HW9 CH30 Solutions

30.17. **Identify:** A current-carrying inductor has a magnetic field inside of itself and hence stores magnetic energy.

(a) Set Up: The magnetic field inside a solenoid is $B = \mu_0 n I$.

Execute: $B = \dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(440)(90.0\ \text{A})}{0.270\ \text{m}} = 0.184\ \text{T}.$

(b) Set Up: The energy density in a magnetic field is $u = \dfrac{B^2}{2\mu_0}$.

Execute: $u = \dfrac{(0.184\ \text{T})^2}{2(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})} = 1.35 \times 10^4\ \text{J/m}^3.$

(c) Set Up: The total stored energy is $U = uV$.

Execute: $U = uV = u(lA) = (1.35 \times 10^4\ \text{J/m}^3)(0.270\ \text{m})(0.600 \times 10^{-4}\ \text{m}^2) = 0.219\ \text{J}.$

(d) Set Up: The energy stored in an inductor is $U = \frac{1}{2}LI^2$.

Execute: Solving for L and putting in the numbers gives

$$L = \frac{2U}{I^2} = \frac{2(0.219\ \text{J})}{(90.0\ \text{A})^2} = 5.40 \times 10^{-5}\ \text{H}.$$

30.23. **Identify:** $i = \mathcal{E}/R(1 - e^{-t/\tau})$, with $\tau = L/R$. The energy stored in the inductor is $U = \frac{1}{2}Li^2$.

Set Up: The maximum current occurs after a long time and is equal to $\mathcal{E}/R$.

Execute: **(a)** $i_{\text{max}} = \mathcal{E}/R$ so $i = i_{\text{max}}/2$ when $(1 - e^{-t/\tau}) = \frac{1}{2}$ and $e^{-t/\tau} = \frac{1}{2}$. $-t/\tau = \ln(\frac{1}{2})$.

$$t = \frac{L \ln 2}{R} = \frac{(\ln 2)(1.25 \times 10^{-3}\ \text{H})}{50.0\ \Omega} = 17.3\ \mu\text{s}.$$

(b) $U = \frac{1}{2}U_{\text{max}}$ when $i = i_{\text{max}}/\sqrt{2}$. $1 - e^{-t/\tau} = 1/\sqrt{2}$, so $e^{-t/\tau} = 1 - 1/\sqrt{2} = 0.2929$.

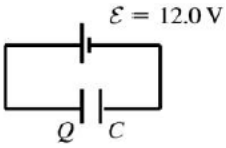
$t = -L\ln(0.2929)/R = 30.7\ \mu\text{s}$.

Evaluate: $\tau = L/R = 2.50 \times 10^{-5}\ \text{s} = 25.0\ \mu\text{s}$. The time in part (a) is 0.692τ and the time in part (b) is 1.23τ.

30.35. **Identify and Set Up:** The angular frequency is given by $\omega = \dfrac{1}{\sqrt{LC}}$. $q(t)$ and $i(t)$ are given by $q = Q\cos(\omega t + \phi)$ and $i = -\omega Q \sin(\omega t + \phi)$. The energy stored in the capacitor is $U_C = \frac{1}{2}CV^2 = q^2/2C$. The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$.

Execute: **(a)** $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(1.50\ \text{H})(6.00 \times 10^{-5}\ \text{F})}} = 105.4\ \text{rad/s}$, which rounds to $105\ \text{rad/s}$. The period is given by $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{105.4\ \text{rad/s}} = 0.0596\ \text{s}.$

(b) The circuit containing the battery and capacitor is sketched in Figure 30.35.



$$\mathcal{E} - \frac{Q}{C} = 0.$$

$$Q = \varepsilon C = (12.5\text{ V})(6.00 \times 10^{-5}\text{ F}) = 7.50 \times 10^{-4}\text{ C}.$$

Figure 30.35

(c) $U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(6.00 \times 10^{-5}\text{ F})(12.5\text{ V})^2 = 4.69 \times 10^{-3}\text{ J}.$

(d) $q = Q\cos(\omega t + \phi)$ (Eq. 30.21).

$q = Q$ at $t = 0$ so $\phi = 0$.

$q = Q\cos\omega t = (7.50 \times 10^{-4}\text{ C})\cos([105.4\text{ rad/s}][0.0235\text{ s}]) = -5.90 \times 10^{-4}\text{ C}.$

The minus sign means that the capacitor has discharged fully and then partially charged again by the current maintained by the inductor; the plate that initially had positive charge now has negative charge and the plate that initially had negative charge now has positive charge.

