We propose a simple model for how the fly sets its path in the post-activation period, which involves comparing a noisy accumulated distance estimate to a stored value.

We model time in discrete steps $t_0, t_1, t_2 \ldots t_k$, starting from t_0, the time of the last food encounter. We assume a constant walking velocity v; that is, the position of the fly updates each time step according to $\theta(t_k) = v \cdot (t_k - t_{k-1})$.

We posit that after each reversal, the fly begins with a reference value, d^*, most simply corresponding to a fixed oscillation range. It stores an accumulated value, d_{est}, which starts at $d_{est}(t_{r_{i-1}}) = 0$, where $t_{r_{i-1}}$ is the timestep of the $i-1$th reversal.

d_{est} updates each time step according to

$$d_{est}(t_k) = d_{est}(t_{k-1}) + v(t_k - t_{k-1}) + \eta_k$$

where η_k represents noise in the accumulation of a traveled distance estimate. The distribution of η_k can depend on $d_{est}(t_{k-1})$, or can be independent and consistent each time step.

With each new update of $d_{est}(t_k)$, the fly compares whether $d_{est}(t_k) > d^*$, and performs a reversal if so, which marks a t_{r_i}, the time of the ith reversal, and a reversal location $\theta(t_{r_i})$. When it reverses, d_{est} is set to 0 and the process begins again.

For every reversal after the first post-food reversal, we have

$$d^* = r_0 + |\theta_{F_1} - \theta_{F_2}| + c_0$$

that is, the reference value is the sum of the inter-food distance $|\theta_{F_1} - \theta_{F_2}|$, plus an overshoot distance c_0, plus r_0. r_0 is the distance travelled from the last food encounter to the first reversal.

It may be the case that after the first reversal, d^* is instead stored as the $|\theta(t_{r_i}) - \theta(t_{r_{i-1}})|$, the previous inter-reversal distance–these two are indistinguishable we the data we have.

Each path can be summarized as a series of reversal locations, $\{\theta(t_{r_0}), \theta(t_{r_1}), \ldots \theta(t_{r_i}) \ldots \theta(t_{r_n})\}$. We seek to demonstrate that the frequencies of reversal locations $\{\theta(t_{r_i})\}$, which we observe in the data, can be accounted for with the model described.

We've already demonstrated this loosely. Now we describe a simple construction for η_k to achieve model-data correspondence.

The simplest construction is to have

$$\eta_k \sim \mathcal{N}(0, \sigma^2)$$

for all k.