Key Parameters' Posterior Sampling Time Analysis

Yifan CHENG y.cheng@u.nus.edu

Portions of Recorded Gibbs Sampler Time for 10 Key Parameters

We first display the first 50 kept post-burn-in MCMC iterations' posterior sampling time (in milliseconds) for 10 key Gibbs sampler steps corresponding to our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`.

```
wd <- paste(projDirec, "simu/mainScalabilityVerificationSimu/m3600T50K5", sep = "/")
setwd(wd)
load("GibbsStepTimeFixedLfullGP.RData"); load("GibbsStepTimeFixedLblock.RData")
load("GibbsStepTimeFixedLsequen.RData"); load("GibbsStepTimeVaryLjSequen.RData")
head(GibbsStepTimeFixedLfullGP, 50)
```

```
##          z  xi theta delta alpha kappa   rho eta upsilon psi
##  [1,] 2682 398   243    58 25241  2985 17885 300      53  54
##  [2,] 2873 408   241    57 24280  2752 18213 299      54  55
##  [3,] 2716 399   233    57 27246  2964 19824 301      55  56
##  [4,] 2637 395   237    56 24267  2626 17303 298      54  54
##  [5,] 2661 400   233    56 25294  2750 18838 304      53  54
##  [6,] 2627 402   237    56 25587  2854 17516 294      53  54
##  [7,] 2649 401   241    55 25413  2955 18547 298      55  55
##  [8,] 2717 401   241    58 25937  2922 18433 300      54  56
##  [9,] 2757 405   241    57 24973  2777 17429 299      54  55
## [10,] 2657 412   244    58 24785  2884 18290 300      54  55
## [11,] 2699 397   236    56 26301  2863 18143 300      55  55
## [12,] 2653 397   235    55 24378  2807 15055 301      56  54
## [13,] 2737 404   236    58 25522  2891 18717 295      53  54
## [14,] 2696 396   236    56 24666  2611 17397 293      53  53
## [15,] 2631 397   233    55 24729  2839 19203 303      54  55
## [16,] 2685 402   236    58 26077  2938 17652 297      55  54
## [17,] 2662 406   237    56 24148  2677 17351 297      53  54
## [18,] 2593 404   234    55 26014  2892 17528 296      53  54
## [19,] 2719 406   236    56 25631  2919 17231 300      54  55
## [20,] 2734 397   235    56 24998  2915 17060 294      54  54
## [21,] 2620 398   237    56 25185  2931 18752 292      53  54
## [22,] 2608 401   231    55 24909  2761 16888 301      54  54
## [23,] 2586 393   231    54 25097  2875 18234 301      53  54
## [24,] 2708 408   236    56 25200  2832 17962 299      53  54
## [25,] 2762 407   235    57 24245  2819 15007 295      52  53
## [26,] 2698 390   237    56 25058  2886 18130 297      53  54
## [27,] 2641 398   235    55 24284  2915 17560 307      55  56
## [28,] 2634 402   240    58 25471  2915 17643 297      53  54
## [29,] 2717 396   235    56 24862  2931 18189 301      54  55
## [30,] 2719 401   235    56 25792  2916 17988 303      57  56
## [31,] 2739 399   247    58 25037  2959 18957 302      54  55
```

```
## [32,] 2678 408   243    58 26268  2805 17380 302      57  57
## [33,] 2605 398   245    57 25127  2961 18744 302      55  56
## [34,] 2631 400   241    57 26036  2894 19564 302      55  56
## [35,] 2704 401   241    57 25012  2852 17672 305      56  57
## [36,] 2747 399   239    58 25662  2968 17766 309      55  56
## [37,] 2659 401   241    60 24726  2869 17499 299      55  56
## [38,] 2666 399   242    58 25049  2910 17290 301      56  55
## [39,] 2606 401   240    58 24789  2939 18671 300      55  56
## [40,] 2737 401   244    57 25217  2657 17698 300      54  55
## [41,] 2753 400   237    57 25775  2810 18767 304      53  54
