Key Parameters' Posterior Sampling Time Analysis

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Portions of Recorded Gibbs Sampler Time for 10 Key Parameters

We first display the first 50 kept post-burn-in MCMC iterations' posterior sampling time (in milliseconds) for 10 key Gibbs sampler steps corresponding to our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`.

```
wd <- paste(projDirec, "simu/mainScalabilityVerificationSimu/m900T30K5", sep = "/")
setwd(wd)
load("GibbsStepTimeFixedLfullGP.RData"); load("GibbsStepTimeFixedLblock.RData")
load("GibbsStepTimeFixedLsequen.RData"); load("GibbsStepTimeVaryLjSequen.RData")
head(GibbsStepTimeFixedLfullGP, 50)
```

```
##         z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 320 54     9     1   459    94 308   8       0   0
##  [2,] 321 55     9     1   464    94 307   9       0   0
##  [3,] 319 55     9     1   459    94 309   9       1   0
##  [4,] 321 55     9     1   454    94 308   8       0   0
##  [5,] 316 54     8     0   454    94 316   8       0   0
##  [6,] 315 54     9     1   454    94 305   9       0   0
##  [7,] 317 57     9     1   463    94 306   9       0   0
##  [8,] 310 54     9     1   457    95 307   9       0   0
##  [9,] 319 55     8     1   456    94 307  10       0   0
## [10,] 321 54     9     1   467    94 308  10       0   0
## [11,] 318 54     9     1   456    95 311  10       1   0
## [12,] 322 54     8     1   457    93 308   9       0   0
## [13,] 321 55     9     1   458    97 308  10       1   0
## [14,] 323 55     9     1   468    94 310   9       0   0
## [15,] 326 55     9     1   454    94 306   9       0   0
## [16,] 321 54     8     1   456    94 316   9       0   0
## [17,] 322 58     9     1   452    94 306   9       0   0
## [18,] 325 57     8     1   454    94 308   8       0   0
## [19,] 320 53     9     1   456    95 310  16       1   0
## [20,] 319 54     8     1   464    94 304   9       0   0
## [21,] 315 54     9     1   462    94 305  10       0   0
## [22,] 314 54     8     1   460    94 309  10       1   0
## [23,] 320 55     9     1   461    94 306   9       0   0
## [24,] 322 54     9     1   456   101 307  10       0   0
## [25,] 326 54     9     1   462    94 307   8       0   0
## [26,] 316 55     9     1   453    93 309   9       0   0
## [27,] 323 54     9     1   452    94 306   9       0   0
## [28,] 321 54     9     1   457    94 312  10       1   0
## [29,] 326 54     8     1   460    94 306   9       0   0
## [30,] 315 54     8     1   460    94 309   8       0   0
## [31,] 320 54     8     0   459    94 307   9       1   0
```

```
## [32,] 318 54     8     1   459    94 313   9       0   0
## [33,] 320 54     9     1   455    94 309   9       1   0
## [34,] 325 54     9     1   455    94 307   9       0   0
## [35,] 327 54     9     1   464   101 309   9       1   0
## [36,] 324 54     9     1   468    94 355   9       1   0
## [37,] 325 55     9     1   459    94 309   9       0   0
## [38,] 321 55     9     1   457    94 311   9       0   0
## [39,] 324 54     8     1   450    94 310   9       1   0
## [40,] 322 57     8     5   466    94 310   8       0   0
## [41,] 317 57     8     1   457    95 309   9       1   1
## [42,] 316 57     8     1   457    94 307  10       1   1
## [43,] 319 58     8     1   460    94 307   9       1   1
## [44,] 316 54     9     1   452    94 306  17       0   0
## [45,] 317 54     9     1   453    94 304   9       0   0
## [46,] 319 54     8     1   450    98 311   9       0   0
## [47,] 319 54     9     1   453    94 307   9       0   0
## [48,] 324 54     9     1   451    94 305   9       0   0
## [49,] 316 54     9     1   455    94 309   9       0   0
## [50,] 312 55     9     1   451    94 308   9       0   0
```

