



COMP 1521

WEEK 7!

CONTENT

01 2'S COMPLIMENT

02 FLOATING POINT NUMBERS

Negative Numbers

to write $-x$ in n bits of data:

$$\text{binary} = 2^n - x$$

2's complement

The result of $2^n - X$ is identical to an operation called 2's complement.

0000 0101 = 5

1111 1011 = -5 $= 2^8 - 5 = 251$

2's complement

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translation:

the negative of a number is just the 2's complement!

2's complement

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To do 2's compliment:

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To do 2's complement:

1. take the 'not' of the binary

0000 0101
1111 1010



invert, or
'not'

2's complement

The result of $2^n - X$ is identical to an operation called 2's complement.

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To do 2's complement:

1. take the 'not' of the binary

0000 0101

1111 1010

invert, or 'not'

<- this is known as the 1's complement!

2. add 1 to the number

1111

1010

1111 1011

+

1

**Taking 2's compliment is much faster than
calculating $2^n - X$ for larger n's!**

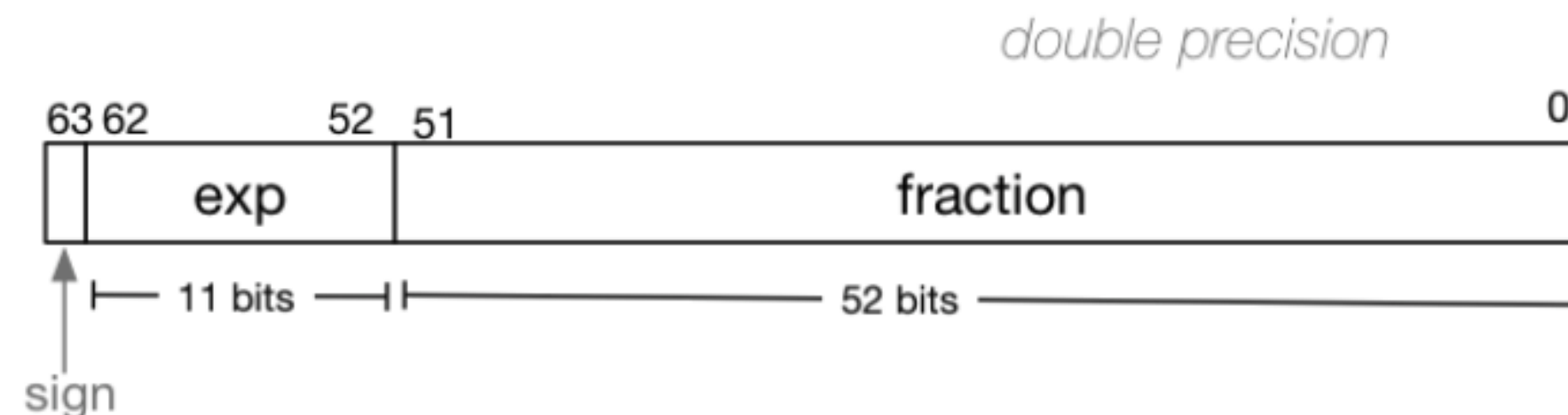
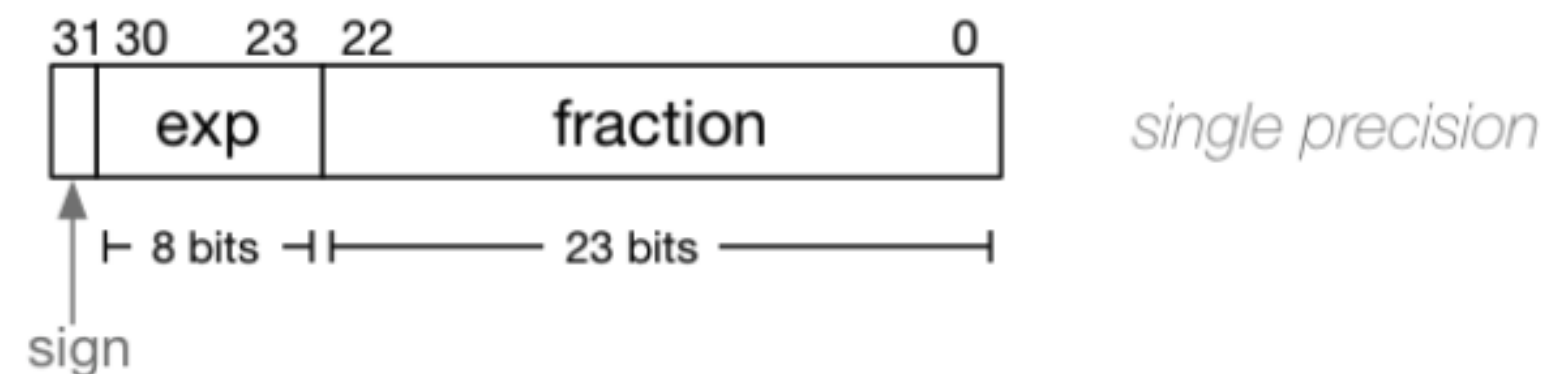
it is equivalent to $*-1$

