Key Parameters' Posterior Sampling Time Analysis

Yifan CHENG   y.cheng@u.nus.edu

Portions of Recorded Gibbs Sampler Time for 10 Key Parameters

We first display the first 50 kept post-burn-in MCMC iterations' posterior sampling time (in milliseconds) for 10 key Gibbs sampler steps corresponding to our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`.

```
wd <- paste(projDirec, "simu/mainScalabilityVerificationSimu/m400T50K5", sep = "/")
setwd(wd)
load("GibbsStepTimeFixedLfullGP.RData"); load("GibbsStepTimeFixedLblock.RData")
load("GibbsStepTimeFixedLsequen.RData"); load("GibbsStepTimeVaryLjSequen.RData")
head(GibbsStepTimeFixedLfullGP, 50)
```

```
##         z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 216 36    19     0    67    17  47  19       0   0
##  [2,] 219 36    19     0    68    17  47  18       0   0
##  [3,] 220 36    19     0    67    17  47  18       0   0
##  [4,] 217 36    18     0    68    17  48  18       0   0
##  [5,] 217 36    19     0    69    17  49  18       0   0
##  [6,] 215 36    19     0    67    17  48  18       0   0
##  [7,] 217 36    18     0    71    17  46  18       0   0
##  [8,] 224 36    18     0    66    17  47  19       0   0
##  [9,] 218 36    20     0    69    17  50  19       0   0
## [10,] 223 36    19     0    66    17  45  19       0   0
## [11,] 218 36    19     0    68    17  48  19       0   0
## [12,] 217 36    19     0    69    17  52  18       0   0
## [13,] 217 36    18     0    68    17  48  18       0   0
## [14,] 206 36    20     0    66    17  48  18       0   0
## [15,] 208 36    19     0    68    17  49  18       0   0
## [16,] 207 36    19     0    68    17  52  18       0   0
## [17,] 218 36    19     0    67    17  46  18       0   0
## [18,] 224 36    19     0    70    17  47  19       0   0
## [19,] 218 36    19     0    68    17  47  19       0   0
## [20,] 220 36    18     0    69    17  49  18       0   0
## [21,] 222 38    20     0    69    17  47  19       0   0
## [22,] 221 36    18     0    66    17  46  19       0   0
## [23,] 226 36    18     0    71    17  47  18       0   0
## [24,] 221 36    19     0    69    18  51  18       0   0
## [25,] 225 36    19     0    67    17  48  18       0   0
## [26,] 220 36    18     0    67    17  46  18       0   0
## [27,] 223 36    19     0    67    17  51  18       0   0
## [28,] 227 36    19     0    68    17  47  18       0   0
## [29,] 217 36    18     0    68    17  50  18       0   0
## [30,] 221 36    19     0    67    17  46  18       0   0
## [31,] 224 36    19     0    66    17  46  19       0   0
```

```
## [32,] 219 36    18     0    66    17  47  19       0   0
## [33,] 221 36    18     0    66    17  47  18       0   0
## [34,] 212 36    19     0    66    17  47  19       0   0
## [35,] 210 36    19     0    69    17  47  19       0   0
## [36,] 211 37    18     0    68    17  47  18       0   0
## [37,] 230 36    18     0    67    17  49  19       0   0
## [38,] 224 36    19     0    66    17  47  19       0   0
## [39,] 216 35    18     0    68    17  52  17       0   0
## [40,] 219 36    18     0    71    17  51  18       0   0
## [41,] 224 36    19     0    67    17  47  19       0   0
## [42,] 222 36    19     0    68    17  48  18       0   0
## [43,] 225 36    19     0    69    17  46  18       0   0
## [44,] 220 36    19     0    70    17  47  18       0   0
## [45,] 225 36    18     0    68    17  47  18       0   0
## [46,] 235 36    18     0    72    17  46  18       0   0
## [47,] 230 36    19     0    67    17  48  18       0   0
## [48,] 226 36    19     0    69    17  48  18       0   0
## [49,] 226 36    19     0    69    17  48  18       0   0
## [50,] 223 36    18     0    71    17  48  18       0   0
```

```
head(GibbsStepTimeFixedLblock, 50)
```

```
