Homework 8

PSTAT 5A: Spring 2023, with Ethan P. Marzban

Instructions

- Please submit your work to Gradescope by no later than **11:59pm on Wednesday, May 31**. As a reminder, late homework will not be accepted.
- Recall that you will be asked to upload a **single** PDF containing your work for *both* the programming and non-programming questions to Gradescope.
  - You can merge PDF files using either Adobe Acrobat, or using adobe's online PDF merger at this link.

Caution

Be aware that some parts may be easier (or, in fact, may *need* to be) computed using Python. If you do use Python for any part, please write down the code you used.

Problem 1: Look at All Those Chickens!

The average weight of an adult male chicken is claimed to be 5.7 lbs. A representative sample of 36 adult male chickens is taken, and it is found that the weights of these sampled chickens have an average of 6.1 lbs and a standard deviation of 0.9 lbs. Suppose that we wish to test the original claims (that the true average weight of an adult male chicken is 5.7 lbs) against a two-sided alternative.

a. Define the parameter of interest.
b. State the null and alternative hypotheses in terms of the parameter you defined in part (a).
c. What distribution do we use when performing our hypothesis test? Be sure to include any/all relevant parameter(s)!
d. Compute the value of the test statistic.
e. Compute the p-value of the test statistic.
f. Conduct the test using the p-value value, and state the conclusions of the test in the context of the problem.
g. Compute the critical value of the test.
h. Conduct the test using critical value, and state the conclusions of the test in the context of the problem.
i. Redo the test, now using an $\alpha = 0.01$ level of significance. Do your conclusions change? If so, state the new conclusions in the context of the problem.

Problem 2: Turn On the Light

GauchoBrite-brand lightbulbs are claimed to burn with an average wattage of 60 Watts. In actuality, the distribution of wattages across all *GauchoBrite*-brand lightbulbs is known to be roughly normal with a standard deviation of 27 Watts. A representative sample of 25 lightbulbs was taken; these 27 lightbulbs had a combined average wattage of 57 Watts.

a. Define the parameter of interest.

b. State the null and alternative hypotheses in terms of the parameter you defined in part (a).

c. What distribution do we use when performing our hypothesis test? Be sure to include any/all relevant parameter(s)!

d. Compute the value of the test statistic.

e. Compute the p-value of the test statistic.

f. Conduct the test using the p-value value, and state the conclusions of the test in the context of the problem.

g. Compute the critical value of the test.

h. Conduct the test using critical value, and state the conclusions of the test in the context of the problem.

Problem 3: Drinking Water

City officials of *Gauchonia* believe that 15% of households in *Gauchonia* have slightly elevated levels of fluoride in their drinking water. To test this claim, a representative sample of 375 households is taken. It is found that 13.6% of households in this sample have elevated levels of fluoride in their drinking water.

a. Check that the success-failure conditions are met.

b. Assuming the null is correct, what is the distribution of the test statistic?

c. Suppose the city officials wish to test their claims against a two-sided alternative at an $\alpha = 0.05$ level of significance. Compute the p-value of the test statistic, and use this to form a conclusion. Be sure to state your conclusion in the context of the problem.

d. Now, suppose the city officials wish to test their claims against a lower-tailed alternative, still at an $\alpha = 0.05$ level of significance. Compute the p-value of the test statistic, and use this to form a conclusion. Be sure to state your conclusion in the context of the problem.

Problem 4: Programming

Part (a): Recap of LaTeX Syntax

> **❗ Task 1**
>
> First, add a second-level header that says `Task 1`. Then, typeset the following set of equations into a Markdown Cell. Pay very close attention to the alignment of equations, and make sure your parentheses display correctly. (Also, you may need to look up how to place a box around text in LaTeX)
>
> $$\begin{aligned} \mathbb{P}(4 \leq X \leq 7) &= \mathbb{P}(X \leq 7) - \mathbb{P}(X \leq 4) \\ &= \mathbb{P}\left(\frac{X-3}{1.4} \leq \frac{7-3}{1.4} \right) - \mathbb{P}\left(\frac{X-3}{1.4} \leq \frac{4-3}{1.4} \right) \\ &= \mathbb{P}\left(\frac{X-3}{1.4} \leq 2.86 \right) - \mathbb{P}\left(\frac{X-3}{1.4} \leq 0.71 \right) \\ &= 0.9979 - 0.7611 = \boxed{0.2368} \end{aligned}$$

Part (b): Numbered Equations

We have not yet talked about how to number equations in LaTeX. The syntax for creating a numbered equation is:

```
\begin{equation}{
  <whatever equation you want}
}\end{equation}
```

For example,

$$f_X(x) = x^2 \tag{1}$$

was created using the syntax

```
\begin{equation}
  f_X(x) = x^2
\end{equation}
```

> **❗ Task 2**
>
> Create a labeled equation (you can use whatever equation you want) in a new Markdown Cell.

> **Note**
>
> Your equation will not appear with a number in your `.ipynb` file; the equation number will only display in your final `.pdf`.

One of the benefits of labeling your equations is that you can reference them later! To create a labeled equation that is referable, use the syntax

```
\begin{equation}{\label{eq:<name>}
  <your equation>
}\end{equation}
```

Then, to reference the equation later, use `\ref{eq:<name>}` where `<name>` is whatever you called your equation. For example:

$$a^2 + b^2 = c^2 \tag{2}$$

was created using

```
\begin{equation}{\label{eq:pyth}}
  a^2 + b^2 = c^2
\end{equation}
```

meaning I can reference equation (2) using the code `\ref{eq:pyth}`.

> **! Task 3**
>
> Create a labeled equation (you can use whatever equation you want) in a new Markdown Cell that is labeled, and then refer to the equation in a markdown cell underneath (just like we did above).

> **Note**
>
> Again, neither the equation number nor the referenced equation number will appear in your `.ipynb` file; they will only appear in your final `.pdf`.