Key Parameters' Posterior Sampling Time Analysis

Yifan CHENG y.cheng@u.nus.edu

Portions of Recorded Gibbs Sampler Time for 10 Key Parameters

We first display the first 50 kept post-burn-in MCMC iterations' posterior sampling time (in milliseconds) for 10 key Gibbs sampler steps corresponding to our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`.

```
wd <- paste(projDirec, "simu/mainScalabilityVerificationSimu/m1600T50K5", sep = "/")
setwd(wd)
load("GibbsStepTimeFixedLfullGP.RData"); load("GibbsStepTimeFixedLblock.RData")
load("GibbsStepTimeFixedLsequen.RData"); load("GibbsStepTimeVaryLjSequen.RData")
head(GibbsStepTimeFixedLfullGP, 50)
```

```
##         z  xi theta delta alpha kappa  rho eta upsilon psi
##  [1,] 715 152    83     4  2250   320 1560  90       3   3
##  [2,] 724 150    83     4  2240   330 1384  87       3   3
##  [3,] 827 151    82     4  2247   314 1379  88       3   3
##  [4,] 712 155    83     4  2269   311 1468  90       3   3
##  [5,] 722 152    83     4  2249   312 1394  88       3   3
##  [6,] 726 151    83     4  2422   308 1499  93       3   3
##  [7,] 725 151    81     3  2238   311 1361  87       3   3
##  [8,] 717 151    81     4  2269   313 1392  89       3   3
##  [9,] 818 150    82     3  2243   312 1418  88       3   3
## [10,] 709 149    81     3  2338   312 1357  90       3   3
## [11,] 713 150    83     4  2213   312 1429  86       3   3
## [12,] 714 149    83     4  2227   308 1484  91       3   3
## [13,] 719 150    81     4  2209   315 1387  89       3   3
## [14,] 707 152    83     4  2272   332 1410  87       3   3
## [15,] 808 149    85     4  2407   320 1461  94       3   3
## [16,] 703 151    83     3  2279   318 1404  88       3   3
## [17,] 710 154    83     4  2307   324 1426  86       3   3
## [18,] 723 150    82     4  2253   317 1385  87       3   3
## [19,] 712 153    83     4  2302   320 1454  88       3   3
## [20,] 710 150    82     4  2247   316 1413  87       3   3
## [21,] 839 150    85     4  2274   320 1377  88       3   3
## [22,] 690 152    83     4  2462   321 1492  84       3   3
## [23,] 709 151    82     4  2293   317 1408  87       3   3
## [24,] 737 155    83     4  2563   312 1374  87       3   3
## [25,] 718 153    84     4  2229   312 1353  87       3   3
## [26,] 720 151    83     4  2257   322 1448  88       3   3
## [27,] 849 152    83     4  2233   318 1586  91       3   3
## [28,] 694 150    82     4  2243   313 1412  92       3   3
## [29,] 716 159    83     4  2240   312 1403  88       3   3
## [30,] 709 149    84     4  2252   313 1380  87       3   3
## [31,] 752 151    89     4  2571   335 1455  93       3   3
```

```
## [32,] 721 152    83     4  2264   313 1401  87        3   3
## [33,] 846 151    83     4  2434   315 1421  87        3   3
## [34,] 707 151    84     4  2250   312 1368  88        3   3
## [35,] 725 154    83     4  2227   332 1421  89        3   3
## [36,] 733 152    85     4  2478   341 1428  87        3   3
## [37,] 727 154    83     4  2215   318 1379  89        3   3
## [38,] 721 152    85     4  2285   318 1483  89        3   3
## [39,] 904 152    84     4  2315   319 1398  87        3   3
## [40,] 687 152    82     4  2266   327 1355  87        3   3
## [41,] 706 150    84     4  2368   315 1345  85        3   3
## [42,] 713 152    83     4  