## [42,] 2630 407   235    56 25198  2889 18424 299      54  55
## [43,] 2615 397   236    55 24991  2778 17383 296      53  54
## [44,] 2619 407   234    56 25489  2913 18800 309      55  56
## [45,] 2730 405   238    57 25175  2889 17437 302      55  56
## [46,] 2705 401   240    57 24769  2900 18331 298      54  55
## [47,] 2643 396   237    57 25448  2937 17896 298      54  55
## [48,] 2623 404   236    56 24332  2953 17370 301      54  55
## [49,] 2636 401   238    57 24714  2970 18374 298      54  56
## [50,] 2695 409   231    56 24601  2852 17487 298      53  54
```

```
head(GibbsStepTimeFixedLblock, 50)
```

```
##          z  xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 2694 425   243    64 22546   169 892 300      56  56
##  [2,] 2567 432   247    60 22163   172 895 302      55  56
##  [3,] 2568 432   242    58 22172   170 873 302      56  56
##  [4,] 2616 427   241    57 22423   171 897 300      55  56
##  [5,] 2738 418   237    57 21755   163 851 294      54  55
##  [6,] 2650 436   246    61 22361   168 869 295      55  56
##  [7,] 2574 417   238    57 22524   171 882 296      56  57
##  [8,] 2635 424   243    59 22426   168 891 309      58  57
##  [9,] 2648 418   239    56 22807   170 880 304      56  56
## [10,] 2537 423   241    57 21780   171 881 298      55  56
## [11,] 2587 440   244    59 22203   170 892 302      56  57
## [12,] 2579 434   244    58 22201   166 863 301      56  57
## [13,] 2618 414   235    57 22465   171 878 301      55  58
## [14,] 2571 423   239    57 22166   166 876 297      55  56
## [15,] 2562 415   246    58 22349   168 883 297      55  55
## [16,] 2597 429   242    57 22655   167 874 304      56  57
## [17,] 2698 431   238    57 21808   168 868 296      56  56
## [18,] 2544 413   240    57 22011   170 896 300      56  57
## [19,] 2565 427   239    58 22109   172 876 302      55  56
## [20,] 2548 427   242    58 22426   172 886 302      56  57
## [21,] 2697 420   239    58 22154   169 866 298      55  55
## [22,] 2501 417   257    60 21867   173 872 298      56  57
## [23,] 2590 422   249    65 22179   167 864 294      56  57
## [24,] 2520 426   247    59 22908   166 892 309      59  58
## [25,] 2747 425   243    58 23610   173 879 312      54  56
## [26,] 2503 417   234    59 22343   169 871 298      56  58
## [27,] 2512 424   239    57 22117   167 902 296      56  56
## [28,] 2615 417   241    58 23568   171 879 303      55  55
## [29,] 2617 415   244    59 22696   173 894 304      56  56
## [30,] 2611 430   244    58 21987   163 934 322      58  58
## [31,] 2612 432   250    60 23005   167 877 295      55  55
## [32,] 2453 411   239    56 22281   171 887 298      56  56
```

```
## [33,] 2652 425   241    58 22585   175 903 309      57  57
## [34,] 2587 432   244    58 22351   172 893 308      56  58
## [35,] 2594 439   245    60 23078   174 899 304      56  56
## [36,] 2483 417   243    60 22346   165 868 291      54  55
## [37,] 2655 422   243    58 22028   171 882 306      57  56
## [38,] 2641 428   245    60 21824   173 880 301      56  57
## [39,] 2530 424   244    58 22204   170 878 301      55  55
## [40,] 2590 418   242    58 23360   170 888 306      56  57
## [41,] 2614 444   247    56 21939   167 862 295      54  55
## [42,] 2551 434   245    58 22042   165 876 298      55  55
## [43,] 2550 430   242    58 22403   172 877 301      56  56
## [44,] 2624 443   249    60 22992   181 863 299      55  56
## [45,] 2731 429   246    61 22646   178 915 308      58  58
## [46,] 2497 431   242    58 22097   176 892 303      56  57
## [47,] 2531 426   245    60 22067   172 903 305      58  59
## [48,] 2521 427   246    66 22737   173 887 300      56  56
## [49,] 2709 426   242    59 22439   172 898 309      56  58
## [50,] 2604 437   247    60 22884   165 859 296      55  56
```

