Key Parameters' Posterior Sampling Time Analysis

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Portions of Recorded Gibbs Sampler Time for 10 Key Parameters

We first display the first 50 kept post-burn-in MCMC iterations' posterior sampling time (in milliseconds) for 10 key Gibbs sampler steps corresponding to our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`.

```
wd <- paste(projDirec, "simu/mainScalabilityVerificationSimu/m1600T30K5", sep = "/")
setwd(wd)
load("GibbsStepTimeFixedLfullGP.RData"); load("GibbsStepTimeFixedLblock.RData")
load("GibbsStepTimeFixedLsequen.RData"); load("GibbsStepTimeVaryLjSequen.RData")
head(GibbsStepTimeFixedLfullGP, 50)
```

```
##         z  xi theta delta alpha kappa  rho eta upsilon psi
##  [1,] 605 104    18     3  2298   310 1425  24       3   3
##  [2,] 569 100    33     3  2381   312 1472  23       3   3
##  [3,] 600 101    33     3  2447   312 1451  23       3   3
##  [4,] 596 105    19     3  2276   311 1354  24       3   3
##  [5,] 599 100    18     3  2507   311 1493  38       3   3
##  [6,] 597 100    19     3  2449   312 1553  39       3   3
##  [7,] 604 105    19     4  2274   311 1428  39       3   3
##  [8,] 606 101    19     3  2269   312 1615  39       3   3
##  [9,] 572 105    19     3  2274   310 1575  41       3   3
## [10,] 589 100    19     3  2286   310 1598  39       3   3
## [11,] 603 101    19     3  2229   312 1398  39       3   3
## [12,] 600 102    17     3  2409   312 1451  39       3   3
## [13,] 601 102    19     4  2327   312 1418  24       3   3
## [14,] 602 107    34     4  2277   338 1418  24       3   3
## [15,] 604 102    34     4  2327   319 1426  24       3   3
## [16,] 736 102    35     3  2292   318 1445  24       3   3
## [17,] 566 100    34     3  2311   317 1467  24       3   3
## [18,] 603 101    34     4  2266   321 1399  24       3   3
## [19,] 597 102    35     3  2418   318 1418  23       3   3
## [20,] 610 101    35     4  2314   312 1396  24       3   3
## [21,] 603 102    34     3  2281   312 1391  24       3   3
## [22,] 608 102    34     4  2490   315 1435  24       3   3
## [23,] 598 101    19     3  2577   312 1445  38       3   3
## [24,] 565  99    19     3  2319   315 1463  39       3   3
## [25,] 603 104    19     3  2288   317 1407  39       3   3
## [26,] 593 101    19     3  2267   312 1612  39       3   3
## [27,] 592 101    19     3  2301   314 1433  39       3   3
## [28,] 588 105    18     3  2346   312 1422  38       3   3
## [29,] 585 100    18     3  2363   314 1375  39       3   3
## [30,] 584 102    19     3  2345   312 1436  38       3   3
## [31,] 706 100    19     3  2401   312 1537  39       3   3
```

```
## [32,] 564 101    19     3  2477   312 1600  38       3   3
## [33,] 587 114    19     4  2234   312 1460  39       3   3
## [34,] 595 101    18     3  2282   311 1589  39       3   3
## [35,] 613 103    19     4  2277   312 1395  39       3   3
## [36,] 604 103    18     3  2427   312 1383  39       3   3
## [37,] 603 102    18     3  2325   312 1372  38       3   3
## [38,] 605 101    19     3  2286   310 1428  39       3   3
## [39,] 568 100    18     3  2402   311 1476  38       3   3
## [40,] 632 104    19     3  2733   321 1426  39       3   3
## [41,] 609 102    19     4  2498   311 1604  45       3   3
## [42,] 601 101    19     3  2444   313 1587  39       3   3
## [43,] 603 102    18     3  2263   312 1601  39       3   3
## [44,] 597 102    19     4  2278   312 1603  40       3   3
## [45,] 594 102    19     3  2261   312 1652  39       3   3
## [46,] 575 101    19     3  2310   315 1713  40       3   3
## [47,] 615 101    19     4  2303   314 1483  39       3   3
## [48,] 599 105    20     3  2344   313 1540  24       3   3
## [49,] 600 101    19     3  2373   312 1562  24       3   3
## [50,] 599 105    19     4  2253   312 1373  39       3   3
```

