Time Series Analysis

Homework Assignment 2

Problem 1

a) We first plot Lake Huron's Annual water level time series, *Xt*[1] as shown in Figure 1 below.

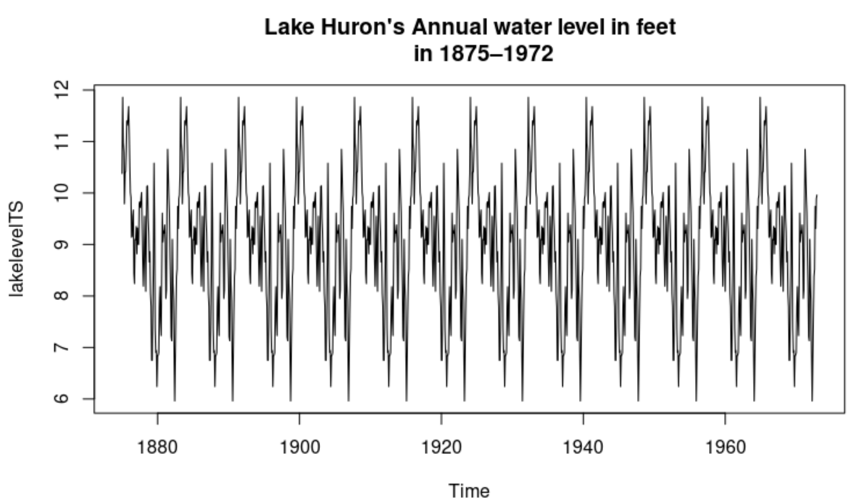


Figure 1

From the first look at the data, it is hard to tell if the seasonality model is multiplicative or additive. However, before decomposing, we cannot clearly see the trend of the time series. We can however see that the seasonality exhibits constant aptitude in that the

[1] Original time series

highest points and lowest are constant over time. I would therefore decompose this time series using additive and the result is in Figure 2.

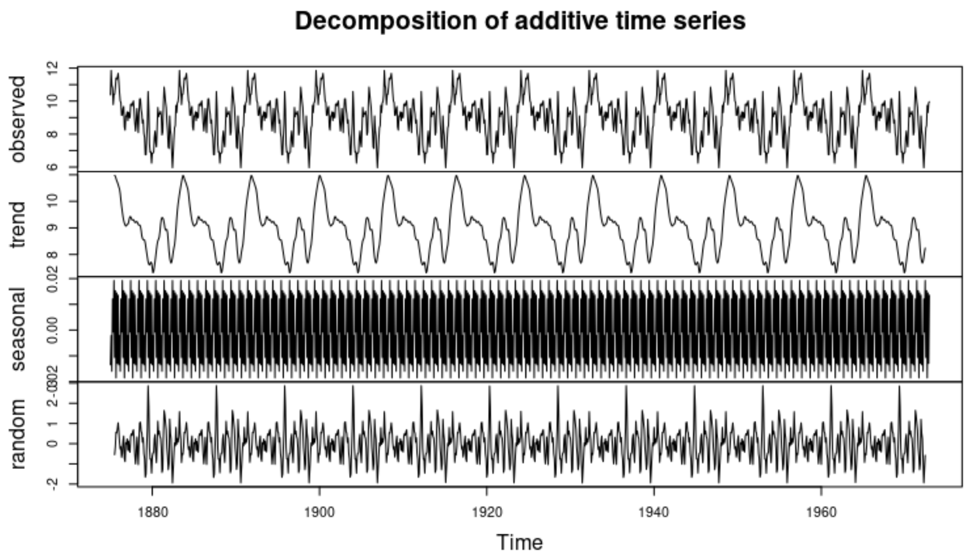


Figure 2

b) The results after fitting the model are;
- $\alpha =$ `12.8838`
- $\beta =$ `-0.002`

The resulting fitted line is shown in Figure 3 as shown below.

