22.41. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charge distributions.

EXECUTE: **(a)** For $r < R, E = 0,$ since these points are within the conducting material. For $R < r < 2R,$ $E = \dfrac{1}{4\pi \epsilon_0}\dfrac{Q}{r^2},$ since the charge enclosed is Q. The field is radially outward. For $r > 2R,$ $E = \dfrac{1}{4\pi \epsilon_0}\dfrac{2Q}{r^2}$ since the charge enclosed is $2Q$. The field is radially outward.

(b) The graph of E versus r is sketched in Figure 22.41.

EVALUATE: For $r < 2R$ the electric field is unaffected by the presence of the charged shell.

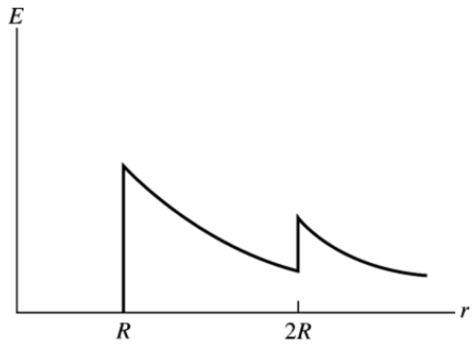


Figure 22.41

22.42. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that is a sphere of radius r and that has the point charge at its center.

EXECUTE: **(a)** For $r < a,$ $E = \dfrac{1}{4\pi \epsilon_0}\dfrac{Q}{r^2},$ radially outward, since the charge enclosed is Q, the charge of the point charge. For $a < r < b,$ $E = 0$ since these points are within the conducting material. For $r > b,$ $E = \dfrac{1}{4\pi \epsilon_0}\dfrac{2Q}{r^2},$ radially inward, since the total enclosed charge is $-2Q$.

(b) Since a Gaussian surface with radius r, for $a < r < b,$ must enclose zero net charge because $E = 0$ inside the conductor, the total charge on the inner surface is $-Q$ and the surface charge density on the inner surface is $\sigma = -\dfrac{Q}{4\pi a^2}$.

(c) Since the net charge on the shell is $-3Q$ and there is $-Q$ on the inner surface, there must be $-2Q$ on the outer surface. The surface charge density on the outer surface is $\sigma = -\dfrac{2Q}{4\pi b^2}$.

(d) The field lines and the locations of the charges are sketched in Figure 22.42a.

(e) The graph of E versus r is sketched in Figure 22.42b.

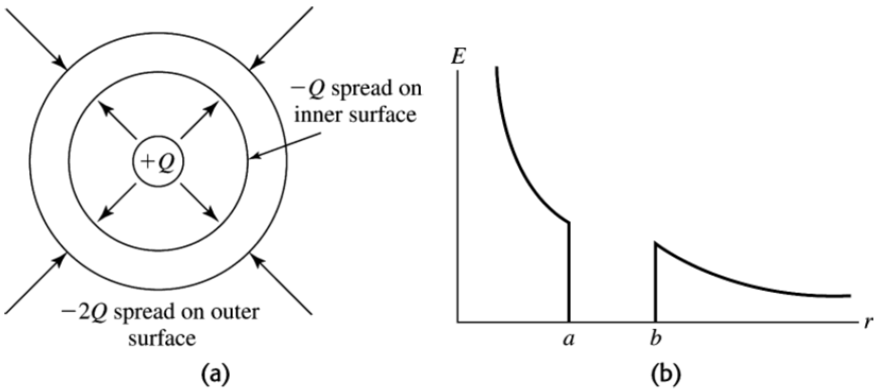


Figure 22.42

22.43. **IDENTIFY:** Apply Gauss's law to a spherical Gaussian surface with radius r. Calculate the electric at the surface of the Gaussian sphere.

(a) SET UP: (i) $r < a$: The Gaussian surface is sketched in Figure 22.43a.

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$.

$Q_{\text{encl}} = 0$; no charge is enclosed.

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ says

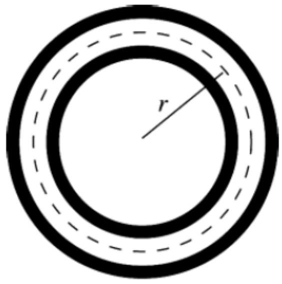
$E(4\pi r^2) = 0$ and $E = 0$.

Figure 22.43a

(ii) $a < r < b$: Points in this region are in the conductor of the small shell, so $E = 0$.

(iii) **SET UP:** $b < r < c$: The Gaussian surface is sketched in Figure 22.43b.

