

CPSC 468 Midterm Review

Chapter 1

0.1 (Turing Machine). A k -tape Turing Machine M is described by a tuple (Γ, Q, δ) . Assume $k \geq 2$, with 1 read-only input tape and $k - 1$ work tapes. The last work tape is assumed to be the output tape.

- Γ the finite alphabet of symbols that M may have on its tapes. Assume that Γ contains at least $\{0, 1, \square, \triangleright\}$.
- Q a finite set of possible states M 's state register may be in. Assume that Q contains a q_{start} and q_{halt} .
- $\delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^{k-1} \times \{L, S, R\}^k$ a transition function for M that takes in the current state and each head's read, and outputs the next state, with $k-1$ writes on all the work tapes, and movement direction for all k tapes.

0.2 (Computing a Function). Let $f : \{0, 1\}^* \rightarrow \{0, 1\}^*$ and $T : \mathbb{N} \rightarrow \mathbb{N}$, with M a TM. We say that M computes f if for every $x \in \{0, 1\}^*$, if M is initialized to the start configuration on input x , then it halts with $f(x)$ on the output tape. We say M computes f in $T(n)$ -time if its computation on every x requires at most $T(|x|)$ steps.

0.3 (Time Constructible). A function $T : \mathbb{N} \rightarrow \mathbb{N}$ is time constructible if $T(n) \geq n$ and there is a TM M that computes the function $x \mapsto \lfloor T(|x|) \rfloor$ in time $T(n)$. $T(n) \geq n$ is to allow the algorithm to read its input. Some time constructible functions are n , $n \log n$, n^2 and 2^n .

0.4. For every $f : \{0, 1\}^* \rightarrow \{0, 1\}$ and time constructible $T : \mathbb{N} \rightarrow \mathbb{N}$, if f is computable in time $T(n)$ by some TM M using alphabet Γ , then it is able to compute the same function using $\{0, 1, \square, \triangleright\}$ in $(c \log_2 |\Gamma|) \cdot T(n)$. This is because we may express each symbol of Γ using $\log |\Gamma|$ binary bits, with some constant c overhead.

0.5. A k -tape TM can have its $k - 1$ work tapes simulated by a single tape by interleaving the k tapes together.

0.6 (Oblivious Turing Machine). An oblivious TM's head movement depends on the length of the input, not the contents of the input. Every TM can be simulated by an oblivious TM.

0.7 (Turing Machine Representation). Every binary string $x \in \{0, 1\}^*$ represents some TM, and every TM is represented by infinite such strings (think: comments in a language). The machine represented by x is denoted M_x .

0.8 (Universal Turing Machine). There exists a TM \mathcal{U} such that for every $x, \alpha \in \{0, 1\}^*$, $\mathcal{U}(x, \alpha) = M_\alpha(x)$, where M_α denotes the TM represented by α . Moreover, if M_α halts on input x within T steps, then $\mathcal{U}_\alpha(x)$ halts within $CT \log T$ steps, where C is a number independent of $|x|$, and depends only on M_α 's alphabet size, number of tapes, and number of states. The cost of simulating any machine M_α has a logarithmic overhead, due to the alphabet size difference between M_α and \mathcal{U} . As \mathcal{U} has a single tape, we do the trick over interleaving M_α 's work tapes together.

0.9 (Uncomputable Function). Define U as follows: for every $\alpha \in \{0, 1\}^*$, if the machine defined by α accepts itself, such that $M_\alpha(\alpha) = 1$, then $U(\alpha) = 0$. In other words $U(\alpha) = 1 - M_\alpha(\alpha)$. There is no such TM that can compute U , because U will always negate it.

0.10 (Halting Problem). A TM H such that $H(\alpha, x) = 1$ if $M_\alpha(x)$ halts, and yields 0 otherwise, does not exist. We can construct a wrapper TM W that invokes H on itself, and performs the opposite. Diagonalization motherfuckers!

0.11 (DTIME). Let $T : \mathbb{N} \rightarrow \mathbb{N}$ be some function. A language L is in $\text{DTIME}(T(n))$ iff there is a deterministic TM that runs in time $c \cdot T(n)$ for some constant $c > 0$ and decides L . This class contains **decision** problems.

0.12 (The Class P). $P = \bigcup_{c \geq 1} \text{DTIME}(n^c)$

0.13 (Church-Turing Thesis). Every physically realizable computation device can be simulated by a TM.

0.14 (Bounds). The asymptotic operators $\{o, O, \Theta, \Omega, \omega\}$ can be thought of as $\{<, \leq, =, \geq, >\}$.

Chapter 2

0.15 (The Class NP). *A language $L \subseteq \{0, 1\}^*$ is in NP if there exists a polynomial $p : \mathbb{N} \rightarrow \mathbb{N}$ and a polynomial time TM M (called the **verifier** for L) s.t. for every $x \in \{0, 1\}^*$, we have $x \in L \iff \exists u \in \{0, 1\}^{p(|x|)}$ s.t. $M(x, u) = 1$. If $x \in L$ and $u \in \{0, 1\}^{p(|x|)}$ satisfy $M(x, u) = 1$, we call u a **certificate** for x (with respect to L and M). NP is the class of languages for which we can tell if u is a solution to the problem $x \in L$ in polynomial time.*

0.16 (EXP). $\text{EXP} = \bigcup_{c > 1} \text{DTIME}(2^{n^c})$.

Chapter 3

Chapter 4

Chapter 5

Chapter 6

Examples