MODELING MODEL ROCKETS: A STUDY IN OPTIMIZING MODEL ROCKETRY

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ABSTRACT

Although rocket science involves advanced mathematics and meticulous calculations, model rocketry is a useful segue into the world of rockets. For this project, the team built eleven unique rockets from different model kits. After measuring the rockets' dimensions, they recreated their rockets in OpenRocket, a program used to simulate model rocket flight. After launching the rockets, experimental apogee was compared to a simulated apogee from OpenRocket and to a calculated theoretical apogee. The team also experimented with different variables to study how they affected apogee - first, by increasing the mass of a chosen rocket, and measuring its effect on the original apogee; next, the shape of the nose cone was varied on a second rocket, and the apogee was again observed and compared. The team found that simulated and experimental apogee were very similar. Increasing mass corresponded with decreasing apogee as predicted. However, the nose cone experiment yielded some unexpected results, as the rocket flew the highest with a cylindrical nose cone. Although they encountered some difficulty along the way, the scholars were successful in capturing the physics that lies behind model rocketry.

INTRODUCTION

Students in the Rocket Science team project used physics, mathematical methods physics, and computer software to study the flight of model rockets. Model rocketry is a popular hobby enjoyed by children and adults alike. Despite its status as a hobby, due to its simplicity and feasibility, model rocketry is useful in studying real rockets in space flight as well as Newtonian physics. Students launched 11 different model rockets, collecting and analyzing data for each flight. Students compared measured experimental results to calculated results found on OpenRocket and predictions derived from Newton's theory.

History of Rocketry

The history of rocketry dates back to Ancient Greece, when in approximately 100 BCE, the aeolipile was designed, a device that used thrust from escaping water vapor to rotate the sphere. The first true rockets are believed to have originated in China. Sources differ on dates of origin and on what constitutes a "true" rocket (1). Howell (2) says that use of true rockets began with firework-like displays for festivals in the 1st century CE, while Stamp (3) maintains that the first rockets were "ground rats," gunpowder-filled tubes from the 12th century that would shoot in all directions on a floor.

Around the year 900, the Chinese began attaching gunpowder-filled bamboo tubes to arrows and firing them at enemies (1). These weapons, called *huo chien* ("fire arrows"), intimidated opponents and spread to India, Japan, and Korea by 1300. Military use of rockets had relatively few advancements until 1804, when William Congreve developed a rocket that was "designed to be launched from ships for the purpose of setting fires on an enemy shoreline" (4). These are the rockets described in Francis Scott Key's poem "The Star-Spangled Banner."

By the 1860s, rockets had fallen into disuse, at least for military purposes. Improvements in conventional field artillery rendered rockets impractical (4). Aside from occasional deployment in the U.S. Civil War, they would seldom be used for wartime purposes again until the mid-20th century and the development of the V-2 and the first intercontinental ballistic missiles (ICBMs), the technology for which is currently only held by the United States, Russia, and China (1,5).

Early rockets were used for two main purposes - military firepower and fireworks displays. Only in the late 19th and early 20th centuries were rockets first viewed as vehicles for transportation and space exploration. In 1903, Russian scientist Konstantin Tsiolkovsky published the "rocket equation" (see Rocket in Deep Space) (1). Other rocket pioneers at this time include Robert Goddard, Hermann Oberth, and Wernher von Braun. The use of rockets for space exploration became more popular following World War II (1).

In 1943, the Jet Propulsion Laboratory (JPL) was established at the California Institute of Technology (Caltech). In its early years, its work involved guided missiles and anti-aircraft equipment for the U.S. Army. In 1957, however, JPL began moving into space research and engineering, playing a major role in the construction of the satellite Explorer 1, the first successfully launched (1958) American satellite. This laboratory continues to cooperate with the American National Aeronautics and Space Administration (NASA), which was formed in 1958 from its precursor, the National Advisory Committee for Aeronautics (NACA), on new space exploration programs (6).

On October 4, 1957, the U.S.-U.S.S.R. space race came to a head with the U.S.S.R.'s successful launch of *Sputnik 1*, which was carried into space on a rocket also named *Sputnik*. Within a year, *(*Sputnik 2) and *(*Sputnik 3), as well as the American *Explorer 1*, were also in Earth's orbit. Around this time, model rocketry became a popular hobby (6,7). Professional rocket technology was further advanced with NASA's Space Shuttle program and the first reusable rockets. Previous rockets, such as the Saturn V, which launched astronauts to the Moon during the Apollo program, were completely disposable, but the Space Shuttles' Solid Rocket Boosters (SRBs) could be retrieved, refurbished, and reused. The SRBs were some of the most powerful solid-fuel rockets ever built, with a thrust at lift-off of 11.8 million Newtons (8).

The conclusion of the Space Shuttle program in 2011 left the Russian Soyuz rockets as the only ones capable of launching humans into space. That gap is quickly being filled, however, by private space agencies. On November 23, 2015, the first VTOL (vertical take-off and landing) was achieved by private space company Blue Origin, supporting the viability of rockets that can be reused without retrieval or refurbishment (9). Rocket technology, far from reaching its apogee, continues to evolve today.

Objective

The main objective was to compare simple rocket theory and advanced computer simulations with experimental data to analyze the effectiveness of each method and verify the main factors that affect rocket flight. To do this, measurable data, such as the apogee of a flight and the time of flight until apogee, were obtained from the OpenRocket simulation. The motion of a rocket was also simplified and derived in several models based on Newtonian physics. These two methods of predicting rocket flight were then analyzed next to the data collected by *VideoPhysics* to determine their accuracy.

