

## Abstract

This paper goes through the procedure of rendering braids twisting around a series of concatenated elbows following a PCC (Piecewise-circular curve).

## Problem Statement

Given a series of points, draw a PCC along those points, render some braids revolving around the PCC, then twist the braids.

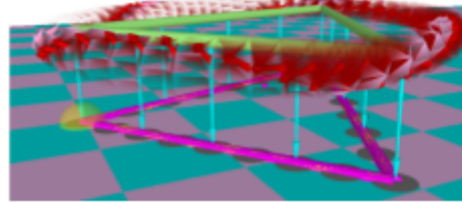
## Implementation

### Modules

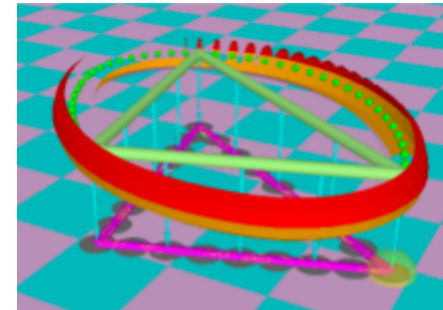
#### Elbow

Depiction of an elbow is the first and primary step. The main elements are:

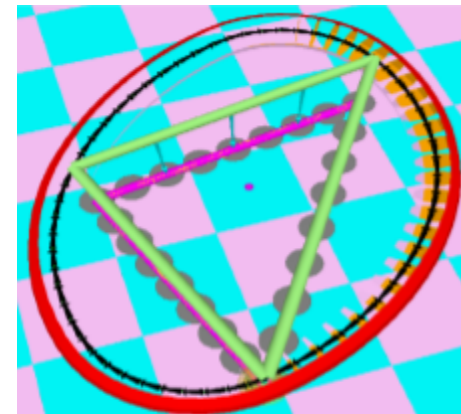
- **Center line:** The line starts and ends at designated points and is denoted by the function  $C(u)$ , which ranges between 0-1. It is generated by algorithm A1 below.
- **Vector K:** A vector normal to the plane generated by two vectors:  $OA$ ,  $OC$ .  $O$  being the origin point, and  $A$ ,  $C$  being the start and end points of the elbow.
- **Vector  $C0\_C1$ :** An "Euler" unit vector pointing from the current  $C(u)$  to  $C(u+1)$ .



The rotating  $K$  vector is depicted by the red arrows within the elbows (colored lattice not rendered)



The green points are the centerline that is surrounded by the rendered elbows



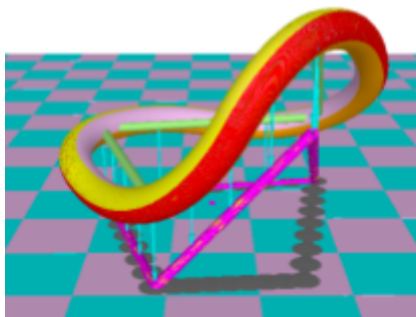
The vector  $C0\_C1$  ( $Cu\_C(u+1)$ ) is depicted by the black arrows

Using these elements, a lattice of quad meshes is constructed using the points  $S(u, v)$ , where 'u' stands for the period along the center line, and 'v' stands for an angle by which  $K$  is rotated around the axis  $C0\_C1$ .

Furthermore, the lattice is colored dependent on the 'v' values, where 0-0.25 is colored yellow, 0.25-0.5 is colored red, 0.5-0.75 is colored orange, and 0.75-1 is colored pink.

### Performer

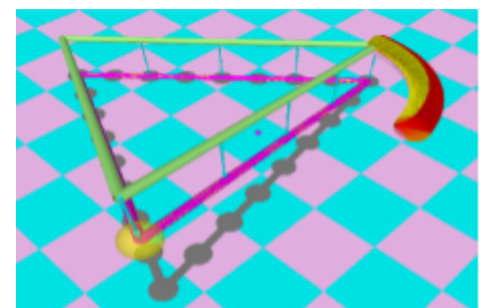
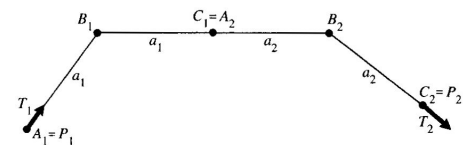
This module was a collaborative effort of the team.



