PRACTICAL - 1

TOPIC: Limits and continuity

1). $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

2). $\lim_{y \to a} \left[\dfrac{\sqrt{9+x} - \sqrt{9}}{y\sqrt{a+y}} \right]$

3). $\lim_{x \to \pi/6} \left[\dfrac{\cos 5x - \sqrt{3}\sin x}{\pi - 6x} \right]$

4). $\lim_{x \to \infty} \dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$

5). Examine the continuity of the following function.

(i). $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, \quad 0 < x \leq \pi/2$

$= \dfrac{\cos x}{\pi - 2x}, \quad \pi/2 < x < \pi$

at $x = \pi$

(ii). $f(x) = \dfrac{x^2-9}{x-3}, \quad 0 < x < 3$

$= x+3, \quad 3 \leq x < 6$

$= \dfrac{x^2+9}{x+3}, \quad 6 \leq x \leq 9$

at $x = 3$ and $x = 6$

6. Find value of k so that the function $f(x)$ is continuous at the indicated point

(i). $f(x) = \frac{1-\cos 4x}{x^2}$, $x < 0$; $x = 0$ } at $x=0$

(ii). $f(x) = (\sec^2 x)^{\cot^2 x}$; $= k$ } at $x=0$

(iii). $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$, $x \neq \pi/3$; $= k$, $x = \pi/3$ } at $x = \pi/3$

7. Discuss the continuity of the following functions which of these function have removable discontinuity ?

(i). $f(x) = \frac{1-\cos 3x}{x \tan x}$, $x \neq 0$; $= 9$, $x = 0$ } at $x=0$

(ii). $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$, $x \neq 0$; $= \pi/60$, $x=0$ } at $x=0$.

(8). If $f(x) = \frac{(e^{x^2} - \cos x)}{x^2}$ for $x \neq 0$ is continuous at $x=0$ find $f(0)$.

(9). If $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \frac{\pi}{2}$ is continuous at $x = \pi/2$ find $f(\pi/2)$.

SOLUTION :-

1). $\lim_{x\to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$

$= \lim_{x\to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \lim_{x\to a} \frac{(a+2x-3x)}{(3a+x-4x)} \cdot \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + 3\sqrt{x}}$

$= \lim_{x\to a} \frac{(a-x)}{(3a-3x)} \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \lim_{x\to a} \frac{(a-x)}{3(a-x)} \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \lim_{x\to a} \frac{1}{3} \cdot \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$= \frac{\sqrt{3a+a} + 2\sqrt{a}}{3\sqrt{3a} + \sqrt{3a}}$

$= \frac{\sqrt{4a} + 2\sqrt{a}}{3\sqrt{3a} + \sqrt{3a}}$

$= \frac{2\sqrt{a} + 2\sqrt{a}}{3 \times 2\sqrt{3a}} = \frac{2\sqrt{a}}{3\sqrt{3a}} = \frac{2}{3\sqrt{3}}$ //

2). $\lim_{y\to 0} \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}}$

$= \lim_{y\to 0} \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+x} + \sqrt{a}}{\sqrt{a+x} + \sqrt{a}}$

$$= \lim_{y\to 0} \frac{(a+y-a)}{(y\sqrt{a+y})(\sqrt{a+y}+\sqrt{a})}$$

$$= \lim_{y\to 0} \frac{y}{y\sqrt{a+y}\cdot(\sqrt{a+y}+\sqrt{a})}$$

$$= \lim_{y\to 0} \frac{1}{\cancel{y\sqrt{a+y}}\sqrt{a}(\sqrt{a}+\sqrt{a})}$$

$$= \lim_{y\to 0} \frac{1}{\sqrt{a}(2\sqrt{a})}$$

$$= \frac{1}{2a}$$

3). $\lim_{x\to\pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

$x - \pi/6 = h \qquad x = h + \pi/6 \qquad h \to 0$

$$= \lim_{h\to 0} \frac{\cos(h+\pi/6) - \sqrt{3}(h+\pi/6)}{\pi - 6(h+\pi/6)}$$

$$= \lim_{h\to 0} \frac{(\cos h\cdot\cos\pi/6 - \sin h\cdot\sin\pi/6) - \sqrt{3}(\sin h\cdot\cos\pi/6 + \cos h\cdot\sin\pi/6)}{-6h}$$

$$= \lim_{h\to 0} \frac{\left[\left(\cos h\frac{\sqrt{3}}{2}\right) - \left(\sin h\frac{1}{2}\right)\right] - \sqrt{3}\left(\sin h\frac{\sqrt{3}}{2} + \cos h\frac{1}{2}\right)}{-6h}$$

$$= \lim_{h\to 0} \frac{\left(\cancel{\cos\frac{\sqrt{3}}{2}h} - \sin\frac{h}{2}\right) - \left(\sin\frac{3h}{2} + \cos\frac{\sqrt{3}}{2}h\right)}{-6h}$$

$$= \lim_{h\to 0} \frac{\frac{-\sin 4h}{2}}{-6h}$$

$$= \lim_{h\to 0} \frac{\sin 2h}{6h}$$

$$= \frac{1}{3}\lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{3}$$

4). $\lim_{x\to\infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$

$$= \lim_{x\to\infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}}$$

$$= \lim_{x\to\infty} \frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$= \lim_{x\to\infty} \frac{8(\sqrt{x^2+3}+\sqrt{x^2+1})}{2(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$= \lim_{x\to\infty} \frac{4(\sqrt{x^2+3})+(\sqrt{x^2+1})}{\sqrt{x^2+5}+\sqrt{x^2-3}}$$

$$= \lim_{x\to\infty} \frac{4\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}}$$

$$= 4\lim_{x\to\infty} \frac{\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}} = 4$$

5). $\therefore f(\pi/2) = \dfrac{\sin(\pi/2)}{\sqrt{1-\cos^2(\pi/2)}}$

i). $f(\pi/2) = 0$

$\therefore f$ at $x = \pi/2$ define

(ii). $\lim_{x\to\pi/2^+} f(x) = \lim_{x\to\pi/2^+} \dfrac{\cos x}{\pi - 2x}$

put $x - \pi/2 = h$

$\therefore x = \pi/2 + h$

as $x \to \pi/2$ $h \to 0^+$

$= \lim_{h\to0^+} \dfrac{\cos(\pi/2+h)}{\pi - 2(\pi/2+h)} = \lim_{h\to0^+} \dfrac{-\sin h}{\pi-\pi-2h}$

$= \lim_{h\to0^+} \dfrac{-\sin h}{-2h} = \dfrac{1}{2}$

(i). $\lim_{x\to\pi/2^-} f(x) = \lim_{x\to\pi/2^-} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$

