Week-1
Mathematics for Data Science - 2
Vectors and Matrices
Assignment

1. Match the matrices in column A with their properties in column B and answer the following question.

	Matrix (Column A)		Properties of matrix (Column B)
a)	$\begin{bmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ 5 & 6 & 7 \end{bmatrix}$	i)	has determinant 0
b)	$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	ii)	is a scalar matrix
c)	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$	iii)	is a lower triangular matrix but not a diagonal matrix
d)	$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$	iv)	is a diagonal matrix but not a scalar matrix

Table: M2W1G1

Which of the following are true? (MSQ)

☐ **Option 1:** a) $\rightarrow$ iii)

☐ Option 2: a) $\rightarrow$ ii)

☐ **Option 3:** b) $\rightarrow$ i)

☐ Option 4: b) $\rightarrow$ ii)

- ☐ **Option 5:** c) $\rightarrow$ iv)
- ☐ Option 6: c) $\rightarrow$ iii)
- ☐ Option 7: d) $\rightarrow$ iv)
- ☐ **Option 8:** d) $\rightarrow$ ii)

Solution:

a) $\begin{bmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ 5 & 6 & 7 \end{bmatrix}$

As all the elements of the matrix, above the diagonal elements are zero, it is a lower triangular matrix.

Also there are elements below the diagonal that are non-zero. So it is not a diagonal matrix.

Hence option (iii) 'is a lower triangular matrix but not a diagonal matrix' is correct.
Option 1: a) $\rightarrow$ iii) is correct.

b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

The determinant of the above matrix is zero. **OR**

By applying the elementary row transformation $R_1 \rightarrow R_1 - R_2$, we have $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. The determinant of this matrix is 0 and note that the row operation $R_1 \rightarrow R_1 - R_2$ does not change the determinant of the matrix and hence the determinant of the original matrix is zero.

So option (i) 'has determinant 0', is correct.
Option 3: b) $\rightarrow$ i) is correct.

c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Only the diagonal elements are non-zero, moreover they are not the same so it is diagonal matrix but not a scalar matrix.

Hence the option (iv) 'is diagonal matrix but not a scalar matrix', is correct.
Option 5: c) $\rightarrow$ iv) is correct.

d) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

As only the diagonal elements are non-zero, moreover they are same i.e. "2" only so it is a scalar matrix.

Hence the option (ii) 'is a scalar matrix', is correct.
Option 8: d) $\rightarrow$ ii) is correct.

2. Match the systems of linear equations in Column A with their number of solutions in column B and their geometric representation in Column C and answer the following question.

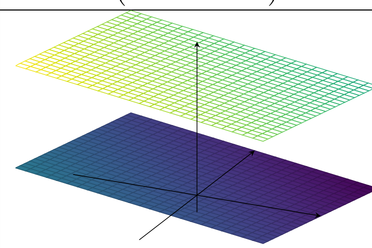
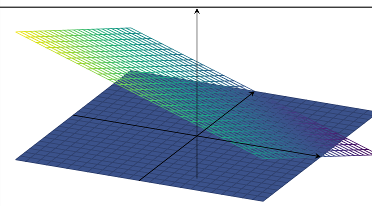
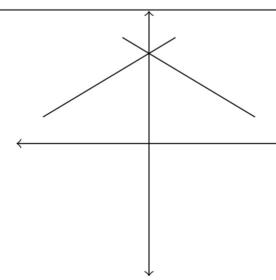
	System of linear equations (Column A)		Number of solutions (Column B)		Geometric representations (Column C)
i)	$x + y = 3$, $x - y = -3$	a)	Infinite solutions	1)	
ii)	$x + y + z = 1$, $x + y + z = 7$	b)	Unique solution	2)	
iii)	$z = 0$, $x + y + z = 1$	c)	No solution	3)	

Table: M2W1G2

Which of the following are true? (MSQ)

☐ **Option 1:** i) $\rightarrow$ b) $\rightarrow$ 3)

☐ Option 2: i) $\to$ a) $\to$ 3)

☐ Option 3: ii) $\to$ c) $\to$ 2)

☐ **Option 4:** ii) $\to$ c) $\to$ 1)

☐ **Option 5:** iii) $\to$ a) $\to$ 2)

☐ Option 6: iii) $\to$ a) $\to$ 1)

Solution:

i) Given $x+y=3$, $x-y=-3$ a system of two linear equations in two unknowns (variables) x and y.

