2021

Theory of Computation

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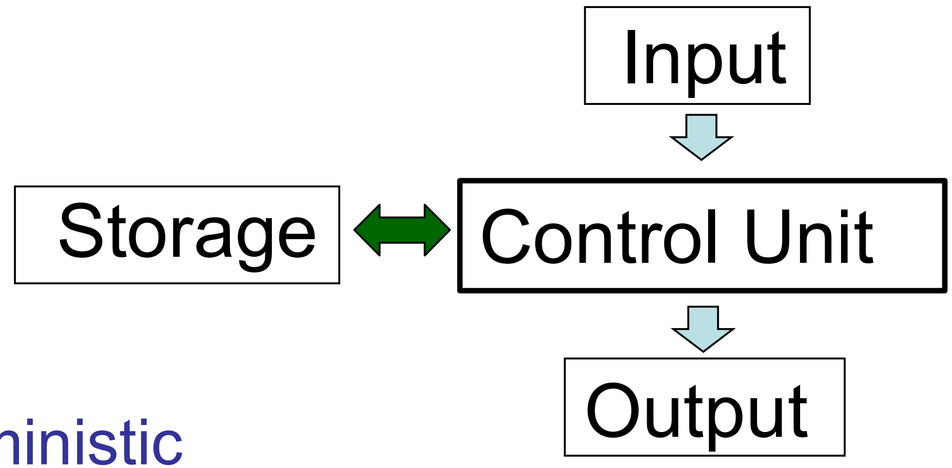
Outline

Automata

Automaton:

An abstract model of a digital computer



•Deterministic V.S. Nondeterministic

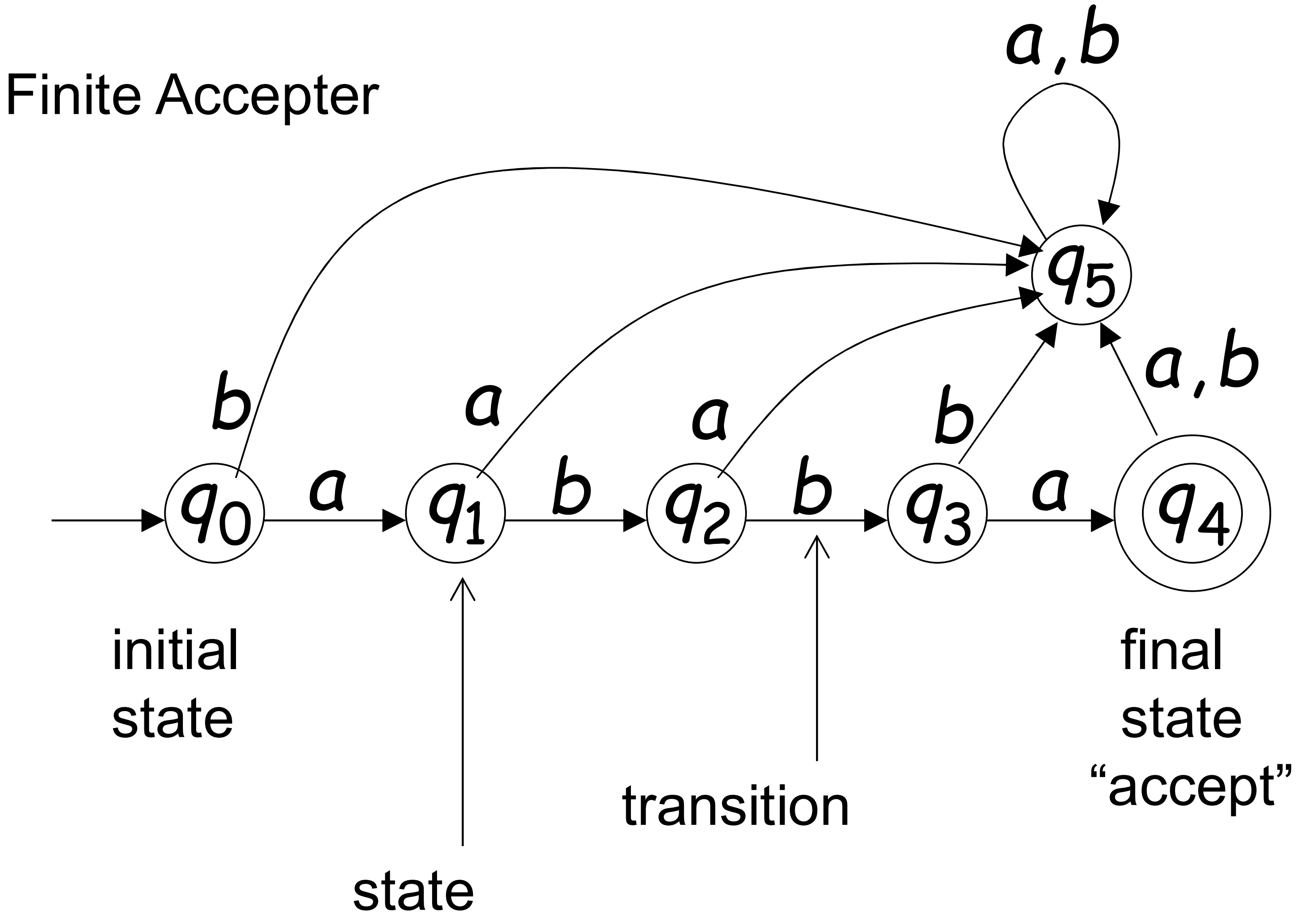
•An automaton whose output is YES or NO

Accepter

•An automaton whose output are strings of symbols

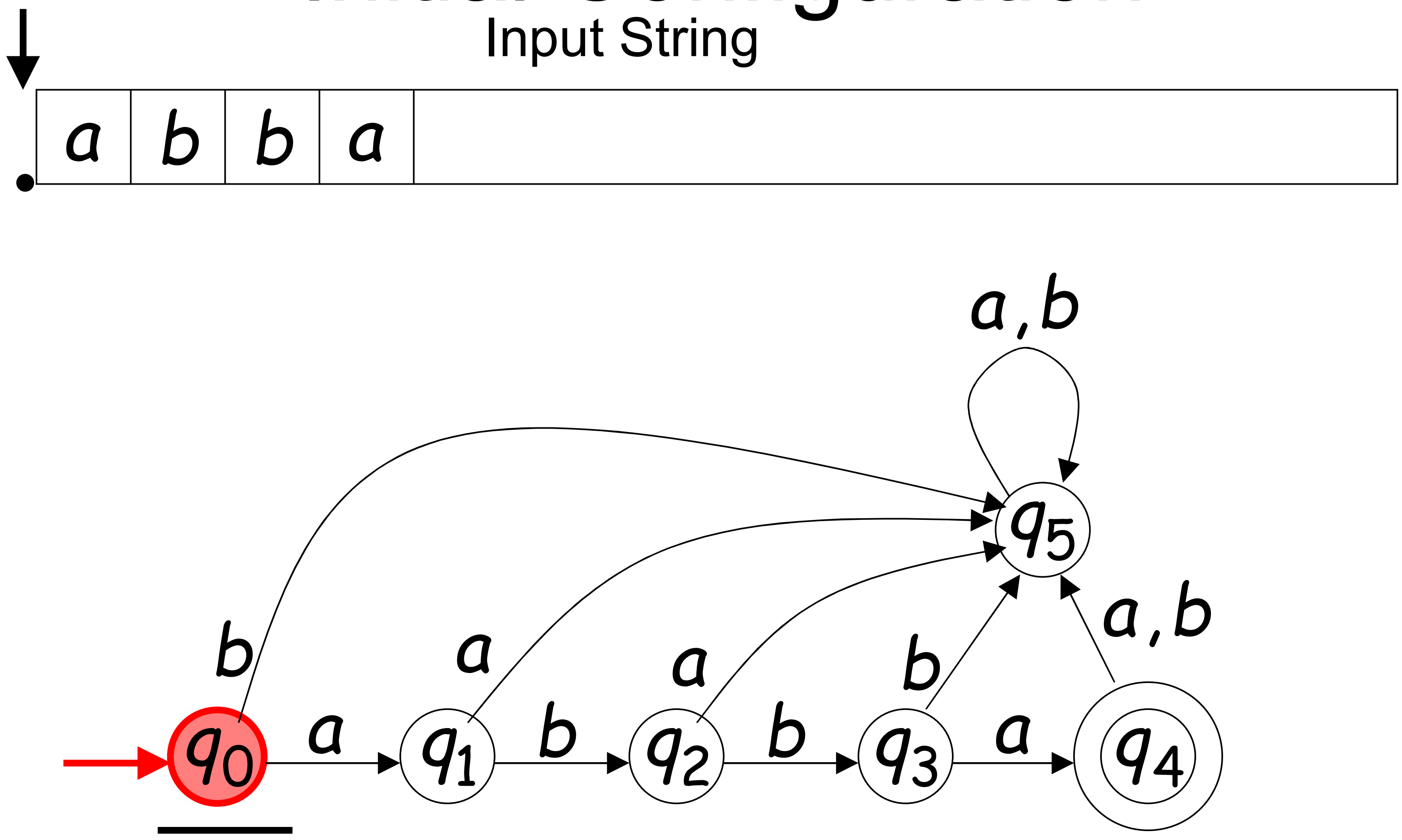
Transducer

Transition Graph

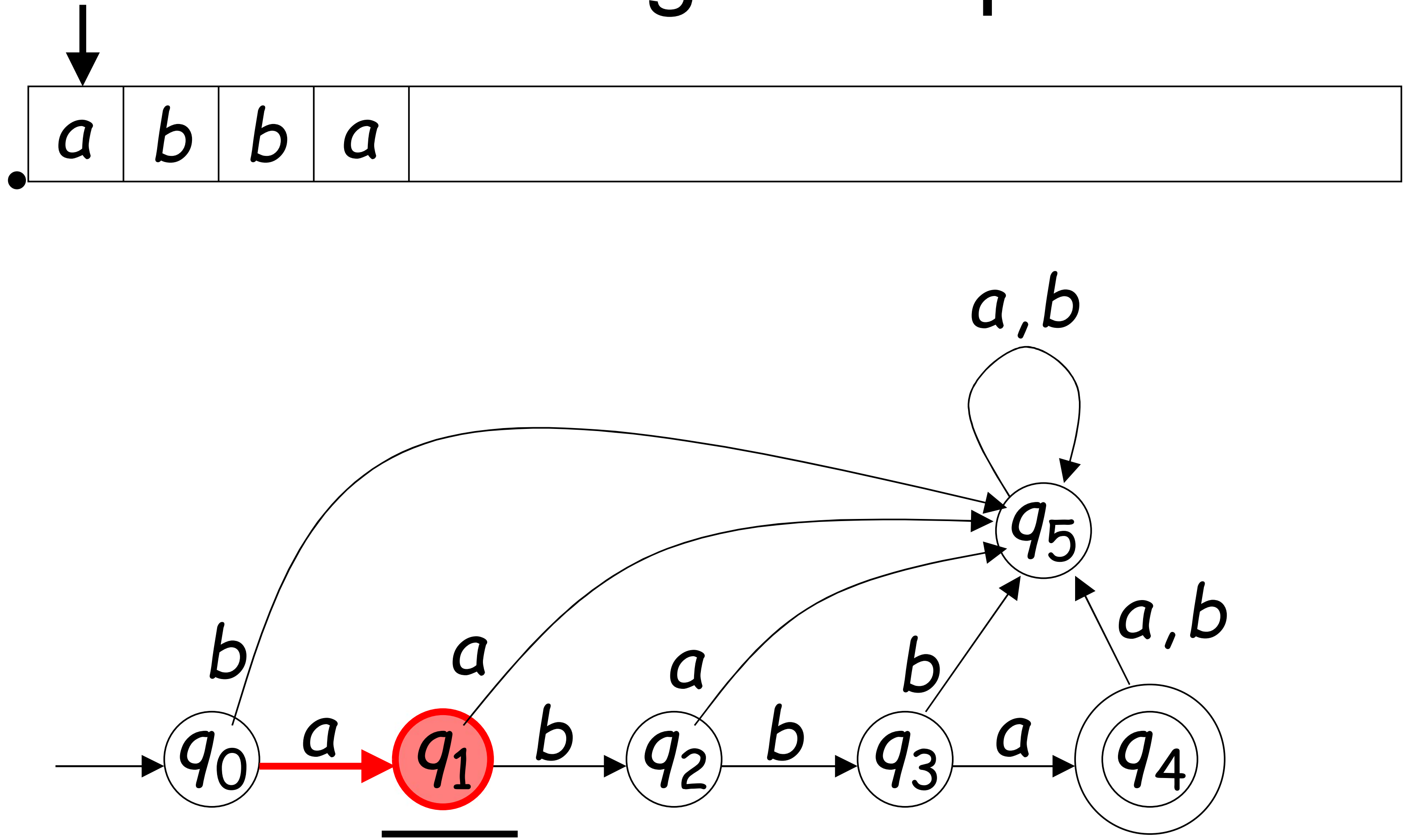


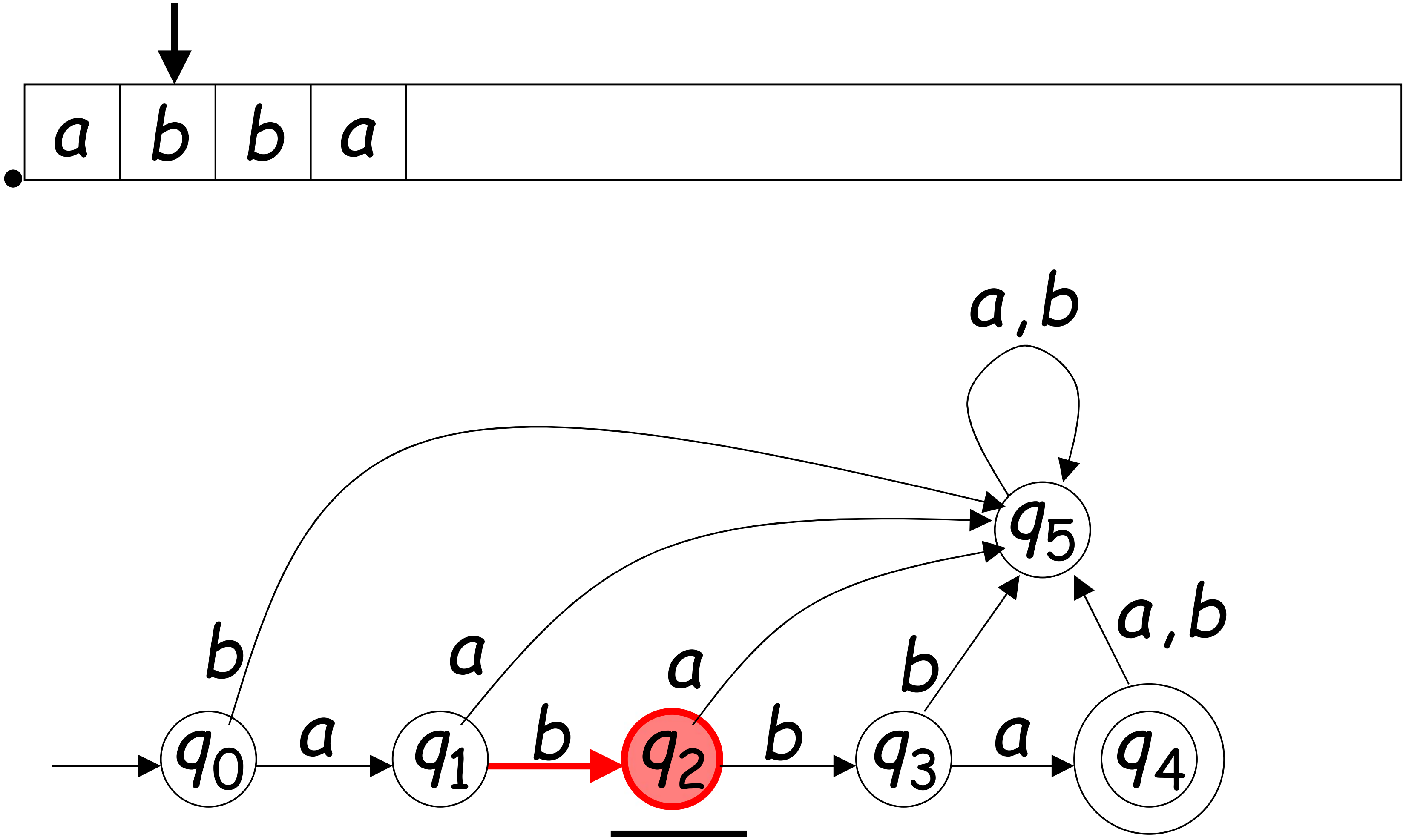
Initial Configuration

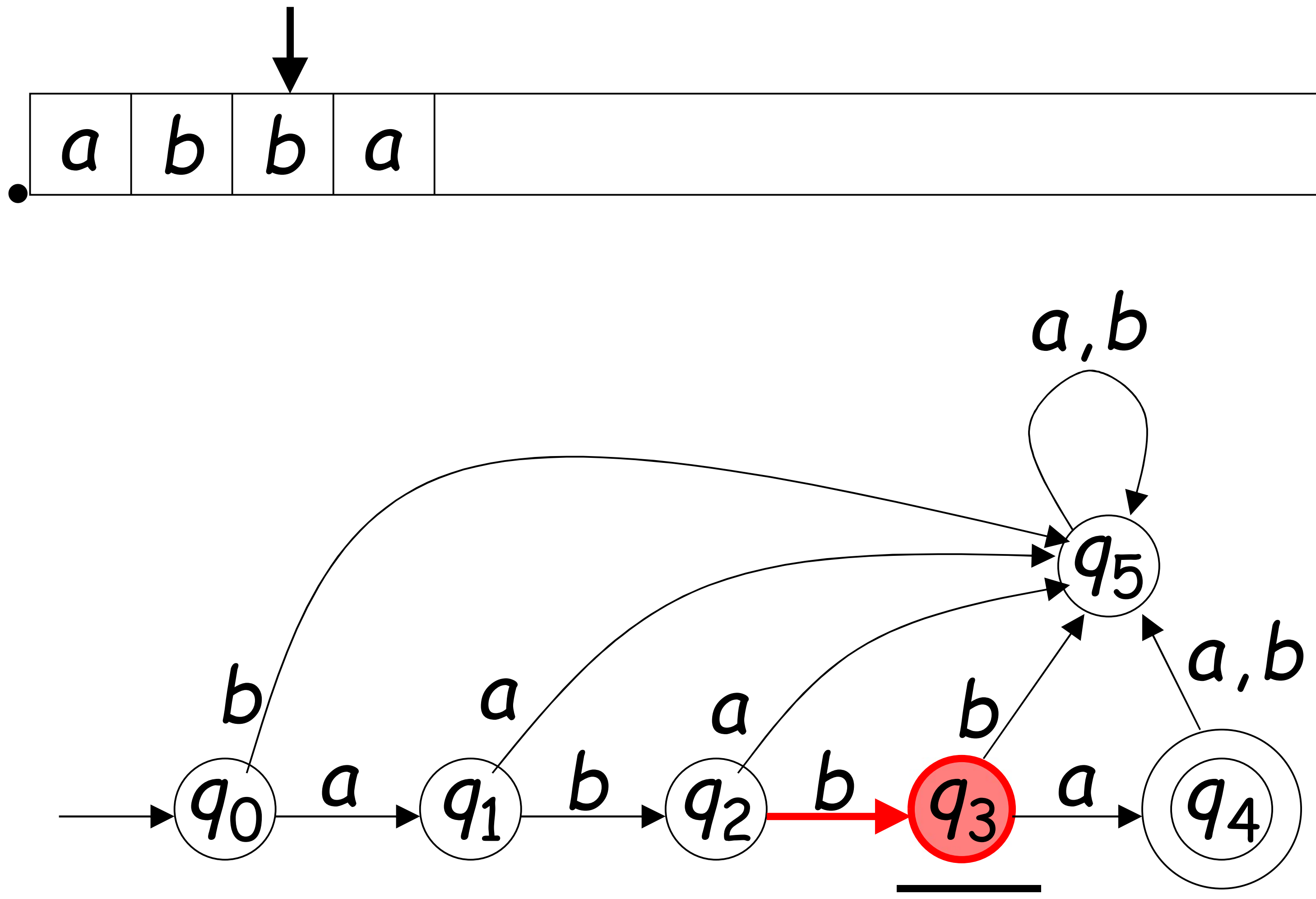
Input String

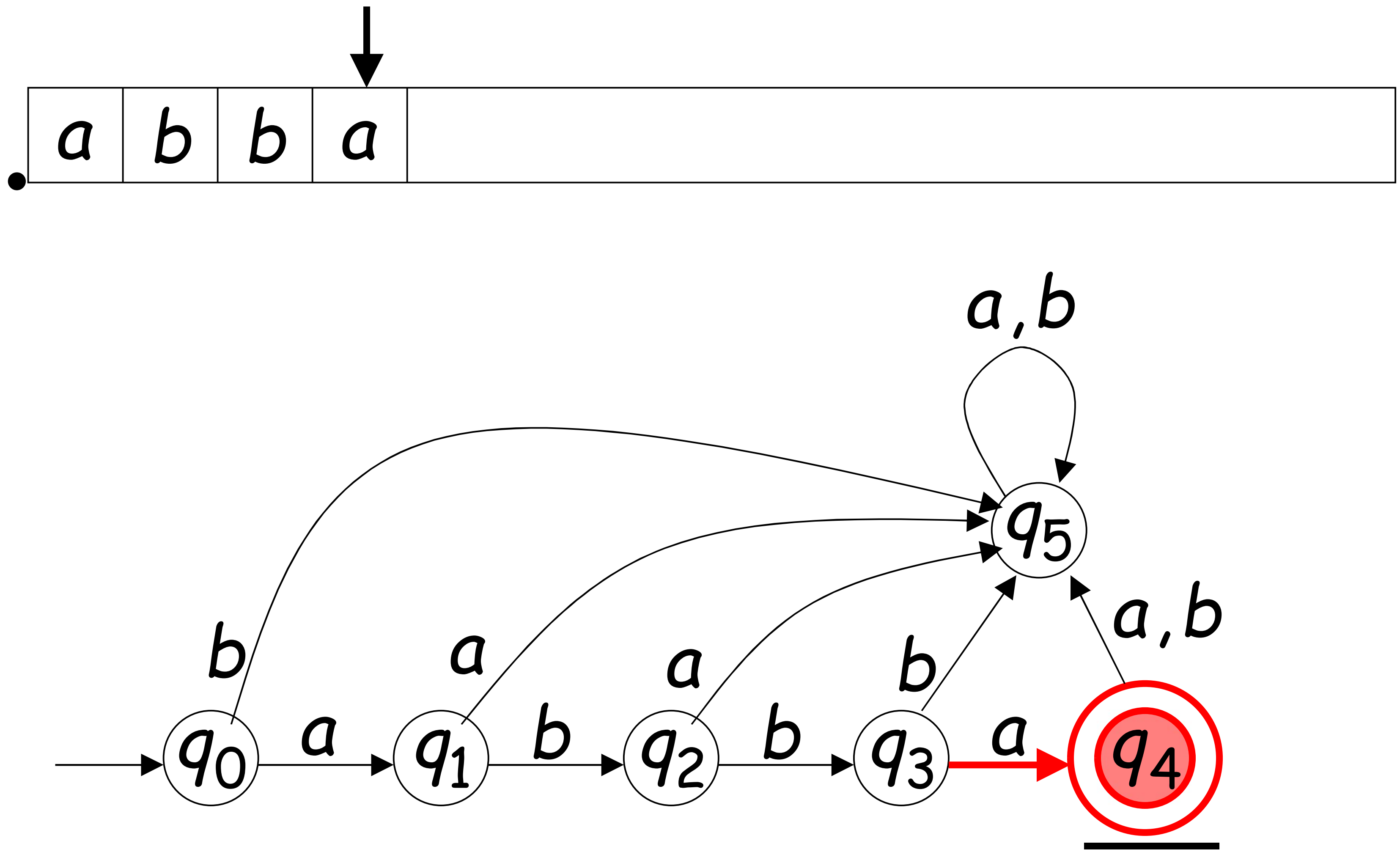


Reading the Input

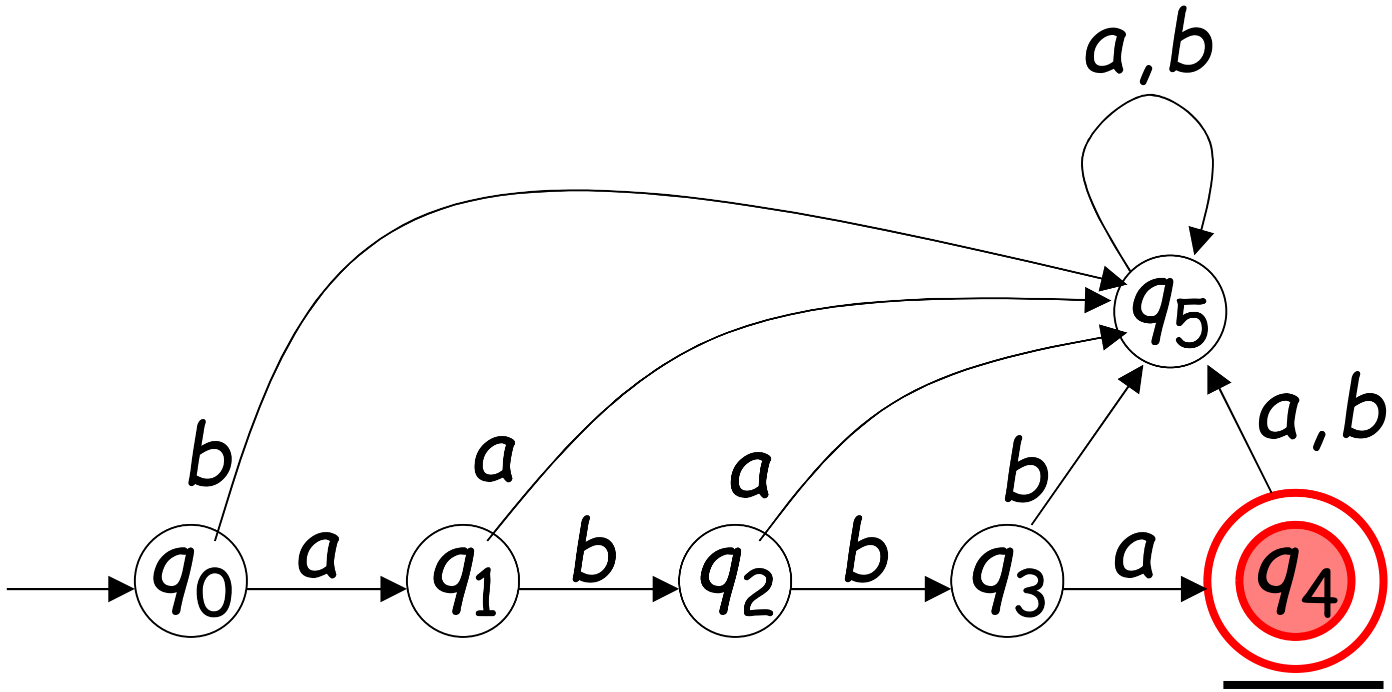



a
b
b
a
a,b
q5
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4


a
b
b
a
a,b
q5
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4

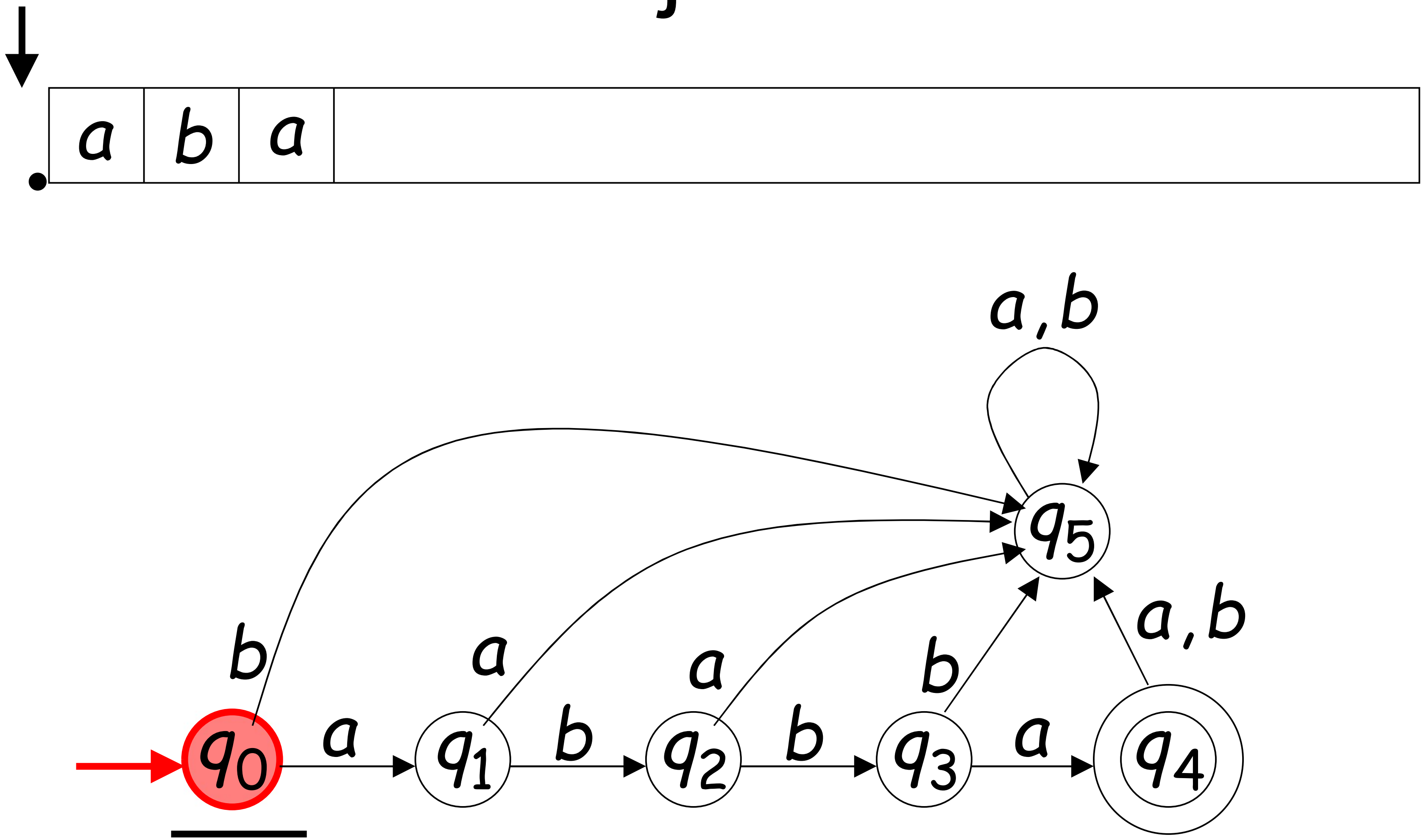

a
b
b
a
a,b
b
a
a
b
a,b
q_0
a
q_1
b
q_2
b
q_3
a
q_4
q_5

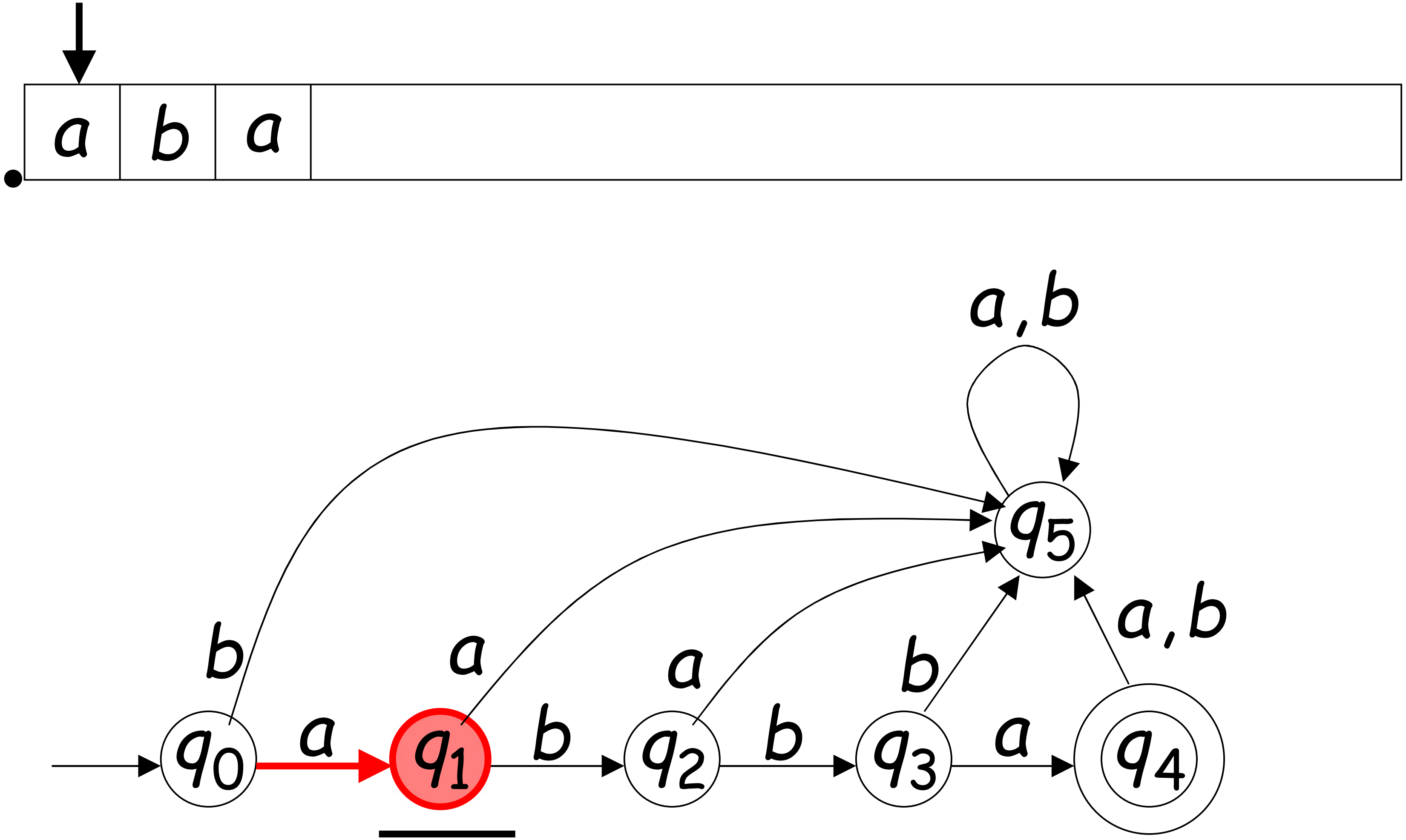
Input finished



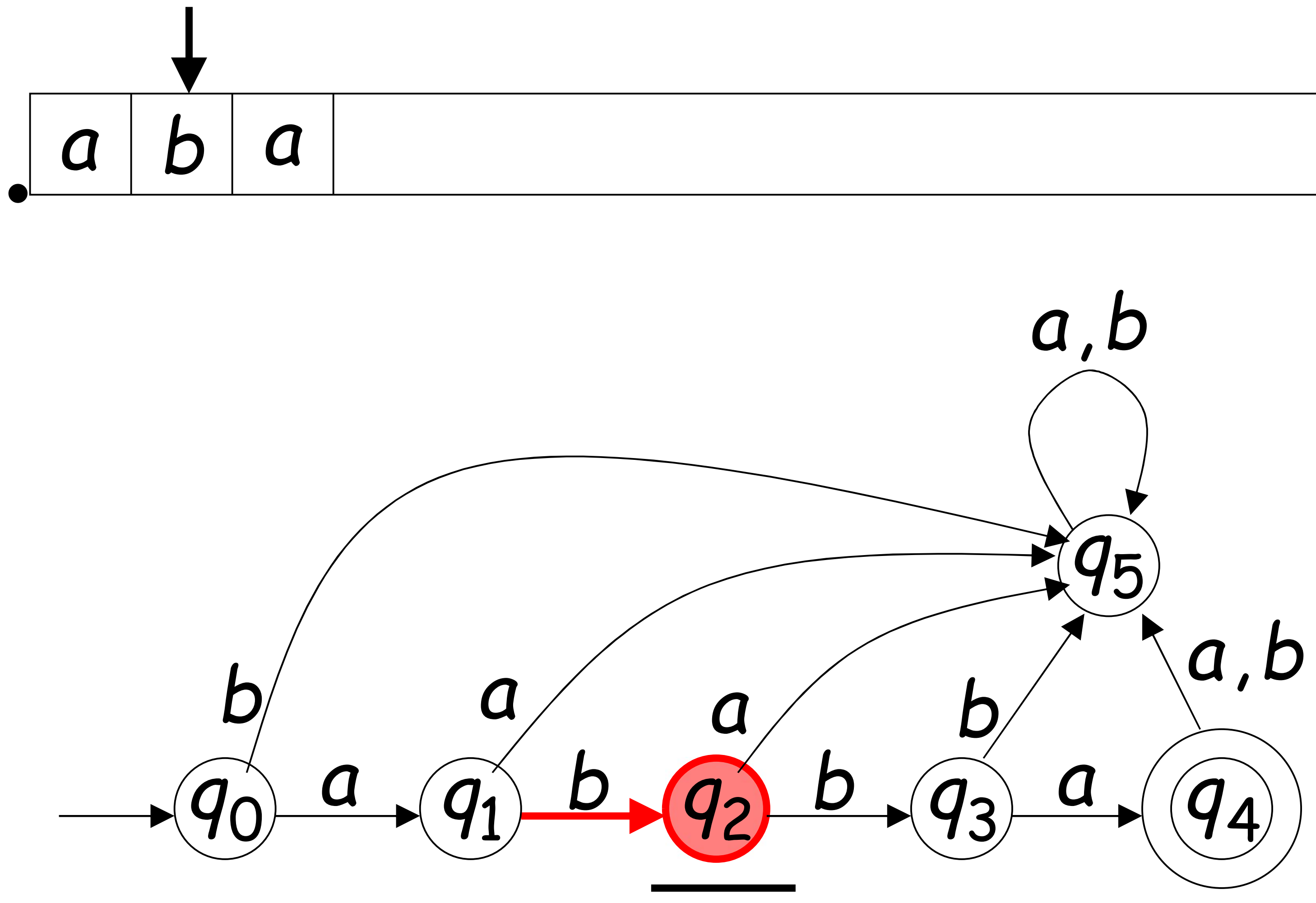
Output: “accept”

