Key Parameters' Posterior Sampling Time Analysis

Yifan CHENG y.cheng@u.nus.edu

Portions of Recorded Gibbs Sampler Time for 10 Key Parameters

We first display the first 50 kept post-burn-in MCMC iterations' posterior sampling time (in milliseconds) for 10 key Gibbs sampler steps corresponding to our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`.

```
wd <- paste(projDirec, "simu/mainScalabilityVerificationSimu/m900T50K5", sep = "/")
setwd(wd)
load("GibbsStepTimeFixedLfullGP.RData"); load("GibbsStepTimeFixedLblock.RData")
load("GibbsStepTimeFixedLsequen.RData"); load("GibbsStepTimeVaryLjSequen.RData")
head(GibbsStepTimeFixedLfullGP, 50)
```

```
##         z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 359 82    44     1   461    95 320  45       1   1
##  [2,] 390 83    44     1   462    94 322  44       1   1
##  [3,] 387 83    44     1   467    95 320  44       1   1
##  [4,] 388 82    44     1   461    94 320  44       1   1
##  [5,] 382 83    44     1   460    95 323  44       1   1
##  [6,] 384 82    45     1   462    95 322  45       1   1
##  [7,] 401 83    45     1   493   105 343  47       1   1
##  [8,] 378 85    43     1   465    94 324  45       1   1
##  [9,] 392 82    44     1   480   101 339  47       1   1
## [10,] 405 84    47     1   485    94 318  45       1   1
## [11,] 510 82    44     1   462    94 321  44       1   1
## [12,] 356 83    44     1   464    96 322  43       1   1
## [13,] 375 87    44     1   466    96 324  44       1   1
## [14,] 379 83    44     1   459    95 327  43       1   1
## [15,] 378 82    43     1   465    95 321  44       1   1
## [16,] 382 83    44     1   470    95 321  45       1   1
## [17,] 402 83    44     1   467    95 322  43       1   1
## [18,] 391 84    44     1   470    96 321  46       1   1
## [19,] 388 83    44     1   467    94 318  44       1   1
## [20,] 383 83    43     1   467    95 322  45       1   1
## [21,] 389 82    44     1   466    95 325  45       1   1
## [22,] 385 83    44     1   465    94 320  45       1   1
## [23,] 492 83    44     1   463    95 323  46       1   1
## [24,] 365 83    44     1   473    95 323  45       1   1
## [25,] 395 82    44     1   472   104 325  44       1   1
## [26,] 389 83    44     1   466    94 320  43       1   1
## [27,] 392 82    44     1   464    95 322  45       1   1
## [28,] 393 83    43     1   463    95 320  44       1   1
## [29,] 392 83    44     1   473    95 326  44       1   1
## [30,] 388 83    44     1   463    95 323  44       1   1
## [31,] 392 83    45     1   469    96 370  44       1   1
```

```
## [32,] 396 83    44     1   473    95 323  44       1   1
## [33,] 391 82    43     1   477   102 332  46       1   1
## [34,] 382 83    44     1   469    98 326  45       1   1
## [35,] 364 83    44     1   477    95 325  46       1   1
## [36,] 372 82    45     1   471    95 319  46       1   1
## [37,] 391 83    44     1   466    95 323  47       1   1
## [38,] 394 83    44     1   481   100 339  47       1   1
## [39,] 393 83    43     1   467    96 323  44       1   1
## [40,] 389 81    43     1   477    96 321  44       1   1
## [41,] 394 82    43     1   466    94 323  44       1   1
## [42,] 386 84    44     1   471    97 370  45       1   1
## [43,] 397 87    45     1   467    97 321  43       1   1
## [44,] 404 83    43     1   469    95 328  44       1   1
## [45,] 391 82    43     1   462    94 319  42       1   1
## [46,] 506 83    44     1   466    95 317  43       1   1
## [47,] 369 82    44     1   464    94 319  44       1   1
## [48,] 383 83    44     1   470    95 323  44       1   1
## [49,] 383 83    48     1   468    97 324  45       1   1
## [50,] 385 83    45     1   473    97 326  45       1   1
```