Solving these equations we get $x=0$, $y=3$. (i.e., Adding these two equations we get $2x=0$, so $x=0$. Substituting $x=0$ in the first equation we have $y=3$.)

As we get only one pair of values for x and y, so the given system of linear equations has a unique solution.
So from Column (B) the option (b) is correct.

Now, in $\mathbb{R}^2$ both the equations represent straight lines and they intersect each other at $x=0$, $y=3$ i.e. $(0,3)$ which is on Y-axis. So from Column (C) the option (3) is correct.

Hence, option 1: i) $\to$ b) $\to$ 3) is correct.

ii) Given $x+y+z=1$, $x+y+z=7$ a system of two linear equations in three unknowns (variables) x, y and z. If there exists a solution (a,b,c) of this system of linear equations then the point (a,b,c) should lie on both the planes, (as these system of linear equations represents plane) which gives us $a+b+c=1$ and $a+b+c=7$ implies $1=7$ which is not true.

Hence there cannot exist any such point. This implies the system of linear equations has no solution. So from Column (B) the option (c) is correct.
Observe that, each equation represents a plane on the coordinate system and the two equations will have no solution if and only if the two planes that represent these two equations are parallel. So from Column (C) the option (1) is correct.

So, option 4: ii) $\to$ c) $\to$ 1) is correct.

iii) Given $z=0$, $x+y+z=1$. $z=0$ denotes the XY-plane and $x+y+z=1$ denotes a plane on the coordinate system.

Substituting $z=0$ in the second equation we get, $x+y=1 \implies y=1-x$, i.e., for any arbitrary value of x say k we get some arbitrary value of y which is $y=1-k$.

$\implies$ Any point of the form $(k,1-k,0)$ will satisfy both the equations, where $k\in\mathbb{R}$.

So, the system of linear equations has infinitely many solutions. So from Column (B), option (a) is correct.

As $z=0$ denotes the XY-plane and $x+y+z=1$ denotes a plane on the coordinate system, intersection of these two planes gives a line (has infinitely many points). So from Column (C) the option (2) is correct.

Hence, option 5: iii) $\to$ a) $\to$ 2) is correct.

3. Let $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix}$. Which of the following options are true for a matrix A, such that $AB = C$? (MSQ)

   - ☐ Such a matrix does not exist.
   - ☐ There is a unique matrix A satisfying this property.
   - ☐ **There are infinitely many such matrices.**
   - ☐ **A should be a 2×3 matrix.**
   - ☐ A should be a 3×2 matrix.

   Solution:

   If A is a matrix of order $m \times n$ and B is a matrix of order $n \times p$, then AB must be a matrix of order $m \times p$.

   Here, B is a matrix of order 3×2, so $n = 3$, $p = 2$ and the resultant matrix $C = AB$ is of order 2×2, so $m = 2$, $p = 2$. Hence A must be a matrix of order 2×3.

   Now, let us take any arbitrary matrix A of order 2×3 and try to see the conditions on the elements of A for which $AB = C$.

   Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

   $AB = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} a_{11} - a_{13} & a_{12} \\ a_{21} - a_{23} & a_{22} \end{bmatrix}$

   So, $\begin{bmatrix} a_{11} - a_{13} & a_{12} \\ a_{21} - a_{23} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix}$

   Comparing the elements we get, $a_{11} - a_{13} = 0 \implies a_{11} = a_{13}$; $a_{12} = 0$; $a_{21} - a_{23} = -1 \implies a_{21} = a_{23} - 1$ and $a_{22} = 2$.

   Hence,
   $$A = \begin{bmatrix} a_{11} & 0 & a_{11} \\ a_{23} - 1 & 2 & a_{23} \end{bmatrix}$$
   where a_{11} and a_{23} can take any real number. So, there are infinitely many such matrices A such that $AB = C$ and the order of the matrix A is 2×3.