Rejection

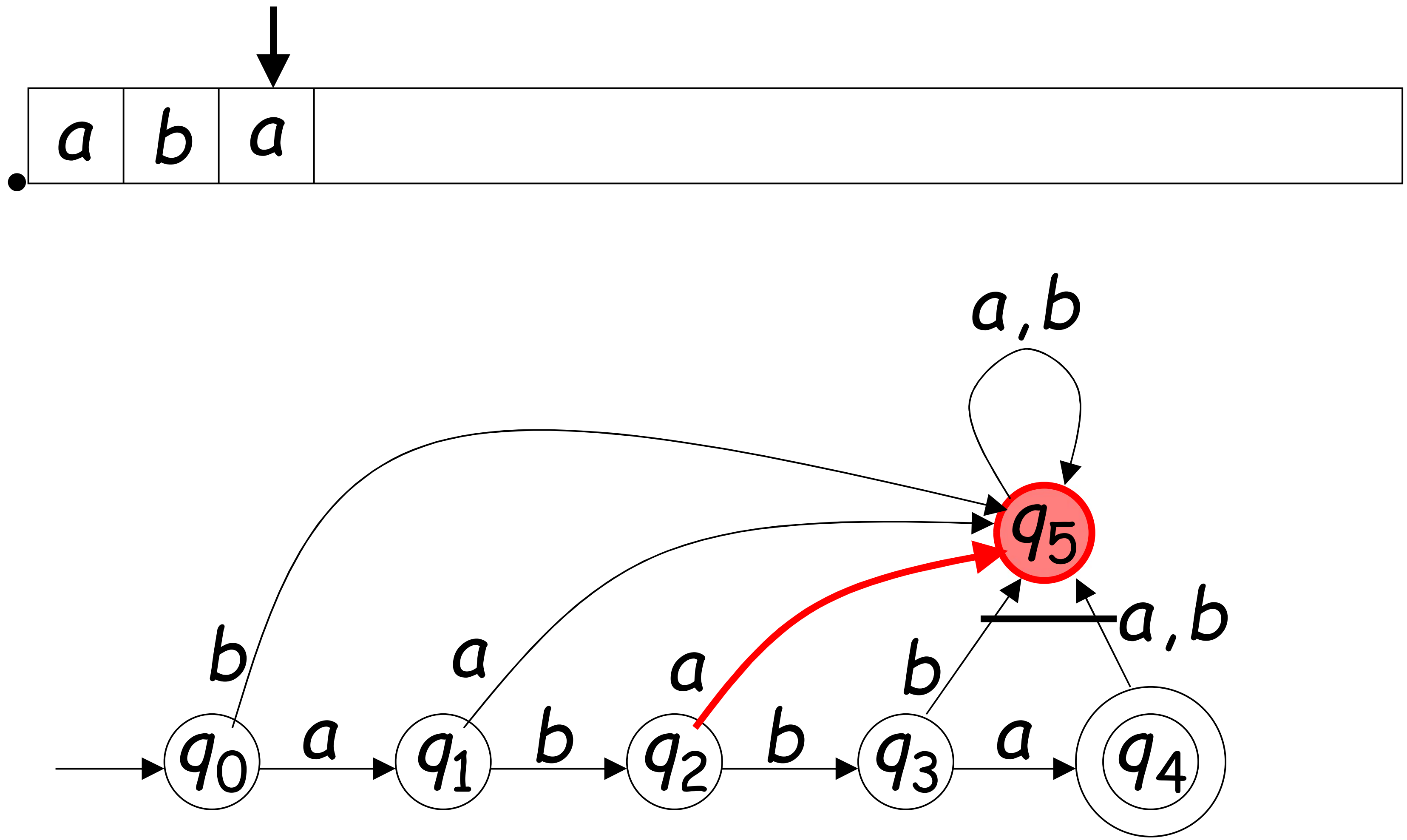




a
b
a
a,b
b
a
a
b
a,b
q_0
a
q_1
b
q_2
b
q_3
a
q_4
q_5

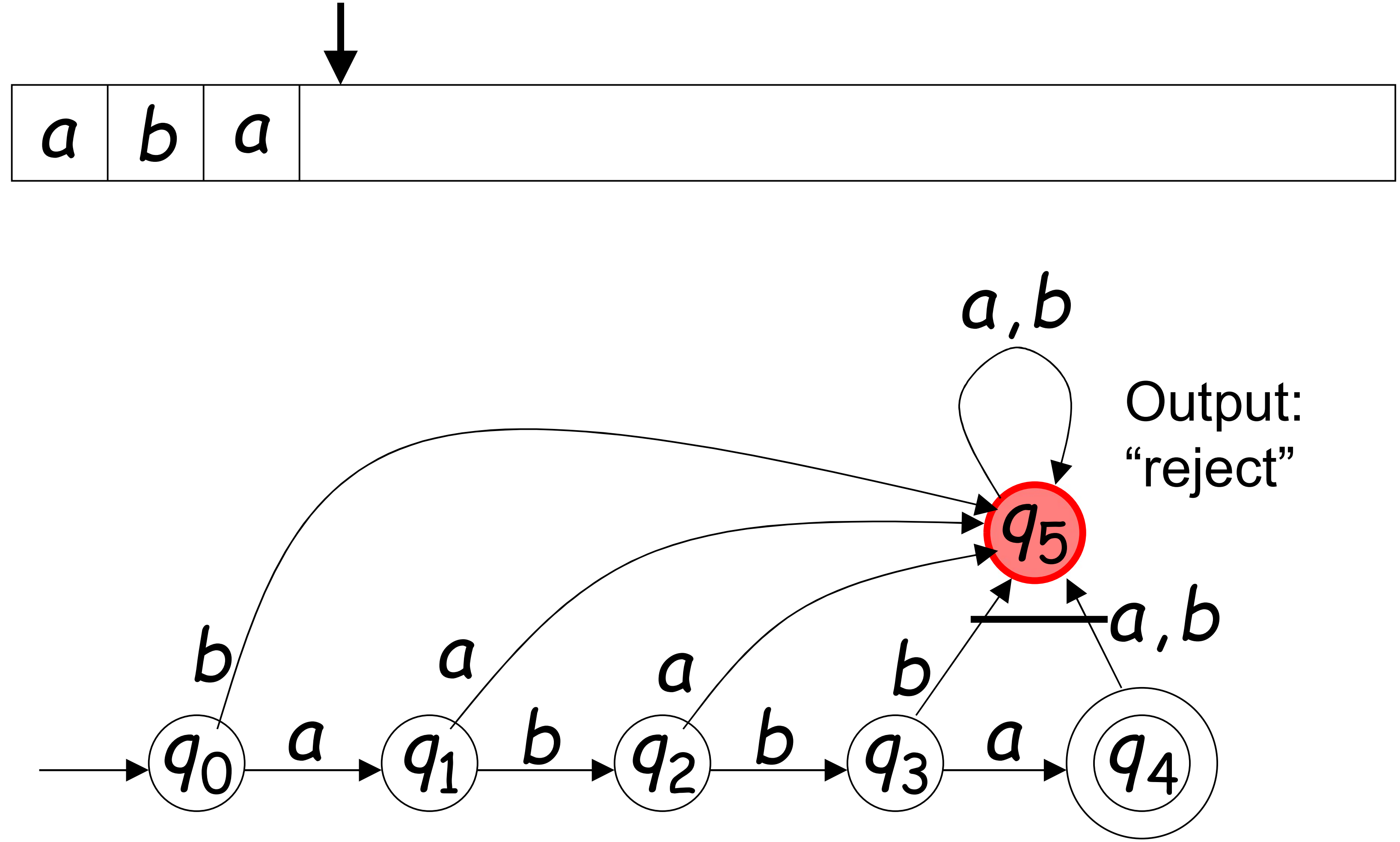


a
b
a
a,b
q_5
b
a
a
b
a,b
q_0
a
q_1
b
q_2
b
q_3
a
q_4

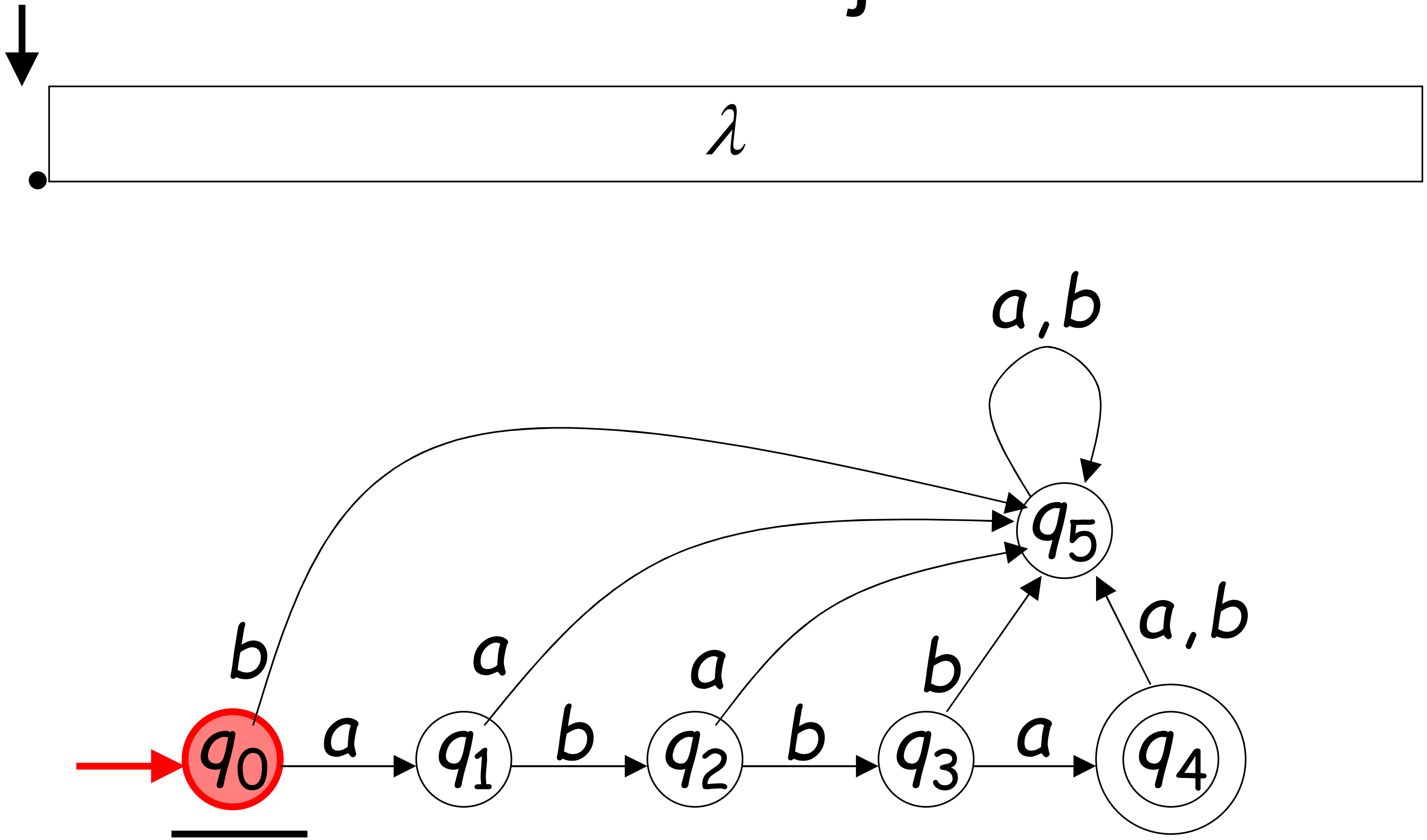


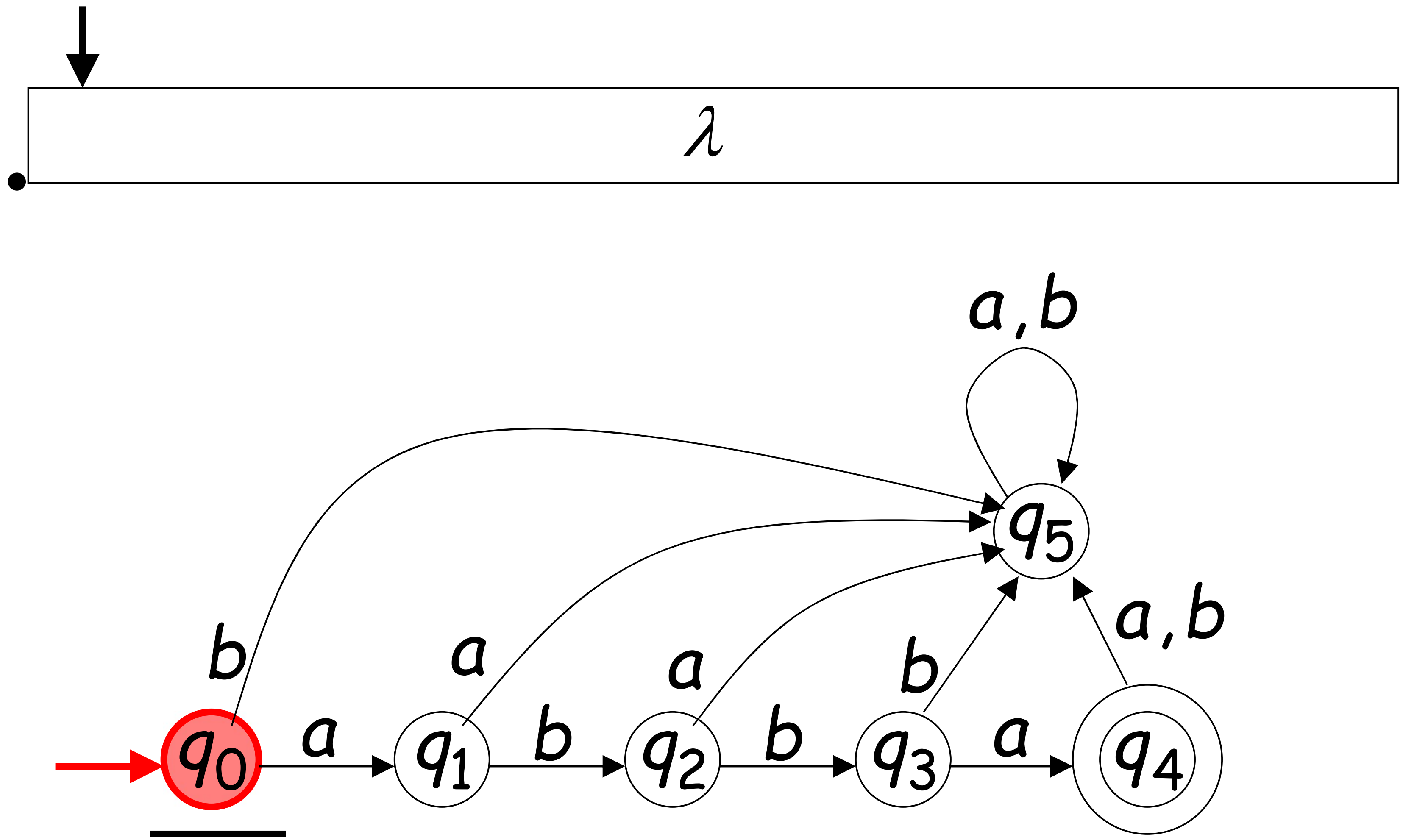
a
b
a
a,b
b
a
a
b
a,b
a
b
b
a
q_0
q_1
q_2
q_3
q_4
q_5

Input finished



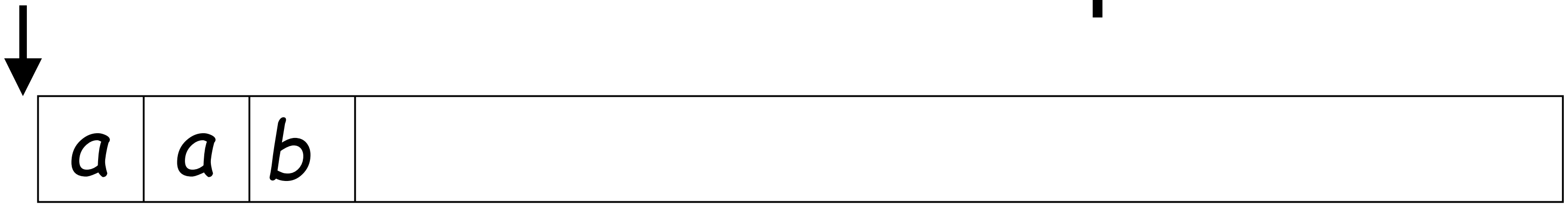
Another Rejection

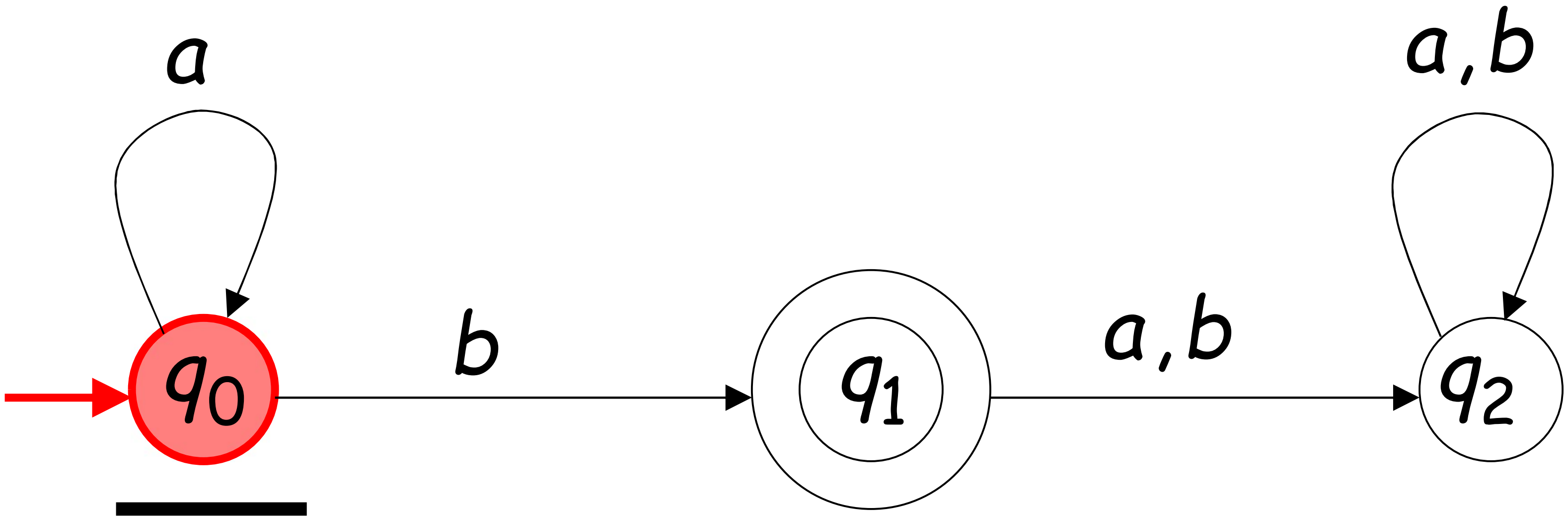


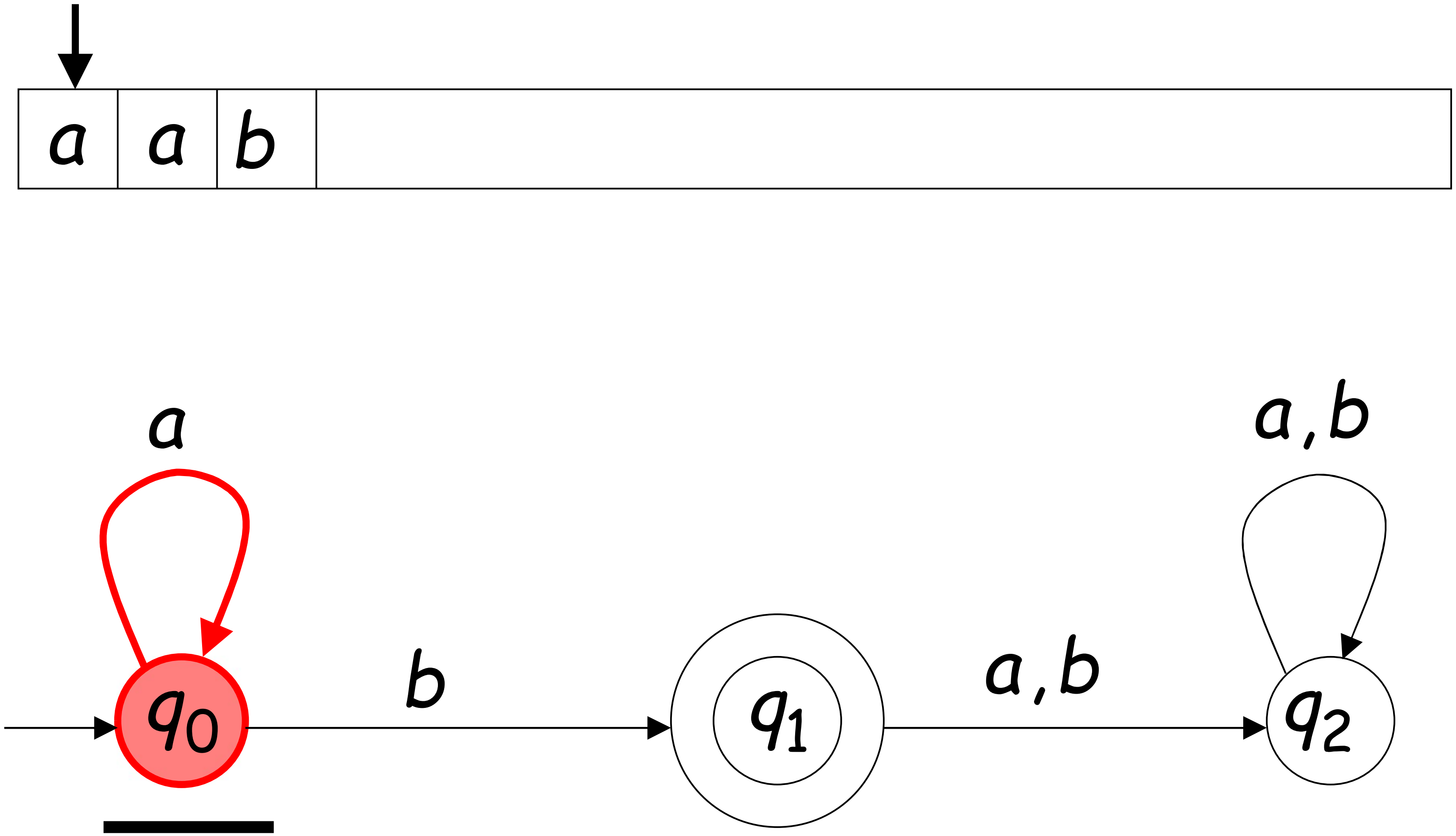


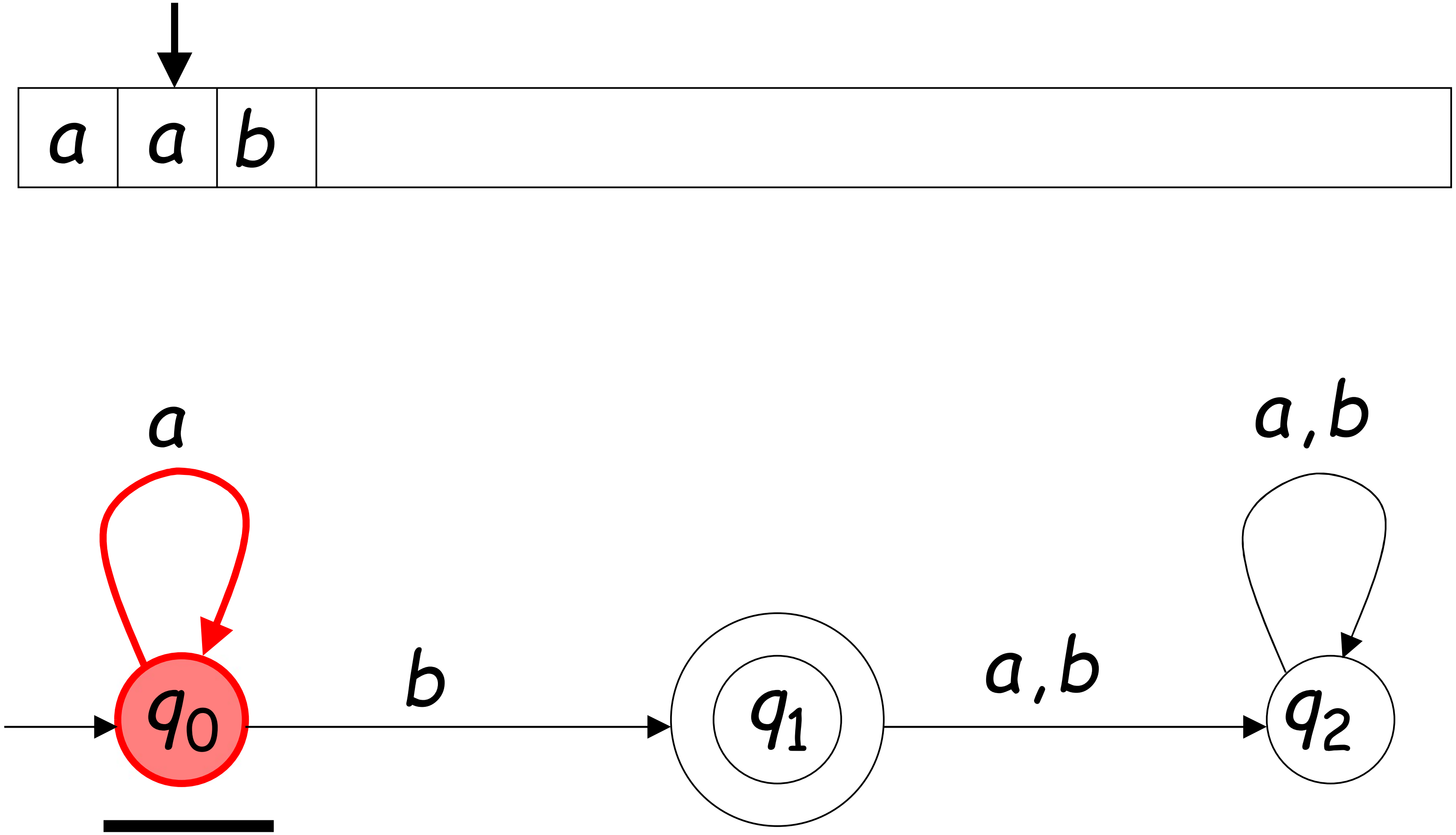
Output:
"reject"