The rendered elbows showcasing the red, orange, pink, and yellow lattices

### Piecewise Circular Curve (PCC)

Given an ordered set of points, and tangential information for each point (expanded on in algorithm A2), an equisided control polygon is constructed for each consecutive pair of points. The polygon is used to construct two circumcircles ( $A1-C1-B2$ ,  $C1-B2-C$ ).. These circumcircles' perimeters are concatenated in such a way that the velocity remains  $G1$  continuous. The line generated is used as the centerline for the shape.



Each edge contains 2 biarcs, depicted above is one of the two biarcs for that edge

### Performer

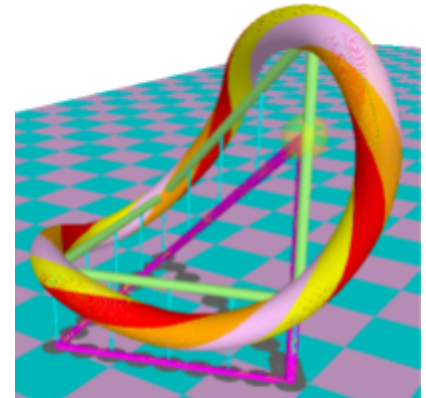
Linh Houang exclusively worked on this module.

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## Twist

The twist goes through several steps:

1. Add a value to the global twist variable (multiple of  $2\pi$ ).
2. Divide the global twist value by the number of elbows in the shape (double the number of points). This provides the local twist for each elbow.
3. Run a preliminary loop over the PCC to calculate the difference between the start rotating vector and the end rotating vector.
4. Divide the difference between the two vectors by the number of elbows, and propagate the reverse extra value to each elbow's local twist to compensate.
5. Per elbow:
  - a. Inherit the previous rotating vector's value (or just the normal for the starting elbow).
  - b. Per  $C(u)$ , rotate the rotating vector around the  $C0\_C1$  vector (calculated each  $C(u)$ ) by the total twist.



*The rendered elbows' lattices have been twisted but remain consistent*

## Performer

The Twist was a collaboration of Abdurrahmane Rikli, and Linh Houang.

## Braids

The braids piggyback off of the rotating vector for each elbow, using it to generate their path: For each  $C(u)$ , a vector is calculated from the rotating vector and the twist value assigned for the braids, then displaced by the tightness value of the braids (radius). This generates a separate vector for the braids to follow.

The different braid algorithms are expanded upon in the algorithm section.

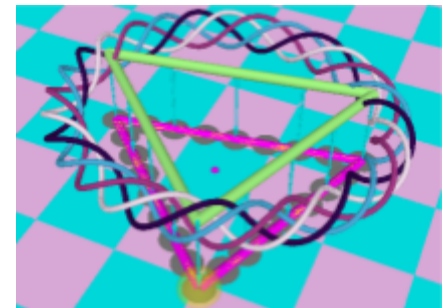
## Performer

The Braids were a collaboration of Abdurrahmane Rikli and Ruth Petit - Bois.

