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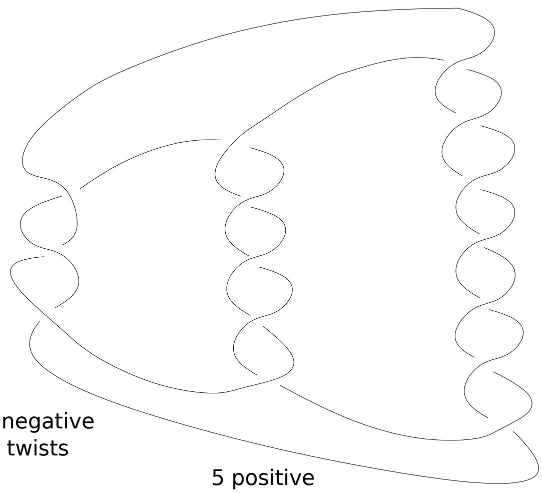
Low-dimensional topology, 4-manifolds, Knot concordance, Topological Quantum Field Theories

Current Research Interests

Topology in four dimensions is particularly interesting because the setting is complicated enough for many intricate phenomena to arise, but in contrast to higher dimensions there is not enough room to untangle and simplify these. Moreover, from dim$=4$ onward the difference between the topological and smooth category emerges; this is demonstrated by the observation that many constructions work topologically but fail to hold smoothly.

To illustrate this, let's start with a knot $K \subset \mathbb{S}^3$ and the question whether K is the boundary of a "nicely" embedded disk in the 4-ball. Freedman has a theorem stating that Alexander polynomial $\Delta_K(t) \doteq 1$ is enough to conclude that the knot in question is **topologically** slice[1]. For example, take a peek at the $(-3, 5, 7)$ pretzel knot which has a trivial Alexander polynomial[2].



Even though Freedman's result tells us that there is a topological slice disk for this particular knot, its Rasmussen s-invariant[3] is nonzero and so it does not have a **smooth** slice disk in $\mathbb{D}^4$. Now, such a creature (a topologically but not smoothly slice knot) gives rise to an exotic smooth structure on $\mathbb{R}^4$, and there are still many open questions regarding this relationship.

I just started working on my PhD this October; topics I want to learn more about include concordance of knots & links and TQFTs.

[1] *Topologically slice* requires the embedding $\mathbb{D}^2 \hookrightarrow \mathbb{D}^4$ of the slice disk with $\partial\mathbb{D}^2 = K \subset \mathbb{S}^3$ to be locally flat, i.e. it can be thickened to $\mathbb{D}^2 \times \mathbb{D}^2 \hookrightarrow \mathbb{D}^4$ and this should restrict to a tubular neighborhood of the knot in $\mathbb{S}^3$.

[2] In case you want to check this: The picture suggests a Seifert surface of genus 1, using this it is not hard to calculate the (self-)linking of homology generators to obtain a Seifert matrix for the knot.

[3] The *s-invariant* can be defined combinatorially using *Khovanov Homology*: That is a homology theory for knots which is motivated from a categorification of the Jones polynomial.