{dt} = \frac{dq/dt}{\epsilon_0 A} = \frac{i_C}{\epsilon_0 A} = \frac{1.80 \times 10^{-3}\ \text{A}}{(8.854 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00 \times 10^{-4}\ \text{m}^2)} = 4.07 \times 10^{11}\ \text{V/m}\cdot\text{s}.$$

Since i_C is constant dE/dt does not vary in time.

(c) $j_D = \epsilon_0 \dfrac{dE}{dt}$ (with ϵ replaced by ϵ_0 since there is vacuum between the plates).

$j_D = (8.854 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.07 \times 10^{11}\ \text{V/m}\cdot\text{s}) = 3.60\ \text{A/m}^2$.

$i_D = j_D A = (3.60\ \text{A/m}^2)(5.00 \times 10^{-4}\ \text{m}^2) = 1.80 \times 10^{-3}$ A; $i_D = i_C$.

EVALUATE: $i_C = i_D$. The constant conduction current means the charge q on the plates and the electric field between them both increase linearly with time and i_D is constant.

29.44. IDENTIFY: The 4.00-cm long left side of the loop is a bar moving in a magnetic field, so an emf is induced across its ends. This emf causes current to flow through the loop, and the external magnetic field exerts a force on the bar due to the current in it. Ohm's law applies to the circuit and Newton's second law applies to the loop.

SET UP: The induced potential across the left-end side is $\mathcal{E} = vBL$, the magnetic force on the bar is $F_{mag} = ILB$, and Ohm's law is $\mathcal{E} = IR$. Newton's second law is $\Sigma\vec{F} = m\vec{a}$. The flux through the loop is decreasing, so the induced current is clockwise. Alternatively, the magnetic force on positive charge in the moving left-end bar is upward, by the right-hand rule, which also gives a clockwise current. Therefore the magnetic force on the 4.00-cm segment is to the left, opposite to $\vec{F}_{ext}$.

EXECUTE: (a) Combining the equations discussed in the set up, the magnetic force on the 4.00-cm bar (and on the loop) is

$F_{mag} = ILB = (\mathcal{E}/R)LB = (vBL/R)LB = v(BL)^2/R$.

Newton's second law gives

$F_{ext} - F_{mag} = ma$.

$ma = F_{ext} - v(BL)^2/R$.

$(0.0240\ \text{kg})a = 0.180\ \text{N} - (0.0300\ \text{m/s})[(2.90\ \text{T})(0.0400\ \text{m})]^2/(0.00500\ \Omega)$.

$a = 4.14\ \text{m/s}^2$.

(b) At terminal speed v_T, $F_{mag} = F_{ext}$.

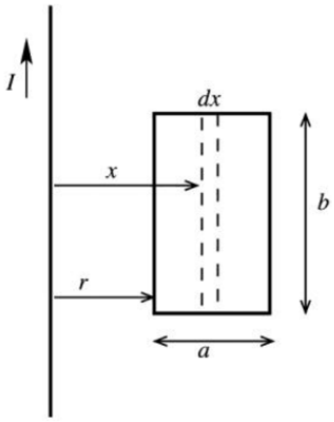
$v_T(BL)^2/R = F_{ext}$.

$v_T = RF_{ext}/(BL)^2 = (0.00500\ \Omega)(0.180\ \text{N})/[(2.90\ \text{T})(0.0400\ \text{m})]^2 = 0.0669\ \text{m/s} = 6.69$ cm/s. The speed is constant thereafter, so the acceleration is zero.

(c) $a = F_{ext}/m = (0.180\ \text{N})/(0.0240\ \text{kg}) = 7.50\ \text{m/s}^2$.

29.49. **(a) IDENTIFY:** (i) $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right|$. The flux is changing because the magnitude of the magnetic field of wire decreases with distance from the wire. Find the flux through a narrow strip of area and integrate over the loop to find the total flux.

SET UP:



Consider a narrow strip of width dx and a dist
x from the long wire, as shown in Figure 29.4
The magnetic field of the wire at the strip is $B = \mu_0 I/2\pi x$. The flux through the strip is $d\Phi_B = Bb\,dx = (\mu_0 Ib/2\pi)(dx/x)$.

Figure 29.49a

EXECUTE: The total flux through the loop is $\Phi_B = \int d\Phi_B = \left(\dfrac{\mu_0 Ib}{2\pi}\right)\displaystyle\int_r^{r+a}\dfrac{dx}{x}$.

