

# Posterior Sampling Time Exploration – $m = 64$ and $T = 1500$

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The overall model fitting time (with `equalTimeDist = TRUE` specified) for our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`, are 2.41 days, 2.41 days, 2.41 days, and 2.35 days, respectively. If we do not take advantage of our tactics for evenly dispersed time points presented in Appendix B of our manuscript by specifying `equalTimeDist = FALSE` instead, we will need more than 3 months to fit the same methods using the same computation resources, as  $T = 1500$  is huge. This corresponds well to what we have discussed in Appendix B regarding our approaches' manifest computational acceleration in Gibbs sampler steps for temporal parameters  $\psi$ ,  $\Upsilon$ , and  $\eta_t$ 's. Since  $m = 64$  is quite small, there aren't any significant differences in the recorded posterior sampling time between our four methods, as expected.

We first display the first 50 kept post-burn-in MCMC iterations' posterior sampling time (in milliseconds) for 10 key Gibbs sampler steps (corresponding to  $\psi$ ,  $\Upsilon$ ,  $\eta_t$ 's,  $z_{jl_j}^o(s_i)$ 's or  $u_j^o(s_i)$ 's,  $\xi_j^o(s_i)$ 's,  $\theta_{jl_j}$ 's,  $\delta_{1:k}$ ,  $\rho$ ,  $\kappa$ , and  $\alpha_{jl_j}^o(s_i)$ 's) for our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`.

```
setwd(wd)
load("GibbsStepTimeFixedLfullGP.RData")
load("GibbsStepTimeFixedLblock.RData")
load("GibbsStepTimeFixedLsequen.RData")
load("GibbsStepTimeVaryLjSequen.RData")
head(GibbsStepTimeFixedLfullGP, 50)
```

##		z	xi	theta	delta	alpha	kappa	rho	eta	upsilon	psi
##	[1,]	125	131	2956	12	14	13	13	3080	21	503
##	[2,]	127	115	3018	12	14	16	13	3087	20	522
##	[3,]	128	123	2957	13	14	13	13	3028	21	511
##	[4,]	125	114	2922	13	14	13	13	3034	20	517
##	[5,]	132	111	2937	13	14	12	13	3185	22	591
##	[6,]	129	113	2919	13	14	13	13	3024	28	552
##	[7,]	125	120	2962	13	14	15	18	3023	31	545
##	[8,]	131	119	2984	12	14	15	18	3035	33	548
##	[9,]	133	118	3021	12	14	15	18	3054	32	556
##	[10,]	126	111	2937	12	14	14	12	3079	21	533
##	[11,]	126	135	2978	13	14	13	13	3153	20	545
##	[12,]	131	163	3001	13	15	13	13	3036	21	554
##	[13,]	130	112	2956	13	14	13	13	2999	20	514
##	[14,]	127	110	2926	13	14	13	13	2976	21	516
##	[15,]	125	111	2910	13	14	13	13	3024	21	520
##	[16,]	129	110	2962	13	14	13	13	3020	21	528
##	[17,]	126	111	2938	12	14	13	13	2998	20	509
##	[18,]	125	112	2960	12	14	13	13	3053	22	521
##	[19,]	132	112	3004	13	15	13	13	3043	21	519
##	[20,]	128	117	3003	13	14	13	13	3087	21	517
##	[21,]	130	115	2941	12	15	13	13	2985	21	511
##	[22,]	131	127	2962	13	15	13	13	3051	20	518
##	[23,]	127	111	2873	13	14	13	13	2956	21	499

```
