1 Intersection and Union Operators in Embedding Space

The operators for intersections $\mathcal{I}$ and unions $\mathcal{U}$ are defined separately for entities, timestamps and attribute values. We first define the operators for intersections. The operators $\mathcal{I}_e$, for entities, $\mathcal{I}_t$, for timestamps, and $\mathcal{I}_x$, for attribute values, should calculate the intersection $[q] = \cap_{i=1}^{n}[q_i]$ for their corresponding part of the feature logic embedding. In order to match the entities to the timestamp and attribute value sets, intersection sets are also calculated for the time part and attribute value part for the intersection sets $\mathcal{I}_e$ and unions $\mathcal{U}_e$. This is also done in the same way for $\mathcal{I}_t$, $\mathcal{U}_t$, $\mathcal{I}_x$, $\mathcal{U}_x$.

$$\begin{aligned}\mathcal{I}_e\left(\mathbf{V}_{q_1},\cdots,\mathbf{V}_{q_n}\right) = \big(&\sum_{i=1}^{n}\alpha_i^e q_{i,f}^e, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^e\},\\ &\sum_{i=1}^{n}\beta_i^e q_{i,f}^t, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^t\},\\ &\sum_{i=1}^{n}\gamma_i^e q_{i,f}^x, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^x\}\big)\end{aligned} \tag{1}$$

$$\begin{aligned}\mathcal{I}_t\left(\mathbf{V}_{q_1},\cdots,\mathbf{V}_{q_n}\right) = \big(&\sum_{i=1}^{n}\alpha_i^t q_{i,f}^e, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^e\},\\ &\sum_{i=1}^{n}\beta_i^t q_{i,f}^t, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^t\},\\ &\sum_{i=1}^{n}\gamma_i^t q_{i,f}^x, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^x\}\big)\end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{I}_x\left(\mathbf{V}_{q_1},\cdots,\mathbf{V}_{q_n}\right) = \big(&\sum_{i=1}^{n}\alpha_i^x q_{i,f}^e, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^e\},\\ &\sum_{i=1}^{n}\beta_i^x q_{i,f}^t, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^t\},\\ &\sum_{i=1}^{n}\gamma_i^x q_{i,f}^x, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^x\}\big)\end{aligned} \tag{3}$$

The operators $\mathcal{U}_e$, for entities, $\mathcal{U}_t$, for timestamps, and $\mathcal{U}_x$, for attribute values, should calculate the union $[q] = \cup_{i=1}^{n}[q_i]$ for their corresponding part of the feature logic embedding.

$$
\begin{aligned}
\mathcal{U}_e\left(\mathbf{V}_{q_1},\cdots,\mathbf{V}_{q_n}\right) = \big(& \sum_{i=1}^{n} \alpha_i^e q_{i,f}^e, \mathbf{OR}_{i=1}^{n}\{q_{i,l}^e\}, \\
& \sum_{i=1}^{n} \beta_i^e q_{i,f}^t, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^t\}, \\
& \sum_{i=1}^{n} \gamma_i^e q_{i,f}^x, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^x\}\big)
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\mathcal{U}_t\left(\mathbf{V}_{q_1},\cdots,\mathbf{V}_{q_n}\right) = \big(& \sum_{i=1}^{n} \alpha_i^t q_{i,f}^e, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^e\}, \\
& \sum_{i=1}^{n} \beta_i^t q_{i,f}^t, \mathbf{OR}_{i=1}^{n}\{q_{i,l}^t\}, \\
& \sum_{i=1}^{n} \gamma_i^t q_{i,f}^x, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^x\}\big)
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\mathcal{U}_x\left(\mathbf{V}_{q_1},\cdots,\mathbf{V}_{q_n}\right) = \big(& \sum_{i=1}^{n} \alpha_i^x q_{i,f}^e, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^e\}, \\
& \sum_{i=1}^{n} \beta_i^x q_{i,f}^t, \mathbf{AND}_{i=1}^{n}\{q_{i,l}^t\}, \\
& \sum_{i=1}^{n} \gamma_i^x q_{i,f}^x, \mathbf{OR}_{i=1}^{n}\{q_{i,l}^x\}\big)
\end{aligned} \tag{6}
$$

For these equations, α_i, β_i and γ_i are weights, **AND** is the conjunction in fuzzy logic: $a \wedge b = \min(a, b)$ and **OR** is the disjunction in fuzzy logic: $a \vee b = \max(a, b)$. The weightings are intended to collect changes in the corresponding logic part and pass them on to the feature part. They are calculated as follows:

$$
\alpha_i^{e,t,x} = \frac{\exp\left(\mathbf{MLP}_1^{e,t,x}\left([q_{i,f}^e; q_{i,l}^e]\right)\right)}{\sum_{j=1}^{n}\exp\left(\mathbf{MLP}_1^{e,t,x}\left([q_{j,f}^e; q_{j,l}^e]\right)\right)} \tag{7}
$$

$$
\beta_i^{e,t,x} = \frac{\exp\left(\mathbf{MLP}_2^{e,t,x}\left([q_{i,f}^t; q_{i,l}^t]\right)\right)}{\sum_{j=1}^{n}\exp\left(\mathbf{MLP}_2^{e,t,x}\left([q_{j,f}^t; q_{j,l}^t]\right)\right)} \tag{8}
$$

$$
\gamma_i^{e,t,x} = \frac{\exp\left(\mathbf{MLP}_3^{e,t,x}\left([q_{i,f}^x; q_{i,l}^x]\right)\right)}{\sum_{j=1}^{n}\exp\left(\mathbf{MLP}_3^{e,t,x}\left([q_{j,f}^x; q_{j,l}^x]\right)\right)} \tag{9}
$$

where $\mathbf{MLP}^{e,t,x}_{1,2,3} : \mathbb{R}^{2d} \to \mathbb{R}^d$ are MLPs and $[\cdot;\cdot]$ is the concatenation. In general, each operator has its own MLPs and parameters and these are not shared between operators.