Q1 iii, iv, v

Q2 ii, v, vi,
vii

Floats (decimals)

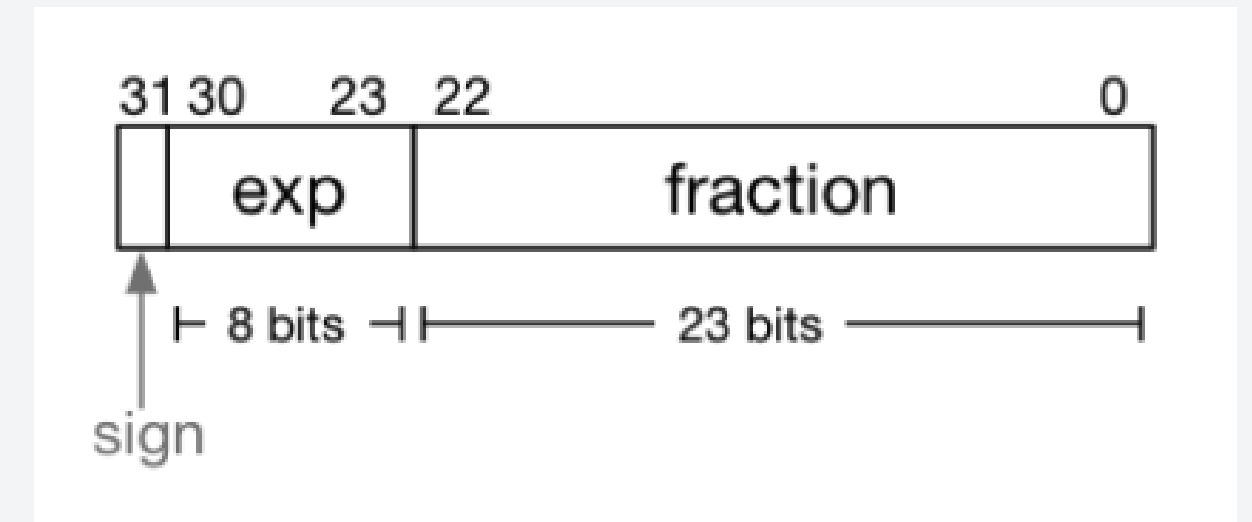
- also known as IEEE 754 standard or IEEE 754-1985
- IEEE 754 single -> float
- IEEE 754 double -> double
- **float** ... typically 32-bit (lower precision, narrower range)
- **double** ... typically 64-bit (higher precision, wider range)
- **long double** ... typically 128-bits (but maybe only 80 bits used)



Floats (decimals)

Overall Formula:

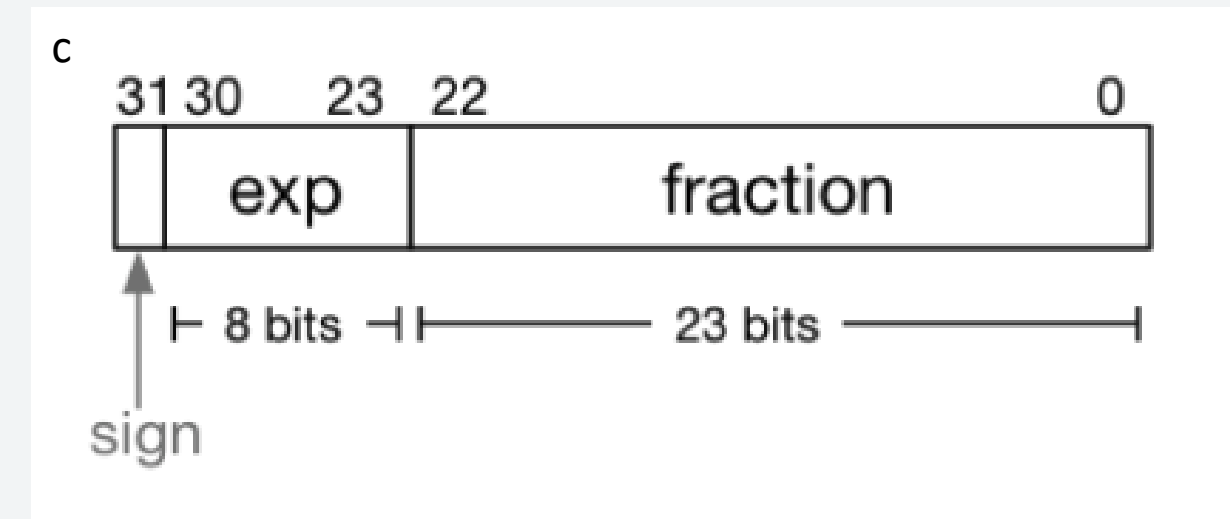
$$(-1)^{\text{sign}} * (1.\text{frac}) * 2^{\text{exp} - 127}$$



Floats (decimals)

1. Sign

- The sign is the easiest .
- 0 -> positive number
- 1 -> negative number



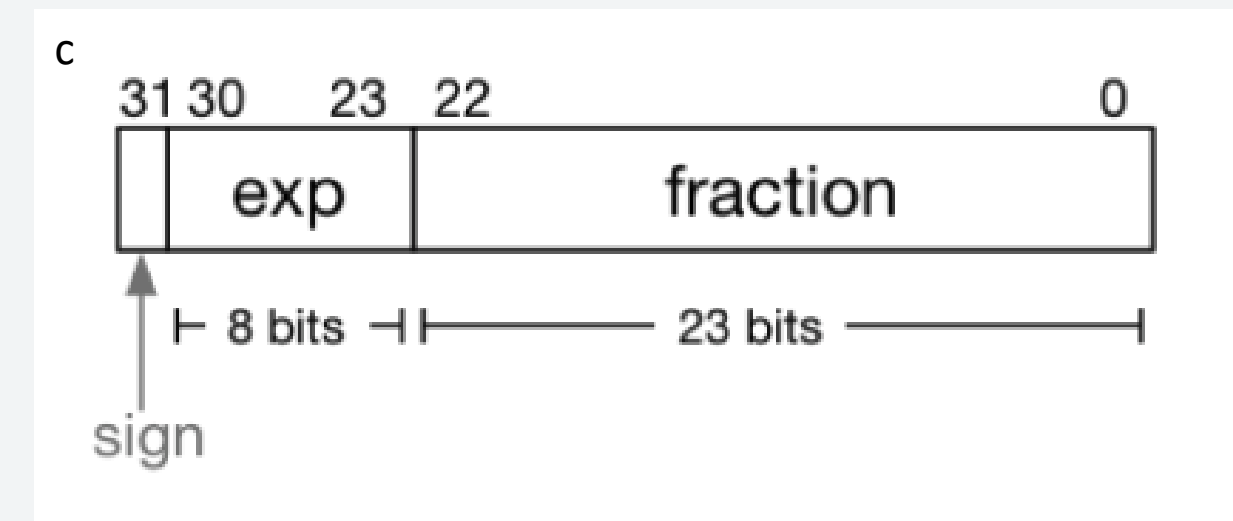
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Floats (decimals)

1. Sign

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Overall Formula:

sign = 0 \Rightarrow $(-1)^0 = 1$ (positive)

sign = 1 \Rightarrow $(-1)^1 = -1$ (negative)

