Key Parameters' Posterior Sampling Time Analysis

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Portions of Recorded Gibbs Sampler Time for 10 Key Parameters

We first display the first 50 kept post-burn-in MCMC iterations' posterior sampling time (in milliseconds) for 10 key Gibbs sampler steps corresponding to our 4 methods, i.e., `fullGPfixedL`, `NNGPblockFixedL`, `NNGPsequenFixedL`, and `NNGPsequenVaryLj`.

```
wd <- paste(projDirec, "simu/mainScalabilityVerificationSimu/m1600T100K5", sep = "/")
setwd(wd)
load("GibbsStepTimeFixedLfullGP.RData"); load("GibbsStepTimeFixedLblock.RData")
load("GibbsStepTimeFixedLsequen.RData"); load("GibbsStepTimeVaryLjSequen.RData")
head(GibbsStepTimeFixedLfullGP, 50)
```

```
##          z  xi theta delta alpha kappa  rho eta upsilon psi
##  [1,] 1488 287   337     4  2499   311 1816 339       3   4
##  [2,] 1465 291   320     4  2544   336 1846 312       3   4
##  [3,] 1486 293   317     4  2304   326 1492 340       4   5
##  [4,] 1499 284   317     4  2454   321 2126 308       3   5
##  [5,] 1456 280   325     4  2224   312 1416 344       3   4
##  [6,] 1456 286   311     4  2475   313 1524 310       3   4
##  [7,] 1470 286   333     4  2300   325 1774 346       4   5
##  [8,] 1489 286   323     5  2597   325 2288 345       4   5
##  [9,] 1452 287   311     4  2471   323 1582 323       4   5
## [10,] 1484 290   340     4  2405   327 2282 335       4   5
## [11,] 1475 285   323     4  2643   326 2097 353       4   5
## [12,] 1448 283   314     4  2517   333 2020 306       3   4
## [13,] 1463 295   333     4  2337   322 1847 327       3   4
## [14,] 1644 284   329     4  2288   314 1929 357       3   4
## [15,] 1476 289   329     4  2385   316 1396 327       4   5
## [16,] 1492 293   312     4  2313   319 1847 352       3   4
## [17,] 1626 284   323     4  2372   324 1810 317       3   4
## [18,] 1499 285   325     4  2356   318 2006 345       3   4
## [19,] 1467 284   313     4  2495   320 1799 312       3   4
## [20,] 1610 285   328     4  2343   315 1827 330       3   5
## [21,] 1469 285   312     4  2526   333 1801 360       3   4
## [22,] 1499 284   313     4  2350   316 1840 317       3   5
## [23,] 1535 290   331     4  2480   322 1668 322       3   4
## [24,] 1504 283   332     4  2328   338 2039 335       4   5
## [25,] 1499 294   314     4  2395   323 1658 309       3   4
## [26,] 1470 286   322     4  2271   321 1815 329       4   4
## [27,] 1482 289   306     4  2353   319 1846 317       3   4
## [28,] 1465 283   317     4  2260   317 1742 324       3   4
## [29,] 1463 288   325     4  2465   324 2070 330       3   4
## [30,] 1462 284   312     4  2380   320 1707 312       3   4
## [31,] 1471 284   327     4  2675   313 1822 329       4   5
```

```
## [32,] 1469 286   312     4  2408   313 1760 313       3   4
## [33,] 1478 288   323     4  2417   315 1351 323       3   4
## [34,] 1648 282   317     4  2299   310 1398 308       3   5
## [35,] 1483 290   330     4  2390   311 1728 364       4   5
## [36,] 1497 282   315     4  2463   322 1722 318       3   5
## [37,] 1627 281   348     4  2284   312 1664 332       3   4
## [38,] 1514 287   318     5  2342   311 1401 318       4   5
## [39,] 1489 285   327     4  2466   315 1881 333       3   5
## [40,] 1617 286   309     4  2291   311 1444 319       3   4
## [41,] 1502 301   330     4  2298   311 1819 324       4   4
## [42,] 1483 283   318     4  2243   311 1321 314       3   5
## [43,] 1538 290   328     4  2219   321 1652 318       3   4
## [44,] 1487 287   340     4  2271   312 1724 351       3   4
## [45,] 1479 283   321     4  2371   311 1696 317       3   4
## [46,] 1477 292   328     4  2659   327 1861 320       4   5
## [47,] 1462 281   336     4  2387   344 1424 330       3   5
## [48,] 1459 282   316     4  2714   314 1512 334       3   5
## [49,] 1477 295   333     4  2227   311 1701 339       3   5
## [50,] 1501 288   330     5  2217   312 1661 322       3   4
```

