TIME SERIES ANALYSIS

PROBLEM 1

a)

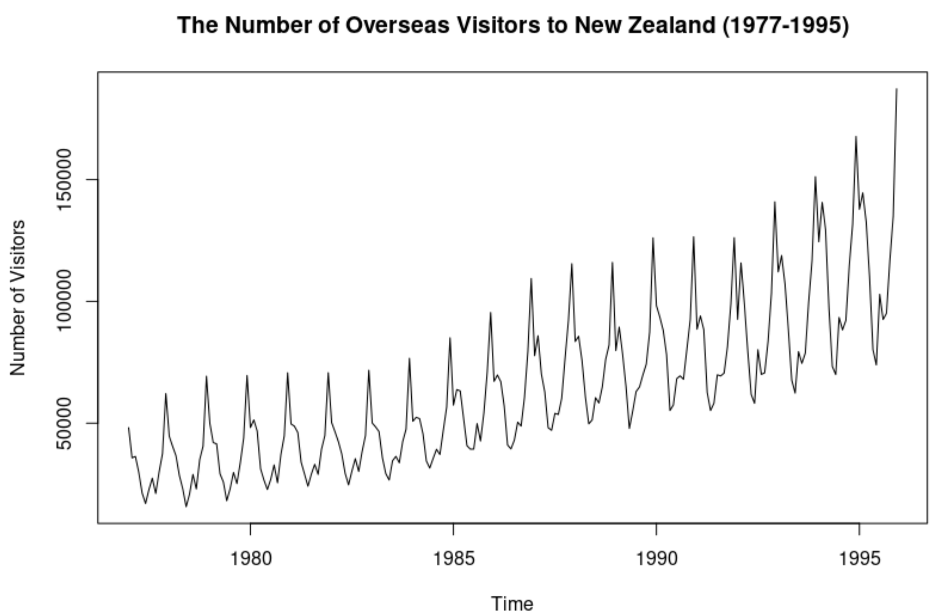

Figure 1

The plot resulting plot X_t is shown in Figure 1. The figure shows that the time series is a multiplicative model. We therefore would need to take the log of the time series before fitting the ARIMA model. The resulting time series is shown in Figure 2.

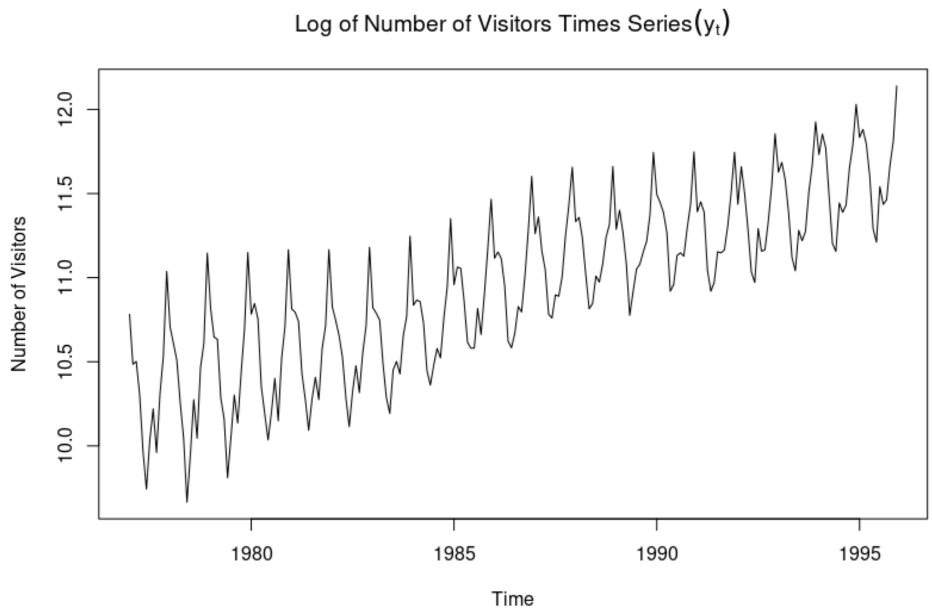

Figure 2

b) The resulting differenced time series is shown in Figure 3. In comparison to Y_t, we can observe that the differencing is removes some bit of seasonality in the data.

