24.48. IDENTIFY: $C = KC_0 = K\epsilon_0 \dfrac{A}{d}$. $V = Ed$ for a parallel plate capacitor; this equation applies whether or not a dielectric is present.

SET UP: $A = 1.0\ \text{cm}^2 = 1.0\times10^{-4}\ \text{m}^2$.

EXECUTE: **(a)** $C = (10)\dfrac{(8.85\times10^{-12}\ \text{F/m})(1.0\times10^{-4}\ \text{m}^2)}{7.5\times10^{-9}\ \text{m}} = 1.18\ \mu\text{F per cm}^2$.

(b) $E = \dfrac{V}{d} = \dfrac{85\ \text{mV}}{7.5\times10^{-9}\ \text{m}} = 1.13\times10^{7}\ \text{V/m}$.

24.54. IDENTIFY and SET UP: We want to estimate the excess charge rubbed onto our head and the resulting voltage when we comb our hair. Treat the head as a sphere and model it as a spherical capacitor.

EXECUTE: **(a)** Estimate: $L \approx 15$ cm = 0.15 m long.

(b) m = (65 μg/cm)(15 cm) $\approx$ 975 μg.

(c) Estimate: N = 25 hairs.

(d) The electric force $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1q_2|}{r^2}$ is the force between the charges at both ends of a hair. $q_1 = q_2 = Q/N$, $r = L$, and the force is twice the weight of the hair, which is $2mg$. Therefore $2mg = \dfrac{1}{4\pi\epsilon_0}\dfrac{(Q/N)^2}{L^2}$. Solve for Q: $Q = \sqrt{(2mg)(4\pi\epsilon_0)L^2N}$. Using m = 975 μg, L = 0.15 m, and N = 25 gives Q = 0.35 μC. The total charge on your head is $2Q$, so Q_{head} = 0.70 μC.

(e) Estimate: Diameter $\approx$ 22 cm, so $R \approx 11$ cm. $C = 4\pi\epsilon_0 R = 4\pi\epsilon_0\,(0.11\ \text{m}) = 12$ pF.

(f) $V = Q/C$ = (0.35 μC)/(12 pF) = 29 kV.

24.57. (a) IDENTIFY: Replace the three capacitors in series by their equivalent. The charge on the equivalent capacitor equals the charge on each of the original capacitors.

SET UP: The three capacitors can be replaced by their equivalent as shown in Figure 24.57a.

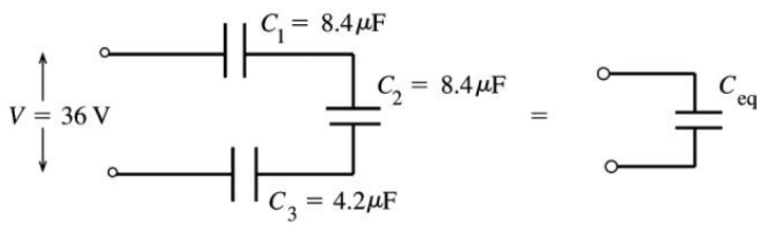

Figure 24.57a

EXECUTE: $C_3 = C_1/2$ so $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} = \dfrac{4}{8.4\ \mu\text{F}}$ and $C_{\text{eq}} = 8.4\ \mu\text{F}/4 = 2.1\ \mu\text{F}$.

$Q = C_{\text{eq}}V = (2.1\ \mu\text{F})(36\ \text{V}) = 76\ \mu\text{C}$.

The three capacitors are in series so they each have the same charge: $Q_1 = Q_2 = Q_3 = 76\ \mu\text{C}$.

EVALUATE: The equivalent capacitance for capacitors in series is smaller than each of the original capacitors.

(b) IDENTIFY and SET UP: Use $U = \frac{1}{2}QV$. We know each Q and we know that $V_1 + V_2 + V_3 = 36$ V.

EXECUTE: $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$.

But $Q_1 = Q_2 = Q_3 = Q$ so $U = \frac{1}{2}Q(V_1 + V_2 + V_3)$.

But also $V_1 + V_2 + V_3 = V = 36$ V, so $U = \frac{1}{2}QV = \frac{1}{2}(76\ \mu\text{C})(36\ \text{V}) = 1.4\times10^{-3}$ J.

EVALUATE: We could also use $U = Q^2/2C$ and calculate U for each capacitor.

(c) IDENTIFY: The charges on the plates redistribute to make the potentials across each capacitor the same.

SET UP: The capacitors before and after they are connected are sketched in Figure 24.57b.

