Posterior Sampling Time Exploration – $m = 64$ and $T = 1000$

Yifan CHENG y.cheng@u.nus.edu

The overall model fitting time (with `equalTimeDist = TRUE` specified) for our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`, are 1.09 days, 1.09 days, 1.09 days, and 1.06 days, respectively. If we do not take advantage of our tactics for evenly dispersed time points presented in Appendix B of our manuscript by specifying `equalTimeDist = FALSE` instead, we will need more than 3 months to fit the same methods using the same computation resources, as $T = 1000$ is huge. This corresponds well to what we have discussed in Appendix B regarding our approaches' manifest computational acceleration in Gibbs sampler steps for temporal parameters ψ, Υ, and $\boldsymbol{\eta}_t$'s. Since $m = 64$ is quite small, there aren't any significant differences in the recorded posterior sampling time between our four methods, as expected.

We first display the first 50 kept post-burn-in MCMC iterations' posterior sampling time (in milliseconds) for 10 key Gibbs sampler steps (corresponding to ψ, Υ, $\boldsymbol{\eta}_t$'s, $z^o_{jl_j}(\boldsymbol{s}_i)$'s or $u^o_j(\boldsymbol{s}_i)$'s, $\xi^o_j(\boldsymbol{s}_i)$'s, θ_{jl_j}'s, $\delta_{1:k}$, ρ, κ, and $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s) for our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`.

```
setwd(wd)
load("GibbsStepTimeFixedLfullGP.RData")
load("GibbsStepTimeFixedLblock.RData")
load("GibbsStepTimeFixedLsequen.RData")
load("GibbsStepTimeVaryLjSequen.RData")
head(GibbsStepTimeFixedLfullGP, 50)
```

```
##         z xi theta delta alpha kappa rho  eta upsilon psi
##  [1,]  72 70  1277     5     7     5   6 1391       9 225
##  [2,]  75 68  1285     5     7     5   6 1322      10 217
##  [3,]  72 71  1324     5     7     5   5 1309       9 214
##  [4,]  72 68  1337     5     7     6   6 1325       9 215
##  [5,]  70 71  1351     5     7     5   5 1405       9 218
##  [6,]  73 69  1290     5     7     5   5 1372       9 217
##  [7,]  75 73  1318     5     7     5   5 1353       9 215
##  [8,]  73 70  1347     5     7     6   6 1352       9 212
##  [9,]  76 68  1349     5     7     6   6 1360       9 210
## [10,]  70 68  1342     5     7     5   5 1361       9 212
## [11,]  73 70  1288     5     7     5   6 1377       9 213
## [12,]  70 69  1313     5     7     5   5 1351      10 214
## [13,]  71 67  1345     5     7     5   6 1318       9 207
## [14,]  72 70  1304     5     7     5   6 1365       9 211
## [15,]  72 69  1279     5     7     6   6 1396       9 207
## [16,]  71 68  1357     5     7     5   6 1351       9 204
## [17,]  67 69  1350     5     7     5   5 1321       9 206
## [18,]  68 69  1304     5     7     5   5 1374       9 208
## [19,]  69 69  1272     5     7     5   6 1419       9 214
## [20,]  71 69  1355     5     6     5   5 1375       9 208
## [21,]  68 68  1378     5     7     5   5 1306       9 208
## [22,]  74 68  1376     5     7     6   6 1380       9 211
## [23,]  72 69  1286     5     7     5   6 1405       9 215
```

```