Hypotheses

Three experiments were devised in which certain aspects of the rockets were modified to alter the outcomes of their launches. In one experiment, the mass of one rocket was modified to see how it would impact launch. It was hypothesized that the more mass the rocket had, the lower its apogee would be. In the second experiment, the nose cone shape of one rocket was altered and its effect on the rocket's apogee was measured. It was hypothesized that the apogee would increase as the nose cones' coefficient of drag decreased. For example, the rocket's apogee would be lower when a cylinder nose cone was used than when the rocket's regular nose cone was used, because the flat face of the cylinder would give the rocket a much larger coefficient of drag than the regular nose cone did. For the third experiment, each rocket was launched with different engines, and it was hypothesized that rockets would reach a higher apogee when launched with an engine that provided more thrust for a longer period of time than the other engines the rocket was flown with.

THEORY: MOTION OF A ROCKET

Essentially, a rocket is a rigid body that loses mass and experiences four main forces: the Earth's gravity, the thrust of the engine, air drag, and lift. Throughout our theoretical derivations, we will treat a rocket as an ideal point mass and neglect forces due to lift. We will consider motion in only one direction (although real rockets will bend, rotate, and "wiggle"). Because we are launching our rockets only hundreds of feet above the surface of the Earth, we will treat the acceleration due to Earth's gravity g as constant ($9.81 m/s^2$). Needless to say, we will also neglect any relativistic effects. We will start with equations governing the motion of rockets in simple cases, and then build our way to more realistic, yet also more complicated, models.

General Equation of Motion

Using Newtonian physics, we will derive the general rocket equation. Let the rocket have mass $m(t)$ and velocity $v(t)$ at time t. Thus, its momentum is $p(t) = mv$. In an infinitesimal time interval dt, the rocket will have ejected a small mass dm_e at a relative exhaust velocity v_e away from the direction in which the rocket is moving. Having ejected mass, the rocket will have increased in speed by dv. The total momentum after this interval is the sum of the momenta of the boosted rocket and ejected mass:

$$p(t + dt) = (m - dm_e)(v + dv) + (v + dv - v_e)dm_e.$$

The change in momentum $p(t+dt)-p(t)$ is

$$\begin{aligned}&(mv + mdv - vdm_e - dm_e dv + vdm_e + dvdm_e - v_e dm_e) - mv \\ &= mdv - v_e dm_e.\end{aligned}$$

But the change in exhaust mass is equal to the loss of the rocket's mass, $dm_e = -dm$, so

$$p(t+dt) - p(t) = mdv + v_e dm. \tag{0.1}$$

Dividing both sides by dt (by now, it should be clear that we are not too concerned with mathematical rigor), we have

$$\frac{p(t+dt)-p(t)}{dt} = m\frac{dv}{dt} + v_e\frac{dm}{dt}.$$

In the limit as dt approaches 0,

$$\frac{dp}{dt} = m\frac{dv}{dt} + v_e\frac{dm}{dt}.$$

But Newton's Laws state that the rate of change in momentum $\frac{dp}{dt}$ is the net external force, which may include gravity and drag, among other things. We'll call the net external force F_{ext} so

$$F_{ext} = m\frac{dv}{dt} + v_e\frac{dm}{dt}. \tag{0.2}$$

The thrust of the rocket is defined to be

$$T = -v_e\frac{dm}{dt}, \tag{0.3}$$

where T is not to be confused with time. Note that $\frac{dm}{dt}$ is negative as the rocket is losing mass, so the thrust actually provides a positive force pushing the rocket. Thus, rearranging terms, we arrive at the general equation of the rocket

$$F_{ext} + T = m\frac{dv}{dt}. \tag{0.4}$$

Now, F_{ext}, m, and T represent the instantaneous values at time t. We have the general equation for a point particle rocketing in one dimension. From (0.3), we can derive $m(t)$ from a general $T(t)$ by integrating

$$m(t) = m_0 - \int_0^t \frac{T}{v_e}dt. \tag{0.5}$$

We can also find the velocity and then position by integrating (0.2) with respect to time,

$$\begin{aligned}v(t) &= v_0 + \int_0^t \frac{F_{ext}(t') + T(t')}{m(t')}dt' \\ z(t) &= z_0 + v_0 t + \int_0^t v(t')dt'.\end{aligned}$$

Thus, in principle, we can numerically integrate this to approximate any motion given the thrust profile, mass profile, and nature of the external forces. First, we will study a few special cases that can be solved exactly.

Rocket in Free Space

We will start with the simplest case, in which we neglect all of these external forces. In this case, we may imagine the rocket in the deep vacuum of space, far away from any gravitating body. We could substitute $F_{ext} = 0$ into (0.2), but we could also use the fact that zero net external force means that momentum is conserved. Thus $p(t + dt) = p(t)$, so by (0.1),

$$\begin{aligned} mdv &= -v_e dm \\ \int_{v_0}^{v(t)} dv &= \int_{m_0}^{m(t)} -v_e \frac{dm}{m}, \end{aligned}$$

where v_0 is the initial velocity, m_0 the initial mass of the rocket (we assume that v_e is constant here, but it varies for real rockets).

$$\Delta v = v_e \ln \frac{m_0}{m(t)}$$

Thus, without net external forces, the velocity of the rocket is governed by the following equation (also known as Tsiolkovsky's equation)

$$v(t) = v_0 + v_e \ln \frac{m_0}{m(t)}, \tag{0.6}$$

Notice that $v(t)$ does not depend on how the mass was released, but only on the thrust velocity and ratio of the boundary state masses m_0 and $m(t)$. Regardless of how long it takes to reach $m(t)$, $v(t)$ is the same. Also, the more mass released, the faster the rockets goes. To maximize height, we ought to maximize the mass ejected. But that does not mean that we should stockpile mass and eject all of it. In practice, there is a limit to how light our rockets can be, and we cannot overload our rockets because then, they may not even lift off!