```
head(GibbsStepTimeFixedLblock, 50)
```

```
##          z  xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 1172 284   350     4  1964    22  93 325       4   5
##  [2,] 1268 284   327     4  2111    22 102 324       4   5
##  [3,] 1135 286   317     4  1949    22  94 318       4   5
##  [4,] 1105 289   332     4  1993    22  96 334       4   5
##  [5,] 1135 288   300     4  2013    21  95 303       3   4
##  [6,] 1261 287   322     4  2044    21  93 317       3   4
##  [7,] 1118 289   306     4  2100    21  96 318       3   4
##  [8,] 1113 290   321     4  2044    21  91 317       3   4
##  [9,] 1119 281   313     4  2125    21  97 322       3   4
## [10,] 1132 289   337     4  1954    21  95 315       3   4
## [11,] 1127 285   323     4  1989    22  96 335       3   4
## [12,] 1102 297   314     4  2197    21  94 321       3   4
## [13,] 1136 292   312     4  1924    22  92 318       3   4
## [14,] 1106 280   319     4  1916    22  96 314       3   4
## [15,] 1108 291   313     4  1952    21  96 313       3   4
## [16,] 1142 283   314     4  1977    26 102 344       3   5
## [17,] 1115 288   316     4  1980    22  95 321       3   5
## [18,] 1124 290   323     4  2127    21  94 314       3   4
## [19,] 1110 284   310     4  1923    21  92 312       3   4
## [20,] 1112 282   330     4  1983    22  93 314       3   4
## [21,] 1235 283   315     4  1981    21  92 308       3   4
## [22,] 1125 285   317     4  1928    22  98 325       3   4
## [23,] 1134 281   314     4  1942    21  94 309       3   4
## [24,] 1116 281   320     4  2345    21  97 346       3   4
## [25,] 1258 289   308     4  1952    22  93 310       3   4
## [26,] 1132 287   321     4  1953    23  95 343       3   4
## [27,] 1130 286   311     4  2158    23  96 314       3   4
## [28,] 1126 287   336     4  1956    23  96 331       4   5
## [29,] 1136 279   318     4  2060    22 100 340       3   4
## [30,] 1115 283   331     4  2030    21  92 323       3   4
## [31,] 1123 287   315     4  1984    22  93 321       3   4
## [32,] 1130 283   315     4  2130    21  95 321       3   4
```

```
## [33,] 1130 289   347     4  2054    22  95 333       4   5
## [34,] 1131 283   324     4  2256    23  97 316       4   4
## [35,] 1132 294   341     4  1996    22  94 343       3   4
## [36,] 1280 290   320     4  1999    22  96 324       3   4
## [37,] 1127 289   315     4  2121    21  95 326       3   4
## [38,] 1127 284   320     4  1966    22  94 318       4   4
## [39,] 1159 281   327     4  2111    22  93 333       3   4
## [40,] 1255 282   317     4  2395    23 106 318       4   5
## [41,] 1114 288   346     4  1955    22  94 325       3   4
## [42,] 1127 288   314     4  2000    22  94 318       3   4
## [43,] 1133 293   318     4  2135    21  97 328       3   4
## [44,] 1108 284   307     4  1935    21  96 304       3   4
## [45,] 1178 285   356     4  2285    22  95 320       3   4
## [46,] 1114 282   308     4  2028    25 102 342       3   4
## [47,] 1120 291   323     4  2006    21  96 324       3   4
## [48,] 1113 282   315     4  2034    21  94 318       3   4
## [49,] 1136 281   328     4  2012    22  96 327       3   4
## [50,] 1100 291   342     4  1947    21  92 320       3   4
```

```
head(GibbsStepTimeFixedLsequen, 50)
```

```
