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GENERATING A RACING LINE FOR AN AUTONOMOUS RACECAR USING PROFESSIONAL DRIVING TECHNIQUES

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ABSTRACT

During a race, professional drivers follow a racing line using specific maneuvers that allow them to utilize as much of the car's tire force as possible. These lines could be used to create trajectories for obstacle avoidance in autonomous vehicles if they could be analytically defined. In fact, many of the techniques described by professional drivers can be expressed by a family of simple curves including straights, clothoids, and constant radius arcs. By comparing different members of this family of curves, different racing techniques can be examined. In particular, the differences between two phase and three phase corners described by professional drivers can be easily captured and analyzed in a single parameter. Experimental results on an autonomous racecar highlight the advantages of two phase cornering over three phase cornering and demonstrate the types of comparisons that can be made with this approach.

Figure 1. *Autonomous Audi TTs on Pikes Peak*

1 INTRODUCTION

For racecar drivers, careful consideration is used when defining a racing line for a given track. Each driver ultimately seeks to minimize the track time by compromising between the shortest distance around a track and a line that results in the fastest speed along the path. Professional racing strategies could be used to create trajectories for obstacle avoidance in autonomous vehicles if there was a way to analytically represent their techniques.

In order to leverage the computational advantage autonomous systems have over their human counterparts, autonomous racing research has generally focused on various optimization algorithms in an attempt to find the fastest path for the car. Mühlmeier and Müller [1] populate a track with a set of initial paths and then use a genetic algorithm to test and evolve the racing line. Gerdts et al. [2] use optimal control and ultimately a moving horizon approach to iterate and converge to a racing line given a set of road boundaries. These algorithms iterate on initial guesses and modify the path using optimization techniques. However, such methods do not attempt to capture the techniques described by professional drivers.

For a professional racecar driver, a single corner or turn can be broken into three phases: turn entry or trail braking, pure cornering, and turn exit or throttle-on-exit. These phases can be described by a family of curves including straights, clothoids, and constant radius arcs. The use of clothoids for general vehicle

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path planning is not uncommon [5], but their use and justification for defining a racing line has yet to be explored. Representing professional racing lines using this family of curves could offer a way of comparing various driving techniques. In particular, different ways that professional drivers coordinate braking and steering can be easily captured and analyzed in a single parameter.

The first section discusses how straights, clothoids and circular arcs can be used to emulate the racing lines professional racecar drivers describe. The second section outlines the specifics of representing and constraining the chosen structure or set of curves. In particular, two parameters are used to control the design of early and late apex turns and two or three phase cornering. The last section analyzes the use of two phase and three phase cornering which demonstrates the types of comparisons that can be made with this approach. To show how the approach can be extended to multiple corners, the paper concludes with the demonstration that a racing line was designed and implemented on a real test track in Colorado where the Pikes Peak International Hill Climb is held (Figure 1).

2 PROFESSIONAL DRIVING TECHNIQUES

A professional driver often separates racing into two objectives. The first is utilizing all the tire friction available to the car at all times [3]. The second is defining and following a successful racing line [4].

Mitchell et al. [3] discuss the concept of maximizing the performance of a car by "driving the traction circle." The traction or friction circle is a model used to describe the limited friction available in a vehicle's tires. There is a tradeoff between steering and braking when negotiating a corner, as illustrated in Figure 2. Driving at the limits implies remaining on the edge of the friction circle but not exceeding its bounds. While negotiating a turn, the friction circle and a racing line can be described in three phases. These three phases are trail braking, maximum cornering, and throttle on exit.

Taruffi [4], who not only won the 1952 Swiss Grand Prix but earned a Doctorate of Industrial Engineering, also describes taking a corner in three phases based on the friction circle. He further described these phases geometrically by connecting a pair of straights to a constant radius arc via two "linking curves." In this paper, clothoids with linearly varying curvature serve as the linking curves Taruffi describes. By plotting the curvature of the path, Figure 3 illustrates how clothoids are used to link straights to constant radius arcs. For the circular arc, the design goal is to minimize the curvature at the apex as much as possible. Figure 4 demonstrates this by approaching a corner from the outside, meeting the inside boundary at the apex and exiting the corner on the outside.

