**23.58.** **IDENTIFY:** We are modeling electrical power lines as conducting cylinders.

**SET UP and EXECUTE:** **(a)** Estimate: About 22 ft ≈ 7.0 m.

**(b)** We want the linear charge density $\lambda$. $\Delta V = \int E_r dr = \int_{0.02\text{ m}}^{7\text{ m}} \frac{\lambda}{2\pi \epsilon_0 r} dr = \frac{\lambda}{2\pi \epsilon_0} \ln(7/0.02)$. Solving for $\lambda$ using $V$ = 22 kV gives $\lambda$ ≈ 210 nC/m which rounds to 200 nC/m.

**(c)** We want $E$. $E = \frac{\lambda}{2\pi \epsilon_0 r}$. Using $\lambda$ =210 nC/m and $r$ = 7 m gives $E$ ≈ 540 V/m.

**23.61.** **(a) IDENTIFY:** The potential at any point is the sum of the potentials due to each of the two charged conductors.

**SET UP:** For a conducting cylinder with charge per unit length $\lambda$ the potential outside the cylinder is given by $V = (\lambda/2\pi \epsilon_0)\ln(r_0/r)$ where $r$ is the distance from the cylinder axis and $r_0$ is the distance from the axis for which we take $V = 0$. Inside the cylinder the potential has the same value as on the cylinder surface. The electric field is the same for a solid conducting cylinder or for a hollow conducting tube so this expression for $V$ applies to both. This problem says to take $r_0 = b$.

**EXECUTE:** For the hollow tube of radius $b$ and charge per unit length $-\lambda$: outside $V = -(\lambda/2\pi \epsilon_0)\ln(b/r)$; inside $V = 0$ since $V = 0$ at $r = b$.

For the metal cylinder of radius $a$ and charge per unit length $\lambda$:

outside $V = (\lambda/2\pi \epsilon_0)\ln(b/r)$, inside $V = (\lambda/2\pi \epsilon_0)\ln(b/a)$, the value at $r = a$.

(i) $r < a$; inside both $V = (\lambda/2\pi \epsilon_0)\ln(b/a)$.

(ii) $a < r < b$; outside cylinder, inside tube $V = (\lambda/2\pi \epsilon_0)\ln(b/r)$.

(iii) $r > b$; outside both the potentials are equal in magnitude and opposite in sign so $V = 0$.

**(b)** For $r = a$, $V_a = (\lambda/2\pi \epsilon_0)\ln(b/a)$.

For $r = b$, $V_b = 0$.

Thus $V_{ab} = V_a - V_b = (\lambda/2\pi \epsilon_0)\ln(b/a)$.

**(c) IDENTIFY and SET UP:** Use $E_r = -\frac{\partial V}{\partial r}$ to calculate $E$.

**EXECUTE:** $E = -\frac{\partial V}{\partial r} = -\frac{\lambda}{2\pi \epsilon_0}\frac{\partial}{\partial r}\ln\left(\frac{b}{r}\right) = -\frac{\lambda}{2\pi \epsilon_0}\left(\frac{r}{b}\right)\left(-\frac{b}{r^2}\right) = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r}$.

**(d)** The electric field between the cylinders is due only to the inner cylinder, so $V_{ab}$ is not changed, $V_{ab} = (\lambda/2\pi \epsilon_0)\ln(b/a)$.

**EVALUATE:** The electric field is not uniform between the cylinders, so $V_{ab} \neq E(b-a)$.

**23.62.** **IDENTIFY:** The wire and hollow cylinder form coaxial cylinders. Problem 23.61 gives $E(r) = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r}$.

**SET UP:** $a = 145 \times 10^{-6}$ m, $b = 0.0180$ m.

**EXECUTE:** $E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r}$ and

$$V_{ab} = E\ln(b/a)r = (2.00 \times 10^4 \text{ N/C})(\ln(0.018\text{ m}/145 \times 10^{-6}\text{ m}))0.012\text{ m} = 1157\text{ V}.$$

**EVALUATE:** The electric field at any $r$ is directly proportional to the potential difference between the wire and the cylinder.

**23.66.** **(a) IDENTIFY:** Calculate the potential due to each thin ring and integrate over the disk to find the potential. $V$ is a scalar so no components are involved.