(e) The current is $i = -\omega Q\sin(\omega t + \phi)$

$i = -(105\text{ rad/s})(7.50 \times 10^{-4}\text{ C})\sin[(105.4\text{ rad/s})(0.0235\text{ s})] = -0.049\text{ A}.$

The negative sign means the current is counterclockwise in Figure 30.15 in the textbook.

or

$\tfrac{1}{2}Li^2 + \dfrac{q^2}{2C} = \dfrac{Q^2}{2C}$ gives $i = \pm\sqrt{\dfrac{1}{LC}}\sqrt{Q^2 - q^2}$ (Eq. 30.26).

$i = \pm(105\text{ rad/s})\sqrt{(7.50 \times 10^{-4}\text{ C})^2 - (-5.90 \times 10^{-4}\text{ C})^2} = \pm 0.049\text{ A},$ which checks.

(f) $U_C = \dfrac{q^2}{2C} = \dfrac{(-5.90 \times 10^{-4}\text{ C})^2}{2(6.00 \times 10^{-5}\text{ F})} = 2.90 \times 10^{-3}\text{ J}.$

$U_L = \tfrac{1}{2}Li^2 = \tfrac{1}{2}(1.50\text{ H})(0.049\text{ A})^2 = 1.80 \times 10^{-3}\text{ J}.$

30.43. **Identify:** This problem involves electromagnetic induction and self-inductance.

Set Up and Execute: $L = \dfrac{\Phi_B}{i}$, $B = \dfrac{\mu_0 I}{2r}$, $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$.. **(a)** We want the self-inductance.

$\Phi_B = BA = \left(\dfrac{\mu_0 I}{2r}\right)(\pi r^2) = \dfrac{\mu_0 \pi r i}{2}$. $L = \dfrac{\Phi_B}{i} = \dfrac{\mu_0 \pi r i/2}{i} = \dfrac{\mu_0 \pi r}{2} = \dfrac{\mu_0 \pi (3.0\text{ cm})}{2} = 59\text{ nH}.$

(b) We want the maximum emf. $|\mathcal{E}| = \dfrac{d\Phi_B}{dt} = \dfrac{d}{dt}\left(\dfrac{\mu_0 \pi r i}{2}\right) = \dfrac{\mu_0 \pi r}{2}\dfrac{di}{dt}$. $i(t) = I_0\sin(2\pi f t)$ gives

$\mathcal{E} = \mu_0 \pi^2 r I_0 f\cos(2\pi f t)$. $\mathcal{E}_{\text{max}} = \mu_0 \pi^2 r I_0 f = \; = \mu_0 \pi^2(0300\text{ m})(1.20\text{ A})(60.0\text{ Hz}) = 26.8\ \mu\text{V}.$

30.46. **Identify:** Follow the steps outlined in the problem.

Set Up: The energy stored is $U = \tfrac{1}{2}Li^2$.

Execute: **(a)** $\mathbf{r}\vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B 2\pi r = \mu_0 i \Rightarrow B = \dfrac{\mu_0 i}{2\pi r}.$

(b) $d\Phi_B = B\,dA = \dfrac{\mu_0 i}{2\pi r} l\,dr.$

(c) $\Phi_B = \displaystyle\int_a^b d\Phi_B = \dfrac{\mu_0 i l}{2\pi}\int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 i l}{2\pi}\ln(b/a).$

(d) $L = \dfrac{N\Phi_B}{i} = l\dfrac{\mu_0}{2\pi}\ln(b/a).$

(e) $U = \dfrac{1}{2}Li^2 = \dfrac{1}{2}l\dfrac{\mu_0}{2\pi}\ln(b/a)i^2 = \dfrac{\mu_0 l i^2}{4\pi}\ln(b/a).$

30.48. IDENTIFY and SET UP: Eq. (30.14) is $i=\frac{\mathcal{E}}{R}(1-e^{-Rt/L}),\ P_R=i^2R,\ \mathcal{E}_L=-L\frac{di}{dt}.$

EXECUTE: **(a)** Using Eq. (30.14) in the power consumed in the resistor gives

$$P_R=i^2R=\left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right]^2R=\frac{\mathcal{E}^2}{R}(1-2e^{-Rt/L}+e^{-2Rt/L}).$$

After a long time, that is $t\to\infty$, the exponential terms all go to zero and the power approaches its maximum value of $\frac{\mathcal{E}^2}{R}$.