Along the straights, the racecar is either at maximum acceleration or deceleration. The entry clothoid is where the driver balances between braking and steering to remain at the limits. This maneuver is called trail braking. The circular arc is where the racecar is experiencing maximum cornering and cannot accelerate or decelerate. The exit clothoid is where the driver steers out of the turn and feathers the throttle to remain at the limits.

Taruffi comments that the most skilled drivers make the connecting curves as long as possible for a given turn, reducing the corner to two phases. This results in a path with a smaller turn-

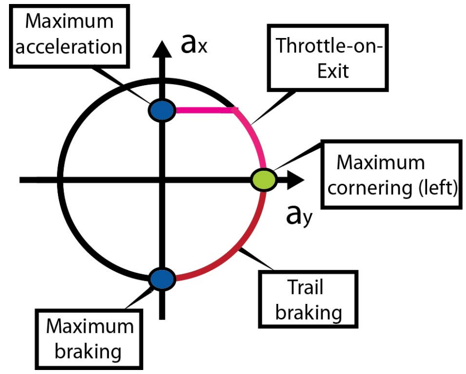

Figure 2. *Maneuvering along the edge of the friction circle*

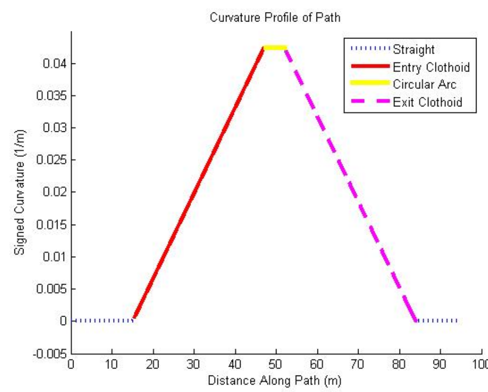

Figure 3. *Curvature of a simple turn*

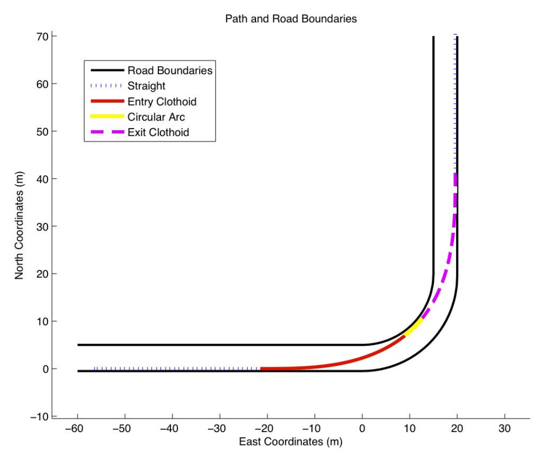

Figure 4. *Racing line of a simple turn*

ing radius at the apex but yields a faster time out of the corner. The key challenge of driving a longer clothoid turn is that it requires more distance to coordinate between longitudinal and lateral control. This extended coordination is difficult to master because slight errors anywhere along the clothoids can cause the car to exceed the limits and begin to slide. The comparison of two and three phase cornering can be analyzed by using a simple family of curves to generate what Taruffi describes.

3 DESIGN PROCESS

Beginning with the road edges of a track, creating a racing line can be divided into two parts. The first is to define the straights along the track. The second involves linking the straights with the three curve structure. The design process discussed assumes that straights along the track have already been found. Figure 5 is a example of what the straights would look like for a simple turn; however, before the straights are connected by the chosen curve structure, it is important to know how the curves are defined.

3.1 THREE PHASE CORNER STRUCTURE

Each turn is designed using two consecutive straights. The turn structure is a sequence of four curves: a straight, an entry clothoid, a constant radius arc, and an exit clothoid as can be seen on Figure 6. A straight also follows the exit clothoid which is considered part of the next curve sequence. When a turn is uniquely defined, the east and north position as a function of distance along the track is described by the following sets of equations:

Straight:

$$\begin{aligned} E_i(s) &= E_0 + s\cos(\psi_i) \\ N_i(s) &= N_0 + s\sin(\psi_i) \\ \psi_i(s) &= \psi_0, \end{aligned} \tag{1}$$

E_0, N_0, and ψ_0 is the initial position and heading of the four curve sequence. s is the distance along the track from (E_0, N_0). The end of the straight and beginning of the entry clothoid is found when $s = L_1 \equiv L_s$ where L_s is the length of the straight.

