AI1103 Assignement 3

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Q53. Given the observations $0.8, 0.71, 0.9, 1.2, 1.68, 1.4, 0.88, 1.62$ from the uniform distribution on $(\theta - 0.2, \theta + 0.8)$ with $-\infty < \theta < \infty$, which of the following is a maximum likelihood estimate for θ ?
1) 0.7
2) 0.9
3) 1.1
4) 1.3

Answer.

$$X \sim U(\theta - 0.2, \theta + 0.8)$$

Let the pdf be denoted by f, then

$$P(x|\theta) = f(x|\theta) = \begin{cases} \frac{1}{(\theta+0.8)-(\theta-0.2)} = 1 & (\theta - 0.2 \leq x \leq \theta + 0.8) \\ 0 & \textit{(Otherwise)} \end{cases}$$

and let the observations be denoted by $\underline{X} = (X_1, X_2, \cdots, X_n)$.

Now, the likelihood function for the observations $(X_1 = x_1, X_2 = x_2, \cdots, X_n = x_n)$ is given by

$$\begin{aligned} L(\underline{x}|\theta) &= P(X_1 = x_1, X_2 = x_2, \cdots, X_n = x_n|\theta) \\ &= P(X = x_1|\theta) \times P(X = x_2|\theta) \times \cdots \times P(X = x_n|\theta) \quad \textit{(i.i.d assumption)} \end{aligned}$$

So,

$$L(\underline{x}|\theta) = \begin{cases} 1 & \textit{when } all(\underline{x}) \geq \theta - 0.2 \textit{ and } all(\underline{x}) \leq \theta + 0.8 \\ 0 & \textit{(Otherwise)} \end{cases}$$

maximising the likelihood implies satisfying the following condition

$$all(\underline{x}) \geq \theta - 0.2 \textit{ and } all(\underline{x}) \leq \theta + 0.8$$

which is same as satisfying

$$min(\underline{x}) \geq \theta - 0.2 \ and \ max(\underline{x}) \leq \theta + 0.8$$

For the given question $min(\underline{x}) = 0.71$ and $max(\underline{x}) = 1.68$
So,

$$0.71 \geq \theta - 0.2 \ and \ 1.68 \leq \theta + 0.8$$

$$\implies 0.91 \geq \theta \ and \ 0.88 \leq \theta$$

Only option that satisfies the condition is the second option 0.9 (Answer)