Key Parameters' Posterior Sampling Time Analysis

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Portions of Recorded Gibbs Sampler Time for 10 Key Parameters

We first display the first 50 kept post-burn-in MCMC iterations' posterior sampling time (in milliseconds) for 10 key Gibbs sampler steps corresponding to our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`.

```
wd <- paste(projDirec, "simu/mainScalabilityVerificationSimu/m400T30K5", sep = "/")
setwd(wd)
load("GibbsStepTimeFixedLfullGP.RData"); load("GibbsStepTimeFixedLblock.RData")
load("GibbsStepTimeFixedLsequen.RData"); load("GibbsStepTimeVaryLjSequen.RData")
head(GibbsStepTimeFixedLfullGP, 50)
```

```
##         z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 228 25     4     0    69    17  47   3       0   0
##  [2,] 359 25     3     0    67    17  46   3       0   0
##  [3,] 230 25     3     0    71    17  47   3       0   0
##  [4,] 226 25     4     0    68    17  47   3       0   0
##  [5,] 227 25     3     0    70    17  47   3       0   0
##  [6,] 225 25     3     0    66    17  47   3       0   0
##  [7,] 230 25     3     0    68    17  50   3       0   0
##  [8,] 233 25     3     0    68    17  47   3       0   0
##  [9,] 226 25     3     0    68    17  47   3       0   0
## [10,] 225 25     3     0    69    17  47   3       0   0
## [11,] 223 25     4     0    67    17  47   3       0   0
## [12,] 225 25     3     0    69    17  47   3       0   0
## [13,] 230 25     3     0    69    17  47   3       0   0
## [14,] 225 25     3     0    69    17  47   3       0   0
## [15,] 228 25     3     0    67    17  48   3       0   0
## [16,] 228 25     3     0    68    17  46   3       0   0
## [17,] 224 25     3     0    67    17  47   3       0   0
## [18,] 227 25     3     0    70    17  47   3       0   0
## [19,] 225 25     3     0    68    17  48   3       0   0
## [20,] 230 25     3     0    68    17  52   3       0   0
## [21,] 225 25     3     0    66    17  47   3       0   0
## [22,] 358 25     3     0    68    17  47   3       0   0
## [23,] 233 25     3     0    69    17  46   3       0   0
## [24,] 227 25     3     0    66    17  48   3       0   0
## [25,] 226 25     3     0    67    17  47   3       0   0
## [26,] 229 25     3     0    67    17  47   3       0   0
## [27,] 232 25     3     0    66    17  46   3       0   0
## [28,] 234 25     4     0    68    17  51   3       0   0
## [29,] 227 25     3     0    70    17  48   3       0   0
## [30,] 229 25     3     0    67    17  47   3       0   0
## [31,] 230 26     3     0    68    17  51   3       0   0
```

```
## [32,] 231 25     3     0    66    17  46   3       0   0
## [33,] 229 25     3     0    68    17  47   3       0   0
## [34,] 230 25     4     0    70    17  47   3       0   0
## [35,] 231 25     4     0    65    17  47   3       0   0
## [36,] 225 25     3     0    69    17  48   3       0   0
## [37,] 227 25     3     0    68    17  47   3       0   0
## [38,] 234 27     3     0    67    17  47   3       0   0
## [39,] 227 26     4     0    67    17  46   3       0   0
## [40,] 227 25     3     0    69    17  47   3       0   0
## [41,] 226 25     3     0    69    17  51   3       0   0
## [42,] 221 25     3     0    69    17  51   3       0   0
## [43,] 233 25     3     0    65    17  47   3       0   0
## [44,] 224 24     3     0    68    17  47   3       0   0
## [45,] 227 25     4     0    68    17  48   3       0   0
## [46,] 228 25     3     0    71    17  47   3       0   0
## [47,] 227 25     3     0    67    17  47   3       0   0
## [48,] 231 25     3     0    68    17  47   3       0   0
## [49,] 227 25     3     0    68    17  47   3       0   0
## [50,] 225 25     3     0    67    17  47   3       0   0
```

