

Assignment 13-Probability and Random Variable

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Download latex code from here-

https://github.com/annu100/AI5002-Probability-and-Random-variables/tree/main.tex/ASSIGNMENT_13

I. GATE-24 SOLUTION

A binary symmetric channel (BSC) has a transition probability of $\frac{1}{8}$. If the binary transmit symbol X is such that $Pr(X = 0) = \frac{9}{10}$, then the probability of error for an optimum receiver will be-

II. SOLUTIONS

Let x_0 and x_1 are two binary transmitted symbols. y_0 and y_1 are recieved symbols.
transition probability= $Pr(y_1|x_0)=Pr(y_0|x_1)$ Given data-

$$\begin{aligned} Pr(x_0) &= \frac{9}{10} \\ Pr(x_1) &= 1 - \frac{9}{10} = \frac{1}{10} \\ Pr(y_1|x_0) &= \frac{1}{8} \\ Pr(y_0|x_1) &= \frac{1}{8} \\ Pr(y_0|x_0) &= 1 - \frac{1}{8} = \frac{7}{8} \\ Pr(y_1|x_1) &= 1 - \frac{1}{8} = \frac{7}{8} \end{aligned}$$

Calculating probability values for MAP criteria -

$$Pr(y_0|x_0) * Pr(x_0) = \frac{7}{8} * \frac{9}{10} = \frac{63}{80} \quad (1)$$

$$Pr(y_0|x_1) * Pr(x_1) = \frac{1}{8} * \frac{1}{10} = \frac{1}{80} \quad (2)$$

$$Pr(y_1|x_0) * Pr(x_0) = \frac{1}{8} * \frac{9}{10} = \frac{9}{80} \quad (3)$$

$$Pr(y_1|x_1) * Pr(x_1) = \frac{7}{8} * \frac{1}{10} = \frac{7}{80} \quad (4)$$

Now according to M.A.P criteria at reciever -

$$Pr(y_0|x_0) * Pr(x_0) > Pr(y_0|x_1) * Pr(x_1)$$

So,when a symbol is recieved is recieved as y_0 ,the decision can be made in favour of x_0 in an optimum way.

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$$Pr(y_1|x_0) * Pr(x_0) > Pr(y_1|x_1) * Pr(x_1)$$

So,when a symbol is recieved is recieved as y_1 ,the decision can be made in favour of x_0 in an optimum way.

So,for the given BSC channel ,with optimum reciver ,both the recieved symbols will be decoded as x_0 .

Hence,the probability of error is equal to probability of transmitting x_1 .

$$\text{so, } Pr(\text{error}) = Pr(x_1) = \frac{1}{10}$$