Entry Clothoid:

$$\begin{aligned} E_i(s) &= E_1 + \textstyle\int_{L_1}^{s} cos(\psi_i)d\psi_i \\ N_i(s) &= N_1 + \textstyle\int_{L_1}^{s} sin(\psi_i)d\psi_i \\ \psi_i(s) &= \psi_1 + \tfrac{(s-L_1)^2}{2RL_{c1}} \end{aligned} \tag{2}$$

E_1, N_1, and ψ_1 is the initial position and heading of the entry clothoid. L_{c1} is the length of the entry clothoid and R is the radius of curvature of the constant radius arc. The end of the clothoid and beginning of the constant radius arc is found when $s = L_2 \equiv L_{c1} + L_s$.

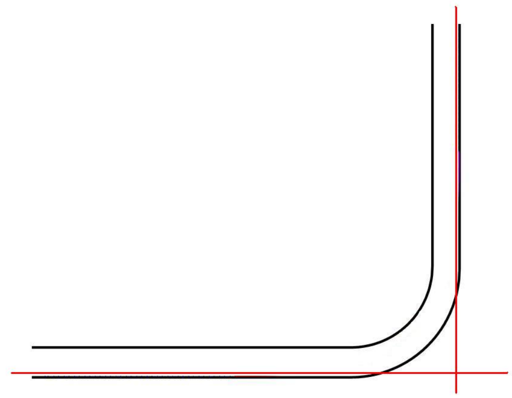

Figure 5. *Straights Defined on a simple turn*

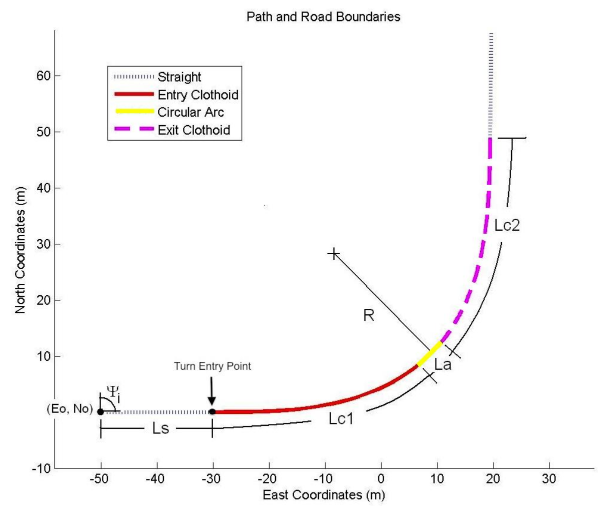

Figure 6. *Parameters that control a turn*

Constant Radius Arc:

$$\begin{aligned} E_i(s) &= E_2 + \textstyle\int_{L_2}^{s} cos(\psi_i)d\psi_i \\ N_i(s) &= N_2 + \textstyle\int_{L_2}^{s} sin(\psi_i)d\psi_i \\ \psi_i(s) &= \psi_2 + \tfrac{s-L_2}{R} \end{aligned} \tag{3}$$

E_2, N_2, and ψ_2 is the initial position and heading of the constant radius arc. L_a is the length of the constant radius arc.The end of the arc and beginning of the exit clothoid is found when $s = L_3 \equiv L_a + L_{c1} + L_s$.

Exit Clothoid:

$$\begin{aligned} E_i(s) &= E_3 + \int_{L_3}^{s} cos(\psi_i) d\psi_i \\ N_i(s) &= N_3 + \int_{L_3}^{s} sin(\psi_i) d\psi_i \\ \psi_i(s) &= \psi_3 + \frac{L_{c2}}{2*R} - \frac{(L_4 - s)^2}{2*R*L_{c2}} \end{aligned} \tag{4}$$

E_3, N_3, and ψ_3 is the initial position and heading of the constant radius arc. L_{c2} is the length of the exit clothoid. The end of the exit clothoid is found when $s = L_4 \equiv L_{c2} + L_a + L_{c1} + L_s$. Note: The east and north position of (2) and (4) can be expressed as a Fresnel powers series.