```
head(GibbsStepTimeFixedLblock, 50)
```

```
##         z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 142 24     3     0    67     3   7   3       0   0
##  [2,] 140 24     3     0    65     3   7   3       0   0
##  [3,] 275 24     3     0    67     3   7   3       0   0
##  [4,] 142 24     3     0    67     3   7   3       0   0
##  [5,] 141 25     3     0    66     3   7   3       0   0
##  [6,] 146 24     3     0    68     3   7   3       0   0
##  [7,] 142 25     3     0    62     3   7   3       0   0
##  [8,] 141 24     3     0    66     3   7   3       0   0
##  [9,] 143 24     4     0    63     3   7   3       0   0
## [10,] 138 24     3     0    66     3   7   3       0   0
## [11,] 142 25     3     0    65     3   7   3       0   0
## [12,] 144 24     3     0    64     3   7   3       0   0
## [13,] 143 24     3     0    65     3   7   3       0   0
## [14,] 148 25     3     0    65     3   7   3       0   0
## [15,] 146 25     3     0    65     3   7   3       0   0
## [16,] 143 25     4     0    66     3   7   3       0   0
## [17,] 144 25     3     0    65     3   7   3       0   0
## [18,] 145 24     3     0    64     3   7   3       0   0
## [19,] 145 24     4     0    65     3   7   3       0   0
## [20,] 146 24     4     0    67     3   7   3       0   0
## [21,] 145 24     3     0    66     3   7   3       0   0
## [22,] 144 27     3     0    66     3   8   3       0   0
## [23,] 144 24     3     0    65     3   7   3       0   0
## [24,] 150 24     4     0    65     3   7   3       0   0
## [25,] 144 24     3     0    63     3   7   3       0   0
## [26,] 146 24     4     0    65     3   7   3       0   0
## [27,] 146 24     3     0    65     3   7   3       0   0
## [28,] 145 24     3     0    65     3   7   3       0   0
## [29,] 143 27     3     0    64     3   7   3       0   0
## [30,] 143 24     3     0    65     3   7   3       0   0
## [31,] 145 25     3     0    67     3   7   3       0   0
## [32,] 146 25     3     0    67     3   7   3       0   0
```

```
## [33,] 145 24     3     0    67     3   7   3       0   0
## [34,] 146 25     3     0    69     3   7   3       0   0
## [35,] 277 24     3     0    65     3   7   3       0   0
## [36,] 148 24     3     0    64     3   7   3       0   0
## [37,] 155 25     3     0    67     3   7   3       0   0
## [38,] 148 24     3     0    67     3   7   3       0   0
## [39,] 146 27     3     0    66     3   7   3       0   0
## [40,] 146 24     3     0    64     3   7   3       0   0
## [41,] 145 24     3     0    66     3   7   3       0   0
## [42,] 144 24     3     0    66     3   7   3       0   0
## [43,] 146 24     4     0    66     3   7   3       0   0
## [44,] 148 24     3     0    66     3   7   3       0   0
## [45,] 155 24     3     0    64     3   7   3       0   2
## [46,] 146 24     4     0    64     3   7   4       0   0
## [47,] 148 24     3     0    63     3   7   3       0   0
## [48,] 146 24     3     0    65     3   7   3       0   0
## [49,] 146 24     3     0    64     3   7   3       0   0
## [50,] 147 24     3     0    64     3   7   3       0   0
```