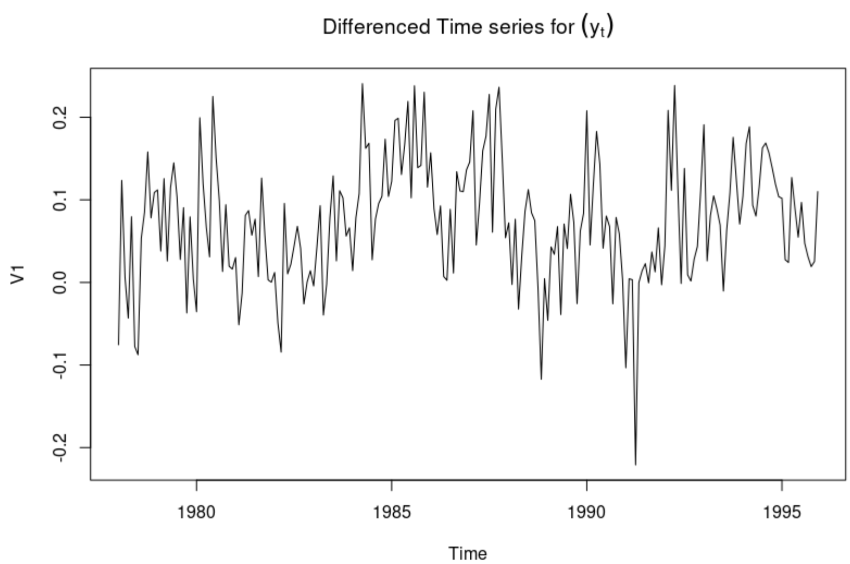

Figure 3

We can test for stationarity of the resulting time series as described below:

Setting: Given a time series X_t
Null Hypothesis : $H_0 \ : \ (X_t)$ is stationary

Alternative Hypothesis (H1):

$H_1 \, : \, (X_t) \, :$ is an intergrated process (has a unit root)

```
> #test stationarity
> kpss.test(diff.Y)  # H0 (= process is stationary) is not rejected

	KPSS Test for Level Stationarity

data:  diff.Y
KPSS Level = 0.20747, Truncation lag parameter = 4, p-value = 0.1
```

Given α = 0.05 and the resulting p-value is 0.1, therefore we do not have enough evidence to reject the null hypothesis. Using this test we can conclude that the time series is stationary.

c) We then difference the already differenced time series and the resulting graph, as shown in Figure 4.

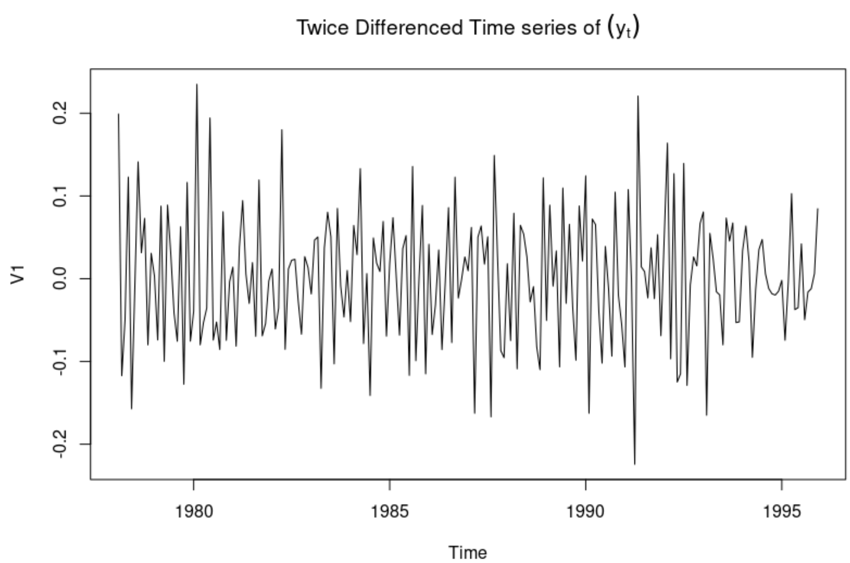

Figure 4

This graph shows the time series to be a bit more stationary than the previous one which was only differenced once. To confirm stationary we perform the following tests.

