CS 698C - Assignment - I

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Part - I

(a) $\Omega = \{HH, HT, TH, TT\}$

(b) We know that event space is collection of all possible subsets of sample space.

$$F = \{ \{HH\}, \{HT\}, \{TH\}, \{TT\}, \{HH, HT\}, \{HH, TH\}, \{HH, TT\}, \{HT, TH\}, \{HT, TT\}, \{TH, TT\}, \{HH, HT, TH\}, \{HH, HT, TT\}, \{HH, TH, TT\}, \{HT, TH, TT\}, \phi, \{HH, HT, TH, TT\} \}$$

Total elements $2^4 = 16$

(c) (i) Given $P(\{HH\}) = a$

$P(\{HT\}) = a$

$P(\{TH\}) = a$

$P(\{TT\}) = a$

a is a number

from II^nd axiom $P\left(\bigcup_i A_i\right) = 1$

So. $\therefore a+a+a+a=1 \Rightarrow a=0.25$

Probabilities of elementary events are equal –

$$P(\{HH\})=0.25$$
$$P(\{HT\})=0.25$$
$$P(\{TH\})=0.25$$
$$P(\{TT\})=0.25$$

(ii) $E=\{HH, HT, TH\}$.

$$P(E)= P(\{HH\})+P(\{HT\})+P(\{TH\})$$
$$=0.75$$

(iii) $E=\{HT, TH\}$

$$P(E)= P(\{HT\})+P(\{TH\})$$
$$=0.50$$

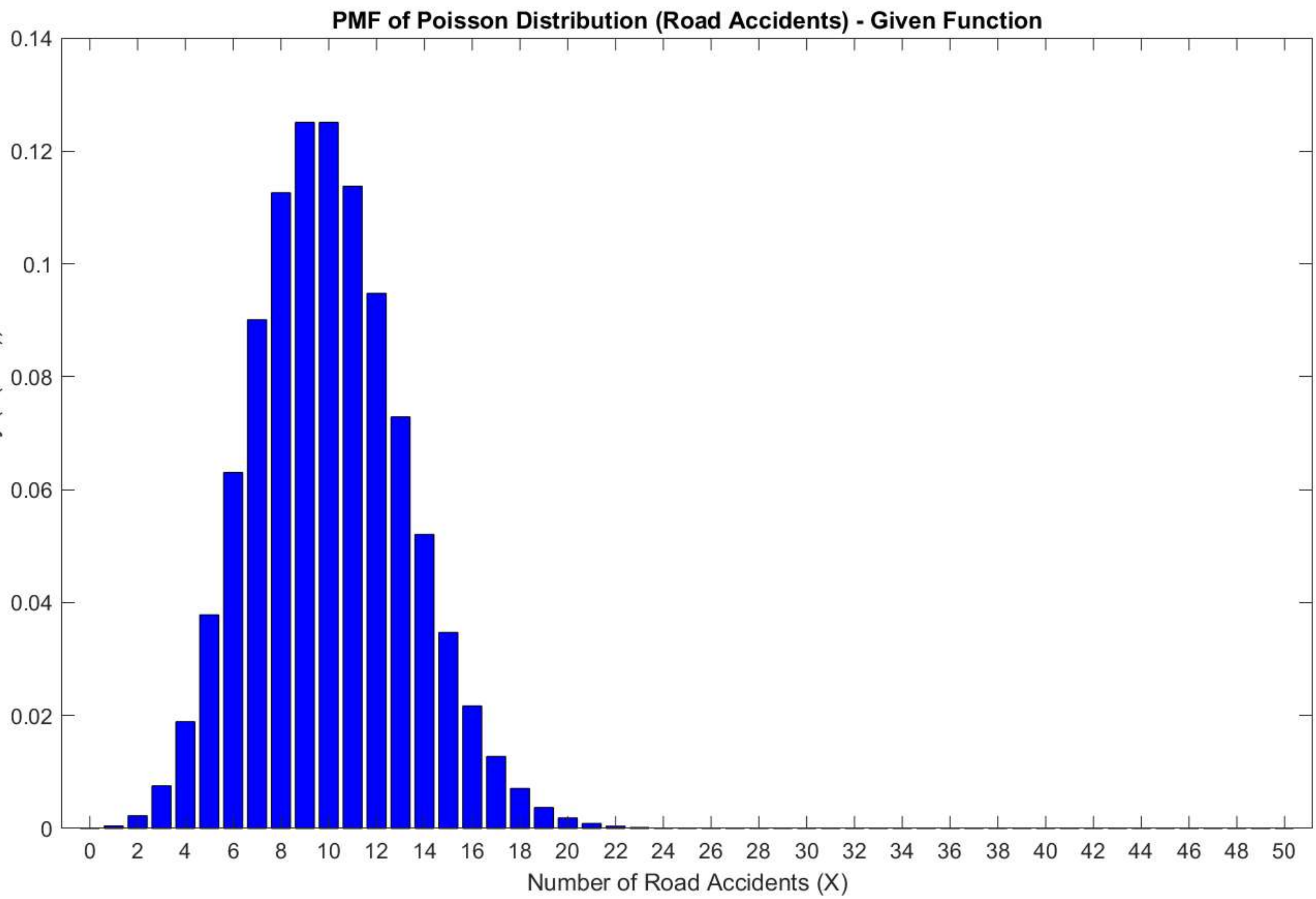
Part-2

2.1 Given – $n=50$, $k=5$, $P=0.9$

So – $f(45, 50, 0.9) = \dfrac{50!}{45!\cdot 5!}(0.9)^{45}(0.1)^{5}$

$f(45, 50, 0.9) = \cancel{0.0087}$ 0.18492

```matlab
% Define the lambda (average) value
lambda = 10;

% Range of possible values for x (number of accidents)
x = 0:50;

% Calculate probabilities using the given function
probabilities = (lambda.^x) .* exp(-lambda) ./ factorial(x);

% Create the plot
figure;  % Create the figure
set(gcf, 'Position', [100 100 1000 600]);  % Set figure size and position

bar(x, probabilities, 'b');  % Blue color for bars
xlabel('Number of Road Accidents (X)');
ylabel('Probability (P(X=x))');  % Clarify y-axis label
title('PMF of Poisson Distribution (Road Accidents) - Given Function');
xticks(0:2:50);  % Show x-axis ticks every other value
grid('y', '--', 'alpha', 0.7);  % Grid on y-axis with dashed lines

% Display the plot
hold off;  % Ensure no previous plots are interfering
```


PMF of Poisson Distribution (Road Accidents) - Given Function
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
Number of Road Accidents (X)

2.2 Given Pmf $f(k,\lambda) = \frac{\lambda^k e^{-\lambda}}{\lambda!}$; $\lambda = 10$

(a) Put $k=0$ $f(k,\lambda) = P(k=0)$

$\Rightarrow$ $P(k=0) = e^{-10}$

(b)