In a Constant Gravitational Field

Now we take into account the acceleration of the rocket due to Earth's gravity, but no drag. If we consider air resistance to be negligible, then this will provide a respectable approximation for the actual flight motion of the rocket. Typically, we can divide the rocket motion into two stages: the thrust period, the time until all of its fuel mass burns out, and the coast period, the time after burnout to the time the rocket reaches maximum height (apogee). First, we solve for the motion during the thrust period. Substituting $F_{ext} = -mg$ into (0.2),

$$\begin{aligned} -mg &= m\frac{dv}{dt} + v_e \frac{dm}{dt} \\ \frac{dv}{dt} &= -\frac{v_e}{m}\frac{dm}{dt} - g. \end{aligned}$$

Integrating with respect to time, we obtain the vertical velocity as a function of time during the thrust period

$$v(t) = v_0 + v_e \ln \frac{m_0}{m(t)} - gt. \tag{0.7}$$

The only (and crucial) difference from (0.6) is the $-gt$ term, which means that the longer it takes to eject the exhaust from the rocket, the slower the rocket will be. Thus, in practice, we want our rockets to eject its mass quickly so that it can accelerate to a very high speed.

Suppose at the time of burnout t_b, all the fuel mass has been ejected, thus concluding the thrust period. The rocket starts at rest, so $v_0 = 0$. Then

$$v(t_b) = v_e \ln \frac{m_0}{m(t_b)}.$$

The change in height is the integral of the vertical velocity (0.7) with respect to time:

$$\Delta z = \int_0^t v(t')dt' = \int_0^t v_0 + v_e \ln \frac{m_0}{m(t')} - gt'dt'.$$

Integrating and rearranging terms,

$$z(t) = z_0 + v_0 t - \frac{gt^2}{2} + \int_0^t v_e \ln \frac{m_0}{m(t')} dt'. \tag{0.8}$$

This resembles the usual parabolic equation of motion, except with the addition of the last term integral. Taking v_e to be constant, we can pull it out of the integral. Suppose further that thrust is constant. As a consequence, by definition (0.3), mass varies linearly during the thrust period:

$$m(t) = m_0 - Mt. \tag{0.9}$$

where the slope constant M is

$$M = \frac{m_0 - m_b}{t_b}.$$

as can be verified by substituting t_b into (0.9). Then (0.8) becomes

$$z(t) = z_0 + v_0 t - \frac{gt^2}{2} + v_e \int_0^t \ln \frac{1}{1 - \frac{M}{m_0} t'} dt'. \tag{0.10}$$

Now the integral in the above equation is

$$\int_0^t - \ln \left(1 - \frac{M}{m_0} t'\right) dt' = t - \frac{m_0 - Mt}{M} \ln \left(1 - \frac{M}{m_0} t\right). \tag{0.11}$$

Substituting (0.9), this is just

$$t - \frac{m(t)}{M} \ln \frac{m(t)}{m_0}. \tag{0.12}$$

Thus, substituting (0.12) into (0.10), we arrive at the complete equation for height during the thrust period:

$$z(t) = z_0 + v_0 t - \frac{gt^2}{2} + v_e \left(t - \frac{m(t)}{M} \ln \frac{m(t)}{m_0}\right). \tag{0.13}$$

After the thrust period begins the coast period, during which the rocket will have constant mass, no thrust, and experience only the acceleration due to Earth's gravity. The equation of motion is then just

$$v(t) = v(t_b) - g(t - t_b) \tag{0.14}$$

where $t \geq t_b$ and $v(t_b)$ is the initial velocity at the start of this coast period. Substituting $v(t_a) = 0$ into (0.14) and letting $v_b = v(t_b)$, we obtain the time of apogee

$$0 = v_b - g(t_a - t_b)$$
$$t_a = \frac{v_b}{g} + t_b.$$

Integrating (0.14), the height as a function of time $\Delta t = t - t_b$ after burnout obeys

$$z(\Delta t) = z_b + v_b \Delta t - g(\Delta t)^2/2$$

By setting $v = 0$ in (0.14), the height of apogee is

$$z_a = z_b + \frac{v_b^2}{2g}.$$

However, using this model, we have very strange predictions, the calculations of which are not necessary to be presented here. For example, we either predict unrealistically high values of apogee such as 500-1000 meters when our rockets typically reached less than 200 meters. If that were not strange enough, sometimes, the model predicted negative values of apogee. Therefore, we must discard this model and consider more forces such as air friction.

Drag and Constant Thrust

Real rockets experience drag, so we will derive a more accurate model. We will use the definition of drag that varies with v^2:

$$D = -\frac{1}{2}\rho A C_D v^2, \tag{0.15}$$

where the minus sign ensures that the force acts opposite to the motion of the rocket. We can absorb all of the coefficients into a single constant, so $D = -bv^2$. Now consider a constant thrust force T propelling the rocket upward, drag force $D = -bv^2$, and constant mass m (over the course of flight, we will suppose that the rocket does not change significantly in mass; we assume this to simplify the derivation). By (0.4) or Newton's Law,

$$m\frac{dv}{dt} = T - mg + D.$$

Divide both sides by m, and let $\beta^2 = T/m - g$ and $\gamma^2 = b/m$ so

$$\frac{dv}{dt} = \beta^2 - \gamma^2 v^2$$
$$\int_0^t dt' = \int_0^{v(t)} \frac{dv}{\beta^2 - \gamma^2 v^2}.$$

We can decompose the right hand side integrand into partial fractions to obtain

$$t = \frac{1}{\beta\gamma} \tanh^{-1}(\gamma v(t)/\beta).$$

Then

$$v(t) = \frac{\beta}{\gamma} \tanh(\beta\gamma t). \tag{0.16}$$

We can integrate (0.16) to get the height as a function of time during the thrust period:

$$z(t) = \frac{\ln(\cosh(\beta\gamma t))}{\gamma^2}. \tag{0.17}$$

After burnout, we have the coast period, during which the rocket obeys

$$m\frac{dv}{dt} = -mg - bv^2$$
$$\frac{dv}{dt} = -g\left(1 + \frac{b}{mg}v^2\right).$$

Setting $\eta^2 = \frac{b}{mg}$,

$$-\frac{1}{g}\int_{v_b}^{v(t)} \frac{dv}{1+\eta^2 v^2} = \int_{t_b}^{t} dt'.$$

Integrating, we have,

$$t - t_b = -\frac{1}{g\eta}\left(\tan^{-1}(\eta v(t)) - \tan^{-1}(\eta v_b)\right).$$