(b) The power in the inductor is

$$P_L=i\mathcal{E}_L=i\left(L\frac{di}{dt}\right)=\left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right]L\frac{d}{dt}\left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right]=\left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right]L\left(\frac{\mathcal{E}}{L}\right)e^{-Rt/L}.$$

$$P_L=\frac{\mathcal{E}^2}{R}(1-e^{-Rt/L})e^{-Rt/L}.$$

(c) $P_L(0)=0$ since $1-e^0=0$. $P_L(t\to\infty)=0$ since $e^{-Rt/L}\to 0$ as $t\to\infty$.

(d) P_L is a maximum when $dP_L/dt=0$. Taking the time derivative of P_L from (b), we have

$$\frac{\mathcal{E}^2}{R}\left[\left(\frac{R}{L}e^{-Rt/L}\right)(e^{-Rt/L})-\frac{R}{L}(1-e^{-Rt/L})(e^{-Rt/L})\right]=\frac{\mathcal{E}^2}{L}e^{-Rt/L}(e^{-Rt/L}-1+e^{-Rt/L})=0.$$

$$2e^{-Rt/L}=1.$$

$t=-(L/R)\ln(1/2)=(L/R)\ln 2.$

At this instant, $e^{-Rt/L}=\frac{1}{2}$. Using the result from (b), we have

$$P_L=\frac{\mathcal{E}^2}{R}(1-e^{-Rt/L})e^{-Rt/L}=\frac{\mathcal{E}^2}{R}\left(1-\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}\frac{\mathcal{E}^2}{R}.$$

(e) $P_\varepsilon=i\mathcal{E}=\frac{\mathcal{E}^2}{R}(1-e^{-Rt/L})$. The maximum power is $\frac{\mathcal{E}^2}{R}$ as $t\to\infty$.

30.50. IDENTIFY and SET UP: Apply $\mathcal{E}_L=-L\frac{di}{dt}$ and $V_R=Ri$. An inductor opposes a change in current through it. Kirchhoff's rules apply.

EXECUTE: **(a)** At the instant the switch is closed, the inductor will not allow any current through it, so all the current goes through R_1. So $i_1=i_2=\mathcal{E}/R_1=(96.0\text{ V})/(12.0\ \Omega)=8.00$ A. $i_3=0$.

(b) After a long time, steady-state is reached, so $di_3/dt=0$ and $\mathcal{E}_L=-L\frac{di_3}{dt}=0$. In this case, the potential across R_1 and across R_2 is 96.0 V. Therefore

$i_2=(96.0\text{ V})/(12.0\ \Omega)=8.00$ A.

$i_3=(96.0\text{ V})/(16.0\ \Omega)=6.00$ A.

$i_1=i_2+i_3=8.00\text{ A}+6.00\text{ A}=14.00$ A.

(c) Apply Kirchhoff's loop rule, giving

$$\mathcal{E}-i_3R_2-L\frac{di_3}{dt}=0.$$

Separating variables and integrating gives

$$\int_0^t -\frac{R_2}{L}dt'=\int_0^{i_3}\frac{1}{i_3'-\mathcal{E}/R_2}di_3'.$$

Carrying out the integration and solving for t gives

$$-\frac{R_2}{L}t=\ln\left(\frac{i_3-\mathcal{E}/R_2}{-\mathcal{E}/R_2}\right).$$

$$t=\frac{L}{R_2}\ln\left(\frac{\mathcal{E}/R_2}{\mathcal{E}/R_2-i_3}\right)=\frac{0.300\text{ H}}{16.0\ \Omega}\ln\left(\frac{96.0\text{ V}}{96.0\text{ V}-(3.00\text{ A})(16.0\ \Omega)}\right)=0.0130\text{ s}=13.0\text{ ms}.$$

(d) $i_2=\mathcal{E}/R_1=(96.0\text{ V})/(12.0\ \Omega)=8.00$ A. $i_1=i_2+i_3=8.00\text{ A}+3.00\text{ A}=11.0$ A.

30.61. **IDENTIFY:** Apply the loop rule to each parallel branch. The voltage across a resistor is given by iR and the voltage across an inductor is given by $L|di/dt|$. The rate of change of current through the inductor is limited.