```r
head(GibbsStepTimeFixedLblock, 50)
```

```
##         z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 586 85    44     1   429     9  28  45       1   1
##  [2,] 560 85    44     1   424     9  30  45       1   1
##  [3,] 596 85    47     1   420     8  28  45       1   1
##  [4,] 569 84    45     1   422     8  29  45       1   1
##  [5,] 583 89    45     1   427     9  28  45       1   1
##  [6,] 587 84    44     1   433     9  27  45       1   1
##  [7,] 571 86    46     1   424     9  29  47       1   1
##  [8,] 571 87    47     1   420     8  27  45       1   1
##  [9,] 574 85    45     1   427     8  28  46       1   1
## [10,] 571 85    45     1   424     8  28  44       1   1
## [11,] 587 85    47     1   426     9  28  45       1   1
## [12,] 568 85    45     1   425     9  30  46       1   1
## [13,] 573 88    45     1   429     8  27  45       1   1
## [14,] 575 84    45     1   420     9  28  46       1   1
## [15,] 582 85    45     1   428     8  28  46       1   1
## [16,] 580 86    45     1   422     9  30  45       1   1
## [17,] 568 85    45     1   427     9  30  45       1   1
## [18,] 571 85    45     1   425     8  28  46       1   1
## [19,] 688 87    43     1   434     9  29  46       1   1
## [20,] 577 85    45     1   428     8  32  45       1   1
## [21,] 560 87    43     1   429     8  28  44       1   1
## [22,] 565 84    44     1   425     8  29  44       1   1
## [23,] 570 85    44     1   426     8  27  43       1   1
## [24,] 582 85    45     1   430     9  27  45       1   1
## [25,] 564 87    46     1   430     9  29  46       1   1
## [26,] 570 85    45     1   427     9  30  46       1   1
## [27,] 692 84    44     1   442     9  26  45       1   1
## [28,] 579 85    46     1   418     8  28  45       1   1
## [29,] 581 85    45     1   422     9  29  46       1   1
## [30,] 578 84    44     1   423     8  28  45       1   1
## [31,] 560 84    44     1   428     8  27  45       1   1
## [32,] 575 85    45     1   439     8  30  45       1   1
```

```
## [33,] 563 85    44     1   421     8  28  45       1   1
## [34,] 572 90    48     1   423     8  28  46       1   1
## [35,] 701 84    45     1   427     8  25  44       1   1
## [36,] 578 85    45     1   423     8  28  45       1   1
## [37,] 578 86    45     1   428     9  27  45       1   1
## [38,] 577 85    44     1   427     9  28  45       1   1
## [39,] 573 85    45     1   418     8  27  45       1   1
## [40,] 577 83    45     1   427     8  28  46       1   1
## [41,] 568 87    45     1   419     9  27  44       1   1
## [42,] 582 85    45     1   425     9  29  45       1   1
## [43,] 573 84    44     1   427     9  31  45       1   1
## [44,] 564 85    45     1   420     9  30  45       1   1
## [45,] 573 84    46     1   428     8  27  45       1   1
## [46,] 571 85    44     1   429     8  27  45       1   1
## [47,] 569 88    44     1   423     8  28  44       1   1
## [48,] 580 84    44     1   426     8  27  45       1   1
## [49,] 567 85    44     1   427     9  28  45       1   1
## [50,] 574 85    45     1   428     8  27  44       1   1
```

