

# HW11 CH32 Solutions

**32.34. IDENTIFY:** We are dealing with electromagnetic waves from a moving magnet. In one cycle, the magnet starts right at the coil, then moves 10 cm away, and then moves back to where it started.

**SET UP and EXECUTE:** (a) One cycle lasts  $\frac{1}{2}$  second. In  $\frac{1}{2}$  cycle,  $\Delta B = B$  and  $\Delta T = \frac{1}{2} \left( \frac{1}{2} \text{ s} \right) = \frac{1}{4} \text{ s}$ .

Using these results and the given numbers gives  $\frac{|\Delta \Phi_B|}{\Delta t} = \frac{A \Delta B}{\Delta t} = \frac{AB}{\Delta t} = \frac{\pi r^2 B}{\Delta t} = 50 \mu\text{Wb/s}$ .

(b) We want the average magnitude of  $E$  within the loop. Using  $\oint E dl = \frac{d\Phi_B}{dt}$  gives  $E 2\pi r = \frac{|\Delta \Phi_B|}{\Delta t}$ .

Solving for  $E$  and using the result from part (a) with  $r = 2.0 \text{ cm}$  gives  $E = \frac{1}{2\pi r} \frac{|\Delta \Phi_B|}{\Delta t} = 400 \mu\text{V/m}$ .

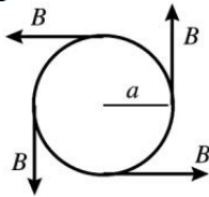
(c) We want the intensity. Eq. (32.28):  $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ . Using  $B = E/c$  this becomes  $S = \frac{E^2}{\mu_0 c}$ . Using  $E = 400 \mu\text{V/m}$ , this gives  $S = 0.42 \text{ nW/m}^2$ .

(d) We want the total power.  $P = SA = S(2\pi r l) = (0.42 \text{ nW/m}^2)(2\pi)(0.0200 \text{ m})(0.10 \text{ m}) = 5.3 \text{ pW}$ .

**32.39. IDENTIFY and SET UP:** In the wire the electric field is related to the current density by  $\vec{E} = \rho \vec{J}$ . Use Ampere's law to calculate  $\vec{B}$ . The Poynting vector is given by  $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$  and  $\vec{P} = \vec{r} \vec{S} \cdot d\vec{A}$  relates the energy flow through a surface to  $\vec{S}$ .

**EXECUTE:** (a) The direction of  $\vec{E}$  is parallel to the axis of the cylinder, in the direction of the current.  $E = \rho J = \rho I / \pi a^2$ . ( $E$  is uniform across the cross section of the conductor.)

(b) A cross-sectional view of the conductor is given in Figure 32.39a; take the current to be coming out of the page.



Apply Ampere's law to a circle of radius  $a$ .

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi a)$$

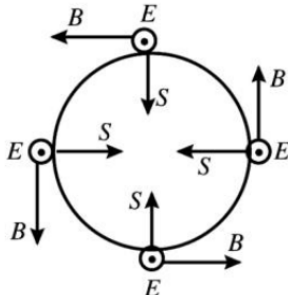
$$I_{\text{encl}} = I.$$

Figure 32.39a

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \text{ gives } B(2\pi a) = \mu_0 I \text{ and } B = \frac{\mu_0 I}{2\pi a}.$$

The direction of  $\vec{B}$  is counterclockwise around the circle.

(c) The directions of  $\vec{E}$  and  $\vec{B}$  are shown in Figure 32.39b.



$$\text{The direction of } \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}.$$

is radially inward.

$$S = \frac{1}{\mu_0} EB = \frac{1}{\mu_0} \left( \frac{\rho I}{\pi a^2} \right) \left( \frac{\mu_0 I}{2\pi a} \right).$$

$$S = \frac{\rho I^2}{2\pi^2 a^3}.$$

Figure 32.39b

**EVALUATE:** (d) Since  $S$  is constant over the surface of the conductor, the rate of energy flow  $P$  is given by  $S$  times the surface of a length  $l$  of the conductor:  $P = SA = S(2\pi al) = \frac{\rho I^2}{2\pi^2 a^3}(2\pi al) = \frac{\rho I^2}{\pi a^2}$ . But  $R = \frac{\rho l}{\pi a^2}$ , so the result from the Poynting vector is  $P = RI^2$ . This agrees with  $P_R = I^2 R$ , the rate at which electrical energy is being dissipated by the resistance of the wire. Since  $\vec{S}$  is radially inward at

the surface of the wire and has magnitude equal to the rate at which electrical energy is being dissipated in the wire, this energy can be thought of as entering through the cylindrical sides of the conductor.