Apply Gauss's law to a spherical Gaussian surface with radius $b < r < c$.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$.

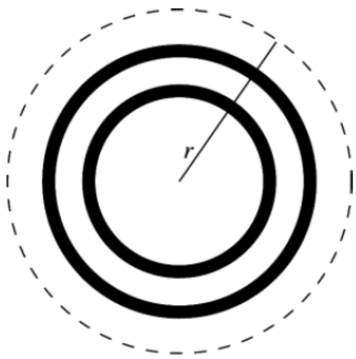
The Gaussian surface encloses all of the small shell and none of tl large shell, so $Q_{\text{encl}} = +2q$.

Figure 22.43b

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{2q}{\epsilon_0}$ so $E = \dfrac{2q}{4\pi \epsilon_0 r^2}$. Since the enclosed charge is positive the electric is radially outward.

(iv) $c < r < d$: Points in this region are in the conductor of the large shell, so $E = 0$.

(v) **SET UP:** $r > d$: Apply Gauss's law to a spherical Gaussian surface with radius $r > d$, as show Figure 22.43c.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$.

The Gaussian surface encloses all of the small shell and all of the large shell, so $Q_{\text{encl}} = +2q + 4q$

Figure 22.43c

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{6q}{\epsilon_0}$.

$E = \dfrac{6q}{4\pi \epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

The graph of E versus r is sketched in Figure 22.43d.

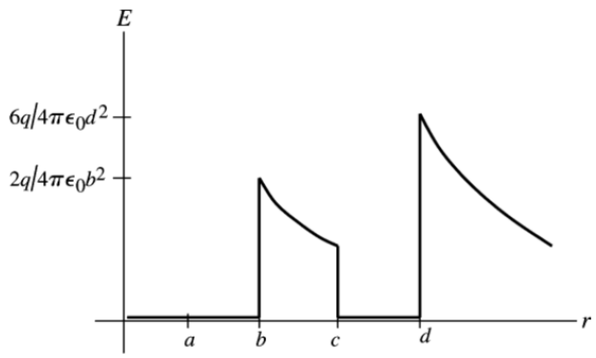


Figure 22.43d

(b) IDENTIFY and SET UP: Apply Gauss's law to a sphere that lies outside the surface of the shell for which we want to find the surface charge.

EXECUTE: (i) charge on inner surface of the small shell: Apply Gauss's law to a spherical Gaussian surface with radius $a < r < b$. This surface lies within the conductor of the small shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$, so there is zero charge on the inner surface of the small shell.

(ii) charge on outer surface of the small shell: The total charge on the small shell is $+2q$. We found in part (i) that there is zero charge on the inner surface of the shell, so all $+2q$ must reside on the outer surface.

(iii) charge on inner surface of large shell: Apply Gauss's law to a spherical Gaussian surface with radius $c < r < d$. The surface lies within the conductor of the large shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses the $+2q$ on the small shell so there must be charge $-2q$ on the inner surface of the large shell to make the total enclosed charge zero.

(iv) charge on outer surface of large shell: The total charge on the large shell is $+4q$. We showed in part (iii) that the charge on the inner surface is $-2q$, so there must be $+6q$ on the outer surface.

22.49. **IDENTIFY:** The charge density inside the cylinder is not uniform but depends on distance from the central axis. We want the electric field both inside and outside the cylinder.

SET UP and EXECUTE: Inside the cylinder ($r \le R$): For the Gaussian surface, choose a cylinder of length L and radius $r < R$ that is coaxial with the charged cylinder. The electric field is perpendicular to the curved surface and parallel to the ends of this surface. The charge density inside the cylinder depends on r, so we must integrate to get the charge q within the Gaussian surface.

$q = \int \rho dV = \int \alpha \left(1 - \dfrac{r}{R}\right) 2\pi r L dr = 2\pi L \alpha r^2 \left(\dfrac{1}{2} - \dfrac{r}{3R}\right)$. Now apply Gauss's law using the cylindrical Gaussian surface. $E(2\pi r L) = \dfrac{2\pi L \alpha r^2 \left(\dfrac{1}{2} - \dfrac{r}{3R}\right)}{\epsilon_0}$. $E = \dfrac{\alpha r}{\epsilon_0}\left(\dfrac{1}{2} - \dfrac{r}{3R}\right)$.

