Lecture Computer Graphics
Assignment 11 – Exercise Exam

Handout date: 6.12.2019

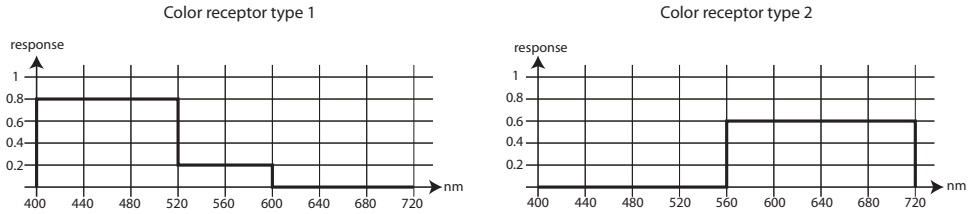
Submission deadline: 13.12.2019 12:00

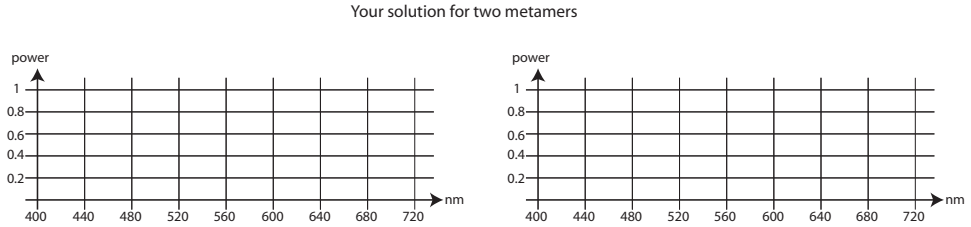
Late submissions are not accepted

1 Colors

(a) **(2 points)** Explain the term *metamer*.

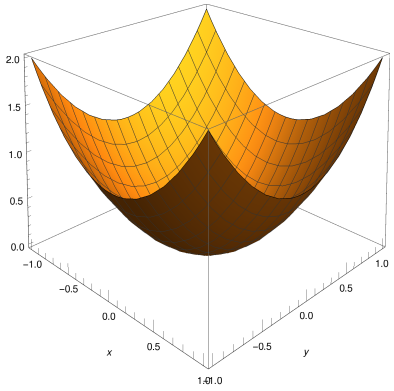
(b) **(2 points)** Assume there is an exotic animal species that has two types of color receptors with response curves as shown below. Sketch two spectral distributions that are metamers for the color perception of this species. Use the empty graphs in the sketch and give a short explanation for your solution. Note that there are many possible solutions.





2 Raytracing

(a) **(3 points)** Solve for the intersection(s) of a generic ray $\mathbf{o} + t\mathbf{d}$ with the paraboloid defined by the graph $z = x^2 + y^2$. Express your answer as the intersection distance value(s) "t" along the ray in terms of the ray direction unit vector $\mathbf{d} = (d_x, d_y, d_z)^T$ and the ray origin $\mathbf{o} = (o_x, o_y, o_z)^T$. You do not need to simplify the expression.



(b) **(3 points)** Derive an expression for a normal vector at an intersection point.

(c) **(2 points)** What goes wrong in evaluating the paraboloid intersection formula when the ray is parallel to the z axis? How can the intersection be calculated in this case?

(d) **(2 points)** If we want to restrict drawing the paraboloid to $-1 \leq x, y \leq 1$ as shown in the figure, what is an efficient way to accomplish this?

3 Lighting and Shading

(a) **(2 points)** Explain the difference between Gouraud and Phong *shading*.

(b) **(1 point)** Which one is computationally more expensive? Why?

4 Triangle Meshes

(a) **(2 points)** How many floating point numbers, n_f, and integers, n_i, are needed to represent a triangle mesh using the face set and indexed face set data structures? Express your answers in terms of the numbers of vertices, edges, and triangles: V, E, and T, respectively).

(b) **(1 point)** Give an example of a triangular mesh that requires less storage in a face set representation than in an indexed face set format.