$= \lim_{x\to\pi/2^-} \dfrac{2\sin x.\cos x}{\sqrt{2\sin^2 x}} = \lim_{x\to\pi/2^-} \dfrac{2\sin x}{\sqrt{2}\sin x}$

$= \dfrac{2}{\sqrt{2}} \lim_{x\to\pi/2} \cos x = 0$

$\therefore$ LHS $\neq$ RHS

$\therefore f$ is not continuous at $x=$

$f(x) = \dfrac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \le x \le 6$

$= \dfrac{x^2-9}{x+3}$ $\quad 6 \le x < 9$

at $x = 3$ & $x = 6$

at $x=3$

$f(3) = \dfrac{x^2-9}{x-3} = 0$

f at $x=3$ define

$\lim_{x\to3^+} f(x) = \lim_{x\to3^+}$

$f(3) = x+3 = 3+3 = 6$

f is defined at $x=3$

(ii). $\lim_{x\to3^+} f(x) = \lim_{x\to3^+} (x+3) = 6$

$\lim_{x\to3^-} f(x) = \lim_{x\to3} \dfrac{x^2-9}{x-3} = \dfrac{(x-3)(x+3)}{(x-3)}$

$\therefore$ LHS = RHS

$\therefore f$ is continuous at $x=3$.

for $x=6$

$f(6) = \dfrac{x^2-9}{x+3} = \dfrac{36-9}{6+3} = \dfrac{27}{9}$

$\lim_{x\to6^+} = \dfrac{x^2-9}{x+3}$

$\lim_{x\to6^+} \dfrac{(x-3)(x+3)}{(x+3)} = \lim_{x\to6^+} (x-3) = 6-3 =$

$\lim_{x\to6} x+3 = 3+6 = 9$ $\quad \therefore$ LHL $\neq$

$\therefore$ function

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(6).
(i). $f(x) = \frac{1-\cos 4x}{x^2}$ $x<0$ } at $x=0$
$= k$ $x=0$

Solⁿ: f is continuous at $x=0$
$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$2 \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = k$ $\quad 2 \lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$2(2)^2 = k$ $\therefore$ $k = 8$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$ } at $x=0$
$= k$ $x=0$
$= 4/3$.

(iii) $f(x) = \frac{\sqrt{3}-\tan x}{\pi - 3x}$ $x \neq \pi/3$ } at $x = \pi/3$
$= k$ $x = \pi/3$

$x - \pi/3 = h$ $\quad \therefore \lim_{x\to\pi/3} f(x) = f(\pi/3)$
$x = h + \pi/3$ $h \to 0$
$f(\pi/2+h) - \sqrt{3}$ $\quad \lim_{x\to\pi/3} \frac{\sqrt{3}-\tan x}{\pi - 3x} = k$

$\lim_{x\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\lim_{h\to 0} \sqrt{3}\left(\frac{\tan\pi/3 + \tan h}{1-\tan\pi/3 . \tan h}\right) / \pi - \pi - 3h$

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$\lim_{h\to 0} \frac{\sqrt{3}(1-\tan\pi/3 . \tan h) - \tan(\pi/3 - \tan h)}{(-3h)(1-\tan\pi/3 . \tan h)}$

$\lim_{h\to 0} \frac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{-3h(1-\sqrt{3}\tan h)}$

$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$

$= \frac{4}{3} \frac{1}{1-\sqrt{3}(0)} - \frac{4}{3}$.

(7).(i). $f(x) = \frac{1-\cos 3x}{x.\tan x}$ $x \neq 0$ } at $x=0$
$x = 0$

$f(x) = \frac{1-\cos 3x}{x \tan x}$

$= \frac{2\sin^2 3/2\, x}{x \tan x}$

$= 2\sin^2 3x/2 \times x^2 / \frac{x . \tan x}{x^2}$

$= 2 \lim_{x\to 0} (3/2)^2 / 1$

$= 2 \times \frac{9}{4} = 9/2$.

$\lim_{x\to 0} f(x) = 9/2 \quad 9 = f(0)$

$\therefore$ f is not cts at $x=0$
$\therefore$ Redine function
$f(x) = \frac{1-\cos x}{x\tan x}$ $x \neq 0$
$9/2$ $x = 0$

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Now, $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$.

(7). (ii) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$ $x \neq 0$ } at $x=0$
$= \pi/6$ $x=0$

at $x=0$

$\lim_{x\to 0} (e^{3\pi}-1) \sin(\pi x/180)/x^2$

$3\lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin \pi x/180}{x}$

$3 \log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$

$\therefore f$ is continuous at $x=0$.

(8). $f(x) = \frac{e^{x^2}-\cos x}{x^2}$ $x=0$

is continuous at $x=0$

$\therefore f$ is continuous at $x=0$

~~$\lim_{x\to 0} f(x) = f(0)$~~

$= \frac{e^{x^2}-\cos x}{x^2} = f(0)$

$\frac{e^{x^2}-\cos x -1+1}{x^2}$

$(e^{x^2}-1) + (1-\cos x)/x^2$

$\frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$

$= \log e + 2\left(\frac{\sin x/2}{x}\right)^2$.

multiply with 2 on Numerator & Denominator

$= 1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$

(9). $f(x) = \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

$f(0)$ is continuous at $x=\pi/2$

$\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$

$= \frac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$

$= \frac{1+\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x})}$

$= \frac{1}{2(\sqrt{2}+\sqrt{2})} = \frac{1}{4\sqrt{2}}$

$f(\pi/2) = \frac{1}{4\sqrt{2}}$ //.

(6). (ii). ~~$f(x) = (\sec^2 x)^{\cot^2 x}$~~ $x=0$ } at $x=0$
$= k$ $x=0$

$f(x) = (\sec^2 x)^{\cot^2 x}$

using

$\sec^2 - \tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

and

$\cot^2 x = 1/\tan^2 x$

$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$

we know that

$\lim_{x\to 0} (1+px)^{1/px} = e$

$\therefore = e$

$\therefore k = e$

PRACTICAL - 2

TOPIC : Derivative

Q1] Show that the following function defined from R to R are differentiable

i). $\cot x$ ii). $\operatorname{cosec} x$ iii). $\sec x$

Q2] If $f(x) = 4x+1$ $x \le 2$

$= x^2+5$ $x > 0$ at $x=2$ then

find f is differentiable or not.

Q3] If $f(x) = 4x+7$ $x < 3$

$= x^2+3x$ ~~$x \le$~~ $x \ge 3$ at $x=3$ then,

find f is differentiable or not

Q4] If $f(x) = 8x-5$ $x \le 2$

$= 3x^2 - 4x + 7$ $x < 2$ at $x=2$ then,

find f is differentiable or not?