2374   325 1392  89        3   3
## [43,] 719 152    81     4  2293   320 1555  92        3   3
## [44,] 720 152    82     4  2279   317 1357  84        3   3
## [45,] 843 150    81     4  2216   316 1340  87        3   3
## [46,] 685 151    84     4  2328   318 1418  87        3   3
## [47,] 721 152    83     4  2231   317 1405  86        3   3
## [48,] 717 152    83     4  2221   317 1369  86        3   3
## [49,] 730 151    83     4  2264   311 1380  88        3   3
## [50,] 723 153    83     4  2269   321 1416  88        3   3
```

```
head(GibbsStepTimeFixedLblock, 50)
```

```
##         z  xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 622 158    83     4  2111    22  94  89       3   3
##  [2,] 634 152    85     4  2028    22  99  89       3   3
##  [3,] 620 153    83     4  2299    22  99  89       3   3
##  [4,] 592 158    93     4  2125    21  96  87       3   3
##  [5,] 617 156    83     4  2099    21  94  88       3   3
##  [6,] 620 151    83     4  1992    21  97  89       3   3
##  [7,] 624 156    82     4  2073    21 100  86       3   3
##  [8,] 626 151    83     4  2158    21  94  95       3   3
##  [9,] 631 150    83     4  2053    25 106  94       3   3
## [10,] 632 150    83     4  2230    22  98  86       3   3
## [11,] 587 149    82     4  2299    21  93  98       3   3
## [12,] 635 152    84     4  2006    21  95  90       3   3
## [13,] 630 155    85     4  1952    21  95  87       3   3
## [14,] 645 155    83     4  2060    22  98  88       3   3
## [15,] 634 152    84     4  2359    25 107  91       3   3
## [16,] 648 153    82     4  2210    23 100  87       3   3
## [17,] 626 152    83     4  1983    21  97  87       3   3
## [18,] 588 152    82     4  2050    22  94  88       3   3
## [19,] 636 163    83     4  2051    22  96  86       3   3
## [20,] 623 155    85     4  2138    21  99  87       3   3
## [21,] 632 151    83     4  2281    21 101  90       3   3
## [22,] 632 153    83     4  2002    22  93  89       3   3
## [23,] 627 152    83     4  1995    21  97  90       3   3
## [24,] 623 152    84     4  1980    21  99  87       3   3
## [25,] 621 154    87     4  2107    21 101  87       3   3
## [26,] 623 151    83     4  2085    21  96  90       3   3
## [27,] 657 153    89     4  2045    22  97  88       3   3
## [28,] 637 153    80     4  2050    21  93  87       3   3
## [29,] 636 154    87     4  2280    24 100  98       3   3
## [30,] 632 150    84     4  2256    21  95  90       3   3
## [31,] 628 153    82     4  2000    22  95  88       3   3
## [32,] 626 154    89     4  2084    25 100  93       3   3
```

```
## [33,] 663 151    87     4  2297    21  95  87       3   3
## [34,] 633 151    83     4  2268    21  96  92       3   3
## [35,] 629 152    83     4  2035    21  96  88       3   3
## [36,] 621 154    85     4  1979    21  99  89       3   3
## [37,] 629 150    83     4  2212    22  93  88       3   3
## [38,] 632 151    84     4  2279    21  97  90       3   3
## [39,] 590 151    83     4  1986    21  95  89       3   3
## [40,] 663 155    84     4  1984    22 101  91       3   3
## [41,] 627 154    81     4  2227    22  99  86       3   3
## [42,] 637 153    84     4  2039    21 105  88       3   3
## [43,] 633 151    82     4  2000    22  96  87       3   3
## [44,] 638 151    83     4  2184    21 100  87       3   3
## [45,] 637 150    83     4  2293    22 103  88       3   3
## [46,] 603 150    83     4  2191    21 103  86       3   3
## [47,] 632 154    81     4  2053    21  97  86       3   3
## [48,] 623 151    81     3  1973    21  99  86       3   3
## [49,] 637 151    80     4  2115    21  95  85       3   3
## [50,] 650 151    83     4  2106    21  93  86       3   3
```

