Modelling and performance analysis of *wireless channel* using *Probability*

PROJECT PRESENTATION-AI5002
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Overview

ABSTRACT

1. Modelling of wireless channels is done in general fading conditions and their performance is analysed using 2 parameters-
   1. Outage Probability
   2. Bit Error Rate
2. Fading in wireless channel is very important like Rayleigh fading for channel estimation and it is simulated in this presentation.

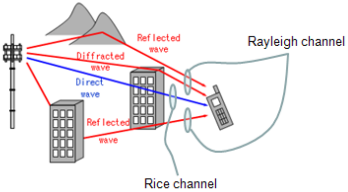


Figure 1: Wireless channel

Q-Function

- *Q function* is a convenient way to express right-tail probabilities for Gaussian random variables
- Mathematically, this can be expressed as:

$$\begin{aligned} Q(x) &= 1 - F_X(x) = 1 - P(X \leq x) && (1) \\ &= P(X > x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt && (2) \end{aligned}$$

Where Does Randomness in Wireless communication Occur?

- Several locations where randomness occurs in the communications system includes:
    - Transmission channel
    - Data generation at information source
    - Clock jitter
    - Processing latency
    - No guiding medium between transmitter and receiver.
    - Multiple signals superpose at the receiver. As a result of destructive interference, strength of signal fades(weakens). This effect is known as fading.
    - ...

Noise Channel Model

$$\text{Binary Information Source} \rightarrow m[n]$$
$$\downarrow$$
$$\text{Transmitter} \rightarrow s(t)$$
$$\downarrow$$
$$\text{Channel} \rightarrow s(t) + n(t)$$
$$\downarrow$$
$$\text{Receiver} \leftarrow r(t)$$
$$\downarrow$$
$$\text{Binary Information Sink} \leftarrow \hat{m}[n]$$

Gaussian Random Variable

- We frequently use **Gaussian random variables** to model noise contribution to transmitted signal
- Gaussian random variable mathematically expressed as:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-((x-\mu)/\sigma)^2/2} \tag{3}$$

where μ is the mean and σ is the standard deviation

AWGN

- **Additive**
  It is because it is added to any noise that might be intrinsic to the information system.
- **White**
  It refers to the idea that the noise has the same power distribution at every frequency.
- **Gaussian**
  It is because it has a normal distribution with mean of 0 as time domain average value sums to 0.

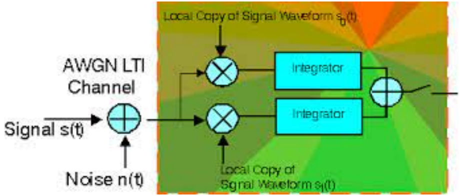
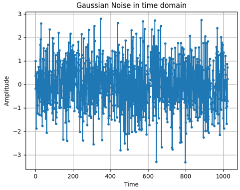
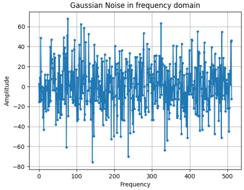


Figure 2: AWGN CHANNEL

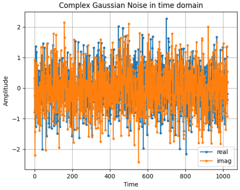
Gaussian Noise Simulation



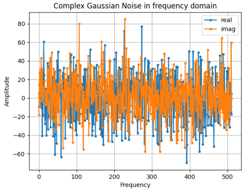
(a) Gaussian Noise in time domain



(b) Gaussian Noise in frequency domain



(c) Complex Gaussian Noise in time domain



(d) Complex Gaussian Noise in frequency domain

Figure 3: Gaussian Noise Simulation

Bivariate Gaussian

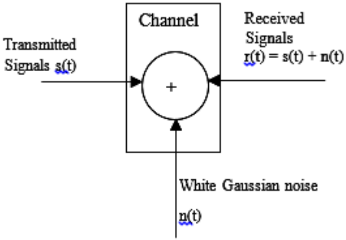
General definition for bivariate Gaussian density with parameters μ_X, μ_Y, σ_X^2, σ_Y^2, and correlation coefficient ρ is given by:

$$f_{XY}(x,y) = \frac{\exp\left(\frac{-1}{2(1-\rho^2)}\left(\left(\frac{x-\mu_X}{\sigma_X}\right)^2 - 2\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right) + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2\right)\right)}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}, \tag{4}$$