   Hence options **"There are infinitely many such matrices." and "A should be a 2×3 matrix." are correct.**

4. Let A be a 2×2 real matrix and let $trace(A)$ denote the sum of the elements in the diagonal of A. Which of the following are true? (MSQ)

   - ☐ $det(A - cI)$ is a polynomial in c of degree 1.
   - ☐ $\boldsymbol{det(A - cI)}$ **is a polynomial in** $\boldsymbol{c}$ **of degree 2.**
   - ☐ $\boldsymbol{det(A - cI) = c^2 - trace(A)c + det(A)}$
   - ☐ $det(A - cI) = c^2 + trace(A)c - det(A)$
   - ☐ $det(A - cI) = trace(A)c - det(A)$
   - ☐ $det(A - cI) = -trace(A)c + det(A)$

   Solution:
   Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$. Then $trace(A) = p + s$ and $det(A) = ps - rq$

   Now, $A - cI = \begin{bmatrix} p & q \\ r & s \end{bmatrix} - c\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} p-c & q \\ r & s-c \end{bmatrix}$

   $$\begin{aligned} det(A - cI) &= (p - c)(s - c) - rq \\ &= ps - cs - cp + c^2 - rq \\ &= c^2 - c(p + s) + ps - rq \\ &= c^2 - trace(A) + det(A) \end{aligned}$$

   So, $det(A-cI)$ is a polynomial in c of degree 2 and $det(A-cI) = c^2 - trace(A) + det(A)$.

   Hence options (2) $det(A - cI)$ **is a polynomial in c of degree 2 and option (3)** $det(A - cI) = c^2 - trace(A) + det(A)$ **are correct.**

5. Suppose there are two types of oranges and two types of bananas available in the market. Suppose 1 kg of each type of orange costs ₹50 and 1 kg of each type of banana costs ₹40. Gargi bought x kg of the first type of each fruit, orange and banana, and y kg of the second type of each fruit, orange and banana. She paid ₹250 for oranges and ₹200 for bananas. Which of the following options are correct with respect to the given information? (MSQ)

   - ☐ **Option 1:** The matrix representation to find x and y can be

   $$\begin{bmatrix} 50 & 50 \\ 40 & 40 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 250 \\ 200 \end{bmatrix}$$

   - ☐ Option 2: The matrix representation to find x and y can be

   $$\begin{bmatrix} 50 & 40 \\ 50 & 40 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 250 \\ 200 \end{bmatrix}$$

☐ **Option 3:** The matrix representation to find x and y can be

$$\begin{bmatrix} 40 & 40 \\ 50 & 50 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 250 \end{bmatrix}$$

☐ **Option 4:** x can be 2 and y can be 3.

☐ **Option 5:** There are infinitely many real values possible for x and y.

☐ Option 6: There are only finitely many real values possible for x and y.

☐ **Option 7:** There are only finitely many natural numbers possible for x and y.

Solution:
Given there are two types of oranges, say type 1 orange, type 2 orange and two types of bananas, say type 1 banana and type 2 banana available in the market, 1 kg of each type of orange costs ₹50 and 1 kg of each type of banana costs ₹40. Gargi bought x kg of the first type of each fruit i.e., x kg of type 1 orange and x kg of type 1 banana, and y kg of the second type of each fruit, i.e., y kg of type 2 orange and y kg of type 2 banana.

She paid ₹250 for oranges i.e., for purchasing x kg of orange of type 1 and y kg of orange of type 2 and ₹200 for bananas i.e., for purchasing x kg of bananas of type 1 and y kg of bananas of type 2.

i.e.,

Gargi brought	Gargi paid
Orange type 1 : x kg Banana type 1 : x kg	Oranges : ₹250
Orange type 2 : y kg Banana type 2 : y kg	Bananas : ₹200
Orange type 1 cost : ₹50 Banana type 1 cost : ₹40	Orange type 2 cost : ₹50 Banana type 2 cost : ₹40

Hence,

$$50x + 50y = 250 \qquad \text{(Equation for oranges)} \qquad (1)$$
$$40x + 40y = 200 \qquad \text{(Equation for Bananas)} \qquad (2)$$

The matrix representation of the above equations are:

$$\begin{bmatrix} 50 & 50 \\ 40 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 250 \\ 200 \end{bmatrix}$$

So option 1 is correct.

If we interchange the order of the equations we get:

$$\begin{aligned} 40x + 40y &= 200 && \text{(Equation for Bananas)} \\ 50x + 50y &= 250 && \text{(Equation for oranges)} \end{aligned}$$

The matrix representation of the above equations are:

$$\begin{bmatrix} 40 & 40 \\ 50 & 50 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 250 \end{bmatrix}$$

So option 3 is correct.