```
head(GibbsStepTimeFixedLsequen, 50)
```

```
##          z  xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 2586 399   240    56   939   162 793 293      54  55
##  [2,] 2659 397   241    59   940   163 793 302      57  58
##  [3,] 2676 402   241    58   957   165 831 302      56  57
##  [4,] 2716 403   240    57   954   163 789 310      55  56
##  [5,] 2639 411   242    57   963   163 815 294      54  54
##  [6,] 2781 411   245    63   968   164 808 293      53  53
##  [7,] 2766 404   241    57   960   165 802 292      53  54
##  [8,] 2644 398   250    58   948   160 810 293      53  54
##  [9,] 2752 397   236    56   941   160 812 293      53  53
## [10,] 2665 400   243    56   943   161 807 300      53  54
## [11,] 2628 395   236    56   932   158 801 298      53  54
## [12,] 2720 403   237    55   945   160 802 295      54  54
## [13,] 2623 396   235    55   936   158 772 286      53  53
## [14,] 2578 400   230    55   943   159 812 289      52  53
## [15,] 2651 402   239    56   951   160 797 297      53  53
## [16,] 2661 400   233    56   955   160 822 324      56  58
## [17,] 2725 414   245    58   954   171 820 303      56  57
## [18,] 2680 409   241    57   954   162 821 299      55  55
## [19,] 2617 404   247    58   945   169 804 297      54  55
## [20,] 2826 400   241    57   947   163 803 299      55  56
## [21,] 2666 401   242    57   951   167 811 301      55  56
## [22,] 2704 408   240    57   949   161 794 293      54  55
## [23,] 2663 402   241    57   952   164 818 298      54  55
## [24,] 2652 411   243    58   950   163 786 299      55  56
## [25,] 2725 400   244    57   947   163 818 294      54  55
## [26,] 2647 396   239    57   945   164 787 295      55  55
## [27,] 2632 407   241    59   949   164 817 299      54  55
## [28,] 2672 407   243    59   952   164 817 301      55  55
## [29,] 2653 397   239    58   951   162 818 297      54  55
## [30,] 2703 403   237    56   957   162 805 297      55  55
## [31,] 2616 408   242    58   957   163 813 296      54  55
## [32,] 2636 410   237    57   944   162 797 293      55  57
## [33,] 2680 398   245    57   953   165 821 295      54  54
```

```
## [34,] 2675 408   243    57   957   164 799 294      54  54
## [35,] 2740 405   263    61   957   168 842 302      56  57
## [36,] 2672 400   242    58   933   164 818 300      55  56
## [37,] 2671 406   241    58   958   171 831 303      55  56
## [38,] 2831 412   247    59   957   167 819 309      57  57
## [39,] 2681 403   238    57   955   164 831 301      56  56
## [40,] 2718 398   245    59   950   164 808 303      56  57
## [41,] 2649 399   239    58   941   163 833 306      56  58
## [42,] 2686 403   242    58   952   165 808 297      55  56
## [43,] 2703 401   242    58   954   164 830 303      55  56
## [44,] 2654 407   247    59   955   170 808 298      55  55
## [45,] 2686 401   243    58   953   166 820 298      55  55
## [46,] 2728 403   247    59   956   169 815 301      55  56
## [47,] 2703 399   242    58   967   163 824 301      55  56
## [48,] 2709 404   249    58   961   166 811 303      56  56
## [49,] 2649 409   246    58   964   160 819 297      54  55
## [50,] 2642 406   238    56   956   165 797 296      54  56
```

