

Challenging problem 8

Annu-EE21RESCH01010

Download latex code from here-

[https://github.com/annu100/AI5002-Probability-and-Random-variables/tree/main.tex/challenging problems](https://github.com/annu100/AI5002-Probability-and-Random-variables/tree/main.tex/challenging%20problems)

I. CHALLENGING PROBLEM 8

Let X_1, X_2, \dots, X_n be independent Poisson random variables with $E[X_i] = \mu_i$. Find the conditional distribution of $X_1, \dots, X_n \mid \sum_{i=1}^n X_i = y$

II. SOLUTIONS

The random variable X_i is distributed as Poisson if the density of X_i is given by

$$E[X_i] = \mu_i$$

Also, $X_i = \mu$

$$f(x_i : \mu_i) = \begin{cases} \frac{e^{-\mu_i} \times \mu_i^{x_i}}{x_i!} & x_i \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{Since}$$

X_1, X_2, \dots, X_n be independent Poisson random variables-

Likelihood function L is required to be calculated and it is given by

$$L = \frac{e^{-\mu_1} \times \mu_1^{x_1}}{x_1!} \times \frac{e^{-\mu_2} \times \mu_2^{x_2}}{x_2!} \dots \frac{e^{-\mu_n} \times \mu_n^{x_n}}{x_n!} \quad (1)$$

$$= \frac{e^{-\mu_n} \times \mu^{\sum_{i=1}^n x_i}}{\prod_{i=1}^n x_i!} \quad (2)$$

$$(3)$$

So, above case is only defined when

$\sum_{i=1}^n X_i = y$ for some y

otherwise $L=0$.

Here L is the conditional distribution of

$$X_1, \dots, X_n \mid \sum_{i=1}^n X_i = y$$

Also sum of two independent Poisson R.V is also R.V with mean as sum of mean of individual random variables which provides for the proof of above expression for L.