```r
head(GibbsStepTimeFixedLsequen, 50)
```

```
##         z xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 530 85    44     1   197     9  32  45       1   1
##  [2,] 537 85    45     1   198     9  30  44       1   1
##  [3,] 536 83    44     1   199     9  33  45       1   1
##  [4,] 523 85    45     1   205     9  30  45       1   1
##  [5,] 533 84    45     1   199     9  32  45       1   1
##  [6,] 533 84    44     1   199     9  36  46       1   1
##  [7,] 529 83    44     1   197     9  33  45       1   1
##  [8,] 530 85    44     1   199     9  32  46       1   1
##  [9,] 533 84    44     1   198     9  31  46       1   1
## [10,] 659 84    45     1   199     9  31  44       1   1
## [11,] 533 85    44     1   197     9  35  45       1   1
## [12,] 529 85    44     1   200     9  35  46       1   1
## [13,] 533 84    44     1   199     9  31  46       1   1
## [14,] 533 88    45     1   198     9  32  45       1   1
## [15,] 523 87    45     1   198     9  32  45       1   1
## [16,] 531 84    45     1   198     9  32  45       1   1
## [17,] 533 84    45     1   199     9  33  46       1   1
## [18,] 531 85    44     1   198     9  33  45       1   1
## [19,] 530 86    45     1   198     9  29  45       1   1
## [20,] 530 84    45     1   198     8  30  46       1   1
## [21,] 530 87    44     1   199     9  33  46       1   1
## [22,] 520 87    45     1   198     9  32  48       1   1
## [23,] 532 88    48     1   196     9  31  45       1   1
## [24,] 527 84    44     1   198     9  32  46       1   1
## [25,] 533 84    44     1   200     9  31  45       1   1
## [26,] 544 84    45     1   201     9  32  46       1   1
## [27,] 671 84    44     1   198     9  31  44       1   1
## [28,] 559 85    47     1   199    10  33  45       1   1
## [29,] 536 84    44     1   207     9  32  47       1   1
## [30,] 543 83    45     1   198     9  33  45       1   1
## [31,] 552 85    45     1   197     9  33  47       1   1
## [32,] 531 85    45     1   196     9  31  44       1   1
## [33,] 525 82    44     1   206     9  33  44       1   1
```

```
## [34,] 517 83    43     1   198     9  32  45        1   1
## [35,] 521 83    45     1   197     9  31  44        1   1
## [36,] 525 86    44     1   196     9  32  45        1   1
## [37,] 520 83    44     1   197     9  31  44        1   1
## [38,] 533 83    44     1   197     9  32  45        1   1
## [39,] 516 83    43     1   199     9  33  45        1   1
## [40,] 524 84    44     1   200     9  30  43        1   1
## [41,] 549 85    44     1   199     9  32  45        1   1
## [42,] 525 84    43     1   200     9  32  42        1   1
## [43,] 529 86    44     1   197     9  31  45        1   1
## [44,] 660 85    44     1   199     9  32  45        1   1
## [45,] 522 84    44     1   198     9  31  44        1   1
## [46,] 529 85    44     1   199     9  32  45        1   1
## [47,] 515 84    45     1   196     9  31  44        1   1
## [48,] 540 88    44     1   199     9  31  45        1   1
## [49,] 544 84    45     1   199     9  32  44        1   1
## [50,] 525 83    44     1   199     9  31  44        1   1
```