Outside the cylinder ($r \ge R$): Use the same Gaussian surface as above except $r > R$. The enclosed charge is just the charge within the cylinder, not the full Gaussian surface. Use the same formula for q that we found above except use $r = R$. This gives $q = 2\pi L \alpha R^2 \left(\dfrac{1}{2} - \dfrac{R}{3R}\right) = \pi L \alpha R^2 / 3$. Now apply Gauss's law.

$E(2\pi r L) = \dfrac{\pi L \alpha R^2}{3 \epsilon_0}$. $E = \dfrac{\alpha R^2}{6 \epsilon_0 r}$.

22.50. **IDENTIFY:** The charge density inside the sphere is not uniform but depends on distance from the center. We want the electric field both inside and outside the sphere. The charge density inside the sphere is $\rho(r)=\rho_0\left(1-\dfrac{r}{R}\right)$. For Gaussian surfaces, choose a sphere of radius r concentric with the charged sphere.

SET UP and EXECUTE: **(a)** Inside the sphere ($r \le R$): We need the charge contained within the Gaussian surface. $q=\int \rho dV=\int \rho_0\left(1-\dfrac{r}{R}\right)4\pi r^2 Ldr = 4\pi\rho_0\left(\dfrac{r^3}{3}-\dfrac{r^4}{4R}\right)$. Now use Gauss's law.

$E(4\pi r^2)=\dfrac{4\pi\rho_0}{\varepsilon_0}\left(\dfrac{r^3}{3}-\dfrac{r^4}{4R}\right)$. $E=\dfrac{\rho_0}{\varepsilon_0}\left(\dfrac{r}{3}-\dfrac{r^2}{4R}\right)$.

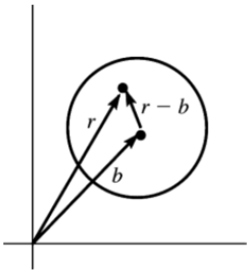
(b) Outside the sphere ($r \ge R$): For q use the same result as in part (a) except let $r = R$, giving $q=\pi\rho_0 R^3/3$. Now use Gauss's law. $E(4\pi r^2)=\dfrac{\pi\rho_0 R}{3\epsilon_0}$. $E=\dfrac{\rho_0 R^3}{12\epsilon_0 r^2}$.

22.53. **(a) IDENTIFY:** Use $\vec{E}(\vec{r})$ from Example (22.9) (inside the sphere) and relate the position vector of a point inside the sphere measured from the origin to that measured from the center of the sphere.

SET UP: For an insulating sphere of uniform charge density ρ and centered at the origin, the electric field inside the sphere is given by $E=Qr'/4\pi\epsilon_0 R^3$ (Example 22.9), where $\vec{r}'$ is the vector from the center of the sphere to the point where E is calculated.

But $\rho=3Q/4\pi R^3$ so this may be written as $E=\rho r/3\epsilon_0$. And $\vec{E}$ is radially outward, in the direction of $\vec{r}'$, so $\vec{E}=\rho\vec{r}'/3\epsilon_0$.

For a sphere whose center is located by vector $\vec{b}$, a point inside the sphere and located by $\vec{r}$ is located by the vector $\vec{r}'=\vec{r}-\vec{b}$ relative to the center of the sphere, as shown in Figure 22.53.



EXECUTE: Thus $\vec{E}=\dfrac{\rho(\vec{r}-\vec{b})}{3\epsilon_0}$.

Figure 22.53

(b) IDENTIFY: The charge distribution can be represented as a uniform sphere with charge density ρ and centered at the origin added to a uniform sphere with charge density $-\rho$ and centered at $\vec{r}=\vec{b}$.

SET UP: $\vec{E}=\vec{E}_{\text{uniform}}+\vec{E}_{\text{hole}}$, where $\vec{E}_{\text{uniform}}$ is the field of a uniformly charged sphere with charge density ρ and $\vec{E}_{\text{hole}}$ is the field of a sphere located at the hole and with charge density $-\rho$. (Within the spherical hole the net charge density is $+\rho-\rho=0$.)

EXECUTE: $\vec{E}_{\text{uniform}}=\dfrac{\rho\vec{r}}{3\epsilon_0}$, where $\vec{r}$ is a vector from the center of the sphere.

$\vec{E}_{\text{hole}}=\dfrac{-\rho(\vec{r}-\vec{b})}{3\epsilon_0}$, at points inside the hole. Then $\vec{E}=\dfrac{\rho\vec{r}}{3\epsilon_0}+\left(\dfrac{-\rho(\vec{r}-\vec{b})}{3\epsilon_0}\right)=\dfrac{\rho\vec{b}}{3\epsilon_0}$.