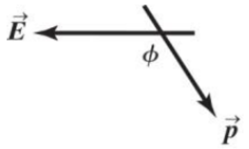
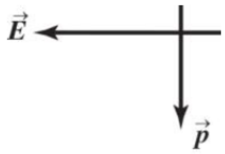
21.56. **IDENTIFY:** An electric dipole is in an external electric field. We want to know about the torque on this dipole and its electric potential energy due to this field.

SET UP: The torque is $\vec{\tau} = \vec{p} \times \vec{E}$ and the potential energy is $U = -\vec{p} \cdot \vec{E} = -pE\cos\phi$.

EXECUTE: **(a)** We want the orientation of the dipole that will produce the maximum torque into the paper. Fig. 21.56a shows the orientation so that the torque will be into the page. The magnitude of the torque is $\tau = pE\sin\phi$, which is a maximum when $\phi = 90°$. Fig. 21.56b shows this orientation, for which $\vec{p}$ is downward. The potential energy at this angle is $U = -pE\cos\phi = -pE\cos 90° = 0$.





Figures 21.56a and 21.56b

(b) We want the orientation so that the torque is zero and the potential energy is a maximum. The magnitude of the torque is $\tau = pE\sin\phi$, which is zero for $\phi = 0°$ or $180°$.

For $\phi = 0°$: $U = -pE\cos 0° = -pE$.

For $\phi = 180°$: $U = -pE\cos 180° = +pE$.

As we can see, the orientation for maximum potential energy is $\phi = 180°$. At this angle, $\vec{p}$ points opposite to the electric field. To find the type of equilibrium, imagine displacing the dipole a small angle from the $\phi = 180°$ equilibrium position. Fig. 21.56c shows the dipole and the electric forces on the dipole. As the figure shows, the torques tend to rotate the dipole *away from* the equilibrium position, so this is an *unstable* equilibrium.

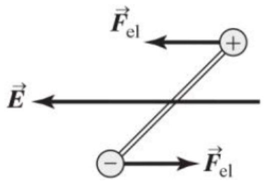


Figure 21.56c

21.83. **IDENTIFY:** Divide the charge distribution into small segments, use the point charge formula for the electric field due to each small segment and integrate over the charge distribution to find the x- and y-components of the total field.

SET UP: Consider the small segment shown in Figure 21.83a.

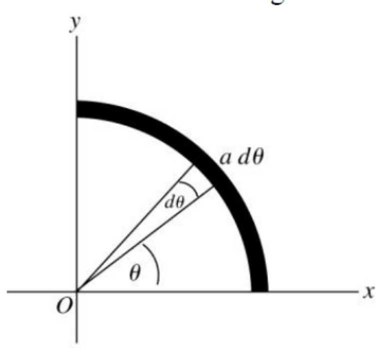
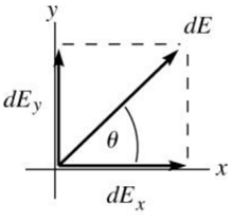


Figure 21.83a

EXECUTE: A small segment that subtends angle $d\theta$ has length $a\,d\theta$ and contains charge $dQ = \left(\frac{a\,d\theta}{\frac{1}{2}\pi a}\right)Q = \frac{2Q}{\pi}d\theta$. ($\frac{1}{2}\pi a$ is the total length of the charge distribution.)

The charge is negative, so the field at the origin is directed toward the small segment. The small segment is located at angle θ as shown in the sketch. The electric field due to dQ is shown in Figure 21.83b, along with its components.



$$dE = \frac{1}{4\pi \epsilon_0} \frac{|dQ|}{a^2}.$$

$$dE = \frac{Q}{2\pi^2 \epsilon_0 a^2} d\theta.$$

Figure 21.83b

$$dE_x = dE\cos\theta = (Q/2\pi^2 \epsilon_0 a^2)\cos\theta d\theta.$$

$$E_x = \int dE_x = \frac{Q}{2\pi^2 \epsilon_0 a^2}\int_0^{\pi/2}\cos\theta d\theta = \frac{Q}{2\pi^2 \epsilon_0 a^2}(\sin\theta\Big|_0^{\pi/2}) = \frac{Q}{2\pi^2 \epsilon_0 a^2}.$$

$$dE_y = dE\sin\theta = (Q/2\pi^2 \epsilon_0 a^2)\sin\theta d\theta.$$

$$E_y = \int dE_y = \frac{Q}{2\pi^2 \epsilon_0 a^2}\int_0^{\pi/2}\sin\theta d\theta = \frac{Q}{2\pi^2 \epsilon_0 a^2}(-\cos\theta\Big|_0^{\pi/2}) = \frac{Q}{2\pi^2 \epsilon_0 a^2}.$$

21.84. **IDENTIFY:** We must add the electric field components of the positive half and the negative half.

SET UP: From Problem 21.83, the electric field due to the quarter-circle section of positive charge has components $E_x = +\frac{Q}{2\pi^2 \epsilon_0 a^2}$, $E_y = -\frac{Q}{2\pi^2 \epsilon_0 a^2}$. The field due to the quarter-circle section of negative charge has components $E_x = +\frac{Q}{2\pi^2 \epsilon_0 a^2}$, $E_y = +\frac{Q}{2\pi^2 \epsilon_0 a^2}$.

EXECUTE: The components of the resultant field are the sum of the x- and y-components of the fields due to each half of the semicircle. The y-components cancel, but the x-components add, giving

$E_x = +\frac{Q}{\pi^2 \epsilon_0 a^2}$, in the $+x$-direction.