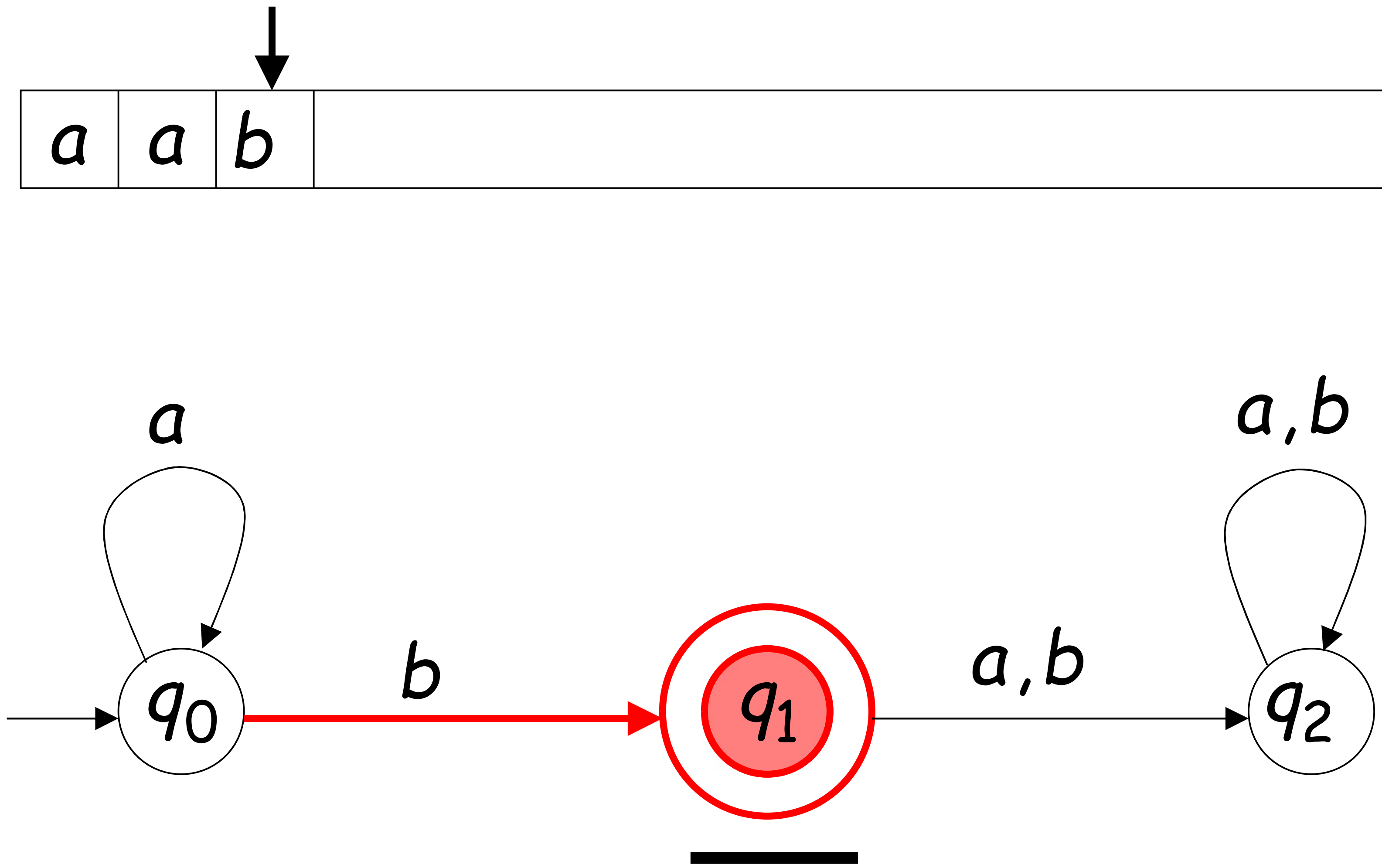
Another Example





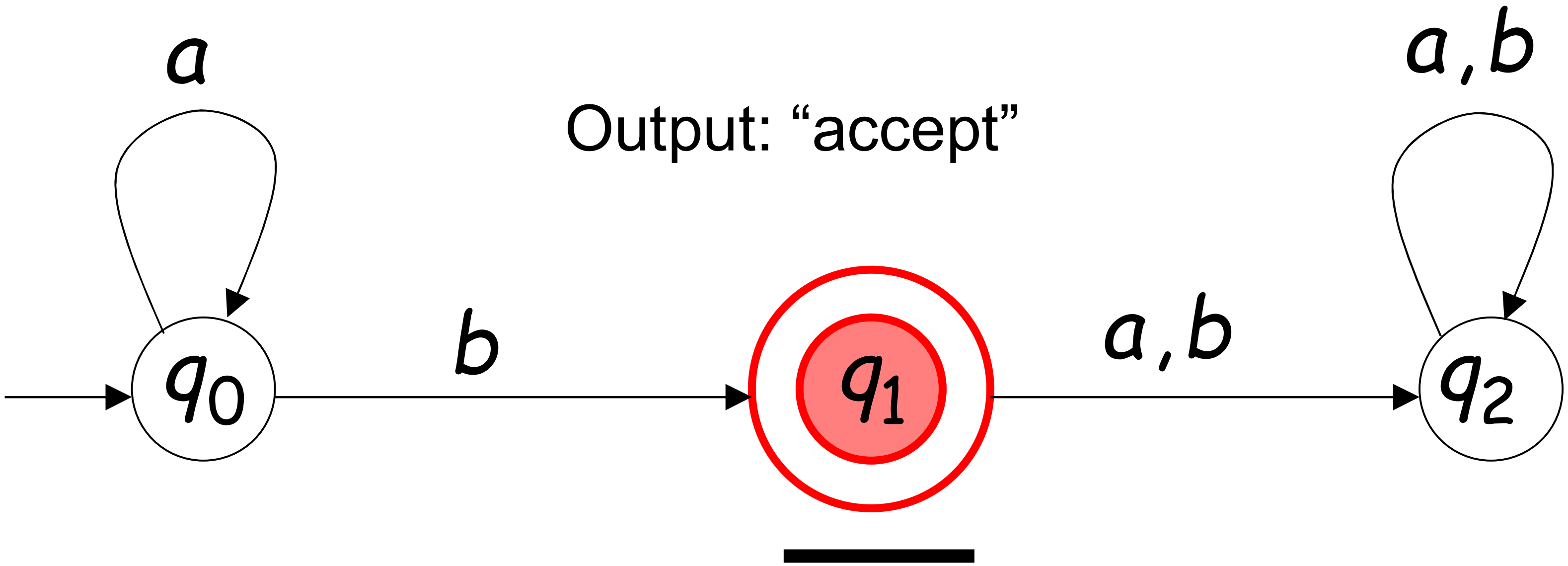

a
a
b
a
b
a,b
a,b
q_0
q_1
q_2


a
a
b
a
a,b
b
a,b
q_0
q_1
q_2

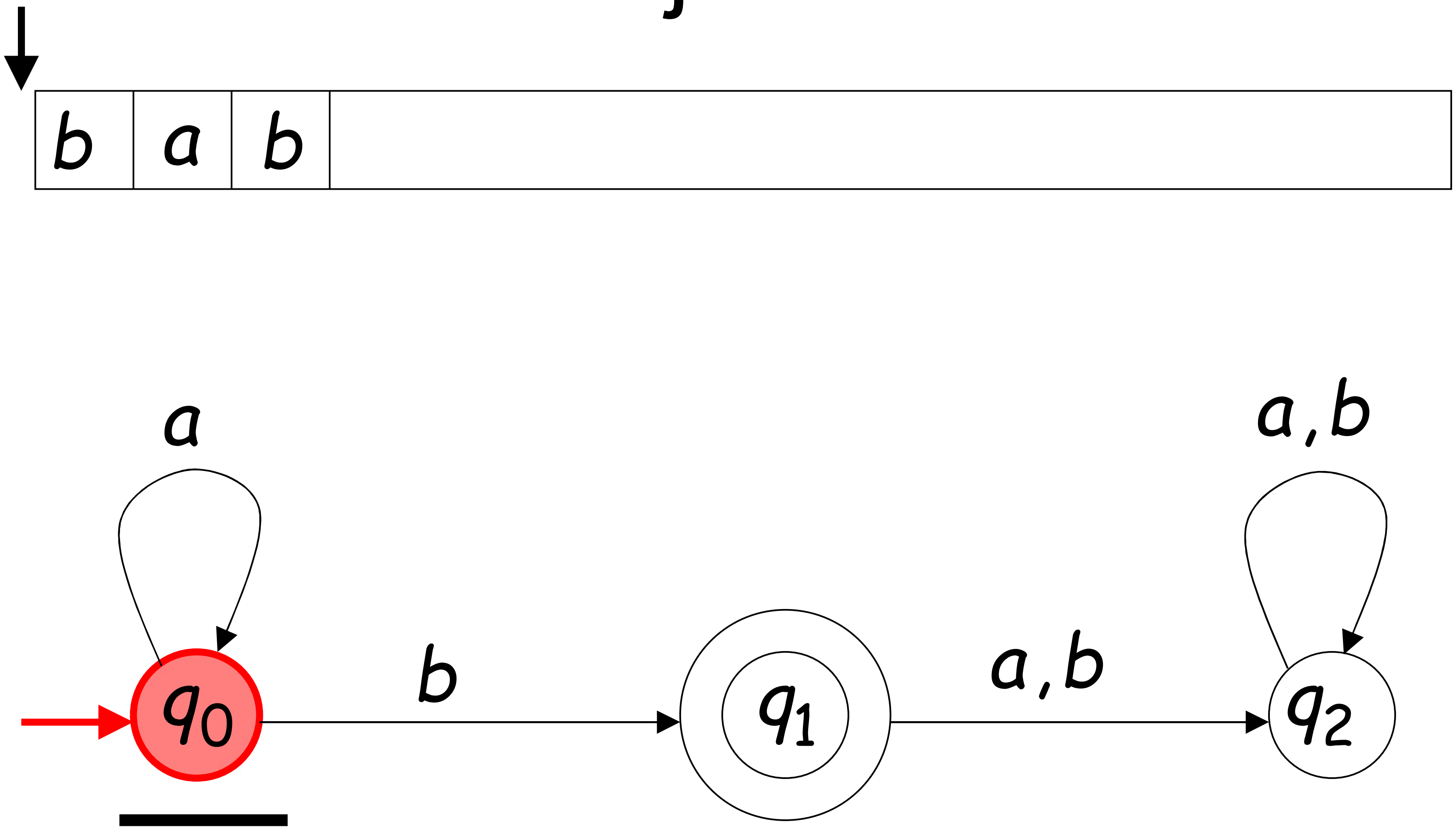


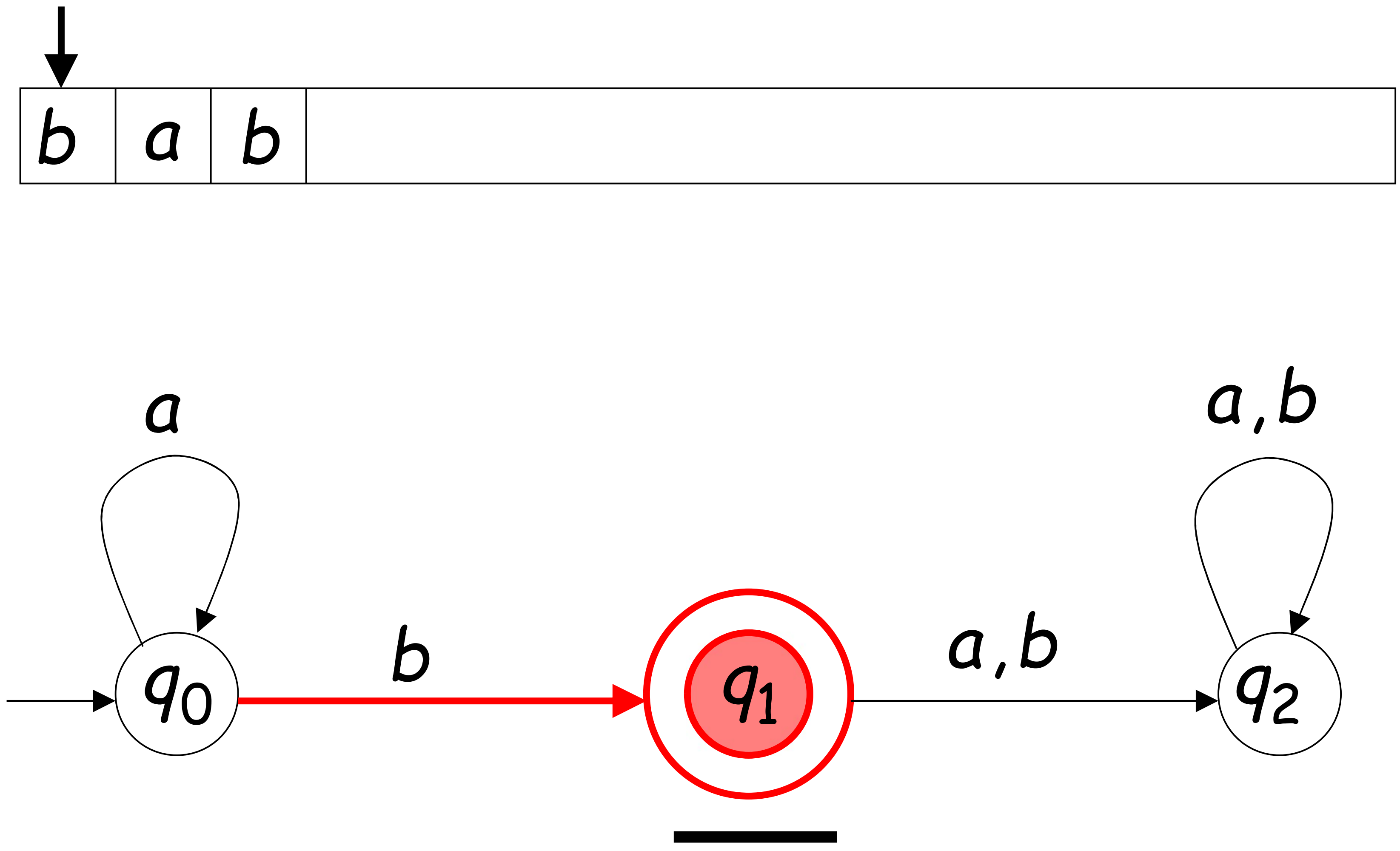
a
a
b
a
a,b
b
a,b
q_0
q_1
q_2

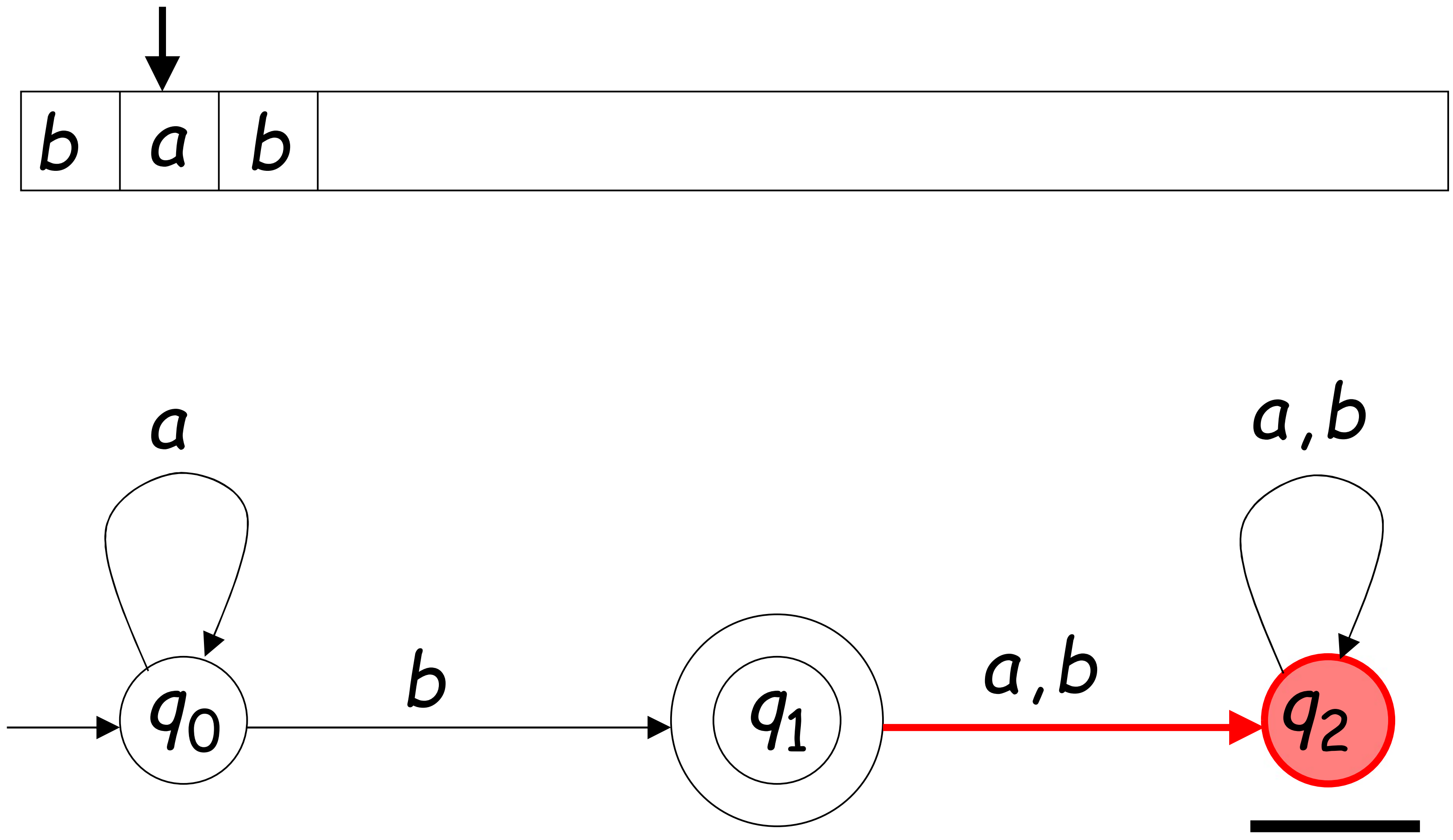
Input finished

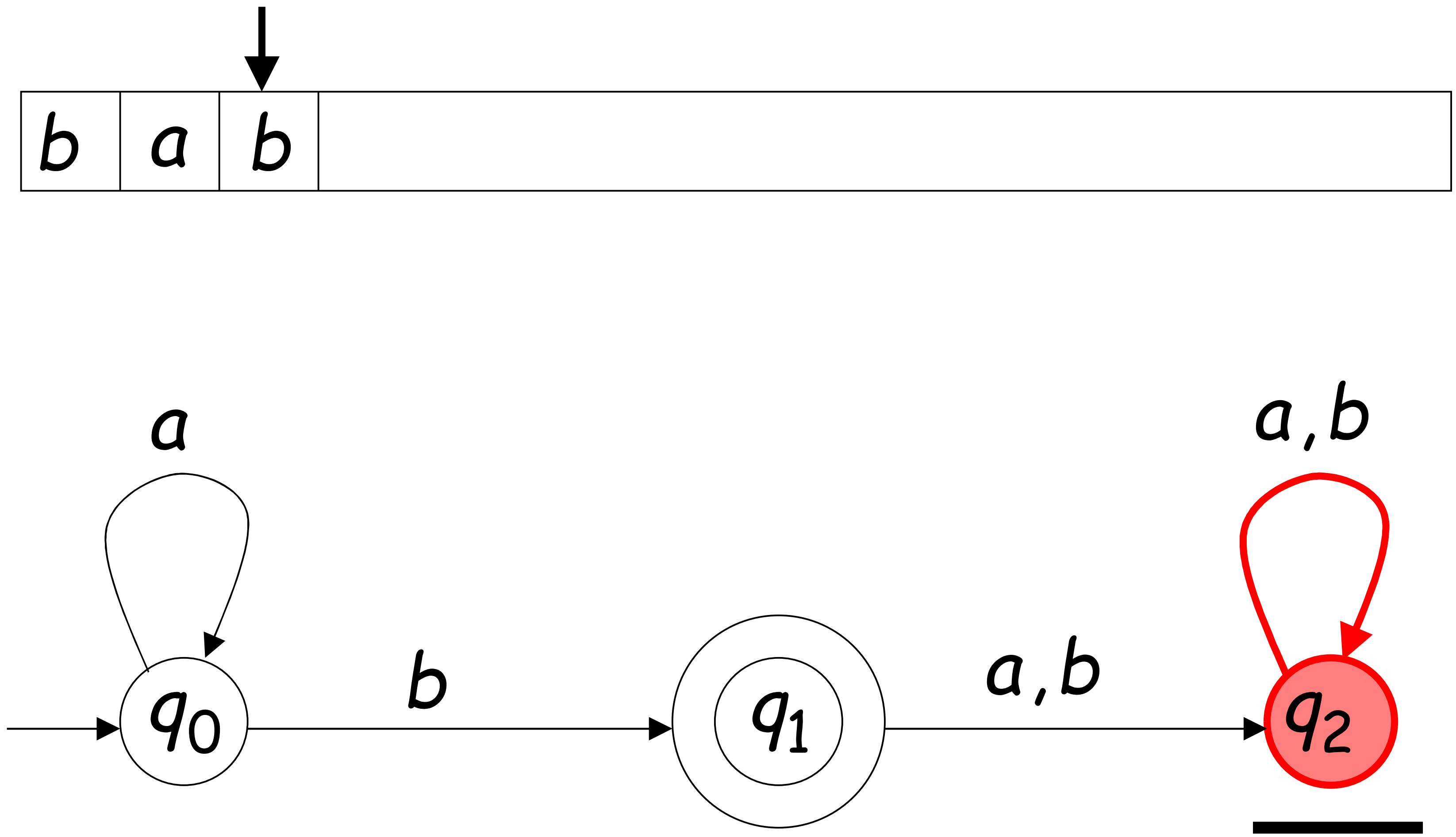


Rejection



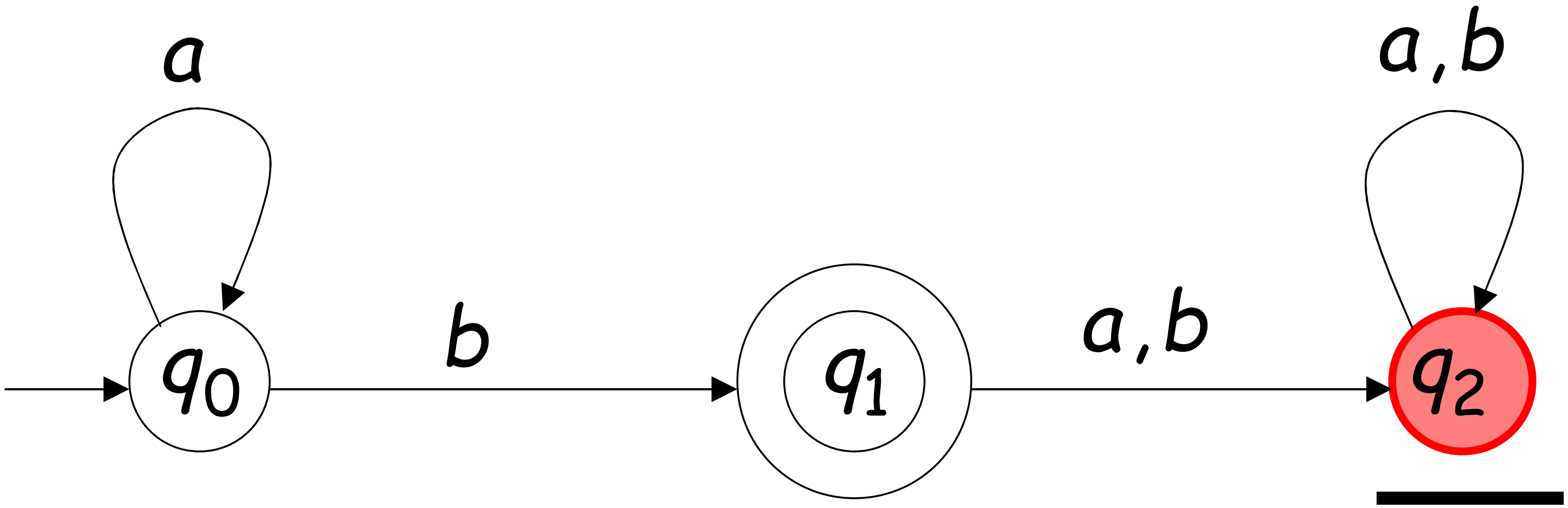

b
a
b
a
a,b
b
a,b
q_0
q_1
q_2


b
a
b
a
a,b
b
a,b
q_0
q_1
q_2



b
a
b
a
b
a,b
a,b
q_0
q_1
q_2

Input finished



Output: “reject”

Trap state

Definition 2.1

Deterministic Finite Accepter (DFA) is define by the 5-tuple

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : a finite set of **internal states**

Σ : a finite set of symbols called **input alphabet**

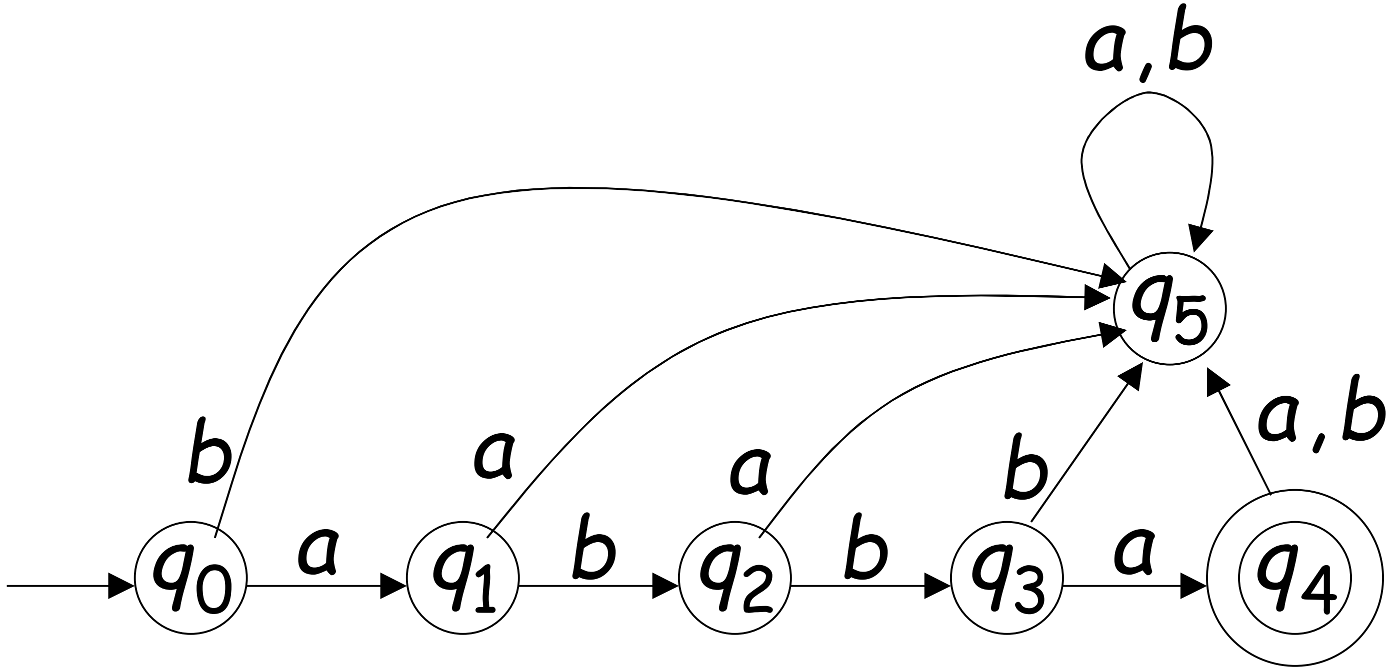
δ : $Q \times \Sigma \rightarrow Q$ called **transition function** (Total function)

