HW7 CH28 Solutions

28.39. IDENTIFY: Apply Ampere's law.

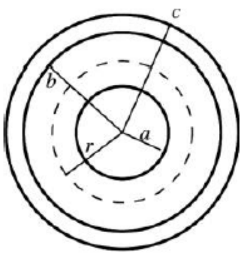
SET UP: To calculate the magnetic field at a distance r from the center of the cable, apply Ampere's law to a circular path of radius r. By symmetry, $\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$ for such a path.

EXECUTE: **(a)** For $a < r < b, I_{\text{encl}} = I \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B2\pi r = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}$.

(b) For $r > c$, the enclosed current is zero, so the magnetic field is also zero.

28.41. IDENTIFY: Apply Ampere's law to calculate $\vec{B}$.

(a) SET UP: For $a < r < b$ the end view is shown in Figure 28.41a.



Apply Ampere's law to a circle of radius r, where $a < r < b$. Take currents I_1 and I_2 to be directed into the page. Take this direction to be positive, so go around the integration path in the clockwise direction.

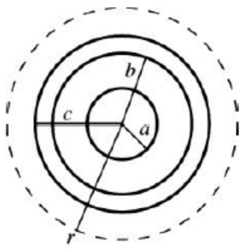
Figure 28.41a

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r), I_{\text{encl}} = I_1$.

Thus $B(2\pi r) = \mu_0 I_1$ and $B = \dfrac{\mu_0 I_1}{2\pi r}$.

(b) SET UP: $\underline{r > c}$: See Figure 28.41b.



Apply Ampere's law to a circle of radius r, where $r > c$. Both currents are in the positive direction.

Figure 28.41b

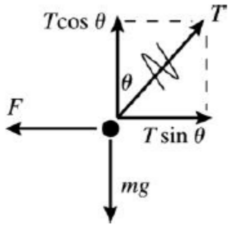
EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r), I_{\text{encl}} = I_1 + I_2$.

Thus $B(2\pi r) = \mu_0 (I_1 + I_2)$ and $B = \dfrac{\mu_0 (I_1 + I_2)}{2\pi r}$.

28.61. **IDENTIFY:** Apply $\Sigma\vec{F}=0$ to one of the wires. The force one wire exerts on the other depends on I so $\Sigma\vec{F}=0$ gives two equations for the two unknowns T and I.

SET UP: The force diagram for one of the wires is given in Figure 28.61.



The force one wire exerts on the other is $F=\left(\dfrac{\mu_0 I^2}{2\pi r}\right)L,$ where $r=2(0.040\text{ m})\sin\theta=8.362\times10^{-3}\text{ m}$ is the distance between the two wires.

Figure 28.61

EXECUTE: $\Sigma F_y=0$ gives $T\cos\theta=mg$ and $T=mg/\cos\theta$.

$\Sigma F_x=0$ gives $F=T\sin\theta=(mg/\cos\theta)\sin\theta=mg\tan\theta$.

And $m=\lambda L$, so $F=\lambda Lg\tan\theta$.

$$\left(\frac{\mu_0 I^2}{2\pi r}\right)L=\lambda Lg\tan\theta.$$

$$I=\sqrt{\frac{\lambda gr\tan\theta}{(\mu_0/2\pi)}}.$$

$$I=\sqrt{\frac{(0.0130\text{ kg/m})(9.80\text{ m/s}^2)(\tan 6.00^\circ)(8.362\times10^{-3}\text{ m})}{2\times10^{-7}\text{T}\cdot\text{m/A}}}=23.7\text{ A}.$$

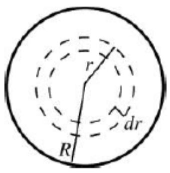
28.62. **IDENTIFY:** Apply $d\vec{B}=\dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l}\times\hat{r}}{r^2}$.

SET UP: The two straight segments produce zero field at P. The field at the center of a circular loop of radius R is $B=\dfrac{\mu_0 I}{2R}$, so the field at the center of curvature of a semicircular loop is $B=\dfrac{\mu_0 I}{4R}$.