```r
head(GibbsStepTimeFixedLblock, 50)
```

```
##         z  xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 582 104    18     3  2139    20  91  23       2   2
##  [2,] 589 101    18     3  1972    21  97  23       3   3
##  [3,] 585 101    18     3  2169    21  92  22       2   2
##  [4,] 583 100    18     3  2204    20  91  22       2   2
##  [5,] 706 101    18     3  1995    20  95  22       3   2
##  [6,] 560 103    18     3  2140    21  93  23       3   3
##  [7,] 575 101    18     3  1964    20  93  23       3   3
##  [8,] 574 100    18     3  2105    20  92  22       2   2
##  [9,] 571 101    18     3  2121    20  94  23       3   3
## [10,] 581 101    18     3  2056    20  97  22       2   2
## [11,] 583 102    18     3  2041    20  94  22       2   2
## [12,] 573 100    18     3  1982    21  97  23       2   3
## [13,] 547 105    18     3  2196    21  94  22       2   2
## [14,] 564 101    18     3  2033    20  95  23       3   3
## [15,] 589 105    18     3  2025    20  89  21       2   2
## [16,] 586 101    18     3  1955    20  93  22       2   2
## [17,] 581 101    18     3  2124    20  95  23       3   3
## [18,] 574 101    18     3  2097    20  92  22       3   2
## [19,] 574 103    18     3  2168    21  96  24       3   3
## [20,] 583 104    19     3  2238    20  92  22       2   2
## [21,] 553 101    18     3  1956    20  96  23       3   2
## [22,] 588 103    18     3  1976    20  98  23       2   2
## [23,] 573 102    19     3  2077    20  94  23       2   2
## [24,] 576 100    18     3  2236    20  96  23       2   2
## [25,] 587 101    18     3  2217    20  94  23       2   3
## [26,] 588 101    18     3  2224    20 103  22       2   2
## [27,] 585 103    18     3  2018    21 101  23       2   3
## [28,] 703 100    17     3  2020    20  89  21       2   2
## [29,] 561 103    18     3  2149    20  91  23       2   2
## [30,] 578 100    18     3  2158    20  95  22       2   2
## [31,] 585 101    17     3  2067    20  95  23       2   2
## [32,] 587 105    18     3  2234    21  93  23       3   3
```

```
## [33,] 581 100    18     3  2066    20  91  23       3   3
## [34,] 593 103    18     3  2162    20  93  22       2   2
## [35,] 722 102    18     3  1994    20 101  23       2   2
## [36,] 559 102    18     3  2210    20  98  22       3   3
## [37,] 591 100    18     3  2033    20  93  23       3   2
## [38,] 598 100    18     3  2259    20  93  23       3   2
## [39,] 594 102    18     4  2026    20  90  23       3   3
## [40,] 587 100    18     3  2005    20  97  22       3   3
## [41,] 589 102    18     3  1983    20  99  23       3   2
## [42,] 579  99    18     3  1979    20  95  23       3   3
## [43,] 540 100    18     3  2177    20  89  22       2   2
## [44,] 572 101    18     3  2115    20  96  23       3   3
## [45,] 574 100    18     3  2246    20  92  23       2   2
## [46,] 595 102    18     3  1961    20  89  23       2   2
## [47,] 577 100    18     3  2001    20  92  23       3   3
## [48,] 583 101    18     3  2024    20  89  22       3   3
## [49,] 579 104    18     3  2187    20 101  22       2   3
## [50,] 718 106    18     3  2032    20  96  22       2   2
```

```
head(GibbsStepTimeFixedLsequen, 50)
```

```