## [24,]  71 69  1304     5     7     5   6 1373      10 213
## [25,]  71 69  1343     5     7     5   6 1321       9 208
## [26,]  70 68  1347     5     6     5   5 1335       9 208
## [27,]  71 68  1318     5     7     6   6 1372       9 212
## [28,]  73 68  1266     5     7     6   6 1399       9 215
## [29,] 212 68  1291     5     7     5   6 1350       9 213
## [30,]  71 70  1328     5     7     5   6 1333       9 212
## [31,]  72 68  1334     5     7     5   5 1331       9 221
## [32,]  74 72  1335     5     7     6   6 1358       9 219
## [33,]  73 70  1326     5     7     5   6 1355       9 213
## [34,]  73 69  1359     5     7     5   5 1372       9 215
## [35,]  71 68  1342     5     7     5   6 1346       9 214
## [36,]  71 69  1320     5     7     5   5 1376       9 219
## [37,]  73 70  1332     5     7     6   6 1363       9 217
## [38,]  72 68  1337     5     7     5   6 1338       9 207
## [39,]  74 68  1301     5     7     6   6 1361       9 210
## [40,]  72 69  1276     5     7     5   6 1389       9 219
## [41,]  72 69  1352     5     7     5   6 1359       9 211
## [42,]  71 69  1329     5     7     5   5 1330       9 210
## [43,]  72 69  1326     5     7     5   5 1363       9 211
## [44,]  72 71  1334     5     7     6   6 1433       9 209
## [45,]  73 70  1287     5     7     5   5 1376       9 212
## [46,]  72 67  1315     5     6     5   5 1331       9 207
## [47,]  69 69  1381     5     7     6   6 1349       9 206
## [48,]  71 73  1389     5     7     6   6 1350       9 208
## [49,]  67 68  1327     5     7     5   6 1329       9 213
## [50,]  69 69  1304     5     7     5   6 1352       9 212
```

```
head(GibbsStepTimeFixedLblock, 50)
```

```
##         z xi theta delta alpha kappa rho  eta upsilon psi
##  [1,]  68 69  1360     5     8     6   6 1363       9 224
##  [2,]  67 70  1361     6     7     6   6 1369      10 216
##  [3,]  69 71  1336     5     7     5   6 1381       9 215
##  [4,]  69 70  1351     5     7     5   5 1365       9 216
##  [5,]  69 69  1333     6     7     6   6 1401       9 219
##  [6,]  69 70  1319     5     7     5   6 1387       9 231
##  [7,]  69 68  1336     5     7     5   5 1351       9 211
##  [8,] 208 70  1359     5     7     6   6 1394      10 213
##  [9,]  67 71  1314     5     7     6   6 1381       9 211
## [10,]  68 69  1285     6     7     6   6 1378      10 214
## [11,]  69 69  1349     6     7     5   6 1304       9 209
## [12,]  70 70  1389     5     8     6   8 1347       9 207
## [13,]  65 71  1309     5     7     5   5 1399       9 209
## [14,]  66 67  1265     5     7     6   6 1405       9 212
## [15,]  69 69  1348     5     7     6   6 1332       9 211
## [16,]  70 69  1375     5     7     5   5 1320       9 210
## [17,]  69 70  1365     5     7     5   6 1371       9 218
## [18,]  70 71  1278     5     7     6   6 1391       9 218
## [19,]  68 70  1282     5     7     5   6 1349       9 213
## [20,]  67 73  1333     5     7     5   5 1298       9 206
## [21,]  67 72  1341     5     7     5   5 1329       9 205
## [22,]  69 70  1307     5     7     6   6 1397       9 215
## [23,]  70 73  1291     6     8     6   6 1343       9 216
## [24,]  68 72  1328     5     7     6   6 1344       9 215
```

```
## [25,]  75 68  1320     5     6     5   5 1358       9 232
## [26,]  69 71  1325     6     7     6   6 1380       9 215
## [27,]  69 70  1343     5     7     6   6 1372      10 230
## [28,]  68 68  1334     5     7     6   6 1376       9 218
## [29,]  68 69  1371     5     7     6   6 1409       9 215
## [30,]  66 73  1345     5     7     5   6 1350       9 215
## [31,]  73 72  1337     6     7     6   6 1414       9 219
## [32,]  68 70  1324     5     7     5   6 1332       9 219
## [33,]  73 77  1446     6     7     5   6 1378       9 224
## [34,]  69 71  1334     5     7     5   5 1401       9 221
## [35,]  68 74  1323     5     7     6   6 1403       9 224
## [36,]  69 73  1323     6     7     6   6 1356       9 230
## [37,]  69 71  1347     5     7     5   5 1382       9 