Temporary lack of self-knowledge in two-sided search and agents selectivity

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Abstract

As technologies immerse, the number of new matching technologies such as platforms for partner or job search grow. However, some people may not fully understand how attractive they would be upon entering the new markets and the behaviour resulting from their unawareness may affect how selective other agents are. I propose a model of two-sided search to investigate how temporary unawareness of own attractiveness lasting for one period effects matching equilibrium and agents selectivity. I find that multiple equilibria with assortative property can arise in such setting and show that agents with high level of attractiveness become more selective as the fraction of people lacking self-knowledge in the market rises.

Keywords: Two-sided matching, Dating market, Matching with frictions

INTRODUCTION

People face a lot of uncertainty about the world, other people's actions and the future every day. However, some degree of uncertainty may come from people themselves as people do not always possess perfectly complete information about their attractiveness, the degree of their skills and abilities. This lack of understanding oneself may introduce different frictions in such markets as a dating market, job market and other matching markets, where matching equilibrium and utility of all agents depends on their understanding of their own type or quality of fit in the case of school and job application markets.

For illustration, with the immersion of technology, more and more people nowadays are using online dating apps in order to search for the other half. The number and the variety of such technologies grow and new entrants of such platforms may not instantly understand how attractive they are to other agents upon entrance: e.g. if all agents assign their preferences over their potential match based on a picture or a short profile on the app, new users may not instantaneously understand how attractive their appearance on the app is to others. The same issue can also be faced by agents moving in the new environments or culture from the one familiar to them, where the preferences of people is not clear and agents may have from very limited to no understanding of their chances in this new dating market due to the lack of understanding about how are they perceived by the others.

Furthermore, new entrants of the job market such as recent university graduates who apply for the job may not understand the level of attractiveness of their resumes to the employers on a certain app or in the certain area. Similar logic is applicable to the high school graduates applying to colleges, who as well might not understand what exactly each school they apply to finds attractive in the application.

Consequently, in the discussed settings it appears that new agents of matching markets may not have any knowledge at all about the level of their attractiveness, however, they may quickly learn the technology and rules of the certain environment and understand their type in the first periods after entrance due to the accessibility of various information resources that is present today. Thus, after learning the peculiarities of the new dating apps or studying the resumes of someone who recently got the job agents may quite quickly understand where they stand in the market in terms of the attractiveness. Nevertheless, their actions even in the short period of unawareness may affect how selective other people on the market with the perfect self-knowledge are. The selectivity of agents is an important factor to consider because it determines the chances of people to get the best outcome: for smart people to get the best job placement and for agents to marry the best possible partner.

Understanding the consequences and the magnitude of the effects of the presence of unaware agents on the market is crucial. Thus, based on the effects on selectivity of the agents some dating apps and other matching platforms designed to match employers with employees for example can manipulate the level of difficulty of the app technology or on contrary put more effort in making those apps more accessible to stimulate better matches.

Most of the papers on the topic of two-sided serach in matching markets focus on the fact that equilibrium match is assortative, but not on to what extent it is so. Moreover, most papers assume that agents possess the complete information about themselves. Thus, in this work I consider a model of two-sided sequential search in the matching market where part of the agents do not know their types for one period and in the following period they learn they attractiveness. This setup introduces a new information friction in the matching market in the form of agents who don't know anything about themselves. The main question I aim to answer is how agents selectivity would react to the changes in the number of unaware participants in the market.

I first show that there can be multiple segregation equilibria where all agents marry in classes such as in the well-studied case without such information frictions. Then I determine the connection between the share of agents who are unaware of their attractiveness and other agents selectivity and show that very attractive agents become more selective.

Related Literature

There is a large body of literature devoted to marriage market, two-sided search and matching with frictions.

First set of papers considers models with focus primarily on search frictions in the marriage markets. Papers such as Burdett and Coles (1997), model search frictions through the discounting factor, meaning that agents don't like to wait forever for the perfect partner and the authors show that agents marry only within classes. Smith (2006) also shows that assortative matching arises in the setting of two-sided search with frictions but under transferable utility.

Chade (2001) demonstrates that the assortative property of equilibrium holds even if search costs are introduced in the additive form instead of through discounting factor and discusses the role of modeling search costs for this assortative equilibrium to arise. Moreover, he focuses on the duration of search and derives the condition for a monotonic relationship between agents type and the time she needs to spend on a market before finding a partner.

Some models as in Adachi (2003) utilize the concept of reservation utility where agents only marry if marriage results in a higher utility that staying single. In this setting, Adachi (2003) links the outcomes of the two-sided search matching to the outcomes of Gale-Shapley mechanism. He demonstrates that if the search frictions vanish, set of equilibrium outcomes in a search model coincides with the set of stable matchings in a corresponding Gale-Shapley marriage problem.

As for other frictions in the two-sided search setting, Chade (2006) studies information frictions which take the form of agents perfectly observing own type and not observing the type of a potential match: in this setting agents do not observe a partner's real type immediately, but rather a noisy signals of that type. Chade (2006) established a positive assorting property of equilibrium in a stochastic case and introduced a concept of acceptance curse, meaning that when agents are accepted by those from the other side of the market, their estimate of a type of a potential partner reduces, so selection is adverse in the sense. Similarly, Antler et al. (2022) build a model of matching-with-search-frictions market with nontransferable utility where agents can still observe each others' type upon meeting, but can't observe actual compatibility, so when agents meet, they don't marry immediately, but they date for some time, during which they learn compatibility with each other and only then they decide whether they want to marry or separate and return to the dating market. Thus, Antler et al. (2022) account for the possibility of divorce and returning to the marriage market after some time. Their results suggest that in this setting matching does not always result in the assortative equilibrium where marriages only happen within classes of agents types and that agents date before marriage for a longer periods of time that it is socially-optimal.