3.2 SOLVING FOR THE PARAMETERS

Each ψ_i equation is used for simplification. Not counting the explicit ψ_i equations, there are eight equations (two from each curve) and eight parameters that uniquely constrain a given turn. E_0, N_0, ψ_0, L_s, L_{c1}, L_a, L_{c2}, and R were chosen as the parameters that will uniquely define a given turn. Each parameter defines the characteristics of how a turn is shaped as can be seen on Figure 6 and can be organized into three groups: initial conditions, independent parameters, and dependent parameters.

3.2.1 INITIAL CONDITIONS E_0, N_0, and ψ_0 are considered the initial conditions of the turn and correspond to the beginning of the racing line or the end of a previous turn. E_0, N_0, and ψ_0, are readily found after the straights are defined along a track.

3.2.2 INDEPENDENT PARAMETERS L_a and L_{c2} can be considered independent parameters. By relating their relative lengths to L_{c1} the shape of a turn can be varied arbitrarily, e.g.

$$\begin{aligned} L_a &= \frac{L_{c1}}{10} \\ L_{c2} &= L_{c1} \ . \end{aligned} \tag{5}$$

How these parameters are related can have a significant effect on the strategy and performance of the racing line. Making L_{c1} larger or smaller than L_{c2} would correspond to late or early apex turns. The length of L_a controls how long a racecar travels at maximum cornering. This parameter can be used to switch between two or three phase turns. If L_a is set to zero then the turn has two phases and reaches maximum cornering for only an instant.

3.2.3 DEPENDENT PARAMETERS The workflow diagram on Figure 7 illustrates how the dependent parameters are solved in order to meet two requirements. The first is to ensure that the end of the turn or exit clothoid is tangent to the following straight in order to properly connect each straight with the next one in the sequence. The second is for the apex of the racing line to meet the inside boundary of the turn in order to minimize the curvature of the turn while remaining within the road boundaries.

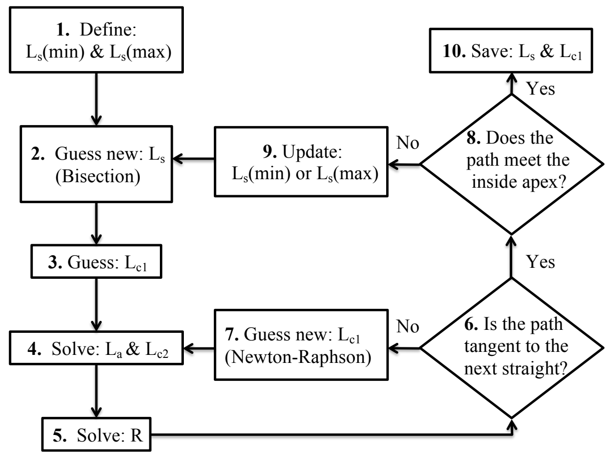

Figure 7. *Flowchart for solving dependent parameters*

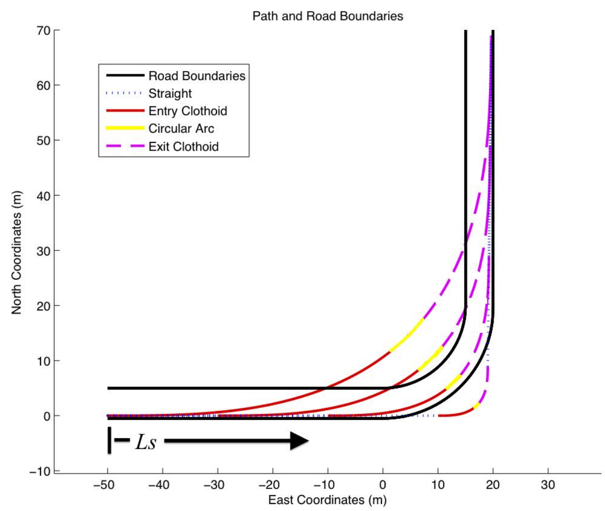

Figure 8. *Varying L_s along a straight*

1. Define L_s(min) and L_s(max) - L_s is the point along the straight where the turn begins. It can vary along a straight as demonstrated in Figure 8. From the figure, one can conclude that L_s is bounded by the beginning of a straight (E_0, N_0) and the point where the two straights intersect. Because the range of L_s is known, bisection is used to find the value of L_s that will allow the apex of the turn to meet the inside boundary.