##         z  xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 560  98    18     3   355    21 101  23       2   2
##  [2,] 530 101    18     3   354    20  99  21       2   2
##  [3,] 555  97    18     3   364    20  99  22       2   2
##  [4,] 563  98    18     3   354    20 100  22       2   2
##  [5,] 573 101    18     3   355    20 100  21       2   2
##  [6,] 559 100    18     3   354    21 102  22       2   2
##  [7,] 556  98    18     3   354    21 101  23       3   3
##  [8,] 561  96    17     3   362    20  98  23       2   2
##  [9,] 558  98    18     3   355    21 102  22       2   2
## [10,] 533 102    18     3   353    22 100  23       2   2
## [11,] 563  99    18     3   355    21 101  23       3   2
## [12,] 558  97    17     3   354    20  99  22       2   2
## [13,] 566  97    17     3   363    21 101  23       3   2
## [14,] 563  98    18     3   355    20 102  23       3   3
## [15,] 562 102    18     3   354    21  99  23       2   2
## [16,] 561  98    17     3   353    21 102  22       3   3
## [17,] 685  99    18     3   352    21 101  22       2   2
## [18,] 540  98    18     3   362    20 103  22       2   2
## [19,] 570  99    18     3   355    21 102  22       2   2
## [20,] 565  98    18     3   354    21 110  23       3   3
## [21,] 568  98    18     4   354    20 106  23       2   2
## [22,] 567  97    17     3   355    20 103  22       3   2
## [23,] 569  99    19     4   357    20  98  22       3   2
## [24,] 574  98    18     3   355    21 101  22       2   2
## [25,] 537  98    18     3   356    20 100  22       2   2
## [26,] 569 102    18     3   355    21  99  23       3   3
## [27,] 568  97    17     3   354    20 103  22       3   3
## [28,] 574  99    18     3   354    21 101  23       3   3
## [29,] 567  98    18     3   352    20  99  22       2   2
## [30,] 564 100    18     3   353    20 101  22       3   2
## [31,] 575  98    18     3   353    21 100  22       2   2
## [32,] 749 102    18     3   355    21 102  22       3   2
## [33,] 547  97    17     3   353    20  99  22       2   2
```

```
## [34,] 562  97    17     3   354    21 103  23       3   3
## [35,] 563  97    17     3   356    21 104  23       3   3
## [36,] 566  96    17     3   357    21 103  21       2   2
## [37,] 566  98    18     3   354    20 101  22       3   3
## [38,] 574  98    17     3   356    20 100  23       3   3
## [39,] 565 102    18     3   354    21 100  22       3   2
## [40,] 679  98    17     3   354    21 101  23       2   2
## [41,] 549  98    18     3   364    21 102  23       3   3
## [42,] 567  99    18     3   357    24 109  23       3   3
## [43,] 568 105    18     3   358    21 102  22       2   2
## [44,] 563 102    18     3   355    21  99  21       2   2
## [45,] 574 100    18     3   353    20  98  21       2   2
## [46,] 577  99    18     3   362    21 100  23       3   2
## [47,] 600 100    18     3   355    20  99  22       2   2
## [48,] 541  97    17     3   355    21 100  21       2   2
## [49,] 566  99    18     3   354    20 102  23       3   2
## [50,] 565  97    18     3   354    20  98  22       2   2
```

```r
head(GibbsStepTimeVaryLjSequen, 50)
```

```
##       u xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 2 16    15     2   579    12  90  24       3   2
##  [2,] 1 17    14     1   573    12  92  23       3   3
##  [3,] 1 17    15     2   578    12  93  22       2   2
##  [4,] 2 17    15     2   585    12  93  22       3   3
##  [5,] 1 17    14     2   578    12  91  21       2   2
##  [6,] 1 17    15     2   595    13  94  22       3   2
##  [7,] 2 17    16     2   542    12  95  23       3   3
##  [8,] 2 17    16     2   580    12  99  22       3   2
##  [9,] 2 20    16     2   578    12  92  22       2   2
## [10,] 2 18    16     3   586    12  86  22       3   2
## [11,] 1 19    14     1   567    12  92  21       2   2
## [12,] 2 18    15     2   581    12  92  22       3   2
## [13,] 1 17    15     2   583    12  91  23       3   3
## [14,] 1 17    15     2   704    12  91  21       