5 Transformations & Projections

5.1 Transformations

(a) **(2 points)** How many basis vectors do you need to define a coordinate system for a two-dimensional plane, and how many for three-dimensional space? What is the problem if you do not use enough basis vectors? Explain why.

(b) **(1 point)** Assume you have an orthonormal basis for the 2D plane with given basis vectors **x** and **y**. Remember that orthonormal means that the basis vectors have unit length and are perpendicular to each other. Given an arbitrary 2D vector **v**, how do you compute the coordinates of **v** in this basis? Make a sketch to explain your answer.

(c) Given a translation and a rotation matrix **T** and **R**,

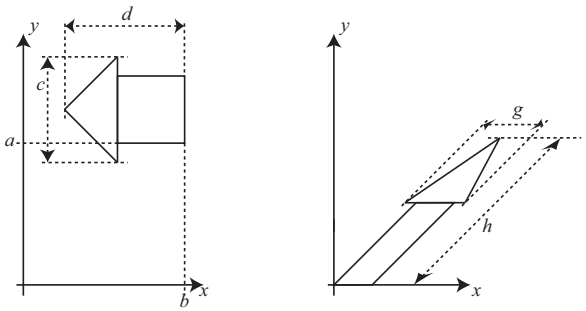
$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & t_{14} \\ 0 & 1 & 0 & t_{24} \\ 0 & 0 & 1 & t_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Write down the sixteen elements of the following matrices:

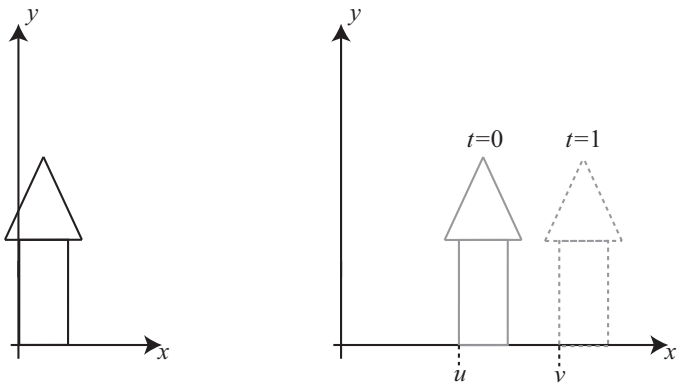
i) **(1 point) TR**

ii) **(1 point)** $\mathbf{R}^{-1}$

(d) **(4 points)** Derive the matrix that performs the transformation of the house from its position on the left to its position on the right in the figure below. Hint: one way to solve is to first derive the transformation without the shear and add the shear as the last step. Note that the shear is by 45 degrees.

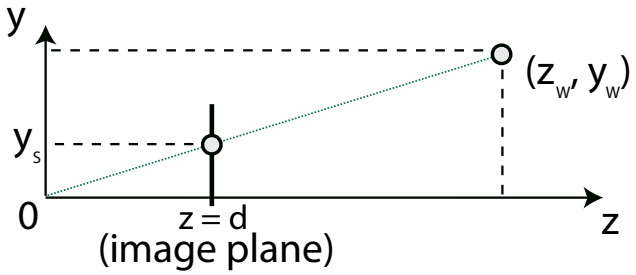


(e) **(2 points)** Consider the figure below. Derive a matrix as a function of a time parameter t, which implements an animation of the house on the left. The animation should translate between the positions indicated with grey and grey dotted lines on the right. For $t = 0$ the house should be transformed to the position in grey lines, and for $t = 1$ the house should be transformed to the position in grey dotted lines. For values $0 < t < 1$, the house should move between these two positions using a horizontal translation.



5.2 Projections

(a) **(2 points)** You are given a 2D set up as shown below. The camera is at the origin looking down z. The image plane is at $z = d$. Derive the *screen space* position y_s of the point (y_w, z_w).



(b) **(2 points)** Generalize the previous question to the 3D setup. That is, derive the expression of the image plane coordinates (x_s, y_s) for the world coordinate point (x_w, y_w, z_w).