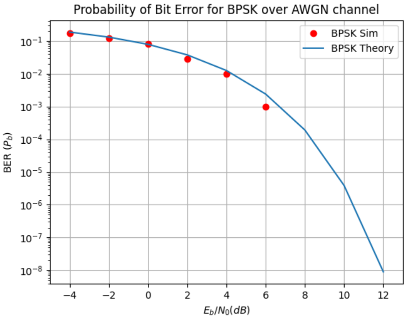
where *correlation coefficient* is defined as:

$$\rho = E\left[\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right)\right]. \tag{5}$$

AWGN Channel Simulation



(a) AWGN CHANNEL



(b) BER versus SNR(Db) for BPSK with **AWGN**

Multipath Fading in wireless communication

- **Multipath fading:**A propagation phenomenon that results in signals reaching the receiver by two or more paths, which we experience in real-world wireless systems

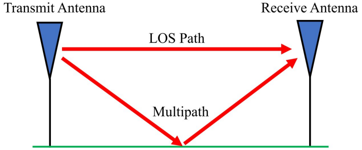


Figure 5: Multipath fading

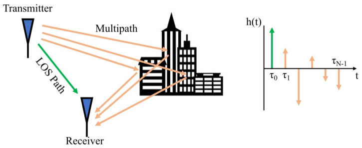


Figure 6: Multipath fading

Types of Fading

There are two types of fading from a time domain perspective

- **Slow Fading**
- **Fast Fading**

There are two types of fading from a frequency domain perspective

- **Frequency Selective Fading**
- **Flat Fading**

Illustration of fading

1. In the figure 7 given below, the red shape shows our signal in the frequency domain
2. black curvy line shows the current channel condition over frequency.
3. Because the narrower signal is experiencing the same channel conditions throughout the whole signal, it's experiencing **flat fading**.
4. The wider signal is very much experiencing frequency **selective fading**.

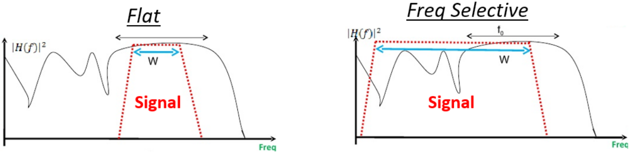


Figure 7: Multipath fading

1. Here is an example of a 16 MHz wide signal that is continuously transmitting.
2. There are several moments in the middle where there's a period of time a piece of signal is missing.
3. This example depicts frequency selective fading, which causes holes in the signal that wipe out some frequencies but not others.

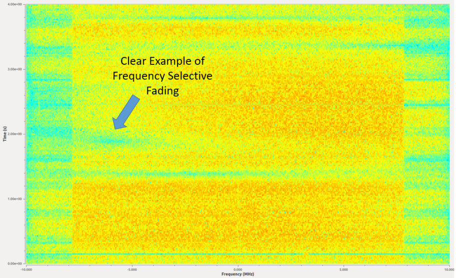


Figure 8: Multipath fading

PDF of Rayleigh

The probability density function of the Rayleigh distribution is

$$f(x;\sigma) = \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)}, \quad x \geq 0, \tag{6}$$

where σ is the scale parameter of the distribution.

Rayleigh Channel simulation

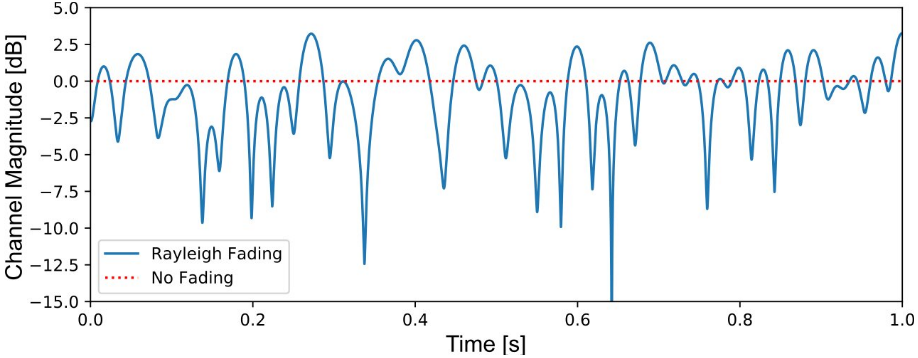


Figure 9: Rayleigh Fading

Rayleigh fading Channel Modelling

The fading channel coefficient h depends on factors

1. attenuation (a_i)
2. time delay(τ_i)