$$(-1)^{\text{sign}} * (1.\text{frac}) * 2^{\text{exp} - 127}$$

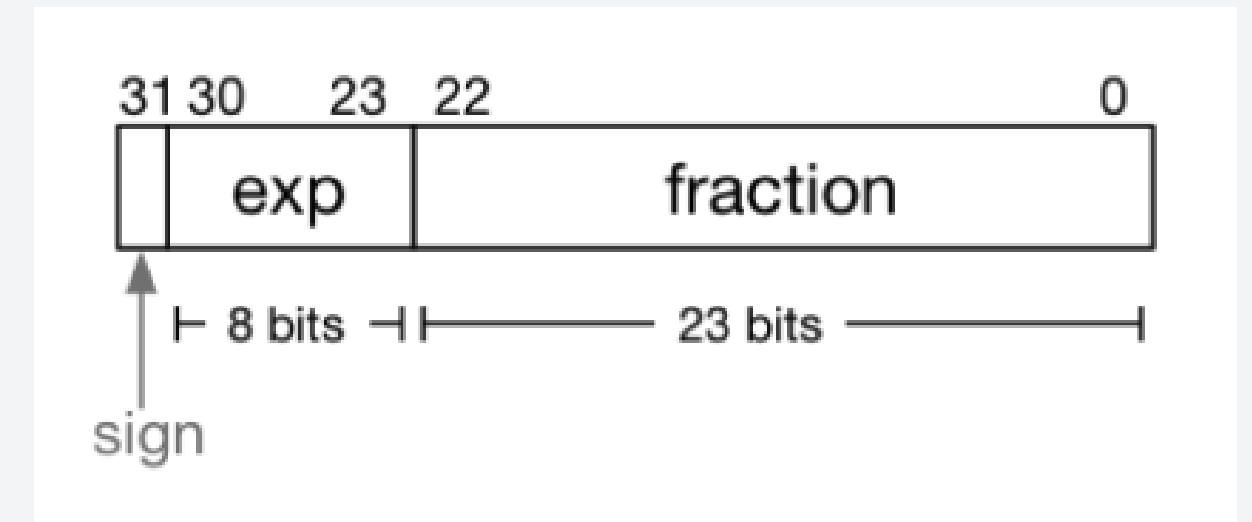
Floats (decimals)

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2. exponent

- exp has a bias of -127 added to it
- $2^{\text{exp} - 127}$



Overall Formula:

$$(-1)^{\text{sign}} * (1.\text{frac}) * 2^{\text{exp} - 127}$$

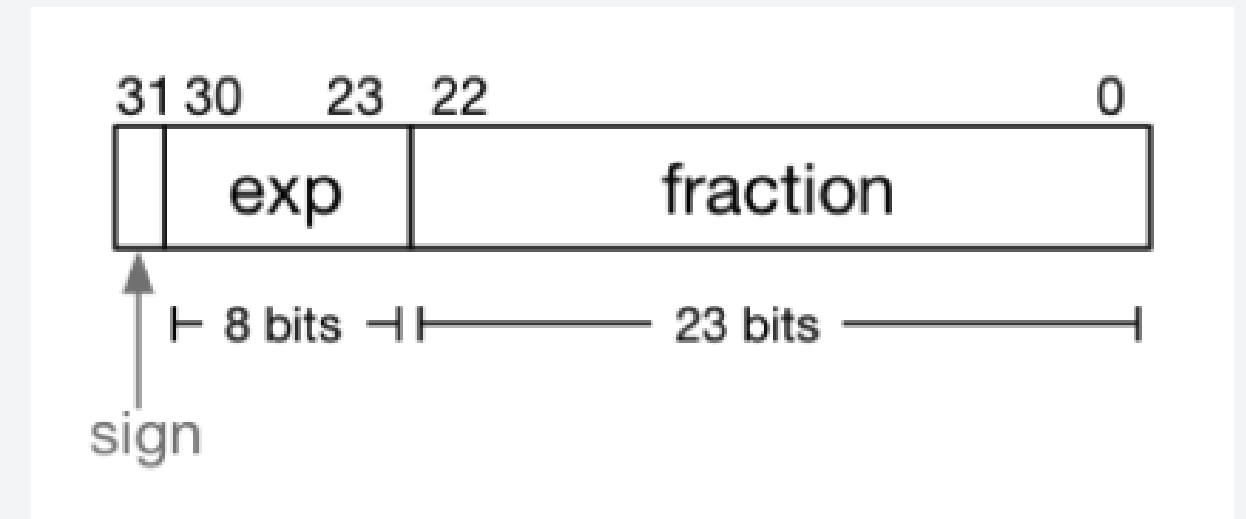
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Overall Formula:

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Exp is an 8-bit **unsigned integer**

The **raw exponent field (exp)** ranges from 0 to 255.

