Compulsory Assignment-Probability and Random Variable

Annu-EE21RESCH01010

Download latex code from here-

https://github.com/annu100/AI5002−Probability−and−Random−variables/tree/main.tex/ASSIGNMENT_8

Download python code from here-

https://github.com/annu100/AI5002−Probability−and−Random−variables/tree/main.py/ASSIGNMENT_8

I. Problem Statement-Problem 3.7

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

II. Solution

Let X_1 denotes the number of heads and X_2 denotes the number of tails that occur when a coin is tossed 6 times.

let n is total number of tosses and p is probability of getting head
p=q=0.5
Clearly, $X_1 \sim Bin(n = 6, p)$
and $X_2 \sim Bin(n = 6, 1 - p = q)$.
$\therefore n - X_2 \sim Bin(6, p)$.
By reproductive property,

$$X_1 + n - X_2 \sim Bin(6 + 6, p) \tag{1}$$

$X = X_1 - X_2$.

$\therefore P(X = x) = \binom{12}{6+x}\frac{1}{2}^{12}, x = -6$ to 6

therefore,X can have any values between -6 to 6.

Convolution of Bernoulli distributions
The convolution of two independent identically distributed Bernoulli random variables is a binomial random variable.

$$\sum_{i=1}^{2} \text{Bernoulli}(p) \sim \text{Binomial}(2, p)$$

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To show this let

$$X_i \sim \text{Bernoulli}(p), \quad 0 < p < 1, \quad 1 \le i \le 2$$

$X_i \sim \text{Bernoulli}(p), \quad 0 < p < 1, \quad 1 \le i \le$ 2and define

$$Y = \sum_{i=1}^{2} X_i Y = \sum_{i=1}^{2} X_i$$

Also, let Z denote a generic binomial random variable:

$$Z \sim \text{Binomial}(2, p), Z \sim \text{Binomial}(2, p)$$

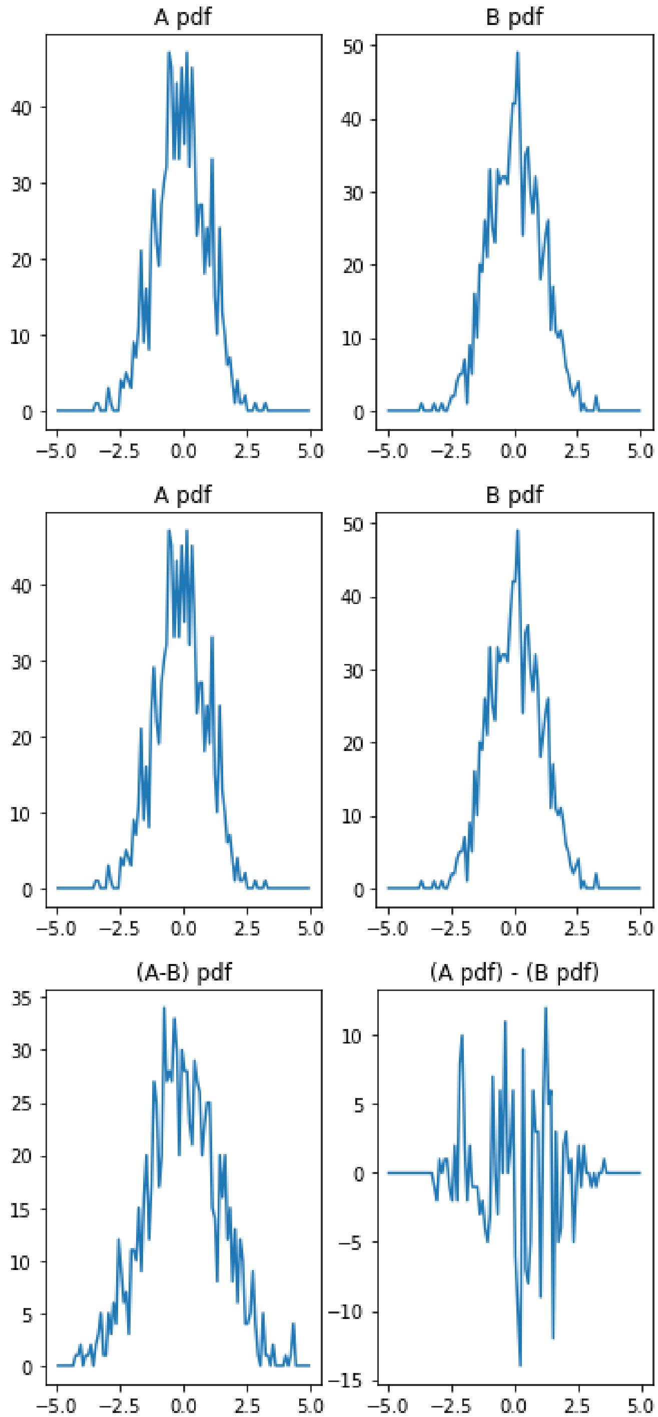
Using probability mass functions
AsX_1 and $X_2$are independent,