<u>KPPS Test</u>

Setting: Given a time series X_t

Null Hypothesis : $H_0 \, : \, (X_t)$ is stationary

Alternative Hypothesis (H1): $H_1 \, : \, (X_t) \, :$ is an intergrated process (has a unit root)

The results of KPSS test are as shown below:

```
        KPSS Test for Level Stationarity

data:  diff2.Y
KPSS Level = 0.027917, Truncation lag parameter = 4, p-value = 0.1
```

Given α = 0.05 and the resulting p-value is 0.1, therefore we do not have enough evidence to reject the null hypothesis. Using this test we can conclude that the time series is stationary.

ADF Test

Setting : $Given\,observations\ (x_t)_{t=1}^{n}\ of\,a\,time\,series\,(X_t)$
supposed to satisfy : $X_t\ =\ \phi X_{t-1}\ +\ \ldots\ +\phi_r X_{t-r}\ +\ Z_t$
$where\ Z_t\ \sim\ WN(0,\,\sigma^2)$

Null Hypothesis : $\mathrm{H}_0\ :\ \phi_1\ =1$
Alternative Hypothesis : $H_1\ :\ \phi_1\ <\ 1$

The result of ADF test is as shown below;

```
        Augmented Dickey-Fuller Test

data:  diff2.Y
Dickey-Fuller = -8.2376, Lag order = 5, p-value = 0.01
alternative hypothesis: stationary
```

For this test, the null hypothesis represents the time series not being stationary and the alternative being that the time series is stationary. Given that α = 0.05 and the resulting p-value is 0.01, we can therefore reject the hypothesis in favour of the alternative being that the time series is stationary.

Since both tests show leaned on the side that the twice differenced time series is stationary, we can therefore conclude as such.

d) We ran the different combinations of the ARMA model that resulted in the following:

```
> #using ARIMA(p,0,q)
> arima(diff2.Y, order = c(0, 0, 0))$aic
[1] -473.1312
> arima(diff2.Y, order = c(0, 0, 1))$aic
[1] -542.3005
> arima(diff2.Y, order = c(1, 0, 0))$aic
[1] -511.7
> arima(diff2.Y, order = c(1, 0, 1))$aic
[1] -542.2935
```

From the result, we take the combination with the smallest AIC value, which is ARIMA(0,0,1) or an equivalent of ARMA(0,1). After fitting the model, the resulting residuals can be investigated using acf function as in Figure 5.

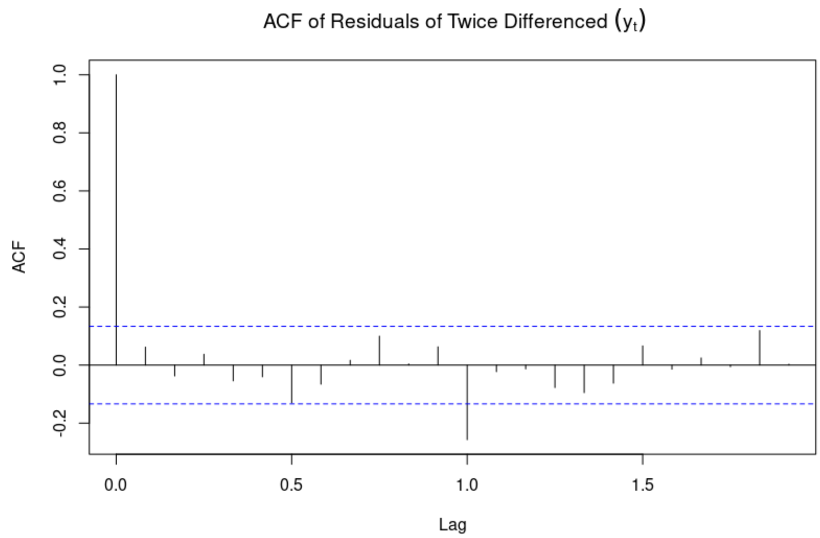

Figure 5

From the ACF results above, we can observe that there are not a lot of correlations in the time series except for 1 lag, showing that the ARMA model fitted was a good model.

The test for independence on the same residual data is a describes below:

Box-Pierce Test

Setting; Given an observation $x_1, \ldots, x_N$ for a given time series X_N where ACF ρ is true but unknown and N > 100, where L is the lag and N is the length of the time series.