There are also some interesting results in two-sided search with behavioral frictions setting. Antler and Bachi (2022) claim that modern technology and dating apps in particular are making search frictions almost negligible and thus, they derive that the share of agents who marry in a symmetric equilibrium converges to zero as market frictions do so. Antler and Bachi (2022) attribute this finding to the fact that agents have coarse reasoning about the extent to which agents from the other side might like them, and therefore they exceed they expectations about the potential match and never marry, so when search frictions are combined with the behavioural frictions, the equilibrium where all agents marry in finite time within certain clusters does not exist anymore.

The most similar to my setting are the works Maruyama (2013) and Maruyama (2018). In both of these papers, author studies two-sided search setting introduced in Burdett and Coles (1997), but under the assumption that agents do not fully understand the level of their attractiveness, they only receive a signal about their type. In both papers the assumption about the finite number of types is introduced: in Maruyama (2013), it is assumed that the number of agents types is limited to two while in Maruyama (2018) this number is n, but only one side of the market lacks knowledge about their type. The results in Maruyama (2018) suggest that agents who lack knowledge about their own type gradually decrease their acceptance threshold over time: they decrease their thresholds after receiving a rejection from the agents on the other side, but don't raise it after receiving acceptance.

The Model

I consider a two-sided search model of the marriage market with nontransferable utility with search frictions in the form of discounting utility over time and with the following information frictions: some agents don't know their type for one period and then they learn it. In this setting agents from two sides of the market meet and can marry or continue search. The set up of the model is described in detail below.

Time: Time is discrete and each period's length equals 1

Agents: There are two sets of agents: set of men and set of women, each of which contains a unit mass of agents. Agents all possess a certain characteristic that defines their attractiveness for other agents. This characteristic is described by a number v, which is independently distributed on $[\underline{v}, \bar{v}]$ according to an atomless distribution $F(.)$ with density $f(.)$. Following Burdett and Coles (1997) and Antler and Bachi (2022), I further refer to this number as agent's pizzazz.

Marriage game: In each period agents from two sides of the market randomly meet in pairs. They perfectly observe the pizzazz of the agent they meet and then they simultaneously announce *Accept* or *Reject*. If both agents announce *Accept*, they marry and leave the market, otherwise they stay in the market and continue the search. I also assume that if the agent is indifferent between announcing *Accept* and *Reject*, she always chooses *Accept*.

Payoffs: If agents marry, they receive each other's pizzazz as payoff: e.g. if a woman with pizzazz w marries a man with pizzazz v, woman receives v and man w. Otherwise the utility of unmatched agent is zero. Agents also discount their future utility with the discount factor $0 < \beta < 1$

Replenishment: If the agents marry and leave the market, I assume they are replaced with the agents with the same pizzazz, so the distribution of pizzazzes is invariant over time. They can be replaced either by the agent who doesn't possess any information about their own pizzazz, but who still knows the distribution of agents pizzazzes and in the second period of agent's life in the market she learns her type and perfectly observes it throughout the entire game with probability γ, which is the same in every period of the game; or she can be replaced by the agent who perfectly observes her type with probability $1-\gamma$, so the fraction of agents who is replaced by agents not knowing their types is constant in every period and known to all agents in the market. Since I'm mostly interested in studying the effects of the number of agents without knowledge about their pizzazzes on agents strategies and selectivity, I utilize this replenishment assumption. Chade (2001), Chade (2006), Eeckhout (1999), Burdett and Wright (1998) and Rubinstein and Wolinsky (1985) also use replenishment assumptions in their works to avoid difficult prove of existence of equilibrium and its characterization. One could use the setting, where agents with certain types arrive to the market according to Poisson process, which is done in Burdett and Coles (1997) and Antler and Bachi (2022) for example, but it would make explicit analysis of the properties of equilibrium and agents optimal threshold more complicated.

Strategies: In my analysis I focus on equilibria in stationary strategies to simplify the analysis. A stationary strategy for agent with pizzazz v who knows her type is given by $\sigma_v(.) : [\underline{v}, \bar{v}] \rightarrow \{1, 0\}$, which is a mapping from pizzazzes of agents on the other side of the market to a decision $1 -$ to accept, or $0 -$ to reject. Strategy for an agent who doesn't know her type is defined by the set of $\{\delta(.), \sigma_v(.)\}$, with $\delta(.) : [\underline{v}, \bar{v}] \rightarrow \{1, 0\}$ and $\sigma_v(.) : [\underline{v}, \bar{v}] \rightarrow \{1, 0\}$ in the similar manner: in the period without information about own types, all such agents behave identically and use the rule $\delta(.)$ to make their decisions about a match with a partner in front of them, whose type they can observe. Then, after learning their type, they start using the rule $\sigma_v(.)$ according to their pizzazz v as the other agents who know their type. Then we can define the sets of agents, for whom $\sigma_v(.) = 1$ and $\delta(.) = 1$: let A_v be the set of agents that agent with pizzazz v is willing to accept as a partner: $A_v(\sigma) = \{x | \sigma_v(x) = 1\}$. For an agent v, who doesn't know her type this set is $B : B(\delta) = \{x | \delta(x) = 1\}$. Consequently, Ω_v and Θ_v are the set of agents who accept v (who knows her type) as a partner among those who know and don't know their type respectively and these sets are defined as $\Omega_v(\sigma) = \{x | \sigma_x(v) = 1\}$ and $\Theta_v(\delta) = \{x | \delta(v) = 1\}$.