## [24,] 127 151 2893 12 14 13 13 2977 20 542
## [25,] 124 138 2911 12 14 13 13 3006 20 543
## [26,] 133 114 2938 12 14 15 13 3023 20 538
## [27,] 126 113 2974 12 14 13 13 3056 21 513
## [28,] 125 115 2984 13 14 13 14 3067 20 511
## [29,] 122 118 2975 13 14 13 13 3077 20 507
## [30,] 120 110 3043 13 15 13 13 3080 20 507
## [31,] 120 115 2961 12 14 13 13 3133 20 514
## [32,] 118 114 2947 12 14 13 13 3013 20 508
## [33,] 120 117 2926 12 14 12 13 2984 20 533
## [34,] 126 120 2904 12 14 13 13 3156 22 553
## [35,] 121 110 2911 12 14 12 13 2981 21 511
## [36,] 129 117 2952 12 14 13 13 2969 20 511
## [37,] 125 110 2940 12 14 13 13 3017 21 516
## [38,] 126 112 2986 12 14 13 13 3058 21 513
## [39,] 125 115 2966 13 15 13 13 3155 21 565
## [40,] 125 110 2939 12 14 12 13 3002 21 513
## [41,] 125 110 2895 12 14 12 13 3015 20 515
## [42,] 123 110 2884 12 14 12 13 2993 20 507
## [43,] 127 113 2915 12 14 12 13 3026 20 504
## [44,] 126 112 2933 12 14 12 13 2982 20 499
## [45,] 126 111 2901 12 14 12 13 3006 20 516
## [46,] 124 112 2928 12 14 13 13 3010 24 547
## [47,] 129 128 2880 12 15 13 13 3009 20 543
## [48,] 125 110 2908 12 14 16 13 2986 21 537
## [49,] 122 144 2896 12 14 13 13 2994 20 545
## [50,] 128 145 2926 12 14 13 13 2996 20 542
```

```
head(GibbsStepTimeFixedLblock, 50)
```

```
##      z xi theta delta alpha kappa rho eta  upsi lon psi
## [1,] 130 117 2925 12 14 14 13 3059 20 529
## [2,] 130 121 2991 12 14 14 13 3046 21 532
## [3,] 136 123 2975 12 14 15 13 2998 21 540
## [4,] 133 112 2919 12 14 13 13 3044 20 507
## [5,] 131 111 2938 12 14 12 13 3017 20 509
## [6,] 133 110 2925 12 14 12 13 3040 21 505
## [7,] 130 110 2964 12 14 13 13 3031 20 501
## [8,] 139 110 2958 12 14 13 13 3072 22 512
## [9,] 138 113 3072 12 14 13 13 3066 21 508
## [10,] 132 111 2991 13 15 13 14 3103 21 535
## [11,] 133 113 2913 12 14 12 13 3028 23 535
## [12,] 130 111 2975 12 14 13 13 3022 21 506
## [13,] 142 111 2984 13 15 13 14 2988 21 510
## [14,] 132 112 2933 13 14 13 13 2986 21 513
## [15,] 129 112 2914 13 14 13 13 3030 21 520
## [16,] 130 115 2972 13 14 12 13 3041 21 516
## [17,] 130 114 2941 13 14 13 13 3012 20 526
## [18,] 132 117 2944 12 14 12 13 3055 21 505
## [19,] 134 125 2987 13 14 16 13 3030 22 527
## [20,] 134 142 2989 13 15 13 13 3115 22 563
## [21,] 144 118 2971 13 15 16 14 3020 27 559
## [22,] 123 140 2936 13 14 13 13 2981 21 556
## [23,] 125 148 2864 12 14 13 13 2930 21 548
## [24,] 122 138 2845 18 14 16 30 2947 34 571
```

```
## [25,] 124 120 2877 12 14 13 13 2977 22 532
## [26,] 127 112 2945 13 14 23 20 3054 33 566
## [27,] 123 122 2939 13 14 16 13 3045 34 568
## [28,] 134 124 2959 13 15 16 13 3020 21 531
## [29,] 131 119 2978 13 14 13 13 3060 20 514
## [30,] 131 113 3041 12 14 12 12 3067 21 525
## [31,] 130 119 2950 12 14 13 13 3113 22 532
## [32,] 135 124 2994 13 15 13 14 3177 22 556
## [33,] 131 131 2933 13 14 13 13 2979 21 540
## [34,] 130 