$P(7<k<10) = P(k=10) - P(k=7)$

$= P(k=8) + P(k=9)$

$\Rightarrow e^{-10}\left(\frac{10^8}{8!} + \frac{10^9}{9!}\right)$

Part-3 pdf $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

(a) Given $\mu=1, \sigma=1, x=0$

$P(x=0) = \frac{1}{\sqrt{2\pi e}} = 0.24198$

(B) Given $\mu=0, \sigma=1, x=1$

$P(x=1) = \frac{1}{\sqrt{2\pi e}} = 0.24198$

(c) $P(x_1 < x < x_2) = 0.3$

$P(x_1 < x < x_3) = 0.45$

(c-i) $x_1 < x_2 < x_3$

In III^{rd} axis

So $P(x_1 < x < x_3) = P(x_1 < x < x_2) + P(x_2 < x < x_3)$

$\Rightarrow \quad P(x_1 < x < x_3) - P(x_2 < x < x_3)$

$= 0.45 - 0.30$

$= \underline{0.15}$

Case-2 $\quad x_2 < x_1 < x_3$

$P(x_2 < x < x_3) = P(x_2 < x < x_1) + P(x_1 < x < x_3)$

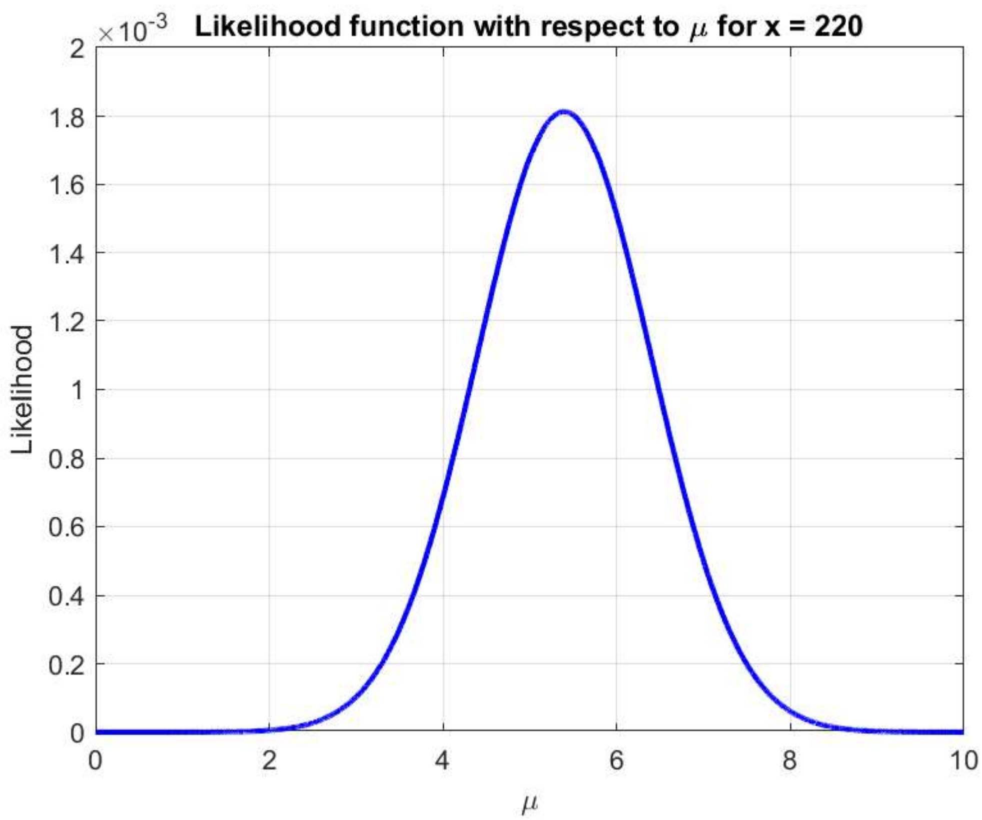
$= 0.45 + 0.30 = \underline{0.75}$

```matlab
%% Question_4 Codes setup

%%    CODE FOR PART A


% Define the recognition time
x = 220;

% Define the range of mu values
mu_values = 0:0.001:10;

% Calculate the probability densities for each mu
probabilities = pdf(x, mu_values);

% Plotting the graph
figure;
plot(mu_values, probabilities, 'b-', 'LineWidth', 2);
xlabel('\mu');
ylabel('Likelihood');
title('Likelihood function with respect to \mu for x = 220');
grid on;

function f = pdf(x, mu)
    f = (1 ./ (x * sqrt(2 * pi))) .* exp(-((log(x) - mu).^2) / 2);
end
```

```matlab
%%    CODE FOR PART B



% Define the observed sample of recognition times
x_observed = [303.25, 443, 220, 560, 880];

% Define the range of values for μ
mu_values = linspace(0, 10, 10000);

% Calculate the likelihood function for each value of μ
likelihood_values = zeros(size(mu_values));
for j = 1:length(mu_values)
    mu = mu_values(j);
    product_term = 1;
    for i = 1:length(x_observed)
        product_term = product_term * x_observed(i);
    end
    product_term = product_term * sqrt(2 * pi)^length(x_observed);
    sum_term = 0;
    for i = 1:length(x_observed)
        sum_term = sum_term + ((log(x_observed(i)) - mu) ^ 2);
    end
    likelihood_values(j) = 1 / (product_term) * exp(-sum_term / 2);
```