q_0 : $q_0 \in Q$ is the **initial state**

F : $F \subseteq Q$ is a set of **final states**

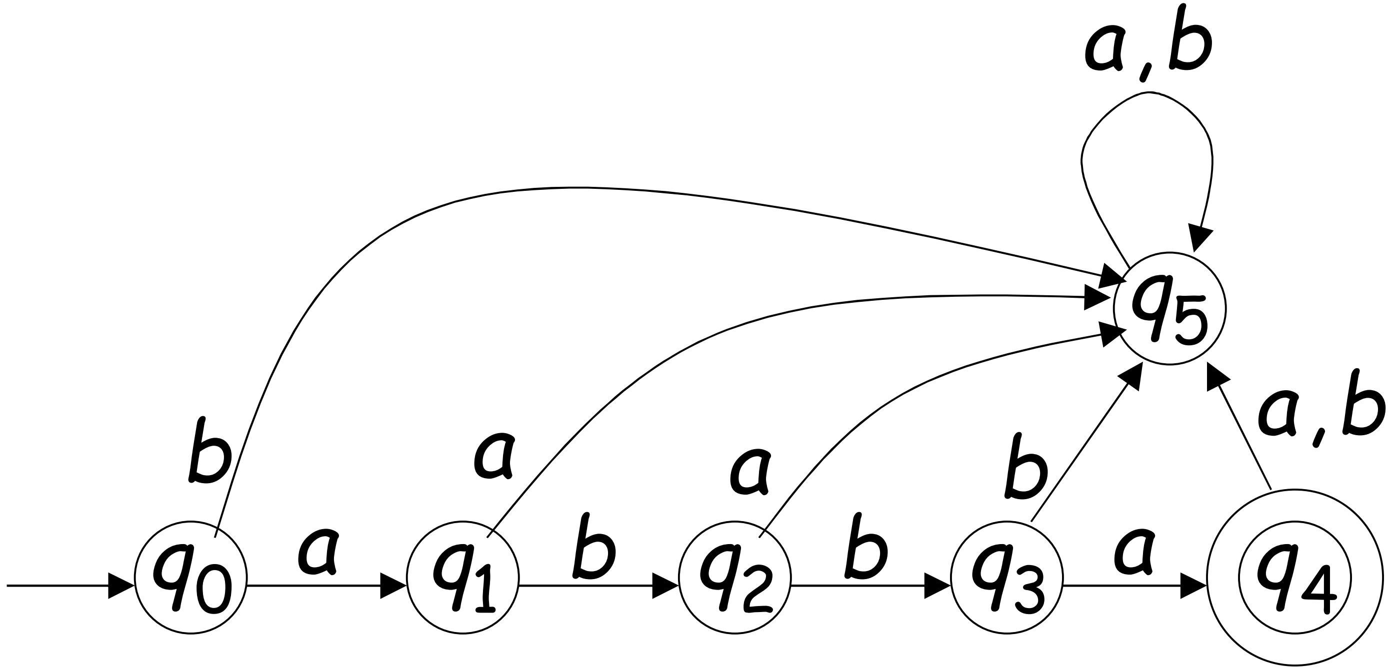
Input Alphabet Σ

$$\Sigma = \{a, b\}$$

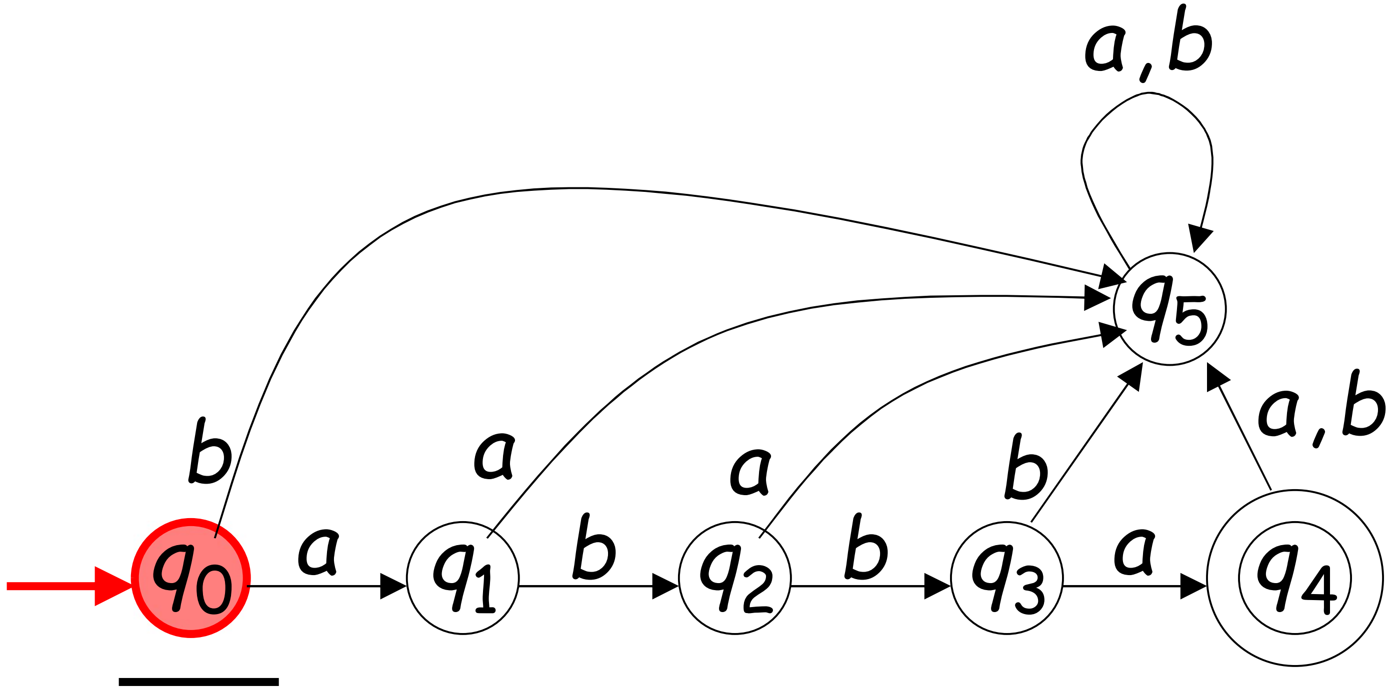


Set of States Q

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

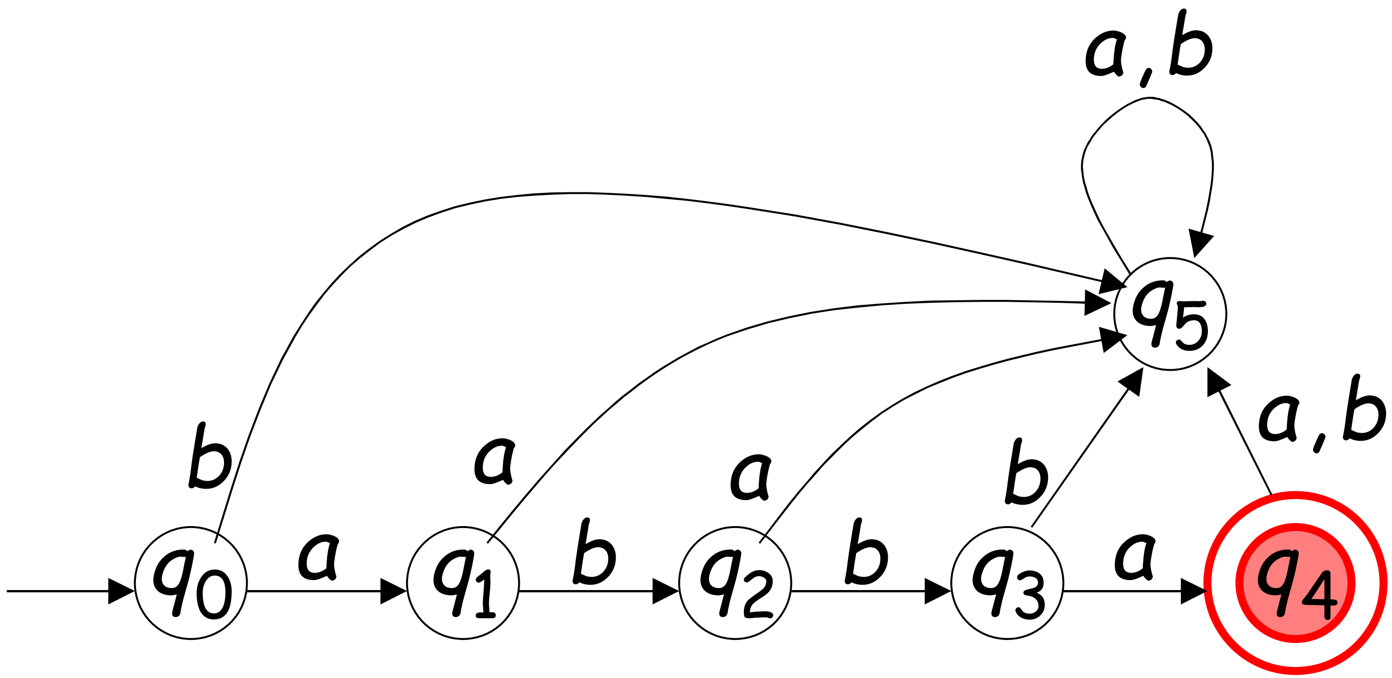


Initial State q_0



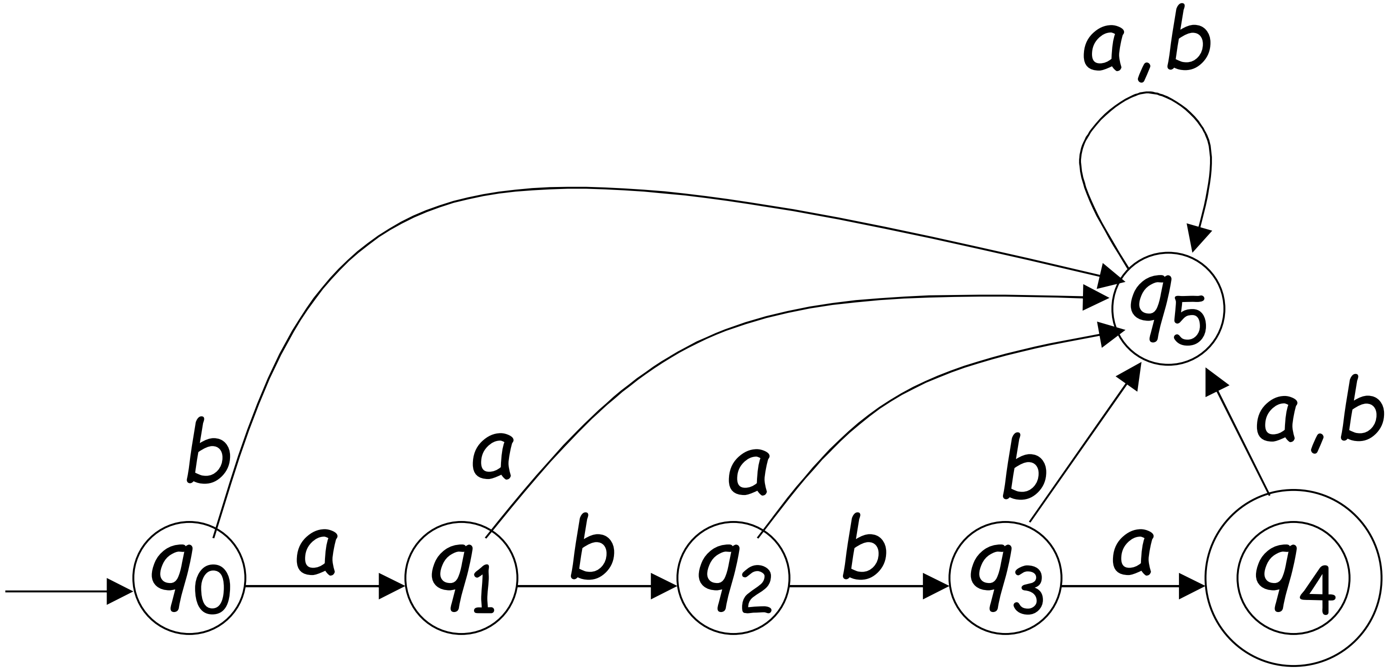
Set of Final States F

$$F = \{q_4\}$$

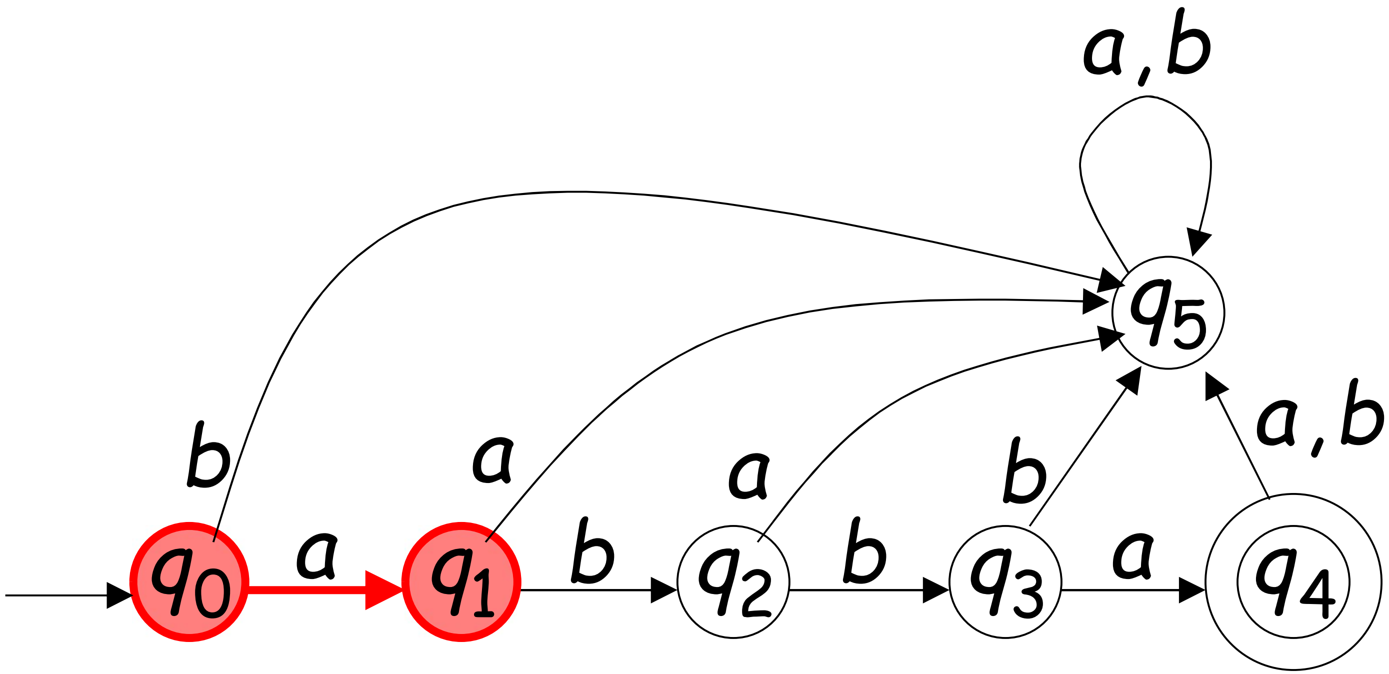


Transition Function δ

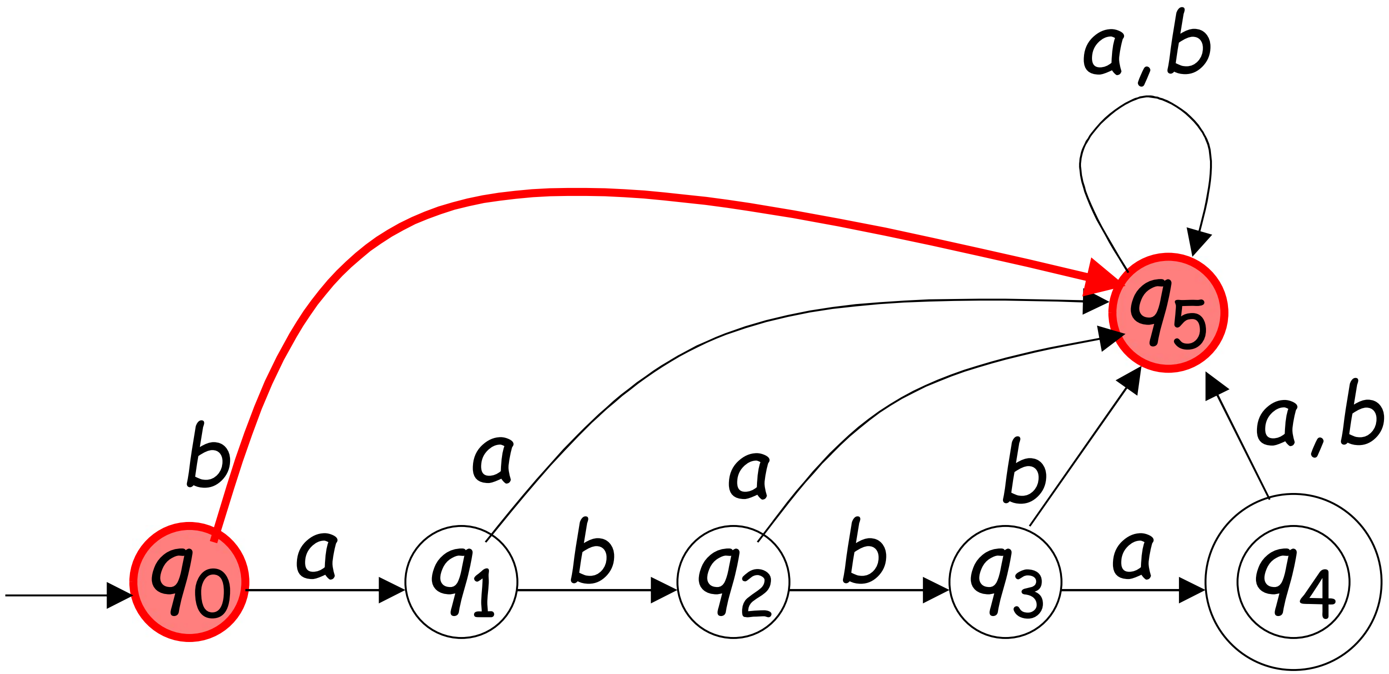
$$\delta : Q \times \Sigma \rightarrow Q$$