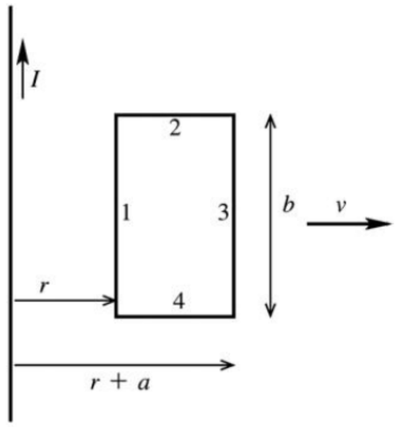
$$\Phi_B = \left(\frac{\mu_0 Ib}{2\pi}\right)\ln\left(\frac{r+a}{r}\right).$$

$$\frac{d\Phi_B}{dt} = \frac{d\Phi_B}{dr}\frac{dr}{dt} = \frac{\mu_0 Ib}{2\pi}\left(-\frac{a}{r(r+a)}\right)v.$$

$$|\mathcal{E}| = \frac{\mu_0 Iabv}{2\pi r(r+a)}.$$

(ii) **IDENTIFY:** $\mathcal{E} = Bvl$ for a bar of length l moving at speed v perpendicular to a magnetic field B. Calculate the induced emf in each side of the loop, and combine the emfs according to their polarity.

SET UP: The four segments of the loop are shown in Figure 29.49b.



EXECUTE: The emf in each side of the loop is

$$\mathcal{E}_1 = \left(\frac{\mu_0 I}{2\pi r}\right)vb,\ \ \mathcal{E}_3 = \left(\frac{\mu_0 I}{2\pi(r+a)}\right)vb,\ \ \mathcal{E}_2 = \mathcal{E}_4 = 0.$$

Figure 29.49b

Both emfs $\mathcal{E}_1$ and $\mathcal{E}_3$ are directed toward the top of the loop so oppose each other. The net emf is

$$\mathcal{E} = \mathcal{E}_1 - \mathcal{E}_3 = \frac{\mu_0 Ivb}{2\pi}\left(\frac{1}{r} - \frac{1}{r+a}\right) = \frac{\mu_0 Iabv}{2\pi r(r+a)}.$$

This expression agrees with what was obtained in (i) using Faraday's law.

(b) (i) **IDENTIFY** and **SET UP:** The flux of the induced current opposes the change in flux.

EXECUTE: $\vec{B}$ is $\otimes$. Φ_B is decreasing, so the flux Φ_{ind} of the induced current is $\otimes$ and the current is clockwise.

(ii) **IDENTIFY** and **SET UP:** Use the right-hand rule to find the force on the positive charges in each side of the loop. The forces on positive charges in segments 1 and 3 of the loop are shown in Figure 29.49c.

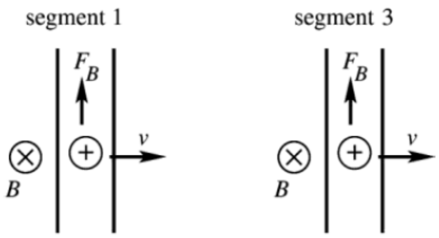


Figure 29.49c

EXECUTE: B is larger at segment 1 since it is closer to the long wire, so F_B is larger in segment 1 and the induced current in the loop is clockwise. This agrees with the direction deduced in (i) using Lenz's law.

(c) EVALUATE: When $v = 0$ the induced emf should be zero; the expression in part (a) gives this. When $a \to 0$ the flux goes to zero and the emf should approach zero; the expression in part (a) gives this. When $r \to \infty$ the magnetic field through the loop goes to zero and the emf should go to zero; the expression in part (a) gives this.

29.52. IDENTIFY: The movement of the rod causes an emf to be induced across its ends, which causes a current to flow through the circuit. The magnetic field exerts a force on this current.

SET UP: The magnetic force is $F_{\text{mag}} = ILB$, the induced emf is $\mathcal{E} = vBL$. $\sum F = ma$ applies to the rod, and $a = dv/dt$.

EXECUTE: The net force on the rod is $F - iLB = ma$. $i = \dfrac{vBL}{R}$. $F - \dfrac{vB^2L^2}{R} = ma$. $F - \dfrac{vB^2L^2}{R} = m\dfrac{dv}{dt}$.

Integrating to find the time gives $\dfrac{F}{m}\displaystyle\int_0^t dt' = \int_0^v \frac{dv'}{1 - \dfrac{v'B^2L^2}{FR}}$, which gives $\dfrac{Ft}{m} = -\dfrac{FR}{B^2L^2}\ln\left(1 - \dfrac{vB^2L^2}{FR}\right)$.