2. Bisection: guess new L_s - This block begins the outer loop that is used to solve for L_s. Each new guess of L_s is half way between the current L_s(min) and L_s(max).

3. Guess L_{c1} - The initial guess of L_{c1} is arbitrary as long as

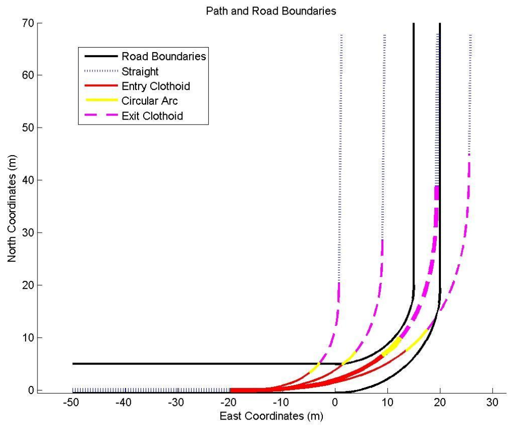

Figure 9. *Varying the curve lengths to ensure tangency*

it is a real, positive number. The guess initializes the inner loop that solves for L_{c1} given a specific L_s.

4. Solve for L_a and L_{c2} - These lengths are readily found using (5).

5. Solve for R - The radius of the circular arc, R, is found from the difference in heading between the two straights:

$$\begin{aligned} \Delta\psi &= \psi_{i+1} - \psi_i \\ &= \frac{L_{c1}}{2R} + \frac{L_a}{R} + \frac{L_{c2}}{2R} \\ \Rightarrow \quad R &= \frac{1}{2\Delta\psi}(L_{c1} + 2L_a + L_{c2}) \end{aligned} \tag{6}$$

6. Is the path tangent to the next straight? - At the end of the exit clothoid, the racing line must be tangent to the straight in the next sequence. The equation for the next straight can be described by:

$$N_{i+1} = E_{i+1} * tan(\psi_{i+1}) + b_{i+1} \tag{7}$$

where b_{i+1} is the location of the intercept on the N axis. The end of the exit clothoid has a final position and heading that can be used to define a line as well. If both intercepts are equal, then the path is tangent to the next straight.

7. Guess new L_{c1} - If the racing line is not tangent to the next straight, Newton-Raphson is used to ensure that the N intercept of the racing line matches b_{i+1} by adjusting the length of L_{c1} and subsequently L_a, and L_{c2}. Figure 9 is an example of varying the curve lengths to fit the racing line to the desired straight.

8. Does the path meet the inside apex? - Figure 4 is an example of the path meeting the inside of the road boundary at the apex. This condition can be determined by calculating the distance from the path to the inside road boundary.

9. Update L_s(min) or L_s(max) - If the turn begins too early the lower bound $L_{s,min}$ is updated otherwise $L_{s,max}$ is updated. Figure 8 is an example of how the solution of L_s would converge.

In summary, a combination of bisection and Newton-Raphson is used to solve for the constraint parameters of each turn along the track. This process is used to both minimize the curvature of each turn and ensure that the path is tangent to the following straight.

3.3 DRIVING AT THE LIMITS

Once the racing line is constructed, a target velocity profile for braking into a corner can be calculated by starting with the required velocity at the constant radius arc. The speed of maximum cornering is dictated by

$$\frac{v^2}{R} = a_y \tag{8}$$

where a_y is the maximum cornering on the friction circle and v is the velocity of the racecar. By integrating the available braking force along the entry clothoid using the friction circle model, a velocity profile can be calculated. From the required velocity at the beginning of the entry clothoid, the braking point on the straight before each turn can be found. As the racecar leaves the apex and exits the turn, the throttle increases as the curvature of the turn decreases. Kritayakirana and Gerdes [6] outline this process using the friction circle model. An important advantage of using the proposed curve structure is that the curvature of each turn can be readily extracted from the racing line making it possible to follow the edge of the friction circle. This again leads to maximizing the total force the tires can use along the track.