```
head(GibbsStepTimeFixedLblock, 50)
```

```
##         z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 329 57     9     1   425     8  28  10       1   1
##  [2,] 335 57     9     1   421     8  26   8       0   0
##  [3,] 330 58     9     1   426     9  27   9       0   0
##  [4,] 328 56     9     1   428     8  28  12       1   0
##  [5,] 308 57     9     1   426     9  29  10       1   0
##  [6,] 309 59     9     1   427     8  27   9       0   0
##  [7,] 330 56     9     1   435     8  26   9       0   0
##  [8,] 334 57     9     1   424     8  26   8       0   0
##  [9,] 332 57     9     1   421     8  27   9       1   1
## [10,] 333 57     9     1   422     9  27   9       0   0
## [11,] 330 58     9     1   425     8  28   9       0   0
## [12,] 328 60     9     1   421     8  27   8       0   0
## [13,] 332 57     9     1   422     8  26   8       0   0
## [14,] 342 58     9     1   424     8  28   9       1   0
## [15,] 329 58     9     1   423     8  28   9       0   0
## [16,] 336 58     9     1   431     9  29  10       1   0
## [17,] 328 56     9     1   423     8  27   9       0   0
## [18,] 334 57     9     1   424     8  27   9       0   0
## [19,] 317 57     9     1   421     8  27   9       0   0
## [20,] 318 57     9     1   420     8  27   9       0   0
## [21,] 330 57     9     1   424     9  27   9       0   0
## [22,] 327 56     9     1   421     8  27   8       0   0
## [23,] 335 57     9     1   425     8  28   9       0   0
## [24,] 332 57     8     1   426     9  29  10       1   0
## [25,] 334 57     9     1   429     9  26   8       0   0
## [26,] 332 58     9     1   426     8  28   9       1   0
## [27,] 326 57     9     1   423     9  28   9       0   0
## [28,] 333 56     9     1   425     8  27   8       0   0
## [29,] 333 57     9     1   426     8  29  10       1   1
## [30,] 337 62     9     1   430     8  26   8       0   0
## [31,] 341 57     8     1   424     9  27   8       0   0
## [32,] 464 57     9     1   425     9  28  10       1   1
```

```
## [33,] 317 57     9     1   418     8  26   7       0   0
## [34,] 318 60     8     1   432    10  30  10       1   0
## [35,] 335 57     9     1   423     8  27   9       0   0
## [36,] 339 57     9     1   424     8  27   9       0   0
## [37,] 339 58     9     1   427     9  27   8       0   0
## [38,] 333 57     9     1   421     8  28   9       0   0
## [39,] 333 61     9     1   430     9  28   9       0   0
## [40,] 339 57     9     1   420     8  27   8       0   0
## [41,] 336 58     9     1   437     8  26   9       0   0
## [42,] 332 57     9     1   426     8  28   9       0   0
## [43,] 344 57     9     1   423     8  28  10       1   1
## [44,] 337 57     9     1   425     8  28   9       1   0
## [45,] 335 57     9     1   423     9  27   9       1   0
## [46,] 320 57     9     1   425     8  27   9       0   0
## [47,] 317 57     8     1   420     8  27   8       0   0
## [48,] 340 58    10     1   423     9  26   8       0   0
## [49,] 330 57     9     1   422     9  28   9       1   0
## [50,] 336 57     9     1   435     8  27   9       1   0
```