Solving for t and putting in the numbers gives

$$t = -\frac{Rm}{B^2L^2}\ln\left(1 - \frac{vB^2L^2}{FR}\right) = -(0.120\text{ kg})(888.9\text{ s/kg})\ln\left(1 - \frac{25.0\text{ m/s}}{(1.90\text{ N})(888.9\text{ s/kg})}\right) = 1.59\text{ s}.$$

29.53. IDENTIFY: Find the magnetic field at a distance r from the center of the wire. Divide the rectangle into narrow strips of width dr, find the flux through each strip and integrate to find the total flux.

SET UP: Example 28.8 uses Ampere's law to show that the magnetic field inside the wire, a distance r from the axis, is $B(r) = \mu_0 Ir/2\pi R^2$.

EXECUTE: Consider a small strip of length W and width dr that is a distance r from the axis of the wire, as shown in Figure 29.53. The flux through the strip is $d\Phi_B = B(r)W\,dr = \dfrac{\mu_0 IW}{2\pi R^2} r\,dr$. The total flux through the rectangle is $\Phi_B = \int d\Phi_B = \left(\dfrac{\mu_0 IW}{2\pi R^2}\right)\int_0^R r\,dr = \dfrac{\mu_0 IW}{4\pi}$.

EVALUATE: Note that the result is independent of the radius R of the wire.

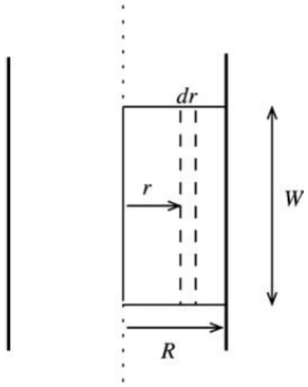


Figure 29.53

29.54. IDENTIFY: Apply Newton's second law to the bar. The bar will experience a magnetic force due to the induced current in the loop. Use $a = dv/dt$ to solve for v. At the terminal speed, $a = 0$.

SET UP: The induced emf in the loop has a magnitude BLv. The induced emf is counterclockwise, so it opposes the voltage of the battery, $\mathcal{E}$.

EXECUTE: **(a)** The net current in the loop is $I = \dfrac{\mathcal{E} - BLv}{R}$. The acceleration of the bar is $a = \dfrac{F}{m} = \dfrac{ILB\sin(90°)}{m} = \dfrac{(\mathcal{E} - BLv)LB}{mR}$. To find $v(t)$, set $\dfrac{dv}{dt} = a = \dfrac{(\mathcal{E} - BLv)LB}{mR}$ and solve for v using the method of separation of variables:

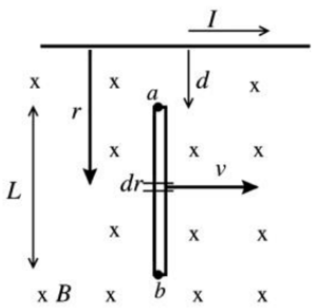
$\int_0^v \dfrac{dv}{(\mathcal{E} - BLv)} = \int_0^t \dfrac{LB}{mR} dt \rightarrow v = \dfrac{\mathcal{E}}{BL}(1 - e^{-B^2L^2t/mR}) = (14\text{ m/s})(1 - e^{-t/6.0\text{ s}})$. The graph of v versus t is sketched in Figure 29.54. Note that the graph of this function is similar in appearance to that of a charging capacitor.

(b) Just after the switch is closed, $v = 0$ and $I = \mathcal{E}/R = 2.4$ A, $F = ILB = 2.074$ N, and $a = F/m = 2.3\text{ m/s}^2$.

(c) When $v = 2.0$ m/s, $a = \dfrac{[12\text{ V} - (2.4\text{ T})(0.36\text{ m})(2.0\text{ m/s})](0.36\text{ m})(2.4\text{ T})}{(0.90\text{ kg})(5.0\ \Omega)} = 2.0\text{ m/s}^2$.

(d) Note that as the speed increases, the acceleration decreases. The speed will asymptotically approach the terminal speed $\dfrac{\mathcal{E}}{BL} = \dfrac{12\text{ V}}{(2.4\text{ T})(0.36\text{ m})} = 14\text{ m/s}$, which makes the acceleration zero.

29.55. **(a)** and **(b)** **IDENTIFY** and **SET UP:**



The magnetic field of the wire is given by $B = \dfrac{\mu_0 I}{2\pi r}$ and varies along the length of the bar. At every point along the bar $\vec{B}$ has direction into the page. Divide the bar up into thin slices, as shown in Figure 29.55a.