```
head(GibbsStepTimeVaryLjSequen, 50)
```

```
##        u xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 56 90   238    55  2158   143 809 301      54  55
##  [2,] 55 89   232    54  2132   141 780 298      54  56
##  [3,] 54 89   242    56  2146   144 803 300      55  55
##  [4,] 55 88   239    55  2159   142 807 301      54  55
##  [5,] 56 89   233    54  2071   146 803 299      55  56
##  [6,] 56 87   237    55  2131   142 775 301      54  55
##  [7,] 55 88   234    55  2252   143 806 307      55  55
##  [8,] 56 88   231    55  2140   143 777 303      54  55
##  [9,] 57 89   235    55  2141   142 803 301      55  58
## [10,] 56 88   235    55  2147   144 810 303      55  56
## [11,] 56 90   241    55  2139   144 783 301      56  57
## [12,] 56 88   242    56  2094   143 819 305      55  56
## [13,] 55 88   236    56  2123   144 788 297      55  56
## [14,] 57 92   239    55  2245   149 812 303      56  56
## [15,] 56 88   237    55  2148   145 804 298      55  56
## [16,] 57 90   239    55  2142   146 786 299      55  56
## [17,] 55 89   239    55  2136   147 816 310      55  55
## [18,] 56 88   239    56  2176   143 787 300      54  55
## [19,] 57 87   234    55  2067   144 775 300      55  56
## [20,] 56 88   236    56  2147   143 808 301      55  56
## [21,] 55 87   237    56  2243   145 795 300      57  56
## [22,] 57 92   238    56  2173   148 788 305      56  56
## [23,] 55 87   236    56  2148   146 806 301      55  56
## [24,] 57 89   239    56  2140   145 774 300      55  56
## [25,] 55 87   233    55  2162   143 825 304      55  56
## [26,] 57 90   239    57  2063   144 790 301      55  57
## [27,] 56 92   242    58  2161   150 809 301      59  56
## [28,] 55 85   238    55  2221   142 799 297      57  56
## [29,] 57 89   233    55  2140   142 774 294      54  57
## [30,] 56 88   233    55  2142   145 810 329      56  57
## [31,] 57 90   237    55  2118   143 787 303      56  55
## [32,] 55 89   233    55  2135   143 799 302      55  56
## [33,] 56 88   239    55  2233   144 811 305      55  55
## [34,] 56 87   235    55  2132   145 793 299      55  56
```

```
## [35,] 56 91   235    56  2263   143 779 301      55  56
## [36,] 59 88   238    56  2131   144 809 300      56  56
## [37,] 56 91   237    56  2141   146 788 302      56  57
## [38,] 56 89   233    55  2178   145 774 300      54  55
## [39,] 57 88   235    55  2139   170 848 318      56  56
## [40,] 54 87   233    55  2200   144 787 294      54  55
## [41,] 57 89   233    55  2176   143 808 303      55  56
## [42,] 54 95   234    54  2265   144 803 297      54  56
## [43,] 56 91   242    55  2153   145 791 298      56  56
## [44,] 55 89   237    56  2158   143 803 301      54  55
## [45,] 56 89   234    55  2106   142 778 300      54  55
## [46,] 56 89   231    55  2139   143 796 299      54  55
## [47,] 55 90   236    55  2181   143 795 297      55  55
## [48,] 57 91   235    58  2178   144 794 298      55  55
## [49,] 56 87   233    55  2269   143 807 300      55  56
## [50,] 58 89   239    55  2149   144 785 299      54  55
```