##          z  xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 1075 290   345     4   364    22 109 324       3   5
##  [2,] 1121 284   337     5   364    23 110 314       3   5
##  [3,] 1073 290   321     4   364    23 107 317       4   5
##  [4,] 1070 282   321     4   366    23 108 324       4   5
##  [5,] 1098 283   322     4   365    23 109 341       4   5
##  [6,] 1088 288   319     4   364    22 105 313       4   4
##  [7,] 1077 285   323     4   364    22 105 319       3   5
##  [8,] 1098 282   335     4   373    22 105 322       3   4
##  [9,] 1080 285   318     4   363    22 106 319       3   4
## [10,] 1082 287   320     4   364    23 105 326       3   5
## [11,] 1074 284   324     4   364    23 106 324       3   5
## [12,] 1077 283   318     4   365    22 105 324       3   4
## [13,] 1060 281   328     4   365    22 105 323       3   4
## [14,] 1060 282   309     4   364    22 105 316       3   4
## [15,] 1087 289   341     5   366    24 108 316       4   5
## [16,] 1091 287   319     4   366    24 105 327       4   5
## [17,] 1076 285   324     4   363    22 105 313       3   4
## [18,] 1077 289   309     4   364    22 107 320       3   4
## [19,] 1069 282   321     4   373    22 107 309       3   4
## [20,] 1088 282   311     4   366    25 107 328       4   5
## [21,] 1087 289   322     4   365    22 105 327       3   5
## [22,] 1076 282   316     4   364    22 105 315       4   4
## [23,] 1073 290   326     4   364    22 106 323       4   4
## [24,] 1062 294   318     4   363    22 105 319       3   4
## [25,] 1072 284   322     4   364    22 105 321       3   5
## [26,] 1087 290   316     4   375    23 107 327       4   5
## [27,] 1085 289   321     4   365    23 108 323       3   4
## [28,] 1080 285   322     4   365    23 107 318       3   4
## [29,] 1075 283   318     4   365    22 106 327       4   5
## [30,] 1083 285   320     4   365    22 106 315       4   5
## [31,] 1081 284   325     4   366    23 107 319       3   5
## [32,] 1075 283   313     4   364    22 105 313       3   4
## [33,] 1095 284   326     4   365    22 113 323       3   4
```

```
## [34,] 1064 288   315     4   363    22 104 321       3   4
## [35,] 1075 285   320     4   364    23 112 312       4   4
## [36,] 1074 289   309     4   366    23 107 326       4   5
## [37,] 1087 284   324     4   369    22 107 315       3   5
## [38,] 1078 285   315     4   365    22 106 320       3   4
## [39,] 1061 285   325     4   364    23 105 313       3   4
## [40,] 1071 287   313     4   364    22 107 327       4   4
## [41,] 1091 287   330     4   366    23 106 322       3   4
## [42,] 1080 300   314     4   364    23 107 328       3   4
## [43,] 1067 287   330     4   363    22 105 312       3   4
## [44,] 1072 286   309     4   374    22 104 319       3   4
## [45,] 1073 284   326     4   365    22 106 317       3   4
## [46,] 1074 290   315     4   364    22 107 329       3   4
## [47,] 1057 289   328     4   362    23 106 326       3   4
## [48,] 1072 284   311     4   364    23 107 325       3   4
## [49,] 1075 285   332     5   364    22 103 325       4   5
## [50,] 1067 286   316     4   364    22  99 312       3   4
```

```
head(GibbsStepTimeVaryLjSequen, 50)
```

```
##       u xi theta delta alpha kappa rho eta upsilon psi
##  [1,] 3 21   321     3   998    15 101 319       4   5
##  [2,] 3 20   325     3   978    14  99 317       3   5
##  [3,] 3 20   320     3   973    14  98 312       4   4
##  [4,] 3 21   321     3   976    14  98 320       3   4
##  [5,] 3 22   316     4   990    14  99 334       4   5
##  [6,] 3 21   312     3  1006    14  99 322       4   5
##  [7,] 3 24   316     4   997    14  97 328       4   5
##  [8,] 3 24   333     3  1037    15  99 327       4   4
##  [9,] 3 21   332     4   988    15 100 339       4   5
## [10,] 3 25   348     3  1003    15  99 331       4   5
## [11,] 3 22   331     3   985    15 100 328       4   5
## [12,] 2 23   351     3  1034    17 102 345       4   5
## [13,] 3 21   327     3  1163    