Equilibrium:

Define $U(v, \sigma_v, \delta)$ to be the expected discounted perceived life-time utility of an agent v, who knows her type and $W(\sigma_v, \delta)$ - of the agent who doesn't know her type. An equilibrium in the marriage game is defined by the set of stationary strategies $\{\sigma_v^*(.), \delta^*(.)\}$ such that for any other $\sigma_v'(.)$ and any v in $[\underline{v}, \bar{v}]$

$$U(v, \sigma_v', \delta^*) \leq U(v, \sigma_v^*, \delta^*) \tag{1}$$

and for any other $\delta'(.)$ and any v in [0,1]

$$W(\sigma_v^*, \delta') \leq W(\sigma_v^*, \delta^*) \tag{2}$$

Expected utility of search:

Every period there is a stationary parameter $\alpha(\gamma)$, which captures the fraction of agents not knowing their type currently participating in the market (As strategies are stationary, the fraction of people marrying in each period should also be so). Notice that α depends on agents strategies as it depends on the probabilities of agents to get married: the more agents leave the market, the more new unaware agents can arrive. So, α depends on agents strategies in a way that with some probability, which is determined by all agents acceptance cutoffs, some of the agents who don't know their types marry and are replaced by the agents with the same type who also don't possess any information about their pizzazz due to the replenishment assumption with probability γ. Further I omit writing $\alpha(.)$ with the dependence on the strategies and γ, θ and simply write α.

Expected discounted perceived life-time utility of an agent v, who knows her type is

$$\begin{aligned} U(v, \sigma_v, \delta) = \alpha \int_{A_v \cap \Theta_v} \frac{x}{1-\beta} f(x)\, \mathrm{d}x + (1-\alpha) \int_{A_v \cap \Omega_v} \frac{x}{1-\beta} f(x)\, \mathrm{d}x + \\ + \beta U(v) \left(\alpha (1 - \int_{A_v \cap \Theta_v} f(x)\, \mathrm{d}x) + (1-\alpha)(1 - \int_{A_v \cap \Omega_v} f(x)\, \mathrm{d}x) \right) \end{aligned} \tag{3}$$

Upon a match with x, agent receives a total discounted utility of $\frac{x}{1-\beta}$ over her lifetime. Thus, $\alpha \int_{A_v \cap \Theta_v} \frac{x}{1-\beta} f(x)\, \mathrm{d}x$ and $(1-\alpha) \int_{A_v \cap \Omega_v} \frac{x}{1-\beta} f(x)\, \mathrm{d}x$ define the expected payoffs conditional on finding a match among the ones who don't know their types and who do divided by the corresponding probabilities of the match, which are given by $\int_{A_v \cap \Theta_v} f(x)\, \mathrm{d}x$ and $\int_{A_v \cap \Omega_v} f(x)\, \mathrm{d}x$ respectively.

The utility in (3) can be further rewritten as

$$U(v, \sigma_v, \delta) = \frac{\frac{\alpha}{1-\beta} \int_{A_v \cap \Theta_v} x f(x)\,\mathrm{d}x + \frac{1-\alpha}{1-\beta} \int_{A_v \cap \Omega_v} x f(x)\,\mathrm{d}x}{\frac{1-\beta}{\beta} + \alpha \int_{A_v \cap \Theta_v} f(x)\,\mathrm{d}x + (1-\alpha) \int_{A_v \cap \Omega_v} f(x)\,\mathrm{d}x} \tag{4}$$

Further for simplicity instead of $\frac{1-\beta}{\beta}$ I write η in the denominator.

For now, let's simply define the discounted perceived life-time utility of an agent who doesn't know her type as W.

Following Antler and Bachi (2022), I assume agents to be using threshold strategies and define $\hat{a}_v$ to be the threshold of a person knowing her type and $\hat{b}$ to be the threshold of an agent who doesn't. Then, $A_v = [\hat{a}_v, \bar{v}]$, $B = [\hat{b}, \bar{v}]$. Finally, define U^* as the expected utility of the search using an optimal strategy of the agent with pizzazz v, who knows her type. Then, $\hat{a}_v = U^*(v)$ as agents accept only those in equilibrium, whose types exceed their expected value from continuing the search.

Before continuing with the analysis, it is important to acknowledge the case of $\alpha = 0$ and state a well-known benchmark result, obtained in various works on the topic such as Bloch and Ryder (2000), Chade (2001) and Burdett and Coles (1997) and briefly mentioned in the literature review. I provide it below without the proof.

Claim 1. *When $\alpha = 0$, there exists a unique partition equilibrium: $[\underline{v}, \bar{v}]$ is divided into finite number of classes by some numbers $0 = v^1 < v^2 .... < v^{N-1} < v^N = 1$ in such way that every agent of type in $[v^i, v^{i+1})$ uses cutoff v^i and agents marry within their classes.*

Claim 1 ensures that all agents marry in finite time and demonstrates the assortative matching in equilibrium. Moreover, in the special case of no search frictions when $\beta = 1$, agents marry agents on the other side of the market with the same pizzazz and there exists a unique stable matching (Bloch and Ryder (2000)).

Returning to the case with information frictions ($\alpha > 0$), we first need to establish the expression for the threshold of agents not knowing their type.

Proposition 1. $\hat{b} = \mathbb{E}[U^*(v)]$*, where expectation is taken with respect to v.*

See Appendix.

The following proposition demonstrates that the threshold of agents with full information about themselves are increasing in their pizzazz.

Proposition 2. *$\hat{a}_v$ is non-decreasing in v*

See Appendix.

Define $a_v = sup(\Omega_v)$ - the agent with the biggest pizzazz an agent with pizzazz v can get in a match among the ones who know their type and similarly $b_v = sup(\Theta_v)$. Now as we have shown that $\hat{a}_v$ is non-decreasing in v, then all agents among those who know their type with pizzazzes below a_v accept v and those with pizzazz above a_v reject v.