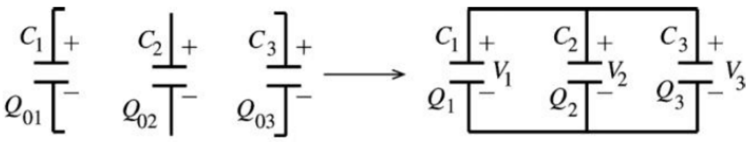

Figure 24.57b

EXECUTE: The total positive charge that is available to be distributed on the upper plates of the three capacitors is $Q_0 = Q_{01} + Q_{02} + Q_{03} = 3(76\ \mu\text{C}) = 228\ \mu\text{C}$. Thus $Q_1 + Q_2 + Q_3 = 228\ \mu\text{C}$. After the circuit is completed the charge distributes to make $V_1 = V_2 = V_3$. $V = Q/C$ and $V_1 = V_2$ so $Q_1/C_1 = Q_2/C_2$ and then $C_1 = C_2$ says $Q_1 = Q_2$. $V_1 = V_3$ says $Q_1/C_1 = Q_3/C_3$ and $Q_1 = Q_3(C_1/C_3) = Q_3(8.4\ \mu\text{F}/4.2\ \mu\text{F}) = 2Q_3$.

Using $Q_2 = Q_1$ and $Q_1 = 2Q_3$ in the above equation gives $2Q_3 + 2Q_3 + Q_3 = 228\ \mu\text{C}$.

$5Q_3 = 228\ \mu\text{C}$ and $Q_3 = 45.6\ \mu\text{C}$, $Q_1 = Q_2 = 91.2\ \mu\text{C}$

Then $V_1 = \dfrac{Q_1}{C_1} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11\ \text{V}$, $V_2 = \dfrac{Q_2}{C_2} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11\ \text{V}$, and $V_3 = \dfrac{Q_3}{C_3} = \dfrac{45.6\ \mu\text{C}}{4.2\ \mu\text{F}} = 11\ \text{V}$.

The voltage across each capacitor in the parallel combination is 11 V.

(d) $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$.

But $V_1 = V_2 = V_3$ so $U = \frac{1}{2}V_1(Q_1 + Q_2 + Q_3) = \frac{1}{2}(11\ \text{V})(228\ \mu\text{C}) = 1.3 \times 10^{-3}\ \text{J}$.

24.60. IDENTIFY: This situation is analogous to having two capacitors C_1 in series, each with separation $\frac{1}{2}(d - a)$.

SET UP: For capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

EXECUTE: **(a)** $C = \left(\dfrac{1}{C_1} + \dfrac{1}{C_1}\right)^{-1} = \frac{1}{2}C_1 = \frac{1}{2}\dfrac{\epsilon_0 A}{(d-a)/2} = \dfrac{\epsilon_0 A}{d-a}$.

(b) $C = \dfrac{\epsilon_0 A}{d-a} = \dfrac{\epsilon_0 A}{d}\dfrac{d}{d-a} = C_0\dfrac{d}{d-a}$.

EVALUATE: **(c)** As $a \to 0$, $C \to C_0$. The metal slab has no effect if it is very thin. And as $a \to d$, $C \to \infty$. $V = Q/C$. $V = Ey$ is the potential difference between two points separated by a distance y parallel to a uniform electric field. When the distance is very small, it takes a very large field and hence a large Q on the plates for a given potential difference. Since $Q = CV$ this corresponds to a very large C.

24.62. IDENTIFY: The electric field energy density is $u = \frac{1}{2}\epsilon_0 E^2$. $U = \dfrac{Q^2}{2C}$.

SET UP: For this charge distribution, $E = 0$ for $r < r_a$, $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $r_a < r < r_b$ and $E = 0$ for $r > r_b$.

Example 24.4 shows that $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

EXECUTE: **(a)** $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\dfrac{\lambda}{2\pi\epsilon_0 r}\right)^2 = \dfrac{\lambda^2}{8\pi^2\epsilon_0 r^2}$.

(b) $U = \int u\,dV = 2\pi L\int u r\,dr = \dfrac{L\lambda^2}{4\pi\epsilon_0}\int_{r_a}^{r_b}\dfrac{dr}{r}$ and $\dfrac{U}{L} = \dfrac{\lambda^2}{4\pi\epsilon_0}\ln(r_b/r_a)$.

(c) $U = \dfrac{Q^2}{2C} = \dfrac{Q^2}{4\pi\epsilon_0 L}\ln(r_b/r_a) = \dfrac{\lambda^2 L}{4\pi\epsilon_0}\ln(r_b/r_a)$. This agrees with the result of part (b).