$$\begin{aligned}
\mathbb{P}[Y = n] &= \mathbb{P}\left[\sum_{i=1}^{2} X_i = n\right] \\
&= \sum_{m\in\mathbb{Z}} \mathbb{P}[X_1 = m] \times \mathbb{P}[X_2 = n - m] \\
&= \sum_{m\in\mathbb{Z}} \left[\binom{1}{m} p^m (1-p)^{1-m}\right]\left[\binom{1}{n-m} p^{n-m} (1-p)^{1-n+m}\right] \\
&= p^n (1-p)^{2-n} \sum_{m\in\mathbb{Z}} \binom{1}{m}\binom{1}{n-m} \\
&= p^n (1-p)^{2-n} \left[\binom{1}{0}\binom{1}{n} + \binom{1}{1}\binom{1}{n-1}\right] \\
&= \binom{2}{n} p^n (1-p)^{2-n} = \mathbb{P}[Z = n]
\end{aligned}$$

Thus,convolution of 2 binomial distributions is also binomial

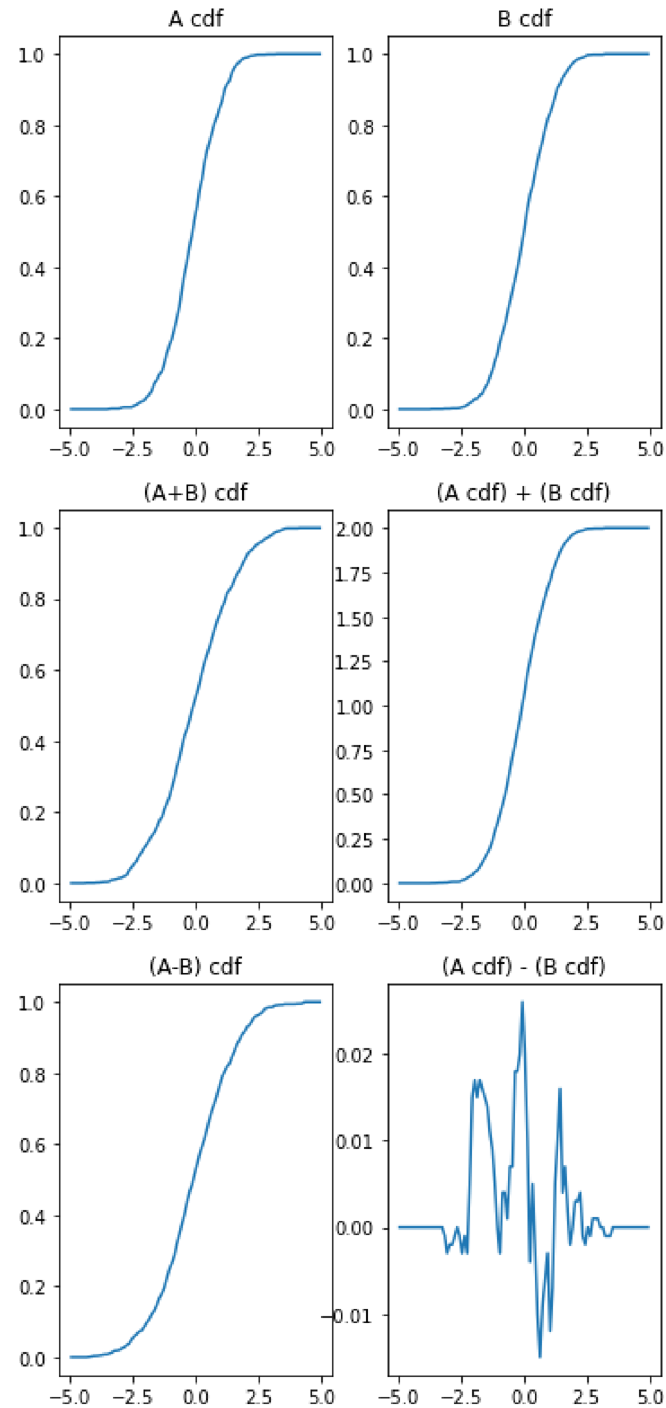
III. Simulations

Question-Plot the sum and difference of 2 bernaulli random variables



A. Simulations-solutions

z=(x-μ)/σ $\sim N(0,1)$
where



μ=n*p
and σ= $\sqrt{n*p(1-p)}$

So,basically normal distributions are approximation of binomial distributions.

PDF of 2 individual binomial random variables are plotted.

PDF of sum of 2 individual random variables and sum of their individual pdfs are plotted and compared.
PDF of difference of 2 individual random variables and difference of their individual pdfs are plotted and compared.
CDF of 2 individual binomial random variables are plotted.

CDF of sum of 2 individual random variables and sum of their individual Cdfs are plotted and compared.
CDF of difference of 2 individual random variables and difference of their individual cdfs are plotted and compared.
If x is a random variable with distribution Bin(n, p), then for sufficiently large n, the distribution of the variable.