Soln:

Q1.

i). $\cot x$

$f(x) = \cot x$

$= \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\cot x - \cot a}{x-a}$

$= \lim_{x\to a} \frac{1/\tan x - 1/\tan a}{x-a}$

$= \lim_{x\to 0} \frac{\tan a - \tan x}{(x-a)\tan x \cdot \tan a}$

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put $x-a=h$ $x=a+h$ as $x\to a$, $h\to 0$

$$Df(h) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(a+h-a).\tan(a+h).\tan a}$$

$$= \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h).\tan a}$$

formula : $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A.\tan B}$

$\tan A - \tan B = \tan(A-B)\,(1+\tan A.\tan B)$

$$= \lim_{h\to 0} \frac{\tan(a-a-h)\cdot(1+\tan a \cdot \tan(a+h))}{h \times \tan(a+h)\tan a.}$$

$$\lim_{h\to 0} = \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$Df(a) = -\cos^2 a$

$\therefore$ f is differentiable $\forall a \in R$

2). $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a.\sin x}$$

put $x-a=h$ $x=a+h$ as $x\to a$

$$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\ \sin a.\sin h}$$

formula :

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right).\sin\left(\frac{a-a-h}{2}\right)}{h\times \sin a.\sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-1/2 \times 2\cos\left(\frac{2a}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a.\operatorname{cosec} a$$

3). $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec(x)-\sec(a)}{x-a}$$

$$= \lim_{x\to a} \frac{1/\cos x - 1/\cos a}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$ put $x-a=h$, $x=a+h$

as $x\to a$ $h\to 0$

$$Df(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h\times \cos a\ \cos(a+h)}$$

Formula: $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$= \lim_{h\to 0} \dfrac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{\cos a \quad \cos(a+h) \times -h/2}$

$= -\frac{1}{2} \times -2 \dfrac{\sin a}{\cos a \times \cos a}$

$= \tan a \sec a$.

Q2. Soln:

L.H.D

$Df(2^-) = \lim_{x\to 2^-} \dfrac{f(x)-f(2)}{x-2}$

$= \lim_{x\to 2^-} \dfrac{4x+1-(4\times 2+1)}{x-2}$

$= \lim_{x\to 2^-} \dfrac{4x+1-9}{x-2}$

$= \lim_{x\to 2^-} \dfrac{4x-8}{x-2}$

$= \lim_{x\to 2^-} \dfrac{4(x-2)}{(x-2)} = 4$.

$Df(2^-) = 4$

R.H.D

$Df(2^+) = \lim_{x\to 2^+} \dfrac{x^2+5-9}{x-2}$

$= \lim_{x\to 2^+} \dfrac{x^2-4}{x^2-2}$

$= \lim_{x\to 2^+} \dfrac{(x+2)(x-2)}{(x-2)}$

$= 2+2 = 4$

$Df(2^+) = 4$ R.H.D = L.H.D

$\therefore f$ is diff. at $x=2$

Q3. Soln: R.H.D :-

$Df(3^+) = \lim_{x\to 3^+} \dfrac{f(x)-f(3)}{x-3}$

$= \lim_{x\to 3^+} \dfrac{x^2+3x+1-(3^2+3-3+1)}{x-3}$

$= \lim_{x\to 3^+} \dfrac{x^2+3x+1-19}{x-3}$

$= \lim_{x\to 3^+} \dfrac{x^2+3x-18}{x-3}$

$= \lim_{x\to 3^+} \dfrac{x(x+6)-3(x+6)}{x-3}$

$= \lim_{x\to 3^+} \dfrac{(x+6)(x-3)}{(x-3)} = 3+6$

$Df(3^+) = 9$

L.H.D :-

$Df(3^-)$

$= \lim_{x\to 3^-} \dfrac{f(x)-f(3)}{x-3}$

$= \lim_{x\to 3^-} \dfrac{4x+7-19}{x-3} = \lim_{x\to 3^-} \dfrac{4x-12}{x-3} = \lim$

$= 4$

$\therefore$ R.H.D $\neq$ L.H.D

$\therefore f$ is not diff. at $x=3$.

Q4. Soln :- $f(2) = 8\times 2 - 5 = 16 - 5 = 11$

R.H.D : $Df(2^+) = \lim_{x\to 2^+} \dfrac{f(x)-f(2)}{x-2}$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$$

$$= 3 \times 2 + 2 = 8$$

$Df(2^+) = 8$.

L.H.D :-

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)} \quad Df(2^-) = 8$$

L.H.D = R.H.D f is diff at $x = 3$.

PRACTICAL-3

* **TOPIC** : App. of Derivative.

1). Find the intervals in which function is increasing or decreasing

i). $f(x) = x^3 - 5x - 11$

ii). $f(x) = x^2 - 4x$

iii). $f(x) = 2x^3 + x^2 - 20x + 4$

iv). $f(x) = x^3 - 27x + 5$

v). $f(x) = 69 - 24x - 9x^2 + 2x^3$

2). Find the intervals in which function is concave upwards

i). $y = 3x^2 - 2x^3$

ii). $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii). $y = x^3 - 27x + 5$

iv). $y = 69 - 24x - 9x^2 + 2x^3$

v). $y = 2x^3 + x^2 - 20x + 4$

Soln:

Q1).

1). $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\therefore$ f is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

+ − +

$-\sqrt{5/3}$ $\sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3})$ $(\sqrt{5/3}, \infty)$

and f is decreasing iff $f(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5}/3) \ldots$

$+ \quad - \quad +$ $\qquad x \in (-\sqrt{5}/3, \sqrt{5}/3)$

$-\sqrt{5}/3 \qquad \sqrt{5}/3$

2). $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$x \in (-\infty, 2)$

3). $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$

$+ \quad - \quad +$ $\qquad x \in (-\infty, -2) \cup (5/3, \infty)$

$-2 \qquad 5/3$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$

$+ \quad - \quad +$ $\qquad x \in$

$-2 \qquad 5/3$

4). $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\therefore f(x)$ is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x-3)(x+3) > 0$

$+ \quad - \quad +$

$-3 \qquad 3$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$

$+ \quad - \quad +$ $\qquad \therefore x$

$-3 \qquad 3$

5). $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore x(x-4) + 1(x-4) > 0$

$\therefore (x-4)(x+1) > 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + x - 4 < 0$

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$\therefore x \in (-1, 4)$

Q2).

i). $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

f is concave upward iff $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\therefore f''(x) > 0$

$x \in (1/2, \infty)$

2. $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore (x-2)(x-1) > 0$

$+ \quad 1 \quad - \quad 2 \quad +$ $\quad x \in (-\infty, 1) \cup (2, \infty)$

3). $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

4). $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0$ $\quad \therefore x > 3/2$

$\therefore x \in (3/2, \infty)$.