Now from the above equations we get,
$50(x + y) = 250 \implies x + y = 5$
$40(x + y) = 200 \implies x + y = 5$
Hence, the solution of the system of linear equations is $(a, 5 - a)$, where a can take any arbitrary real number but in this context, both of them should be positive i.e., $a \geq 0$, $5 - a \geq 0 \implies 5 \geq a \implies 0 \leq a \leq 5$. Hence, a can be any real number in between 0 and 5.
Clearly, there are infinitely many real values possible for x and y and also $x = 2$ and $y = 3$ can be one possible solution.

So option (4) and (5) are also correct.

As solutions are of the form $(a, 5 - a)$ and $0 \leq a \leq 5$ so, there will be finitely many natural numbers possible as solutions they are :
$x = 0$, $y = 5$
$x = 1$, $y = 4$
$x = 2$, $y = 3$
$x = 3$, $y = 2$
$x = 4$, $y = 1$
$x = 5$, $y = 0$
So, Option (7) is correct.

6. Suppose $det(3A) = n \times det(A)$ for any 3×3 real matrix A. What is the value of n?
[Answer: 27]
Solution:
If any real number c is multiplied to a row of a square matrix M, then the determinant of the new matrix will be c times the determinant of the matrix M.

Now let c is a real number and A is a square matrix, cA means c is multiplied with all the elements of matrix A.

If A is a square matrix of order $p \times p$, then A has p rows. Hence $det(cA) = c^p det(A)$.

Here $c = 3$ and $p = 3$ substituting these values, we get $det(3A) = 3^3 det(A) = 27 det(A)$.
So, $n = 27$.

7. Suppose $A = \begin{bmatrix} 2019 & 100 & 2119 \\ 2020 & 200 & 2220 \\ 2021 & 300 & 2321 \end{bmatrix}$. What will be the value of $det(A)$? [Answer: 0]

Solution:

We know that adding a scalar multiple of one row (column) with another row does not change the value of determinant of a matrix.

$$\begin{vmatrix} 2019 & 100 & 2119 \\ 2020 & 200 & 2220 \\ 2021 & 300 & 2321 \end{vmatrix} \xrightarrow{R_3 - R_2} \begin{vmatrix} 2019 & 100 & 2119 \\ 2020 & 200 & 2220 \\ 1 & 100 & 101 \end{vmatrix} \xrightarrow{R_2 - R_1} \begin{vmatrix} 2019 & 100 & 2119 \\ 1 & 100 & 101 \\ 1 & 100 & 101 \end{vmatrix}$$

As two rows of the last matrix are identical, the determinant will be zero.

Hence, $det(A) = 0$.

8. Let A be a square matrix such that $A^2 = A$. If $(I + A)^3 - 1A = I + mA$, then find the value of m. (NAT)

Solution: Given $A^2 = A \implies A.A^2 = A.A \implies A^3 = A^2 = A$

Now,

$$\begin{aligned} (I + A)^3 - 1.A &= I + m.A \\ I^3 + 3I^2A + 3IA^2 + A^3 - 1.A &= I + m.A \\ I + 3.A + 3.A^2 + A^3 - 1.A &= I + m.A \\ 3.A + 3.A + A - 1.A &= m.A \\ 7.A - 1.A &= m.A \\ m &= 7 - 1 = 6 \end{aligned}$$

9. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, then what will be the determinant of A ? (NAT)

Solution: Given $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$.

Note if each element of any row (or column) of a matrix is multiplied by the same number them the value of determinant is multiplied by that number.

Adding a scalar multiple of one row with another does not change the determinant of a matrix.

$$|A| = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} \xrightarrow{R_3 - R_1} \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 2 & 2 & 2 \end{vmatrix} \xrightarrow{R_1 - R_2} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 2 & 2 \end{vmatrix} = 2 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$

As two rows of the last matrix are identical, the determinant will be zero.

Hence, $det(A) = 0$**.**

10. Let A be a square matrix of order 3 and B be a matrix that is obtained by adding the first row of A to the third row of A and adding 3 times the second row of A to the first row of A. If $det(A) = 1$, then find out the value of $det(2A^2B^{-1})$.

