⊛ Is set of Odd numbers with binary operations (+), i.e., $\langle O, + \rangle$ an abelian group? If not explain the reasons with necessary notations.

Answer:

We denote the structure as

$\langle O, + \rangle$, where $O = \{\ldots, -3, -1, 1, 3, \ldots\}$

Checking the group axioms one by one:

① Closure:

For $a, b \in O$, $a+b$ is even.

proof:

let, $a = 2m+1$, ~~and~~ $b = 2n+1 \in O$

~~$b = 2n+1 \in$~~ ~~$m = \{$~~ m, n can be any integer

so,

$a+b = (2m+1) + (2n+1)$

$= 2(m+n+1)$

which is always an even integer, not in O.

so this fails closure (O is not closed under $+$).

② Associativity:

Addition on integers is associative:

$(a+b)+c = a+(b+c)$.

So the condition holds.

③ **Identity:** For a group there must exist $e \in O$ such that $a+e=a$ for all $a \in O$

The additive identity ~~is 0~~ in $\mathbb{Z}$ is 0.
But $0 \notin O$ (since 0 is even).
No identity element in O.

④ **Inverse Element:**
Each $a \in O$ must have $b \in O$ such that $a+b=0$.
For example, ~~if~~ $1 \in O$, the inverse element $-1 \in O$.
But since, $0 \notin O$, we already failed the identity test, so inverses don't exist in this structure.

⑤ **Commutativity:**
Integer addition are commutative:
for all $a, b \in S$

$a+b=b+a$. It holds.

Conclusion:
The set of odd integers under addition $\langle O; + \rangle$ is not an Abelian group because:
- Not closed under addition.
- ~~At~~ Has no identity element (0 is not odd).
Consequently it fails the inverse property.

Even though associativity and commutativity holds, the first two failures are enough.

so, $\langle O, + \rangle$ is not an abelian group.