2   2
## [15,] 1 17    15     2   553    13  99  23       3   3
## [16,] 1 17    15     2   574    12  94  22       2   2
## [17,] 1 17    16     2   572    12  98  23       3   2
## [18,] 2 20    17     2   608    14 102  23       3   3
## [19,] 2 19    16     2   619    14  96  23       3   3
## [20,] 1 17    15     2   573    12  89  22       2   2
## [21,] 2 18    16     2   578    12  87  22       2   2
## [22,] 1 16    14     2   695    13  95  23       3   3
## [23,] 1 16    15     2   542    12  91  22       2   2
## [24,] 1 17    14     1   573    12  96  23       3   3
## [25,] 1 19    16     2   570    12  88  22       2   2
## [26,] 1 17    14     1   571    12  95  22       2   3
## [27,] 1 17    15     1   571    13  94  22       3   2
## [28,] 1 17    14     2   570    13  93  22       3   3
## [29,] 1 17    15     2   566    12  85  21       2   2
## [30,] 1 17    15     2   704    12  93  23       3   3
## [31,] 1 17    14     1   532    13  91  22       3   2
## [32,] 2 18    15     2   570    12  93  22       2   2
## [33,] 1 17    15     2   565    13  93  23       3   3
## [34,] 2 17    15     2   568    12  90  22       2   2
```

```
## [35,] 1 17    15     2    572    12  91  21        2   2
## [36,] 1 16    15     1    568    12  93  22        3   3
## [37,] 1 17    14     1    562    12  88  23        3   3
## [38,] 1 18    16     2    701    13  91  22        2   2
## [39,] 1 16    14     1    539    14  96  24        3   3
## [40,] 2 18    15     2    570    13  92  23        3   3
## [41,] 1 17    16     2    571    13  95  22        2   2
## [42,] 1 18    16     2    579    13  93  23        3   3
## [43,] 2 18    16     3    595    14  95  23        3   3
## [44,] 2 17    16     3    583    13  93  23        2   2
## [45,] 2 18    15     2    587    13  89  22        3   2
## [46,] 1 18    14     1    712    13  96  22        3   2
## [47,] 1 17    15     1    555    13  92  23        3   3
## [48,] 1 17    16     2    581    13  96  24        3   3
## [49,] 2 18    16     2    583    14  94  23        3   3
## [50,] 1 17    14     1    586    14  93  22        3   3
```

As expected, there aren't any significant differences between our 4 methods regarding posterior sampling time for the 3 temporal parameters ψ, Υ, and $\boldsymbol{\eta}_t$'s.

Posterior Sampling Time Summary Statistics

We then present vital posterior sampling time summary statistics for the 7 spatial-related parameters ($z^o_{jl_j}(\boldsymbol{s}_i)$'s or $u^o_j(\boldsymbol{s}_i)$'s, $\xi^o_j(\boldsymbol{s}_i)$'s, θ_{jl_j}'s, $\delta_{1:k}$, ρ, κ, and $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s) to showcase the manifest scalability improvements brought about by our 3 novelties, i.e., slice sampling, spatial NNGP, and sequential updates.

```
apply(GibbsStepTimeFixedLfullGP[,1:7], 2, summary)
```

```
##                z       xi  theta   delta    alpha    kappa      rho
## Min.    546.0000  97.0000 17.000  2.0000 2175.000 305.0000 1325.000
## 1st Qu. 591.0000 100.0000 19.000  3.0000 2267.000 312.0000 1402.000
## Median  599.0000 101.0000 32.000  3.0000 2339.000 316.0000 1455.000
## Mean    604.8164 101.5026 26.311  3.2974 2358.661 316.5146 1477.033
## 3rd Qu. 607.0000 102.0000 34.000  4.0000 2430.000 319.0000 1533.000
## Max.    789.0000 253.0000 45.000 13.0000 2940.000 356.0000 2191.000
```

```
apply(GibbsStepTimeFixedLblock[,1:7], 2, summary)
```

```
##               z       xi  theta  delta    alpha  kappa      rho
## Min.    515.000  95.0000 15.000 2.0000 1831.000 18.000  81.0000
## 1st Qu. 565.000  99.0000 18.000 3.0000 1976.000 20.000  90.0000
## Median  574.000 100.0000 18.000 3.0000 2063.000 20.000  94.0000
## Mean    582.528 100.3484 18.127 3.2024 2083.491 20.362  93.8578
## 3rd Qu. 591.000 102.0000 19.000 3.0000 2171.000 21.000  97.0000
## Max.    775.000 238.0000 21.000 4.0000 2690.000 26.000 117.0000
```