Solution:

Given A is a square matrix of order 3, and B is a matrix that is obtained by adding the first row of A to the third row of A and adding 3 times the second row of A to the first row of A. Observe that the determinant will not be changed if we perform any row operations of the above mentioned type. Thus $det(B) = det(A)$.

Now $det(B^{-1}) = \frac{1}{det(B)} = \frac{1}{det(A)}$, $det(AB) = det(A)\ det(B)$ and $det(kA) = k^n\ det(A)$ where A is any square matrix of order n and k is any scalar.

Thus we have

$$\begin{aligned} det(2A^2B^{-1}) &= 2^3 det(A^2B^{-1}) \\ &= 8det(A^2)\ det(B^{-1}) \\ &= 8det(A)\ det(A)\ \frac{1}{det(B)} \\ &= 8det(A)\ det(A)\ \frac{1}{det(A)} \\ &= 8(1),\ \text{since } det(A) = 1 \\ &= 8. \end{aligned}$$

11. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then what will be the value of the sum of the diagonal elements of A^4?

Solution: Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Let $\text{tr}(A)$ denote the trace of a matrix A, i.e., the sum of all diagonal elements of the matrix A.

Thus $\text{tr}(A) = 1 + 1 + 1 = 3$. Now

$$\begin{aligned} A^2 = A.A &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+1+1 & 1+1+1 & 1+1+1 \\ 1+1+1 & 1+1+1 & 1+1+1 \\ 1+1+1 & 1+1+1 & 1+1+1 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} \end{aligned}$$

Thus $\text{tr}(A^2) = 3 + 3 + 3 = 9 = 3^2 = (tr(A))^2$. Also

$$\begin{aligned} A^3 = A^2.A &= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} . \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3+3+3 & 3+3+3 & 3+3+3 \\ 3+3+3 & 3+3+3 & 3+3+3 \\ 3+3+3 & 3+3+3 & 3+3+3 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix} \end{aligned}$$

Thus $\text{tr}(A^3) = 9 + 9 + 9 = 27 = 3^3 = (tr(A))^3$
Similarly, $\text{tr}(A^4) = 27 + 27 + 27 = 81 = 3^4 = (tr(A))^4$

12. Let $A = [\alpha_{ij}]$ be a square matrix of order 3, where $\alpha_{ij} = i + j$. Find $det(A)$.

Solution: Given $A = [\alpha_{ij}]$ be a square matrix of order 3, where $\alpha_{ij} = i+j$, $1 \leq i, j \leq 3$.

So, $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$.

Note that adding a scalar multiple of one row with another does not change the determinant of the matrix.

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \xrightarrow{R_3 - R_2} \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Since two rows of the last matrix are identical, the determinant will be zero.

Hence, $det(A) = 0$.

Comprehension Type Question:

Suppose there are three families F_1, F_2, F_3 living in different cities and they pay ₹x_1, ₹x_2, ₹x_3 per unit respectively for electricity consumption each month. In January 2021, the electricity consumption by F_1, F_2, and F_3 is 30 units, 20 units, and 25 units, respectively. In February 2021, it is 20 units, 35 units, and 25 units, respectively. In March 2021, it is 20 units, 10 units, and 15 units, respectively. The total amount paid by the three families together for electricity consumption in January, February, and March is ₹670, ₹730, and ₹400 respectively.

Answer the following questions using this given data.

13. If we want to find x_1, x_2, x_3 by solving a system of linear equations represented by the matrix form $Ax = b$, where $x = (x_1, x_2, x_3)^T$, and $b = \begin{bmatrix} 670 \\ 730 \\ 400 \end{bmatrix}$, then which of the following options are correct? (MSQ)

   - ☐ **Option 1:** The first row of the matrix A is $\begin{bmatrix} 30 & 20 & 25 \end{bmatrix}$.
   - ☐ Option 2: The second row of the matrix A is $\begin{bmatrix} 20 & 35 & 10 \end{bmatrix}$.
   - ☐ **Option 3:** The third row of the matrix A is $\begin{bmatrix} 20 & 10 & 15 \end{bmatrix}$.
   - ☐ Option 4: The third row of the matrix A is $\begin{bmatrix} 20 & 10 & 25 \end{bmatrix}$.