```
head(GibbsStepTimeVaryLjSequen, 50)
```

```
##       u xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 0 13    43     0   391     4  25  43       1   1
##  [2,] 0 12    40     0   382     4  26  43       1   1
##  [3,] 0 12    41     1   383     4  25  43       1   1
##  [4,] 0 12    40     1   380     4  25  42       1   1
##  [5,] 0 14    42     1   390     4  26  46       1   1
##  [6,] 0 12    41     1   502     4  25  42       1   1
##  [7,] 0 12    40     0   379     4  25  42       1   1
##  [8,] 0 12    42     1   385     4  24  41       0   1
##  [9,] 0 12    40     0   385     4  25  42       1   1
## [10,] 0 12    40     0   385     4  25  42       1   1
## [11,] 0 12    41     0   379     4  25  42       1   1
## [12,] 0 12    40     0   375     4  26  44       1   1
## [13,] 0 12    42     1   390     4  25  43       1   1
## [14,] 0 12    41     1   390     4  26  43       1   1
## [15,] 0 12    41     0   379     4  26  43       1   1
## [16,] 0 12    42     1   384     4  25  42       0   1
## [17,] 0 13    42     1   386     4  25  42       1   1
## [18,] 0 12    39     0   516     4  28  44       1   1
## [19,] 0 12    41     0   384     4  25  44       1   1
## [20,] 0 14    41     0   381     4  25  43       1   1
## [21,] 0 12    42     1   388     4  26  43       1   1
## [22,] 0 12    42     0   377     4  24  42       1   0
## [23,] 0 12    43     1   385     4  25  43       1   1
## [24,] 0 12    42     1   379     4  25  42       1   1
## [25,] 0 12    40     0   378     4  25  42       0   0
## [26,] 0 12    42     1   386     4  25  44       1   1
## [27,] 0 12    42     1   402     5  27  47       1   1
## [28,] 0 13    41     0   406     4  25  42       1   1
## [29,] 0 14    41     1   384     4  25  41       0   0
## [30,] 0 12    41     1   382     4  26  42       1   1
## [31,] 0 11    42     1   383     4  26  43       1   1
## [32,] 0 12    44     1   384     4  25  43       1   1
## [33,] 0 11    40     0   380     4  25  42       1   1
## [34,] 0 12    40     1   382     4  24  44       1   1
```

```
## [35,] 0 11     41      1     383      4   26   41         1    0
## [36,] 0 12     41      1     385      4   26   44         1    1
## [37,] 0 12     41      0     384      4   25   44         1    1
## [38,] 0 12     42      1     377      4   23   43         0    0
## [39,] 0 12     44      1     391      4   27   44         1    1
## [40,] 0 12     42      0     388      4   26   46         1    1
## [41,] 0 12     41      0     380      4   26   43         1    1
## [42,] 0 12     41      0     397      5   24   47         1    1
## [43,] 0 12     43      1     385      4   28   46         1    1
## [44,] 0 12     40      0     382      4   27   44         1    1
## [45,] 0 12     42      1     383      4   26   43         1    1
## [46,] 0 12     42      0     378      4   24   41         0    0
## [47,] 0 12     42      1     391      4   26   44         1    1
## [48,] 0 12     41      1     386      4   25   43         1    1
## [49,] 0 12     41      1     384      4   26   44         1    1
## [50,] 0 12     42      1     381      4   23   41         1    1
```

