CH26 Solutions

26.41. **IDENTIFY:** The capacitors, which are in parallel, will discharge exponentially through the resistors.

SET UP: Since V is proportional to Q, V must obey the same exponential equation as Q, $V = V_0 e^{-t/RC}$. The current is $I = (V_0/R)\, e^{-t/RC}$.

EXECUTE: **(a)** Solve for time when the potential across each capacitor is 10.0 V:

$t = -RC \ln(V/V_0) = -(80.0\ \Omega)(35.0\ \mu\text{F}) \ln(15/50) = 3370\ \mu\text{s} = 3.37\ \text{ms}.$

(b) $I = (V_0/R)\, e^{-t/RC}$. Using the above values, with $V_0 = 50.0$ V, gives $I = 0.188$ A.

26.47. **IDENTIFY:** In both cases, simplify the complicated circuit by eliminating the appropriate circuit elements. The potential across an uncharged capacitor is initially zero, so it behaves like a short circuit. A fully charged capacitor allows no current to flow through it.

(a) SET UP: Just after closing the switch, the uncharged capacitors all behave like short circuits, so any resistors in parallel with them are eliminated from the circuit.

EXECUTE: The equivalent circuit consists of $50\ \Omega$ and $25\ \Omega$ in parallel, with this combination in series with $75\ \Omega$, $15\ \Omega$, and the 100-V battery. The equivalent resistance is $90\ \Omega + 16.7\ \Omega = 106.7\ \Omega$, which gives $I = (100\ \text{V})/(106.7\ \Omega) = 0.937$ A.

(b) SET UP: Long after closing the switch, the capacitors are essentially charged up and behave like open circuits since no charge can flow through them. They effectively eliminate any resistors in series with them since no current can flow through these resistors.

EXECUTE: The equivalent circuit consists of resistances of $75\ \Omega$, $15\ \Omega$, and three 25-Ω resistors, all in series with the 100-V battery, for a total resistance of $165\ \Omega$. Therefore $I = (100\text{V})/(165\ \Omega) = 0.606$ A.

26.48. **IDENTIFY:** Both the charge and energy decay exponentially, but not with the same time constant since the energy is proportional to the *square* of the charge.

SET UP: The charge obeys the equation $Q = Q_0 e^{-t/RC}$ but the energy obeys the equation $U = Q^2/2C = (Q_0 e^{-t/RC})^2/2C = U_0 e^{-2t/RC}$.

EXECUTE: **(a)** The charge is reduced by half: $Q_0/2 = Q_0 e^{-t/RC}$. This gives $t = RC \ln 2 = (225\ \Omega)(12.0\ \mu\text{F})(\ln 2) = 1.871\ \text{ms}$, which rounds to 1.87 ms.

(b) The energy is reduced by half: $U_0/2 = U_0 e^{-2t/RC}$. This gives $t = (RC \ln 2)/2 = (1.871\ \text{ms})/2 = 0.936\ \text{ms}$.

26.49. **IDENTIFY:** When the capacitor is fully charged the voltage V across the capacitor equals the battery emf and $Q = CV$. For a charging capacitor, $q = Q(1 - e^{-t/RC})$.

SET UP: $\ln e^x = x$.

EXECUTE: **(a)** $Q = CV = (5.90\times10^{-6}\ \text{F})(28.0\ \text{V}) = 1.65\times10^{-4}\ \text{C} = 165\ \mu\text{C}$.

(b) $q = Q(1 - e^{-t/RC})$, so $e^{-t/RC} = 1 - \dfrac{q}{Q}$ and $R = \dfrac{-t}{C\ln(1 - q/Q)}$. After

$$t = 3\times10^{-3}\ \text{s}: R = \frac{-3\times10^{-3}\ \text{s}}{(5.90\times10^{-6}\ \text{F})(\ln(1-110/165))} = 463\ \Omega.$$

(c) If the charge is to be 99% of final value: $\dfrac{q}{Q} = (1 - e^{-t/RC})$ gives

$t = -RC\ln(1 - q/Q) = -(463\ \Omega)(5.90\times10^{-6}\ \text{F})\ln(0.01) = 0.0126\ \text{s} = 12.6\ \text{ms}.$

26.70. IDENTIFY and SET UP: Just after the switch is closed the charge on the capacitor is zero, the voltage across the capacitor is zero and the capacitor can be replaced by a wire in analyzing the circuit. After a long time the current to the capacitor is zero, so the current through R_3 is zero. After a long time the capacitor can be replaced by a break in the circuit.