4 ANALYZING TWO AND THREE PHASE CORNERS

A set of experiments was constructed to test the observations made by Taruffi that longer connecting curves for a turn will result in faster track times. Specifically, a three phase turn as described before was compared to the a two phase turn. Another way of expressing a two phase turn is that it contains a corner entry and exit with an instantaneous apex. An oval constructed from three phase turns and an oval with two phase turns were designed to take a racing line of the same oval track as seen on Figures 10 and 11 respectively. Two tests were run where the friction circle limit in the controller was set to 0.4g and 0.9g. The test with a lower friction value was used to approach an idealized case were the racecar would not approach the physical limits. The higher friction value test was used to validate that the physical friction limits along the path can be tracked, and the trend in track times is consistent with the results of the first test.

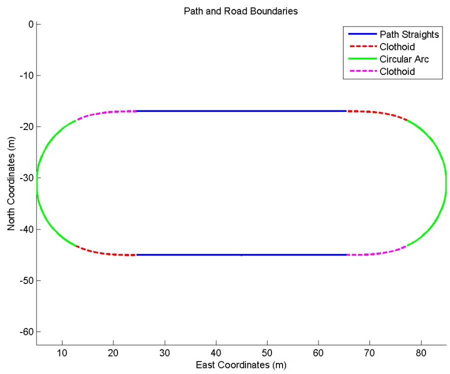

Figure 10. *Oval with three phase turns*

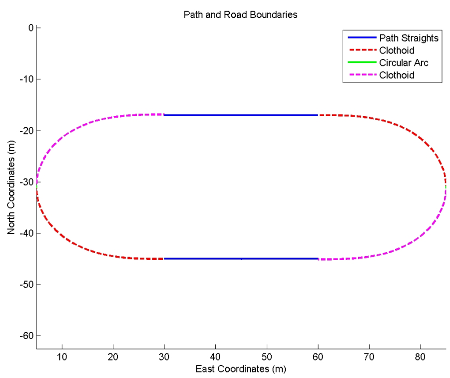

Figure 11. *Oval with two phase turns*

4.1 EXPERIMENTAL AUTONOMOUS RACECAR

The Autonomous Audi TTS (four-wheel drive) in Figure 1, was used for testing on an open, paved surface. A controller designed by Kritayakirana and Gerdes [6] sends commands to the electronic power steering motor, active brake booster, and throttle by-wire of the vehicle. The vehicle is equipped with a Differential Global Positioning System (DGPS) and inertial sensors (INS), from which vehicle position and other states can be obtained.

4.2 EXPERIMENTAL RESULTS

In the case where the friction limit was assumed to be 0.4g, the elapsed time for a single lap was evaluated 16 times on each oval. For the oval with three phase turns, the average measured time was 23.521 seconds with a standard deviation of 0.2102 seconds, and the average time for the oval with two phase turns was 22.722 seconds with a standard deviation of 0.1383 seconds. From these results, the oval with two phase turns has an average advantage of .8 seconds. For the test where the friction was assumed to be .9g, the average measured time of the oval with three phase turns was 14.170 seconds with a standard deviation of 0.1584 seconds, and the average time for the oval with

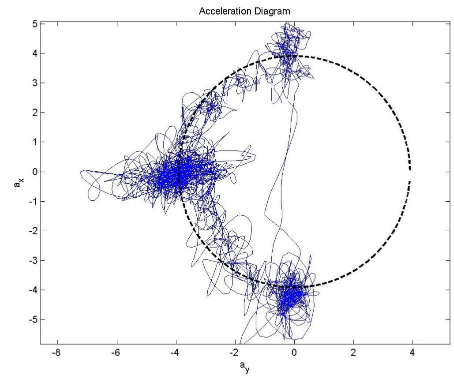

Figure 12. *Acceleration diagram of oval with three phase turns superimposed on a 0.4g friction circle*