##          z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 191 37    20     0    64     3   7  19       0   0
##  [2,] 187 37    20     0    66     3   9  19       0   0
##  [3,] 186 37    20     0    64     3   7  19       0   0
##  [4,] 186 36    19     0    64     3   7  18       0   0
##  [5,] 186 37    19     0    64     3   7  19       0   0
##  [6,] 190 37    20     0    65     3   8  19       0   0
##  [7,] 188 41    20     0    63     3   7  19       0   0
##  [8,] 180 36    19     0    63     3   7  19       0   0
##  [9,] 174 37    19     0    65     3   8  19       0   0
## [10,] 175 37    19     0    66     3   7  19       0   0
## [11,] 180 37    19     0    63     3   7  19       0   0
## [12,] 184 37    19     0    64     3   7  19       0   0
## [13,] 183 36    19     0    66     3   7  19       0   0
## [14,] 183 37    20     0    67     3   7  19       0   0
## [15,] 187 36    19     0    64     3   7  19       0   0
## [16,] 186 37    20     0    66     3   7  19       0   0
## [17,] 186 37    19     0    65     3   7  19       0   0
## [18,] 184 36    19     0    65     3   7  19       0   0
## [19,] 183 37    20     0    65     3   7  19       0   0
## [20,] 183 36    20     0    67     3   7  19       0   0
## [21,] 183 36    20     0    66     3   7  19       0   0
## [22,] 186 37    19     0    64     3   7  19       0   0
## [23,] 189 37    19     0    65     3   7  19       0   0
## [24,] 186 36    20     0    69     3   8  21       0   0
## [25,] 184 36    19     0    65     3   7  19       0   0
## [26,] 186 37    20     0    66     3   7  19       0   0
## [27,] 190 37    19     0    65     3   7  19       0   0
## [28,] 188 37    20     0    63     3   7  19       0   0
## [29,] 186 36    19     0    64     3   7  19       0   0
## [30,] 192 38    20     0    64     3   7  19       0   0
## [31,] 182 36    19     0    64     3   7  19       0   0
## [32,] 175 36    19     0    63     3   7  19       0   0
```

```
## [33,] 174 36    19     0    66     3   7  19       0   0
## [34,] 175 37    19     0    63     3   7  19       0   0
## [35,] 176 37    19     0    64     3   7  19       0   0
## [36,] 189 40    19     0    67     3   9  19       0   0
## [37,] 186 37    20     0    65     3   7  19       0   0
## [38,] 187 36    19     0    65     3   7  18       0   0
## [39,] 183 36    19     0    64     3   7  18       0   0
## [40,] 185 37    19     0    65     3   7  19       0   0
## [41,] 184 37    20     0    64     3   8  19       0   0
## [42,] 184 37    19     0    64     3   7  19       0   0
## [43,] 184 36    19     0    64     3   7  19       0   0
## [44,] 191 36    19     0    64     3   7  19       0   0
## [45,] 186 37    19     0    65     3   7  19       0   0
## [46,] 184 37    19     0    65     3   7  19       0   0
## [47,] 185 36    19     0    65     3   7  18       0   0
## [48,] 182 39    19     0    65     3   7  19       0   0
## [49,] 186 36    19     0    64     3   7  19       0   0
## [50,] 186 36    19     0    64     3   7  19       0   0
```

```
head(GibbsStepTimeFixedLsequen, 50)
```

```
##         z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 193 39    20     0    86     3   7  19       0   0
##  [2,] 189 36    19     0    86     3   7  19       0   0
##  [3,] 193 37    19     0    87     3   7  19       0   0
##  [4,] 189 36    19     0    87     3   7  19       0   0
##  [5,] 193 37    20     0    87     3   7  19       0   0
##  [6,] 198 36    20     0    87     3   7  19       0   0
##  [7,] 188 36    20     0    87     3   8  19       0   0
##  [8,] 192 36    19     0    87     3   7  19       0   0
##  [9,] 192 36    20     0    87     3   7  19       0   0
## [10,] 191 36    19     0    87     3   7  19       0   0
## [11,] 196 36    20     0    87     3   7  19       0   0
## [12,] 196 40    19     0    87     3   7  19       0   0
## [13,] 197 37    20     0    86     3   8  19       0   0
## [14,] 196 36    19     0    87     3   8  20       0   0
## [15,] 189 36    19     0    87     3   7  19       0   0
## [16,] 193 36    20     0    87     3   7  19       0   0
## [17,] 194 36    19     0    92     3   8  21       0   0
## [18,] 319 36    19     0    85     3   7  19       0   0
## [19,] 182 36    20     0    87     3   7  19       0   0
## [20,] 180 36    20     0    87     3   7  19       0   0
## [21,] 183 36    19     0    87     3   7  19       0   0
## [22,] 197 36    20     0    87     3   7  19       0   0
## [23,] 193 36    19     0    87     3   7  19       0   0
## [24,] 195 37    19     0    87     3   7  19       0   0
## [25,] 194 36    20     0    88     3   7  19       0   0
## [26,] 197 37    20     0    87     3   7  20       0   0
## [27,] 191 36    19     0    87     3   7  18       0   0
## [28,] 197 36    19     0    87     3   7  18       0   0
## [29,] 201 37    21     0    87     3   8  19       0   0
## [30,] 191 36    19     0    87     3   8  19       0   0
## [31,] 192 36    19     0    87     3   9  19       0   0
## [32,] 191 37    20     0    87     3   8  19       0   0
## [33,] 191 36    19     0    92     3   8  19       0   0
```