```
head(GibbsStepTimeFixedLsequen, 50)
```

```
##         z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 145 24     4     0    87     3   8   3       0   0
##  [2,] 143 24     3     0    87     3   7   3       0   0
##  [3,] 146 24     3     0    87     3   7   3       0   0
##  [4,] 147 24     3     0    87     3   7   3       0   0
##  [5,] 145 24     3     0    87     3   7   3       0   0
##  [6,] 146 24     3     0    87     3   7   3       0   0
##  [7,] 146 24     3     0    88     3   7   3       0   0
##  [8,] 146 25     3     0    88     3   7   3       0   0
##  [9,] 144 24     3     0    87     3   7   3       0   0
## [10,] 146 24     3     0    87     3   7   3       0   0
## [11,] 146 24     3     0    87     3   7   3       0   0
## [12,] 150 24     3     0    87     3   7   3       0   0
## [13,] 149 28     4     0    87     3   7   3       0   0
## [14,] 141 24     3     0    87     3   7   3       0   0
## [15,] 145 25     3     0    87     3   7   3       0   0
## [16,] 146 24     3     0    88     3   7   3       0   0
## [17,] 144 24     3     0    87     3   7   3       0   0
## [18,] 144 24     3     0    87     3   7   3       0   0
## [19,] 145 25     3     0    87     3   7   3       0   0
## [20,] 150 24     3     0    91     3   7   3       0   0
## [21,] 144 24     3     0    87     3   7   3       0   0
## [22,] 143 24     3     0    88     3   8   3       0   0
## [23,] 276 24     3     0    87     3   7   3       0   0
## [24,] 144 24     3     0    87     3   9   3       0   0
## [25,] 144 24     4     0    87     3   7   3       0   0
## [26,] 145 24     3     0    87     3   7   3       0   0
## [27,] 153 24     3     0    87     3   8   3       0   0
## [28,] 146 24     3     0    87     3   7   3       0   0
## [29,] 144 24     3     0    87     3   7   3       0   0
## [30,] 147 24     3     0    87     3   7   3       0   0
## [31,] 145 24     3     0    87     3   7   3       0   0
## [32,] 143 24     3     0    87     3   7   3       0   0
## [33,] 145 24     4     0    87     3   7   3       0   0
```

```
## [34,] 146 24     3     0    96     3   7   3       0   0
## [35,] 146 24     3     0    87     3   7   3       0   0
## [36,] 142 24     4     0    87     3   7   3       0   0
## [37,] 145 24     3     0    87     3   7   3       0   0
## [38,] 144 24     3     0    87     3   7   3       0   0
## [39,] 146 24     3     0    87     3   7   3       0   0
## [40,] 145 24     3     0    87     3   7   3       0   0
## [41,] 145 24     3     0    87     3   7   3       0   0
## [42,] 146 24     3     0    87     3   7   3       0   0
## [43,] 142 24     3     0    87     3   7   3       0   0
## [44,] 150 24     3     0    87     3   8   3       0   0
## [45,] 144 24     3     0    87     3   7   3       0   0
## [46,] 145 24     3     0    87     3   7   3       0   0
## [47,] 145 25     3     0    87     3   7   3       0   0
## [48,] 145 24     3     0    87     3   7   3       0   0
## [49,] 145 24     3     0    87     3   7   3       0   0
## [50,] 145 24     3     0    88     3   7   3       0   0
```

```
head(GibbsStepTimeVaryLjSequen, 50)
```

```