Figure 29.55a

EXECUTE: The emf $d\mathcal{E}$ induced in each slice is given by $d\mathcal{E} = \vec{v} \times \vec{B} \cdot d\vec{l}$. $\vec{v} \times \vec{B}$ is directed toward the wire, so $d\mathcal{E} = -vB\,dr = -v\left(\dfrac{\mu_0 I}{2\pi r}\right)dr$. The total emf induced in the bar is

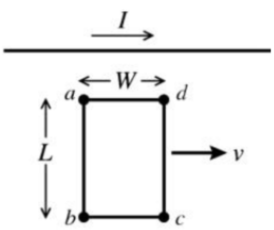
$$V_{ba} = \int_a^b d\mathcal{E} = -\int_d^{d+L}\left(\frac{\mu_0 I v}{2\pi r}\right)dr = -\frac{\mu_0 I v}{2\pi}\int_d^{d+L}\frac{dr}{r} = -\frac{\mu_0 I v}{2\pi}\left[\ln(r)\right]_d^{d+L}.$$

$$V_{ba} = -\frac{\mu_0 I v}{2\pi}(\ln(d+L) - \ln(d)) = -\frac{\mu_0 I v}{2\pi}\ln(1+L/d).$$

EVALUATE: The minus sign means that V_{ba} is negative, point a is at higher potential than point b. (The force $\vec{F} = q\vec{v} \times \vec{B}$ on positive charge carriers in the bar is towards a, so a is at higher potential.) The potential difference increases when I or v increase, or d decreases.

(c) **IDENTIFY:** Use Faraday's law to calculate the induced emf.

SET UP: The wire and loop are sketched in Figure 29.55b.

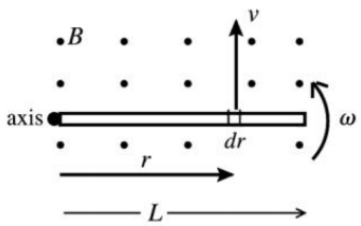


EXECUTE: As the loop moves to the right the magnetic flux through it doesn't change. Thus $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = 0$ and $I = 0$.

Figure 29.55b

29.57. **IDENTIFY:** Use the expression for motional emf to calculate the emf induced in the rod.

SET UP: **(a)** The rotating rod is shown in Figure 29.57a.



The emf induced in a thin slice is $d\mathcal{E} = \vec{v} \times \vec{B} \cdot d\vec{l}$.

Figure 29.57a

EXECUTE: Assume that $\vec{B}$ is directed out of the page. Then $\vec{v} \times \vec{B}$ is directed radially outward and $dl = dr$, so $\vec{v} \times \vec{B} \cdot d\vec{l} = vB\,dr$.

$v = r\omega$ so $d\mathcal{E} = \omega Br\,dr$.

The $d\mathcal{E}$ for all the thin slices that make up the rod are in series so they add:

$$\mathcal{E} = \int d\mathcal{E} = \int_0^L \omega Br\,dr = \tfrac{1}{2}\omega BL^2 = \tfrac{1}{2}(8.80\ \text{rad/s})(0.650\ \text{T})(0.240\ \text{m})^2 = 0.165\ \text{V}.$$

EVALUATE: $\mathcal{E}$ increases with ω, B, or L^2.

(b) SET UP and EXECUTE: No current flows so there is no IR drop in potential. Thus the potential difference between the ends equals the emf of 0.165 V calculated in part (a).

(c) SET UP: The rotating rod is shown in Figure 29.57b.

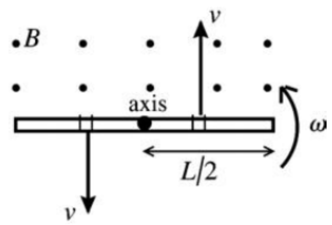


Figure 29.57b

EXECUTE: The emf between the center of the rod and each end is $\mathcal{E} = \tfrac{1}{2}\omega B(L/2)^2 = \tfrac{1}{4}(0.165\ \text{V}) = 0.0412\ \text{V}$, with the direction of the emf from the center of the rod toward each end. The emfs in each half of the rod thus oppose each other and there is no net emf between the ends of the rod.

EVALUATE: ω and B are the same as in part (a) but L of each half is $\tfrac{1}{2}L$ for the whole rod. $\mathcal{E}$ is proportional to L^2, so is smaller by a factor of $\tfrac{1}{4}$.