Test Problem;

$$H_0 : \rho(1) = \ldots = p(L) = 0$$
$$H_1 : \rho(1), \ldots, p(L) \neq 0$$

Turning Point Test:

Given observation $r_1, \ldots, r_N$;

Test problem ;

H_0 : $r_{1,} \ldots, r_n$ are iid

H_1 : $r_{1,} \ldots, r_n$ are not iid

Difference Sign Test:

Given observation $r_1, \ldots, r_N$;

Test problem ;

H_0 : $r_{1,} \ldots, r_n$ are iid

H_1 : $r_{1,} \ldots, r_n$ are not iid

The results of the result above are shown in the code snippet below;

```
> #Test for independence
> Box.test(arma_resid, type = "Box-Pierce", lag = 20) # H0 = (TS is iid)
ejected

        Box-Pierce test

data:  arma_resid
X-squared = 29.495, df = 20, p-value = 0.07845

> turning.point.test(arma_resid) # H0 = (TS is iid) is rejected

        Turning Point Test

data:  arma_resid
statistic = -2.4365, n = 215, p-value = 0.01483
alternative hypothesis: non randomness

> difference.sign.test(arma_resid) # H0 = (TS is iid) is not rejected

        Difference Sign Test

data:  arma_resid
statistic = 0.4714, n = 215, p-value = 0.6374
alternative hypothesis: nonrandomness

>
```

Given the value of alpha $=$ 0.05, we can arrive at the following conclusion:

- In Box-Pierce test, we fail to reject the null hypothesis since p-value = 0.07 which greater than the alpha 0.05.

- In Turning Point test, the null hypothesis is rejected. This is because p-value = 0.01 which is less than the alpha value.
- Lastly, in Difference sign test, we also fail to reject the null hypothesis as p-value = 0.64, and larger than the alpha.

In conclusion, since we had one test rejecting the null hypothesis that supported the residuals values ais not iid, we can therefore reject the null hypothesis in favour of the alternative to conclude that the residuals are not iid.

e) After different combinations of $\mathrm{ARIMA(p,1,q)(P,1,Q)_{12}}$ on the time series Y_t, the lowest, the combination with the lowest AIC, with a value of `-565.2817` was ARIMA(1,1,1,0,1,1). When the model is fitted, the ACF of the resulting residuals were as shown in Figure 6.

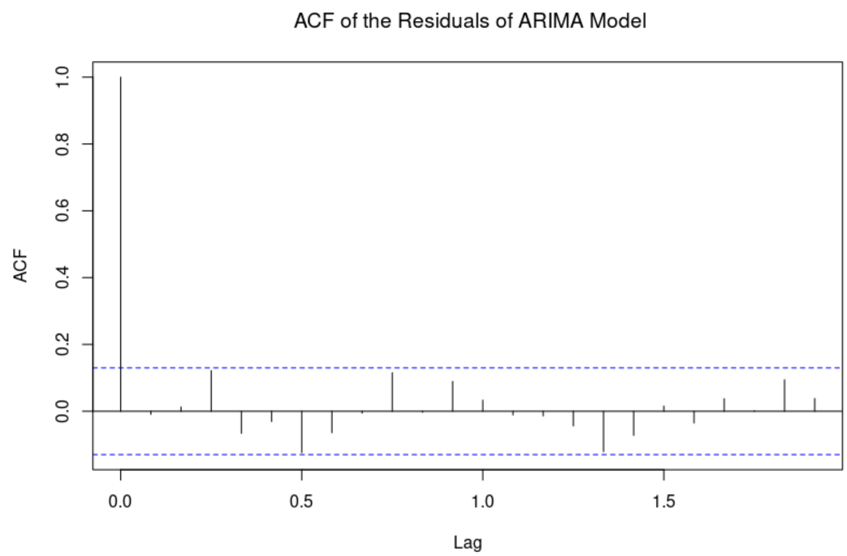

Figure 6

From the ACF plot above, we can see there is no significant correlation and therefore can deduce that the model was well fit.