Modelling the distribution of the fading channel coefficient

$$
\begin{aligned}
h &= \sum_{i=0}^{L-1} a_i \exp\left(-j2\pi f_c \tau_i\right) && (7)\\
&= \sum_{i=0}^{L-1} a_i \cos\left(2\pi f_c \tau_i\right) - j \sum_{i=0}^{L-1} a_i \sin\left(2\pi f_c \tau_i\right) && (8)\\
&= X + jY && (9)
\end{aligned}
$$

where X=$\sum_{i=0}^{L-1} a_i \cos\left(2\pi f_c \tau_i\right)$ and Y=$-\sum_{i=0}^{L-1} a_i \sin\left(2\pi f_c \tau_i\right)$

Joint Pdf of X and Y

$$X, Y \sim \mathcal{N}(\mu, \sigma^2) \tag{10}$$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp \frac{-(x-\mu)^2}{2\sigma^2} \tag{11}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} exp \frac{-(y-\mu)^2}{2\sigma^2} \tag{12}$$

Assuming X and Y are independent random variables and substituting $\mu = 0$ for simplification

$$f_{XY}(x, y) = \frac{1}{2\pi\sigma^2} exp\Big(-(x^2 + y^2)\Big) \tag{13}$$

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Distribution of the fading channel in terms of its Amplitude and phase using Jacobian

$$h = x + jy = ae^{j\phi} \tag{14}$$

$$a = \sqrt{x^2 + y^2}, \phi = \tan^{-1}\frac{y}{x} \tag{15}$$

$$f_{A,\Phi}(a,\phi) = f_{XY}(x,y)\left|J_{XY}\right| \tag{16}$$

$$\left|J_{XY}\right| = \begin{vmatrix} \frac{\partial x}{\partial a} & \frac{\partial y}{\partial a} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} \end{vmatrix} = \begin{vmatrix} \cos\phi & \sin\phi \\ -a\sin\phi & a\cos\phi \end{vmatrix} \tag{17}$$

from (13),(15) and (17) we get

$$f_{A,\Phi}(a,\phi) = \frac{a}{2\pi\sigma^2}\exp\left(\frac{-a^2}{2\sigma^2}\right) \tag{18}$$

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Marginal distribution of A

$$f_A(a) = \int_{-\pi}^{\pi} f_{A,\Phi}(a,\phi)\, d\phi \tag{19}$$

$$= \int_{-\pi}^{\pi} \frac{a}{2\pi\sigma^2} \exp\left(\frac{-a^2}{2\sigma^2}\right) d\phi \tag{20}$$

$$= \frac{a}{\sigma^2} \exp\left(\frac{-a^2}{2\sigma^2}\right) \tag{21}$$

1. The coefficient follows the Rayleigh distribution and is fading in nature.
2. It is therefore called as a Rayleigh fading channel.

Performace Analysis of wireless channels

Outage Probability

Outage probability can be easily computed if we know the probability distribution characteristics of the fading.

$P_{out}(R) = Pr((log(1 + |h|^2 \times SNR) < R)$

Error rate

In digital transmission, the number of bit errors is the number of received bits of a data stream over a communication channel that have been altered due to noise, interference, distortion or bit synchronization errors. The bit error rate (BER) is the number of bit errors per unit time.

Simulation Results-Outage Probability

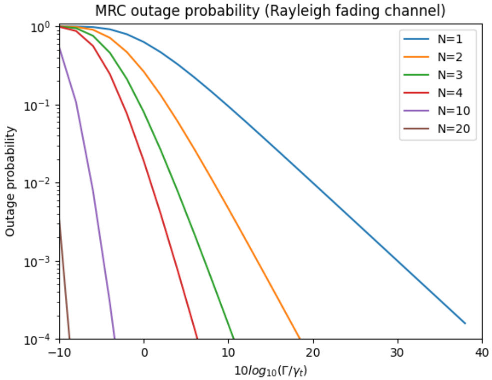

Figure 10: Outage probability of MRC processing in Rayleigh fading channel

Simulation Results-Error Rate

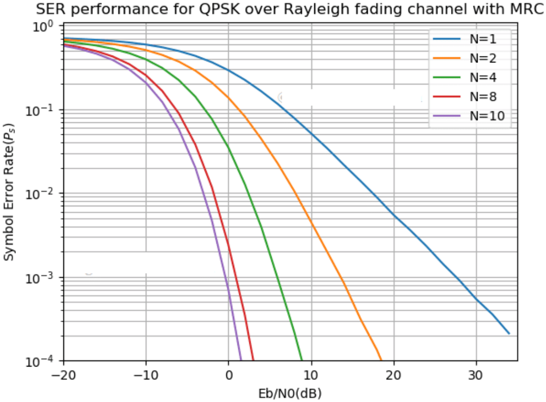

Figure 11: Symbol error rate Vs EbN0 for QPSK over i.i.d Rayleigh flat fading channel with MRC processing at the receiver