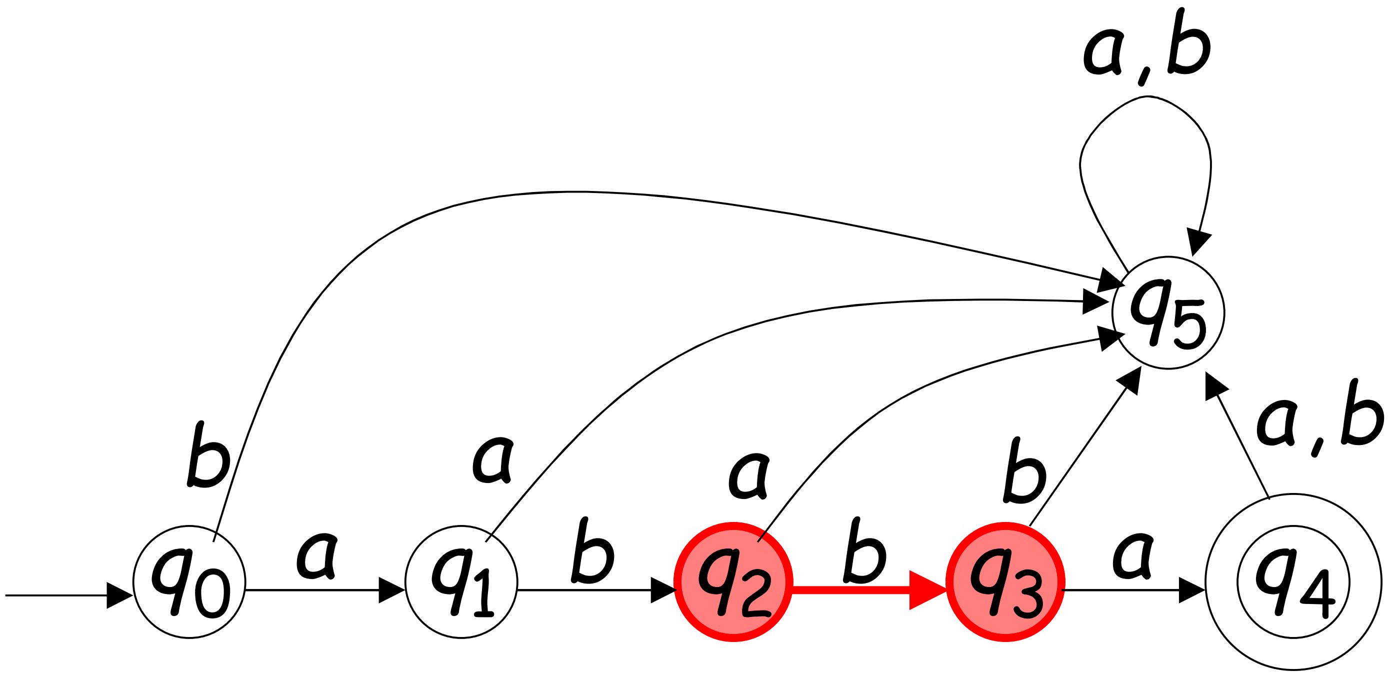
$$\delta(q_0, a) = q_1$$



$$\delta(q_0, b) = q_5$$

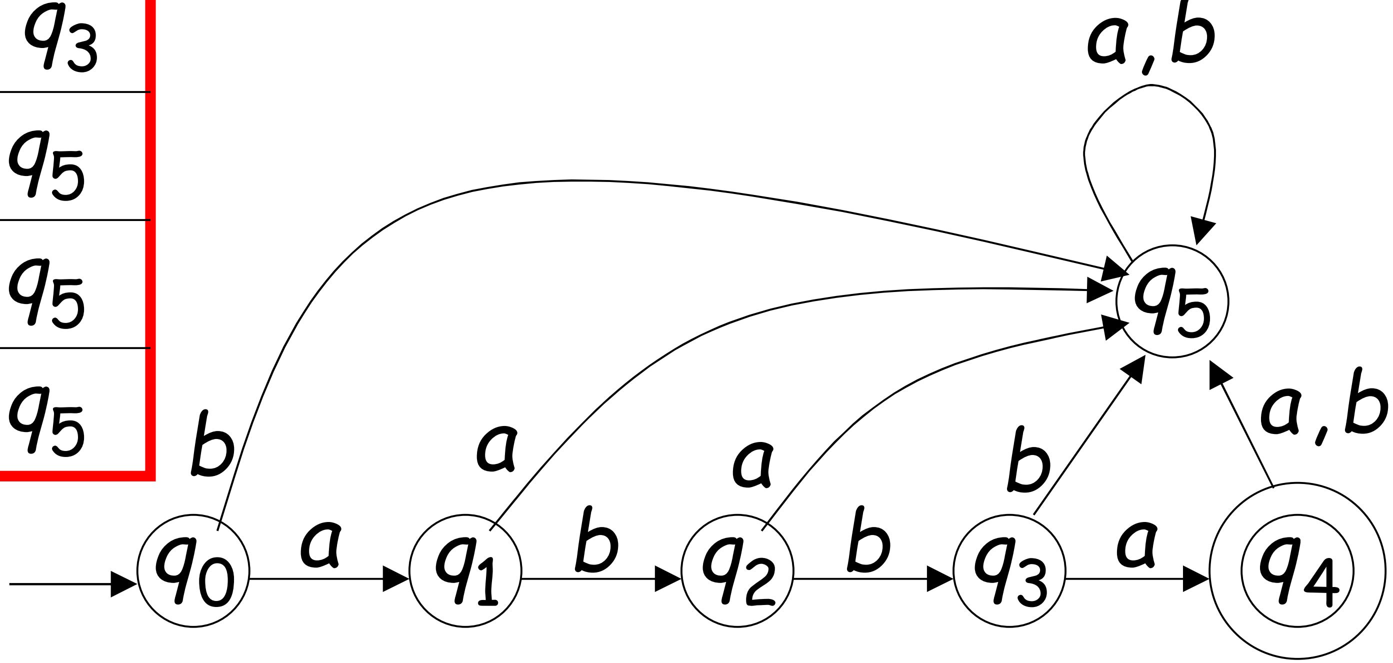


$$\delta(q_2, b) = q_3$$



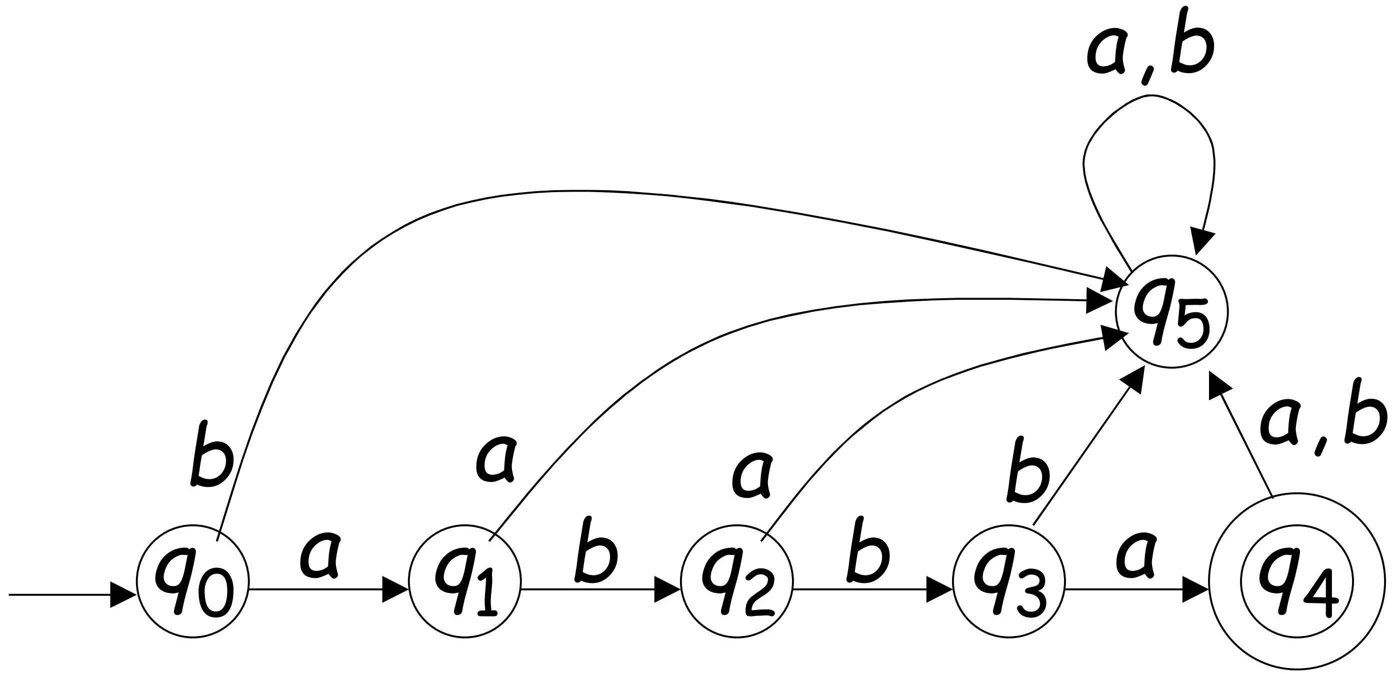
Transition Function δ

δ	a	b
q_0	q_1	q_5
q_1	q_5	q_2
q_2	q_5	q_3
q_3	q_4	q_5
q_4	q_5	q_5
q_5	q_5	q_5

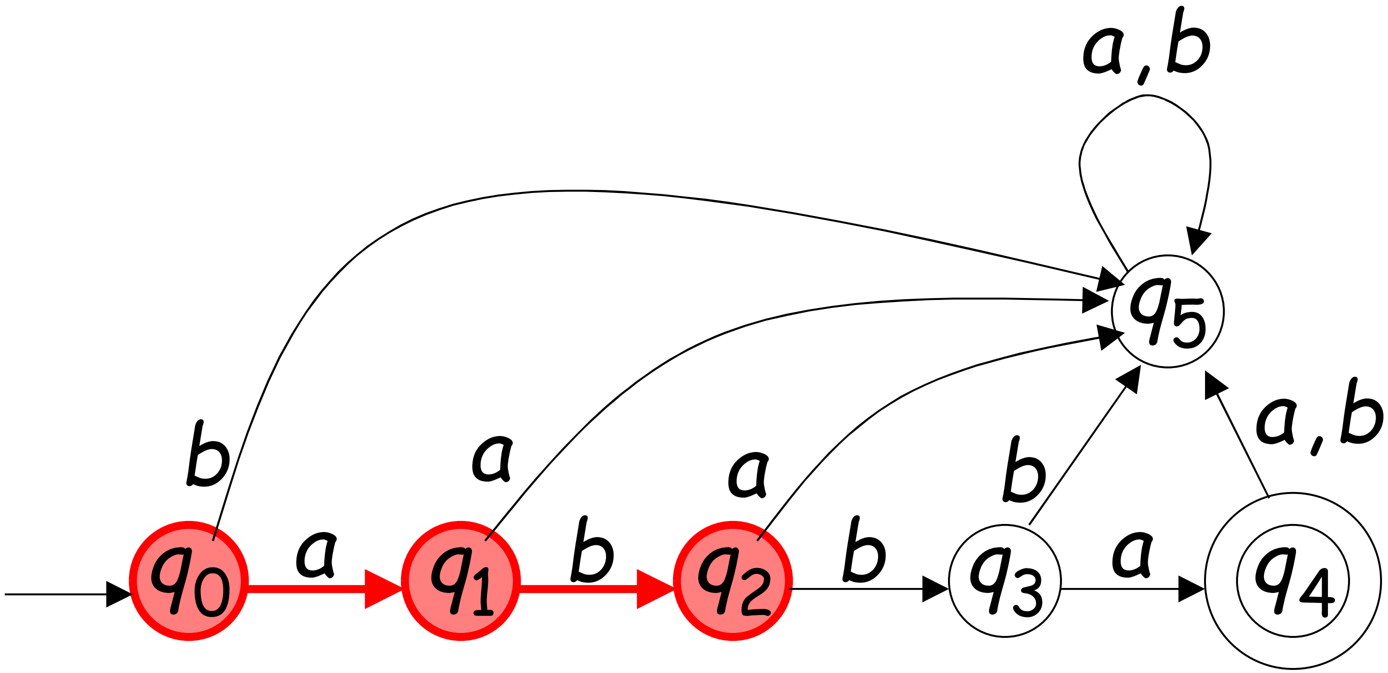


Extended Transition Function $\delta *$

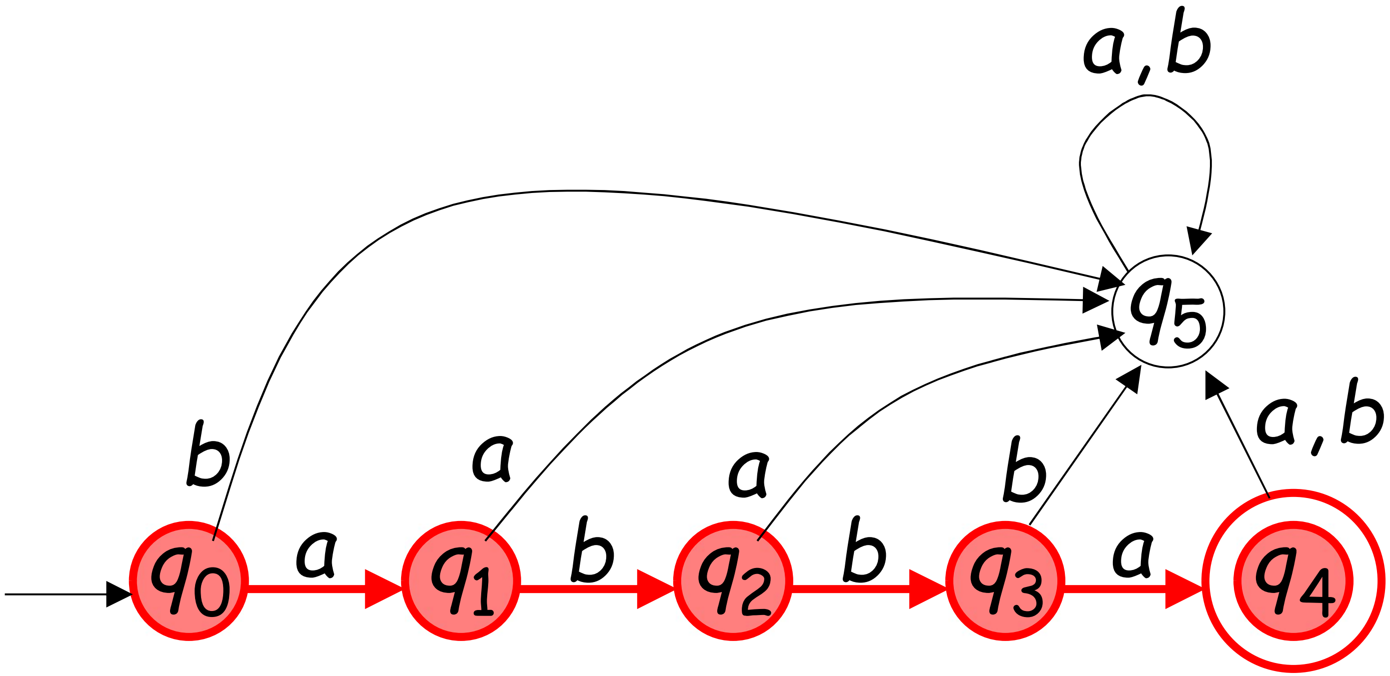
$$\delta^* : Q \times \Sigma^* \to Q$$



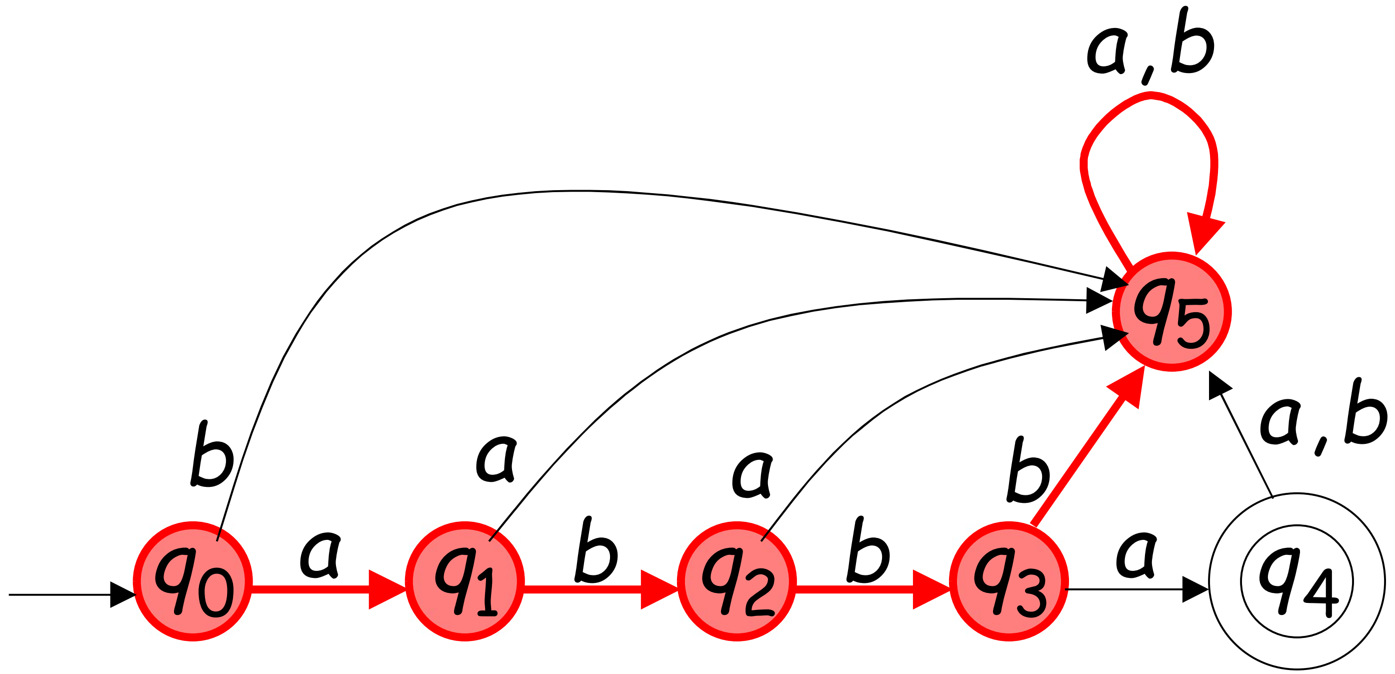
$$\delta^*(q_0, ab) = q_2$$



$$\delta^*(q_0, abba) = q_4$$



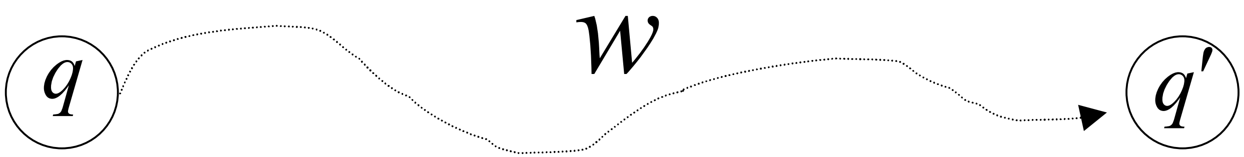
$$\delta^*(q_0, abbbaa) = q_5$$



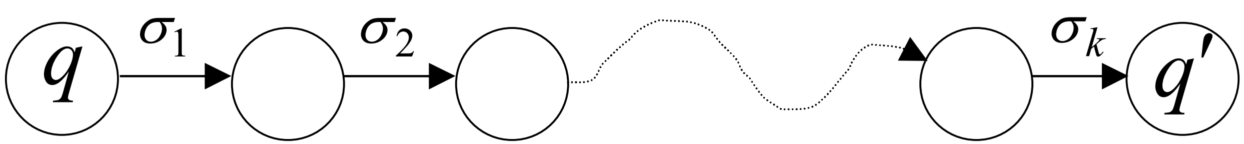
Observation: If there is a walk from q to q' with label w

Theorem 2.1

iff $\qquad \delta^*(q,w)=q'$

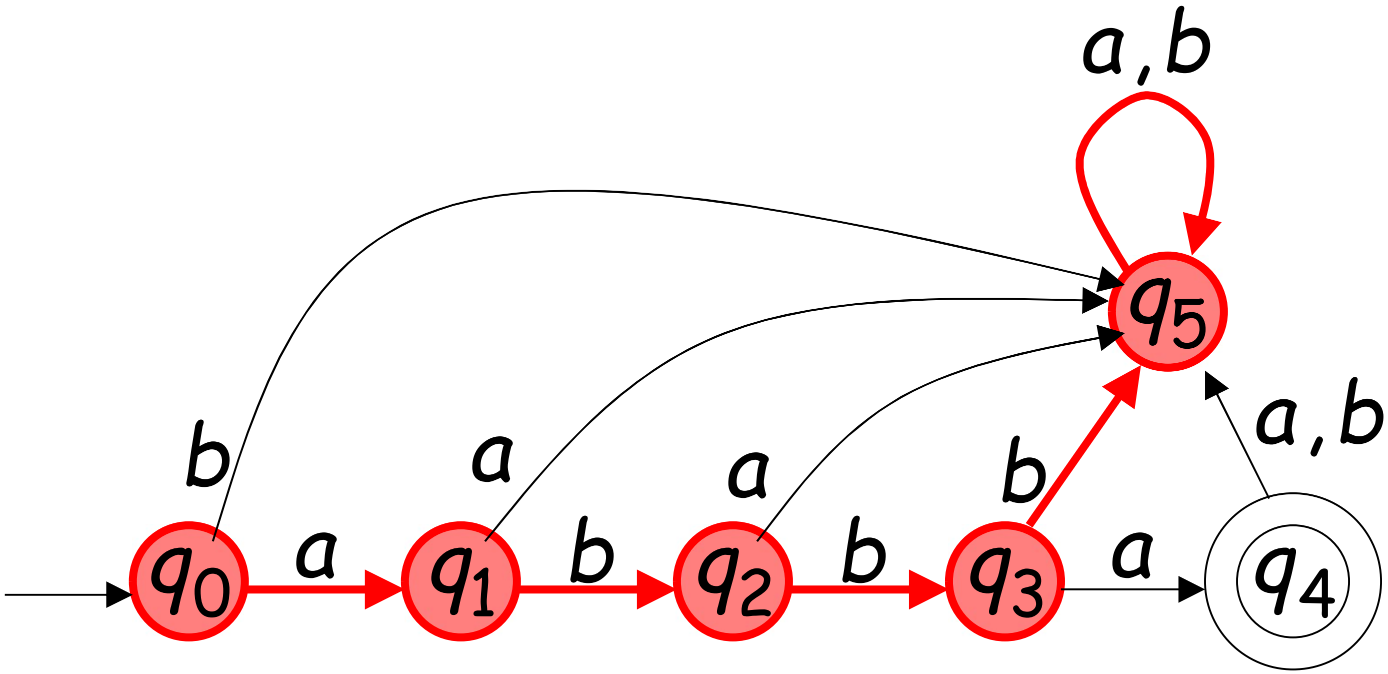


$$w=\sigma_1\sigma_2\cdots\sigma_k$$



Example: There is a walk from q_0 to q_5 with label $abbbaa$

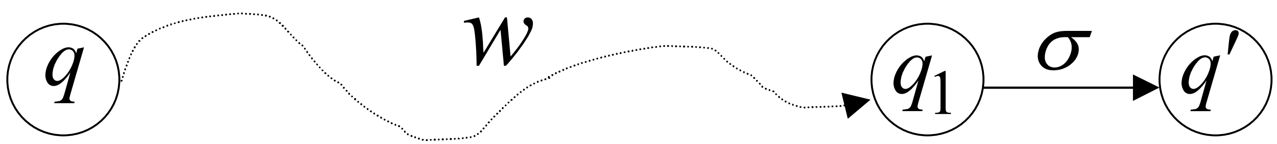
$$\delta^*(q_0, abbbaa) = q_5$$



Recursive Definition

$$\delta^*(q, \lambda) = q$$

$$\delta^*(q, w\sigma) = \delta(\delta^*(q, w), \sigma)$$



$$\left.\begin{array}{l} \delta^*(q, w\sigma) = q' \\ \\ \delta(q_1, \sigma) = q' \end{array}\right\} \Rightarrow \delta^*(q, w\sigma) = \delta(q_1, \sigma)$$

$$\left.\begin{array}{l} \delta^*(q, w\sigma) = \delta(q_1, \sigma) \\ \\ \delta^*(q, w) = q_1 \end{array}\right\} \Rightarrow \delta^*(q, w\sigma) = \delta(\delta^*(q, w), \sigma)$$

$$\delta^*(q_0, ab) =$$

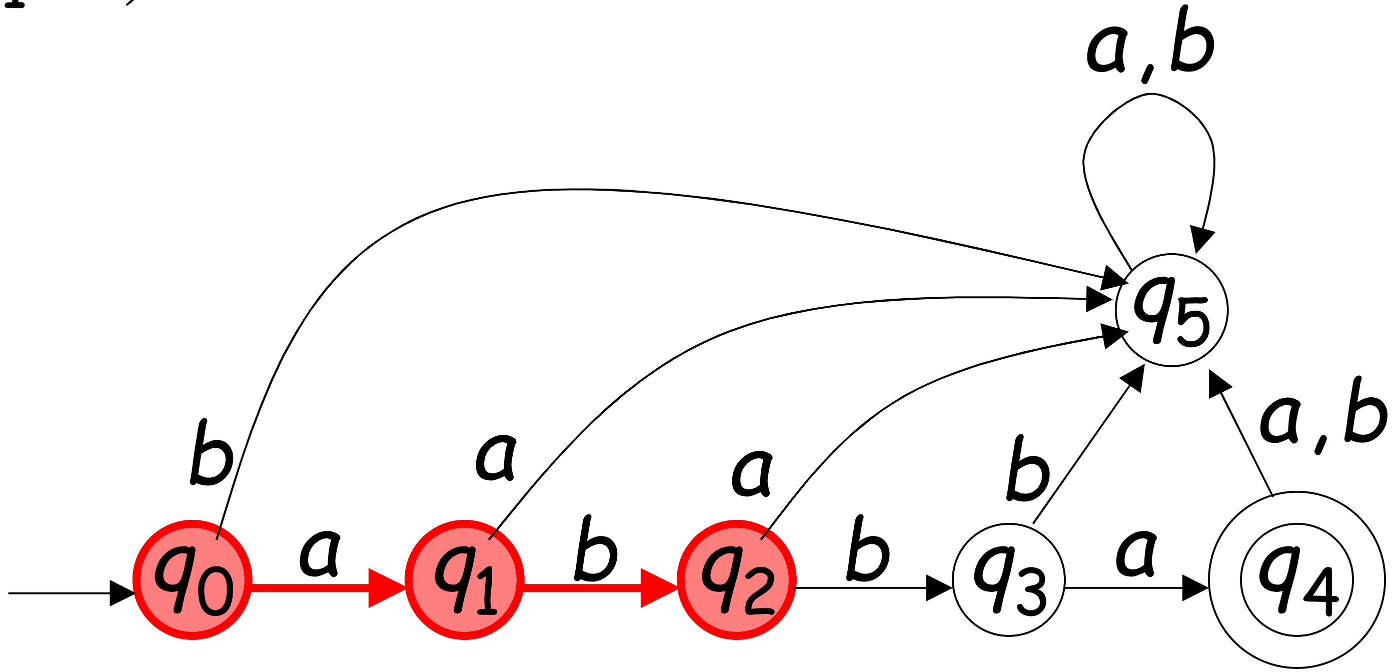
$$\delta(\delta^*(q_0, a), b) =$$

$$\delta(\delta(\delta^*(q_0, \lambda), a), b) =$$

$$\delta(\delta(q_0, a), b) =$$

$$\delta(q_1, b) =$$

$$q_2$$



Languages Accepted by DFAs

Take DFA M

Definition:

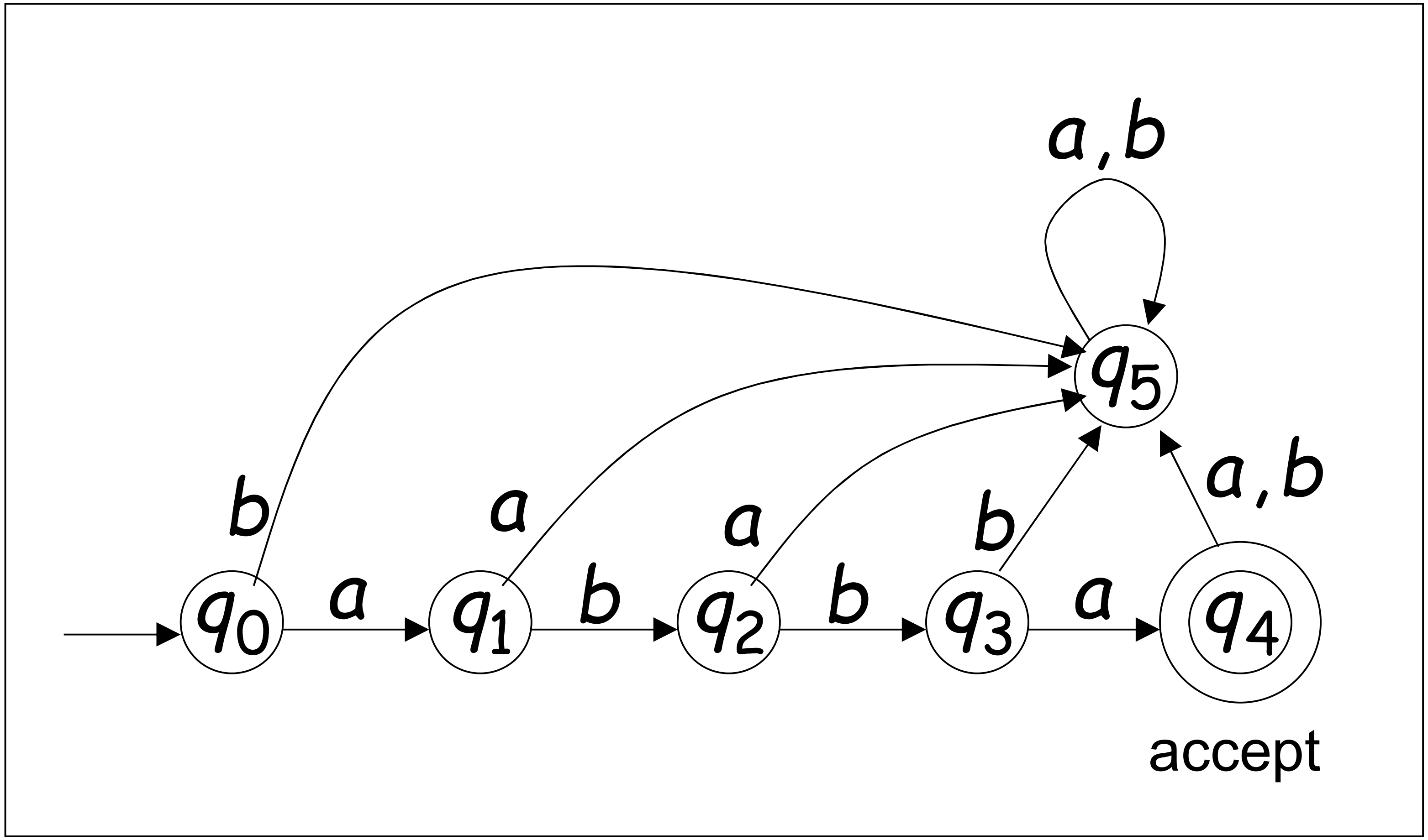
- The language $L(M)$ contains all input strings accepted by M