As expected, there aren't any significant differences between our 4 methods regarding posterior sampling time for the 3 temporal parameters ψ, Υ, and $\boldsymbol{\eta}_t$'s.

Posterior Sampling Time Summary Statistics

We then present vital posterior sampling time summary statistics for the 7 spatial-related parameters ($z^o_{jl_j}(\boldsymbol{s}_i)$'s or $u^o_j(\boldsymbol{s}_i)$'s, $\xi^o_j(\boldsymbol{s}_i)$'s, θ_{jl_j}'s, $\delta_{1:k}$, ρ, κ, and $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s) to showcase the manifest scalability improvements brought about by our 3 novelties, i.e., slice sampling, spatial NNGP, and sequential updates.

```
apply(GibbsStepTimeFixedLfullGP[,1:7], 2, summary)
```

```
##                z        xi    theta    delta    alpha   kappa      rho
## Min.    2479.000  390.0000 229.0000  54.0000 23132.00 2211.00 13126.00
## 1st Qu. 2797.750  407.0000 242.0000  58.0000 25254.75 2897.00 17832.00
## Median  3693.000  531.0000 299.0000  71.0000 34987.00 3561.50 23116.50
## Mean    3819.395  542.3482 336.3604  72.5452 35893.15 3683.99 25448.17
## 3rd Qu. 4253.250  613.0000 392.0000  82.0000 41807.25 4150.25 30968.50
## Max.    8598.000 1240.0000 832.0000 189.0000 66029.00 6711.00 53315.00
```

```
apply(GibbsStepTimeFixedLblock[,1:7], 2, summary)
```

```
##                z       xi    theta   delta    alpha    kappa     rho
## Min.    2390.000 401.0000 230.0000 54.0000 21291.00 159.0000 837.000
## 1st Qu. 2554.000 418.0000 240.0000 58.0000 22031.75 168.0000 874.000
## Median  2611.000 425.0000 243.0000 58.0000 22346.00 170.0000 884.000
## Mean    2618.505 425.6026 242.8858 58.4904 22407.43 170.5392 883.873
## 3rd Qu. 2679.000 432.0000 246.0000 59.0000 22713.00 172.0000 893.000
## Max.    2927.000 582.0000 271.0000 69.0000 24919.00 200.0000 970.000
```

```
apply(GibbsStepTimeFixedLsequen[,1:7], 2, summary)
```

```
##                z       xi    theta  delta     alpha    kappa      rho
## Min.    2497.000 386.0000 228.0000 54.000  921.0000 156.0000 763.0000
## 1st Qu. 2649.000 400.0000 239.0000 57.000  948.0000 162.0000 801.0000
## Median  2685.000 404.0000 241.0000 57.000  954.0000 164.0000 814.0000
## Mean    2692.092 403.9276 241.8212 57.489  953.6206 164.9444 814.5748
## 3rd Qu. 2731.000 407.0000 244.0000 58.000  959.0000 166.0000 827.0000
## Max.    2977.000 554.0000 281.0000 70.000 1038.0000 196.0000 907.0000
```

```
apply(GibbsStepTimeVaryLjSequen[,1:7], 2, summary)
```

```
##                u      xi    theta   delta    alpha   kappa      rho
## Min.    52.0000  83.000 225.0000 52.0000 2014.000 137.000 753.0000
## 1st Qu. 55.0000  87.000 232.0000 54.0000 2125.000 142.000 783.0000
## Median  55.0000  88.000 234.0000 55.0000 2143.000 143.000 794.0000
## Mean    55.5604  88.806 234.8754 55.0086 2154.113 143.551 793.9108
## 3rd Qu. 56.0000  89.000 237.0000 56.0000 2174.000 145.000 804.0000
## Max.    67.0000 236.000 272.0000 62.0000 2391.000 171.000 866.0000
```

The results correspond well to what we have deduced in Appendix H of our manuscript.

- Compared to their `fullGPfixedL` counterparts, `NNGPblockFixedL`'s Gibbs sampler steps corresponding to ρ and κ are evidently accelerated by our **spatial NNGP prior**;
- The only Gibbs sampler step time that should clearly differ between `NNGPblockFixedL` and `NNGPsequenFixedL` is the step updating all $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s, which result from whether we adopt our **sequential updating method** or not. Since $m = 3600$ here is big, `NNGPsequenFixedL` is more than 20 times faster than `NNGPblockFixedL` for the posterior sampling step corresponding to $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s;
- Thanks to our **slice sampling approach**, `NNGPsequenVaryLj`'s Gibbs sampler steps for $u^o_j(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s are significantly faster than `NNGPsequenFixedL`'s Gibbs sampler steps for $z^o_{jl_j}(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s. It turns out that `NNGPsequenVaryLj`'s Gibbs sampler step for $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s is slower than its `NNGPsequenFixedL` counterpart, indicating that inefficiencies caused by case discussion, calculating all required upper or lower bounds, and rejection sampling outweigh acceleration brought about by slice sampling's ensured non-increasing posterior samples for L_j's through the MCMC iterations.

We finally calculate standard deviations for the 7 spatial-related parameters' posterior sampling time across all kept post-burn-in MCMC iterations.

```
round(apply(GibbsStepTimeFixedLfullGP[,1:7], 2, sd), 5)
```

```
##           z          xi       theta       delta       alpha       kappa
##  1201.10917   145.18009   109.89246    16.82124 11066.41063   920.00325
##         rho
##  8954.58930
```

```
round(apply(GibbsStepTimeFixedLblock[,1:7], 2, sd), 5)
```

```
##         z        xi     theta     delta     alpha     kappa       rho
##  84.91665  11.03039   4.41378   1.54376 512.54549   3.58567  14.67497
```

```
round(apply(GibbsStepTimeFixedLsequen[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 65.38256  7.52426  4.94183  1.28896 10.61606  4.60573 19.06303
```

```
round(apply(GibbsStepTimeVaryLjSequen[,1:7], 2, sd), 5)
```

```
##        u       xi    theta    delta    alpha    kappa      rho
##  1.15578  7.22701  4.25482  1.09652 50.96995  2.96587 14.50897
```