SET UP: With S closed the circuit is sketched in Figure 30.61a.

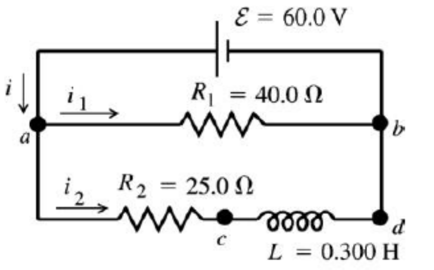


Figure 30.61a

The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is closed the current in the inductor has not had time to increase from zero, so $i_2 = 0$.

EXECUTE : **(a)** $\varepsilon - v_{ab} = 0$, so $v_{ab} = 65.0$ V.

(b) The voltage drops across R, as we travel through the resistor in the direction of the current, so point a is at higher potential.

(c) $i_2 = 0$ so $v_{R_2} = i_2R_2 = 0$.

$\varepsilon - v_{R_2} - v_L = 0$ so $v_L = \varepsilon = 65.0$ V.

(d) The voltage rises when we go from b to a through the emf, so it must drop when we go from a to b through the inductor. Point c must be at higher potential than point d.

(e) After the switch has been closed a long time, $\dfrac{di_2}{dt} \to 0$ so $v_L = 0$. Then $\varepsilon - v_{R_2} = 0$ and $i_2R_2 = \varepsilon$

so $i_2 = \dfrac{\varepsilon}{R_2} = \dfrac{65.0 \text{ V}}{30.0\ \Omega} = 2.17$ A.

SET UP: The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is opened again the current through the inductor hasn't had time to change and is still $i_2 = 2.40$ A. The circuit is sketched in Figure 30.61b.

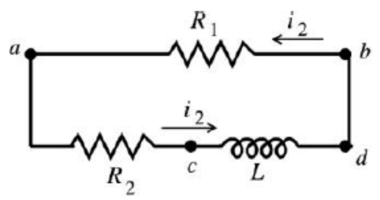


Figure 30.61b

EXECUTE: The current through R_1 is $i_2 = 2.40$ A in the direction b to a.

Thus $v_{ab} = -i_2R_1 = -(2.17 \text{ A})(37.0\ \Omega)$.

$v_{ab} = -80.2$ V.

(f) Point where current enters resistor is at higher potential; point b is at higher potential.

(g) $v_L - v_{R_1} - v_{R_2} = 0$.

$v_L = v_{R_1} + v_{R_2}$.

$v_{R_1} = -v_{ab} = 80.2$ V; $v_{R_2} = i_2R_2 = (2.17 \text{ A})(30.0\ \Omega) = 65.0$ V.

Then $v_L = v_{R_1} + v_{R_2} = 80.2 \text{ V} + 65.0 \text{ V} = 145.2$ V.

As you travel counterclockwise around the circuit in the direction of the current, the voltage drops across each resistor, so it must rise across the inductor and point d is at higher potential than point c. The current is decreasing, so the induced emf in the inductor is directed in the direction of the current. Thus, $v_{cd} = -145.2$ V.

(h) Point d is at higher potential.

30.63. **IDENTIFY and SET UP:** The circuit is sketched in Figure 30.63a. Apply the loop rule. Just after S_1 is closed, $i = 0$. After a long time i has reached its final value and $di/dt = 0$. The voltage across a resistor depends on i and the voltage across an inductor depends on di/dt.

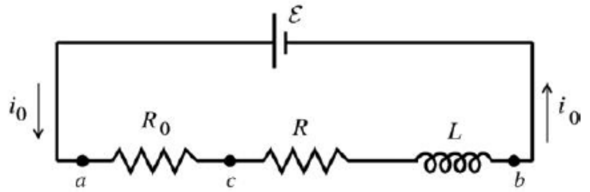


Figure 30.63a

EXECUTE: **(a)** At time $t = 0$, $i_0 = 0$ so $v_{ac} = i_0 R_0 = 0$. By the loop rule $\mathcal{E} - v_{ac} - v_{cb} = 0$ so $v_{cb} = \mathcal{E} - v_{ac} = \mathcal{E} = 36.0$ V. ($i_0 R = 0$ so this potential difference of 36.0 V is across the inductor and is an induced emf produced by the changing current.)