```r
head(GibbsStepTimeFixedLsequen, 50)
```

```
##          z  xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 689 148    78     3   356    21 103  84       3   3
##  [2,] 691 148    79     4   361    21 103  84       3   3
##  [3,] 682 148    78     3   356    22 102  84       3   3
##  [4,] 688 149    79     3   357    21 100  84       3   3
##  [5,] 804 146    78     3   357    21 101  83       3   3
##  [6,] 654 147    83     3   356    21 100  83       3   2
##  [7,] 682 148    79     4   363    21 102  85       3   3
##  [8,] 689 150    80     4   356    20  98  82       3   3
##  [9,] 696 152    79     4   356    20 100  83       3   3
## [10,] 692 151    81     4   357    20  97  82       3   3
## [11,] 686 152    80     4   363    21  99  82       3   2
## [12,] 653 151    80     3   355    20 100  83       3   2
## [13,] 684 148    78     4   355    20  98  82       3   2
## [14,] 680 158    79     4   356    21  99  83       3   3
## [15,] 686 150    79     3   364    21 102  84       3   3
## [16,] 694 148    79     4   355    21 100  84       3   3
## [17,] 707 152    80     4   355    21  99  86       3   3
## [18,] 646 148    80     4   357    21 109  89       3   3
## [19,] 690 148    78     3   358    21 107  84       3   3
## [20,] 676 153    80     4   357    21  99  84       3   3
## [21,] 690 149    80     4   356    21 103  84       3   3
## [22,] 691 146    78     4   357    21 102  85       3   3
## [23,] 694 148    78     4   357    21 101  83       3   3
## [24,] 666 149    80     4   357    21 101  84       3   3
## [25,] 690 149    79     3   357    21 104  86       3   3
## [26,] 706 149    79     4   357    21 100  84       3   3
## [27,] 710 147    79     4   357    21 101  83       3   3
## [28,] 685 148    79     4   356    20 100  84       3   3
## [29,] 701 149    81     4   359    21 103  85       3   3
## [30,] 798 148    79     4   356    21 102  85       3   3
## [31,] 686 149    80     4   357    20 101  84       3   3
## [32,] 691 147    80     4   357    21 100  84       3   3
## [33,] 688 148    79     3   357    21  98  83       3   3
```

```
## [34,] 686 149    86     4   356    22 104  86        3   3
## [35,] 685 147    79     3   357    21 103  85        3   3
## [36,] 794 150    79     3   356    21 102  87        3   3
## [37,] 661 147    79     3   357    21 101  84        3   3
## [38,] 684 147    80     3   357    21 104  84        3   3
## [39,] 702 148    83     3   362    22 106  88        3   3
## [40,] 693 148    83     3   356    23 103  84        3   3
## [41,] 697 148    80     4   356    21 110  89        3   3
## [42,] 829 147    79     3   357    21 103  83        3   3
## [43,] 655 156    80     4   356    20 102  82        3   2
## [44,] 693 151    81     4   363    21 105  85        3   3
## [45,] 724 149    90     3   357    24 108  89        3   3
## [46,] 692 150    80     4   355    21 103  83        3   3
## [47,] 696 149    80     4   356    20 101  83        3   3
## [48,] 686 146    78     3   359    20  98  82        3   3
## [49,] 654 148    81     4   357    21 101  87        3   3
## [50,] 686 148    83     4   356    21 100  84        3   3
```