Thus, we can rewrite U^*, using the equation obtained in (4) as

$$U^*(v) = \frac{\frac{\alpha}{1-\beta}\int_{\hat{a}_v}^{b_v} xf(x)\,\mathrm{d}x + \frac{1-\alpha}{1-\beta}\int_{\hat{a}_v}^{a_v} xf(x)\,\mathrm{d}x}{\eta + \alpha[F(b_v) - F(\hat{a}_v)] + (1-\alpha)[F(a_v) - F(\hat{a}_v)]} \tag{5}$$

Notice that as the threshold $\hat{b}$ is the same for all agents who don't know their type, then for agents with $v \geq \hat{b}$ we have $b_v = 1$ and for $v < \hat{b}$ we have $b_v = 0$.

Thus, for agents who know their type with pizzazz $v \geq \hat{b}$ (define those *high-types*) life-time discounted utility under optimal strategy is given by

$$U_h^*(v) = \frac{\frac{\alpha}{1-\beta}\int_{\hat{a}_v}^{\bar{v}} xf(x)\,\mathrm{d}x + \frac{1-\alpha}{1-\beta}\int_{\hat{a}_v}^{a_v} xf(x)\,\mathrm{d}x}{\eta + \alpha[1 - F(a_v)] + [F(a_v) - F(\hat{a}_v)]} \tag{6}$$

and for those with pizzazz $v < \hat{b}$ (define those *low-types*) it can be written as

$$U_l^*(v) = \frac{\frac{1-\alpha}{1-\beta}\int_{\hat{a}_v}^{a_v} xf(x)\,\mathrm{d}x}{\eta + (1-\alpha)[F(a_v) - F(\hat{a}_v)]} \tag{7}$$

From (4) since for them set Θ_v is empty. Notice that the monotonicity of thresholds implies that there exist such x and $\hat{a}_x$ that for all y such that $\hat{a}_y \geq \hat{a}_x$ the following holds: $y \geq \hat{b} = \mathbb{E}[U^*(v)]$, so those agents are accepted by the agents not knowing their pizzazz and for all $\hat{a}_y < \hat{a}_x$ we would have $y < \hat{b} = \mathbb{E}[U^*(v)]$. Define this $\hat{a}_x$ as a^*. Hence, we can write the problem of a high-type agent with pizzazz v knowing her type: she chooses her acceptance cutoff maximizing her expected life-time discounted utility:

$$U_h^*(v) = \max_{a^* \leq \hat{a}_v \leq a_v} \frac{\frac{\alpha}{1-\beta}\int_{\hat{a}_v}^{\bar{v}} xf(x)\,\mathrm{d}x + \frac{1-\alpha}{1-\beta}\int_{\hat{a}_v}^{a_v} xf(x)\,\mathrm{d}x}{\eta + \alpha[1 - F(a_v)] + [F(a_v) - F(\hat{a}_v)]} \tag{8}$$

Here the condition for an agent to be a high-type ($v \geq \hat{b}$) is rewritten in the form of thresholds as $a^* \leq \hat{a}_v$. Note that agent v maximizes her utility given the optimal strategies of other agents, thus as the distribution of pizzazzes v is continuous, single agent can determine whether she is high or low type, since other agents strategies determine $\mathbb{E}[U^*(v)]$.

Similarly, a low-type agent with pizzazz v solves

$$U_l^*(v) = \max_{\substack{0 \leq \hat{a}_v \leq a_v \\ \hat{a}_v < a^*}} \frac{\frac{1-\alpha}{1-\beta} \int_{\hat{a}_v}^{a_v} x f(x)\,\mathrm{d}x}{\eta + (1-\alpha)[F(a_v) - F(\hat{a}_v)]} \tag{9}$$

Before moving on to characterizing the solutions of agents maximization problems in (8) and (9), first let us show that the constraint $\hat{a}_v \leq a_v$ never binds in equilibrium.

Proposition 3. *In equilibrium $\hat{a}_v < a_v$*

See Appendix.

The following proposition establishes the existence of a unique solution of problems in (8) and (9).

Proposition 4. *If $\hat{a}_v > a^*$ there is a unique solution $\hat{a}_v$ to the problem given by (8). If $\hat{a}_v < a^*$ there is a unique solution $\hat{a}_v$ to the problem given by (9).*

See Appendix.

Proposition 4 defines the equations characterizing equilibrium thresholds for high and low types respectively, which are given by:

$$\alpha \int_{a_v}^{\bar{v}} (x - \hat{a}_v) f(x)\,\mathrm{d}x + \int_{\hat{a}_v}^{a_v} (x - \hat{a}_v) f(x)\,\mathrm{d}x - \eta \hat{a}_v = 0 \tag{10}$$

when $\alpha \int_{a_v}^{\bar{v}} (x - a_v) f(x)\,\mathrm{d}x - \eta a_v < 0$ and $\alpha \int_{a_v}^{\bar{v}} (x - a^*) f(x)\,\mathrm{d}x + \int_{a^*}^{a_v} (x - a^*) f(x)\,\mathrm{d}x - \eta a^* > 0$ and otherwise optimal threshold is a^* for high-type agents and

$$\int_{\hat{a}_v}^{a_v} (x - \hat{a}_v) f(x)\,\mathrm{d}x - \eta \hat{a}_v = 0 \tag{11}$$

when $\int_{a^*}^{a_v}(x-a^*)f(x)\,\mathrm{d}x - \eta a^* < 0$ and a^* otherwise for the low-type agents. Notice that optimal thresholds of low-type agents do not depend on α, only in a sense that α determines whether agent will fall in a low-type category.

Now we can show that there are segregation equilibria, where agents marry within classes as in $\alpha = 0$ case.