16 103 337       4   5
## [14,] 3 21   316     3   978    14  99 317       4   5
## [15,] 4 24   338     4  1004    15 100 328       3   5
## [16,] 3 24   326     3   977    14  97 332       4   4
## [17,] 3 20   318     3   980    14  97 332       4   5
## [18,] 3 22   317     3  1151    14  98 328       4   5
## [19,] 3 20   324     3  1001    14  98 329       4   5
## [20,] 3 23   321     3   993    14  98 330       4   4
## [21,] 3 20   322     3   983    15 103 328       4   5
## [22,] 3 20   328     3   979    15  98 322       4   5
## [23,] 3 21   332     3   979    14  99 340       4   5
## [24,] 3 20   327     3   982    14  99 323       4   5
## [25,] 3 20   325     3   978    15 101 322       4   4
## [26,] 3 21   330     3   996    14 100 325       4   4
## [27,] 3 21   320     3  1004    14  99 321       3   5
## [28,] 3 21   332     3   968    14  98 320       4   5
## [29,] 3 21   330     3   985    14  99 319       4   5
## [30,] 3 21   325     4   990    14  99 320       4   5
## [31,] 3 20   319     3  1001    14  97 324       4   5
## [32,] 3 21   324     3  1128    15  99 322       4   5
## [33,] 3 22   323     3   976    14  99 336       3   4
## [34,] 3 21   321     4  1000    14 101 328       4   4
```

```
## [35,] 3 23   319     3   989    14  97 321       3   4
## [36,] 3 20   322     3   979    14  97 326       3   4
## [37,] 3 21   317     3  1106    14  97 324       4   5
## [38,] 3 21   309     3   981    14  97 323       3   4
## [39,] 3 20   312     3  1007    15  99 324       4   5
## [40,] 4 23   310     3  1001    15  98 327       4   5
## [41,] 3 21   314     3   976    14  98 324       4   4
## [42,] 3 21   318     3   970    14  97 319       4   4
## [43,] 3 21   332     3   984    15 101 325       4   5
## [44,] 3 21   325     3   982    14 101 331       4   4
## [45,] 3 20   319     3  1010    15  99 327       4   5
## [46,] 2 20   324     3   979    14  97 320       4   4
## [47,] 4 25   327     3  1024    15 101 323       4   5
## [48,] 3 20   320     3   986    14  96 315       3   4
## [49,] 3 22   330     3  1022    17  98 310       3   4
## [50,] 3 24   322     4   975    15  96 314       4   5
```

As expected, there aren't any significant differences between our 4 methods regarding posterior sampling time for the 3 temporal parameters ψ, Υ, and $\boldsymbol{\eta}_t$'s.

Posterior Sampling Time Summary Statistics

We then present vital posterior sampling time summary statistics for the 7 spatial-related parameters ($z^o_{jl_j}(\boldsymbol{s}_i)$'s or $u^o_j(\boldsymbol{s}_i)$'s, $\xi^o_j(\boldsymbol{s}_i)$'s, θ_{jl_j}'s, $\delta_{1:k}$, ρ, κ, and $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s) to showcase the manifest scalability improvements brought about by our 3 novelties, i.e., slice sampling, spatial NNGP, and sequential updates.

```
apply(GibbsStepTimeFixedLfullGP[,1:7], 2, summary)
```

```
##                z       xi    theta  delta    alpha    kappa      rho
## Min.    1378.000 278.0000 300.0000  4.000 2161.000 308.0000 1301.000
## 1st Qu. 1445.000 283.0000 313.0000  4.000 2223.750 311.0000 1638.000
## Median  1465.000 285.0000 316.0000  4.000 2279.000 316.0000 1685.000
## Mean    1484.252 286.3702 317.6382  4.032 2319.485 317.7624 1675.367
## 3rd Qu. 1502.000 288.0000 321.0000  4.000 2377.000 321.0000 1774.250
## Max.    1755.000 426.0000 368.0000 12.000 3043.000 386.0000 2362.000
```

```
apply(GibbsStepTimeFixedLblock[,1:7], 2, summary)
```

```
##                z       xi    theta  delta    alpha  kappa     rho
## Min.    1074.000 274.0000 293.0000 4.0000 1889.000 20.000  88.000
## 1st Qu. 1125.000 283.0000 314.0000 4.0000 1999.000 21.000  94.000
## Median  1140.000 286.0000 319.0000 4.0000 2087.000 22.000  97.000
## Mean    1155.679 286.3756 320.3912 4.0894 2124.528 21.976  97.143
## 3rd Qu. 1162.000 289.0000 326.0000 4.0000 2216.000 22.000  99.000
## Max.    1369.000 434.0000 384.0000 5.0000 2725.000 30.000 117.000
```