## Algorithms

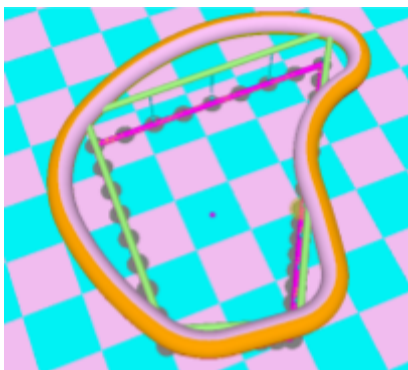
### A1:

```
float centerLineLeft = radius*cos(t*alpha);
float centerLineRight = radius*sin(t*alpha);
vec centerLineCalcLeft = V(centerLineLeft, I);
vec centerLineCalcRight = V(IxK).mul(centerLineRight);
vec centerLineVec = centerLineCalcLeft.add(centerLineCalcRight);
pt centerLinePoint = P(0,centerLineVec);
return centerLinePoint;
```



*Example of a braid pattern produced by the program*

To get the centerline point, take in the parameter  $t$  (being an arbitrary distance between the center points). Multiply the radius of the elbows by the cosine of the parameter  $t$  by the angle (now referenced as  $\mathbf{a}$ ) and multiply the radius of the elbows by the sine of the parameter  $t$  by the angle (now referenced as  $\mathbf{b}$ ). Multiply the result of  $\mathbf{a}$  by the vector  $I$  ( $I$  being the vector containing the point of the origin to the start point). Multiply the result  $\mathbf{b}$  by the cross product of  $I$  and  $K$  (being the normal). Add these vectors together and the center point is the addition of the origin and the new calculated vector.



*Image depicting the First Method of*

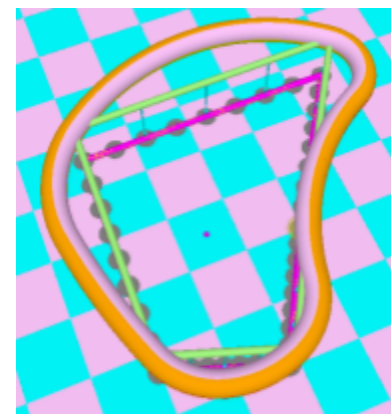
### A2:

#### *Tangent estimation:*

In the project Linh implemented three tangents estimations algorithms:

**First Method:** By using the previous edge

**Second Method:** Using the method in the PCC paper



*Image depicting the Second*

**Third Method:** By interpolating the tangent information obtained by Algorithm One using LPM morph. The pseudo code for Three is as followed:

```
Tangents = Using algorithm One to get tangent information for all vertices.
For all vertices, tangent at vertex i:
    InterpolatedTangent[i] = LPM(Tangents[i - 1], Tangents[i + 1])
```

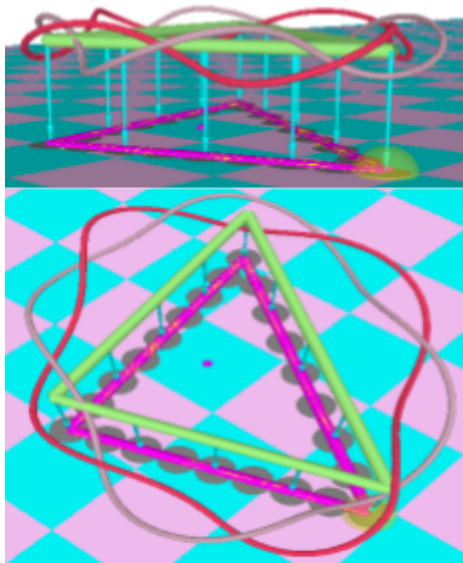
### Braids

```
pt[] points = new pt[numOfThreads];
float delta = 2 * PI / numOfThreads;
vec vector_displace = V(reference);
pt point_displaced = new pt(); point_displaced.setTo(origin);
vec angle;
```

Initiate an array of points to return. Get an angle magnitude to iterate over for each sector. Initiate a reference vector that aligns with the rotating vector for the elbow. Initiate a point that aligns with C(u).



Image depicting the Third Method of Tangent Estimation (curves upon itself, but still connected)



Examples of Double Helix Pattern (Side & Top)

### DOUBLE\_HELIX

```
For i to numOfBraids{
    vector_displace = lrot(reference, axis, i * delta);
    point_displaced.add(vector_displace);
    points[i] = P(point_displaced);
    vector_displace = V(reference);
    point_displaced = point_displaced.setTo(origin);
}
```

Subdivide the circle into n sectors, then animate a path for each sector. The "weaving" only becomes apparent after twisting.

