

Compulsory Assignment-Probability and Random Variable

Annu-EE21RESCH01010

Download latex code from here-

<https://github.com/annu100/AI5002-Probability-and-Random-variables/tree/main.tex/>
Compulsory Assignment

Download python code from here-

<https://github.com/annu100/AI5002-Probability-and-Random-variables/tree/main.py/>
Compulsory Assignment

I. PROBLEM STATEMENT-PROBLEM 3.7

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X ?

II. SOLUTION

Let X_1 denotes the number of heads and X_2 denotes the number of tails that occur when a coin is tossed 6 times.

let n is total number of tosses and p is probability of getting head

$p=q=0.5$

Clearly, $X_1 \sim \text{Bin}(n = 6, p)$

and $X_2 \sim \text{Bin}(n = 6, 1 - p = q)$.

$\therefore n - X_2 \sim \text{Bin}(6, p)$.

By reproductive property,

$$X_1 + n - X_2 \sim \text{Bin}(6 + 6, p) \quad (1)$$

$$X = X_1 - X_2.$$

$$\therefore P(X = x) = \binom{12}{6+x} \frac{1}{2}^{12}, x = -6 \text{ to } 6$$

therefore, X can have any values between -6 to 6.

Convolution of Bernoulli distributions

The convolution of two independent identically distributed Bernoulli random variables is a binomial random variable.

$$\sum_{i=1}^2 \text{Bernoulli}(p) \sim \text{Binomial}(2, p)$$

To show this let

$$X_i \sim \text{Bernoulli}(p), \quad 0 < p < 1, \quad 1 \leq i \leq 2$$

and define

$$Y = \sum_{i=1}^2 X_i$$

Also, let Z denote a generic binomial random variable:

$$Z \sim \text{Binomial}(2, p)$$

Using probability mass functions

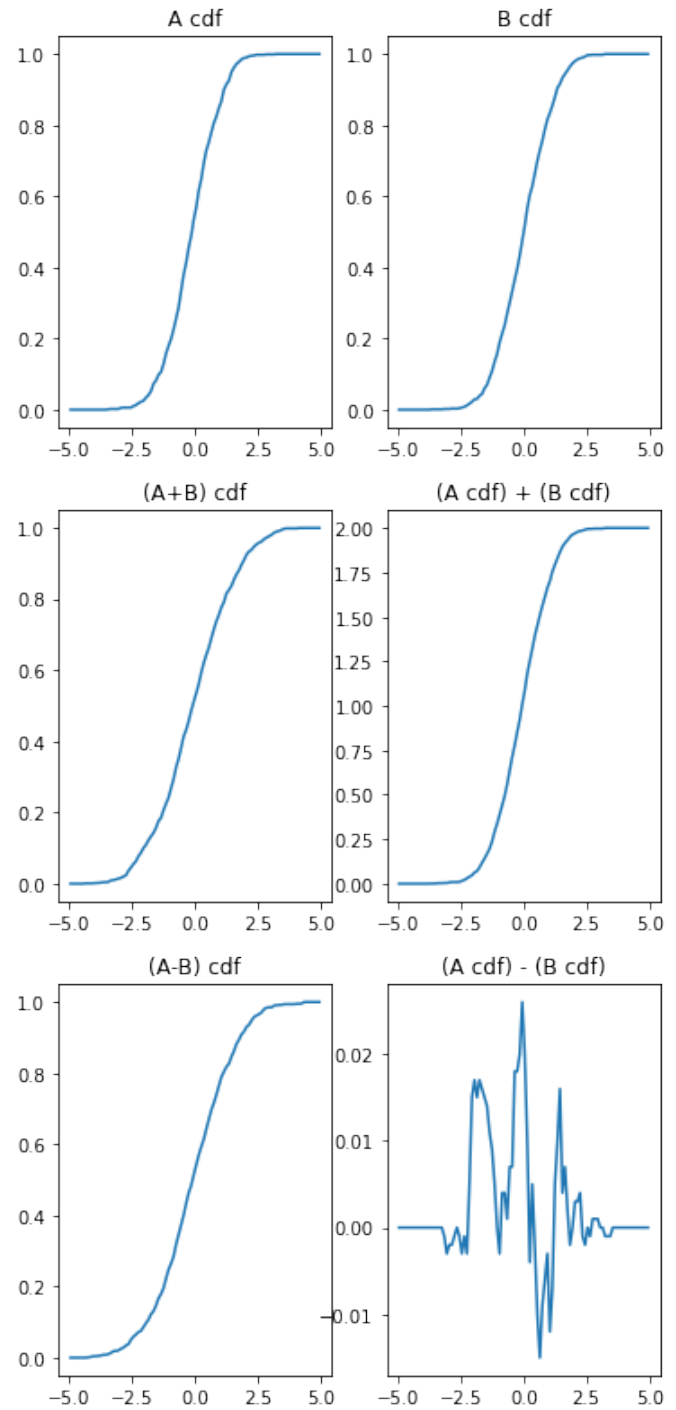
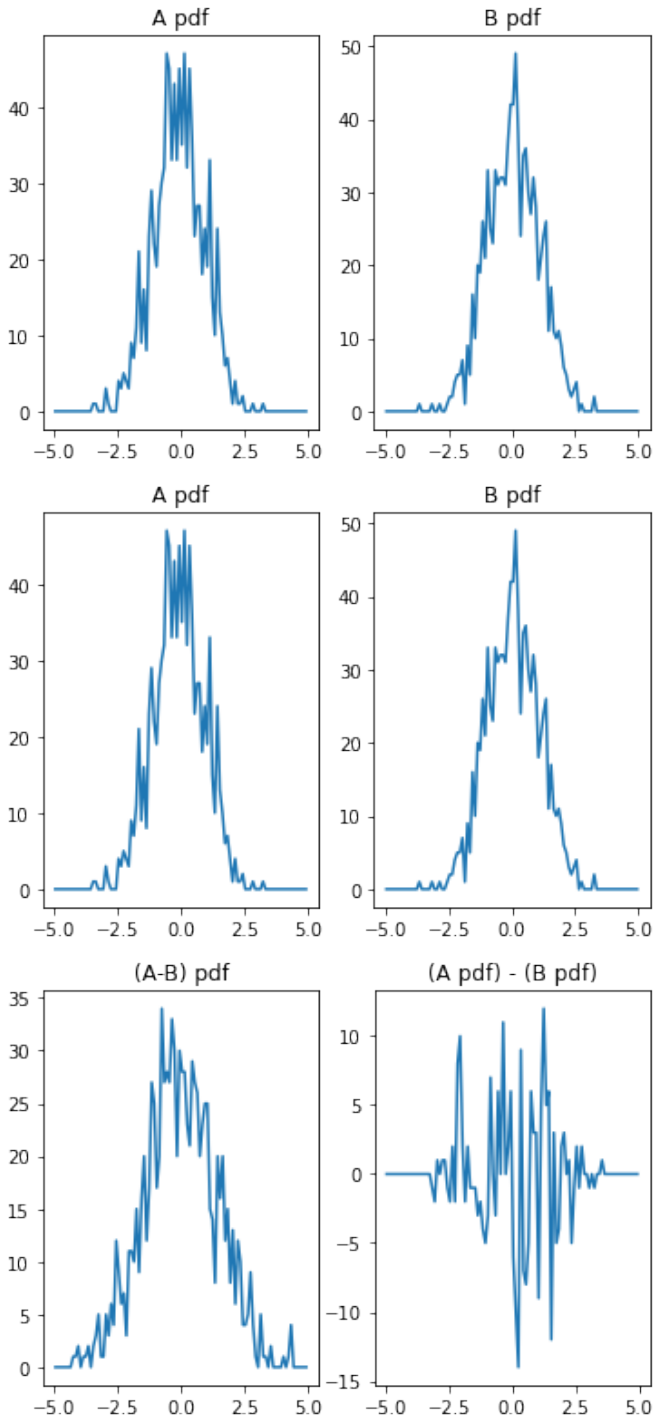
As X_1 and X_2 are independent,