150 2977 12 14 13 13 2960 20 549
## [35,] 130 146 2936 13 14 13 13 2993 21 548
## [36,] 127 148 2956 12 14 12 12 2956 21 557
## [37,] 141 148 2938 13 15 13 13 3030 21 543
## [38,] 133 159 2962 13 15 13 13 3009 20 539
## [39,] 135 161 2998 13 15 13 13 3065 21 568
## [40,] 135 150 2916 13 14 12 13 2973 20 541
## [41,] 129 117 2913 13 14 13 13 3025 21 518
## [42,] 131 118 2869 12 14 12 12 2977 20 507
## [43,] 129 110 2840 12 14 12 13 2980 20 506
## [44,] 132 111 2850 12 14 13 13 2933 20 498
## [45,] 131 109 2849 12 14 13 13 2981 20 507
## [46,] 130 114 2917 12 14 13 13 2987 20 500
## [47,] 136 116 2910 13 14 13 13 3015 20 515
## [48,] 130 109 2905 12 14 13 13 3022 20 513
## [49,] 133 111 2931 13 14 13 13 3030 21 523
## [50,] 131 110 2922 12 14 13 13 3000 21 517
```

```
head(GibbsStepTimeFixedLsequen, 50)
```

```
##      z xi theta delta alpha kappa rho eta  upsilon psi
## [1,] 114 114 3002 13 21 15 13 3057 21 540
## [2,] 112 112 2957 13 20 15 13 3073 21 553
## [3,] 113 112 2946 12 20 15 13 3015 21 539
## [4,] 119 110 2943 12 20 16 13 3027 20 533
## [5,] 113 109 2921 12 20 16 13 2956 20 537
## [6,] 118 110 2882 12 20 16 13 2955 20 538
## [7,] 120 113 2907 12 20 15 13 2999 20 541
## [8,] 121 112 2902 12 20 13 13 3010 21 508
## [9,] 122 109 2946 12 20 12 13 3006 20 505
## [10,] 119 112 2948 12 20 13 13 2986 20 492
## [11,] 122 112 2964 12 20 13 13 3065 20 494
## [12,] 119 113 3020 13 21 13 13 3102 22 517
## [13,] 120 119 2999 13 21 13 13 3085 21 502
## [14,] 121 113 3008 13 20 13 13 3082 21 513
## [15,] 122 115 2959 13 20 13 13 3047 20 512
## [16,] 123 111 2920 13 21 13 13 3006 21 508
## [17,] 121 110 2934 12 21 13 13 3025 20 513
## [18,] 121 116 2935 12 21 13 13 3028 21 519
## [19,] 120 110 2957 12 20 13 13 3050 22 511
## [20,] 121 112 3003 12 20 13 13 3050 21 512
## [21,] 123 112 3012 13 21 13 13 3102 21 505
## [22,] 117 110 3056 13 20 13 13 3081 21 512
## [23,] 118 111 2923 12 20 13 13 3104 22 548
## [24,] 118 113 2959 13 20 13 13 3027 21 514
## [25,] 118 112 2934 12 20 13 13 3000 21 510
```

```
## [26,] 121 121 2962 12 20 13 13 2994 21 517
## [27,] 121 111 3037 13 20 13 13 2987 21 517
## [28,] 120 111 2949 12 20 12 13 2965 20 512
## [29,] 129 110 2956 13 20 13 14 3043 21 513
## [30,] 122 110 2965 13 21 14 14 3029 20 516
## [31,] 122 111 2988 13 21 13 14 3048 21 512
## [32,] 117 112 2932 13 21 13 13 3011 21 504
## [33,] 119 116 2942 13 20 13 13 3042 22 522
## [34,] 119 113 2866 13 20 13 13 2979 21 521
## [35,] 120 113 2822 12 20 12 13 2958 20 505
## [36,] 122 113 2852 13 21 13 13 2935 21 531
## [37,] 124 114 2856 13 21 13 13 2963 21 521
## [38,] 121 113 2865 12 20 13 13 3003 21 505
## [39,] 120 113 2899 13 21 13 13 3004 21 513
## [40,] 120 112 2882 13 20 13 13 3013 21 514
## [41,] 264 114 2984 13 21 13 13 3072 22 535
## [42,] 115 115 2939 13 21 13 13 3019 23 555
## [43,] 115 117 2960 13 20 13 13 3018 20 519
## [44,] 113 112 2953 13 20 13 13 3001 21 519
## [45,] 116 117 2978 13 21 13 13 3035 21 514
## [46,] 115 110 2913 12 20 13 13 3003 21 506
## [47,] 116 111 2907 13 20 13 13 2980 20 509
## [48,] 124 113 2926 12 20 13 13 3005 21 514
## [49,] 118 116 2877 13 20 13 13 3047 22 510
## [50,] 120 112 2869 13 20 13 14 3048 22 515
```