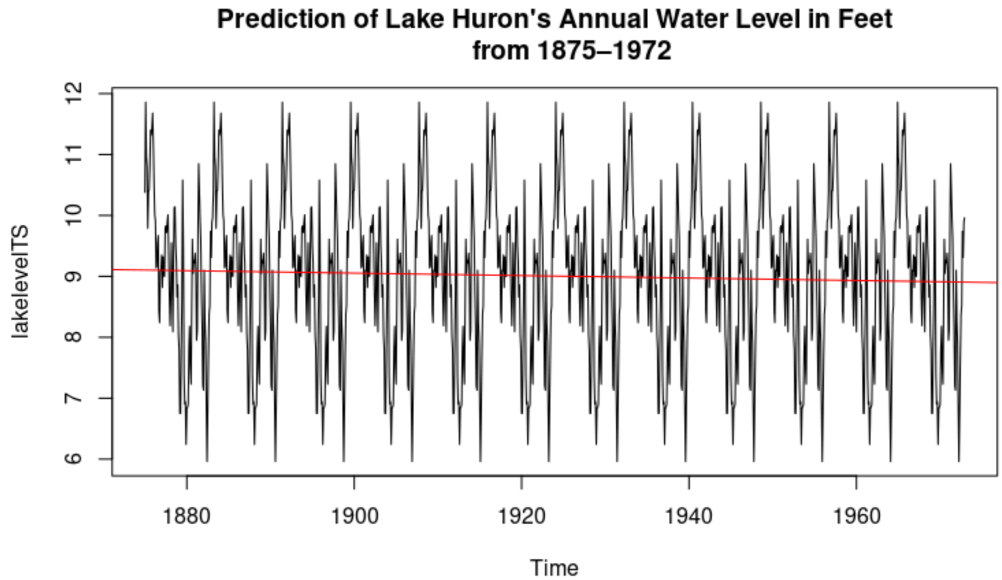


Figure 3

The fitted line appears to be fitted in the middle of the graph right about the mean of the time series which is ~ 9 and a standard deviation of ~ 1.3. This is expected as the values of the time series fluctuate above and below the average or mean value over time.

c) We then calculate the resulting residuals *Rt*[2] from the linear model procedure and plotted the results in Figure 4. It can be observed that the graph from the residuals looks similar to the original time series, with a difference in the scale. This is because the residuals are a result of the difference between the original time series data and the fitted data and the difference in scale is ~ 9 similar to the mean of the original time series.

[2] Residual from the difference of observed data and the fitted values

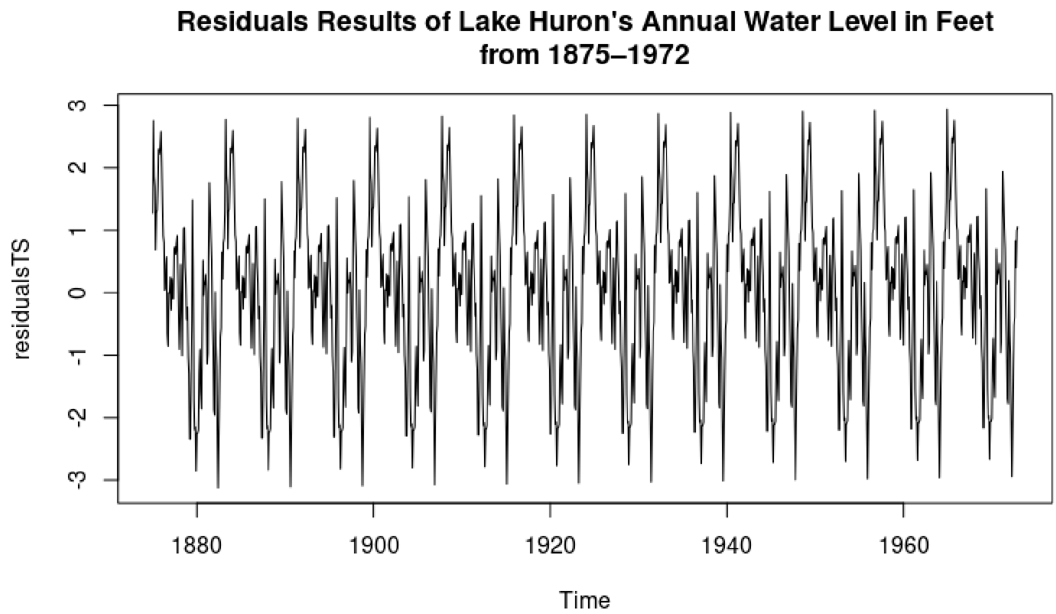


Figure 5

d) The mean and standard deviation comparisons rounded off to 4 decimal places are as follows;

Time Series	Mean	Standard Deviation
Xt	9.0041	1.3121
Rt	2.709e-17	1.3109

We can observe that the means of the two time series are different with *Xt* having a larger mean than *Rt* because the *Rt* is a result of the difference between *Xt* and fitted values. However, the two time series have almost a similar standard deviation.