(b) After a long time $\dfrac{di_0}{dt} \to 0$ so the potential $-L\dfrac{di_0}{dt}$ across the inductor becomes zero. The loop rule gives $\mathcal{E} - i_0(R_0 + R) = 0$.

$$i_0 = \frac{\mathcal{E}}{R_0 + R} = \frac{36.0\text{ V}}{50.0\ \Omega + 150\ \Omega} = 0.180\text{ A}.$$

$v_{ac} = i_0 R_0 = (0.180\text{ A})(50.0\ \Omega) = 9.0\text{ V}.$

Thus $v_{cb} = i_0 R + L\dfrac{di_0}{dt} = (0.180\text{ A})(150\ \Omega) + 0 = 27.0\text{ V}.$ (Note that $v_{ac} + v_{cb} = \mathcal{E}$.)

(c) $\mathcal{E} - v_{ac} - v_{cb} = 0.$

$$\mathcal{E} - iR_0 - iR - L\frac{di}{dt} = 0.$$

$$L\frac{di}{dt} = \mathcal{E} - i(R_0 + R) \text{ and } \left(\frac{L}{R + R_0}\right)\frac{di}{dt} = -i + \frac{\mathcal{E}}{R + R_0}.$$

$$\frac{di}{-i + \mathcal{E}/(R + R_0)} = \left(\frac{R + R_0}{L}\right)dt.$$

Integrate from $t = 0$, when $i = 0$, to t, when $i = i_0$:

$$\int_0^{i_0} \frac{di}{-i + \mathcal{E}/(R + R_0)} = \frac{R + R_0}{L}\int_0^t dt = -\ln\left[-i + \frac{\mathcal{E}}{R + R_0}\right]_0^{i_0} = \left(\frac{R + R_0}{L}\right)t, \text{ so}$$

$$\ln\left(-i_0 + \frac{\mathcal{E}}{R + R_0}\right) - \ln\left(\frac{\mathcal{E}}{R + R_0}\right) = -\left(\frac{R + R_0}{L}\right)t.$$

$$\ln\left(\frac{-i_0 + \mathcal{E}/(R + R_0)}{\mathcal{E}/(R + R_0)}\right) = -\left(\frac{R + R_0}{L}\right)t.$$

Taking exponentials of both sides gives $\dfrac{-i_0 + \mathcal{E}/(R + R_0)}{\mathcal{E}/(R + R_0)} = e^{-(R + R_0)t/L}$ and $i_0 = \dfrac{\mathcal{E}}{R + R_0}(1 - e^{-(R + R_0)t/L}).$

Substituting in the numerical values gives $i_0 = \dfrac{36.0\text{ V}}{50\ \Omega + 150\ \Omega}(1 - e^{-(200\ \Omega/4.00\text{ H})t}) = (0.180\text{ A})(1 - e^{-t/0.020\text{ s}}).$

At $t \to 0, i_0 = (0.180\text{ A})(1 - 1) = 0$ (agrees with part (a)). At $t \to \infty, i_0 = (0.180\text{ A})(1 - 0) = 0.180$ A (agrees with part (b)).

$$v_{ac} = i_0 R_0 = \frac{\mathcal{E}R_0}{R+R_0}(1-e^{-(R+R_0)t/L}) = 9.0\ \text{V}(1-e^{-t/0.020\ \text{s}}).$$

$$v_{cb} = \mathcal{E} - v_{ac} = 36.0\ \text{V} - 9.0\ \text{V}(1-e^{-t/0.020\ \text{s}}) = 9.0\ \text{V}(3.00+e^{-t/0.020\ \text{s}}).$$

At $t \to 0, v_{ac} = 0, v_{cb} = 36.0\ \text{V}$ (agrees with part (a)). At $t \to \infty, v_{ac} = 9.0\ \text{V}, v_{cb} = 27.0\ \text{V}$ (agrees with part (b)). The graphs are given in Figure 30.63b.

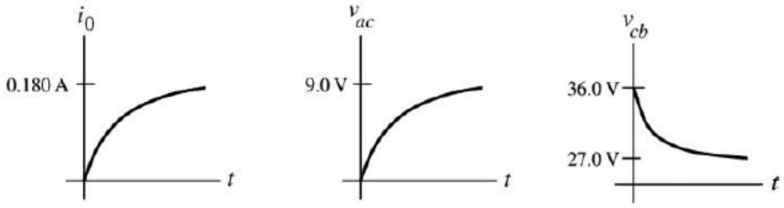


Figure 30.63b