Theorem 1. *There can exist multiple segregation equilibria where marriages take place within each class - a unique partition of $[\underline{v}, \bar{v}]$ given by some numbers $0 = v^1 < v^2 .... < v^{N-1} < v^N = 1$ in such way that every agent who knows her type in $[v^i, v^{i+1})$ uses cutoff v^i and agents marry within their classes. Agents who don't know their type use $\hat{b} = \mathbb{E}[\hat{a_v}]$ as their threshold, where expectation is taken w.r.t. all possible values of v.*

See Appendix.

We see that under the setting with information frictions, where some agents don't know their type, agents also marry in clusters and equilibria have an assortative property. However, when for high-type agents everyone from their clusters are available since they are accepted by those who don't know their pizzazzes, low-type agents are not accepted by those agents. Thus, even though they accept everyone from the cluster they belong to and can marry only agents from that cluster, some of those agents may not know their type and not accept them. Thus, they can't always mate with all types in their cluster and they become worse off as the marriage time might take longer especially under high values of α.

Following theorem establishes a relationship between some agents selectivity and α.

Theorem 2. *In equilibrium partition, the boundaries of agents with $v \geq \hat{b}$ are increasing in α, while the boundaries of agents with $v < \hat{b}$ are decreasing in α*

See Appendix.

Theorem 2 provides the main result of the analysis: agents with high pizzazz become more selective in the presence of agents who don't know anything about themselves. This makes those who are also of high type worse-off since their best possible match is also shrinking in response.

Notice that low-type agents become less selective on contrary, but there might arise different equilibria, when with change of alpha more agents become high or low types. It is difficult to explicitly derive the change in a^* as the cluster boundaries are defined implicitly and the cluster boundaries of high types might either increase more that the boundaries of low type decrease or less in response to an increase in α, which can result in different position of a^* since it is defined as a threshold of an agent with pizzazz $\mathbb{E}[U^*(v)]$.

The analysis of the properties that the distribution of pizzazzes and β need to satisfy is tough due to the presence of the expectation term in the analysis. While it is possible to derive conditions for $F(v^i) - F(v^{i+1})$ to be increasing in types, where v^i, v^{i+1} are one equilibrium class boundaries, as was done in Chade (2001) for example to gain insights about clusters size comparison, the exact boundaries needed to be assessed for the understanding of the exact position of $\mathbb{E}[\hat{a_v}]$ on the set of types, which are hard to derive explicitly.

Conclusions

In this work I analysed a model with temporary lack of self-knowledge of some fraction of the agents in two-sided search matching market setting. While some previous work only considered market with learning when analysing imperfect self-knowledge, I focused on the specific case where agents do not anything about themselves and do not receive any signal about their type upon entering the market, but they quickly learn their type and proceed with the search as the agents with full information about the level of their attractiveness. New technologies allowing people to match online and meet people with different cultures reinforce the reasonableness of such assumption, as people can learn the peculiarities of the market and where they stand in it rather quickly given the availability of various information resources.

My results suggest that even under the replenishment assumption, which usually yields the unique matching equilibrium like in Chade (2001), Chade (2006) and Antler and Bachi (2022), in the setting with agents lacking their own pizzazzes multiple segregation equilibria can arise. In all those equilibria, agents are partitioned in classes and marry with types lying only within the class their pizzazz belongs to and thus possess the assortative property. However, those agents with pizzazzes below the expected value of all thresholds among people of all types (with the full information about themselves) can't always get a match with all types in their class due to the the presence of people without knowledge about themselves, who do not accept agents with types below the expected value of thresholds, thus they are worse off as the period of search might take longer for them under large values of α.

As for the selectivity of agents, my results suggest that in all matching equilibria agents with high pizzazzes become selective as the fraction of lacking self-knowledge agents increases. However, the boundaries of such high types can vary among equilibria, thus under an assumption of general form of distribution different sets of agents can become more selective after such increase due to the existence of multiple equilibria. For the same reason different agents might turn out to be low types, who become less selective in the presence of more self-unaware agents on the market. They are thus ensuring their marriage as some people from their cluster don't accept them anymore due to lack of self-knowledge. This results suggest, that in general agents with very high pizzazzes give less chances to those with lower and agents of lower types thus end up having worst prospects in the matching market when the fraction of agents not knowing their type increases. Furthermore, agents with low pizzazzes now accept more people in their partner and they have to do so to avoid ending up alone. These results have potential for practical policy implementation as people with average pizzazzes would have worse chances of getting matched to one of the best partner or get into one of the best schools. Thus, if we are talking about job and college applications, the designers of the apps should devote more time to simplify their technology as much as possible to minimize the fraction of agents lacking self-knowledge if they want to achieve diversity.

As for the further directions of research, some condition on the distribution of pizzazzes and discounting factor could potentially be derived in the future to better understand the exact structure of each arising matching equilibrium and potentially prove the uniqueness of it under more restrictive assumptions. Moreover, one could extend the model by dropping the replenishment assumption and model the matching market where the entrance of new agents happens in the flow with some given rule and thus study the properties of matching equilibria and agents changes in their level of selectivity.

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APPENDIX

Proof of Proposition 1:

Agent who doesn't know her type faces the following problem: after observing the pizzazz w of the agent from the other side of the market, she needs to decide, whether to accept or reject her and continue search.