```
head(GibbsStepTimeFixedLsequen, 50)
```

```
##         z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 330 55     9     1   195     9  26   9       0   0
##  [2,] 329 55     9     1   195     9  26   9       1   0
##  [3,] 330 54     9     1   195     9  26   8       0   0
##  [4,] 327 55     9     1   194     9  26   9       0   0
##  [5,] 334 55     9     1   194     9  26   9       0   0
##  [6,] 329 54     9     1   200     8  25   8       0   0
##  [7,] 326 54     8     1   195     9  27   9       0   0
##  [8,] 329 55     8     1   195     9  26   9       0   0
##  [9,] 332 54     9     1   200    10  26   9       0   0
## [10,] 450 57     8     1   194     8  25   8       0   0
## [11,] 310 55     9     1   195     9  27   9       0   0
## [12,] 304 55     9     1   195     9  26   9       0   0
## [13,] 330 54     8     1   196     9  27   9       0   0
## [14,] 323 54     9     1   196     9  26   9       0   0
## [15,] 326 54     9     1   195     9  27   9       0   0
## [16,] 328 54     9     1   195     9  26   8       0   0
## [17,] 326 54     8     1   195     9  26   9       0   0
## [18,] 335 55     9     1   195     9  26   9       0   0
## [19,] 326 54     8     1   195     8  27   9       1   1
## [20,] 328 54     9     1   195     9  26   9       0   0
## [21,] 322 54     8     1   194     9  26   8       0   0
## [22,] 326 54     8     1   195     9  27   9       0   0
## [23,] 326 54     9     1   195     9  26   9       0   0
## [24,] 443 54     8     1   195     9  26   9       0   0
## [25,] 310 58     9     1   194     8  26   9       0   0
## [26,] 312 53     9     1   194     9  27   9       1   0
## [27,] 324 54     9     1   194     9  26  10       0   0
## [28,] 326 54     8     1   195     9  26   9       0   0
## [29,] 329 55     9     1   194     9  27   9       0   0
## [30,] 325 55     8     1   195     8  27  10       0   0
## [31,] 324 55     9     1   199     8  26   9       0   0
## [32,] 314 57     8     1   194     8  26   9       0   0
## [33,] 314 55     8     1   195     9  27   9       1   0
```

```
## [34,] 320 54     9     1   198     9  25   8       0   0
## [35,] 316 54     8     1   195     8  26   8       0   0
## [36,] 319 55     9     1   195     9  27   9       0   0
## [37,] 315 54     9     1   194     9  26   9       0   0
## [38,] 441 55     9     1   195     9  25   9       0   0
## [39,] 297 54     8     1   194     8  26   9       1   0
## [40,] 314 54     9     1   195     9  26   9       0   0
## [41,] 327 54     9     1   195     8  25   8       0   0
## [42,] 326 54     8     1   195     9  26   9       0   0
## [43,] 328 54     9     1   195     8  26   9       0   0
## [44,] 324 54     8     1   194     9  27   9       1   0
## [45,] 322 54     9     1   194     9  26   9       0   0
## [46,] 319 54     8     1   195     8  25   8       0   0
## [47,] 330 57    10     1   194     8  26   9       0   0
## [48,] 319 54     8     1   195     9  27  10       1   0
## [49,] 320 54     9     1   194     9  26   9       0   0
## [50,] 324 54     8     1   195     8  25   8       0   0
```

```
head(GibbsStepTimeVaryLjSequen, 50)
```

```