The **actual exponent (e = exp - 127)** would *mathematically* range from: -127 to 128

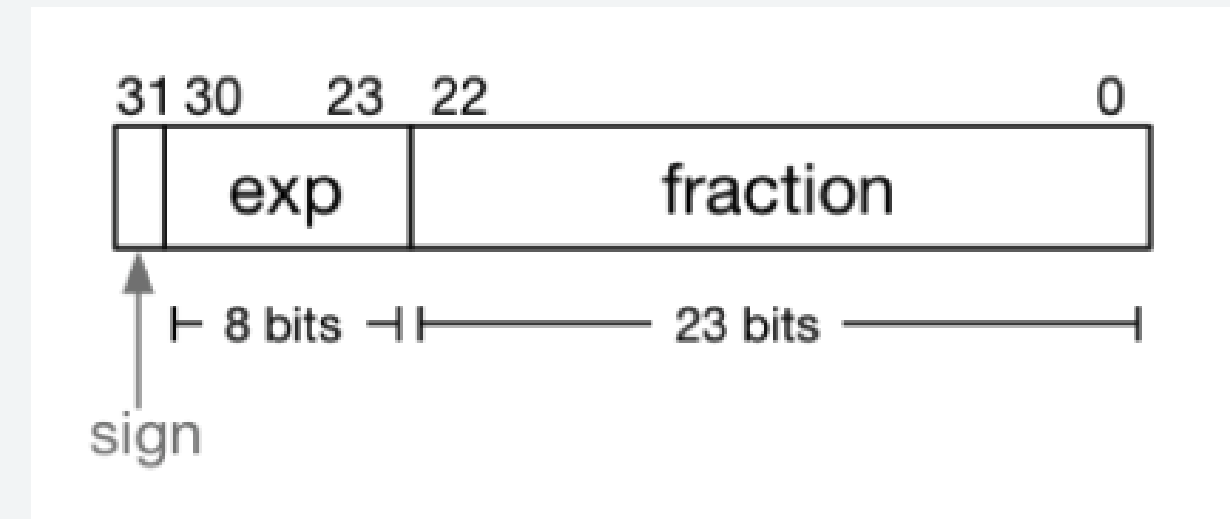
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Overall Formula:

3. fraction

- the fraction is concatenated to the end of 1
- e.g. for a fraction of “0101 1010”,
- 1.frac means 1.0101 1010 (binary decimal)

$$(-1)^{\text{sign}} * (1.\text{frac}) * 2^{\text{exp} - 127}$$

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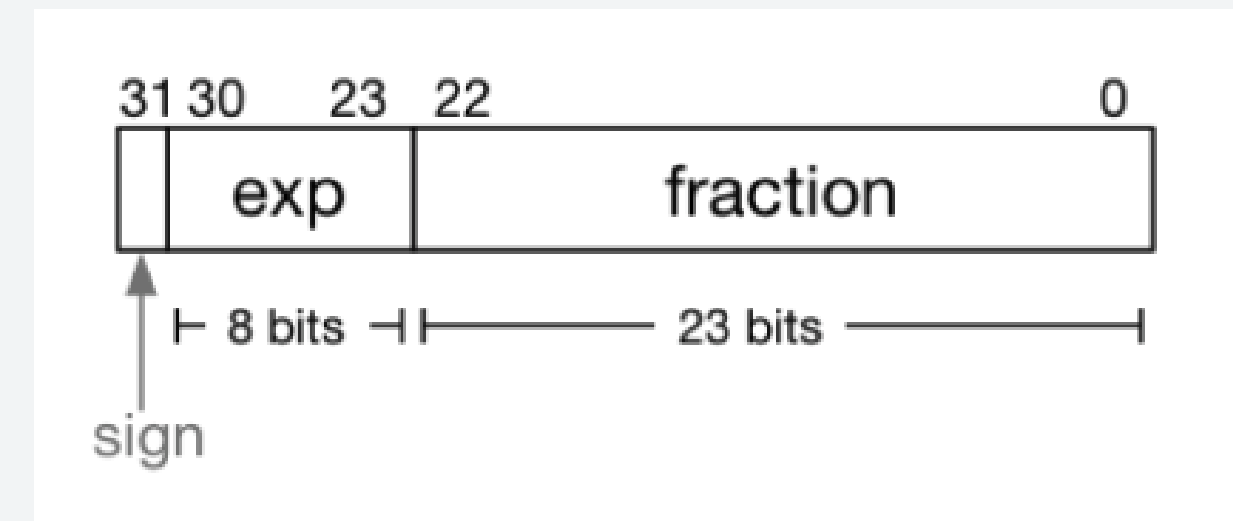
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1.101 is