EXECUTE: The semicircular loop of radius a produces field out of the page at P and the semicircular loop of radius b produces field into the page. Therefore, $B=B_a-B_b=\dfrac{1}{2}\left(\dfrac{\mu_0 I}{2}\right)\left(\dfrac{1}{a}-\dfrac{1}{b}\right)=\dfrac{\mu_0 I}{4a}\left(1-\dfrac{a}{b}\right)$, out of page.

28.65. **(a) IDENTIFY:** Consider current density J for a small concentric ring and integrate to find the total current in terms of α and R.

SET UP: We can't say $I=JA=J\pi R^2$, since J varies across the cross section.



To integrate J over the cross section of the wire, divide the wire cross section up into thin concentric rings of radius r and width dr, as shown in Figure 28.65.

Figure 28.65

EXECUTE: The area of such a ring is dA, and the current through it is $dI=J\,dA$; $dA=2\pi r dr$ and

$dI=J\,dA=\alpha r(2\pi r\,dr)=2\pi\alpha r^2 dr.$

$I=\int dI=2\pi\alpha\int_0^R r^2 dr=2\pi\alpha(R^3/3)$ so $\alpha=\dfrac{3I}{2\pi R^3}$.

(b) IDENTIFY and SET UP: (i) $r\le R$.

Apply Ampere's law to a circle of radius $r<R$. Use the method of part (a) to find the current enclosed by Ampere's law path.

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = B\oint dl = B(2\pi r)$, by the symmetry and direction of $\vec{B}$. The current passing through the path is $I_{\text{encl}} = \int dl$, where the integration is from 0 to r.

$I_{\text{encl}} = 2\pi\alpha\int_0^r r^2 dr = \dfrac{2\pi\alpha r^3}{3} = \dfrac{2\pi}{3}\left(\dfrac{3I}{2\pi R^3}\right)r^3 = \dfrac{Ir^3}{R^3}$. Thus $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives

$B(2\pi r) = \mu_0\left(\dfrac{Ir^3}{R^3}\right)$ and $B = \dfrac{\mu_0 I r^2}{2\pi R^3}$.

(ii) **IDENTIFY and SET UP:** $r \ge R$.

Apply Ampere's law to a circle of radius $r > R$.

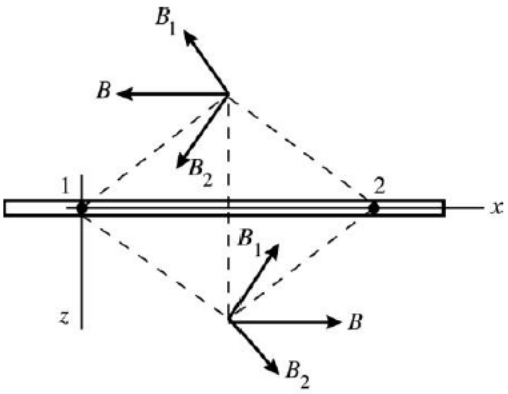
EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = B\oint dl = B(2\pi r)$.

$I_{\text{encl}} = I$; all the current in the wire passes through this path. Thus $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives

$B(2\pi r) = \mu_0 I$ and $B = \dfrac{\mu_0 I}{2\pi r}$.

28.69. **IDENTIFY:** Use what we know about the magnetic field of a long, straight conductor to deduce the symmetry of the magnetic field. Then apply Ampere's law to calculate the magnetic field at a distance a above and below the current sheet.

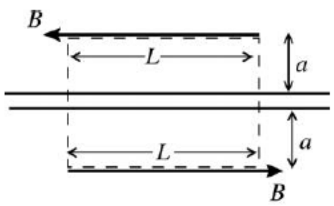
SET UP: Do parts (a) and (b) together.



Consider the individual currents in pairs, where the currents in each pair are equidistant on either side of the point where $\vec{B}$ is being calculated. Figure 28.69a shows that for each pair the z-components cancel, and that above the sheet the field is in the $-x$-direction and that below the sheet it is in the $+x$-direction.

Figure 28.69a

Also, by symmetry the magnitude of $\vec{B}$ a distance a above the sheet must equal the magnitude of $\vec{B}$ a distance a below the sheet. Now that we have deduced the symmetry of $\vec{B}$, apply Ampere's law. Use a path that is a rectangle, as shown in Figure 28.69b.



$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

Figure 28.69b

I is directed out of the page, so for I to be positive the integral around the path is taken in the counterclockwise direction.