##       u xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 0  8     7     0   289     4  22   9       0   0
##  [2,] 0  8     7     0   296     4  23  10       1   0
##  [3,] 0  8     7     0   289     4  23  10       0   0
##  [4,] 0  8     7     0   283     4  22   9       0   0
##  [5,] 0  8     7     0   284     4  21   8       0   0
##  [6,] 0  8     7     0   288     4  22   9       0   0
##  [7,] 0  8     8     0   290     4  23  10       1   0
##  [8,] 0  8     8     1   293     4  22   9       0   0
##  [9,] 0  8     6     0   291     4  23   9       1   1
## [10,] 0 10     8     0   286     4  22   9       0   0
## [11,] 0  8     6     0   290     4  23   9       1   0
## [12,] 0  8     7     0   294     4  23   9       0   0
## [13,] 0  8     6     0   285     4  23   9       0   0
## [14,] 0  8     7     0   289     4  22   9       0   0
## [15,] 0  8     7     0   289     4  22   9       0   0
## [16,] 0  8     8     1   292     4  24  10       1   1
## [17,] 0  8     7     0   287     4  22   8       0   0
## [18,] 0  8     6     0   289     4  22   8       0   0
## [19,] 0  8     7     0   296     4  22   9       0   0
## [20,] 0  8     8     0   294     4  23  10       1   0
## [21,] 0  8     8     0   292     4  23  10       1   0
## [22,] 0  8     6     0   283     4  22   9       0   0
## [23,] 0  8     7     0   287     4  23  10       1   0
## [24,] 0  8     7     0   291     4  23  10       1   0
## [25,] 0  9     7     0   289     4  22   9       0   0
## [26,] 0  8     6     0   288     4  22   9       0   0
## [27,] 0  8     7     0   298     4  23  10       1   0
## [28,] 0  8     7     0   292     4  23  10       1   0
## [29,] 0  8     8     0   294     4  22   9       0   0
## [30,] 0  8     7     0   295     4  23   9       0   0
## [31,] 0  8     7     0   292     4  23   9       0   0
## [32,] 0  8     6     0   288     4  23  10       1   0
## [33,] 0  8     7     0   288     4  22   9       0   0
## [34,] 0  8     8     0   293     4  22  10       0   0
```

```
## [35,] 0  8     7     0   287     4  22   9       0   0
## [36,] 0  8     7     0   297     4  23   9       1   0
## [37,] 0  8     8     0   290     4  23  10       1   0
## [38,] 0  8     6     0   291     4  22   8       0   0
## [39,] 0  8     6     0   297     4  23  10       0   0
## [40,] 0  8     6     0   288     4  23  10       1   1
## [41,] 0  8     8     1   294     4  24  10       1   0
## [42,] 0  8     7     0   288     4  22   9       0   0
## [43,] 0  8     8     0   287     4  22   9       0   0
## [44,] 0  8     8     0   285     4  22   9       0   0
## [45,] 0  8     7     0   290     4  23   9       0   0
## [46,] 0  8     7     0   287     4  22   9       0   0
## [47,] 0  8     6     0   291     4  22   9       0   0
## [48,] 0  8     8     0   293     4  23  10       1   0
## [49,] 0  8     6     0   421     4  23  10       1   1
## [50,] 0  8     8     0   297     4  22  10       1   0
```

As expected, there aren't any significant differences between our 4 methods regarding posterior sampling time for the 3 temporal parameters ψ, Υ, and $\boldsymbol{\eta}_t$'s.

Posterior Sampling Time Summary Statistics

We then present vital posterior sampling time summary statistics for the 7 spatial-related parameters ($z^o_{jl_j}(\boldsymbol{s}_i)$'s or $u^o_j(\boldsymbol{s}_i)$'s, $\xi^o_j(\boldsymbol{s}_i)$'s, θ_{jl_j}'s, $\delta_{1:k}$, ρ, κ, and $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s) to showcase the manifest scalability improvements brought about by our 3 novelties, i.e., slice sampling, spatial NNGP, and sequential updates.