Her expected payoff if she accepts is given by $P(\text{w accepts})w + P(\text{w rejects})\mathbb{E}[U^*(v)]$, where expectation is taken with respect to the possible type v that the agent may have in the next period. Then, if she rejects, it is simply the continuation value of search given by $\mathbb{E}[U^*(v)]$ (we don't need β before the expectation since utility $U(v)$ is already a discounted lifetime utility). Since agents use threshold strategies, then $P(\text{w accepts})$ is given by $\alpha P(v > \hat{b}) + (1-\alpha)P(v > \hat{a}_v)$ and $P(\text{w rejects}) = 1 - [\alpha P(v > \hat{b}) + (1-\alpha)P(v > \hat{a}_v)]$. Then the agent announces *Accept* i.f.f. the following inequality is satisfied:

$$P(\text{w accepts})w + P(\text{w rejects})\mathbb{E}[U^*(v)] \geq \mathbb{E}[U^*(v)] \tag{12}$$

which comes down to

$$w \geq \mathbb{E}[U^*(v)] \tag{13}$$

Proof of Proposition 2:

The ideas of this proof are based on those of Lemma 2 in Chade (2001).

Consider two agents with pizzazzes w, v with $w > v$. Then we need to show that $U^*(w) > U^*(v)$. As $w > v$, $\Omega_v \subseteq \Omega_w$ and $\Theta_v \subseteq \Theta_w$ as more agents would be willing to accept an agent with the higher pizzazz. Then, recall that $A_v = [\hat{a}_v, \bar{v}]$ and $A_w = [\hat{a}_w, \bar{v}]$. Therefore, $A_v \cap \Omega_v \subseteq A_v \cap \Omega_w$ and consequently $(A_v \cap \Omega_w)^c \subseteq (A_v \cap \Omega_v)^c$. Then as $U^*(v)$ is given by

$$\begin{aligned} U^*(v) = \alpha \int_{A_v \cap \Theta_v} \frac{x}{1-\beta} f(x)\,\mathrm{d}x + (1-\alpha) \int_{A_v \cap \Omega_v} \frac{x}{1-\beta} f(x)\,\mathrm{d}x + \\ + \beta U^*(v) \left(\alpha \int_{(A_v \cap \Theta_v)^c} f(x)\,\mathrm{d}x + (1-\alpha) \int_{(A_v \cap \Omega_v)^c} f(x)\,\mathrm{d}x \right) \end{aligned} \tag{14}$$

and as $x \geq U^*$ for $x \in [\hat{a}_v, \bar{v}]$ since $U_v^* = \hat{a}_v$. Then

$$\alpha \int_{A_v \cap \Theta_v} \frac{x}{1-\beta} f(x)\,\mathrm{d}x \leq \alpha \int_{A_v \cap \Theta_w} \frac{x}{1-\beta} f(x)\,\mathrm{d}x \tag{15}$$

$$(1-\alpha) \int_{A_v \cap \Omega_v} \frac{x}{1-\beta} f(x)\,\mathrm{d}x \leq (1-\alpha) \int_{A_v \cap \Omega_w} \frac{x}{1-\beta} f(x)\,\mathrm{d}x \tag{16}$$

and then

$$\frac{\alpha}{1-\beta} \int_{A_v \cap \Theta_w} x f(x)\,\mathrm{d}x - \frac{\alpha}{1-\beta} \int_{A_v \cap \Theta_v} x f(x)\,\mathrm{d}x \geq \\ \geq \beta\alpha \int_{(A_v \cap \Theta_w)^c} U^*(v) f(x)\,\mathrm{d}x - \beta\alpha \int_{(A_v \cap \Theta_v)^c} U^*(v) f(x)\,\mathrm{d}x \tag{17}$$

as $x \geq U^*$ and $\frac{\alpha}{1-\beta} > \alpha\beta$, while $(A_v \cap \Theta_w) \cup (A_v \cap \Theta_w)^c = [\underline{v}, \bar{v}]$.

Similarly,

$$\frac{1-\alpha}{1-\beta} \int_{A_v \cap \Omega_w} x f(x)\,\mathrm{d}x - \frac{1-\alpha}{1-\beta} \int_{A_v \cap \Omega_v} x f(x)\,\mathrm{d}x \geq \\ \geq \beta(1-\alpha) \int_{(A_v \cap \Omega_w)^c} U^*(v) f(x)\,\mathrm{d}x - \beta(1-\alpha) \int_{(A_v \cap \Omega_v)^c} U^*(v) f(x)\,\mathrm{d}x \tag{18}$$

Hence, we obtained the following inequality:

$$U^*(v) \leq \alpha \int_{A_v \cap \Theta_w} \frac{x}{1-\beta} f(x)\,\mathrm{d}x + (1-\alpha) \int_{A_v \cap \Omega_w} \frac{x}{1-\beta} f(x)\,\mathrm{d}x + \\ + \beta U^*(v) \left(\alpha \int_{(A_v \cap \Theta_w)^c} f(x)\,\mathrm{d}x + (1-\alpha) \int_{(A_v \cap \Omega_w)^c} f(x)\,\mathrm{d}x \right) \tag{19}$$

Applying the same logic, it is straightforward to show that the following inequality also holds:

$$U^*(w) \geq \alpha \int_{A_v \cap \Theta_w} \frac{x}{1-\beta} f(x)\,\mathrm{d}x + (1-\alpha) \int_{A_v \cap \Omega_w} \frac{x}{1-\beta} f(x)\,\mathrm{d}x + \\ + \beta U^*(v) \left(\alpha \int_{(A_v \cap \Theta_w)^c} f(x)\,\mathrm{d}x + (1-\alpha) \int_{(A_v \cap \Omega_w)^c} f(x)\,\mathrm{d}x \right) \quad (20)$$

Therefore, we get $U^*(w) \geq U^*(v)$, which is the same as $\hat{a}_w \geq \hat{a}_v$, which completes the proof.

