Annika Hultin

Sorting Algorithm Comparisons

For this experiment I implemented the famous Insertion Sort, Merge Sort, and Quicksort algorithms to determine which algorithm is the most efficient. I tested each algorithm 10 times on input sizes ranging from 1,000 to 100,000 and evaluated the average number of comparisons it took for the algorithm to sort the sequence.

The first algorithm I tested was Insertion Sort. I used an array-based implementation of this algorithm where there was a sorted portion of the array, the front end, and an unsorted portion, the back end. I "inserted" each value of the sequence array into the sorted portion using a while loop, and slowly the array would be completely sorted. This algorithm is effective on small input sizes, but it quickly becomes inefficient as the input size grows. This can be seen in my data because the number of comparisons for each input size is drastically larger than the number of comparisons for Merge Sort and Quicksort. The theoretical running time of Insertion Sort is O(c*n^2), and I found the value of c to be 0.25. This indicates that the running time of my implementation of Insertion Sort was a little faster than the expected n^2 because my constant c was very small. However, Insertion Sort is still significantly slower than Merge Sort or Quicksort.

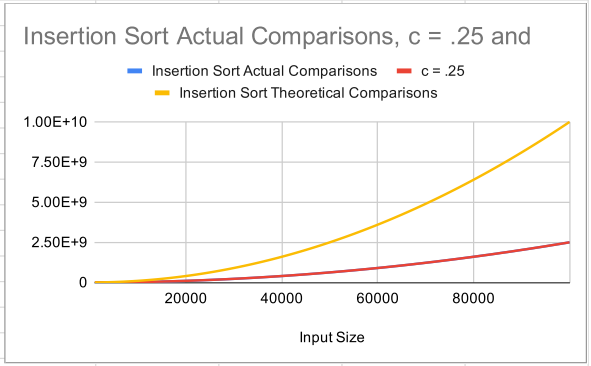
I implemented the Merge Sort algorithm using two separate methods called mergeSort and merge. The mergeSort method handled the recursive calls to mergeSort to split the array in half until there was only one element in each half of the array. Then I used the merge method to merge the smaller arrays back together, sorting the values as they were put back into the original array. This implementation uses the same number of comparisons for all 10 runs on each input size because the array gets split the same number of times and merge must compare the same

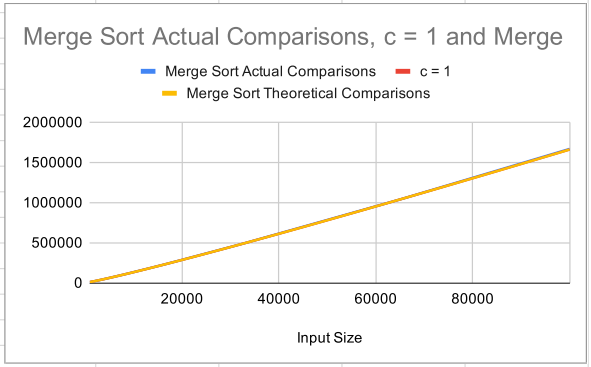
number of elements for each input size. Traditionally Merge Sort has a running time of O(c*nlogn), and I computed my c value to be 1. At each input size, my implementation of Merge Sort performed roughly nlogn comparisons to sort the given sequence.

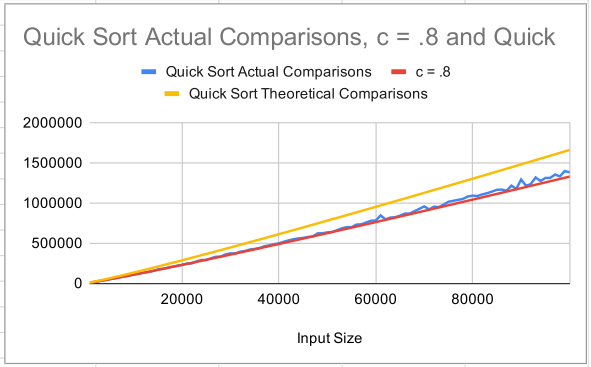
I implemented my Quicksort algorithm using a pivot value equal to the last element in the given sequence. I then used three while loops to partition the values to the correct side of the pivot. One while loop contained two sub-while loops that compared each value left of the pivot's index to the pivot and compared each value right of the pivot's index to the pivot, swapping any elements that were greater or less than the pivot to the correct side of the pivot. When the outer while loop finished, the left index equaled the pivot's correct index, and I performed a swap to place the pivot in the correct spot in the array. I then recursively repeated this process until the entire array was sorted. Traditionally Quicksort has a running time of O(c*nlogn), and I computed my c value to be 0.8. This indicates that the algorithm is running slightly faster than expected, outperforming both Merge Sort and Insertion Sort.