```r
head(GibbsStepTimeVaryLjSequen, 50)
```

```
##       u xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 3 17    79     3   787    13  95  84       3   3
##  [2,] 3 18    77     3   768    13  93  84       3   3
##  [3,] 2 15    76     3   769    13  95  84       3   3
##  [4,] 3 17    82     3   793    13  93  87       3   3
##  [5,] 2 15    78     3   763    13  93  85       3   3
##  [6,] 2 17    78     3   777    13  96  84       3   3
##  [7,] 1 15    78     3   763    13  95  84       3   3
##  [8,] 2 16    77     3   776    13  93  84       3   3
##  [9,] 1 15    77     3   753    12  90  85       3   3
## [10,] 3 19    78     3   774    13  94  83       3   3
## [11,] 3 17    78     3   778    13  92  83       3   3
## [12,] 1 15    75     3   763    13  90  84       3   3
## [13,] 3 17    79     3   766    12  90  83       3   3
## [14,] 3 18    78     3   907    13  93  84       3   3
## [15,] 2 16    79     3   781    13  95  84       3   3
## [16,] 2 15    78     3   764    12  91  84       3   3
## [17,] 2 18    78     3   779    13  95  84       3   3
## [18,] 2 16    77     3   785    13  93  86       3   3
## [19,] 2 17    81     3   816    15 101  88       3   3
## [20,] 2 16    78     3   898    13  97  83       3   3
## [21,] 2 16    78     3   769    13  94  84       3   3
## [22,] 3 17    79     3   770    13  94  84       3   3
## [23,] 3 17    78     3   769    13  93  83       3   3
## [24,] 2 18    78     3   773    13  94  83       3   3
## [25,] 2 15    76     3   788    13  95  85       3   3
## [26,] 2 15    77     3   898    13  91  85       3   2
## [27,] 2 16    78     3   771    13  94  85       3   3
## [28,] 2 16    76     3   783    13  95  85       3   3
## [29,] 3 16    78     3   765    13  91  83       3   3
## [30,] 3 16    78     3   772    13  94  83       3   3
## [31,] 2 16    78     3   784    14  98  85       3   3
## [32,] 2 15    77     3   891    13  97  84       3   3
## [33,] 2 18    80     3   759    12  92  84       3   3
## [34,] 3 16    77     3   771    13  94  84       3   3
```

```
## [35,] 2 16    78     3    768    13  92  84        3   3
## [36,] 3 16    78     3    777    13  93  83        3   3
## [37,] 3 16    77     3    776    13  91  82        3   3
## [38,] 1 15    76     3    909    13  94  84        3   3
## [39,] 3 20    82     3    819    15  97  84        3   3
## [40,] 2 15    76     3    771    13  93  84        3   3
## [41,] 2 15    77     3    767    13  98  85        3   3
## [42,] 2 15    78     3    763    12  90  84        3   2
## [43,] 3 16    79     3    767    13  91  83        3   3
## [44,] 3 19    79     3    911    15  99  88        3   3
## [45,] 1 17    80     3    825    16 100  90        3   3
## [46,] 3 17    78     3    765    12  90  83        3   3
## [47,] 3 18    82     3    803    15 101  89        3   3
## [48,] 2 16    77     3    791    13  95  84        3   3
## [49,] 2 16    76     3    765    12  93  84        3   3
## [50,] 3 16    77     3    903    13  96  85        3   3
```