**32.40. IDENTIFY:** The changing magnetic field of the electromagnetic wave produces a changing flux through the wire loop, which induces an emf in the loop. The wavelength of the wave is much greater than the diameter of the loop, so we can treat the magnetic field as being uniform over the area of the loop.

**SET UP:**  $\Phi_B = B\pi r^2 = \pi r^2 B_{\max} \cos(kx - \omega t)$ , taking  $x$  for the direction of propagation of the wave.

Faraday's law says  $|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right|$ . The intensity of the wave is  $I = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{c}{2\mu_0} B_{\max}^2$ , and  $f = \frac{c}{\lambda}$ .

**EXECUTE:**  $|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| = \omega B_{\max} \sin(kx - \omega t) \pi r^2$ .  $|\mathcal{E}|_{\max} = 2\pi f B_{\max} \pi r^2$ .

$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{6.90 \text{ m}} = 4.348 \times 10^7 \text{ Hz}$ . Solving  $I = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{c}{2\mu_0} B_{\max}^2$  for  $B_{\max}$  gives

$$B_{\max} = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.0187 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}}} = 1.252 \times 10^{-8} \text{ T}.$$

$$|\mathcal{E}|_{\max} = 2\pi(4.6888 \times 10^7 \text{ Hz})(1.252 \times 10^{-8} \text{ T})\pi(0.076 \text{ m})^2 = 6.69 \times 10^{-2} \text{ V} = 66.9 \text{ mV}.$$

**32.42. IDENTIFY:** This problem involves an  $L$ - $R$ - $C$  ac circuit, electromagnetic waves, and Faraday's law.

**SET UP:**  $\mathcal{E} = -N \frac{d\Phi_B}{dt}$ ,  $B = E/c$ ,  $k = \omega/c$ .  $B_x$  must have the same mathematical form as  $E_y$ .

**EXECUTE:** (a) We want the flux.  $dA = adz$ .  $\Phi_B = \int_{-a/2}^{a/2} B_x adz = aB_{\max} \int_{-a/2}^{a/2} \cos(kx - \omega t) dz$ . This gives

$$\Phi_B = \frac{aB_{\max}}{k} [\sin(ka/2 - \omega t) + \sin(ka/2 + \omega t)] = \frac{aE_{\max}}{\omega} [2\sin(ka/2)\cos\omega t].$$

(b) We want the magnitude of the emf. Using the result from part (a) we get  $\mathcal{E} = N \frac{d\Phi_B}{dt} =$

$$\frac{d}{dt} \left( \frac{aE_{\max}}{\omega} [2\sin(ka/2)\cos\omega t] \right) = 2NaE_{\max} \sin(ka/2) \sin\omega t.$$

(c) We want  $C$ . At resonance,  $2\pi f_0 = \omega_0 = 1/\sqrt{LC}$ . Solve for  $C$  and put in the given numbers using  $f_0 =$

$$4.00 \text{ MHz and } L = 78.0 \mu\text{H}. \text{ The result is } C = \frac{1}{L(2\pi f_0)^2} = 20.3 \text{ pF}.$$

(d) We want the rms current. The circuit is at resonance, so  $i = V/Z = V/R$ . (Note that we are using  $i$  instead of  $I$  for the current amplitude so as not to confuse it with the intensity  $I$ .) Using the result from part (b) gives  $V = \mathcal{E}_{\max} = 2NaE_{\max}$ . Now find  $E_{\max}$  using the intensity  $I = \frac{1}{2} \epsilon_0 c E_{\max}^2$ . This gives

$$E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}}, \text{ so } V = 2NaE_{\max} = 2Na\sqrt{\frac{2I}{\epsilon_0 c}}. \text{ Thus } i = \frac{V}{R} = \frac{2Na}{R}\sqrt{\frac{2I}{\epsilon_0 c}}. \text{ Finally } i_{\text{rms}} = i/\sqrt{2}, \text{ so we get}$$

$$i_{\text{rms}} = \frac{2Na}{R\sqrt{2}}\sqrt{\frac{2I}{\epsilon_0 c}} = \frac{2Na}{R}\sqrt{\frac{I}{\epsilon_0 c}}. \text{ Using the numbers in the problem gives } i_{\text{rms}} = 19.4 \text{ A.}$$