Quicksort is the most efficient algorithm according to my experimental results. For each input size it performed on average less comparisons than the other two algorithms. Quicksort also does not use any secondary structure to sort the sequence, unlike Merge Sort, so while Merge Sort and Quicksort have very similar running times, Quicksort is slightly more efficient than Merge Sort. The experimental data for each algorithm agrees with the Big-O because I was able to find a c value so that the actual number of comparisons was similar to the theoretical number of comparisons. The Big-O predicted comparisons did not match Quicksort as well as the other algorithms because Quicksort has an expected running time of O(nlogn) and a worst case of O(n^2). Therefore we can see slight deviations from the theoretical running time curve in the Quicksort graph, but on average it runs in O(nlogn). (See data and graphs below)

Input Size	Insertion Sort Act	c = .25	Insertion Sort Th	Merge Sort Actua	c = 1	Merge Sort Theo	Quick Sort Actua	c = .8	Quick Sort Theoretical Comparisons
1000	255025.1	250000	1000000	9976	9965.784285	9965.784285	7594.6	7972.627428	9965.784285
2000	1004056.8	1000000	4000000	21952	21931.56857	21931.56857	16561.6	17545.25486	21931.56857
3000	2242508.4	2250000	9000000	34904	34652.24036	34652.24036	27332.2	27721.79228	34652.24036
4000	4010522.9	4000000	16000000	47904	47863.13714	47863.13714	37722.9	38290.50971	47863.13714
5000	6266318.1	6250000	25000000	61808	61438.5619	61438.5619	50018	49150.84952	61438.5619
6000	9015843.4	9000000	36000000	75808	75304.48071	75304.48071	60004	60243.58457	75304.48071
7000	12266626.5	12250000	49000000	89808	89411.97445	89411.97445	72006.9	71529.57956	89411.97445
8000	15981103.1	16000000	64000000	103808	103726.2743	103726.2743	83952.9	82981.01942	103726.2743
9000	20162110.7	20250000	81000000	118616	118221.3836	118221.3836	94484.6	94577.10686	118221.3836
10000	24926652.9	25000000	100000000	133616	132877.1238	132877.1238	106237	106301.699	132877.1238
11000	30218590.6	30250000	121000000	148616	147677.3749	147677.3749	118966.6	118141.8999	147677.3749
12000	36055044.3	36000000	144000000	163616	162608.9614	162608.9614	131106.5	130087.1691	162608.9614
13000	42235704.7	42250000	169000000	178616	177660.912	177660.912	139636.1	142128.7296	177660.912
14000	48941647	49000000	196000000	193616	192823.9489	192823.9489	151938	154259.1591	192823.9489
15000	56203346.5	56250000	225000000	208616	208090.1232	208090.1232	172261.1	166472.0986	208090.1232
16000	63786007.9	64000000	256000000	223616	223452.5486	223452.5486	180142.2	178762.0388	223452.5486
17000	72102783.6	72250000	289000000	239232	238905.2011	238905.2011	192772.5	191124.1609	238905.2011
18000	81230175.2	81000000	324000000	255232	254442.7671	254442.7671	206632.6	203554.2137	254442.7671
19000	90140952.7	90250000	361000000	271232	270060.5242	270060.5242	217975.8	216048.4193	270060.5242
20000	99987148.4	100000000	400000000	287232	285754.2476	285754.2476	229483.5	228603.3981	285754.2476
21000	110556903.5	110250000	441000000	303232	301520.1359	301520.1359	245127.5	241216.1087	301520.1359