As expected, there aren't any significant differences between our 4 methods regarding posterior sampling time for the 3 temporal parameters ψ, Υ, and $\boldsymbol{\eta}_t$'s.

Posterior Sampling Time Summary Statistics

We then present vital posterior sampling time summary statistics for the 7 spatial-related parameters ($z^o_{jl_j}(\boldsymbol{s}_i)$'s or $u^o_j(\boldsymbol{s}_i)$'s, $\xi^o_j(\boldsymbol{s}_i)$'s, θ_{jl_j}'s, $\delta_{1:k}$, ρ, κ, and $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s) to showcase the manifest scalability improvements brought about by our 3 novelties, i.e., slice sampling, spatial NNGP, and sequential updates.

```r
apply(GibbsStepTimeFixedLfullGP[,1:7], 2, summary)
```

```
##                z       xi   theta delta    alpha    kappa      rho
## Min.    347.000  81.0000 41.0000 1.000 450.0000  93.0000 309.0000
## 1st Qu. 379.000  82.0000 43.0000 1.000 462.0000  94.0000 319.0000
## Median  385.000  82.0000 44.0000 1.000 465.0000  95.0000 322.0000
## Mean    389.276  82.7366 43.8958 1.008 466.1494  95.9408 323.9814
## 3rd Qu. 392.000  83.0000 44.0000 1.000 469.0000  97.0000 326.0000
## Max.    562.000 217.0000 56.0000 3.000 499.0000 113.0000 391.0000
```

```r
apply(GibbsStepTimeFixedLblock[,1:7], 2, summary)
```

```
##               z       xi  theta  delta    alpha  kappa     rho
## Min.    480.00  81.0000 41.000 1.0000 410.0000  8.000 25.0000
## 1st Qu. 501.75  83.0000 44.000 1.0000 419.0000  8.000 27.0000
## Median  514.00  84.0000 44.000 1.0000 422.0000  8.000 28.0000
## Mean    533.31  84.2434 44.263 1.0006 423.1224  8.467 28.1102
## 3rd Qu. 563.00  85.0000 45.000 1.0000 426.0000  9.000 29.0000
## Max.    757.00 225.0000 53.000 2.0000 454.0000 12.000 35.0000
```

```r
apply(GibbsStepTimeFixedLsequen[,1:7], 2, summary)
```

```
##                 z       xi   theta  delta    alpha   kappa     rho
## Min.    504.0000  82.0000 42.0000 1.0000 195.0000  8.0000 26.0000
## 1st Qu. 528.0000  84.0000 44.0000 1.0000 198.0000  9.0000 31.0000
## Median  533.0000  84.0000 44.0000 1.0000 199.0000  9.0000 32.0000
## Mean    541.5196  84.5774 44.3988 1.0018 198.8446  9.0628 31.7456
## 3rd Qu. 540.0000  85.0000 45.0000 1.0000 199.0000  9.0000 33.0000
## Max.    728.0000 224.0000 56.0000 2.0000 218.0000 13.0000 42.0000
```

```r
apply(GibbsStepTimeVaryLjSequen[,1:7], 2, summary)
```

```
##                u       xi  theta  delta    alpha  kappa     rho
## Min.    0e+00  11.0000 39.000 0.0000 374.0000 4.0000 23.0000
## 1st Qu. 0e+00  12.0000 42.000 1.0000 393.0000 4.0000 26.0000
## Median  0e+00  12.0000 43.000 1.0000 403.0000 4.0000 27.0000
## Mean    4e-04  12.1992 42.588 0.8306 406.3786 4.0716 26.6904
## 3rd Qu. 0e+00  12.0000 43.000 1.0000 409.0000 4.0000 28.0000
## Max.    1e+00 146.0000 51.000 2.0000 582.0000 7.0000 32.0000
```

The results correspond well to what we have deduced in Appendix H of our manuscript.

- Compared to their `fullGPfixedL` counterparts, `NNGPblockFixedL`'s Gibbs sampler steps corresponding to ρ and κ are evidently accelerated by our **spatial NNGP prior**;
- The only Gibbs sampler step time that should clearly differ between `NNGPblockFixedL` and `NNGPsequenFixedL` is the step updating all $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s, which result from whether we adopt our **sequential updating method** or not. Since $m = 900$ here is quite big, `NNGPsequenFixedL` is considerably faster than `NNGPblockFixedL` for the posterior sampling step corresponding to $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s;
- Thanks to our **slice sampling approach**, `NNGPsequenVaryLj`'s Gibbs sampler steps for $u^o_j(\boldsymbol{s}_i)$'sand $\xi^o_j(\boldsymbol{s}_i)$'s are significantly faster than `NNGPsequenFixedL`'s Gibbs sampler steps for $z^o_{jl_j}(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s. It turns out that `NNGPsequenVaryLj`'s Gibbs sampler step for $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s is slower than its `NNGPsequenFixedL` counterpart, indicating that inefficiencies caused by case discussion, calculating all required upper or lower bounds, and rejection sampling outweigh acceleration brought about by slice sampling's ensured non-increasing posterior samples for L_j's through the MCMC iterations.

We finally calculate standard deviations for the 7 spatial-related parameters' posterior sampling time across all kept post-burn-in MCMC iterations.

```r
round(apply(GibbsStepTimeFixedLfullGP[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 27.59543  2.26986  0.90350  0.10926  6.17183  2.25865  9.12389
```

```r
round(apply(GibbsStepTimeFixedLblock[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 44.20064  3.66448  0.99721  0.02449  5.85637  0.59075  1.43320
```

```r
round(apply(GibbsStepTimeFixedLsequen[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 32.43286  3.07966  0.95055  0.04239  2.03078  0.31949  1.49363
```

```r
round(apply(GibbsStepTimeVaryLjSequen[,1:7], 2, sd), 5)
```

```
##        u       xi    theta    delta    alpha    kappa      rho
##  0.02000  2.70736  1.32477  0.37568 28.33927  0.26549  1.33075
```