```
## [34,] 202 39    21     0    87     3   9  20        0   0
## [35,] 191 37    20     0    87     3   8  19        0   0
## [36,] 194 36    19     0    87     3   7  19        0   0
## [37,] 193 37    20     0    87     3   9  20        0   0
## [38,] 189 36    19     0    87     3   8  19        0   0
## [39,] 196 37    19     0    87     3   7  20        0   0
## [40,] 192 37    19     0    86     3   7  19        0   0
## [41,] 190 36    19     0    87     3   8  19        0   0
## [42,] 181 36    19     0    87     3   8  19        0   0
## [43,] 182 37    19     0    87     3   8  19        0   0
## [44,] 184 36    19     0    87     3   8  19        0   0
## [45,] 190 37    20     0    87     3   7  19        0   0
## [46,] 191 36    19     0    87     3   8  19        0   0
## [47,] 193 36    19     0    87     3   8  19        0   0
## [48,] 195 36    20     0    86     3   8  19        0   0
## [49,] 192 36    19     0    85     3   8  19        0   0
## [50,] 192 36    19     0    87     3   8  19        0   0
```

```
head(GibbsStepTimeVaryLjSequen, 50)
```

```
##       u xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 0  6    19     0   206     1   6  19       0   0
##  [2,] 0  6    18     0   206     1   6  19       0   0
##  [3,] 0  6    19     0   208     1   6  19       0   0
##  [4,] 0  6    20     0   206     1   5  19       0   0
##  [5,] 0  6    19     0   205     1   5  19       0   0
##  [6,] 0  6    19     0   209     1   5  19       0   0
##  [7,] 0  8    19     0   203     1   5  19       0   0
##  [8,] 0  6    19     0   207     1   5  19       0   0
##  [9,] 0  6    18     0   208     1   5  19       0   0
## [10,] 0  6    19     0   210     1   6  20       0   0
## [11,] 0  6    18     0   207     1   6  19       0   0
## [12,] 0  6    19     0   213     1   6  20       0   0
## [13,] 0  6    19     0   206     1   6  19       0   0
## [14,] 0  6    19     0   207     1   5  20       0   0
## [15,] 0  6    18     0   337     1   6  19       0   0
## [16,] 0  6    18     0   194     1   5  19       0   0
## [17,] 0  6    18     0   192     1   5  19       0   0
## [18,] 0  6    18     0   194     1   5  19       0   0
## [19,] 0  6    19     0   220     1   6  19       0   0
## [20,] 0  6    18     0   206     1   6  19       0   0
## [21,] 0  6    18     0   206     1   6  19       0   0
## [22,] 0  6    19     0   204     1   6  18       0   0
## [23,] 0  6    18     0   206     1   6  19       0   0
## [24,] 0  6    18     0   210     1   6  19       0   0
## [25,] 0  6    19     0   208     1   6  19       0   0
## [26,] 0  6    18     0   207     1   7  19       0   0
## [27,] 0  6    18     0   207     1   7  19       0   0
## [28,] 0  6    19     0   207     1   7  19       0   0
## [29,] 0  6    18     0   205     1   7  19       0   0
## [30,] 0  6    18     0   203     1   7  19       0   0
## [31,] 0  6    18     0   207     1   6  19       0   0
## [32,] 0  6    18     0   207     1   7  20       0   0
## [33,] 0  6    18     0   212     1   6  19       0   0
## [34,] 0  6    18     0   213     1   8  20       0   0
```

```
## [35,] 0  6    19     0   214     1   7   20        0    0
## [36,] 0  6    19     0   207     1   7   19        0    0
## [37,] 0  6    18     0   207     1   6   19        0    0
## [38,] 0  6    19     0   206     1   7   20        0    0
## [39,] 0  6    17     0   195     1   6   19        0    0
## [40,] 0  6    17     0   193     1   6   19        0    0
## [41,] 0  6    19     0   198     1   6   19        0    0
## [42,] 0  6    19     0   208     1   7   19        0    0
## [43,] 0  6    18     0   204     1   6   18        0    0
## [44,] 0  6    17     0   206     1   6   18        0    0
## [45,] 0  6    18     0   205     1   5   18        0    0
## [46,] 0  6    19     0   209     1   5   19        0    0
## [47,] 0  6    18     0   205     1   5   19        0    0
## [48,] 0  6    18     0   208     1   5   19        0    0
## [49,] 0  6    18     0   206     1   5   19        0    0
## [50,] 0  6    18     0   207     1   5   19        0    0
```