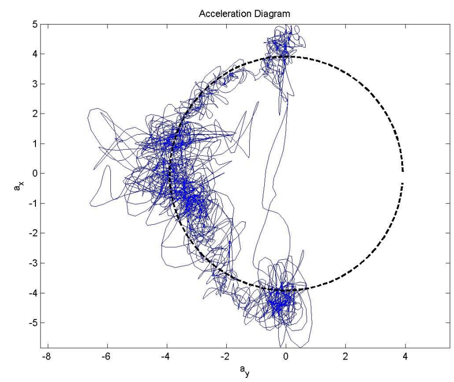

Figure 13. *Acceleration diagram of oval with two phase turns superimposed on a 0.4g friction circle*

two phase turns was 13.664 seconds with a standard deviation of 0.1442 seconds. When the friction limit was assumed to be 0.9g, there was an average advantage of 0.506 seconds for the oval with two phase turns. For both tests, the average time advantage was much larger than the variability in track time which illustrates that the length of the constant radius arc creates a significant increase in track time.

4.3 VALIDATION OF RESULTS

To ensure a proper comparison, the ability of the racecar to reach maximum performance for each racing line was analyzed. Measurements from the INS were used to construct acceleration diagrams illustrated on Figures 12 and 13 for a single lap. Notice that the acceleration diagram of the three phase oval (Figure 12) with the longer constant radius segments remains in the region of maximum cornering longer than the oval with two phase turns. This is expected because the constant radius path is designed to be the point of maximum cornering on the acceleration diagram. It was also confirmed that for the 0.4g test there were no limitations of acceleration or deceleration. Figures 14 and 15 represent

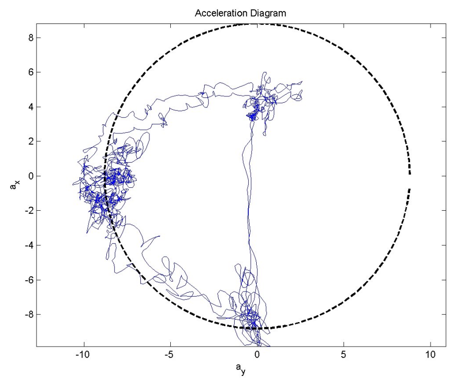

Figure 14. *Acceleration diagram of oval with three phase turns superimposed on a 0.9g friction circle*

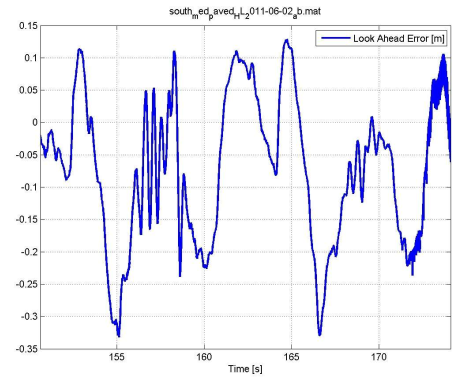

Figure 16. *Lateral and lookahead error for the oval with three phase turns*

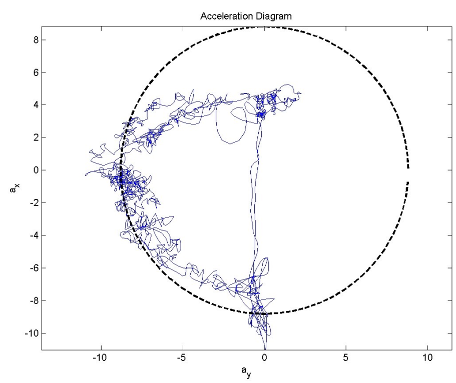

Figure 15. *Acceleration diagram of oval with two phase turns superimposed on a 0.9g friction circle*