(c) **(2 points)** Is this transformation affine? (briefly justify your answer)

(d) **(2 points)** Explain how homogeneous coordinates can be used to represent the perspective transformation above in matrix form?

6 Textures & Shadows

(a) **(1 point)** What does the quantity stored at each texel of a shadow map represent?

(b) **(1 point)** How could you generate a shadow map for a directional light source? The light rays of a directional light source are parallel and don't originate from one point

(c) **(1 point)** What is different when you need to handle an omnidirectional point light source?

(d) **(2 points)** In a fragment shader that has access to a shadow map, how do you determine if the current fragment is in the shadow or not? Show using GLSL or pseudocode.

(e) **(1 point)** How can you handle multiple light sources, how do their contributions have to be combined?

7 Fractals & L-Systems

Symbols used in this section: ω - initial state of a system, θ - rotation angle. Symbols: F - draw forward, $+$ counter-clockwise rotation, $-$ clockwise rotation, [push state to stack,] pop state.

7.1 L-system Grammar Expansion

(a) **(1 point)** The *Sierpiński arrowhead curve* system has the following rules:

$$\begin{aligned} \omega &= R \\ R &\rightarrow L - R - L \\ L &\rightarrow R + L + R \end{aligned}$$

How fast does the number of symbols in the expansion grow with the iteration number n? Mark exactly one choice.
☐ linear $O(n)$
☐ polynomial $O(n^p)$
☐ exponential $O(k^n)$

(b) **(2 points)** Let us consider the fractal plant L-system:

$$\begin{aligned} \omega &= X \\ X &\rightarrow F[+X]F[-X] + X \\ F &\rightarrow FF \end{aligned}$$

At iteration 5, the expansion of this system contains $F_5 = 422$ instances of symbol F and $X_5 = 243$ instances of symbol X. What will be the numbers F_7, X_7 of these symbols at iteration 7? (can be expressed analytically with respect to F_5, X_5)

(c) **(1 point)** One of the following L-system specification contains an error (making it impossible to draw properly). Which one and why?

(A)

$$\begin{aligned}\omega &= X\\ X &\rightarrow F[+X]F-X]+X\\ F &\rightarrow FF\end{aligned}$$

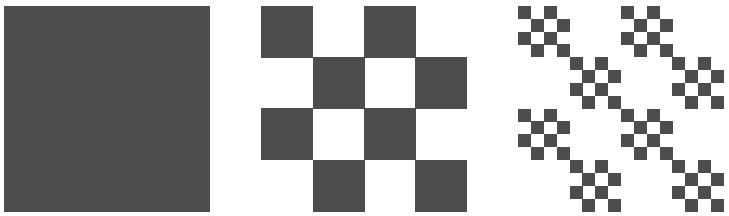
(B)

$$\begin{aligned}\omega &= X\\ X &\rightarrow F+F-F+F\\ F &\rightarrow F\end{aligned}$$

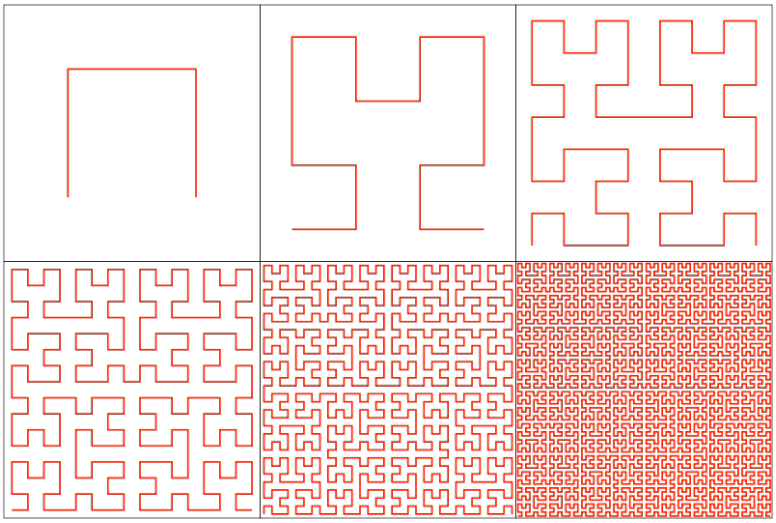
7.2 Fractal dimension

Calculate the fractal (Hausdorff) dimension. (numerical value not needed, analytical solution is enough)