**SET UP:** Consider a thin ring of radius $y$ and width $dy$. The ring has area $2\pi y\,dy$ so the charge on the ring is $dq = \sigma(2\pi y\,dy)$.

**EXECUTE:** The result of Example 23.11 then says that the potential due to this thin ring at the point on the axis at a distance $x$ from the ring is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{x^2+y^2}} = \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{y\,dy}{\sqrt{x^2+y^2}}.$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{y\,dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+y^2}\right]_0^R = \frac{\sigma}{2\epsilon_0}(\sqrt{x^2+R^2}-x).$$

**EVALUATE:** For $x \gg R$ this result should reduce to the potential of a point charge with $Q = \sigma\pi R^2$.

$$\sqrt{x^2+R^2} = x(1+R^2/x^2)^{1/2} \approx x(1+R^2/2x^2) \text{ so } \sqrt{x^2+R^2}-x \approx R^2/2x.$$

Then $V \approx \dfrac{\sigma}{2\epsilon_0}\dfrac{R^2}{2x} = \dfrac{\sigma\pi R^2}{4\pi\epsilon_0 x} = \dfrac{Q}{4\pi\epsilon_0 x}$, as expected.

**(b) IDENTIFY and SET UP:** Use $E_x = -\dfrac{\partial V}{\partial x}$ to calculate $E_x$.

**EXECUTE:** $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\sigma}{2\epsilon_0}\left(\dfrac{x}{\sqrt{x^2+R^2}}-1\right) = \dfrac{\sigma x}{2\epsilon_0}\left(\dfrac{1}{x}-\dfrac{1}{\sqrt{x^2+R^2}}\right).$

**23.67.** **IDENTIFY:** We must integrate to find the total energy because the energy to bring in more charge depends on the charge already present.

**SET UP:** If $\rho$ is the uniform volume charge density, the charge of a spherical shell of radius $r$ and thickness $dr$ is $dq = \rho 4\pi r^2\,dr$, and $\rho = Q/(4/3\ \pi R^3)$. The charge already present in a sphere of radius $r$ is $q = \rho(4/3\ \pi r^3)$. The energy to bring the charge $dq$ to the surface of the charge $q$ is $Vdq$, where $V$ is the potential due to $q$, which is $q/4\pi\epsilon_0 r$.

**EXECUTE:** The total energy to assemble the entire sphere of radius $R$ and charge $Q$ is sum (integral) of the tiny increments of energy.

$$U = \int Vdq = \int \frac{q}{4\pi\epsilon_0 r}dq = \int_0^R \frac{\rho\frac{4}{3}\pi r^3}{4\pi\epsilon_0 r}(\rho 4\pi r^2 dr) = \frac{3}{5}\left(\frac{1}{4\pi\epsilon_0}\frac{Q^2}{R}\right)$$

where we have substituted $\rho = Q/(4/3\ \pi R^3)$ and simplified the result.

**23.68.** **IDENTIFY:** Divide the rod into infinitesimal segments with charge $dq$. The potential $dV$ due to the segment is $dV = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{r}$. Integrate over the rod to find the total potential.

**SET UP:** $dq = \lambda dl$, with $\lambda = Q/\pi a$ and $dl = a\,d\theta$.

**EXECUTE:** $dV = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{r} = \dfrac{1}{4\pi\epsilon_0}\dfrac{\lambda\,dl}{a} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{\pi a}\dfrac{dl}{a} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q\,d\theta}{\pi a}.$

$$V = \frac{1}{4\pi\epsilon_0}\int_0^\pi \frac{Q\,d\theta}{\pi a} = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}.$$

**23.72. IDENTIFY:** We are dealing with a charged bar in an external electric field.

**SET UP and EXECUTE:** **(a)** We want $V$ as a function of $y$. $E$ is constant so $V_y - V_0 = -E_y y,$ which gives $V(y) = V_0 - Ey.$