   Solution:
   As per the given data we have,

Months (in 2021)	Electric F_1	Consumption F_2	(in units) F_3	Total payments by $F_1, F_2 \& F_3$ together
January	30	20	25	670
February	20	35	25	730
March	20	10	15	400

F_1, F_2, F_3 live in different cities and they pay ₹x_1, ₹x_2, ₹x_3 per unit respectively for electricity consumption each month, that is, F_1 pays ₹x_1, F_2 pays ₹x_2, F_3 pays ₹x_3. Hence we have a system of three equations in three unknowns.

$$\begin{aligned} 30x + 20y + 25z &= 670 \\ 20x + 35y + 25z &= 730 \\ 20x + 10y + 15z &= 400 \end{aligned}$$

The matrix representation of the above system in the form of $AX = B$ is:

$$\begin{bmatrix} 30 & 20 & 25 \\ 20 & 35 & 25 \\ 20 & 10 & 15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 670 \\ 730 \\ 400 \end{bmatrix}$$

where $A = \begin{bmatrix} 30 & 20 & 25 \\ 20 & 35 & 25 \\ 20 & 10 & 15 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 670 \\ 730 \\ 400 \end{bmatrix}$.

Therefore, the first row of the matrix A is $\begin{bmatrix} 30 & 20 & 25 \end{bmatrix}$.

The second row of the matrix A is $\begin{bmatrix} 20 & 35 & 25 \end{bmatrix}$.

The third row of the matrix A is $\begin{bmatrix} 20 & 10 & 15 \end{bmatrix}$.

Hence, options 1, and 3 are correct.

14. If x' is the solution of $Ax = b$, where $x' = (x'_1, x'_2, x'_3)^T$, then find the value of $x'_1 + x'_2 + x'_3$.

Solution:

Given $x' = (x'_1, x'_2, x'_3)^T$, be the solution of $AX = B$.

$\implies Ax' = B$.

$\implies \begin{bmatrix} 30 & 20 & 25 \\ 20 & 35 & 25 \\ 20 & 10 & 15 \end{bmatrix} \begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix} = \begin{bmatrix} 670 \\ 730 \\ 400 \end{bmatrix}$.

We have a system of three equations in three unknowns.

$$30x_1' + 20x_2' + 25x_3' = 670 \tag{1}$$
$$20x_1' + 35x_2' + 25x_3' = 730 \tag{2}$$
$$20x_1' + 10x_2' + 15x_3' = 400 \tag{3}$$

Subtracting 1 from 2, we get

$$-10x_1' + 15x_2' = 60$$
$$\implies -2x_1' + 3x_2' = 12 \tag{4}$$

Now, $6 \times (2) - 10 \times (3)$ gives

$$120x_1' + 210x_2' + 150x_3' = 4380$$
$$200x_1' + 100x_2' + 150x_3' = 4000$$
$$\implies -80x_1' + 110x_2' = 380 \tag{5}$$

$4 \times (4) - (5)$ gives

$$-8x_1' + 12x_2' = 48$$
$$-8x_1' + 11x_2' = 38$$
$$\implies x_2' = 10 \tag{6}$$

Substituting the value of x_2' in 4, we get,

$$-2x_1' + 3(10) = 12$$
$$\implies -2x_1' + 30 = 12$$
$$\implies -2x_1' = -18$$
$$\implies x_1' = 9$$

Finally, substituting the values of x_1' and x_2' in 1, we get

$$30(9) + 20(10) + 25x_3' = 670$$
$$\implies 270 + 200 + 25x_3' = 670$$
$$\implies 25x_3' = 200$$
$$\implies x_3' = 8$$

Thus $x_1' + x_2' + x_3' = 9 + 10 + 8 = 27$.

15. Which of the following are correct? (MSQ)

☐ **Option A:**
$det(A) = 30 \times det\left(\begin{bmatrix}35 & 25\\10 & 15\end{bmatrix}\right) - 20 \times det\left(\begin{bmatrix}20 & 25\\20 & 15\end{bmatrix}\right) + 25 \times det\left(\begin{bmatrix}20 & 35\\20 & 10\end{bmatrix}\right)$

☐ **Option B:**
$det(A) = 30 \times det\left(\begin{bmatrix}35 & 25\\10 & 15\end{bmatrix}\right) + 20 \times det\left(\begin{bmatrix}25 & 20\\15 & 20\end{bmatrix}\right) + 25 \times det\left(\begin{bmatrix}20 & 35\\20 & 10\end{bmatrix}\right)$