e) In order to get a first impression of the dependencies in the time series, we can use the autocorrelation function to graphically display this as in Figure 5 and Figure 6 below. Figure 5 shows the result for *Xt* and Figure 6 shows the result for *Rt.* In both cases the correlograms are similar. Most of the lag corresponds to a statistically significant correlation, meaning they extend outside the confidence level. We can also observe that the confidence points in the graph form an oscillating pattern. In the first parts of the positive correlation, we can see a decay in the correlation as the lag increases. It then shifts to the negative correlation side forming a bell shape and finally shifting again to the positive correlation side. We can predict that the patterns oscillate on both sides from this observation. We can then conclude this data exhibits autoregressive patterns.

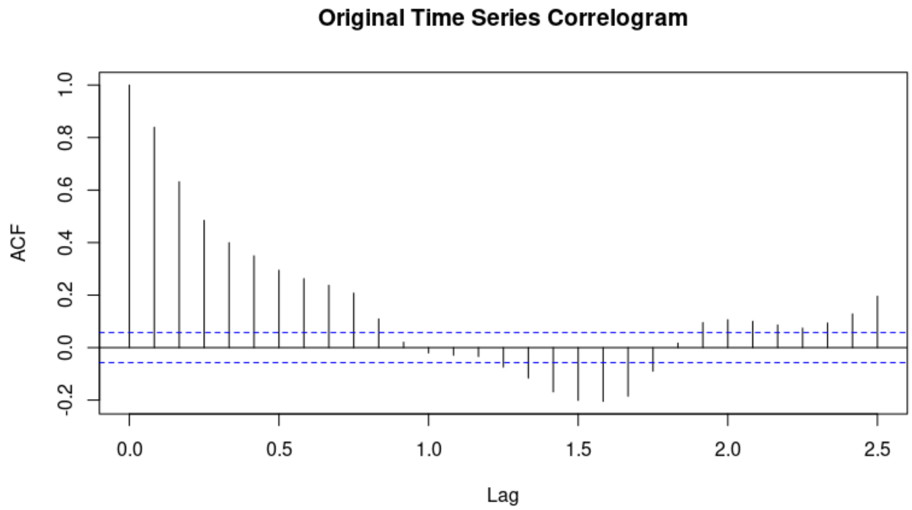


Figure 5

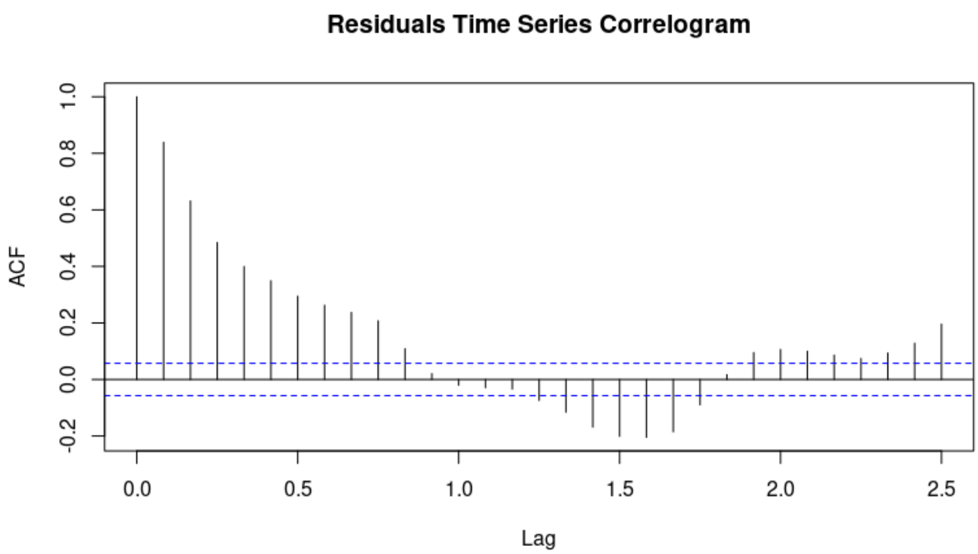


Figure 6

f) We are going to check the independence of the realisation of *Rt* using the following testing techniques.

1) **Box-Pierce Test**

- ***Problem setting:*** Given our residual observations $r_1,\ldots,r_n$ represented as R_t having a true ACP ρ and $N > 100$, $L>1$, with N representing the length of the time series data and L is the number of lags.
- ***Test Problem***:
  - ❖ H_0: ρ(1) … ρ(L) = 0
  - ❖ H_1: For ρ(1) … ρ(L), one results to number that is not zero.