We can then find the time of apogee by setting $v = 0$

$$t_a = t_b + \frac{1}{g\eta}\tan^{-1}(\eta v_b). \tag{0.18}$$

Rearranging (0.18) and setting $C = \tan^{-1}(\eta v_b)$, we can solve for velocity as a function of time during the coast period

$$v(t) = \frac{1}{g\eta}\tan(g\eta(t - t_b) + C).$$

Integrating this with respect to time, we get the height during the coast period

$$z(t) = z_b - \frac{1}{g^2\eta^2}\ln(\cos(g\eta(t - t_b) + C)). \tag{0.19}$$

Plugging in the time of apogee given by (0.18), the height of apogee is

$$z_a = z_b - \frac{1}{g^2\eta^2}\ln(\cos(\tan^{-1}(2\eta v_b)))). \tag{0.20}$$

We can compare the theoretical apogee to the apogee measured by experiment as shown in Table I. We found the percent error using

$$\text{Percent Error} = \left| \frac{A_{\text{theoretical}} - A_{\text{experimental}}}{A_{\text{theoretical}}} \right|. \tag{0.21}$$

To calculate the theoretical z_a, we had to substitute several numerical values: all of the rockets in the Table I used A8-3 engines (see Appendix A for information on engine classification), so we assume that the thrust period for each rocket lasted $t_b = .5$ seconds and that the average thrust was 3.18 N. We take $g = 9.81\ m/s^2$. The mass varied for each rocket, but once known, we were able to calculate $\beta = \sqrt{T/m - g}$. We took air density to be $\rho = 1.225\ kg/m^3$, and the coefficient of drag to be a constant $C_D = .75$ (though they actually differed for each rocket). The body-tube radius r also varied for each rocket. Once known, we could then calculate $b = \rho\pi r^2 C_D/2$ and $\gamma = \sqrt{b/m}$. Substituting these parameters and t_b into (0.16), we found the velocity at burnout v_b. We could also calculate $\eta = \sqrt{b/(mg)}$, and from (0.18), found the time of apogee t_a. Substituting t_a and v_b into (0.20), we finally computed the height of apogee z_a.

A8-3 Rockets	Theoretical Apogee (m)	Experimental Apogee (m)	Percent Error
Saket	63.9	38.5	39.7%
Alex P	39.9	39.4	1.13%
Alex C	59.1	49.6	16.1%
Jake	61.9	73.3	18.4%
Kevin	77.2	46.6	39.6%
Mike	68.9	55.7	19.2%
Annie L	61.8	60.8	1.57%
Annie Z	67.0	87.1	30.0%
Alan	43.6	29.5	32.3%
Audrey	52.4	43.9	16.3%
Matt	47.2	34.8	26.2%
Average	58.4	50.8	13.1%

Table I: Theoretical Apogees vs. Experimental Apogees

Two of our rockets (Alex P's and Annie L's) reached an apogee that differed from the theoretical apogee by around 1%, comparable to the best simulated predictions of Table II. Overall, though, the average percent error 13.1% was higher than, but not too far off from, that of the simulated predictions, with some theoretical apogees differing as much as 40% from measured results. One reason for the variation is that our drag model treated both thrust and mass as non-zero constants, an assumption that is completely unrealistic and even mathematically inconsistent. The range in percent error can be explained by the fact that our rockets were assembled from different kits, systematic variations such as engine performance and wind, and the high sensitivity of the equations to initial conditions. Still, the fact that the theoretically predictions were not too far from measurements, some even very close (the 1% errors), merits Newton's Theory and the drag model. Even without considering varying mass and thrust, rigid body dynamics, and aerodynamics, we could build a fair model.

We could make our models fancier and more realistic. However, when we add more parameters, the calculations become unwieldy and we lose the ability to write exact closed formulas. Henceforth, it is more economical to resort to computer-aided numerical integration, which is faster and based on more accurate models.

Drag from Nose Cone Skin Friction

The skin friction coefficient C_f is the friction caused by air flow around the rocket and is determined by the body and fin surface area in contact with the airflow. It can be defined by:

$$C_f = \frac{2D_f}{\rho v_0^2 A} \tag{0.22}$$

where C_f is the drag coefficient (10). C_f can also be calculated using Reynolds number, the ratio of inertial force over viscous force. For our purposes, only the approximate value of incorrectly sprayed aircraft paint will apply, as we spray-painted over the balsa wood of our rockets. This value is 200 micrometers. Because our rocket does not reach the speeds that would require calculating Reynolds number, the skin friction coefficient in our case can be found with the equation

$$C_f = 0.032 \cdot \left(\frac{R_s}{L}\right)^{0.2}$$

where R_s would be 200 μm (10).

At subsonic speeds, the drag on the nose cone would be significantly smaller than that of skin friction. Still, the shape of the nose cone will affect the flight of a rocket, sometimes even in a way that would result in negative drag coefficients, producing a small reduction in drag.

The nose cone drag can be approximated by the square of the sine to the joint angle ϕ, giving any nose cone shape that smoothly transitions to the body a pressure drag of zero.

The theoretical pressure drag (10) of the nose cones are:

$$\begin{aligned} C_{conical} &= 0.2 \\ C_{ogive} &= -0.05 \\ C_{cylindrical} &= 0.8 \end{aligned}$$

The drag coefficient is usually complex and based on many variables, and therefore tends to be calculated experimentally.

Thrust Curves

To explore Newton's Laws further, we estimated a net force curve for a generic 50g model rocket using an Estes A8-3 engine. We did this by analyzing the A8-3 thrust curve found on the

Estes website and linearizing the graph by performing regressions on small collections of consecutive points, making sure each line had an r-value of above 0.90 (11). These regressions were then combined into a piecewise function and graphed using Desmos. Moreover, we subtracted out the force of gravity on the rocket and multiplied the net force equation by 0.98, as an approximation for drag (12). Figure 1 shows our completed net force approximation.