EXECUTE: **(a)** Ignoring the capacitor for the moment, the equivalent resistance of the two parallel resistors is $\frac{1}{R_{\text{eq}}} = \frac{1}{6.00\ \Omega} + \frac{1}{3.00\ \Omega} = \frac{3}{6.00\ \Omega}$; $R_{\text{eq}} = 2.00\ \Omega$. In the absence of the capacitor, the total current in the circuit (the current through the $8.00\text{-}\Omega$ resistor) would be $i = \frac{\mathcal{E}}{R} = \frac{42.0\text{ V}}{8.00\ \Omega + 2.00\ \Omega} = 4.20\text{ A}$, of which 2/3, or 2.80 A, would go through the $3.00\text{-}\Omega$ resistor and 1/3, or 1.40 A, would go through the 6.00-Ω resistor. Since the current through the capacitor is given by $i = \frac{V}{R}e^{-t/RC}$, at the instant $t = 0$ the circuit behaves as through the capacitor were not present, so the currents through the various resistors are as calculated above.

(b) Once the capacitor is fully charged, no current flows through that part of the circuit. The 8.00-Ω and the 6.00-Ω resistors are now in series, and the current through them is $i = \mathcal{E}/R = (42.0\text{ V})/(8.00\ \Omega +$

$6.00\ \Omega) = 3.00\text{ A}$. The voltage drop across both the $6.00\text{-}\Omega$ resistor and the capacitor is thus $V = iR = (3.00\text{ A})(6.00\ \Omega) = 18.0\text{ V}$. (There is no current through the $3.00\text{-}\Omega$ resistor and so no voltage drop across it.) The charge on the capacitor is $Q = CV = (4.00 \times 10^{-6}\text{ F})(18.0\text{ V}) = 7.2 \times 10^{-5}\text{ C}$.

26.71. IDENTIFY: We have a capacitor that contains a dielectric and is in a series circuit with a resistor and a battery.

SET UP and EXECUTE: **(a)** We want the charge. $Q_0 = CV_0$. $C = \frac{\epsilon_0 A}{d}$ = 1.18 pF using the given A and d. Q_0 = (1.18 pF)(10.0 V) = 11.8 pC.

(b) The target variable is the current. $V_C = Ed = (E_0/K)d = V_0/K$. For the complete circuit $\mathcal{E} = RI + V_C = RI + V_0/K = RI + \mathcal{E}/K$. $I = \frac{\mathcal{E}}{R}\left(1 - \frac{1}{K}\right) = \frac{10.0\text{ V}}{10.0\ \Omega}\left(1 - \frac{1}{12.0}\right) = 0.917\text{ A}$.

(c) We want the initial energy in the capacitor. $U_C = \frac{1}{2}CV_C^2 = \frac{1}{2}(KC_0)\left(\frac{V_0}{K}\right)^2 = \frac{U_0}{K} = 4.92\text{ pJ}$. (This result also tells us that the stored energy before the dielectric was inserted was U_0 = (12.0)(4.92 pJ) = 59.0 pJ.)

(d) We want the final energy in the capacitor. $U_f = \frac{1}{2}CV_f^2 = \frac{1}{2}(KC_0)\mathcal{E}^2 = KU_0$.

$\Delta U = KU_0 - U_0 = (12.0)(59.0\text{ pJ}) - 4.92\text{ pJ} = 703\text{ pJ}$.

(e) We want the total energy supplied by the battery. $U_{\mathcal{E}} = \int P_{\mathcal{E}}dt = \int i\mathcal{E}dt$. $i = I_0 e^{-t/RC} = I_0 e^{-t/RKC_0}$.

Therefore $U_{\mathcal{E}} = \int_0^\infty I_0 e^{-t/RKC_0}\mathcal{E}dt = I_0 KRC_0\mathcal{E} = (0.917\text{ A})(12.0)(10.0\ \Omega)(1.18\text{ pF})(10.0\text{ V})$ = 1298 pJ, which rounds to 1300 pJ.