As expected, there aren't any significant differences between our 4 methods regarding posterior sampling time for the 3 temporal parameters ψ, Υ, and $\boldsymbol{\eta}_t$'s.

Posterior Sampling Time Summary Statistics

We then present vital posterior sampling time summary statistics for the 7 spatial-related parameters ($z^o_{jl_j}(\boldsymbol{s}_i)$'s or $u^o_j(\boldsymbol{s}_i)$'s, $\xi^o_j(\boldsymbol{s}_i)$'s, θ_{jl_j}'s, $\delta_{1:k}$, ρ, κ, and $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s) to showcase the manifest scalability improvements brought about by our 3 novelties, i.e., slice sampling, spatial NNGP, and sequential updates.

```
apply(GibbsStepTimeFixedLfullGP[,1:7], 2, summary)
```

```
##                 z       xi   theta  delta    alpha    kappa      rho
## Min.    640.0000 145.0000 78.0000  2.000 2138.000 303.0000 1296.000
## 1st Qu. 704.0000 150.0000 82.0000  4.000 2217.000 311.0000 1366.000
## Median  719.0000 151.0000 83.0000  4.000 2281.000 314.0000 1422.000
## Mean    723.9746 151.4386 82.9466  3.869 2306.249 315.5866 1442.891
## 3rd Qu. 731.0000 152.0000 84.0000  4.000 2367.000 319.0000 1495.000
## Max.    939.0000 288.0000 98.0000 11.000 2979.000 376.0000 2132.000
```

```
apply(GibbsStepTimeFixedLblock[,1:7], 2, summary)
```

```
##                 z       xi    theta delta    alpha   kappa      rho
## Min.    574.0000 148.0000  79.0000 3.000 1935.000 20.0000  87.0000
## 1st Qu. 623.0000 151.0000  82.0000 4.000 2028.000 21.0000  94.0000
## Median  630.0000 152.0000  83.0000 4.000 2093.000 21.0000  96.0000
## Mean    635.5834 152.5778  83.4584 3.963 2113.644 21.5504  96.4844
## 3rd Qu. 637.0000 153.0000  84.0000 4.000 2186.000 22.0000  98.0000
## Max.    824.0000 323.0000 111.0000 7.000 2558.000 27.0000 116.0000
```

```
apply(GibbsStepTimeFixedLsequen[,1:7], 2, summary)
```

```
##                 z       xi   theta  delta   alpha   kappa      rho
## Min.    617.0000 142.0000  75.000 3.0000 347.000 19.0000  90.0000
## 1st Qu. 688.0000 149.0000  81.000 4.0000 357.000 21.0000 102.0000
## Median  713.0000 153.0000  83.000 4.0000 365.000 22.0000 106.0000
## Mean    714.0948 152.0324  82.454 3.8754 362.944 21.8626 105.1648
## 3rd Qu. 725.0000 154.0000  84.000 4.0000 366.000 22.0000 108.0000
## Max.    911.0000 309.0000 101.000 6.0000 409.000 27.0000 122.0000
```

```r
apply(GibbsStepTimeVaryLjSequen[,1:7], 2, summary)
```

```
##                u        xi  theta  delta    alpha   kappa      rho
## Min.    1.0000  13.0000 73.000 2.0000 739.0000 12.0000  86.0000
## 1st Qu. 2.0000  14.0000 77.000 3.0000 757.0000 13.0000  92.0000
## Median  2.0000  15.0000 77.000 3.0000 764.0000 13.0000  93.0000
## Mean    2.1136  15.0362 77.637 2.9812 774.4342 12.9298  93.5652
## 3rd Qu. 3.0000  16.0000 78.000 3.0000 774.0000 13.0000  95.0000
## Max.    3.0000 154.0000 90.000 5.0000 971.0000 18.0000 105.0000
```

The results correspond well to what we have deduced in Appendix H of our manuscript.

- Compared to their `fullGPfixedL` counterparts, `NNGPblockFixedL`'s Gibbs sampler steps corresponding to ρ and κ are evidently accelerated by our **spatial NNGP prior**;
- The only Gibbs sampler step time that should clearly differ between `NNGPblockFixedL` and `NNGPsequenFixedL` is the step updating all $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s, which result from whether we adopt our **sequential updating method** or not. Since $m = 1600$ here is big, `NNGPsequenFixedL` is a few times faster than `NNGPblockFixedL` for the posterior sampling step corresponding to $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s;
- Thanks to our **slice sampling approach**, `NNGPsequenVaryLj`'s Gibbs sampler steps for $u^o_j(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s are significantly faster than `NNGPsequenFixedL`'s Gibbs sampler steps for $z^o_{jl_j}(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s. It turns out that `NNGPsequenVaryLj`'s Gibbs sampler step for $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s is slower than its `NNGPsequenFixedL` counterpart, indicating that inefficiencies caused by case discussion, calculating all required upper or lower bounds, and rejection sampling outweigh acceleration brought about by slice sampling's ensured non-increasing posterior samples for L_j's through the MCMC iterations.

We finally calculate standard deviations for the 7 spatial-related parameters' posterior sampling time across all kept post-burn-in MCMC iterations.

```r
round(apply(GibbsStepTimeFixedLfullGP[,1:7], 2, sd), 5)
```

```
##         z        xi     theta     delta     alpha     kappa       rho
##  39.80364   4.22239   1.90758   0.48403 113.15502   6.94603 100.41901
```

```r
round(apply(GibbsStepTimeFixedLblock[,1:7], 2, sd), 5)
```

```
##         z        xi     theta     delta     alpha     kappa       rho
##  35.78841   5.12094   2.05005   0.20109 104.90547   1.04004   3.59271
```

```r
round(apply(GibbsStepTimeFixedLsequen[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 42.36489  5.93313  2.68815  0.33451  6.78764  1.16413  4.42447
```

```r
round(apply(GibbsStepTimeVaryLjSequen[,1:7], 2, sd), 5)
```

```
##        u       xi    theta    delta    alpha    kappa      rho
##  0.68585  3.55795  1.74041  0.14160 35.41980  0.61396  2.43847
```