To find the height versus time graph, we integrated each individual linear piece of the net force function to receive a momentum curve. Subsequently, we integrated the momentum function piece by piece and divided it by mass to get height as a function of time. Since the thrust curve only displayed values during the time of firing, we extended the ending of the graph assuming only gravity was acting on the rocket. Figure 2 shows our approximated height graph.

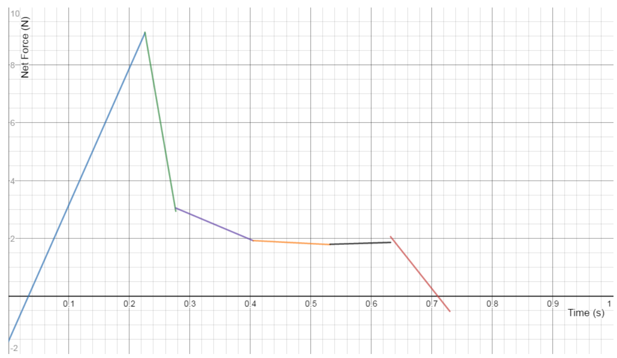

Figure 1: Net Force on a Generic 50 gram Rocket vs. Time
The figure here displays the net force on a 50 gram rocket (gravity subtracted from thrust) as a function of the time while the engine is firing. While the graph of thrust would be continuous in reality, we converted it to a piecewise function in linear sections while remaining mostly accurate for ease of integration. We also assume constant mass, as the change of mass during this period is mostly negligible.

Figure 2: Height of a Generic 50 gram Rocket vs. Time
This figure displays the height of a 50 gram rocket as a function of time. After dividing the mass out, integrating and extending the graph past the fire time, this figure was the result. We again assume constant mass.

METHODOLOGY

Rocket Construction

Each author built his or her own model rocket from a different kit. In total, eleven rockets were built, all with unique mass, length, fin number and shape, nose cone shape, and material. Materials used include cardboard, plastic, and balsa wood. Also, this is the first year that we used a 3D printer to design parts. Ten nose cone models were designed and three were printed using the Math department Makerbot printer. Due to scheduling and air-force clearance problems, we had time to test only the three: ogive, conical, and cylindrical. The other seven unused models–which included elliptical, parabolic–can be used by future scholars.

Launch Preparation and Launching

On the 18th of July, engines were weighed, the boundaries of the launch site measured, and locations for the launch pad and clinometers determined (see Section 4.3, Collection of Data).

On the 19th of July, clearance to launch from Morristown Airport Air Traffic Control was obtained. This was necessary because the launch site was within a five-mile radius of the airport. On the 20th of July, the rockets were transported to a field on Drew University's campus and launched. Each of the eleven rockets was launched twice on this day. Every rocket was launched first with an Estes A8-3 engine, and later with either an Estes B6-2 or B4-4 engine, depending on the size of the rocket. (See Appendix A) Data was gathered for time of flight (from launch to parachute deployment, or, in the case of the rocket without a parachute, to break-up) and for altitude at parachute deployment or break-up (with a clinometer). For more information on data collection, see Collection of Data.

Launching continued on the 22nd of July at a new location (for logistical purposes). One rocket was launched four times, with extra mass added (with lead shot) each time to determine the added mass's effect on rocket performance. In addition, a second rocket was launched three times, with a differently shaped nose cone each time (conical, ogive, and flat cylindrical). Each of the preceding trials was done with an Estes A8-3 engine. After their launches, two more rockets were launched with Estes B6-4 engines, and a final two launched with C6-5 engines. These latter two launches yielded no reliable data, as the rockets rose too high to track.

Collection of Data

After a launch site was chosen, three points from which to measure the height of the rocket were designated (these points were located either 150 or 200 ft from the launch pad, depending on the field). In order to do this, clinometers, which give the angle from the surface of the ground to some point (in this case, the rocket), were used. These angles were recorded along with their relative distances to each other, when the rocket deployed its parachute.

The height z of the rocket can be found using the three different angles of incline ϕ, each with their own coordinate (h, k). Using these three different values of (ϕ, h, k) in a system of equations allowed the (x, y, z) intersection point of the possible locations for each angle to be calculated.

$$z(x, y) = \cot\left(\frac{\pi}{2} - \phi\right)\sqrt{(x-h)^2 + (y-k)^2}$$

The $\cot(\frac{\pi}{2} - \phi)$ term provides the height when multiplied by the distance to the rocket. Therefore, the z-coordinate in this equation gives us the height of the rocket. However, if there are three different points, there will be three different equations. Therefore, by setting the three equations equal to each other, a point where they all have the same height can be determined. If each (ϕ, h, k) was its own graph, the height would be the z of the intersection point (Fig. 3).

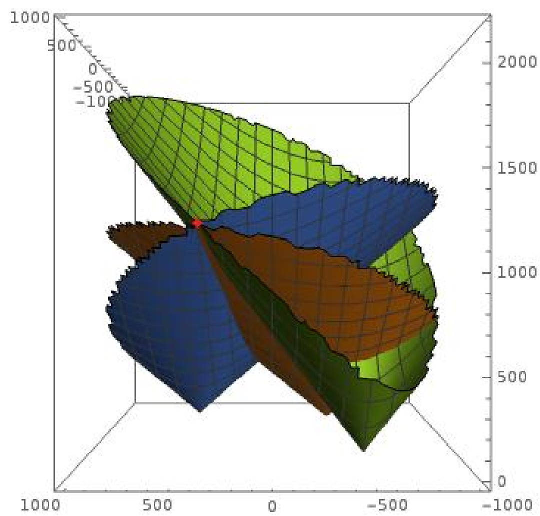

Figure 3: Mathematica 3D Plot of Cone Intersection
This is the graph of the aforementioned functions of height as a function of the angles and distance. We graphed each of the three functions with the different values from the three different points of measurement using Mathematica and found the intersection of all functions, which gave us the height.