(f) We want the energy dissipated in the resistor. $U_R = \int P_R dt = \int i^2 R dt \int_0^\infty \left(I_0 e^{-t/RKC_0}\right)^2 dt = \frac{I_0^2 R^2 C_0 K}{2}$ = 595 pJ using the given numbers.

26.75. IDENTIFY: With S open and after equilibrium has been reached, no current flows and the voltage across each capacitor is 18.0 V. When S is closed, current I flows through the $6.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors.

SET UP: With the switch closed, a and b are at the same potential and the voltage across the $6.00\text{-}\Omega$ resistor equals the voltage across the $6.00\text{-}\mu\text{F}$ capacitor and the voltage is the same across the $3.00\text{-}\mu\text{F}$ capacitor and $3.00\text{-}\Omega$ resistor.

EXECUTE: **(a)** With an open switch: $V_{ab} = \varepsilon = 18.0\text{ V}$.

(b) Point a is at a higher potential since it is directly connected to the positive terminal of the battery.

(c) When the switch is closed $18.0\text{ V} = I(6.00\,\Omega + 3.00\,\Omega)$. $I = 2.00\text{ A}$ and

$V_b = (2.00\text{ A})(3.00\,\Omega) = 6.00\text{ V}$.

(d) Initially the capacitor's charges were $Q_3 = CV = (3.00\times10^{-6}\text{ F})(18.0\text{ V}) = 5.40\times10^{-5}\text{ C}$ and

$Q_6 = CV = (6.00\times10^{-6}\text{ F})(18.0\text{ V}) = 1.08\times10^{-4}\text{ C}$. After the switch is closed

$Q_3 = CV = (3.00\times10^{-6}\text{ F})(18.0\text{ V} - 12.0\text{ V}) = 1.80\times10^{-5}\text{ C}$ and

$Q_6 = CV = (6.00\times10^{-6}\text{ F})(18.0\text{ V} - 6.0\text{ V}) = 7.20\times10^{-5}\text{ C}$. Both capacitors lose $3.60\times10^{-5}\text{ C}$ $= 36.0\,\mu\text{C}$.

26.82. IDENTIFY: This problem involves a capacitor in an R-C circuit. We need to use Kirchhoff's rules.

SET UP: Refer to Fig. 26.82 with the problem in the textbook.

EXECUTE: **(a)** We want V_{out}. After a long time, the capacitor is fully charged, so $V_{\text{out}} = V_R = IR$. $I = \mathcal{E}/5R$, so $V_{\text{out}} = R(\mathcal{E}/5R) = \mathcal{E}/5 = (15\text{ V})/5 = 3.0\text{ V}$.

(b) We want the time constant τ_{ch} during charging, which is with S open. The resistance in the circuit is $4R$, so $\tau_{ch} = 4RC$.

(c) We want the time constant τ_d during discharging, which is with S closed. Apply Kirchhoff's rules. The current choices are: I_1 is downward through R; I_2 is upward through C, and I_4 is downward through $4R$.

<u>Loop 1</u>: Clockwise through the small circuit with R and C: $I_1R = q/C$.

<u>Loop 2</u>: Clockwise around the outside of the circuit: $\mathcal{E} - 4RI_4 - q/C = 0$.

<u>Junction rule</u>: $I_4 = I_1 - I_2$.

The capacitor is discharging, so $I_2 = -dq/dt$.

Combining these equations gives $\dfrac{dq}{dt} = \dfrac{\mathcal{E}}{4R} - \dfrac{q}{4RC/5}$. From this result we see that $\tau_d = 4RC/5$.

(d) During the charging-discharging cycle, we want the time between successive 10.0 V output voltages across the capacitor. In one complete cycle, the potential difference across the capacitor discharges from 10.0 V to 5.0 V and then recharges from 5.0 V back to 10.0 V. The time constants in the two parts of the cycle are *not* the same.

<u>Discharging</u>: Using the result of part (c), solve $\dfrac{dq}{dt} = \dfrac{\mathcal{E}}{4R} - \dfrac{q}{4RC/5}$. The circuit discharges from 10.0 V to 5.0 V, so the initial voltage across the capacitor is $V_{0,d} = 10.0\text{ V}$. Separate variables and integrate.