$$1 * 2^0 + 1 * 2^{-1} + 0 * 2^{-2} + 1 * 2^{-3}$$

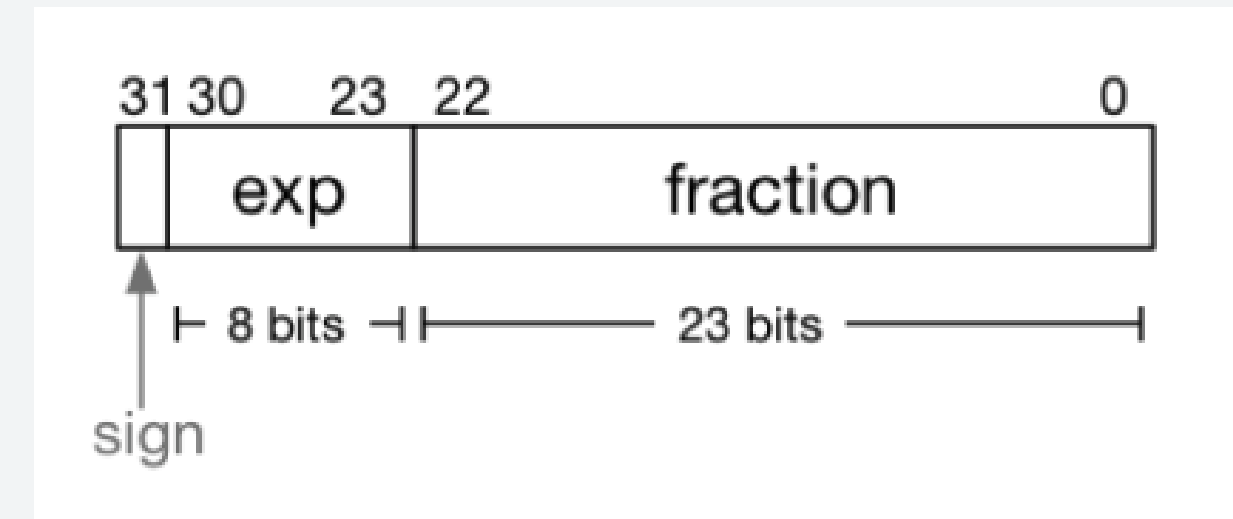


Overall Formula:

$$(-1)^{\text{sign}} * (1.\text{frac}) * 2^{\text{exp} - 127}$$

Floats (decimals)

```
$ ./explain_float_representation -96.125
-96.125 is represented in IEEE-754 single-precision by these bits:
11000010110000000010000000000000
sign | exponent | fraction
   1 | 10000101 | 100000001000000000000000
sign bit = 1
sign = -
raw exponent    = 10000101 binary
                  = 133 decimal
actual exponent = 133 - exponent_bias
                  = 133 - 127
                  = 6
number = -1.100000001000000000000000 binary * 2**6
        = -1.50195 decimal * 2**6
        = -1.50195 * 64
        = -96.125
```



Overall Formula:

$$(-1)^{\text{sign}} * (1.\text{frac}) * 2^{\text{exp} - 127}$$

Floats (decimals)

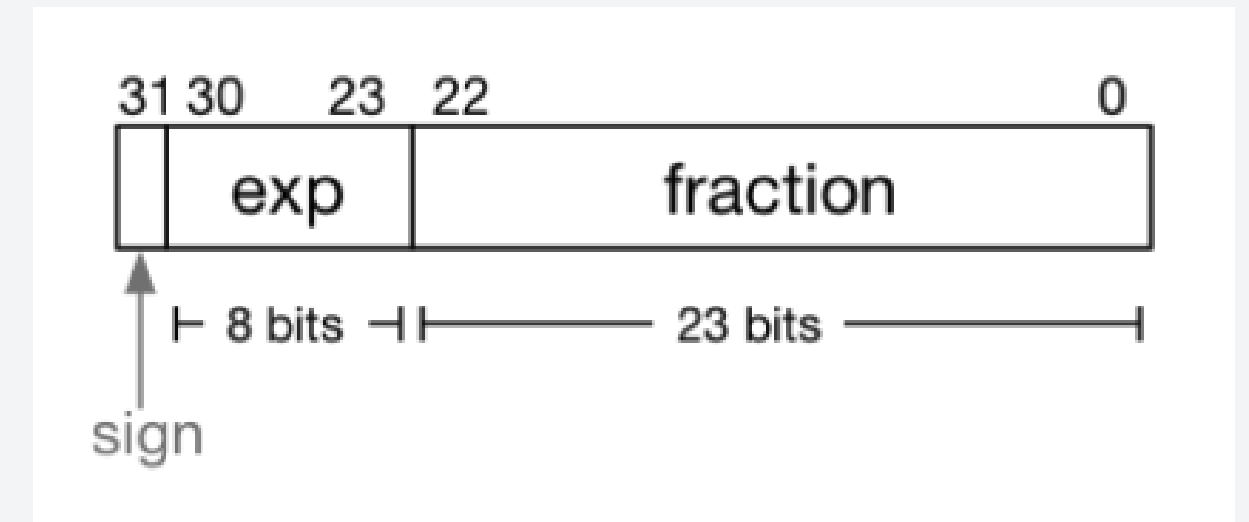
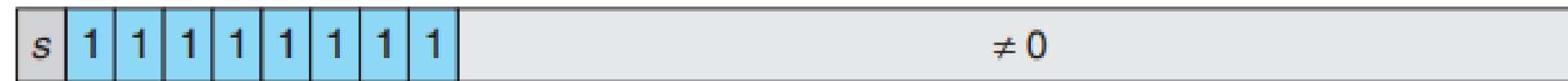
Special Cases:

3a. Infinity



3b. NaN

NaN = Not a Number, such as when you try to divide by 0.



+ and - inf are both defined! (so pay attention to the sign)

NaN is usually not defined as + or -

Floats (decimals)

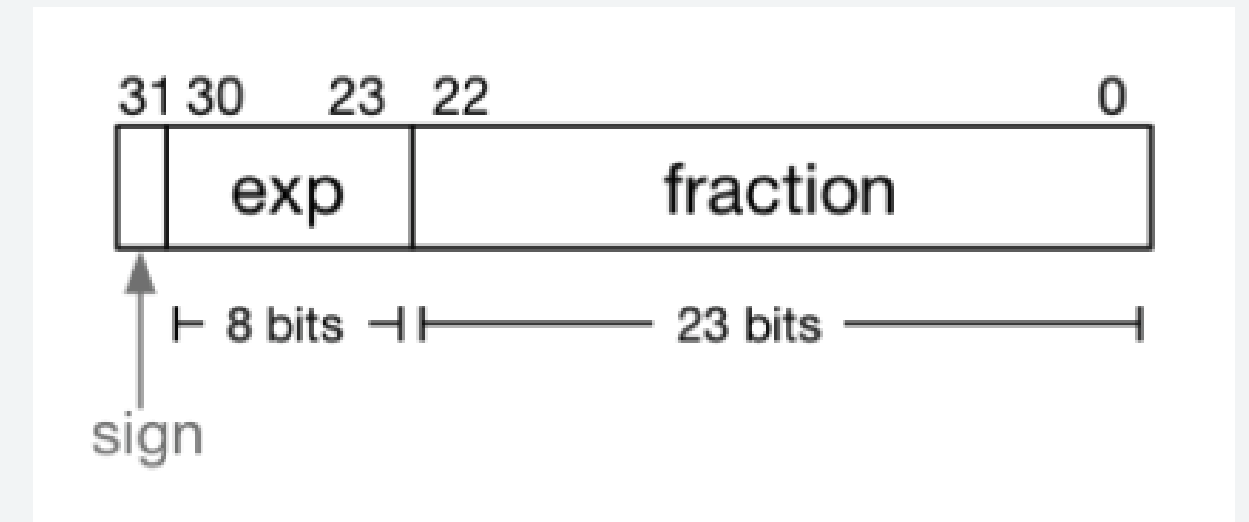
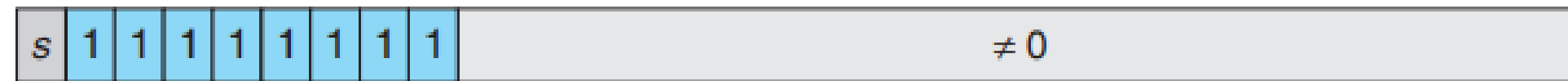
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The trick is to always check if the exp bits \geq 0xFF!

q4 a->f

working backwards

how to make a number K into float form?

We need to first express the number k as $(1 + \text{frac}) \times 2^n$. To work out the fraction, we divide k by the largest 2^n that is smaller than k .

q5

Labs - extract the components of a
float