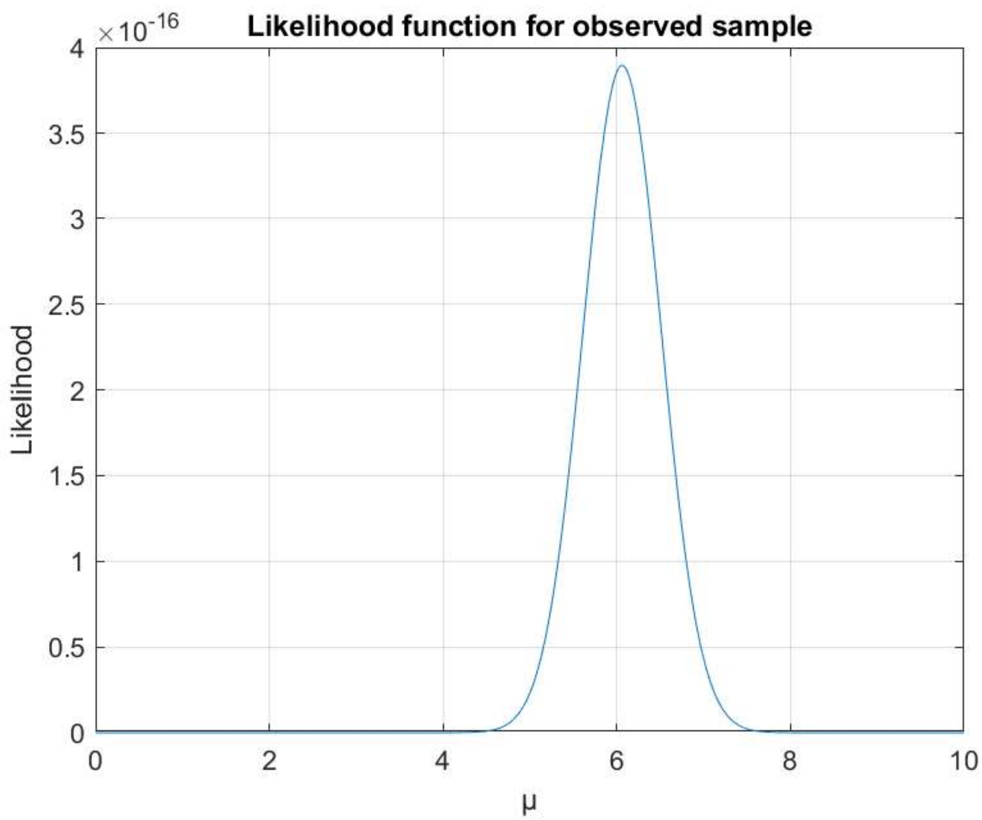
```
end

% Plot the likelihood fmnction
plot(mu_values, likelihood_values)
xlabel('µ')
ylabel('Likelihood')
title('Likelihood function for observed sample of recognition times')
grid on

% Define the probability density function
function density = f(x, mu)
    n = length(x);
    product_term = 1;
    for i = 1:n
        product_term = product_term * x(i);
    end
    density = 1 / (product_term * sqrt(2 * pi)^n);
    sum_term = 0;
    for i = 1:n
        sum_term = sum_term + ((log(x(i)) - mu) ^ 2);
    end
    density = density * exp(-sum_term / 2);
end
                              %%  CODE FOR PART C


% Find the maximum likelihood value and its index
[max_likelihood, max_index] = max(likelihood_values);
% Corresponding value of µ for maximum likelihood
mu_max_likelihood = mu_values(max_index);

fprintf('Approximate value of µ for maximum likelihood: %.8f\n', mu_max_likelihood);

% this comes to  6.06060606 %
```

Part A



Likelihood function with respect to μ for x = 220
$\times 10^{-3}$
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Likelihood
0
2
4
6
8
10
μ

Part B



For Part C, I got mu around 6.060606, given code above