219
## [38,]  69 70  1372     5     7     8   6 1335       9 220
## [39,]  66 69  1333     5     7     5   6 1345       9 225
## [40,]  68 75  1366     5     7     5   6 1408       9 227
## [41,]  66 68  1321     5     7     5   5 1412      10 221
## [42,]  66 73  1345     5     7     6   6 1388       9 224
## [43,]  68 68  1396     6     7     6   6 1362       9 214
## [44,]  68 73  1337     5     7     6   6 1362       9 214
## [45,]  69 70  1367     5     7     6   6 1383       9 221
## [46,]  67 71  1359     5     7     6   6 1397      10 217
## [47,]  67 68  1377     5     7     6   6 1369       9 213
## [48,]  67 68  1323     5     7     6   6 1395       9 214
## [49,]  65 70  1303     5     7    10   6 1398       9 224
## [50,]  68 79  1342     6     7     6   6 1358       9 227
```

```
head(GibbsStepTimeFixedLsequen, 50)
```

```
##        z xi theta delta alpha kappa rho  eta upsilon psi
##  [1,] 73 68  1318     5    12     5   5 1315       9 211
##  [2,] 72 68  1324     5    13     5   6 1351       9 215
##  [3,] 72 69  1317     5    13     6   6 1363       9 216
##  [4,] 72 69  1332     5    13     6   6 1353       9 211
##  [5,] 72 68  1306     5    13     5   6 1396       9 212
##  [6,] 71 71  1380     6    13     6   6 1419       9 225
##  [7,] 72 69  1326     6    13     6   6 1433      10 226
##  [8,] 71 72  1348     4    13     5   5 1337       9 210
##  [9,] 72 70  1329     5    13     5   5 1322       9 212
## [10,] 73 67  1328     5    13     6   6 1360       9 213
## [11,] 73 68  1299     5    13     6   6 1400       9 222
## [12,] 73 68  1313     5    13     5   6 1399       9 218
## [13,] 72 68  1356     5    13     6   6 1344       9 213
## [14,] 70 67  1367     5    13     5   6 1347       9 212
## [15,] 73 72  1383     5    13     5   6 1359       9 218
## [16,] 72 68  1305     5    13     6   6 1424       9 214
## [17,] 73 70  1316     5    13     6   6 1449       9 214
## [18,] 72 68  1316     5    13     6   6 1359       9 218
## [19,] 72 68  1322     5    13     5   5 1363       9 217
## [20,] 74 68  1347     5    13     6   6 1381       9 217
## [21,] 74 67  1321     5    13     6   6 1416       9 220
## [22,] 74 72  1297     5    13     5   6 1391       9 214
## [23,] 75 74  1318     5    13     6   6 1380       9 217
## [24,] 74 69  1326     5    13     6   6 1345       9 215
## [25,] 74 74  1306     5    13     6   6 1341       9 220
```

```
## [26,] 74 69  1291     5    12     5   5 1306       9 215
## [27,] 75 68  1300     5    13     6   6 1359       9 218
## [28,] 75 71  1326     5    13     6   6 1362       9 223
## [29,] 78 71  1313     5    13     6   6 1399       9 225
## [30,] 75 71  1323     5    13     5   5 1383       9 216
## [31,] 75 71  1297     5    13     6   6 1351       9 214
## [32,] 76 69  1316     5    13     6   6 1378       9 216
## [33,] 75 74  1275     5    13     6   6 1354       9 225
## [34,] 75 75  1274     6    13     5   6 1326       9 222
## [35,] 75 68  1293     6    13     5   6 1329       9 217
## [36,] 74 71  1390     6    13     6   6 1365       9 211
## [37,] 78 71  1380     5    13     5   5 1368       9 217
## [38,] 77 68  1310     6    13     6   6 1402       9 213
## [39,] 76 68  1321     5    13     6   6 1394       9 214
## [40,] 76 69  1341     5    13     6   6 1315      10 211
## [41,] 74 69  1347     5    13     5   5 1283       9 214
## [42,] 77 69  1306     5    13     6   6 1361       9 215
## [43,] 76 70  1273     5    13     5   6 1349       9 217
## [44,] 76 70  1301     5    13     6   6 1388       9 229
## [45,] 77 69  1369     6    13     6   6 1356       9 221
## [46,] 80 70  1395     6    14     6   6 1404      10 224
## [47,] 76 70  1363     5    13     5   6 1335       9 210
## [48,] 76 69  1351     5    13     6   6 1381       9 211
## [49,] 76 73  1260     5    13     6   6 1398       9 221
## [50,] 76 71  1295     5    13     5   5 1367       9 211
```