```
head(GibbsStepTimeVaryLjSequen, 50)
```

```
##          u xi theta delta alpha kappa rho  eta  upsilon psi
## [1,] 12 37 2940 12 37 13 13 3038 21 533
## [2,] 12 37 2902 12 40 12 12 3038 20 515
## [3,] 12 37 2933 12 40 13 13 3053 21 527
## [4,] 12 41 2931 12 39 12 12 3040 20 513
## [5,] 12 37 2968 12 37 12 13 3058 21 499
## [6,] 12 37 2967 12 41 12 13 3044 21 508
## [7,] 12 36 2972 13 40 13 13 3089 21 509
## [8,] 12 45 2990 13 39 13 13 2978 20 529
## [9,] 12 38 2943 13 38 12 13 3066 21 522
## [10,] 12 37 2956 13 42 12 13 3023 22 530
## [11,] 13 37 2902 12 40 12 13 3018 21 515
## [12,] 13 39 2935 12 38 12 13 3015 21 514
## [13,] 13 39 2966 12 40 12 13 3016 21 520
## [14,] 12 39 2967 12 41 13 13 3051 20 510
## [15,] 12 37 3003 13 41 12 13 3036 20 490
## [16,] 12 37 3014 13 39 13 13 3024 22 502
## [17,] 12 37 2944 12 41 13 13 2943 21 511
## [18,] 12 37 2826 13 42 12 13 3055 21 520
## [19,] 13 40 3002 12 39 12 12 3060 21 533
## [20,] 12 37 2945 12 37 13 13 3061 21 517
## [21,] 13 46 2965 13 41 13 13 3082 20 504
## [22,] 12 42 3023 13 40 13 13 3115 21 506
## [23,] 13 38 2997 13 39 12 13 3091 21 498
## [24,] 13 38 2955 12 37 13 13 3051 21 514
## [25,] 12 39 2919 13 41 12 13 2924 21 531
## [26,] 12 39 3024 13 42 12 13 3060 21 530
```

```
## [27,] 12 40 3003 12 39 12 13 3076 21 521
## [28,] 12 40 2971 12 37 12 13 3019 22 521
## [29,] 12 44 2957 13 42 13 13 3059 20 519
## [30,] 13 43 3003 13 40 13 13 3086 21 515
## [31,] 12 38 2988 12 38 13 13 3056 20 498
## [32,] 12 38 2984 12 40 13 13 3051 20 490
## [33,] 13 38 2974 12 42 13 13 2997 20 503
## [34,] 13 38 2940 12 40 12 13 2985 20 518
## [35,] 13 43 2877 13 38 13 13 3038 22 565
## [36,] 12 39 2820 13 42 14 13 3038 21 532
## [37,] 12 37 3050 13 41 13 13 3021 21 530
## [38,] 12 37 2890 12 38 12 12 3050 20 520
## [39,] 12 39 2920 12 37 12 13 3040 20 487
## [40,] 12 37 2935 12 41 13 13 3027 21 502
## [41,] 12 46 2998 12 39 12 12 2983 21 512
## [42,] 12 37 2830 13 39 13 13 2939 20 518
## [43,] 12 38 3002 13 38 13 13 3077 21 514
## [44,] 12 37 3044 12 41 12 13 3018 21 523
## [45,] 12 36 2957 12 40 12 13 3045 21 528
## [46,] 12 46 2992 12 39 12 13 3089 20 530
## [47,] 13 41 2997 12 45 13 13 3089 21 507
## [48,] 12 37 3038 12 40 13 13 3078 21 501
## [49,] 12 38 3021 13 40 13 13 2979 20 503
## [50,] 13 38 2981 12 38 12 13 2959 22 507
```

We then present vital posterior sampling time summary statistics for the 10 key parameters.