```
apply(GibbsStepTimeFixedLsequen[,1:7], 2, summary)
```

```
##                z       xi   theta  delta    alpha   kappa      rho
## Min.    515.0000  94.0000 16.0000 2.0000 344.0000 19.0000  86.0000
## 1st Qu. 561.0000  97.0000 17.0000 3.0000 353.0000 20.0000  99.0000
## Median  569.0000  98.0000 18.0000 3.0000 353.0000 21.0000 101.0000
## Mean    574.3044  98.4202 17.8028 3.0554 353.8104 20.5866 101.1084
## 3rd Qu. 577.0000  99.0000 18.0000 3.0000 355.0000 21.0000 103.0000
## Max.    755.0000 235.0000 34.0000 5.0000 370.0000 28.0000 120.0000
```

```r
apply(GibbsStepTimeVaryLjSequen[,1:7], 2, summary)
```

```
##              u       xi    theta delta    alpha   kappa      rho
## Min.    1.0000  16.0000 13.0000 1.000 532.0000 11.0000  80.0000
## 1st Qu. 1.0000  17.0000 15.0000 2.000 591.0000 12.0000  94.0000
## Median  2.0000  17.0000 16.0000 2.000 599.0000 12.0000  95.0000
## Mean    1.5622  17.5796 15.6692 2.307 604.7246 12.4996  95.6168
## 3rd Qu. 2.0000  18.0000 16.0000 3.000 608.0000 13.0000  97.0000
## Max.    5.0000 155.0000 30.0000 4.000 780.0000 17.0000 114.0000
```

The results correspond well to what we have deduced in Appendix H of our manuscript.

- Compared to their `fullGPfixedL` counterparts, `NNGPblockFixedL`'s Gibbs sampler steps corresponding to ρ and κ are evidently accelerated by our **spatial NNGP prior**;
- The only Gibbs sampler step time that should clearly differ between `NNGPblockFixedL` and `NNGPsequenFixedL` is the step updating all $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s, which result from whether we adopt our **sequential updating method** or not. Since $m = 1600$ here is big, `NNGPsequenFixedL` is a few times faster than `NNGPblockFixedL` for the posterior sampling step corresponding to $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s;
- Thanks to our **slice sampling approach**, `NNGPsequenVaryLj`'s Gibbs sampler steps for $u^o_j(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s are significantly faster than `NNGPsequenFixedL`'s Gibbs sampler steps for $z^o_{jl_j}(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s. It turns out that `NNGPsequenVaryLj`'s Gibbs sampler step for $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s is slower than its `NNGPsequenFixedL` counterpart, indicating that inefficiencies caused by case discussion, calculating all required upper or lower bounds, and rejection sampling outweigh acceleration brought about by slice sampling's ensured non-increasing posterior samples for L_j's through the MCMC iterations.

We finally calculate standard deviations for the 7 spatial-related parameters' posterior sampling time across all kept post-burn-in MCMC iterations.

```r
round(apply(GibbsStepTimeFixedLfullGP[,1:7], 2, sd), 5)
```

```
##         z       xi     theta     delta     alpha     kappa       rho
##  34.87194  3.99530   7.52383   0.63074 107.25374   6.07160  95.92855
```

```r
round(apply(GibbsStepTimeFixedLblock[,1:7], 2, sd), 5)
```

```
##         z       xi     theta     delta     alpha     kappa       rho
##  36.15967  3.63043   0.83774   0.41069 137.17129   0.81406   4.73241
```

```r
round(apply(GibbsStepTimeFixedLsequen[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 33.46874  4.06474  1.03079  0.23736  3.07342  0.77086  3.63396
```

```r
round(apply(GibbsStepTimeVaryLjSequen[,1:7], 2, sd), 5)
```

```
##        u       xi    theta    delta    alpha    kappa      rho
##  0.56903  2.93015  0.76960  0.56534 34.07729  0.62584  3.83944
```