The beginning of each rocket's first launch and a few full launches were also filmed to analyze the flights with video tracking software. Vernier's Video Physics is an iOS application that can produce a distance v time graph from a video, given a scale of measurement and an axis. Though the program is capable of automatically tracking moving objects, the software is finicky and often loses track of the object or tracks incorrectly. Rather than using the automatic tracker, the rockets' paths were traced frame by frame through their respective videos. Unfortunately, the apogee collected by this program was slightly inaccurate, likely due to camera perspective.

COMPUTER SIMULATION

The model rocket launches were simulated using the open source OpenRocket simulator, which allows for a variety of variables to be changed, giving realistic models (via Runge-Kutta numerical integration methods to solve the general equations of motion). The dimensions and masses of our rockets were measured, and the rockets were built in OpenRocket. We display an example of one scholar's rocket, created in the OpenRocket software in Figure 4. Using the A8-3 Estes engine provided in the simulation and default wind speed, expected turbulence, etc., the simulated apogees were found and recorded. Upon retrieving experimental data, the apogees of the simulated launches were compared to those of the experiments, and the simulator was found to be fairly accurate (Table II).

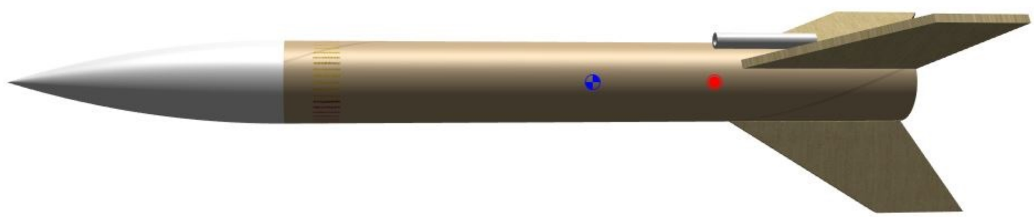

Figure 4: OpenRocket's 3D Finished Simulated Rocket–Annie Z's.
OpenRocket allows the user to generate a three-dimensional image of the simulated rocket once the parts and their dimensions are given to the user. It also shows the center of pressure, as shown by the red dot, and center of gravity, as shown by the blue dot, which are the points at which the lift and weight act on the rocket respectively.

To compare our simulated and experimental apogees, we found the percent error using (0.21) except we use the "simulated" apogee in place of the "theoretical" apogee.

A8-3 Rockets	Simulated Apogee (m)	Experimental Apogee (m)	Percent Error
Jake's	67.6	73.3	8.49%
Matt's	33.0	34.8	5.58%
Annie Z's	88.0	87.0	1.07%
Alex P's	34.0	39.4	15.96%
Kevin's	44.2	46.6	5.50%
Audrey's	46.6	43.8	5.84%
Annie L's	67.5	60.8	9.93%
Average	54.41	55.1	7.48%

Table II: Simulated Apogees vs. Experimental Apogees

The accuracy of the computer simulations to the experimental rocket launch apogees were extremely close, some as close as 1.07% error (Table II). All except for Alex P's rocket had a percent error under 10%. According to the trials, OpenRocket, even with various shapes, sizes, and masses of rockets, performed accurately in predicting apogee. The overall average of all the rockets' percent error was 7.48%, which is quite accurate.

The graphs below in Figure 5 and Figure 6 shows the height vs time for the simulated launch in OpenRocket (blue dots) and data points from VideoPhysics, gathered from the experimental launch (red dots).

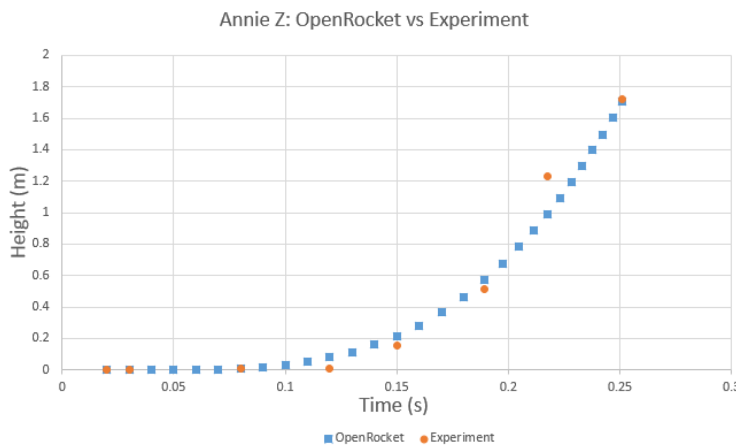

Figure 5: Graph of Annie Z's OpenRocket's Simulated Launch vs Experimental Launch with Data Points Pulled from VideoPhysics

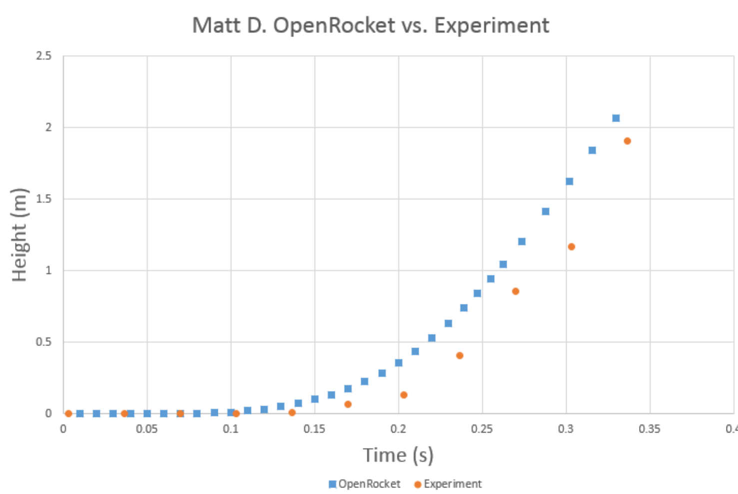

Figure 6: Graph of Matt D's OpenRocket's Simulated Launch vs Experimental Launch with Data Points Pulled from VideoPhysics

EXPERIMENTAL DATA AND ANALYSIS

Control Group

These rockets were launched with a standard motor and no variations from the model's instructions, and the data from each launch were recorded (Table III).