To also test for independence, we run the same test, Box-pierce test , Turning Point test and Difference sign test as in describe in question (d) above resulting in the following;

```
> Box.test(arima_model_residual, type = "Box-Pierce", lag = 20) # H0 = (TS is ii
d) is not rejected

        Box-Pierce test

data:  arima_model_residual
X-squared = 19.862, df = 20, p-value = 0.4666

>
> turning.point.test(arima_model_residual) # H0 = (TS is iid) is not rejected

        Turning Point Test

data:  arima_model_residual
statistic = -0.89362, n = 228, p-value = 0.3715
alternative hypothesis: non randomness

>
> difference.sign.test(arima_model_residual) # H0 = (TS is iid) is not rejected

        Difference Sign Test

data:  arima_model_residual
statistic = 1.259, n = 228, p-value = 0.208
alternative hypothesis: nonrandomness
```

From this we can following condition given that alpha = 0.05

- In Box-Pierce test, the p-value = 0.4666, and we fail to reject the null hypothesis
- In Turning Point test, the p-value = 0.3715 and we, therefore, fail to reject the null hypothesis.
- In Difference sign test, the p-value = 0.208 and we therefore fail to reject the null hypothesis.

In conclusion, we can conclude that the residuals from the ARIMA model is iid.

Problem 2:

1. $X_t = X_{t-1} - 0.25X_{t-2} + Z_t + 0.45Z_{t-1}$

a) Determine the order p, d and q of the ARIMA process.

$$X_t - X_{t-1} + 0.25X_{t-2} = Zt + 0.45Zt - 1$$

$$\Phi(x) = 1 - x + 0.25x^2$$
$$x_1 = x_2 = 2$$

The polynomial has roots outside the complex unit circle,
$p = 2,\ d = 0$

$$\Theta(x) = 1 + 0.45x$$
$$x_1 = -\frac{20}{9}$$

The above polynomial has a outside the complex unit circle,
therefore $q = 1$

The resulting model is ***ARIMA(2,0,1)***

b) The process is stationary because the polynomial Φ(x) has both roots outside the complex unit circle.
c) The process is ARMA causal as well because the polynomial Φ(x) has both roots outside the complex unit circle.
d) The process is ARMA invertible because Θ(x) has all roots outside the complex unit circle
e) Figure 7 and Figure 8 show the plots for the theoretical acf and pacf respectively