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x < -1/6$

$\therefore f''(x) \ngtr 0$

$\therefore$ There exists ~~in~~ interval $(-1/6, \infty)$

AR
27/01/2020

PRACTICAL-4

* TOPIC: App. of derivative & Newton's Method

Q1). Find max & min values ~~of~~ for the foll:-

i). $f(x) = x^2 + \frac{16}{x^2}$

ii). $f(x) = 3 - 5x^3 + 3x^5$

iii). $f(x) = x^3 - 3x^2 + 1$ $[-1/2, 4]$

iv). $f(x) = 2x^3 - 3x^2 - 12x + 1$ $[-2, 3]$

Q2). Find the root of the foll. equation by Newton
(Take 4 derivations only)
Correct upto 4 decimal.

i). $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

ii). $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii). $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

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2)

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^2$

$\therefore x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/2^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x = 2$, and $x = -2$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

Consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60$

$= 30 > 0$ $\therefore f$ has minimum value

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$ $\therefore f$ has maximum value at

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore f$ has maximum value 5 at $x = -1$ and has minimum value 1 at $x = 1$

(iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x - 2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$f_1(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$
$= 0.0879 - 1.0272 - 55$
$= -55.9393$

$\therefore x_3 = x_2 - f(x_2) / f^1(x_2)$
$= 0.1712 + 0.0011 / 55.9393$
$= \underline{0.1712}$

$\therefore$ The root of the equation is $\underline{0.1712}$

(ii) $f(x) = x^3 - 4x - 9$ $[2,3)$
$f^1(x) = 3x^2 - 4$
$f(2) = 23 - 4(2) - 9$
$= 8 - 8 - 9$
$= -9$
$f(3) = 33 - 4(3) - 9$
$= 27 - 12 - 9$
$= 6$

Let $x_0 = 3$ be the initial approximation
$\therefore$ By Newton's Method,
$x_{n+1} = x_n - f(x_n) / f_1(x_n)$
$x_1 = x_0 - f(x_0) / f_1(x_0)$
$= 3 - 6/23$
$= 2.7392$
$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$
$= 20.5528 - 10.9568 - 9$
$= 0.596$
$f_1(x_1) = 3(2.7392)^2 - 4$
$= 22.5096 - 4$
$= 18.5096$
$x = x_1 - f(x_1) / f'(x_1)$
$= 2.7392 - 0.596 / 18.5096$
$= 2.7071$
$f(x_2) = (2.7071)^3 - 4(2.7071)$
$= 19.8386 - 10.8284$
$= 0.0102$
$f'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$= 17.9851$
$= 2.7071 - \frac{0.0102}{17.9851}$
$= 2.7071 - 0.0056 = \underline{2.7015}$
$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9 = -0.0901$
$f'(3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.$
$x_4 = 2.7015 + 0.0901 / 17.8943 = 2.7015$
$= 2.7065$

(3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1,2]$
$f_1(x) = 3x^2 - 3.6x - 10$
$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= -1.8 - 10 + 17$
$= 6.2$
$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17 = -2.2$
Let $x_0 = 2$ be initial approximat
Newton's Method:
$x_{n+1} = x_n - f(x_n) / f_1(x_n)$
$x_1 = x - f(x_2) / f_1(x_2)$

$f_1(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$
$= 0.0879 - 1.0272 - 55$
$= -55.9393$

$\therefore x_3 = x_2 - f(x_2) / f_1(x_2)$
$= 0.1712 + 0.0011 / 55.9393$
$= 0.1712$

$\therefore$ The root of the equation is 0.1712

(ii) $f(x) = x^3 - 4x - 9$ $[2, 3]$
$f_1(x) = 3x^2 - 4$
$f(2) = 2^3 - 4(2) - 9$
$= 8 - 8 - 9$
$= -9$
$f(3) = 3^3 - 4(3) - 9$
$= 27 - 12 - 9$
$= 6$

Let $x_0 = 3$ be the initial approximation
$\therefore$ By Newton's Method,
$x_{n+1} = x_n - f(x_n) / f_1(x_n)$
$x_1 = x_0 - f(x_0) / f_1(x_0)$
$= 3 - 6/23$
$= 2.7392$
$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$
$= 20.5528 - 10.9568 - 9$
$= 0.596$
$f_1(x_1) = 3(2.7392)^2 - 4$
$= 22.5096 - 4$
$= 18.5096$
$x_2 = x_1 - f(x_1) / f'(x_1)$
$= 2.7392 - 0.596 / 18.5096$

2.7071
$f(x_2) = (2.7071)^3 - 4(2.7071)$
$= 19.8386 - 10.8284$
$= 0.0102$

$f_1(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$= 17.9851$
$= 2.7071 - \frac{0.0102}{17.9851}$
$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9 = -0.0901$
$f_1(x_3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.89$
$x_4 = 2.7015 + 0.0901 / 17.8943 = 2.7015 +$
$= 2.7065$

$f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$
$f_1(x) = 3x^2 - 3.6x - 10$
$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= -1.8 - 10 + 17$
$= 6.2$
$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approxim
Newton's Method.
$x_{n+1} = x_n - f(x_n) / f_1(x_n)$
$x_1 = x - f(x_2) / f_1(x_2)$

$f_1(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$
$= 0.0879 - 1.0272 - 55$
$= \underline{-55.9393}$

$\therefore x_3 = x_2 - f(x_2) / f^1(x_2)$
$= 0.1712 + 0.0011 / 55.9393$
$= \underline{0.1712}$

$\therefore$ The root of the equation is $\underline{\underline{0.1712}}$

(ii) $f(x) = x^3 - 4x - 9$ $[2, 3)$
$f'(x) = 3x^2 - 4$
$f(2) = 2^3 - 4(2) - 9$
$= 8 - 8 - 9$
$= -9$
$f(3) = 3^3 - 4(3) - 9$
$= 27 - 12 - 9$
$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's Method,
$x_{n+1} = x_n - f(x_n) / f^1(x_n)$
$x_1 = x_0 - f(x_0) / f^1(x_0)$
$= 3 - 6 / 23$
$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$
$= 20.5528 - 10.9568 - 9$
$= 0.596$

$f_1(x_1) = 3(2.7392)^2 - 4$
$= 22.5096 - 4$
$= 18.5096$

$x = x_1 - f(x_1) / f'(x_1)$
$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$
$= 19.8386 - 10.8284$
$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$= 17.9851$
$= 2.7071 - \frac{0.0102}{17.9851}$
$= 2.7071 - 0.00556 = \underline{\underline{2.7015}}$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9 = -0.0901$