(a) **(2 points)** This pattern:



(b) **(2 points)** The Hilbert Curve



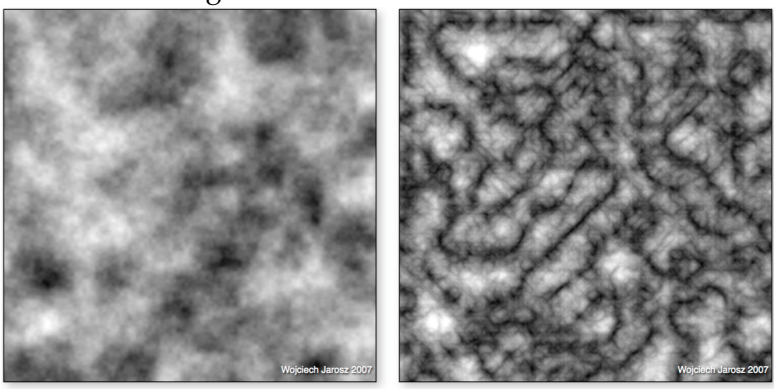
8 Procedural Modeling

(a) **(1 point)** What makes Perlin noise band-limited?

(b) **(1 point)** What method is used to make perlin noise reproducible without using a lot of memory?

(c) **(1 point)** Let $perlin\_noise(x,y,z)$ be the function for calculating Perlin Noise in the coordinate (x,y,z). How can you define a function $f(x,y,z)$ that returns Perlin noise of double frequency?

(d) **(1 point)** What is the difference between Fractional Brownian motion (FBm) and Turbulence when calculating the noise?



FBm Turbulence

9 Bézier Curves

The figure below shows the control points of a Bézier curve.

(a) **(1 point)** What is the degree of the curve?

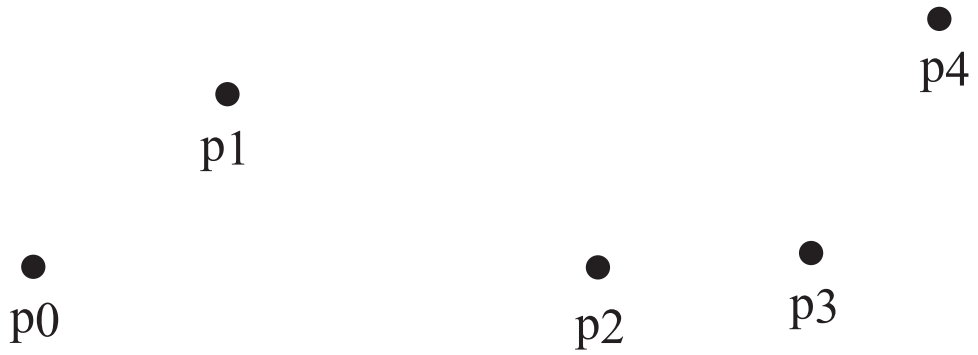
(b) **(2 points)** Sketch the convex hull of the original control points. Explain the convex hull property.

(c) **(1 point)** Explain the symmetry property.

(d) **(1 point)** Sketch the calculation of a point on the curve for the parameter value $t = 0.75$ (approximately) using the Casteljau algorithm.

(e) **(1 point)** The point at $t = 0.75$ splits the curve into two segments. Indicate the new control points of the first segment that goes from $t = 0$ to $t = 0.75$.

(f) **(2 points)** Give pseudo-code for an algorithm to draw the curve as a sequence of line segments using adaptive subdivision. The algorithm should be based on recursive curve splitting.



10 What to hand in

Electronic version of the document (.pdf), either created electronically or hand-written and scanned, and a `readme.txt` file with each team member's full name, and brief comments of difficulties encountered.

Note on grading: We will convert exam points into exercise points by normalizing the total amount to 10 points as usual.