22000	120810384	121000000	484000000	319232	317354.7499	317354.7499	246455.9	253883.7999	317354.7499
23000	132310189.6	132250000	529000000	335232	333254.9635	333254.9635	272020.8	266603.9708	333254.9635
24000	143956538.1	144000000	576000000	351232	349217.9228	349217.9228	290289	279374.3383	349217.9228
25000	156216604.2	156250000	625000000	367232	365241.0119	365241.0119	289927.7	292192.8095	365241.0119
26000	168920906.7	169000000	676000000	383232	381321.8241	381321.8241	312025.3	305057.4593	381321.8241
27000	182551295.7	182250000	729000000	399232	397458.1382	397458.1382	330076.5	317966.5106	397458.1382
28000	195577098	196000000	784000000	415232	413647.8978	413647.8978	331915.7	330918.3182	413647.8978
29000	209912016.7	210250000	841000000	431232	429889.1931	429889.1931	360412.8	343911.3545	429889.1931
30000	224979745.3	225000000	900000000	447232	446180.2464	446180.2464	372819.2	356944.1971	446180.2464
31000	239927265.2	240250000	961000000	463232	462519.3984	462519.3984	371359	370015.5188	462519.3984
32000	255859252.3	256000000	1024000000	479232	478905.0971	478905.0971	393015.7	383124.0777	478905.0971
33000	272347244.7	272250000	1089000000	495464	495335.8873	495335.8873	402000.3	396268.7099	495335.8873
34000	288590601.3	289000000	1156000000	512464	511810.4023	511810.4023	420278.1	409448.3218	511810.4023
35000	306242021.2	306250000	1225000000	529464	528327.3556	528327.3556	427724.6	422661.8844	528327.3556
36000	323919386.4	324000000	1296000000	546464	544885.5343	544885.5343	438260.7	435908.4274	544885.5343
37000	342079194.7	342250000	1369000000	563464	561483.7931	561483.7931	461250.4	449187.0344	561483.7931
38000	360753999.5	361000000	1444000000	580464	578121.0483	578121.0483	471877.5	462496.8387	578121.0483
39000	380937171.8	380250000	1521000000	597464	594796.2736	594796.2736	486767.2	475837.0189	594796.2736
40000	399494294.3	400000000	1600000000	614464	611508.4952	611508.4952	498197.7	489206.7961	611508.4952
41000	419831887.5	420250000	1681000000	631464	628256.7879	628256.7879	516696.7	502605.4303	628256.7879
42000	441336618.2	441000000	1764000000	648464	645040.2717	645040.2717	534558.8	516032.2174	645040.2717
43000	462810906.4	462250000	1849000000	665464	661858.1087	661858.1087	548233.6	529486.487	661858.1087
44000	483644613	484000000	1936000000	682464	678709.4997	678709.4997	559327.2	542967.5998	678709.4997
45000	506635492.5	506250000	2025000000	699464	695593.6821	695593.6821	565136.7	556474.9457	695593.6821
46000	529853727.3	529000000	2116000000	716464	712509.9271	712509.9271	577047.5	570007.9417	712509.9271
47000	552096211.9	552250000	2209000000	733464	729457.5374	729457.5374	583518.8	583566.0299	729457.5374
48000	576560467.1	576000000	2304000000	750464	746435.8457	746435.8457	622205.9	597148.6766	746435.8457
49000	600229310.2	600250000	2401000000	767464	763444.2123	763444.2123	623365	610755.3698	763444.2123
50000	625157100.9	625000000	2500000000	784464	780482.0237	780482.0237	633866.9	624385.619	780482.0237
51000	650539200.9	650250000	2601000000	801464	797548.691	797548.691	639584.6	638038.9528	797548.691