$f'(3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901 / 17.8943 = 2.7015 + 0.0050$
$= 2.7065$

(3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$
$f_1(x) = 3x^2 - 3.6x - 10$
$f(1) = (1)^3 - 1.8(7)^2 - 10(1) + 17$
$= -1.8 - 10 + 17$
$= 6.2$
$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approximation B.
Newton's Method.
$x_{n+1} = x_n - f(x_n) / f_1(x_n)$
$x^1 = x - f(x_2) / f_1(x_2)$

$= 2 - 2.2/5.2$
$= 2 - 0.4230 = \underline{1.577}$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$
$= 3.9219 - 4.4764 - 15.77 + 17$
$= \underline{0.6755}$

$f_1(x) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -\underline{8.2164}$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$
$= 1.577 + 0.6755/8.2164$
$= 1.577 + 0.0822$
$= \underline{1.6592}$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) +$
$= 4.5677 - 4.9553 - 16.592 + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$x_3 = x_2 - f(x_2)/f'(x_2)$
$= 1.6592 + 0.0204/7.7143$
$= 1.6592 + 0.0026$
$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$

$f_1(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2847 - 5.9824 - 10$
$= -\underline{7.6977}$

$x_4 = x_3 - f(x_3)/f_1(x_3)$
$= 1.6618 + \frac{0.0004}{7.6977}$
$= \underline{1.6618}$

PRACTICAL - 5

TOPIC : Integration

Q1) Solve the following integration.

i) $\frac{dx}{\sqrt{x^2+2x-3}}$

ii) $\int (4e^{3x}+1)\, dx$

iii) $\int (2x^2-3\sin x+5\sqrt{x})\, dx$

iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\, dx$

v) $\int t^7 \sin(2t^4)\, dt$

vi). $\int \sqrt{x}\cdot(x^2-1)\, dx$

vii). $\int 1/x^3 \sin(1/x^2)\, dx$

viii). $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\, dx$

ix). $\int e^{\cos^2 x}\sin 2x\, dx$

x). $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

~~1). $\int \frac{1}{x^2+2x-3}\, dx$~~

1). $\int \frac{1}{x^2+2x-3}\, dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}}\, dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}}$

Substitute

put $x+1 = t$

$dx = \frac{1}{t} \times dt$ where $t=1$ $t = x+1$

$\int \frac{1}{\sqrt{t^2-4}}\, dt$

using,

$\int \frac{1}{\sqrt{x^2-a^2}}\, dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+c$ //

2). $\int (4e^{3x}+1)\, dx$

$= \int 4e^{3x}\, dx + \int 1\, dx$

$= 4\int e^{3x}\, dx + \int 1\, dx$ # $\int e^{ax}\, dx = \frac{1}{a}\times e^{ax}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$ //

3/. $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10 \times \sqrt{x}}{3} + C$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

4/. $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}}\,dx$

split the ~~deno~~ denominator.

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2}\,dx + \int 3x^{1/2}\,dx + \int \frac{4}{x^{1/2}}\,dx$

$= \frac{x^{5/2+1}}{5/2 + 1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5/. $\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4}\,du$

$= t^4 \sin(2t^4) \times \frac{1}{8}\,du = \frac{t^4 \times \sin(2t^4)\,du}{8}$

Substitute t^4 with $u/2$

$= \int \frac{u/2 \times \sin(u)\,du}{2}$

$= \int \frac{u \times \sin(u)}{16}\,du$

$= \frac{1}{16} \int u \times \sin(u)\,du$

$\int u\,dv = uv - \int v\,du$

where $u = u$

$dv = \sin(u) \times du$

$du = 1\,du$ $\quad V = -\cos(u)$

$= \frac{1}{16}(u \times (-\cos(u)) - \int -\cos(u)\,du$

$\int \cos x\,dx = \sin(x)$

$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$

Return the substitution $u = 2t^4$

$= \frac{1}{16} \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$.

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

$$= \frac{-1}{-1/3\, t^{2/3-1}} = \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$$

$= 3\sqrt[3]{t}$ //.

Return substitution $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + c$ //.

vi). $\int \sqrt{x}\,(x^2-1)\,dx$

$= \int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\, dx$

$= \int x^{5/2} - x^{1/2}\, dx$

$= \int x^{5/2}\, dx - \int x^{1/2}\, dx$

$$I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7}$$

$$= \frac{2x^3\sqrt{x}}{7} //.$$

$$I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3},$$

$$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c //.$$

iii). $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\, da$

$$= \int \frac{\cos x}{\sin x^{2/3}}\, dx$$

Put $t = \sin(x)$

$t = \cos x$

$$= \int \frac{\cos.(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)}\, dt$$

$$= \frac{1}{\sin x^{3/2}}\, dt$$

$$= \frac{1}{t^{2/3}}\, dt$$

(x). $\int \frac{x^2-2x}{x^3-3x^2+1}\, dx$

Put $x^3-3x^2+1 = dt$

$$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x}\, dt$$

$$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x}\, dt$$

$$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3}\, dt$$

$$= \int \frac{1}{3(x^3-3x^2+1)}\, dt = \int \frac{1}{3t}\, dt.$$

$= \frac{1}{3} \int 1/t\, dt \qquad \int 1/x\, dx = \ln|x|$

$= 1/3 \times \ln|t| + c$

$= \frac{1}{3} \times \ln(|x^3-3x^2+1|) + c$ //.

AV 06/01/2020

PRACTICAL - 6

* Topic :- Application of integration & Numerical Integration.

1. Find the length of the following curve.

Q1. $x = t\sin t$ $y = 1-\cos t$ $t \in [0, 2\pi]$
for t belong to $[0, 2\pi]$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$x = t - \sin t$$

$$\frac{dx}{dt} = t - \cos t$$

$$y = 1 - \cos t$$

$$\frac{dy}{dt} = 0 - (-\sin t)$$

$$\frac{dy}{dt} = \sin t$$

$$L = \int_0^{2\pi} \sqrt{(t - \cos t + t\sin t^2) + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$\int_0^{2\pi} 2\left|\sin\frac{t}{2}\right| dt \quad \because \sin^2\frac{t}{2} = \frac{1-\cos t}{2}.$$

$$= \int_0^{2\pi} 2\sin\frac{t}{2}\, dt$$

$$= \int_0^{2\pi}\left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$$

$$= 4+4$$

$$= \boxed{8}.$$

Q.3] $y = \sqrt{4-x^2} \qquad x \in [-2,2]$

Sol^n $2 = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$\frac{dy}{dx} = 2\int_0^2 \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= 2\int_0^2 \sqrt{1+\frac{x^2}{4-x^2}}\, dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\left(\sin^{-1}(x/2)\right)_0^2$$

$$= 2\pi$$

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3]. $y = x^{3/2}$ in $[0,4]$