##       u xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 0  4     2     0   184     1   5   3       0   0
##  [2,] 0  4     2     0   187     1   5   3       0   0
##  [3,] 0  4     2     0   186     1   6   3       0   0
##  [4,] 0  4     2     0   186     1   6   3       0   0
##  [5,] 0  4     2     0   185     1   5   3       0   0
##  [6,] 0  4     2     0   186     1   6   3       0   0
##  [7,] 0  4     3     0   185     1   5   3       0   0
##  [8,] 0  4     2     0   193     1   5   3       0   0
##  [9,] 0  4     2     0   188     1   6   3       0   0
## [10,] 0  4     2     0   188     1   6   3       0   0
## [11,] 0  4     2     0   187     1   5   3       0   0
## [12,] 0  4     2     0   196     1   5   3       0   0
## [13,] 0  4     2     0   188     1   6   3       0   0
## [14,] 0  4     2     0   188     1   5   3       0   0
## [15,] 0  4     3     0   187     1   5   3       0   0
## [16,] 0  4     3     0   187     1   5   3       0   0
## [17,] 0  4     2     0   181     1   5   3       0   0
## [18,] 0  4     2     0   184     1   5   3       0   0
## [19,] 0  4     3     0   184     1   5   3       0   0
## [20,] 0  4     2     0   182     1   5   3       0   0
## [21,] 0  4     2     0   185     1   6   3       0   0
## [22,] 0  4     3     0   184     1   5   3       0   0
## [23,] 0  4     2     0   184     1   5   3       0   0
## [24,] 0  4     2     0   184     1   5   3       0   0
## [25,] 0  4     2     0   187     1   5   3       0   0
## [26,] 0  4     2     0   183     1   5   3       0   0
## [27,] 0  4     2     0   186     1   5   3       0   0
## [28,] 0  4     2     0   185     1   5   3       0   0
## [29,] 0  4     2     0   187     1   6   3       0   0
## [30,] 0  4     2     0   187     1   6   3       0   0
## [31,] 0  4     2     0   192     1   5   3       0   0
## [32,] 0  4     2     0   185     1   5   3       0   0
## [33,] 0  4     2     0   186     1   5   3       0   0
## [34,] 0  4     2     0   184     1   5   3       0   0
```

```
## [35,] 0  4     2     0   186     1   5   3       0   0
## [36,] 0  4     3     0   186     1   5   3       0   0
## [37,] 0  4     2     0   184     1   5   3       0   0
## [38,] 0  4     2     0   185     1   5   3       0   0
## [39,] 0  4     3     0   184     1   5   3       0   0
## [40,] 0  4     2     0   183     1   5   3       0   0
## [41,] 0  4     2     0   186     1   5   3       0   0
## [42,] 0  4     3     0   318     1   5   3       0   0
## [43,] 0  6     3     0   183     1   5   3       0   0
## [44,] 0  4     3     0   185     1   6   3       0   0
## [45,] 0  4     2     0   184     1   5   3       0   0
## [46,] 0  4     2     0   184     1   5   3       0   0
## [47,] 0  4     2     0   187     1   6   3       0   0
## [48,] 0  4     2     0   185     1   5   3       0   0
## [49,] 0  4     3     0   188     1   6   3       0   0
## [50,] 0  4     2     0   192     1   6   3       0   0
```

As expected, there aren't any significant differences between our 4 methods regarding posterior sampling time for the 3 temporal parameters ψ, Υ, and $\boldsymbol{\eta}_t$'s.

Posterior Sampling Time Summary Statistics

We then present vital posterior sampling time summary statistics for the 7 spatial-related parameters ($z^o_{jl_j}(\boldsymbol{s}_i)$'s or $u^o_j(\boldsymbol{s}_i)$'s, $\xi^o_j(\boldsymbol{s}_i)$'s, θ_{jl_j}'s, $\delta_{1:k}$, ρ, κ, and $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s) to showcase the manifest scalability improvements brought about by our 3 novelties, i.e., slice sampling, spatial NNGP, and sequential updates.