A8-3 Rockets	Apogee (m)	Time before Apogee (s)
Saket's	38.5	3.67
Alan's	29.4	3.53
Alex C's	49.6	3.30
Mike's	55.6	3.83
Jake's	73.3	2.38
Matt's	34.8	3.34
Annie Z's	87.0	3.66
Alex P's	39.4	3.66
Kevin's	46.6	3.65
Audrey's	43.9	3.30
Annie L's	60.8	3.18

Table III: Control Group Launches

Changing Thrust

Changing from A class engines to B class engines increased the impulse applied to the rocket. Apogee of each launch increased as a result, shown by much higher data values (Table IV). Because the clinometer has a maximum angle measurement of 70°, some apogees could not be calculated. However, it is certain that they lie above 127 m.

Name	Engine Used	Apogee (m)	Time to Apogee (s)
Annie Z	B4-4	Too high for measurement	7.86
Alex C	B4-4	115	5.44
Saket	B4-4	Too high for measurement	5.49
Annie L	B4-4	Too high for measurement	6.41
Kevin	B6-4	162	3.90
Jake	B6-4	101	4.81
Alex P	B6-2	85.6	2.40
Kevin	B6-2	97.1	2.53
Mike	B6-2	106	2.89
Alan	B6-2	47.1	1.38
Matt	B6-2	53.2	1.73

Table IV: Changing Thrust

Changing Mass Experiment

An important element of rocket science is the mass of the rocket itself. It was hypothesized that a negative correlation between mass and rocket apogee should exist. To test this, the following scatterplot in Figure 7 analyzes the relationship between the masses of each rocket and their respective apogees with an Estes A8-3 Engine.

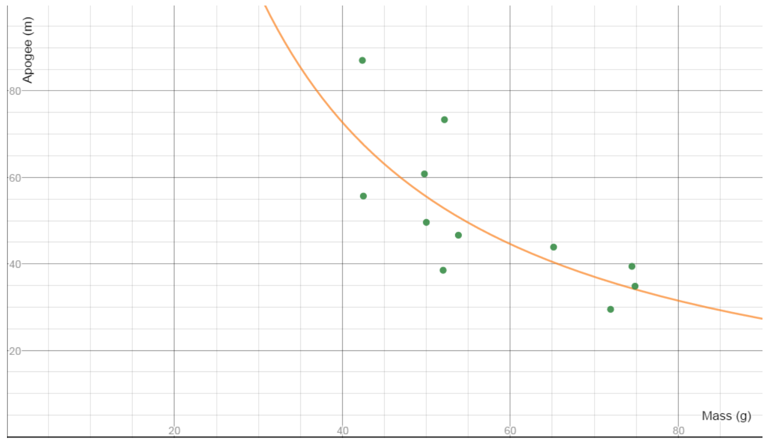

Figure 7: Scatterplot and Best Fit Curve Comparing Masses of Rockets vs Apogee
The figure displays a scatterplot, where each data point represents an individual launch, with mass and apogee corresponding to the position along the axes. The curve is the result of log-log (power function) regression on the data points.

A clear negative correlation between mass of the rocket and apogee was found by performing a log-log regression. The correlation had an r value of $-.78$, relatively strong, and an r^2 value of $.62$, which means that 62% of the variation in apogee can be explained by the best fit curve between mass and apogee. Although this means there are other factors involved in apogee variation, it shows that mass is also a significant factor.

To further analyze the negative trend between mass and apogee, a single rocket was launched multiple times with an A8-3 engine, each time with different added mass in the form of lead shot. Results were recorded in Table V:

Mass (g)	Apogee (m)
37.0	62.3
42.3	45.7
48.6	45.2
54.1	43.6

Table V: Varied Mass of Single Rocket vs. Apogee

Using these data, another scatterplot was constructed (Fig. 8). Similar to the previous scatterplot, this graph displays data points indicating launches with different masses of the rocket (additional mass was placed in the nose cone) and with specific apogees. The curve results from the same type of regression as in Figure 7. This plot further confirmed the negative correlation between mass of a rocket and the apogee of flight. Keeping with the regression of the previous data set, log-log regression was performed again. It returned an r value of -0.85 and an r^2 value of 0.722, indicating a strong correlation and implying that 72.2% of the variation could be explained by the best fit curve. A substantial percentage of change in apogee across consecutive

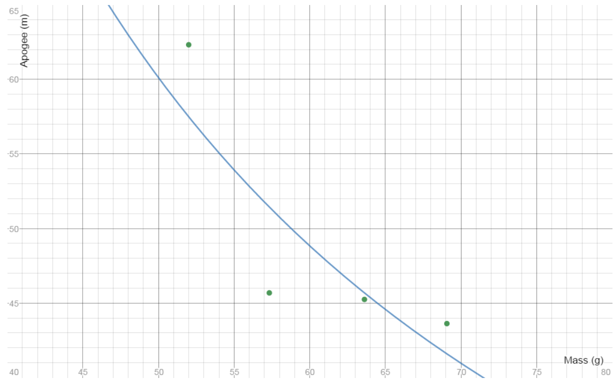

Figure 8: Scatterplot and Best Fit Curve Comparing Varied Mass of Single Rocket vs Apogee

launches is explained by the change in mass, thereby validating the hypothesis that an increase in mass will induce a decrease in apogee.

Changing Nose Cone

Annie Z's rocket was used for all nose cone alterations. All launches were performed with A8-3 engines, and the results recorded in Table VI. The data confirmed that the nose cone designed by the model rocket company was more effective than the 3D-printed ones. Unexpectedly, the cylindrical nose cone, which was flat in the front, performed better than the others that were 3D printed. One would expect it to do worse because of its inferior aerodynamics. It is possible that the discrepancy was caused by errors such as a sudden updraft in the wind during the launch, or human error such as accidental tilting in the other launches, which would lower the apogee of the rocket.