Proof of Proposition 3:

Consider a high-type agent with pizzazz v who knows her type. If she uses threshold $\hat{a}_v < a_v$, her utility reads

$$U_1^*(v) = \frac{\frac{\alpha}{1-\beta} \int_{\hat{a}_v}^{\bar{v}} x f(x)\,\mathrm{d}x + \frac{1-\alpha}{1-\beta} \int_{\hat{a}_v}^{a_v} x f(x)\,\mathrm{d}x}{\eta + \alpha[1 - F(a_v)] + [F(a_v) - F(\hat{a}_v)]} \quad (21)$$

If a high-type agent with pizzazz v uses cutoff $\hat{a}_v = a_v$, then she will only mate with the agents who don't know their type with non-zero probability. Thus, her utility shrinks to

$$U_2^*(v) = \frac{\frac{\alpha}{1-\beta} \int_{\hat{a}_v}^{\bar{v}} x f(x)\,\mathrm{d}x}{\eta + \alpha[1 - F(a_v)]} = \frac{\frac{\alpha}{1-\beta} \int_{a_v}^{1} x f(x)\,\mathrm{d}x}{\eta + \alpha[1 - F(a_v)]} \quad (22)$$

Then, $U_1^*(v) - U_2^*(v)$ is given by

$$U_1^*(v) - U_2^*(v) = \frac{\eta}{1-\beta} \int_{\hat{a}_v}^{\bar{v}} x f(x)\,\mathrm{d}x > 0 \quad (23)$$

Hence, in equilibrium $\hat{a}_v < a_v$.

Proof of Proposition 4:

Let us first show the existence of a unique solution of (8). As both constraints are non-binding ($\hat{a}_v > a^*$), the first order condition simply yields

$$\alpha \int_{a_v}^{\bar{v}} (x - \hat{a}_v) f(x)\,\mathrm{d}x + \int_{\hat{a}_v}^{a_v} (x - \hat{a}_v) f(x)\,\mathrm{d}x - \eta \hat{a}_v = 0 \quad (24)$$

Hence, threshold of high-type agent v is determined by the equation (24). Notice that despite the fact that α depends on all agents thresholds, taking all other agents optimal strategies as given, a change in single agent's strategy wouldn't change alpha. Therefore, while maximizing her utility, each agent treats α as a constant parameter. Following the logic in Chade (2001), I define the left hand side of (24) as $H(\hat{a}_v)$ and manipulate its properties to determine the existence of a unique solution. At a point a_v, we have

$$H(a_v) = \alpha \int_{a_v}^{\bar{v}} (x - a_v) f(x)\,\mathrm{d}x - \eta a_v \tag{25}$$

and at a point a^*, we get

$$H(a^*) = \alpha \int_{a_v}^{\bar{v}} (x - a^*) f(x)\,\mathrm{d}x + \int_{a^*}^{a_v} (x - a^*) f(x)\,\mathrm{d}x - \eta a^* \tag{26}$$

If we take a derivative of $H(\hat{a}_v)$ w.r.t. $\hat{a}_v$, we will get

$$H'(\hat{a}_v) = -\alpha \int_{a_v}^{\bar{v}} f(x)\,\mathrm{d}x - \int_{\hat{a}_v}^{a_v} f(x)\,\mathrm{d}x - \eta < 0 \tag{27}$$

Therefore, $H(\hat{a}_v)$ is decreasing everywhere. Thus, if $H(a^*) > 0$ and $H(a_v) < 0$ then there exists a unique point $\hat{a}_v^*$, solving (24) and (27) serves as a second order condition reinforcing the fact that $\hat{a}_v^*$ is indeed yield maximum of the objective function(notice that $H(0) = \alpha \int_{a_v}^{1} x f(x)\,\mathrm{d}x + \int_0^{a_v} x f(x)\,\mathrm{d}x > 0$, so $H(\hat{a}_v)$ isn't negative everywhere on $[\underline{v}, \bar{v}]$). Otherwise, the optimal threshold $\hat{a}_v^*$ equals a^* since $H'(\hat{a}_v)$ is negative.

Now let's consider problem in (9) of a low-type agent. First order condition yields

$$(1 - \alpha) \int_{\hat{a}_v}^{a_v} (x - \hat{a}_v) f(x)\,\mathrm{d}x - \eta \hat{a}_v = 0 \tag{28}$$

Similarly, define left hand side of (28) as $K(\hat{a}_v)$. Then we have

$$K(0) = (1 - \alpha) \int_0^{a_v} x f(x)\,\mathrm{d}x > 0 \tag{29}$$

and

$$K(a^*) = (1-\alpha)\int_{a^*}^{a_v}(x-a^*)f(x)\,\mathrm{d}x - \eta a^* \tag{30}$$

Taking the derivative of $K(\hat{a}_v)$ w.r.t. $\hat{a}_v$ yields

$$K'(\hat{a}_v) = -(1-\alpha)\int_{\hat{a}_v}^{a_v} f(x)\,\mathrm{d}x - \eta < 0 \tag{31}$$

Again, we showed that $K'(\hat{a}_v)$ is negative everywhere on $[0,1]$. As in the case of a high-type agent, if $K(a^*) < 0$, there exits a unique $\hat{a}_v^*$ solving (28), otherwise $\hat{a}_v^* = a^*$.

Therefore, I have shown that problems given in (8) and (9) of high and low type agents knowing their type have a unique solution.