Sol^n. $f'(x) = \frac{3}{2}x^{1/2}$

$$[f'(x)]^2 = \frac{9}{4}x$$

$$l = \int_a^b \sqrt{1+[f'(x)]^2}\, dx$$

$$= \int_0^x \sqrt{1+\frac{9}{4}x}\, dx$$

put $u = 1+\frac{9}{4}x$, $du = \frac{9}{4}dx$

$$L = \int_1^{1+\frac{9}{4}x} \frac{4}{9}\sqrt{u}\, du = \left[\frac{4}{9} - \frac{2}{3}(u^{3/2})\right]_1^{1+\frac{9}{4}x}$$

$$= \frac{8}{27}\left[\left(1+\frac{9x}{4}\right) - 1\right]$$

4]. $x = 3\sin t$ + $y = 3\cos t$

Sol^n. $\frac{dx}{dt} = 3\cos t$

$$\frac{dy}{dt} = -3\sin t$$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$\int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\ dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\ dt$$

$$= 3\int_0^{2\pi} x\ dt$$

$$= 3[x]_0^{2\pi}$$

$$= 3[2\pi - 0]$$

$\therefore L = 6\pi$ units

5]. $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y = [1, 2]$

Sol^n. $\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\ dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\ dy$$

$$= \frac{1}{2}\int_1^2 y^2\ dy + \frac{1}{2}\int_1^2 y^{-2}\ dy$$

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$$\frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$\frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$$

$$= \frac{17}{12} \text{ units}$$

1). $\int_0^2 e^{x^2}\,dx$ with $n=4$

Sol^n $\int_0^2 e^{x^2}\,dx = 16.4526$

In each case the width of the sub interval be

$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

and so the sub intervals will be $[0, 0.5]$, $[0.5, 1]$ $[1, 1.5]$, $[1.5, 2]$

By simpson rule

$$\int_0^2 e^{x^2}\,dx = \frac{1/2}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$$

$$\approx \frac{1/2}{3}\left(e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^2}\right.$$

$$\approx 17.3536$$

2). $\int_0^4 x^2\,dx$ $n=4$

$$\Delta x = \frac{4-0}{\ } = 1$$

$$\int_a^b f(x)dx = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$= \frac{1}{3}\left[y(0) + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$$

$$= \frac{1}{3}\left[0^3 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$$

$$= \frac{64}{3}$$

3). $\int_0^{\pi/3} \sqrt{\sin x\, dx}$ $n = 6$

Sol^n. $\Delta x = \frac{b-a}{n}$

$$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

x	0	$\pi/8$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$
y	0	0.4167	0.584	0.707	0.801	0.87
	y_0	y_1	y_2	y_3	y_4	y_5

$$\int_0^{\pi/3} \sqrt{\sin x}\ dx = \frac{\Delta x}{3}\left(y_0 + 4(y_1 + y_3 + \tfrac{1}{3}) + 2(y_2 + y_4)\right)$$

$$= \frac{\pi/18}{3}\left(0 + 4(0.4167 + 0.707 + 0.875) + 0.930\right)$$

$$\cancel{x = 2(0.584 + 0.801) + 0.930}$$

$$x = 0.681$$

AK 09/01/2020

PRACTICAL-7

Q1. Calculate the following differential equations

(1). $x\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = 1/x$ $\quad Q(x) = e^x/x$

If $e^{\int 1/x\, dx}$

x

$$y(IF) = \int Q(x)\ (IF)dx + c$$

$$= \int \frac{e^x}{x} x\ dx + c$$

$$= \int e^x\ dx + c$$

$$xy = e^x + c$$

(2). $e^x\frac{dy}{dx} + 2e^x y = 1$

Soln: $\frac{dy}{dx} + \frac{2e^x}{e^x} = \frac{1}{e^x}$

$$\frac{dy}{dx} + 2y = \frac{1}{e^x}$$

$$\frac{dy}{dx} + 2y = e^{-x}$$

$P(x) = 2 \quad \cancel{Q(x)} = e^{-x}$

$\cancel{\int f(x) = dx}$

$$If = e^{\int 2dx}$$

$$= e^{2x}$$

$$y(IF) = \int Q(x)(IF)\,dx + c.$$

$= \int e^{x}\, dx + c$

$= y \cdot e^{2x}$

$= e^{x} + C.$

3). $x \dfrac{dy}{dx} = \dfrac{\cos x}{x} - 2y$

Soln:- $\dfrac{dy}{dx} + \dfrac{2y}{x} = \dfrac{\cos x}{x^2}$

$P(x) = 2(x)$ $\quad Q(x) = \dfrac{\cos x}{x^2}$

$IF = e^{\int P(x)\, dx}$

$= e^{\int 2x\, dx}$

$y(IF) = \int Q(x)\,(IF)\, dx + c$

$= \int \dfrac{\cos x}{x^2} - x^2\, dx + C$

$= \int \cos x + c$

$\therefore x^2 y = \sin x + c$

4). $x \cdot \dfrac{dy}{dx} + 3y = \dfrac{\sin x}{x^2}$

Soln: $\dfrac{dy}{dx} + \dfrac{3y}{x} = \dfrac{\sin x}{x^3}$

$P(x) = 3/2$ $\quad Q(x) = \sin x / x^3$

$P(x) = \int 3/x\, dx$

$= x^3$

$IF = e^{\int P(x)\, dx}$

$= x^3$

$y(IF) = \int Q\, f(x)\,(IF)\, dx + c$

$= \int \dfrac{\sin x}{x^3} \cdot x^3\, dx + c$

$= \int \sin x + c$

$x^3 y = -\cos x + c$

5). $e^{2x} \dfrac{dy}{dx} + 2e^{2x} y = 2x$

Soln:- $\dfrac{dy}{dx} + 2y = \dfrac{2x}{e^{2x}}$

$P(x) = 2$

$Q(x) = \dfrac{2x}{e} = 2xe^{-2x}$

$(IF) = e^{\int P(x)\, dx}$

$= e^{\int 2\, dx}$

$= e^{2x}$

$y(IF) = \int Q(x)\,(IF)\, dx + c$

$= \int 2xe^{-2x} e^{2x} + c$

$y e^{2x} = \int 2x + c = x^2 + c$ //.