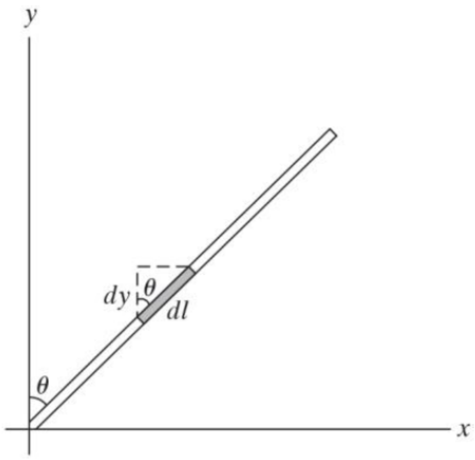


**Figure 23.72**

**(b)** We want the potential energy. Refer to Fig. 23.72. $U = \int V dq.$ $dq = \lambda dl$ with $\lambda = Q/L,$ and

$dl \cos\theta = dy.$ $U = \int_0^{L\cos\theta} (V_0 - Ey)(Q/L)\dfrac{dy}{\cos\theta} = \left.\dfrac{\dfrac{Q}{L}\left(V_0 y - \dfrac{Ey^2}{2}\right)}{\cos\theta}\right|_0^{L\cos\theta} = Q\left(V_0 - \dfrac{EL\cos\theta}{2}\right).$

**(c)** We want $V_0$ so $U = 0$ when $\theta = 0°$. $Q\left(V_0 - \dfrac{EL\cos\theta}{2}\right) = 0$ gives $V_0 = EL/2$.

**(d)** We want the angular speed $\omega$ at $\theta = 0°$ if the bar is released from rest at $\theta = 90°$. Energy conservation gives $U_0 + K_0 = U_{90} + K_{90}$. We can neglect gravity, so $\dfrac{1}{2}I\omega^2 = QV_0$. Using the result from (c) and $I = 1/3\ ML^2$ gives $\omega = \sqrt{\dfrac{3QE}{ML}}.$

**(e)** We want the frequency $f$ of oscillation. Apply $\sum \tau_z = I\alpha_z$ . $-QE\dfrac{L}{2}\sin\theta = \dfrac{1}{3}ML^2\dfrac{d^2\theta}{dt^2}.$ This gives $\dfrac{d^2\theta}{dt^2} = -\left(\dfrac{3QE}{2ML}\right)\sin\theta \approx -\left(\dfrac{3QE}{2ML}\right)\theta$ for small amplitude oscillations. $\omega = \sqrt{\dfrac{3QE}{2ML}}$ , so $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{3QE}{2ML}}.$

**23.81. IDENTIFY:** We are dealing with a heart cell that is modeled as a cylindrical shell of charge.

**SET UP:** $E = \dfrac{\lambda}{2\pi \epsilon_0 r}$.

**EXECUTE:** **(a)** We want the charge $Q$. Using $E = \dfrac{\lambda}{2\pi \epsilon_0 r}$ with $\lambda = Q/L$, we have

$\Delta V = \int E_r dr = \int \dfrac{\lambda}{2\pi \epsilon_0} dr = \int_a^b \dfrac{Q/L}{2\pi \epsilon_0 r} dr = \dfrac{Q/L}{2\pi \epsilon_0} \ln(b/a)$. Solving for $Q$ and using $\Delta V = 90.0$ mV, $a$ = 9.0 $\mu$m, $b$ = 10.0 $\mu$m, and $L$ = 100 $\mu$m gives $Q = 4.75 \times 10^{-15}$ C.

**(b)** We want the electric field just inside the membrane at $r = a = 9.0$ $\mu$m. Using $E = \dfrac{Q/L}{2\pi \epsilon_0 a}$ gives $E$ = 94.9 kV/m.

**(c)** We want to find out how much charged moved across the cell wall. The potential went from $-90.0$ mV to $+20.0$ mV, so $\Delta V = 110$ mV. Using information from (a) gives $Q = \dfrac{(2\pi \epsilon_0)(\Delta V)L}{\ln(b/a)}$.

With $\Delta V = 110$ mV and the given numbers, we get $Q = 5.81 \times 10^{-15}$ C.

**(d)** We want to know how many $Na^+$ ions crossed the boundary. $N = \dfrac{5.81 \times 10^{-15} \text{ C}}{1.60 \times 10^{-19} \text{ C/ion}} = 36,300.$