Proof of Theorem 1:

Again, my computations follow the logic presented in Chade (2001). I show the existence of equilibrium by construction. First, let us focus on agents who know the values of their pizzazzes and thus the threshold of those who don't will be uniquely defined. Consider an agent with type 1 who knows her type. She has the highest pizzazz and is accepted by all agents on the other side of the market. Let's fix some a^*. Thus, for her $a_1 = \bar{v}$. Then, her threshold is determined by

$$\int_{\hat{a}_1}^{\bar{v}}(x-\hat{a}_1)f(x)\,\mathrm{d}x - \eta\hat{a}_1 = 0 \tag{32}$$

or a^* if $H > 0$ (since $H(1) < 0$ for an agent of type 1). Then we have the set of agents $[\hat{a}_1, 1]$, who agent with pizzazz 1 is willing to accept. The monotonicity of thresholds (Proposition 2) implies that all agents in this set are accepted by everyone. If $\hat{a}_1 > a^*$, the partition continues. Now consider agent with type $\hat{a}_1 - \epsilon$. For small $\epsilon > 0$ all agents in $[\underline{v}, \hat{a}_1)$ accept agent with pizzazz $\hat{a}_1 - \epsilon$. Thus, $\lim_{\epsilon\to 0} a_{\hat{a}_1-\epsilon} = \hat{a}_1$ and the equation determining the threshold of this agent is given by

$$\alpha\int_{\hat{a}_{\hat{a}_1-\epsilon}}^{\bar{v}}(x-\hat{a}_{\hat{a}_1-\epsilon})f(x)\,\mathrm{d}x + \int_{\hat{a}_{\hat{a}_1-\epsilon}}^{\hat{a}_1}(x-\hat{a}_{\hat{a}_1-\epsilon})f(x)\,\mathrm{d}x - \eta\hat{a}_{\hat{a}_1-\epsilon} = 0 \tag{33}$$

Similarly, after obtaining the threshold in a similar manner, we have a set of $[\hat{a}_{\hat{a}_1-\epsilon}, \hat{a}_1)$, where all agents with pizzazzes in this set are accepted by all agents on the other side of the market (who know their types) with pizzazzes $\underline{v} \leq v < \hat{a}_1$.

We can apply exactly the same logic of construction for the low-type agents starting from a^*. Since each agent chooses her strategy taking as given the optimal strategies of all other agents and one agent's strategy can't alter α, here α is treated as a constant which is the same for everyone.

To gain further intuition about why the process stops, notice that all thresholds of high type agents depend on primitives of the model such as η and α. Thus, under the distribution $f(.)$ each threshold $\hat{a}_v$ we obtain in the iteration process can be characterized by the parameters η, α and k, where k is the step of iteration. Then fix k and define $\hat{a}_v(k, \eta, \alpha)$ to be the threshold obtained on a k-th step of iteration for the high type agents. As agents with pizzazz $\mathbb{E}[U^*(v)]$ are high type due to tie break rule, where agents accept when they are indifferent, their thresholds are determined by (24) as for the high-type. Thus, a^* is determined by (24) and can be defined by the same parameters of the model and written as $\hat{a}^*_v(k, \eta, \alpha)$.

Then as the iteration process starts at $\hat{a}^*_v(k, \eta, \alpha)$ for low-types, similarly all their thresholds are determined by the parameters η and $\hat{a}^*_v(k, \eta, \alpha)$. Thus, low type thresholds can also be determined only by η, α and k. Thus, we can calculate the $\mathbb{E}[U^*(v)]$, which is the expectation of the threshold over all possible pizzazzes and it is defined by only the η, α and k apart from the distribution of pizzazzes. Thus, we can determine k from the equation

$$\mathbb{E}[\hat{a}_v] = \hat{a}^*_v(k, \eta, \alpha) \tag{34}$$

and that is the switching point for the construction of low-types clusters. Even though solutions to (8) and (9) are unique, the solution to (34) is not necessarily unique, depending on the form of $F(.)$ and other primitives of the model. As the expectation of all agents thresholds depends on the size of clusters apart from the threshold, a few values of k can yield $\mathbb{E}[\hat{a}_v] = \hat{a}^*_v(k, \eta, \alpha)$.

Proof of Theorem 2:

Recall that high type's agents with pizzazz v threshold is determined by

$$\alpha \int_{a_v}^{1} (x - \hat{a}_v) f(x)\, \mathrm{d}x + \int_{\hat{a}_v}^{a_v} (x - \hat{a}_v) f(x)\, \mathrm{d}x - \eta \hat{a}_v = 0 \tag{35}$$

We thus can determine the relationship between $\hat{a}_v$ and α by implicit differentiation.

Hence, the derivative of $\hat{a}_v$ w.r.t. α is given by $\hat{a}'_v(\alpha)$

$$\hat{a}'_v(\alpha) = -\frac{H'_\alpha(\hat{a}_v)}{H'_{\hat{a}_v}(\hat{a}_v)} = -\frac{\int_{a_v}^{\bar{v}}(x-\hat{a}_v)f(x)\,\mathrm{d}x}{-\alpha[1-F(a_v)]-[F(a_v)-F(\hat{a}_v)]-\eta} \tag{36}$$

As for $[a_v, \bar{v}]$ $x > a_v$, we have that the numerator of (36) is positive, while denominator is negative. Hence, $\hat{a}_v(\alpha) > 0$ and thresholds of high-types increase in α

Similarly, as thresholds of low-type's agent with pizzazz v is given by

$$(1-\alpha)\int_{\hat{a}_v}^{a_v}(x-\hat{a}_v)f(x)\,\mathrm{d}x - \eta\hat{a}_v = 0 \tag{37}$$

Then for these agents the derivative of $\hat{a}_v$ w.r.t. α is given by $\hat{a}_v(\alpha)$

$$\hat{a}'_v(\alpha) = -\frac{K'_\alpha(\hat{a}_v)}{K'_{\hat{a}_v}(\hat{a}_v)} = -\frac{-\int_{a_v}^{a_v}(x-\hat{a}_v)f(x)\,\mathrm{d}x}{-(1-\alpha)[F(a_v)-F(\hat{a}_v)]-\eta} \tag{38}$$

which is negative.