As expected, there aren't any significant differences between our 4 methods regarding posterior sampling time for the 3 temporal parameters ψ, Υ, and $\boldsymbol{\eta}_t$'s.

Posterior Sampling Time Summary Statistics

We then present vital posterior sampling time summary statistics for the 7 spatial-related parameters ($z^o_{jl_j}(\boldsymbol{s}_i)$'s or $u^o_j(\boldsymbol{s}_i)$'s, $\xi^o_j(\boldsymbol{s}_i)$'s, θ_{jl_j}'s, $\delta_{1:k}$, ρ, κ, and $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s) to showcase the manifest scalability improvements brought about by our 3 novelties, i.e., slice sampling, spatial NNGP, and sequential updates.

```
apply(GibbsStepTimeFixedLfullGP[,1:7], 2, summary)
```

```
##                 z       xi  theta delta  alpha   kappa     rho
## Min.     168.0000  35.0000 15.000 0e+00 62.000 16.0000 44.0000
## 1st Qu.  198.0000  36.0000 18.000 0e+00 67.000 17.0000 48.0000
## Median   212.0000  36.0000 19.000 0e+00 68.000 17.0000 50.0000
## Mean     211.5018  35.9726 18.431 6e-04 67.723 17.0162 49.7274
## 3rd Qu.  219.0000  36.0000 19.000 0e+00 69.000 17.0000 52.0000
## Max.     381.0000 167.0000 23.000 1e+00 77.000 21.0000 60.0000
```

```
apply(GibbsStepTimeFixedLblock[,1:7], 2, summary)
```

```
##                 z       xi   theta delta   alpha  kappa     rho
## Min.     160.0000  35.0000 16.0000 0e+00 60.0000 2.0000  6.0000
## 1st Qu.  175.0000  36.0000 18.0000 0e+00 63.0000 3.0000  7.0000
## Median   178.0000  36.0000 19.0000 0e+00 64.0000 3.0000  7.0000
## Mean     180.8608  36.1242 18.7772 2e-04 63.7966 2.9856  7.1076
## 3rd Qu.  182.0000  36.0000 19.0000 0e+00 65.0000 3.0000  7.0000
## Max.     347.0000 176.0000 23.0000 1e+00 73.0000 6.0000 11.0000
```

```
apply(GibbsStepTimeFixedLsequen[,1:7], 2, summary)
```

```
##                 z       xi   theta delta   alpha  kappa   rho
## Min.     158.0000  34.0000 16.0000     0 82.0000 2.0000  7.00
## 1st Qu.  176.0000  35.0000 18.0000     0 85.0000 3.0000  7.00
## Median   181.0000  35.0000 19.0000     0 85.0000 3.0000  7.00
## Mean     183.3338  35.6472 18.6408     0 85.1594 3.0002  7.53
## 3rd Qu.  185.0000  36.0000 19.0000     0 85.0000 3.0000  8.00
## Max.     343.0000 171.0000 23.0000     0 95.0000 4.0000 11.00
```

```r
apply(GibbsStepTimeVaryLjSequen[,1:7], 2, summary)
```

```
##          u       xi  theta delta    alpha kappa    rho
## Min.     0   5.0000 16.000     0 185.0000     1 5.0000
## 1st Qu.  0   6.0000 18.000     0 202.0000     1 5.0000
## Median   0   6.0000 18.000     0 205.0000     1 6.0000
## Mean     0   6.1232 18.242     0 207.3494     1 5.7782
## 3rd Qu.  0   6.0000 19.000     0 208.0000     1 6.0000
## Max.     0 141.0000 21.000     0 373.0000     1 8.0000
```