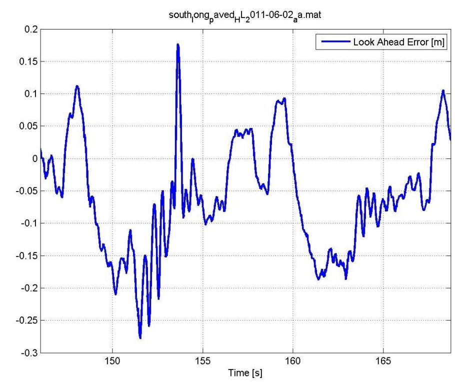

Figure 17. *Lateral and lookahead error for the oval with two phase turns*

the measured accelerations for the 0.9g test which confirm that all tests were able to track the approximated friction circle limits. For the 0.9g test, a limit on acceleration from the engine is apparent because the acceleration data does not reach the top of the friction circle. The Audi was also able to remain at a defined friction limit with a reasonable tracking error (Figures 16 and 17) for the 0.4g friction test. The 0.9g friction test yielded larger tracking error because 0.9g was overestimating the friction available on parts of the track.

As can be seen on Figure 18, the lap time for the 0.9g case increased during the test. This increase in lap time is consistent with the tires overheating and losing grip over the course of the test.

4.4 APPLICATION OF THE DESIGN PROCESS

The design process was implemented on a real world race track to determine its practical application. The track where the Pikes Peak International Hill Climb is held was used for this purpose. Figures 19 and 20 are samples of the path on the mountain. The initial design of the racing line was much more conservative than typical to allow room for any unexpected errors. The design process covered 12.42 miles with over 156 turns. The figures illustrate that all the straights defined along the track were successfully linked together by the proposed curve structure. Ultimately, the autonomous Audi TTs was able to successfully climb the mountain using the constructed racing line.

5 CONCLUSION

This paper demonstrates how a set of simple curves can be used to express and analyze professional driving techniques. The governing equations were discussed as well as the fact that two parameters control specific characteristics of a turn. Two and three phase turns can be expressed by a single parameter; early and late apex turns are described with one parameter also. Using a simple family of curves to emulate a racing line offers a way of analyzing the techniques professional drivers describe. By using this process to emulate the two and three phase cornering Taruffi discusses, vehicle testing demonstrated that longer clothoids can lead to faster lap times.

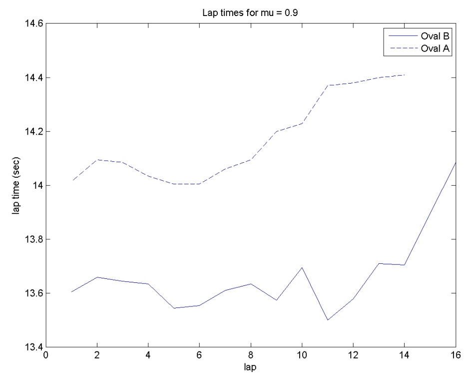

Figure 18. *Lap times for the 0.9g friction limit*

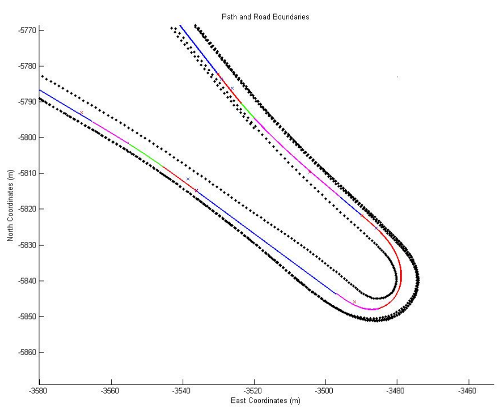

Figure 19. *Switchback on Pikes Peak*

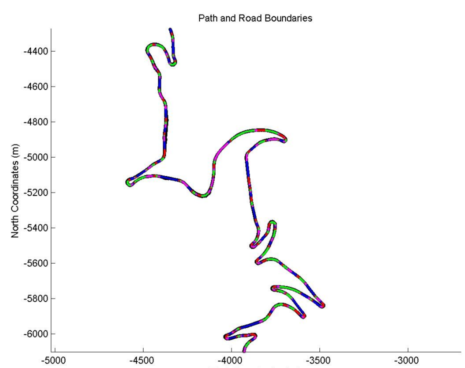

Figure 20. *Overhead view of part of the Pikes Peak racing line*

5.1 FUTURE WORK

Future work involves updating the racing line during a race when the limits are exceeded or when an obstacle is detected. It will also involve studying the specific advantages to early or late apex turns.

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