24.64. IDENTIFY: The capacitor is equivalent to two capacitors in parallel, as shown in Figure 24.64.

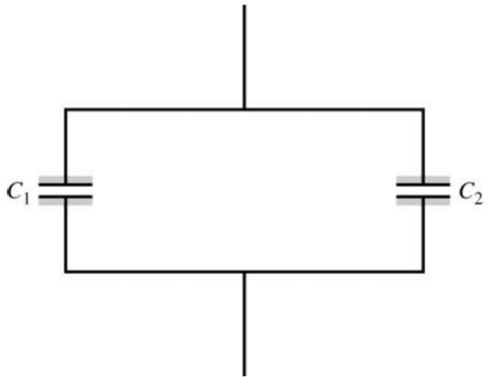

Figure 24.64

SET UP: Each of these two capacitors have plates that are 12.0 cm by 6.0 cm. For a parallel-plate capacitor with dielectric filling the volume between the plates, $C = K\epsilon_0\dfrac{A}{d}$. For two capacitors in parallel, $C = C_1 + C_2$. The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

EXECUTE: **(a)** $C = C_1 + C_2$.

$$C_2 = \epsilon_0\frac{A}{d} = \frac{(8.854\times10^{-12}\ \text{F/m})(0.120\ \text{m})(0.060\ \text{m})}{4.50\times10^{-3}\ \text{m}} = 1.42\times10^{-11}\ \text{F}.$$

$C_1 = KC_2 = (3.40)(1.42\times10^{-11}\ \text{F}) = 4.83\times10^{-11}\ \text{F}$. $C = C_1 + C_2 = 6.25\times10^{-11}\ \text{F} = 62.5\ \text{pF}$.

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2}(6.25\times10^{-11}\ \text{F})(18.0\ \text{V})^2 = 1.01\times10^{-8}\ \text{J}$.

(c) Now $C_1 = C_2$ and $C = 2(1.42\times10^{-11}\ \text{F}) = 2.84\times10^{-11}\ \text{F}$.

$U = \frac{1}{2}CV^2 = \frac{1}{2}(2.84\times10^{-11}\ \text{F})(18.0\ \text{V})^2 = 4.60\times10^{-9}\ \text{J}$.

24.66. **IDENTIFY:** The system is equivalent to two capacitors in parallel. One of the capacitors has plate separation d, plate area $w(L-h)$ and air between the plates. The other has the same plate separation d, plate area wh and dielectric constant K.

SET UP: Define K_{eff} by $C_{\text{eq}} = \dfrac{K_{\text{eff}} \epsilon_0 A}{d}$, where $A = wL$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$.

EXECUTE: **(a)** The capacitors are in parallel, so $C = \dfrac{\epsilon_0 w(L-h)}{d} + \dfrac{K\epsilon_0 wh}{d} = \dfrac{\epsilon_0 wL}{d}\left(1 + \dfrac{Kh}{L} - \dfrac{h}{L}\right)$.

This gives $K_{\text{eff}} = \left(1 + \dfrac{Kh}{L} - \dfrac{h}{L}\right)$.

(b) For petrol, with $K = 1.95$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{4}\right) = 1.24$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{2}\right) = 1.48$;

$\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{3L}{4}\right) = 1.71$.

(c) For methanol, with $K = 33$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{4}\right) = 9$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \dfrac{L}{2}\right) = 17$;

$\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \dfrac{3L}{4}\right) = 25$.

(d) This kind of fuel tank sensor will work best for methanol since it has the greater range of K_{eff} values.

24.71. **IDENTIFY:** This problem involves a capacitor with dielectric inside.

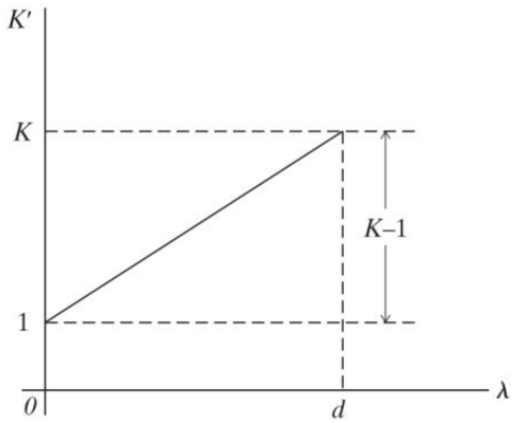

Figure 24.71a

SET UP: The dielectric constant K' is not uniform within the capacitor. We first need to find an equation for K'. At the first plate, the $K' = 1$, at the second plate $K' = K$, and it varies linearly. Call x the distance of any point from the first plate. Using this information, sketch the graph of K' versus x, as shown in Fig. 24.71a. Using the slope-intercept form, we see that the slope of this line is $(K-1)/d$ and the x-intercept is 1. The equation is $K' = \left(\dfrac{K-1}{d}\right)x + 1$.