The results correspond well to what we have deduced in Appendix H of our manuscript.

- Compared to their `fullGPfixedL` counterparts, `NNGPblockFixedL`'s Gibbs sampler steps corresponding to ρ and κ are evidently accelerated by our **spatial NNGP prior**;
- The only Gibbs sampler step time that should clearly differ between `NNGPblockFixedL` and `NNGPsequenFixedL` is the step updating all $\alpha_{jl_j}^{o}(\boldsymbol{s}_i)$'s, which result from whether we adopt our **sequential updating method** or not. Since $m = 400$ is quite small here, `NNGPblockFixedL` is slightly faster than `NNGPsequenFixedL` for the posterior sampling step corresponding to $\alpha_{jl_j}^{o}(\boldsymbol{s}_i)$'s. As m gets larger, `NNGPsequenFixedL` will be much faster than `NNGPblockFixedL` for the same step;
- Thanks to our **slice sampling approach**, `NNGPsequenVaryLj`'s Gibbs sampler steps for $u_j^{o}(\boldsymbol{s}_i)$'s and $\xi_j^{o}(\boldsymbol{s}_i)$'s are significantly faster than `NNGPsequenFixedL`'s Gibbs sampler steps for $z_{jl_j}^{o}(\boldsymbol{s}_i)$'s and $\xi_j^{o}(\boldsymbol{s}_i)$'s. It turns out that `NNGPsequenVaryLj`'s Gibbs sampler step for $\alpha_{jl_j}^{o}(\boldsymbol{s}_i)$'s is slower than its `NNGPsequenFixedL` counterpart, indicating that inefficiencies caused by case discussion, calculating all required upper or lower bounds, and rejection sampling outweigh acceleration brought about by slice sampling's ensured non-increasing posterior samples for L_j's through the MCMC iterations.

We finally calculate standard deviations for the 7 spatial-related parameters' posterior sampling time across all kept post-burn-in MCMC iterations.

```r
round(apply(GibbsStepTimeFixedLfullGP[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 24.41466  2.01769  0.81128  0.02449  1.72188  0.40196  2.38617
```

```r
round(apply(GibbsStepTimeFixedLblock[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 19.27167  3.34446  0.63596  0.01414  1.45686  0.13927  0.34705
```

```r
round(apply(GibbsStepTimeFixedLsequen[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 19.84517  2.81530  0.63926  0.00000  1.08325  0.04243  0.64279
```

```r
round(apply(GibbsStepTimeVaryLjSequen[,1:7], 2, sd), 5)
```

```
##        u       xi    theta    delta    alpha    kappa      rho
##  0.00000  3.28588  0.63554  0.00000 19.83958  0.00000  0.69333
```