```
apply(GibbsStepTimeFixedLsequen[,1:7], 2, summary)
```

```
##                z       xi    theta delta    alpha   kappa      rho
## Min.    1033.000 279.0000 299.0000 4.000 362.0000 21.0000  91.0000
## 1st Qu. 1063.000 284.0000 316.0000 4.000 364.0000 22.0000 105.0000
## Median  1073.000 286.0000 321.0000 4.000 365.0000 22.0000 106.0000
## Mean    1090.681 286.3248 321.6182 4.052 365.9912 22.5796 106.6914
## 3rd Qu. 1090.000 288.0000 326.0000 4.000 366.0000 23.0000 108.0000
## Max.    1311.000 429.0000 375.0000 6.000 414.0000 30.0000 125.0000
```

```
apply(GibbsStepTimeVaryLjSequen[,1:7], 2, summary)
```

```
##             u       xi    theta delta    alpha  kappa     rho
## Min.    2.000  18.0000 299.0000 3.000  956.000 13.000  93.000
## 1st Qu. 3.000  20.0000 315.0000 3.000  978.000 14.000  97.000
## Median  3.000  21.0000 321.0000 3.000  986.000 14.000  98.000
## Mean    3.056  21.3178 320.9168 3.132 1002.139 14.403  98.423
## 3rd Qu. 3.000  22.0000 326.0000 3.000 1000.000 15.000  99.000
## Max.    4.000 191.0000 369.0000 5.000 1230.000 18.000 111.000
```

The results correspond well to what we have deduced in Appendix H of our manuscript.

- Compared to their `fullGPfixedL` counterparts, `NNGPblockFixedL`'s Gibbs sampler steps corresponding to ρ and κ are evidently accelerated by our **spatial NNGP prior**;
- The only Gibbs sampler step time that should clearly differ between `NNGPblockFixedL` and `NNGPsequenFixedL` is the step updating all $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s, which result from whether we adopt our **sequential updating method** or not. Since $m = 1600$ here is big, `NNGPsequenFixedL` is a few times faster than `NNGPblockFixedL` for the posterior sampling step corresponding to $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s;
- Thanks to our **slice sampling approach**, `NNGPsequenVaryLj`'s Gibbs sampler steps for $u^o_j(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s are significantly faster than `NNGPsequenFixedL`'s Gibbs sampler steps for $z^o_{jl_j}(\boldsymbol{s}_i)$'s and $\xi^o_j(\boldsymbol{s}_i)$'s. It turns out that `NNGPsequenVaryLj`'s Gibbs sampler step for $\alpha^o_{jl_j}(\boldsymbol{s}_i)$'s is slower than its `NNGPsequenFixedL` counterpart, indicating that inefficiencies caused by case discussion, calculating all required upper or lower bounds, and rejection sampling outweigh acceleration brought about by slice sampling's ensured non-increasing posterior samples for L_j's through the MCMC iterations.

We finally calculate standard deviations for the 7 spatial-related parameters' posterior sampling time across all kept post-burn-in MCMC iterations.

```
round(apply(GibbsStepTimeFixedLfullGP[,1:7], 2, sd), 5)
```

```
##         z        xi     theta     delta     alpha     kappa       rho
##  59.44248   5.57123   7.66915   0.38132 125.37100   8.52805 178.28102
```

```
round(apply(GibbsStepTimeFixedLblock[,1:7], 2, sd), 5)
```

```
##         z        xi     theta     delta     alpha     kappa       rho
##  51.24662   5.52770   9.76289   0.28535 159.38157   1.03113   3.81139
```

```
round(apply(GibbsStepTimeFixedLsequen[,1:7], 2, sd), 5)
```

```
##        z       xi    theta    delta    alpha    kappa      rho
## 48.93822  4.62463  8.83625  0.22295  3.07106  1.07140  3.19985
```

```
round(apply(GibbsStepTimeVaryLjSequen[,1:7], 2, sd), 5)
```

```
##        u       xi    theta    delta    alpha    kappa      rho
##  0.42059  4.86147  8.51906  0.33912 45.08198  0.70618  2.11541
```