Einstein-Wiener-Khinchin Theorem

- In particular, we are interested in the *power spectral density* (PSD) of a signal, which is related to the autocorrelation function via the *Einstein-Wiener-Khintchine* (EWK) Relations:

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f\tau} d\tau \tag{22}$$

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) e^{j2\pi f\tau} df \tag{23}$$

- Relating the PSD between input $x(t)$ and output $y(t)$ of a system $h(t)$, we have the following very important result:

$$S_Y(f) = |H(f)|^2 S_X(f) \tag{24}$$

PSD Example

- Find $S_X(f)$ of the following random process:

$$x(t) = A\cos(2\pi f_c t + \Theta),$$

where Θ is uniformly distributed over the interval $[-\pi, \pi]$.

- First solve for $R_X(\tau)$ by using:

$$R_X(\tau) = E\{x(t+\tau)x(t)\} = \frac{1}{2}A^2\cos(2\pi f_c \tau)$$

- Then solve for the PSD using EWK relations:

$$\begin{aligned}
S_X(f) &= \int_{-\infty}^{\infty} R_X(\tau)e^{-j2\pi f\tau}d\tau = \int_{-\infty}^{\infty}\frac{1}{2}A^2\cos(2\pi f_c\tau)e^{-j2\pi f\tau}d\tau \\
&= \int_{-\infty}^{\infty}\frac{1}{4}A^2\left(e^{j2\pi f_c\tau} + e^{-j2\pi f_c\tau}\right)e^{-j2\pi f\tau}d\tau \\
&= \frac{A^2}{4}(\delta(f-f_c) + \delta(f+f_c))
\end{aligned}$$

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Anatomy of a Typical Digital Communication System

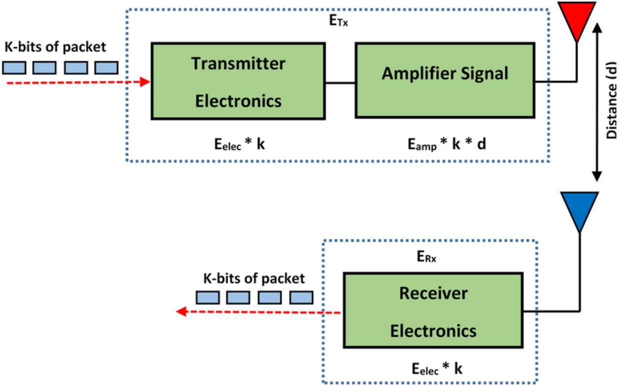

Figure 12: Digital Communication System Block Diagram

$$C = B \log_2(1 + SNR) \qquad [\mathrm{b/s}] \tag{25}$$

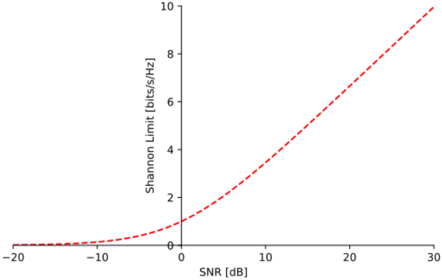

Figure 13: Shannan limit $\frac{C}{B}$ versus SNR

Filtered AWGN Channel

- We know that the power spectral density of white Gaussian noise is equal to:

$$S_N(f) = \mathbb{F}\{R_n(\tau)\} = \int_{-\infty}^{\infty} R_n(\tau)e^{-j2\pi f\tau}d\tau = \frac{N_0}{2} \tag{26}$$