$$(6).\ \sec^2 x \cdot \tan y\, dx + \sec^2 y \tan y\, dy = 0.$$

Soln:-

$$\sec^2 x \cdot \tan y\, dx = -\sec^2 y \cdot \tan x\, dy$$

$$\frac{\sec^2 x}{\tan x}\, dx = \frac{-\sec^2 x}{\tan y}\, dy$$

$$\int \frac{\sec^2 x}{\tan x}\, dx = -\int \frac{\sec^2 x}{\tan y}\, dy$$

$$\int \frac{\sec^2 x}{\tan x}\, dx = -\int \frac{\sec^2 x}{\tan y}\, dy$$

$$\log|\tan x| = -\log|\tan y| + C$$

$$\log|\tan x + \tan y| = C$$

$$\tan x + \tan y = e^c$$

$$(7).\ \frac{dy}{dx} = \sin^2(x-y+1)$$

put $x - y + 1 = v$

$$\therefore 1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$1 - \frac{dv}{dx} = \frac{dy}{dx}$$

$$1 - \frac{dv}{dx} = \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 x$$

$$\frac{dv}{ds^2} = dx$$

$$\int \sec^2 x\, dv = \int dx$$

$$\tan v = x + c$$

$$\therefore \tan(x-y+1) = x + c$$

$$(8).\ \frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

put $2x + 3y = v$

$$2 + 3\frac{dy}{dx} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$= \frac{1}{3}(dv/dx - 2) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= \frac{3(v+1)}{(v+2)} = \int \frac{v+2}{v+1}\, d$$

$$= \int \frac{v+1}{v}\, dx + \int \frac{1}{v+1}\, dv = \int 3$$

$$v \log|v| = 3x + C.$$

$$2x + 3y + \log|2x+3y+1| = 3$$

$$3y = x - \log|2x+3y+1| + c$$

PRACTICAL-8.

* TOPIC :- Euler's Method.

Q1. $dx/dy = y + e^x - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ?$

Soln :-

$f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	82021	98215
4	2	9.8215		

$\therefore y(2) = 9.8215$

Q2. $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$y_0 = 0$, $y_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$ //

Q3. $\frac{dy}{dx} = \sqrt{x/y}$ $y(0) = 1$ $h = 0.2$ find $y(1) = ?$ 6n

$x_0 = 0$ $y(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7696	1.5051
5	1	1.5051		

$\therefore y(1) = 1.5051$

Q4. $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$ $h = 0.5$

$h = 0.25$

$y_0 = 2$ $x_0 = 1$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	7.754	4
1	1.5	~~4~~	7.75	7.874
2	2	7.875		

$y(2) = 7.875$ //

(Q4). $y_0 = 2$ $x_0 = 1$ $h = 0.25$
(2).

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	19.3360
2	1.5	4.4218	59.6569	299.9960
3	1.75	19.3360	1122.6426	
4	2	299.9960		

$y(2) = 299.9960$

(Q5). $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1) = 1$ $h = 0.2$

$x_0 = 1$ $y_0 = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1, 2) = 3.6$

20/01/2020

PRACTICAL-9

(Q1).
(i). $\lim_{(x,y) \to (-4,-1)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

At $(-4, -1)$, Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(-4)^3 - 3(-1) + (-1)^2 - 1}{-4(-1) + 5}$

$= \frac{-64 + 3 + 1 - 1}{4 + 5}$

$= \frac{-61}{9}$

(ii). $\lim_{(x,y) \to (2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

At $(2, 0)$, Denominator $\neq 0$

$\therefore$ By applying limit,

$= \frac{(0+1)((2)^2 + 0 - 4(2))}{2 + 0}$

$= \frac{1(4 + 0 - 8)}{2}$

$= \frac{-4}{2}$

$= 2$

(iii). $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

At $(1,1,1)$, denominator $= 0$

$\therefore \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{\cancel{(x-yz)}(x+yz)}{x^2\cancel{(x-yz)}}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

On Applying limit:-

$= \frac{1+1(1)}{(1)^2}$

$= 2$.

(Q2). (i). $f(x,y) = xye^{x^2+y^2}$

$\therefore fx = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x)$

$\therefore fx = 2xye^{x^2+y^2}$

$\therefore fx = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$\therefore fy = 2yxe^{x^2+y^2}$

(ii). $f(x,y) = e^x.\cos y$

$fx = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(e^x \cos y)$

$\therefore fx = e^x \cos y$

$\therefore fy = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^x \cos y)$

$\therefore fy = -e^x \sin y$

(iii). $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$fx = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore fx = 3x^2y^2 - 6xy$

$fy = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore fy =$

(Q3). (i). $f(x,y) = \frac{2x}{1+y^2}$

~~Sol~~ $f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{d}{dx}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{2+2y^2-0}{(1+y^2)^2}$$

$$= \frac{2\cancel{(1+y^2)}}{\cancel{(1+y^2)}(1+y)^2}$$

$$= \boxed{\frac{2}{1+y^2}}.$$

At (0,0)

$$= \frac{2}{1+0}$$

$$= 2$$

$f_y = \frac{d}{dy}\left(\frac{2x}{1+y^2}\right)$

$$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}.$$

At (0,0),

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= \boxed{0}.$$

(4). (i). $f(x,y) = \frac{y^2-xy}{x^2}$

$$\therefore f_x = \frac{x^2 \frac{d}{dx}(y^2-xy) - (y^2-xy)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2-xy)(2x)}{x^4}$$

$$= \frac{-x^2y - 2x(y^2-xy)}{x^2}$$

$f_y = \frac{2y-x}{x^2}$

$$f_{xx} = \frac{d}{dx}\left(\frac{-x^2y - 2x(xy^2-xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{d}{dx}(-x^2y-2xy^2+2x^2y)\right)}{(x^4)^2},$$

$f_{yy} = \frac{d}{dy}\left(\frac{2y-x}{x^2}\right)$

$= \frac{2-0}{x^2} = \frac{2}{x^2}$ //.

$f_{xy} = \frac{d}{dy}\left(\frac{-x^2y-2xy^2+2x^2y}{x^4}\right)$

$= \frac{-x^2-4xy+2x^2}{x^4}$

$f_{yx} = \frac{d}{dx}\left(\frac{2y-x}{x^2}\right)$

$= \frac{x^2\frac{d}{dx}(2y-x) - (2y-x)\frac{d}{dx}(x^2}{(x^2)^2}$

$= \frac{-x^2-4xy+2x^2}{x^4}$

from ③ & ④ :

$f_{xy} - f_{yx}$.

(ii). $f(x,y) = x^3+3x^2y^2 - \log(x^2+1)$

$f_x = \frac{d}{dx}(x^3+3x^2y^2 - \log(x^2+1))$

$= 3x^2+6xy^2 - \frac{2x}{x^2+1}$

$f_y = \frac{d}{dy}(x^3+3x^2y^2 - \log(x^2+1))$

$= 0+6x^2y - 0$

$= 6x^2y$.