Observation 1:

We noticed that no matter the lag value we chose, the p-value remained the same. We then choose our lag as 2. The result of this test is represented as;

```
> ##Box-Pierce test
> Box.test(residualsTS, lag = 2, type = "Box-Pierce") # Decision : Reject H0

        Box-Pierce test

data:  residualsTS
X-squared = 1295.3, df = 2, p-value < 2.2e-16
```

We obtain the p-value as 2.2e-16. Given that our $\alpha = 0.05$, we, therefore, reject the null hypothesis H_0 in favour of the alternative hypothesis H_1.

2) **Turning Point Test:**

- **Problem setting:** Our residual observations $r_1,\ldots,rn$ is represented as *Rt.*
- ***Test Problem***:
  - ❖ H_0: Each observation in $r_1,\ldots,rn$ is independent of the value in the previous observation
  - ❖ H_1: Each observation in the time series is dependent on the previous observation.

Observation 2:

The results of the Turning point test are shown the code snip below.

```
>
>
> turning.point.test(residualsTS) # Reject H0

        Turning Point Test

data:  residualsTS
statistic = -19.288, n = 1176, p-value < 2.2e-16
alternative hypothesis: non randomness
```

We can observe that we obtain the p-value as 2.2e-16. Given that our $\alpha = 0.05$, we, therefore, reject the null hypothesis H_0 in favour of the alternative hypothesis H_1 as we did in our previous test.

3) **Difference Sign Test:**

- **Problem setting:** Our residual observations $r_1,\ldots,r_n$ is represented as *Rt.*
- ***Test Problem***:
    - H_0: Each observation in $r_1,\ldots,r_n$ is independent of the value in the previous observation
    - H_1: Each observation in the time series is dependent on the previous observation.

Observations 3:

The result of the Difference Sign test is shown in the code snippet below:

```
> difference.sign.test(residualsTS)

        Difference Sign Test

data:  residualsTS
statistic = -0.050486, n = 1176, p-value = 0.9597
alternative hypothesis: nonrandomness
```

We can observe that we obtain the p-value as 0.9597. Given that our $\alpha = 0.05$, we, therefore, do not reject the null hypothesis H_0 with regards to this test.

Conclusion:

We have conducted 3 tests resulting in 2 of the tests leading to rejecting the null hypothesis in favour of the alternative hypothesis and 1 in not rejecting the null hypothesis. Therefore since we have at least one result for rejecting the hypothesis and not enough evidence to support that each observation in the residual time series is independent of the previous observation, we therefore reject the null hypothesis and overall conclude that each observation in the residual time series is not independent of its predecessor.

Problem 2

a) The resulting generated time series is represented in Figure 7.