Nose cone shape	Apogee (m)	Time before Apogee (s)	Mass of Nose Cone (g)
Unaltered nose cone	87.1	3.66	5.23
Conical	59.5	1.68	7.75
Ogive	66.4	1.57	3.22
Cylinder	73.6	2.55	6.00

Table VI: Launch Patterns with Varied Nose Cones

CONCLUSION

This project was considered a success by those involved. Students in the group cooperated not only to assist one another while working through advanced Newtonian physics, but also to launch model rockets and record data.

In the first experiment, where rocket mass was altered (see Changing Mass Experiment), it was hypothesized that mass and apogee would be inversely proportional. After each rocket's mass and apogee were plotted on a graph, a best fit curve was created. This curve shows that generally as mass increased, apogee decreased. In addition, the mass of a single rocket was altered, and apogee was recorded. The lightest rocket, 37.00 grams, had the highest apogee at 62.315 meters. The hypothesized trend then continued for 3 more masses. The launch with the most mass (54.07 grams) had the smallest apogee (43.626 meters). Thus, the hypothesis was supported. However, some other results did not entirely agree with Newtonian physics and the hypotheses. For the experiment in which the nose cone shape was altered, it was hypothesized that as the coefficient of drag increased, apogee would decrease. This hypothesis was supported by Newtonian physics equations. When the experiment was conducted (see Changing Nose Cone), it was expected that the ogive nose cone, which had the smallest coefficient of drag (-0.05) would perform better than the two others. Although the ogive nose cone shape resulted in a higher apogee than the conical shape (coefficient of drag 0.2), the rocket performed even better with the cylinder nose cone — unanticipated, since the cylinder had the highest coefficient of drag (0.8).

As humans conducted most of the experiment, human error will account for a part of the error. Since we built and painted our rockets with no prior experience, minor errors during construction may have lead to small discrepancies in rocket flight, such as increased drag from rough paint. Rockets were simulated on OpenRocket by different people, so it is possible that there were mistakes in the creation of rocket simulations. Instead of measuring the wind speed or humidity during the launches, which OpenRocket accounts for, the program's default settings were used. The clinometer that was used to measure apogee also depended on human reaction time, though error was minimized as possible by laying low to the ground. The three angles collected were triangulated, so displacement from the launch pad was not an error. For the nose cone experiment, the ogive nose cone was 3D printed very roughly and so bumps would have affected the ogive flight as compared to the conical and cylindrical one.

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REFERENCES

[1] Benson T, compiler. Brief History of Rockets [Internet]. 2014 [cited 2016 July 27]. Available from: https://www.grc.nasa.gov/www/k-12/TRC/Rockets/history_of_rockets.html

[2] Howell E. 2015 May 2. Rockets: A History [Internet]. Space.com; [cited 2016 Jul 28]. Available from: http://www.space.com/29295-rocket-history.html

[3] Stamp J. 2013 Feb. The History of Rocketry [Internet]. Smithsonian Magazine; [cited 2016 Jul 28].

Available from: http://www.smithsonianmag.com/innovation/the-history-of-rocket-science-4078981/?no-ist

[4] Early Rocketry [Internet]. Eugene(OR):University of Oregon; [cited 2016 Jul 28].
Available from: http://abyss.uoregon.edu/ js/space/lectures/lec01.html

[5] ICBM. [accessed 2016 Jul 28].
Available from: https://www.britannica.com/technology/icbm

[6] Gruntman M. Blazing the Trail: The Early History of Spacecraft and Rocketry [Internet]. Reston(VA):American Institute of Aeronautics and Astronautics; 2003 [cited 2016 Jul 28].
Available from: http://site.ebrary.com.ezproxy.drew.edu/lib/drew/reader.action?docID=10516693

[7] NASA Administrator, compiler. Explorer 1 Fast Facts [Internet]. 2008 [cited 2016 July 27].
Available from: https://www.nasa.gov/mission_pages/explorer/fast-facts.html

[8] Wilson J, editor. Solid Rocket Boosters [Internet]. National Aeronautics and Space Administration; [2006 Mar 5, cited 2016 Jul 28].
Available from: http://www.nasa.gov/returntoflight/system/system_SRB.html

[9] Meyer R. The First Reusable Rocket Is Here. The Atlantic. 2015 Nov 24 [accessed 2016 Jul 29].
Available from: http://www.theatlantic.com/technology/archive/2015/11/the-first-reusable-rocket/417525/

[10] Niskanen S. Development of an Open Source model rocket simulation software [Internet]. Helsinki University of Technology; 2009 [cited 2016 July 28].
Available from: http://openrocket.sourceforge.net/thesis.pdf

[11] Estes A8 [Internet]. 2000 October 3. National Association of Rocketry; [cited 2016 Jul 28].
Available from: http://www.nar.org/SandT/pdf/Estes/A8.pdf

[12] Peraire J, Widnall S [Internet]. Cambridge(MA):Massachusetts Institute of Technology; 2008 [cited 2016 July 28].
Available from: http://ocw.mit.edu/courses/aeronautics-and-astronautics/16-07-dynamics-fall-2009/lecture-notes/MIT16_07F09_Lec14.pdf

[13] Estes Rocket Lab [Internet]. Estes-Cox Corp.; 2008 [cited 2016 July 28].
Available from: http://www2.estesrockets.com/pdf/Model%20Rocket%20Engines%20%20Igniters.pdf

APPENDICES

APPENDIX A: A NOTE ON MODEL ROCKET ENGINE DESIGNATIONS

To aspiring model rocketeers, the wide array of engine designations can be confusing. This section aims to eliminate that confusion. Each engine designation is in the format [letter][x]-[y], where the letter represents the engine's impulse, x is the average thrust in Newtons, and y is the time in seconds after consumption of the fuel before the parachute deploys (13).