```
head(GibbsStepTimeVaryLjSequen, 50)
```

```
##       u xi theta delta alpha kappa rho  eta upsilon psi
##  [1,] 5 20  1411     5    32     5   5 1373       9 212
##  [2,] 5 19  1374     5    29     6   6 1335       9 214
##  [3,] 5 19  1367     6    33     5   6 1366       9 213
##  [4,] 6 20  1296     5    31     5   6 1383       9 215
##  [5,] 4 17  1360     5    29     5   5 1327       9 213
##  [6,] 5 18  1354     6    32     6   6 1356       9 214
##  [7,] 5 19  1308     5    32     6   6 1393       9 214
##  [8,] 5 19  1355     5    30     5   5 1338       9 216
##  [9,] 5 19  1421     5    29     6   6 1370       9 218
## [10,] 5 19  1298     5    33     5   5 1419       9 211
## [11,] 5 18  1335     6    32     6   6 1406      10 211
## [12,] 5 19  1450     5    29     5   5 1346       9 211
## [13,] 5 19  1308     5    32     5   6 1409       9 217
## [14,] 6 19  1312     5    32     6   6 1391       9 213
## [15,] 5 18  1391     5    30     5   5 1344       9 213
## [16,] 5 19  1438     5    31     6   7 1446       9 215
## [17,] 5 19  1323     5    31     5   6 1403       9 217
## [18,] 5 19  1352     5    30     6   6 1340       9 214
## [19,] 6 19  1416     6    32     5   6 1380       9 210
## [20,] 6 19  1296     5    33     5   6 1309       9 214
## [21,] 5 18  1320     5    31     5   5 1322      10 206
## [22,] 5 21  1457     5    33     6   6 1388       9 215
## [23,] 5 19  1248     5    33     6   6 1444       9 229
## [24,] 5 18  1397     5    30     4   5 1290       9 205
## [25,] 5 19  1331     5    29     5   5 1359       9 208
## [26,] 5 18  1320     5    32     6   6 1418       9 218
```

```
## [27,] 6 19  1318     5    32     6   6 1448       9 216
## [28,] 5 19  1405     5    30     5   6 1334       9 216
## [29,] 5 19  1331     5    33     5   6 1476       9 213
## [30,] 5 18  1369     6    34     6   6 1413       9 214
## [31,] 5 19  1418     5    30     5   5 1324       9 216
## [32,] 5 17  1317     5    29     5   6 1373       9 210
## [33,] 5 19  1328     5    33     6   6 1367       9 214
## [34,] 5 19  1373     5    31     6   6 1334       9 212
## [35,] 5 19  1348     5    30     6   6 1345      10 213
## [36,] 5 19  1239     5    32     5   6 1441       9 214
## [37,] 6 19  1428     5    31     6   6 1292       9 205
## [38,] 5 19  1307     5    30     5   6 1364       9 207
## [39,] 5 19  1296     5    32     6   6 1425      10 216
## [40,] 5 19  1337     5    34     6   6 1373       9 214
## [41,] 5 20  1394     5    30     6   6 1364       9 214
## [42,] 5 19  1349     5    28     4   4 1352       9 214
## [43,] 5 19  1283     5    32     6   6 1424       9 213
## [44,] 5 20  1392     5    31     6   6 1346       9 208
## [45,] 5 19  1304     5    29     5   6 1398       9 208
## [46,] 6 20  1336     5    32     6   6 1384       9 212
## [47,] 5 19  1393     6    32     6   6 1354       9 211
## [48,] 5 19  1392     5    29     5   6 1362       9 217
## [49,] 5 18  1237     5    31     5   5 1440       9 216
## [50,] 5 20  1371     5    31     6   6 1321       9 217
```

We then present vital posterior sampling time summary statistics for the 10 key parameters.