EXECUTE: Since $\vec{B}$ is parallel to the sheet, on the sides of the rectangle that have length $2a$, $\oint \vec{B}\cdot d\vec{l} = 0$. On the long sides of length L, $\vec{B}$ is parallel to the side, in the direction we are integrating around the path, and has the same magnitude, B, on each side. Thus $\oint \vec{B}\cdot d\vec{l} = 2BL$. n conductors per unit length and current I out of the page in each conductor gives $I_{\text{encl}} = InL$. Ampere's law then gives $2BL = \mu_0 InL$ and $B = \frac{1}{2}\mu_0 In$.

28.70. **IDENTIFY:** Find the vector sum of the fields due to each sheet.

SET UP: Problem 28.69 shows that for an infinite sheet $B = \frac{1}{2}\mu_0 In$. If I is out of the page, $\vec{B}$ is to the left above the sheet and to the right below the sheet. If I is into the page, $\vec{B}$ is to the right above the sheet and to the left below the sheet. B is independent of the distance from the sheet. The directions of the two fields at points P, R and S are shown in Figure 28.70.

EXECUTE: **(a)** Above the two sheets, the fields cancel (since there is no dependence upon the distance from the sheets).

(b) In between the sheets the two fields add up to yield $B = \mu_0 nI$, to the right.

(c) Below the two sheets, their fields again cancel (since there is no dependence upon the distance from the sheets).

EVALUATE: The two sheets with currents in opposite directions produce a uniform field between the sheets and zero field outside the two sheets. This is analogous to the electric field produced by large parallel sheets of charge of opposite sign.

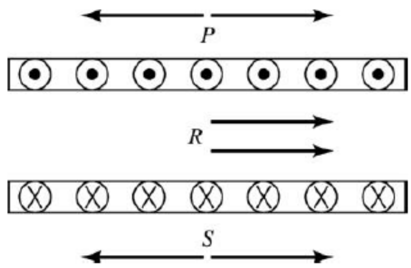


Figure 28.70

28.71. **IDENTIFY:** A charged cylindrical shell is rotating. This motion produces a magnetic field which exerts a torque on a very small disk at its midpoint.

SET UP and **EXECUTE:** **(a)** We want the current. $I = \dfrac{\Delta Q}{\Delta t}$. In one full rotation, charge Q_1 passes through in time T_1 which is the period of rotation of the cylinder. Thus $\Delta Q = Q_1$ and $\Delta T = T_1 = 2\pi/\omega_1$. So $I = \dfrac{Q_1}{(2\pi/\omega_1)} = \dfrac{Q_1\omega_1}{2\pi}$.

(b) We want B. Apply Ampere's law: $\oint \vec{B} \cdot d\hat{l} = \mu_0 I_{\text{encl}}$. The rectangular path of integration is similar to the one used in Example 28.9. The inner segment ab is on the axis of the cylinder, is equidistant from its ends, and has length $l << H$. $B = 0$ outside the cylinder, and the field is perpendicular to bc and da.

Using the current from part (a), $\oint \vec{B} \cdot d\hat{l} = \mu_0 I_{\text{encl}}$ gives us $Bl = \mu_0 I \dfrac{l}{H} = \mu_0 \dfrac{Q_1\omega_1}{2\pi}\dfrac{l}{H}$. Solving for B and realizing that the field points along the $+z$-axis, we have $\vec{B} = \dfrac{\mu_0\omega_1 Q_1}{2\pi H}\hat{k}$.

(c) We want the torque on the disk. $\tau = \mu B \sin\phi$. Since the disk is very small, we can treat the magnetic field as uniform over its surface and equal to the field at the center of the cylinder. Using the given magnetic moment, our result from (b), and $\phi = \theta$, we get

$$\tau = \left(\frac{1}{4}Q_2\omega_2 R_2^2\right)\left(\frac{\mu_0 Q_1\omega_1}{2\pi H}\right)\sin\theta = \frac{\mu_0 Q_1 Q_2 \omega_1\omega_2 R_2^2}{8\pi H}\sin\theta.$$

(d) We want the angular momentum of the disk. $L = I\omega = \dfrac{1}{2}MR_2^2\omega_2$.