```
apply(GibbsStepTimeFixedLfullGP[,1:7], 2, summary)
```

```
##               z      xi  theta delta   alpha   kappa     rho
## Min.    199.000 24.0000 2.0000 0e+00 63.0000 16.0000 45.0000
## 1st Qu. 213.000 24.0000 3.0000 0e+00 66.0000 17.0000 47.0000
## Median  216.000 25.0000 3.0000 0e+00 67.0000 17.0000 50.0000
## Mean    219.643 24.7242 3.0336 4e-04 67.0654 17.0116 48.8606
## 3rd Qu. 220.000 25.0000 3.0000 0e+00 68.0000 17.0000 50.0000
## Max.    364.000 31.0000 7.0000 2e+00 74.0000 21.0000 57.0000
```

```
apply(GibbsStepTimeFixedLblock[,1:7], 2, summary)
```

```
##                z       xi  theta delta   alpha  kappa    rho
## Min.    132.0000  24.0000 2.0000     0 62.0000 2.0000 7.0000
## 1st Qu. 142.0000  24.0000 3.0000     0 64.0000 3.0000 7.0000
## Median  145.0000  24.0000 3.0000     0 65.0000 3.0000 7.0000
## Mean    147.0992  24.4538 3.1326     0 65.0538 2.9992 7.0378
## 3rd Qu. 149.0000  25.0000 3.0000     0 66.0000 3.0000 7.0000
## Max.    301.0000 164.0000 7.0000     0 72.0000 4.0000 9.0000
```

```
apply(GibbsStepTimeFixedLsequen[,1:7], 2, summary)
```

```
##               z      xi  theta delta   alpha  kappa     rho
## Min.    135.0000  23.000 2.0000 0e+00 85.0000 3.0000  7.0000
## 1st Qu. 143.0000  24.000 3.0000 0e+00 87.0000 3.0000  7.0000
## Median  146.0000  24.000 3.0000 0e+00 87.0000 3.0000  7.0000
## Mean    148.0586  24.247 3.1088 6e-04 87.1808 3.0006  7.1372
## 3rd Qu. 149.0000  24.000 3.0000 0e+00 87.0000 3.0000  7.0000
## Max.    294.0000 163.000 7.0000 2e+00 97.0000 4.0000 10.0000
```

```
apply(GibbsStepTimeVaryLjSequen[,1:7], 2, summary)
```

```
##         u       xi theta delta    alpha kappa   rho
## Min.    0   4.0000 2.000     0 175.0000     1  5.00
## 1st Qu. 0   4.0000 2.000     0 183.0000     1  5.00
## Median  0   4.0000 2.000     0 185.0000     1  5.00
## Mean    0   4.1488 2.297     0 188.1246     1  5.28
## 3rd Qu. 0   4.0000 3.000     0 188.0000     1  6.00
## Max.    0 142.0000 3.000     0 350.0000     1 10.00
```

The results correspond well to what we have deduced in Appendix H of our manuscript.

- Compared to their `fullGPfixedL` counterparts, `NNGPblockFixedL`'s Gibbs sampler steps corresponding to ρ and κ are evidently accelerated by our **spatial NNGP prior**;
- The only Gibbs sampler step time that should clearly differ between `NNGPblockFixedL` and `NNGPsequenFixedL` is the step updating all $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s, which result from whether we adopt our **sequential updating method** or not. Since $m = 400$ is quite small here, `NNGPblockFixedL` is slightly faster than `NNGPsequenFixedL` for the posterior sampling step corresponding to $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s. As m gets larger, `NNGPsequenFixedL` will be much faster than `NNGPblockFixedL` for the same step;
- Thanks to our **slice sampling approach**, `NNGPsequenVaryLj`'s Gibbs sampler steps for $u^o_j(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s are significantly faster than `NNGPsequenFixedL`'s Gibbs sampler steps for $z^o_{jl_j}(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s. It turns out that `NNGPsequenVaryLj`'s Gibbs sampler step for $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s is slower than its `NNGPsequenFixedL` counterpart, indicating that inefficiencies caused by case discussion, calculating all required upper or lower bounds, and rejection sampling outweigh acceleration brought about by slice sampling's ensured non-increasing posterior samples for L_j's through the MCMC iterations.

We finally calculate standard deviations for the 7 spatial-related parameters' posterior sampling time across all kept post-burn-in MCMC iterations.

```
round(apply(GibbsStepTimeFixedLfullGP[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 21.74262  0.77785  0.33180  0.02828  1.57925  0.30836  2.16650
```

```
round(apply(GibbsStepTimeFixedLblock[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 15.88370  2.11167  0.37235  0.00000  1.34211  0.04000  0.20194
```

```
round(apply(GibbsStepTimeFixedLsequen[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 16.49360  2.06999  0.31966  0.03162  0.99755  0.02449  0.36222
```

```
round(apply(GibbsStepTimeVaryLjSequen[,1:7], 2, sd), 5)
```

```
##        u       xi    theta    delta    alpha    kappa      rho
##  0.00000  2.75677  0.45698  0.00000 18.68339  0.00000  0.45743
```