```
round(apply(GibbsStepTimeFixedLfullGP, 2, summary), 2)
```

```
##              z    xi   theta delta alpha kappa   rho     eta upsilon    psi
## Min.     58.00 66.00 1226.00  4.00  5.00  4.00  4.00 1264.00     8.0 202.00
## 1st Qu.  65.00 69.00 1308.00  5.00  7.00  5.00  6.00 1341.00     9.0 212.00
## Median   67.00 70.00 1334.00  5.00  7.00  6.00  6.00 1368.00     9.0 216.00
## Mean     68.58 70.27 1336.99  5.17  6.95  5.58  5.76 1370.64     9.1 217.39
## 3rd Qu.  69.00 71.00 1363.00  5.00  7.00  6.00  6.00 1399.00     9.0 221.00
## Max.    241.00 88.00 1497.00 16.00 18.00 16.00 12.00 1560.00    21.0 260.00
```

```
round(apply(GibbsStepTimeFixedLblock, 2, summary), 2)
```

```
##              z    xi  theta delta alpha kappa   rho     eta upsilon    psi
## Min.     61.00 66.00 1217.0  4.00  5.00   4.0  4.00 1259.00    8.00 201.00
## 1st Qu.  68.00 68.00 1308.0  5.00  7.00   5.0  5.00 1340.00    9.00 212.00
## Median   69.00 70.00 1336.0  5.00  7.00   5.0  6.00 1369.00    9.00 216.00
## Mean     70.44 69.94 1339.1  5.21  6.95   5.5  5.73 1371.95    9.14 217.04
## 3rd Qu.  71.00 71.00 1366.0  5.00  7.00   6.0  6.00 1399.00    9.00 220.00
## Max.    220.00 84.00 1625.0 11.00 12.00  13.0 11.00 1779.00   19.00 276.00
```

```
round(apply(GibbsStepTimeFixedLsequen, 2, summary), 2)
```

```
##              z    xi   theta delta alpha kappa   rho     eta upsilon    psi
## Min.     62.00 66.00 1211.00  4.00 11.00  4.00  4.00 1253.00    8.00 199.00
## 1st Qu.  68.00 68.00 1306.00  5.00 13.00  5.00  6.00 1341.00    9.00 212.00
## Median   70.00 70.00 1335.00  5.00 13.00  6.00  6.00 1369.00    9.00 216.00
## Mean     71.12 70.35 1337.21  5.28 13.04  5.79  5.94 1371.91    9.34 217.59
## 3rd Qu.  72.00 71.00 1366.00  5.00 13.00  6.00  6.00 1399.00    9.00 221.00
## Max.    238.00 88.00 1510.00 11.00 21.00 14.00 12.00 1582.00   20.00 264.00
```

```
round(apply(GibbsStepTimeVaryLjSequen, 2, summary), 2)
```

```
##            u    xi   theta delta  alpha kappa rho     eta upsilon    psi
## Min.    4.00 16.00 1211.00  4.00  27.00  3.00 4.0 1256.00    8.00 200.00
## 1st Qu. 5.00 18.00 1306.00  5.00  30.00  5.00 5.0 1340.00    9.00 209.00
## Median  5.00 19.00 1332.00  5.00  31.00  5.00 6.0 1366.00    9.00 212.00
## Mean    5.02 18.62 1335.18  5.12  31.36  5.19 5.5 1368.49    9.08 212.53
## 3rd Qu. 5.00 19.00 1361.00  5.00  32.00  6.00 6.0 1394.00    9.00 215.00
## Max.    6.00 23.00 1521.00  7.00 177.00  7.00 7.0 1559.00   12.00 257.00
```

```
round(apply(GibbsStepTimeFixedLfullGP, 2, sd), 3)
```

```
##       z      xi   theta   delta   alpha   kappa     rho     eta upsilon     psi
##  13.890   2.358  41.458   0.454   0.398   0.692   0.486  42.446   0.419   7.241
```

```
round(apply(GibbsStepTimeFixedLblock, 2, sd), 3)
```

```
##       z      xi   theta   delta   alpha   kappa     rho     eta upsilon     psi
##  11.803   2.209  42.878   0.471   0.389   0.652   0.514  44.384   0.528   7.118
```

```
round(apply(GibbsStepTimeFixedLsequen, 2, sd), 3)
```

```
##       z      xi   theta   delta   alpha   kappa     rho     eta upsilon     psi
##  12.174   2.834  43.041   0.784   0.734   0.920   0.770  43.219   1.227   8.063
```

```
round(apply(GibbsStepTimeVaryLjSequen, 2, sd), 3)
```

```
##       u      xi   theta   delta   alpha   kappa     rho     eta upsilon     psi
##   0.335   0.805  40.928   0.400   7.774   0.586   0.595  39.929   0.296   5.718
```