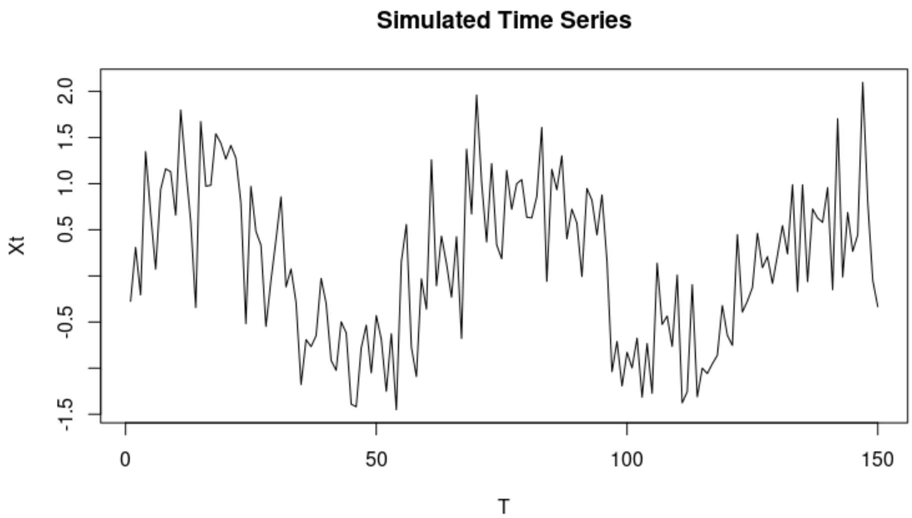


Figure 8

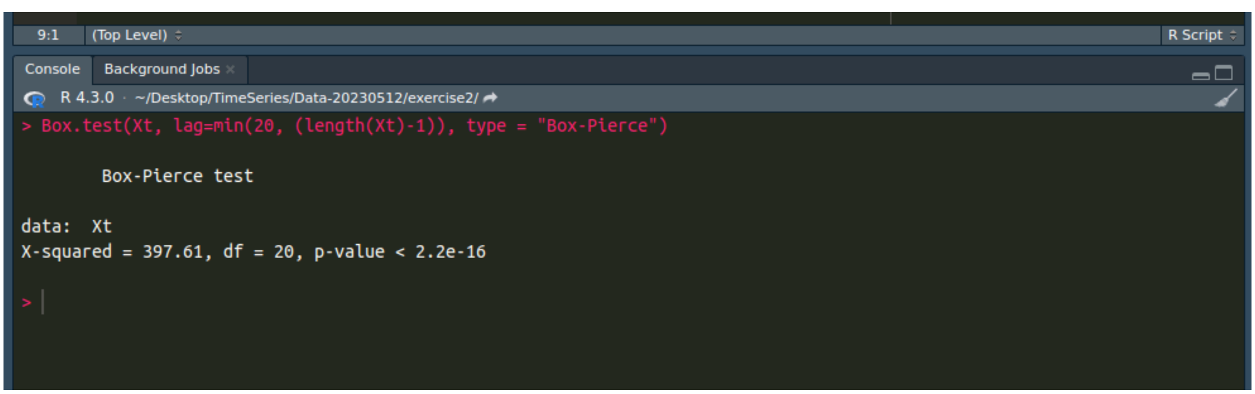
b)

1) **Box-Pierce Test**

- ***Problem setting:*** Given our residual observations $r_1,\ldots,r_{□}$ represented as $R_{□}$ having a true ACP ρ and $N > 100$, $L>1$, with N representing the length of the time series data and L is the number of lags.
- ***Test Problem***:
  - ❖ H_0: $\rho(1) \ldots \rho(L) = 0$
  - ❖ H_1: For $\rho(1) \ldots \rho(L)$, one results to a number that is not zero.

Observation 1:

The result of the Box-Pierce test is shown in the code snippet below;



We obtain the p-value as 2.2e-16. Given that our $\alpha = 0.05$, we, therefore, reject the null hypothesis H_0 in favour of the alternative hypothesis H_1.

2) **Turning Point Test:**

- **Problem setting:** Our residual observations $x_1,\ldots,x_n$ is represented as *Xt*.
- ***Test Problem***:
  - H_0: Each observation in $x_1,\ldots,x_n$ are iid.
  - H_1: Each observation in *Xt* is not iid.

The result of this test is shown in the code snippet below;

```
10:1   (Top Level) ÷                                                    R Script ÷
Console   Background Jobs ×
R 4.3.0 · ~/Desktop/TimeSeries/Data-20230512/exercise2/
> turning.point.test(Xt)

        Turning Point Test

data:  Xt
statistic = 1.2339, n = 150, p-value = 0.2172
alternative hypothesis: non randomness

> |
```

We can observe that we obtain the p-value as 0.2172. Given that $\alpha = 0.05$ we, therefore, do not reject the null hypothesis H_0 with regards to this test.

3) **Difference Sign Test:**

- **Problem setting:** Our residual observations $r_1,\ldots,r_n$ is represented as *Xt*.
- ***Test Problem***:
  - H_0: Each observation in $x_1,\ldots,x_n$ are iid.
  - H_1: Each observation in *Xt* is not iid.

```
11:1   (Top Level) ÷                                                    R Script ÷
Console   Background Jobs ×
R 4.3.0 · ~/Desktop/TimeSeries/Data-20230512/exercise2/
> difference.sign.test(Xt)

        Difference Sign Test

data:  Xt
statistic = -1.5505, n = 150, p-value = 0.121
alternative hypothesis: nonrandomness

> |
```

The results from this test show that the p-value is 0.121 is greater than the value of α which is 0.05. We, therefore, do not reject the null hypothesis H_0.

Conclusion:
From the test above we obtain 2 tests that favour in not rejecting the null hypothesis while 1 rejects the null hypothesis in favour of the alternative hypothesis. Since we have one test resulting in rejecting the null hypothesis, we, therefore, reject the hypothesis in favour of the alternative hypothesis and conclude that the time series *Xt* is not identical and independent data (iid).