```
apply(GibbsStepTimeFixedLfullGP[,1:7], 2, summary)
```

```
##                z       xi   theta  delta    alpha    kappa     rho
## Min.    292.0000  52.0000  6.0000 0.0000 439.0000  91.0000 297.000
## 1st Qu. 316.0000  54.0000  8.0000 1.0000 452.0000  94.0000 306.000
## Median  321.0000  54.0000  9.0000 1.0000 455.0000  94.0000 308.000
## Mean    324.7298  54.7586  8.6664 0.9486 455.3278  93.9964 308.928
## 3rd Qu. 325.0000  55.0000  9.0000 1.0000 458.0000  94.0000 310.000
## Max.    480.0000 191.0000 18.0000 5.0000 480.0000 103.0000 364.000
```

```
apply(GibbsStepTimeFixedLblock[,1:7], 2, summary)
```

```
##                z       xi   theta  delta    alpha   kappa     rho
## Min.    293.0000  55.0000  7.0000 0.0000 410.0000  8.0000 25.0000
## 1st Qu. 326.0000  57.0000  9.0000 1.0000 420.0000  8.0000 26.0000
## Median  333.0000  57.0000  9.0000 1.0000 424.0000  8.0000 27.0000
## Mean    335.7746  57.1688  8.8824 0.9938 423.2304  8.1494 27.1156
## 3rd Qu. 339.0000  57.0000  9.0000 1.0000 426.0000  8.0000 28.0000
## Max.    481.0000 190.0000 18.0000 5.0000 449.0000 10.0000 33.0000
```

```
apply(GibbsStepTimeFixedLsequen[,1:7], 2, summary)
```

```
##                z      xi   theta  delta    alpha   kappa     rho
## Min.    288.0000  53.000  7.0000 0.0000 193.0000  8.0000 25.0000
## 1st Qu. 318.0000  55.000  9.0000 1.0000 195.0000  9.0000 26.0000
## Median  325.0000  56.000  9.0000 1.0000 199.0000  9.0000 27.0000
## Mean    328.3926  55.633  8.8414 0.9842 197.8944  8.8828 26.7362
## 3rd Qu. 332.0000  56.000  9.0000 1.0000 200.0000  9.0000 27.0000
## Max.    496.0000 196.000 17.0000 5.0000 232.0000 11.0000 37.0000
```

```
apply(GibbsStepTimeVaryLjSequen[,1:7], 2, summary)
```

```
##         u       xi  theta  delta    alpha  kappa     rho
## Min.    0   7.0000 5.0000 0.0000 256.0000 4.0000 20.0000
## 1st Qu. 0   8.0000 6.0000 0.0000 275.0000 4.0000 22.0000
## Median  0   8.0000 7.0000 0.0000 279.0000 4.0000 22.0000
## Mean    0   8.1984 6.9156 0.0406 282.4816 4.0032 22.1778
## 3rd Qu. 0   8.0000 7.0000 0.0000 284.0000 4.0000 23.0000
## Max.    0 145.0000 9.0000 1.0000 446.0000 6.0000 29.0000
```

The results correspond well to what we have deduced in Appendix H of our manuscript.

- Compared to their `fullGPfixedL` counterparts, `NNGPblockFixedL`'s Gibbs sampler steps corresponding to ρ and κ are evidently accelerated by our **spatial NNGP prior**;
- The only Gibbs sampler step time that should clearly differ between `NNGPblockFixedL` and `NNGPsequenFixedL` is the step updating all $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s, which result from whether we adopt our **sequential updating method** or not. Since $m = 900$ here is quite big, `NNGPsequenFixedL` is considerably faster than `NNGPblockFixedL` for the posterior sampling step corresponding to $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s;
- Thanks to our **slice sampling approach**, `NNGPsequenVaryLj`'s Gibbs sampler steps for $u^o_j(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s are significantly faster than `NNGPsequenFixedL`'s Gibbs sampler steps for $z^o_{jl_j}(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s. It turns out that `NNGPsequenVaryLj`'s Gibbs sampler step for $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s is slower than its `NNGPsequenFixedL` counterpart, indicating that inefficiencies caused by case discussion, calculating all required upper or lower bounds, and rejection sampling outweigh acceleration brought about by slice sampling's ensured non-increasing posterior samples for L_j's through the MCMC iterations.

We finally calculate standard deviations for the 7 spatial-related parameters' posterior sampling time across all kept post-burn-in MCMC iterations.

```
round(apply(GibbsStepTimeFixedLfullGP[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 24.69713  3.94520  0.76709  0.34869  5.08322  1.15596  6.88755
```

```
round(apply(GibbsStepTimeFixedLblock[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 25.96119  2.21099  0.50222  0.10081  5.04615  0.37082  0.99380
```

```
round(apply(GibbsStepTimeFixedLsequen[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 27.01919  4.09195  0.53170  0.14817  3.01620  0.35650  1.06002
```

```
round(apply(GibbsStepTimeVaryLjSequen[,1:7], 2, sd), 5)
```

```
##        u       xi    theta    delta    alpha    kappa      rho
##  0.00000  3.84884  0.72179  0.19738 22.55208  0.06317  0.84447
```