52000	675633844.1	676000000	2704000000	818464	814643.6481	814643.6481	661095.8	651714.9185	814643.6481					
53000	702219586.5	702250000	2809000000	835464	831766.3512	831766.3512	684787	665413.0809	831766.3512					
54000	729572690.6	729000000	2916000000	852464	848916.2765	848916.2765	699421.6	679133.0212	848916.2765					
55000	755185671.4	756250000	3025000000	869464	866092.9199	866092.9199	698834.3	692874.3359	866092.9199					
56000	784499071.4	784000000	3136000000	886464	883295.7956	883295.7956	730978	706636.6365	883295.7956					
57000	812549842.8	812250000	3249000000	903464	900524.435	900524.435	734801.5	720419.548	900524.435					
58000	839561461.1	841000000	3364000000	920464	917778.3862	917778.3862	757049	734222.709	917778.3862					
59000	870267661.8	870250000	3481000000	937464	935057.2127	935057.2127	779443.8	748045.7702	935057.2127					
60000	900184038.5	900000000	3600000000	954464	952360.4928	952360.4928	781933	761888.3943	952360.4928					
61000	928424620.4	930250000	3721000000	971464	969687.819	969687.819	845505.4	775750.2552	969687.819					
62000	960975419.4	961000000	3844000000	988464	987038.7969	987038.7969	796383	789631.0375	987038.7969					
63000	992350433.9	992250000	3969000000	1005464	1004413.045	1004413.045	818259.2	803530.4361	1004413.045					
64000	1023517742	1024000000	4096000000	1022464	1021810.194	1021810.194	823824	817448.1554	1021810.194					
65000	1055794034	1056250000	4225000000	1039464	1039229.886	1039229.886	842118.5	831383.9091	1039229.886					
66000	1089232193	1089000000	4356000000	1056928	1056671.775	1056671.775	868464.5	845337.4197	1056671.775					
67000	1121481514	1122250000	4489000000	1074928	1074135.523	1074135.523	868684.6	859308.4183	1074135.523					
68000	1157843184	1156000000	4624000000	1092928	1091620.805	1091620.805	898483.5	873296.6436	1091620.805					
69000	1189317963	1190250000	4761000000	1110928	1109127.303	1109127.303	927342.9	887301.8425	1109127.303					
70000	1225534409	1225000000	4900000000	1128928	1126654.711	1126654.711	957881.2	901323.7689	1126654.711					
71000	1260486894	1260250000	5041000000	1146928	1144202.73	1144202.73	920421.8	915362.1838	1144202.73					
72000	1295073397	1296000000	5184000000	1164928	1161771.069	1161771.069	954527.5	929416.8549	1161771.069					
73000	1333084939	1332250000	5329000000	1182928	1179359.446	1179359.446	949143.1	943487.5565	1179359.446					
74000	1367972044	1369000000	5476000000	1200928	1196967.586	1196967.586	979519.6	957574.0689	1196967.586					
75000	1406704974	1406250000	5625000000	1218928	1214595.223	1214595.223	1015458.5	971676.1785	1214595.223					
76000	1444111340	1444000000	5776000000	1236928	1232242.097	1232242.097	1027780	985793.6773	1232242.097					
77000	1481318545	1482250000	5929000000	1254928	1249907.954	1249907.954	1038894.7	999926.3628	1249907.954					
78000	1521549327	1521000000	6084000000	1272928	1267592.547	1267592.547	1051228	1014074.038	1267592.547					
79000	1559327668	1560250000	6241000000	1290928	1285295.638	1285295.638	1080848.6	1028236.51	1285295.638					
80000	1601172889	1600000000	6400000000	1308928	1303016.99	1303016.99	1091430	1042413.592	1303016.99					
81000	1641208993	1640250000	6561000000	1326928	1320756.377	1320756.377	1087973.1	1056605.102	1320756.377					
82000	1683430736	1681000000	6724000000	1344928	1338513.576	1338513.576	1107748.9	1070810.861	1338513.576					
83000	1721805683	1722250000	6889000000	1362928	1356288.368	1356288.368	1122431	1085030.695	1356288.368					
84000	1763719072	1764000000	7056000000	1380928	1374080.543	1374080.543	1142705.4	1099264.435	1374080.543					
85000	1807125915	1806250000	7225000000	1398928	1391889.894	1391889.894	1164728.6	1113511.915	1391889.894					
86000	1853106592	1849000000	7396000000	1416928	1409716.217	1409716.217	1167195.3	1127772.974	1409716.217					
87000	1891079084	1892250000	7569000000	1434928	1427559.317	1427559.317	1153262.2	1142047.454	1427559.317					
88000	1934282612	1936000000	7744000000	1452928	1445418.999	1445418.999	1216533.2	1156335.2	1445418.999					
89000	1980304614	1980250000	7921000000	1470928	1463295.077	1463295.077	1175266.7	1170636.061	1463295.077					
90000	2025289397	2025000000	8100000000	1488928	1481187.364	1481187.364	1292856.1	1184949.891	1481187.364					
91000	2070390936	2070250000	8281000000	1506928	1499095.682	1499095.682	1214842.3	1199276.546	1499095.682					
92000	2113682691	2116000000	8464000000	1524928	1517019.854	1517019.854	1236595.2	1213615.883	1517019.854					
93000	2161713610	2162250000	8649000000	1542928	1534959.708	1534959.708	1318626	1227967.766	1534959.708					
94000	2206720119	2209000000	8836000000	1560928	1552915.075	1552915.075	1273971.5	1242332.06	1552915.075					
95000	2252546550	2256250000	9025000000	1578928	1570885.79	1570885.79	1312081.7	1256708.632	1570885.79					
96000	2305340827	2304000000	9216000000	1596928	1588871.691	1588871.691	1312450.2	1271097.353	1588871.691					
97000	2354302432	2352250000	9409000000	1614928	1606872.621	1606872.621	1354631.7	1285498.097	1606872.621					
98000	2400291155	2401000000	9604000000	1632928	1624888.425	1624888.425	1331417.8	1299910.74	1624888.425					
99000	2450384790	2450250000	9801000000	1650928	1642918.95	1642918.95	1398528.7	1314335.16	1642918.95					
100000	2500154622	2500000000	10000000000	1668928	1660964.047	1660964.047	1381787.8	1328771.238	1660964.047					