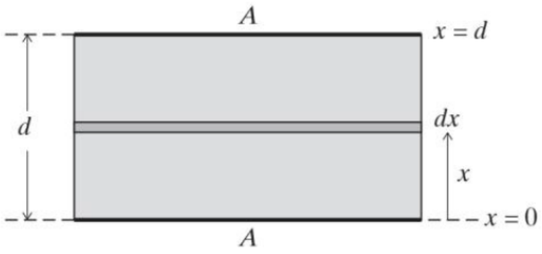

Figure 24.71b

EXECUTE: (a) We want to find the capacitance of this device. Since K' depends only on the distance x from the first plate, break the dielectric into thin slabs of area A and thickness dx as shown in

Fig. 24.71b. Using the equation $C = \dfrac{\epsilon_0 AK}{d}$, the capacitance of a single slab is $dC = \dfrac{\epsilon_0 AK}{dx}$. These slabs are all in series with each other, so we apply the equation $1/C_{eq} = 1/C_1 + 1/C_2 + \ldots$ But in this case each capacitance to be added is infinitesimal, so we must integrate to get the sum. Doing so gives $\dfrac{1}{C} = \int \dfrac{1}{dC} = \int_0^d \dfrac{dx}{K' \epsilon_0 A} = \dfrac{1}{\epsilon_0 A}\int_0^d \dfrac{dx}{1+(K-1)x/d} = \dfrac{d}{\epsilon_0 A(K-1)} \ln\left[1+(K-1)x/d\right]\Big|_0^d$. Evaluating at the two limits and rearranging gives $C = \dfrac{\epsilon_0 A(K-1)}{d \ln K}$.

(b) We want to find C when $K = 1$. Putting $K = 1$ into our result gives $\dfrac{0}{0}$, which is indeterminate. So to evaluate the limit, we need to use L'Hopital's rule, which gives

$C = \lim\limits_{K \to 1}\left[\dfrac{\epsilon_0 A(K-1)}{d \ln K}\right] = \lim\limits_{K \to 1}\left[\dfrac{\epsilon_0 A}{d}\dfrac{d(K-1)/dK}{d(\ln K)/dK}\right] = \left(\dfrac{\epsilon_0 A}{d}\right)\left(\dfrac{1}{1/K}\right) = \dfrac{K\epsilon_0 A}{d}$. We recognize this result as the capacitance of a parallel-plate capacitor filled with uniform material having dielectric constant K.

24.72. IDENTIFY: The system can be considered to be two capacitors in parallel, one with plate area $L(L-x)$ and air between the plates and one with area Lx and dielectric filling the space between the plates.

SET UP: $C = \dfrac{K\epsilon_0 A}{d}$ for a parallel-plate capacitor with plate area A.

EXECUTE: **(a)** $C = \dfrac{\epsilon_0}{D}\left[(L-x)L + xKL\right] = \dfrac{\epsilon_0 L}{D}\left[L+(K-1)x\right]$.

(b) $dU = \tfrac{1}{2}(dC)V^2$, where $C = C_0 + \dfrac{\epsilon_0 L}{D}(-dx + dxK)$, with $C_0 = \dfrac{\epsilon_0 L}{D}\left[L+(K-1)x\right]$. This gives

$dU = \tfrac{1}{2}\left(\dfrac{\epsilon_0 L dx}{D}(K-1)\right)V^2 = \dfrac{(K-1)\epsilon_0 V^2 L}{2D}dx.$

(c) If the charge is kept constant on the plates, then $Q = \dfrac{\epsilon_0 LV}{D}\left[L+(K-1)x\right]$ and

$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}C_0 V^2\left(\dfrac{C}{C_0}\right)$. $U \approx \dfrac{C_0 V^2}{2}\left(1 - \dfrac{\epsilon_0 L}{DC_0}(K-1)dx\right)$ and $\Delta U = U - U_0 = -\dfrac{(K-1)\epsilon_0 V^2 L}{2D}dx.$

(d) Since $dU = -Fdx = -\dfrac{(K-1)\epsilon_0 V^2 L}{2D}dx$, the force is in the opposite direction to the motion dx, meaning that the slab feels a force pushing it out.

EVALUATE: **(e)** When the plates are connected to the battery, the plates plus slab are not an isolated system. In addition to the work done on the slab by the charges on the plates, energy is also transferred between the battery and the plates. Comparing the results for dU in part (c) to $dU = -Fdx$ gives

$F = \dfrac{(K-1)\epsilon_0 V^2 L}{2D}.$