### SINE

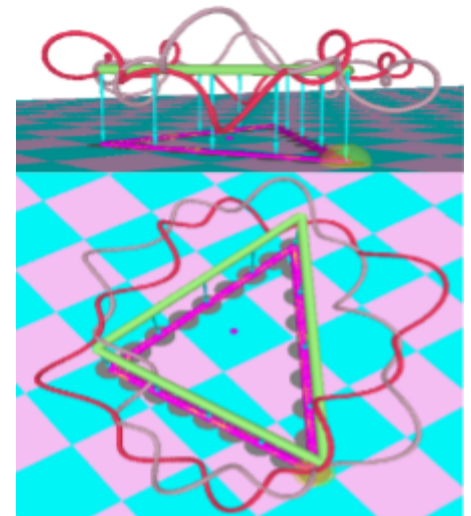
```
For i to numOfBraids{
    vector_displace = lrot(vector_displace, axis, i * delta); // Vector to start point
    angle = vector_displace;
```

```
    vector_displace = lrot(vector_displace, axis, delta); // Vector to end point (and next sector's start point)
    point_displaced.add(angle); // Move point to start point
    vector_displace.sub(angle); // Vector for point to oscillate on
    point_displaced.add(vector_displace.mul(sin(period * 2 * PI)));
    // Displace point by oscillated value
    points[i] = P(point_displaced);
    vector_displace = V(reference); // Reset vector
    point_displaced = point_displaced.setTo(origin); // Reset point
}
```

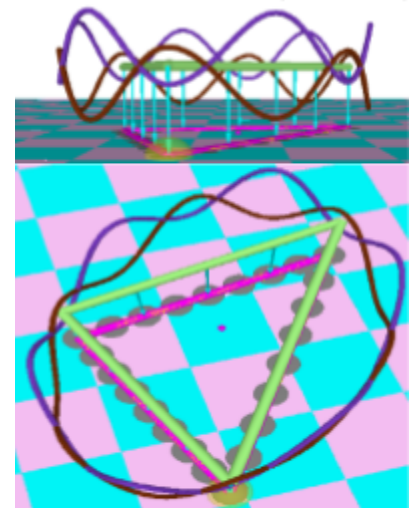
Subdivide the circle into n sectors, then have each braid oscillate at the edge of the section in a Sine arc.

### COSINE

```
For i to numOfBraids{
    angle = vector_displace.rotate(i * delta, reference, referenceOrthogonal); // Vector to start point
```



Examples of Sine Pattern (Side & Top)



Examples of Cosine Single Pattern



```

        vector_displace.rotate(delta, reference, referenceOrthogonal); // Vector to end point (and
next sector's start point)

        point_displaced.add(angle); // Move point to start point
        vector_displace.sub(angle); // Vector for point to oscillate on
        point_displaced.add(vector_displace.mul(cos(period * 2 * PI))); // Displace point by
oscillated value

        points[i] = point_displaced;
        vector_displace = V(reference); // Reset vector
        point_displaced = point_displaced.setTo(origin); // Reset point
    }

```

Subdivide the circle into  $n$  sectors, then have each braid oscillate at the edge of the section in a Cosine arc.

### SINECOSINESINGLE

```

points = new pt[2];
points[0] = P(point_displaced.add(vector_displace.mul(sin(period * 2 *
PI))));
point_displaced = point_displaced.setTo(origin);
points[1] = P(point_displaced.add(referenceOrthogonal.mul(cos(period * 2 *
PI))));

```

Draw a Sine arc vertically and a cosine arc horizontally.