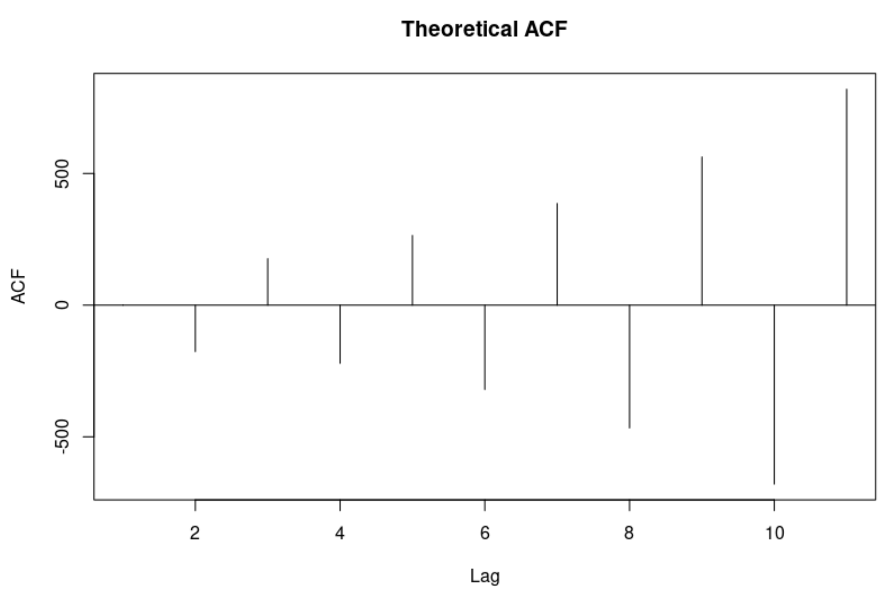

Figure 7

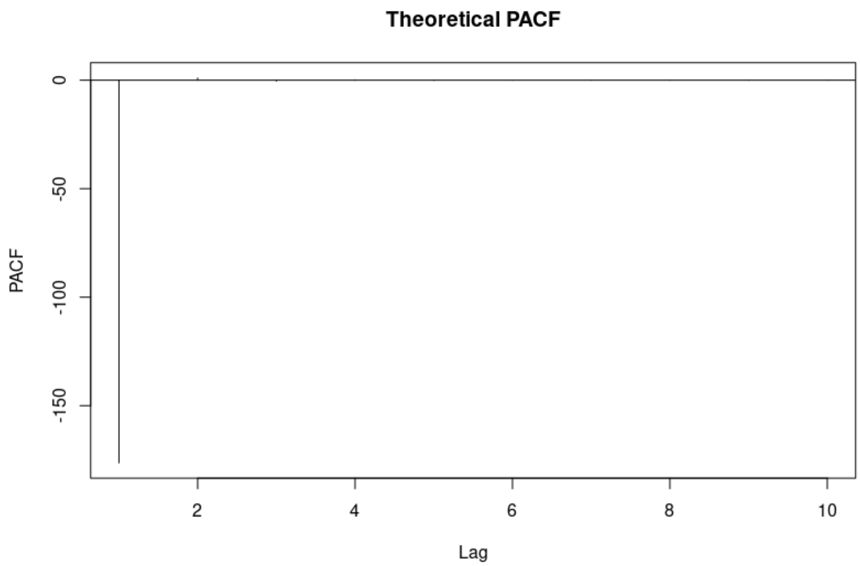

Figure 8

2. $X_t = 0.6X_{t-1} + 0.4X_{t-2} + Z_t + Z_{t-1}$

a) Determine the order p, d and q of the ARIMA process.

$X_t - 0.6X_{t-1} - 0.4X_{t-2} = Z_t + Z_{t-1}$

$\Phi(x) = 1 - 0.6x - 0.4x^2$
$x_1 = -2.5$
$x_2 = 1$

The polynomial has root $x_1 = -2.5$ lies outside the complex unit circle and $x_2 = 1$
$p = 1, d = 1$

$\Theta(x) = 1 + x$
$x_1 = -1$
The above polynomial has its root inside the complex unit circle, therefore $q = 0$

The resulting model is ARIMA(1,1,0)

b) The process is not stationary as the polynomial $\Phi(x)$ has roots inside the complex unit circle.

c) The process is not an ARMA causal process because the polynomial $\Phi(x)$ has roots inside the complex unit circle.

d) The process is not an ARMA invertible process as the polynomial $\Theta(x)$ has its root inside the complex unit circle

3. $X_t = X_{t-1} + Z_t + 0.49Z_{t-2}$

a) Determine the order p, d and q of the ARIMA process.

$X_t - X_{t-1} = Z_t + 0.49Z_{t-1}$

$\Phi(x) = 1 - x$
$x = 1$

The polynomial has root $x_1 = 1$ lies
inside the complex unit circle
$p = 0,\ d = 1$

$\Theta(x) = 1 + 0.45x^2$
$x_1 = +1.42857i$
$x_2 = -1.42857i$

The above polynomial has its root outside the complex unit circle,
therefore $q = 2$

The resulting model is ***ARIMA(0,1,2)***

b) The process is not stationary because the polynomial Φ(x) has root inside the complex unit circle.
c) The process is not an ARMA casual process as the polynomial Φ(x) has its root inside the complex unit circle.
d) The process is an ARMA invertible process as the polynomial Θ(x) has its root outside the complex unit circle.

4. $Xt = -2.5X_{t-1} + 6X_{t-2} + Z_t - Z_{t-1} + 0.16_{t-2}$

a) Determine the order p, d and q of the ARIMA process.

$X_t + 2.5X_{t-1} - 6X_{t-2} = Z_t - Z_{t-1} + 0.16X_{t-2}$

$\Phi(x) = 1 + 2.5x = 6x^2$
$x_1 = -0.25\,,\ x_2 = 0.6666$

The polynomial has both of its root lying
inside the complex unit circle
$p = 0,\ d = 2$

$\Theta(x) = 1 - x + 0.16$
$x_1 = 5$
$x_2 = 1.25$

The above polynomial has its root outside the complex unit circle,
therefore $q = 2$

The resulting model is ***ARIMA(0,2,2).***

b) The process is not stationary because the polynomial Φ(x) has its root inside the complex unit circle.
c) The process is not an ARMA casual process as the polynomial Φ(x) has its root inside the complex unit circle.
d) The process is an ARMA invertible process as the polynomial Θ(x) has its root outside the complex unit circle.