```
round(apply(GibbsStepTimeFixedLfullGP, 2, summary), 2)
```

```
##           z      xi  theta delta alpha kappa  rho      eta  epsilon  psi
## Min.    114.00 106.00 2712.00 11.00 13.00 12.00 12.00 2774.00   19.00 475.00
## 1st Qu. 124.00 112.00 2891.00 12.00 14.00 12.00 13.00 2974.00   20.00 505.00
## Median 126.00 114.00 2926.50 12.00 14.00 13.00 13.00 3008.00   21.00 513.00
## Mean   128.04 115.96 2929.54 12.38 14.18 13.04 13.15 3011.56   21.06 517.59
## 3rd Qu. 129.00 117.00 2964.00 13.00 14.00 13.00 13.00 3044.00   21.00 528.00
## Max.   286.00 165.00 3165.00 28.00 31.00 34.00 45.00 3263.00   47.00 604.00
```

```
round(apply(GibbsStepTimeFixedLblock, 2, summary), 2)
```

```
##           z      xi  theta delta alpha kappa  rho      eta  epsilon  psi
## Min.    111.00 107.00 2741.00 11.00 13.00 12.00 12.00 2819.00   19.00 473.00
## 1st Qu. 124.00 112.00 2884.00 12.00 14.00 12.00 13.00 2972.00   20.00 507.00
## Median 128.00 114.00 2921.00 12.00 14.00 13.00 13.00 3006.00   21.00 518.00
## Mean   129.45 116.85 2923.71 12.61 14.35 13.65 13.33 3009.43   21.13 522.28
## 3rd Qu. 131.00 119.00 2960.00 13.00 14.00 14.00 13.00 3043.00   21.00 537.00
## Max.   280.00 161.00 3193.00 30.00 27.00 28.00 43.00 3279.00   48.00 614.00
```

```
round(apply(GibbsStepTimeFixedLsequen, 2, summary), 2)
```

```
##           z      xi  theta delta alpha kappa  rho      eta  epsilon  psi
## Min.    112.00 106.00 2704.00 12.00 19.00 12.00 12.00 2747.00   19.00 463.00
## 1st Qu. 123.00 111.00 2882.00 12.00 20.00 13.00 13.00 2972.75   20.00 508.00
## Median 126.00 114.00 2917.00 12.00 20.00 13.00 13.00 3006.00   21.00 523.00
## Mean   128.39 118.12 2920.04 13.77 21.58 15.05 15.06 3010.67   22.46 527.44
## 3rd Qu. 130.00 122.00 2955.00 13.00 21.00 16.00 13.00 3045.00   21.00 543.00
## Max.   294.00 166.00 3187.00 29.00 36.00 37.00 38.00 3258.00   43.00 618.00
```

```
round(apply(GibbsStepTimeVaryLjSequen, 2, summary), 2)
```

```
##           u      xi    theta delta  alpha kappa   rho    eta  epsilon    psi
## Min.    11.00 34.00 2695.00 11.00  35.00 11.00 11.00 2798.00   19.00 466.00
## 1st Qu. 12.00 37.00 2902.00 12.00  38.00 12.00 13.00 2986.00   20.00 498.00
## Median 12.00 38.00 2939.00 12.00  40.00 12.00 13.00 3024.00   21.00 508.00
## Mean   12.26 38.31 2937.98 12.34  40.08 12.49 12.81 3020.22   20.62 508.71
## 3rd Qu. 12.00 39.00 2976.00 13.00  41.00 13.00 13.00 3058.00   21.00 518.00
## Max.   16.00 57.00 3173.00 15.00 184.00 16.00 15.00 3271.00   25.00 585.00
```

```
round(apply(GibbsStepTimeFixedLfullGP, 2, sd), 3)
```

```
##          z      xi    theta    delta  alpha  kappa   rho    eta  epsilon    psi
## 16.505   7.541  55.614   0.771   0.927   1.491   1.601  53.926   2.521  17.848
```

```
round(apply(GibbsStepTimeFixedLblock, 2, sd), 3)
```

```
##          z      xi    theta    delta  alpha  kappa   rho    eta  epsilon    psi
## 15.826   8.240  56.833   1.708   1.608   2.235   2.521  53.998   2.587  19.541
```

```
round(apply(GibbsStepTimeFixedLsequen, 2, sd), 3)
```

```
##          z      xi    theta    delta  alpha  kappa   rho    eta  epsilon    psi
## 16.288   9.934  55.152   3.833   3.867   4.334   5.573  55.426   4.809  23.018
```

```
round(apply(GibbsStepTimeVaryLjSequen, 2, sd), 3)
```

```
##          u      xi    theta    delta  alpha  kappa   rho    eta  epsilon    psi
##   0.478   2.608  58.438   0.479   8.906   0.528   0.421  58.706   0.627  15.162
```