### SINECOSINESERIES

```

delta = PI / numofThreads; // Redistribute the delta for half the circle instead
for(int i = 0; i < numofThreads*2; i+=2){
    // Sine sector
        vector_displace = lrot(vector_displace, axis, i * delta); //
Vector to start point
        angle = vector_displace;
        vector_displace = lrot(vector_displace, axis, numofThreads * delta); // Vector to end point
        point_displaced.add(angle); // Move point to start point
        vector_displace.sub(angle); // Vector for point to oscillate on
        point_displaced.add(vector_displace.mul(sin(period * 2 * PI))); //
Displace point by oscillated value
        points[i] = P(point_displaced);
        vector_displace = V(reference); // Reset vector
        point_displaced = point_displaced.setTo(origin); // Reset point
        // Cosine sector
        vector_displace = lrot(vector_displace, axis, (i+1) * delta); //
Vector to start point
        angle = vector_displace;
        vector_displace = lrot(vector_displace, axis, numofThreads * delta);
// Vector to end point
        point_displaced.add(angle); // Move point to start point
        vector_displace.sub(angle); // Vector for point to oscillate on
        point_displaced.add(vector_displace.mul(cos(period * 2 * PI))); //
Displace point by oscillated value
        points[i+1] = P(point_displaced);
        vector_displace = V(reference); // Reset vector
        point_displaced = point_displaced.setTo(origin); // Reset point
    }
}

```

Oscillates the path from one sector to the next-opposite sector, alternating between a sine wave and a cosine wave.

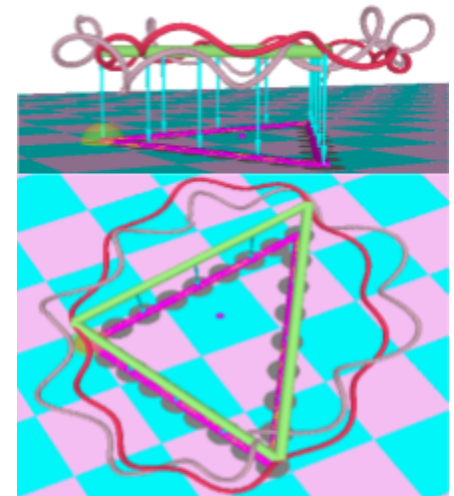
### SINEALT

```

points = new pt[8];
for(int i = 0; i < 8; i+=2){
    vector_displace = lrot(reference, axis, (i * PI / 2) + sin(period * 2 * PI) * PI);
}

```

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*Examples of SineCosine Single Pattern (Side & Top)*



*Examples of SineCosine Series Pattern (Side & Top)*

```

points[i] = P(point_displaced.add(vector_displace));
point_displaced = point_displaced.setTo(origin); // Reset point
vector_displace = V(reference); // Reset vector
vector_displace = lrot(reference, axis, PI / 6);
point_displaced.add(vector_displace);
angle = lrot(reference, axis, (i * PI / 2) * 4 * PI / 6);
angle.sub(vector_displace);
points[i+1] = P(point_displaced.add(angle.mul(cos(period * 2 * PI))));
}

```

## Extra Functionality

Keyboard Key	Action	Keyboard Key	Action
<    >	Twist Elbow Clockwise    Counter Clockwise	y	Turn the Radius of the Elbows to 5 instead of 20
[    ]	Twist Original Black Path P Clockwise    Counterclockwise	1	Add / Remove the rendering buffer for increased performance
/	Reset Elbow Twists to 0	-	Tighten the twists/braids
h	Switch Tangent Methods	2	Increase the number of paths
g	Render Original Black path P	s	Decrease the number of paths
j	Render the Biarc (Or anything to do with the Elbows)	;	Show colored quads/Lattice Structure
\	Show Rotating K ( Red Arrows)	o	Show CenterLine (green points)
v	Switch between path patterns		

## Credits

Abdurrahmane Rikli wrote the first draft. Linh Hoang & Ruth Petit - Bois edited. The code was written by all, with notation indicating who wrote what. Screenshots and video taken by Ruth Petit - Bois.