- $L(M)$= { strings that drive M to a final state}

Example

$$L(M) = \{abba\}$$

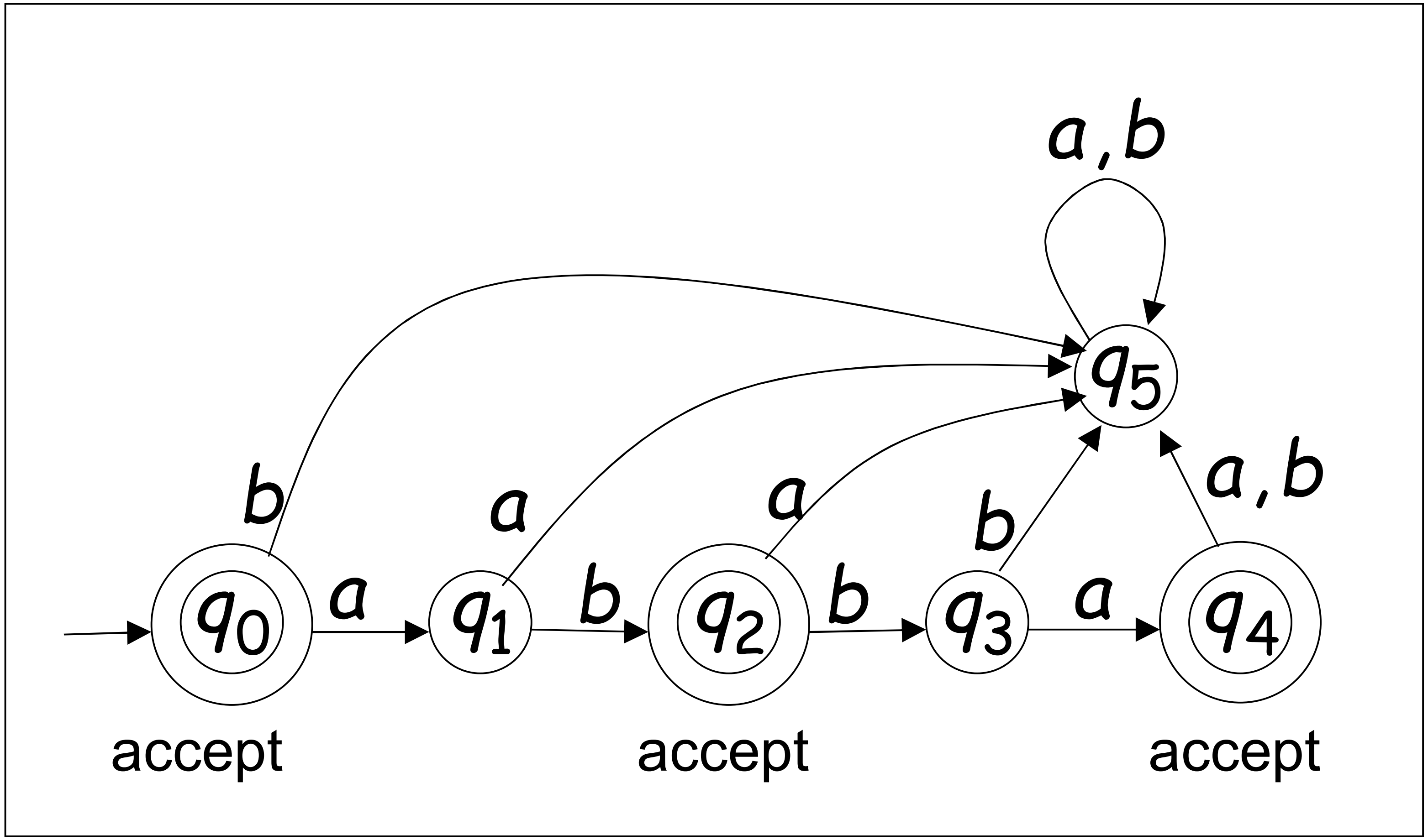
M



Another Example

$$L(M)=\{\lambda, ab, abba\}$$

M



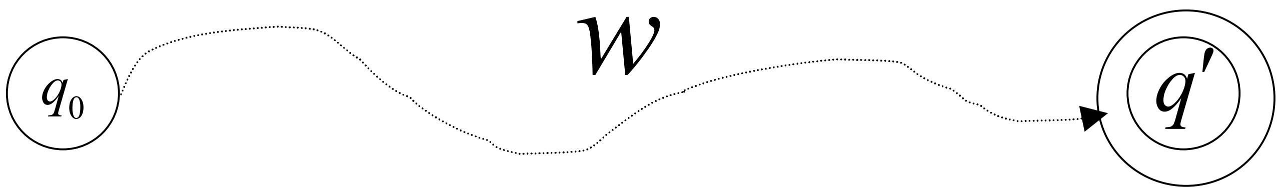
Formally

For a DFA $M = (Q, \Sigma, \delta, q_0, F)$

Language accepted by M :

$$L(M) = \{ w \in \Sigma^* : \delta^*(q_0, w) \in F \}$$

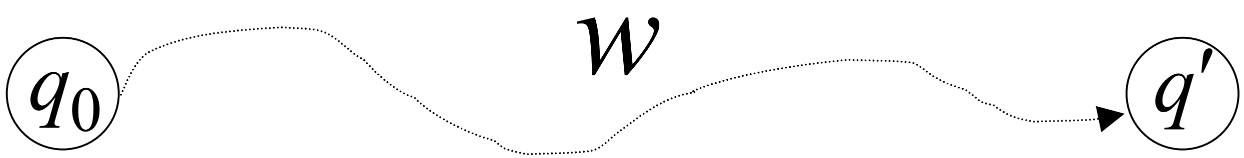
$q' \in F$



Observation

- Language rejected by M :

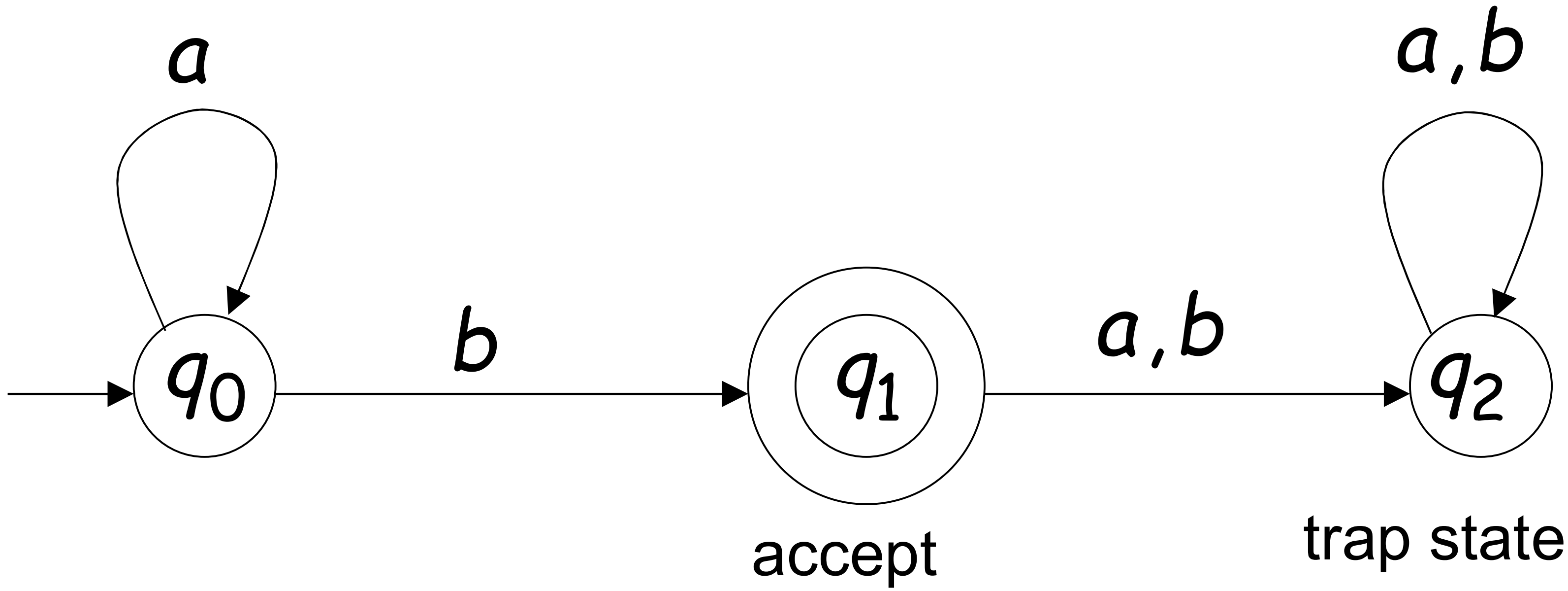
$$\overline{L(M)} = \{ w \in \Sigma^* : \delta^*(q_0, w) \notin F \}$$



$q' \notin F$

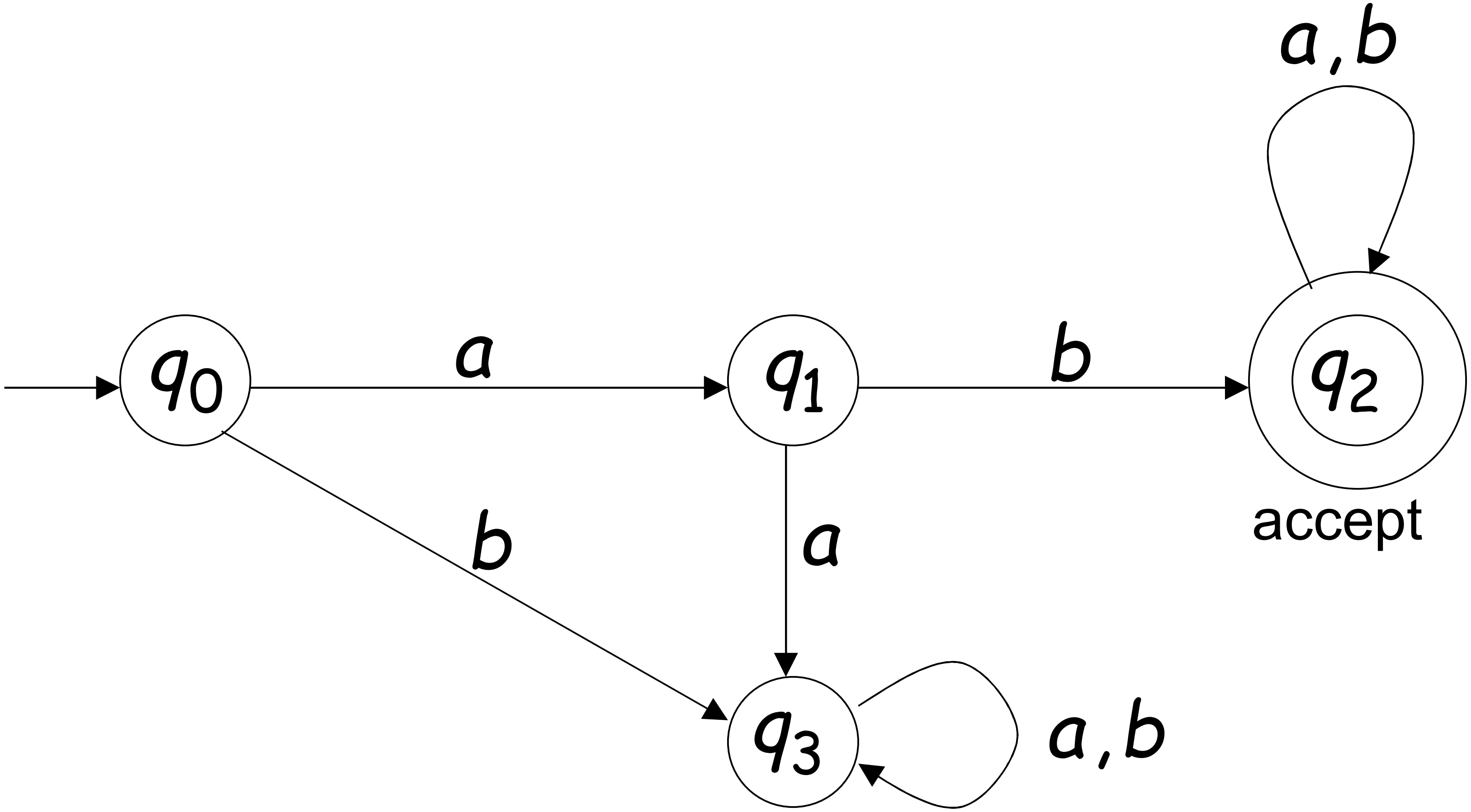
Example 2.2 $M=(Q,\Sigma,\delta,q_0,F)$

$$L(M)=\{a^n b : n \geq 0\}$$



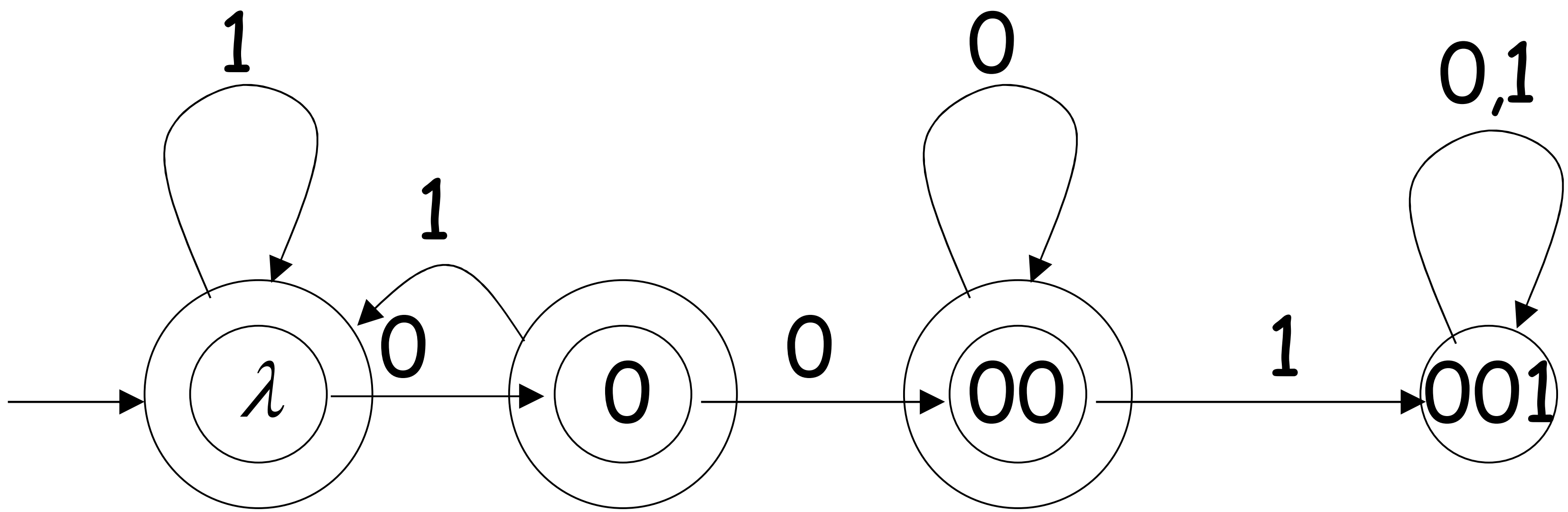
Example 2.3

$L(M)$ = { all strings with prefix ab }



Example 2.4

$L(M)$ = { all strings without substring 001 }



Regular Languages

A language L is regular iff there exists some DFA M such that $L = L(M)$

All regular languages form a language family

Examples of regular languages:

$\{abba\}$ $\{\lambda, ab, abba\}$ $\{a^n b : n \geq 0\}$

{ all strings with prefix ab }

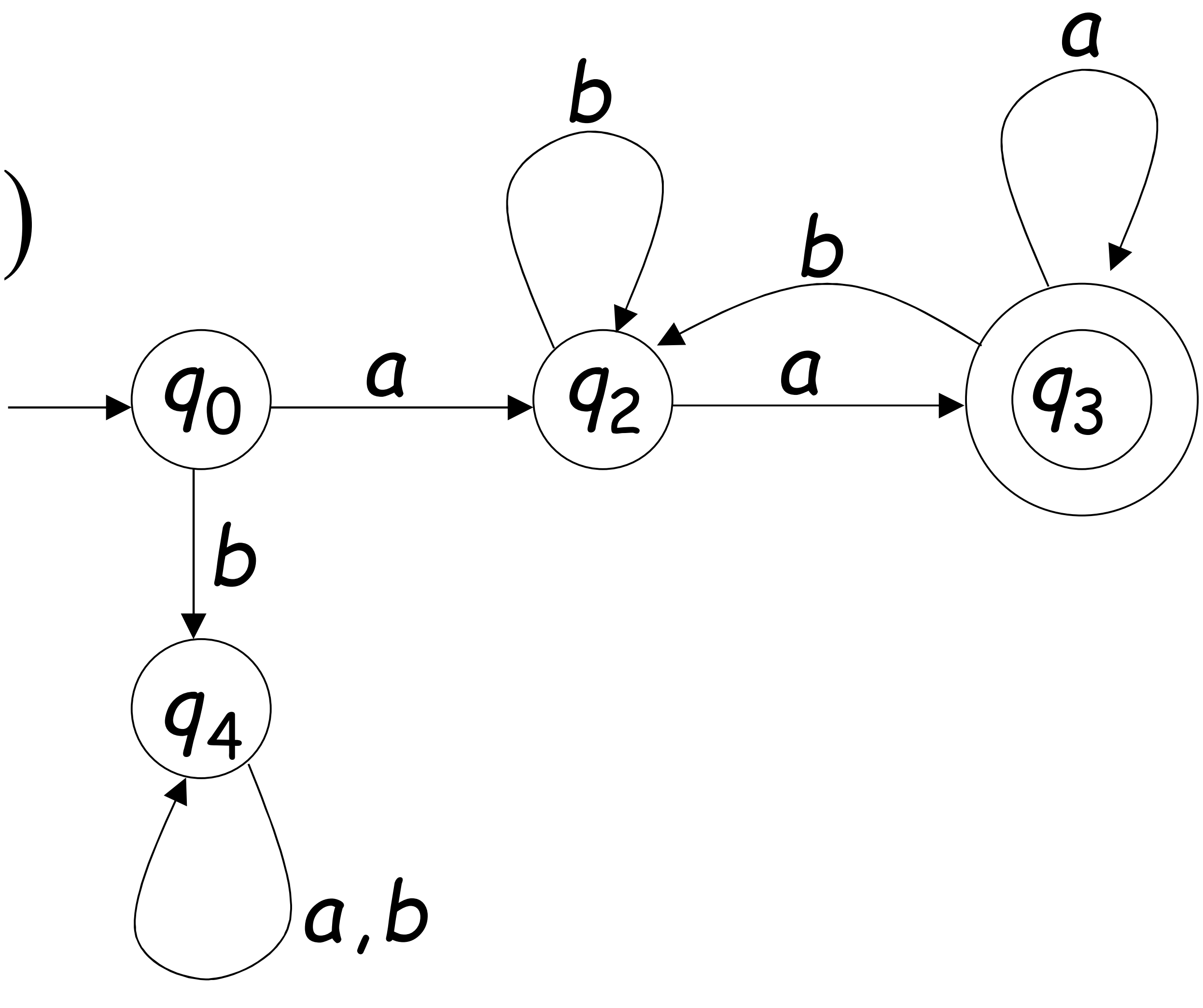
{ all strings without substring 001 }

There exists DFA that accept these Languages

Example 2.5

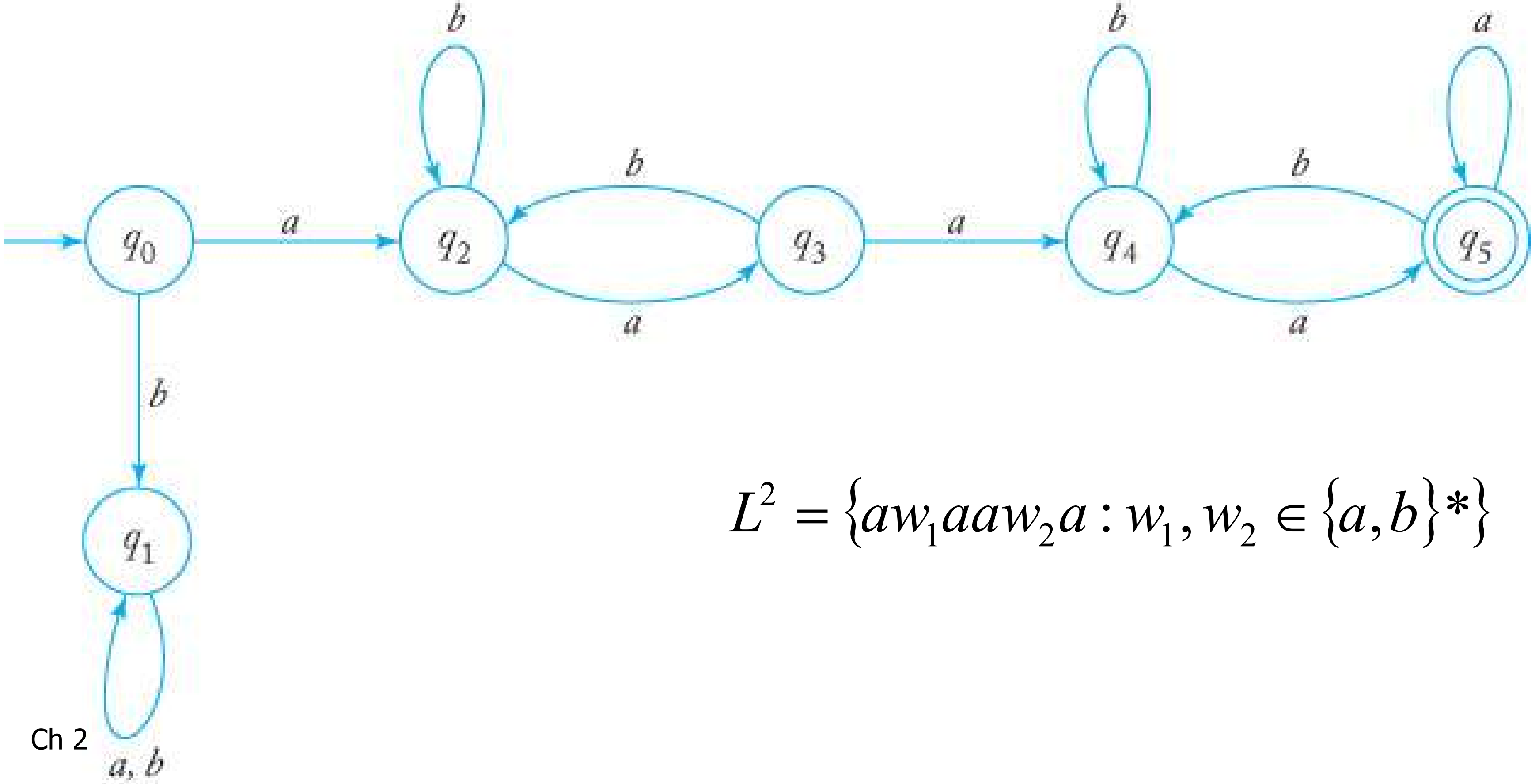
The language $L = \{awa : w \in \{a,b\}^*\}$ is regular:

$L = L(M)$



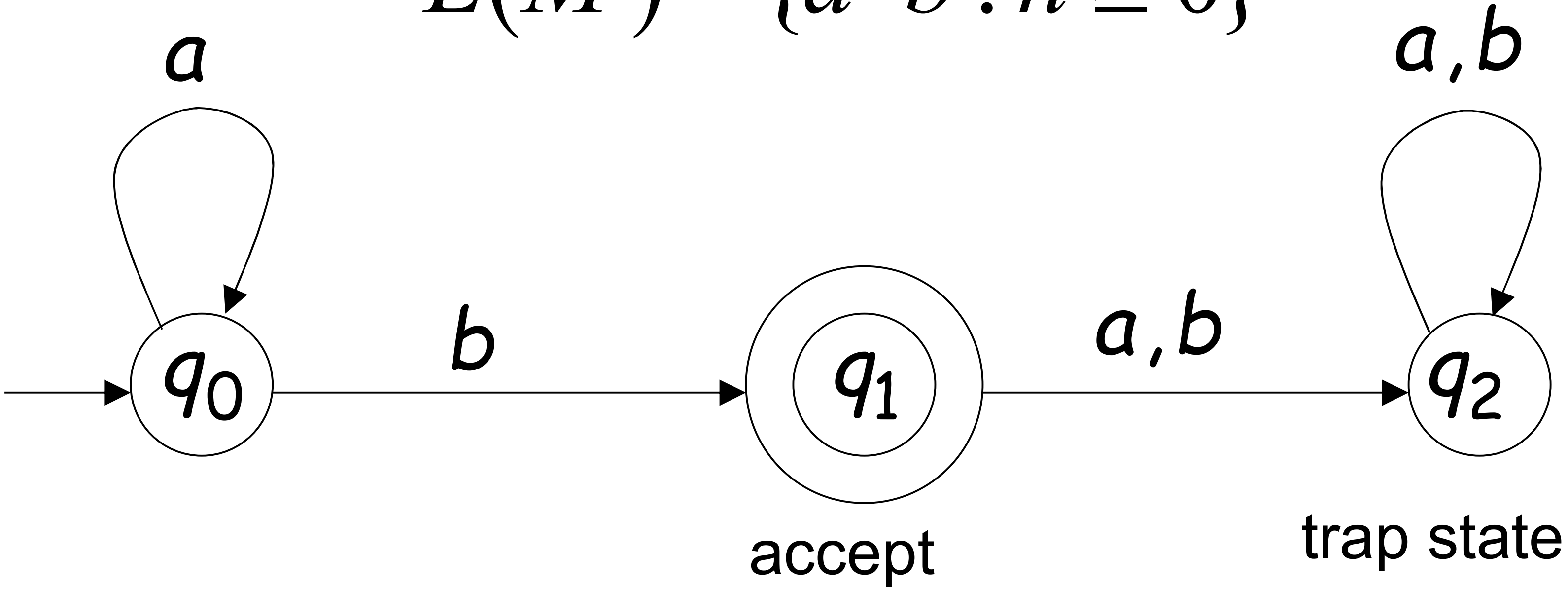
Example 2.6

The language $L=\{awa : w \in \{a,b\}^*\}$
is regular, how about L^2?



$$L^2=\{aw_1aaw_2a : w_1, w_2 \in \{a,b\}^*\}$$

$$L(M) = \{a^n b : n \geq 0\}$$



$$\boxed{L = \{a^n b^n : n \geq 0\}} \ ?$$

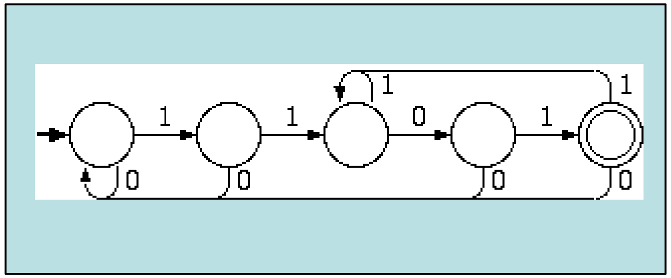
There exist languages which are not Regular:

There is no DFA that accepts such a language

(we will prove this later in the class)

DFA Recap

Input String



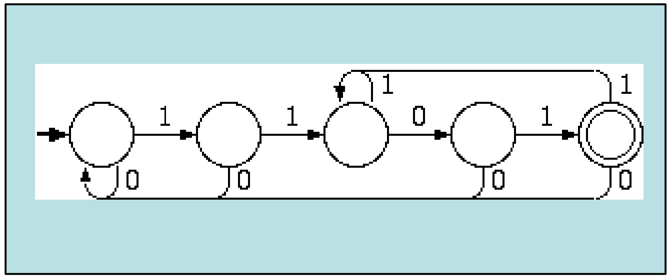
DFA

Accept or Reject

- A machine with finite number of states, some states are accepting states, others are rejecting states
- At any time, it is in one of the states
- It reads an input string, one character at a time

DFA Recap

Input String



Accept or Reject

DFA

- After reading each character, it moves to another state depending on what is read and what is the current state
- If reading all characters, the DFA is in an accepting state, the input string is accepted.
- Otherwise, the input string is rejected.

Definition 2.1

Deterministic Finite Accepter (DFA) is define by the 5-tuple

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : a finite set of **internal states**

Σ : a finite set of symbols called **input alphabet**

δ : $Q \times \Sigma \rightarrow Q$ called **transition function**

q_0 : $q_0 \in Q$ is the **initial state**

F : $F \subseteq Q$ is a set of **final states**

Regular Languages

A language L is regular iff there exists some DFA M such that $L = L(M)$

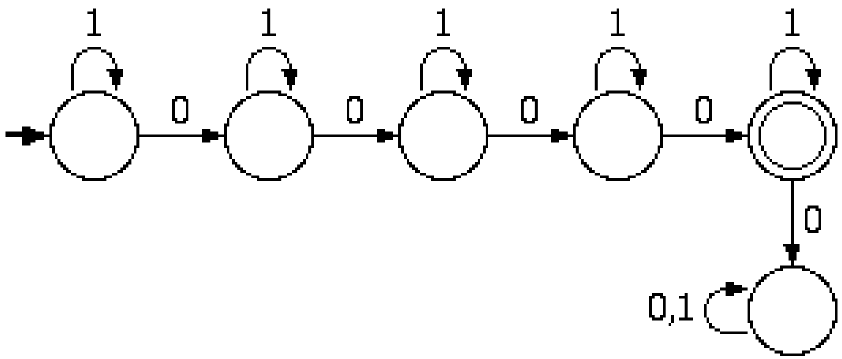
Definition:

– The language $L(M)$ contains all input strings accepted by a DFA M

– $L(M)$= { strings that drive M to a final state}

More Examples

- All strings that contain exactly 4 "0"s.
- All strings containing exactly 4 "0" s and at least 2 "1" s.
- All strings of length at most five.
- All strings ending in "1101".
- All strings whose binary interpretation is divisible by 5.
- All strings that contain the substring 0101.
- All strings that don’t contain the substring 110.



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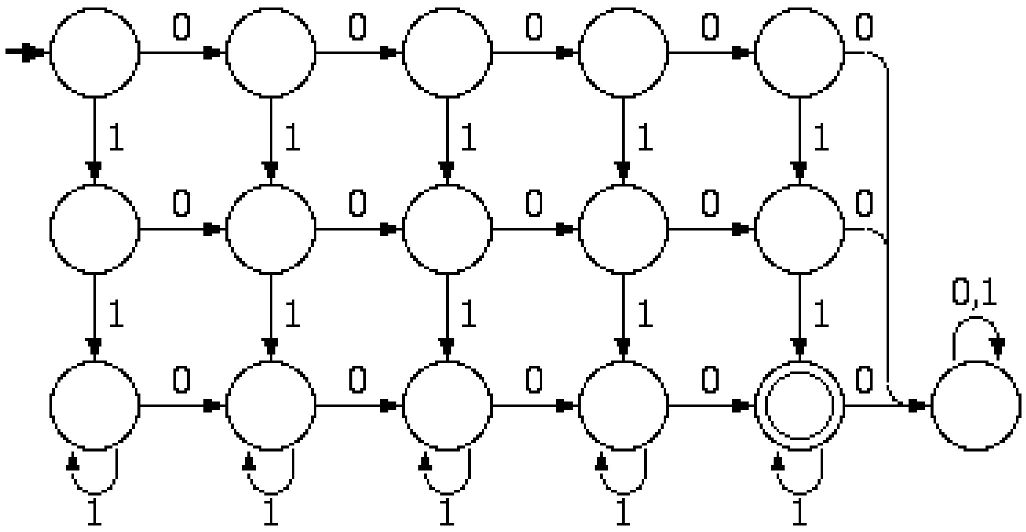

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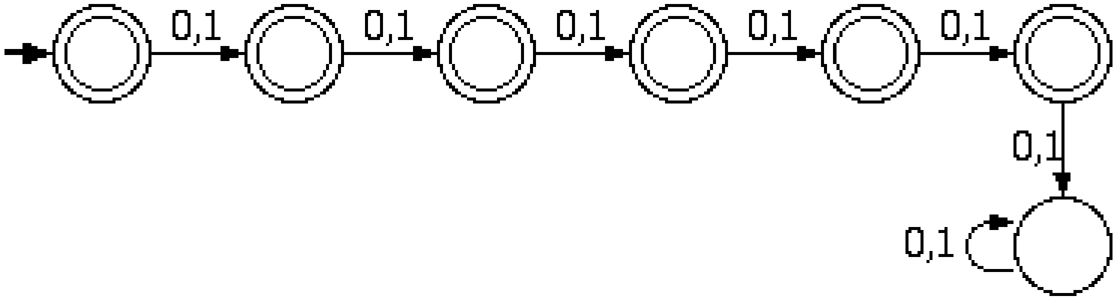

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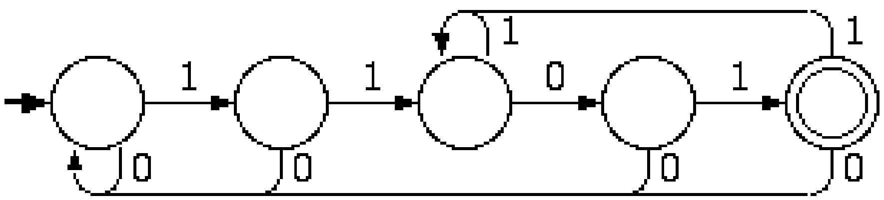

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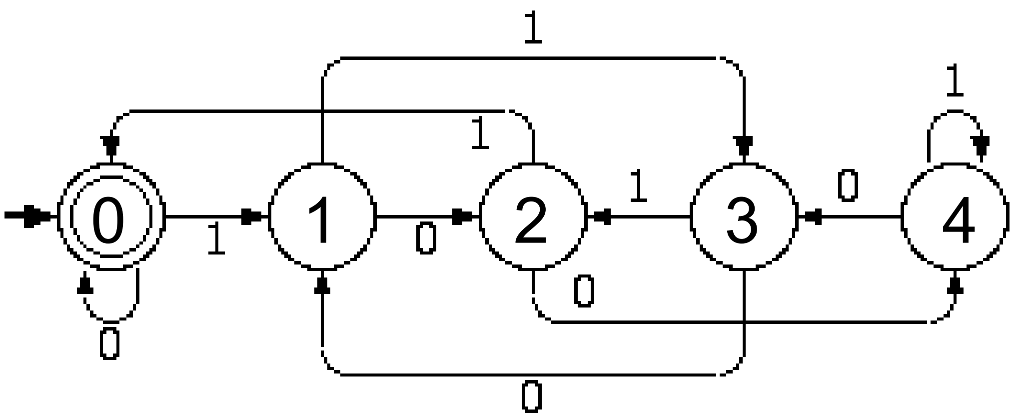

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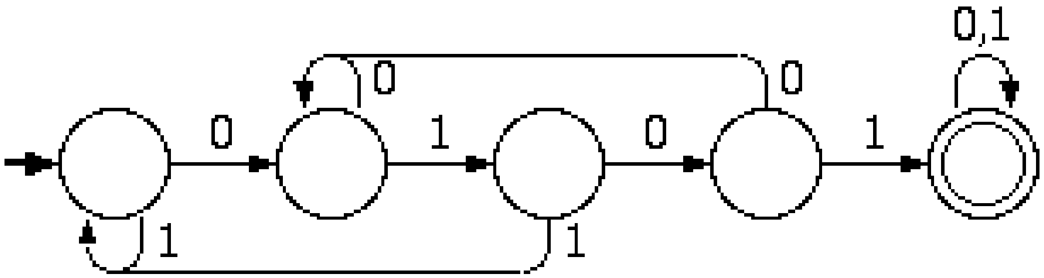

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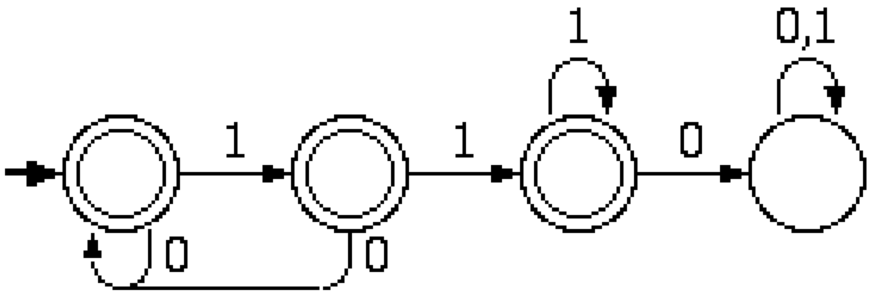
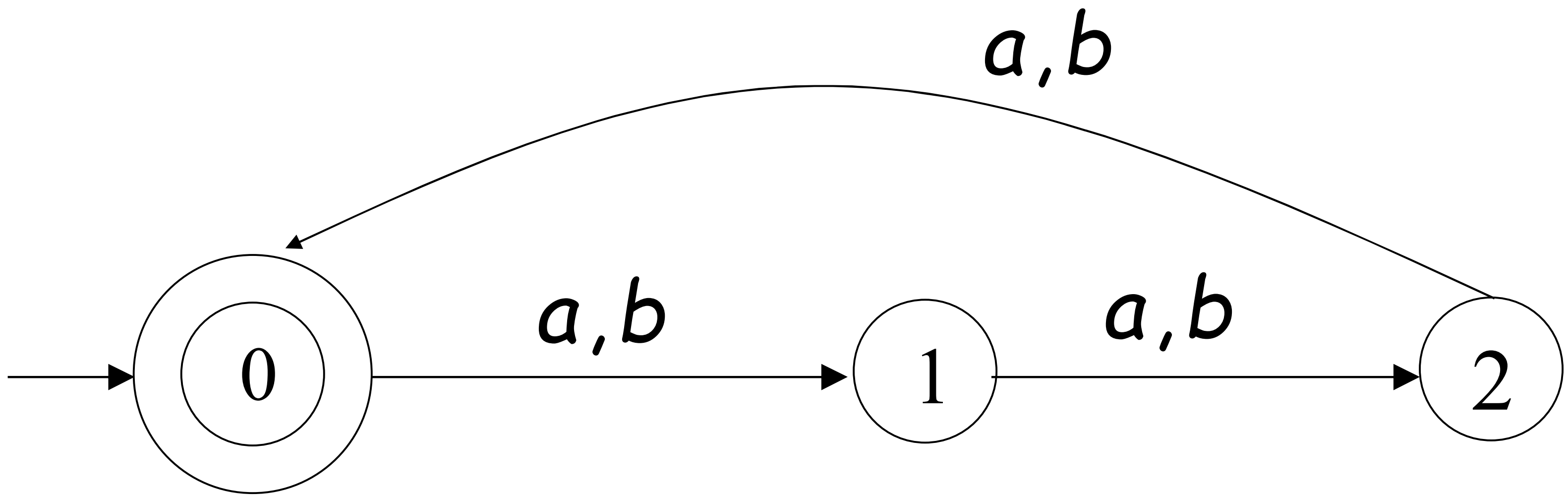


Exercise 2.1.7

Find DFAs for the following languages on ∑={a,b}

(a) L = {w: |w| mod 3 = 0}

(b) L = {w: n_a(w) mod 3 > n_b(w) mod 3}



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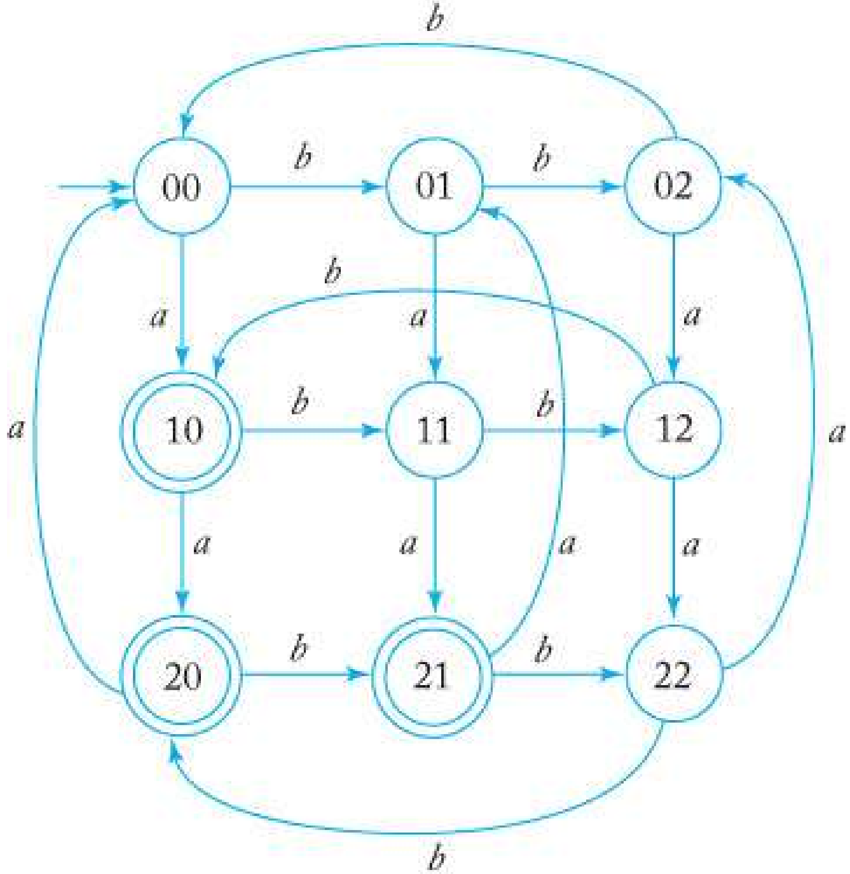


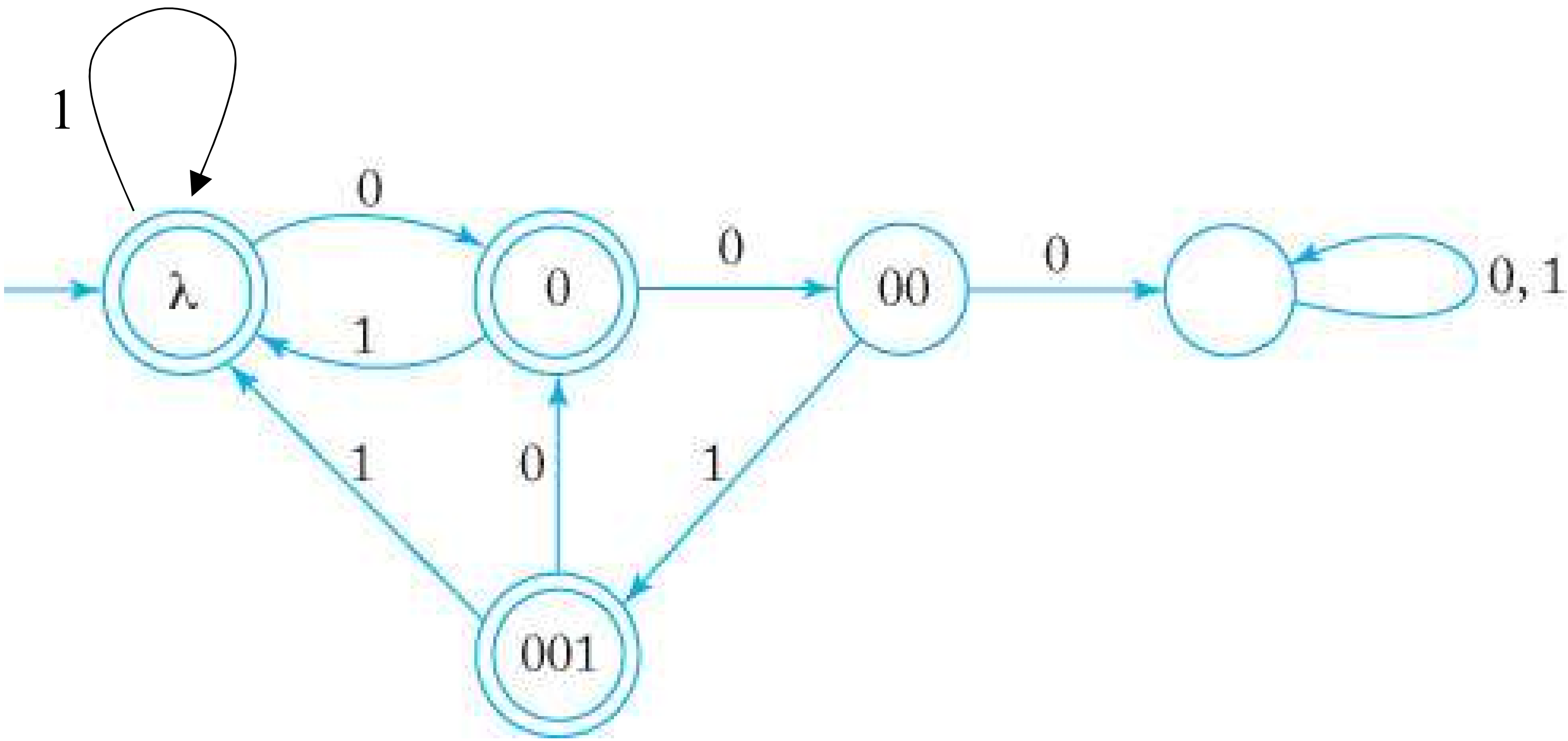
Exercise 2.1.9

(a)Every 00 is followed immediately by a 1.

Ex: 101, 0010,0010011001 ϵ L

0001 and 00100 ∉ L

$\Sigma = \{0,1\}$



Questions?

Short Quiz

- All strings that start with 0 and have odd length or start with 1 and have even length.
- All strings where every odd position is a 1.

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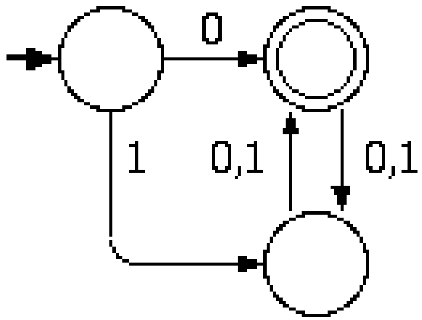

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