$f_{xx} = 6x+6y^2 - \left(\frac{x^2+1\frac{d}{dx}(2x) - 2x\frac{d}{dx}(x^2+1)}{(x^2+1)^2}\right)$

$= 6x+6y^2 - \left(\frac{2(x^2+1)-4x^2}{(x^2+1)^2}\right)$ — ①.

$f_{yy} = \frac{d}{dx}(6x^2y)$

$= 6x^2$ — ②.

$f_{xy} = \frac{d}{dx}\left(3x^2+6xy^2 - \frac{2x}{x^2+1}\right)$

$= 0+12xy - 0$

$= 12xy$

$f_{yx} = \frac{d}{dx}(6x^2y)$

$= 12xy$

From ③ & ④,

$\therefore f_{xy} = f_{yx}$

(iii). $f(x,y) = \sin(xy) + e^{x+y}$

$\to f_x = y\cos(xy) \cancel{+e^{x+y}}(1)$

$= y\cos(xy) + e^{x+y}$

$\therefore f_{xx} = \frac{d}{dx}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$

$= -y^2\sin(xy) + e^{x+y}$

(Q5).

(i). $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\to f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\quad f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\quad = \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\quad f_y$ at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y+1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$\cancel{\frac{2\sqrt{2}+1}{\sqrt{2}}x} + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

Q.1). Find the directional derivative of the following function at given points a & in the direction given vector.

(i). $f(x,y) = x+2y-3$ $\quad a = (1,-1)$ $\quad u = 3i-j$

Here, $u = 3i-j$ is not a unit vector

$|u| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$.

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = 1+2(-1)-3 = -1-$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(\cancel{1+\frac{3}{\sqrt{10}}}\right), \left(-1-\frac{h}{\sqrt{10}}\right)-3$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right)$

$= \frac{1+3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$ //

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$$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu)-f(a)}{h}$$

$$= \lim_{h\to 0} = \frac{4 + h/\sqrt{10} + 4}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}} //$$

$f(x) = y^2 - 4x + 1$ $a = (3,4)$ $u = i + 5j$

~~Here~~ Here $u = i + 5j$ is not a unit vector

$$|\vec{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}.$$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$f(a) = f(3,4) = 4^2 - 4(3) + 1 = 5$$

$$f(a+hu) = \cancel{f}(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f_{x,y}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1 -$$

$$= \frac{25h^2}{26} + \frac{}{\sqrt{26}} - \frac{4h}{\sqrt{20}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36h}{26}}{h}$$

$$\frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$$

$$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}} //$$

(Q2). Find gradient vector for the following function a

(i). $f(x,y) = x^4 + y^x$, $a = (1,1)$

$$f_x = y \cdot x^{y-1} + y^x \log y$$

$$f_y = x^y \log x + y^x \log y$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= (\cancel{yx^{y-1}} + y^2 \log y, x^3 \log x + xy^x$$

$$f(1,1) = (1+0, 1+0)$$

$$= (1,1) //$$

(ii). $f(x,y) = (\tan^{-1} x) \cdot y^2$ $a = (1,-1)$

$$f_x = \frac{1}{1+x^2} \cdot y^2$$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, \; 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2}, \; \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \; \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, \; \frac{-\pi}{2}\right)$ //

(Q5). Find the equation of tangent & normal to each of the following using curves at given points.

(i). $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} . x$

$(x_0, y_0) = (1,0)$ $\therefore x_0 = 1$ & $y_0 = 0$.

eqn of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \; 2(1) = e^0 \; 0$

$= 1(2) + 0$

$= 2$ //.

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 . 1$

$= 0 + 1.1$

$= 1$ //.

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0 - 1$ $\therefore$ It is the required eqn of tangent

eqn of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) = 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1,0)$

$= 1 + 2(0) \; 0 + d = 0$

$d + 1 = 0$

$\therefore \boxed{d = -1}$

(Q4). Find the eqn of tangent & normal li[...] to each of the following surface.

(i). $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2,1,0)$ $\therefore x_0 = 2, \; y_0 = 1$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$
$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$
$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

Eqn of tangent
$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0)$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$
$= 4x - 8 + 3y - 3 = 0$
$4x + 3y - 11 = 0$ → This is required eqn of the tangent

Eqn of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{dz}$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

(Q5). Find the local maxima and minima for the following

(i). $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$
$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$
$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$
$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y = -2 \rightarrow (1)$

$f_y = 0$
$2y - 3x - 4 = 0$
$2y - 3x = 4 \rightarrow (2)$

multiply eqn 1 with 2
$4x - 2y = -4$
$2y - 3x = 4$
$x = 0$

By substituting,
$2(0) - y = -2$
$-y = -2$
$\therefore y = 2$

$\therefore$ critical points are $(0,2)$

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

Here $r > 0$
$= rt - s^2$
$= 6(2) - (-3)^2$
$= 3 > 0$

$\therefore f$ has max at $(0,2)$
$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2$

$\therefore 0+4-0+0-8$
$= -4$

(ii). $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0 \rightarrow (1)$

$f_y = 0$

$3x^2 - 2y = 0 \rightarrow (2)$

mul. (1) and (2).
× ×
(3) (4)

$12x^2 + 9y = 0$
$-12x^2 - 8y = 0$

$\therefore y = 0$

Substituting.

$4x^2 + 3(0) = 0$
$4x^2 = 0$
$x = 0$

critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$
$t = f_{yy} = 0 - 2 = -2$
$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$
r at $(0,0)$
$= 24(0) + 6(0) = 0$
$\therefore r = 0$
$rt - s^2 = 0(-2) - (0)^2$
$= 0 - 0 = 0$
$r = 0$ & $rt - s^2 = 0$
(Nothing to say)

$f(x,y)$ at $(0,0)$
$2(0)^4 + 3(0)^2(0) - 0$
$= 0 + 0 - 0$
$= 0$

(iii). $f(x,y) = x^2 - y^2 + 2x + 8y - 70$
$f_x = 2x + 2$
$f_y = -2y + 8$
$f_x = 0 \quad \therefore \quad 2x + 2 = 0$
$x = \frac{-2}{2} = -1 \quad \therefore x$

$f_y = 0 \quad -2y + 8 = 0$
$y = \frac{-8}{-2}$
$\therefore y = 4$

$\therefore$ critical points are $(-1, 4)$.

$r = f_{xx} = 2$
$t = f_{yy} = -2$
$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$
$= -4 - 0$
$= -4 < 0$

$f(x, y)$ at $(-1, 4)$

$(1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$
$= 17 + 30 - 70$
$= 33$

A
27/11/2020