$$\begin{aligned} \mathbb{P}[Y = n] &= \mathbb{P}\left[\sum_{i=1}^2 X_i = n\right] \\ &= \sum_{m \in \mathbb{Z}} \mathbb{P}[X_1 = m] \times \mathbb{P}[X_2 = n - m] \\ &= \sum_{m \in \mathbb{Z}} \left[\binom{1}{m} p^m (1-p)^{1-m} \right] \left[\binom{1}{n-m} p^{n-m} (1-p)^{1-n+m} \right] \\ &= p^n (1-p)^{2-n} \sum_{m \in \mathbb{Z}} \binom{1}{m} \binom{1}{n-m} \\ &= p^n (1-p)^{2-n} \left[\binom{1}{0} \binom{1}{n} + \binom{1}{1} \binom{1}{n-1} \right] \\ &= \binom{2}{n} p^n (1-p)^{2-n} = \mathbb{P}[Z = n] \end{aligned}$$

Thus, convolution of 2 binomial distributions is also binomial

III. SIMULATIONS

Question-Plot the sum and difference of 2 bernauli random variables



A. Simulations-solutions

$$z = (x - \mu) / \sigma \sim N(0, 1)$$

where

$$\mu = n * p$$

$$\text{and } \sigma = \sqrt{n * p(1 - p)}$$

So, basically normal distributions are approximation of binomial distributions.

PDF of 2 individual binomial random variables are plotted.

PDF of sum of 2 individual random variables and sum of their individual pdfs are plotted and compared.

PDF of difference of 2 individual random variables and difference of their individual pdfs are plotted and compared.

CDF of 2 individual binomial random variables are plotted.

CDF of sum of 2 individual random variables and sum of their individual Cdfs are plotted and compared.

CDF of difference of 2 individual random variables and difference of their individual cdfs are plotted and compared.

If x is a random variable with distribution $\text{Bin}(n, p)$, then for sufficiently large n , the distribution of the variable.