$\displaystyle\int \frac{dq}{\mathcal{E}/4R - q/(4RC/5)} = \int dt$ gives $\ln\left(\dfrac{\mathcal{E}}{4R} - \dfrac{q}{4RC/5}\right) = t + K$, where K is a constant of integration.

Putting this result into exponential form gives $K'e^{-5t/4RC} = \dfrac{\mathcal{E}}{4R} - \dfrac{q}{4RC/5}$, where K' is a constant. When $t = 0$, $q = Q_0$, which gives $K' = \dfrac{\mathcal{E}}{4R} - \dfrac{Q_0}{4RC/5}$. Using this result, $V = q/C$, and $V_{0,d} = Q_0/C$ gives

$\left(\frac{\mathcal{E}}{4R}-\frac{V_{0,d}}{4R/5}\right)e^{-5t/4RC}=\frac{\mathcal{E}}{4R}-\frac{V}{4R/5}$. Solving for V (and calling it V_d) and simplifying, we get

$V_d=\frac{\mathcal{E}}{5}+\left(V_{0,d}-\frac{\mathcal{E}}{5}\right)e^{-5t/4RC}$.

(Check: At $t = 0$, $\mathcal{E}$ = 15 V and $V_{0,d}$ = 10.0 V, which gives V_d = 10 V, as it should. For $t\to\infty$ we have V_d = 3.0 V, which agrees with our result in part (a).)

Charging: The circuit is a simple series circuit containing the battery, the capacitor, and a resistance $4R$. It charges from 5.0 V to 10.0 V, so $V_{0,ch}$ = 5.0 V. Applying Kirchhoff's loop rule gives $\frac{\mathcal{E}}{4R}-\frac{q}{4RC}=\frac{dq}{dt}$.

We solve this differential equation as we did for discharging. Separate variables and integrate, using the initial condition that $V = V_{0,ch}$ when $t = 0$. Carrying out these steps and solving for V_{ch} gives

$V_{ch}=\mathcal{E}-\left(\mathcal{E}-V_{0,ch}\right)e^{-t/4RC}$. (Check: At $t = 0$, V_{ch} = 15 V – (15 V – 5.0 V) = 5.0 V, as we should get. As $t\to\infty$, $V_{ch}\to\mathcal{E}$ as it should.) Now we find the time to charge the capacitor from 5.0 V to 10.0 V and to discharge it from 10.0 V to 5.0 V.

Charging from 5.0 V to 10.0 V: Using $V_{ch}=\mathcal{E}-\left(\mathcal{E}-V_{0,ch}\right)e^{-t/4RC}$ with $\mathcal{E}$ = 15 V, $V_{0,ch}$ = 5.0 V, and V_{ch} = 10.0 V, we have 10.0 V = 15 V – (15 V – 5.0 V)$e^{-t/4RC}$. Solving for the charging time t_{ch} gives

$t_{ch}=4RC\ln 2$.

Discharging from 10.0 V to 5.0 V: Use $V_d=\frac{\mathcal{E}}{5}+\left(V_{0,d}-\frac{\mathcal{E}}{5}\right)e^{-5t/4RC}$ with $\mathcal{E}$ = 15 V, $V_{0,d}$ = 10.0 V, and V_d = 5.0 V, we have 5.0 V = 3.0 V + (10.0 V – 3.0 V)$e^{-5t/4RC}$. Solving for the discharge time t_d gives

$t_d=\frac{4RC}{5}\ln(7/2)$.

The total time T for one cycle is $T = t_d + t_{ch} = \frac{4RC}{5}\ln(7/2)+4RC\ln 2$. Simplifying gives

$T=\frac{4RC}{5}\left(\ln 7+4\ln 2\right)$.

(e) We want the frequency f of operation. Using $f = 1/T$ with R = 10.0 kΩ and C = 10.0 μF, we have T = 4(10.0 kΩ)(10.0 μF)(ln 7 + 4 ln 2)/5 = 0.3775 s. $f = 1/T$ = 1/(0.3775 s) = 2.65 Hz.