- When this noise is passed through an LTI system with impulse response $h(t)$, the output power spectral density will be defined by the Einstein-Wiener-Khinchin (EWK) Theorem, namely:

$$S_Y(f) = |H(f)|^2 S_N(f) \tag{27}$$

Average Bit Energy

- The symbol energy is then $E_s = E_{-s} = A^2 T = \frac{A^2}{R_b}$
  - Notice how E_s decreases as R_b increases
- We define the energy per bit as:

$$\bar{E}_b = P(1) \cdot \int_{-\infty}^{\infty} s_1^2(t)dt + P(0) \cdot \int_{-\infty}^{\infty} s_2^2(t)dt \tag{28}$$

where $P(1)$ is the probability that the bit is a "1", and $P(0)$ is the probability that the bit is a "0"

- Suppose $s_1(t) = s(t)$ and $s_2(t) = -s(t)$, then:

$$\bar{E}_b = E_s\{P(1) + P(0)\} = E_s = \int_{-\infty}^{\infty} s^2(t)dt = A^2 T \tag{29}$$

Signal Vectors

- Let $\phi_j(t)$ be an orthnormal set of functions on the time interval $[0, T]$ such that:

$$\int_0^T \phi_i(t)\phi_j(t)dt = \begin{cases} 1 & i = j \\ 0 & \text{otherwise} \end{cases}$$

- Let $s_i(t)$ be the modulation signal that we can represent in terms of the orthonormal functions:

$$s_i(t) = \sum_{k=1}^{N} s_{ik}\phi_k(t) \tag{30}$$

which can be represented by the vector:

$$s_i(t) \rightarrow \mathbf{s}_i = (s_{i1},\ s_{i2},\ s_{i3}, \ldots s_{iN}) \tag{31}$$

Vector Manipulations

- To find s_{ik}, solve:

$$\int_0^T s_i(t)\phi_l(t)dt = \sum_{k=1}^{N} s_{ik} \int_0^T \phi_k(t)\phi_l(t)dt = s_{il} \tag{32}$$

- The vector dot product between $s_i(t)$ and $s_j(t)$ is equal to:

$$\int_0^T s_i(t)s_j(t)dt = \mathbf{s}_i \cdot \mathbf{s}_j = \rho_{ij} \tag{33}$$

while the energy of a signal $s_i(t)$ is equal to:

$$E_{s_i} = \int_0^T s_i^2(t)dt = \mathbf{s}_i \cdot \mathbf{s}_i = \|\mathbf{s}_i\|^2 \tag{34}$$

Euclidean Distance

- To compute the *Euclidean Distance* using signal space vectors, we need to solve:

$$
\begin{aligned}
d^2_{\min} &= \int_0^T \Delta s^2_{ij}(t)dt = \int_0^T (s_i(t) - s_j(t))^2 dt \\
&= \|\mathbf{s}_i - \mathbf{s}_j\|^2 = (\mathbf{s}_i - \mathbf{s}_j) \cdot (\mathbf{s}_i - \mathbf{s}_j) \\
&= E_{s_i} + E_{s_j} - 2\rho_{ij}
\end{aligned}
$$

where:

$$
\rho_{ij} = \int_0^T s_i(t)s_j(t)dt = \mathbf{s}_i \cdot \mathbf{s}_j \tag{35}
$$

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Solving for the Power Efficiency

- Choose orthonormal basis functions $\phi_i(t)$, $i = 1, 2, \ldots, k$, where k is the dimension of the signal space
- Find $\mathbf{s}_i$, $i = 1, 2, \ldots, M$ where $\mathbf{s}_i = (s_{i1},\ s_{i2}, \ldots s_{ik})$ and $s_{ij} = \int_0^T s_i(t)\phi_j(t)dt$
- Consequently, solve for:

$$d^2_{\min} = \min_{i \neq j} \|\mathbf{s}_i - \mathbf{s}_j\|^2$$

$$\bar{E}_s = \frac{1}{M}\sum_{i=1}^{M} \|\mathbf{s}_i\|^2$$

$$\bar{E}_b = \bar{E}_s / \log_2(M)$$

$$\varepsilon_p = d^2_{\min} / \bar{E}_b$$

Conclusion

1. This paper presentation has considered wireless communication in a general fading environment, which can be reduced to other types of fading environments like Rayleigh.
2. Rayleigh fading channel is simulated
3. Obtained closed-form expressions for PDF and CDF of SNR for the interference-limited system case.
4. Some of the performance measures are evaluated and discussed in the presentation.
5. Generalisation of communication system is analysed.

Reference

IEEE paper on Modelling and performance analysis of Wireless channel Performance analysis of wireless communication system in general fading environment subjected to shadowing and interference

THANK YOU.......