☐ **Option C:**
$det(A) = -20 \times det\left(\begin{bmatrix}20 & 25\\10 & 15\end{bmatrix}\right) + 35 \times det\left(\begin{bmatrix}30 & 25\\20 & 15\end{bmatrix}\right) - 25 \times det\left(\begin{bmatrix}30 & 20\\20 & 10\end{bmatrix}\right)$

☐ Option D:
$det(A) = 20 \times det\left(\begin{bmatrix}20 & 25\\10 & 15\end{bmatrix}\right) - 35 \times det\left(\begin{bmatrix}30 & 25\\20 & 15\end{bmatrix}\right) + 25 \times det\left(\begin{bmatrix}30 & 20\\20 & 10\end{bmatrix}\right)$

Solution:
Here $A = \begin{bmatrix}\mathbf{30} & \mathbf{20} & \mathbf{25}\\20 & 35 & 25\\20 & 10 & 15\end{bmatrix}$. So
$det(A) = 30 \times det\left(\begin{bmatrix}35 & 25\\10 & 15\end{bmatrix}\right) - 20 \times det\left(\begin{bmatrix}20 & 25\\20 & 15\end{bmatrix}\right) + 25 \times det\left(\begin{bmatrix}20 & 35\\20 & 10\end{bmatrix}\right)$

Hence option A is correct.

$$\begin{aligned}
det(A) &= 30 \times \begin{vmatrix}35 & 25\\10 & 15\end{vmatrix} - 20 \times \begin{vmatrix}20 & 25\\20 & 15\end{vmatrix} + 25 \times \begin{vmatrix}20 & 35\\20 & 10\end{vmatrix}\\
&= 30(35 \times 15 - 10 \times 25) - 20(20 \times 15 - 25 \times 20) + 25(20 \times 10 - 20 \times 25)\\
&= 30(525 - 250) - 20(300 - 500) + 25(200 - 700)\\
&= 30(275) - 20(-200) + 25(-500)\\
&= 8250 + 4000 - 12500 = -250
\end{aligned}$$

Hence $det(A) = -250$.

As we know that interchanging the columns of a matrix changes the sign of the determinant, we have
$det(A) = 30 \times det\left(\begin{bmatrix}35 & 25\\10 & 15\end{bmatrix}\right) - 20 \times (-det\left(\begin{bmatrix}20 & 25\\20 & 15\end{bmatrix}\right)) + 25 \times det\left(\begin{bmatrix}20 & 35\\20 & 10\end{bmatrix}\right)$
Hence, $det(A) = 30 \times det\left(\begin{bmatrix}35 & 25\\10 & 15\end{bmatrix}\right) + 20 \times det\left(\begin{bmatrix}20 & 25\\20 & 15\end{bmatrix}\right) + 25 \times det\left(\begin{bmatrix}20 & 35\\20 & 10\end{bmatrix}\right)$

Hence option B is correct.

Similarly, expanding for the determinant of A along the second row, we get $det(A) = -20 \times det\left(\begin{bmatrix}20 & 25\\10 & 15\end{bmatrix}\right) + 35 \times det\left(\begin{bmatrix}30 & 25\\20 & 15\end{bmatrix}\right) - 25 \times det\left(\begin{bmatrix}30 & 20\\20 & 10\end{bmatrix}\right)$

Thus option C is correct.

For option D, we expand along the second column of A.

$$\begin{aligned}
det(A) &= 20 \times det\left(\begin{bmatrix} 20 & 25 \\ 10 & 15 \end{bmatrix}\right) - 35 \times det\left(\begin{bmatrix} 30 & 25 \\ 20 & 15 \end{bmatrix}\right) + 25 \times det\left(\begin{bmatrix} 30 & 20 \\ 20 & 10 \end{bmatrix}\right) \\
&= 20(20 \times 15 - 10 \times 25) - 35(30 \times 15 - 20 \times 25) + 25(30 \times 10 - 20 \times 20) \\
&= 20(300 - 250) - 35(450 - 500) + 25(300 - 400) \\
&= 20(50) - 35(-50) + 25(-100) \\
&= 1000 + 1750 - 2500 = 250 \neq det(A)
\end{aligned}$$

Hence, option D is not correct