Assignment 3 -Probability and Random Variable

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Download Python code from here

https://github.com/annu100/AI5002−Probability−and−Random−variables/blob/main/ASSIGNMENT_3/Assignment_3_Bayes.py

Download latex code from here-

https://github.com/annu100/AI5002−Probability−and−Random−variables/blob/main/ASSIGNMENT_3/main.tex

I. Problem Statement-Problem 2.10

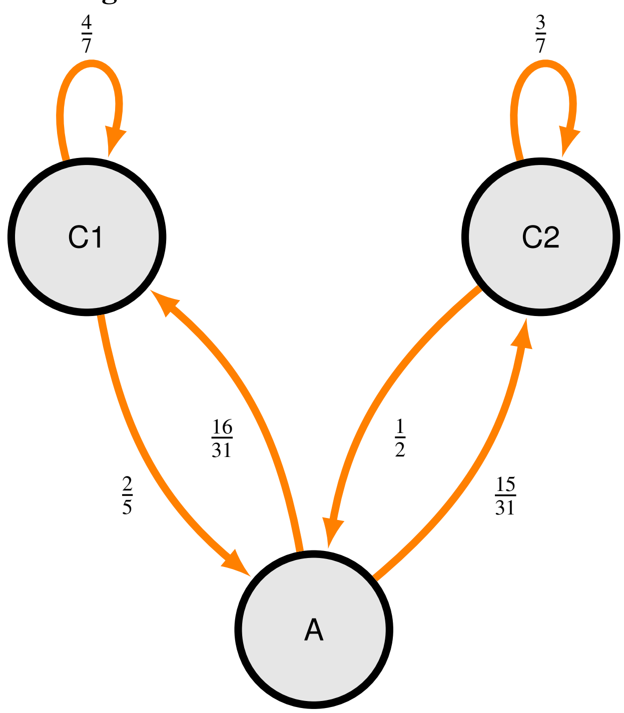
Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

II. Solutions

Bag 1 contains 3 red and 4 black balls.
Bag 2 contains 4 red and 5 black balls.

let C1: Event of transferring black ball from bag 1 to 2
let C1: Event of transferring red ball from bag 1 to 2
let A : Event that the ball drawn from 2 is red after the transfer of a ball from bag 1 to bag 2.

Using markov chain



Note that:Above drawn markov chain is 3 states first order time homogeneous markov chain as transition probabilities are depending only on last one state.
$Pr(A|C1)$:Representing transition probability for going state A from state C1.
$Pr(A|C2)$:Representing transition probability for going state A from state C2.
Pr(C1):Representing probability to remain in C1.
Pr(C2):Representing probability to remain in C2.

$$Pr(C1) = \frac{4}{7}$$
$$Pr(C2) = \frac{3}{7}$$
$$Pr(A|C1) = \frac{4}{10} = \frac{2}{5}$$
$$Pr(A|C2) = \frac{5}{10} = \frac{1}{2}$$

From Baye's theoram

$$\begin{aligned} Pr(\text{Drawn ball is red}) &= P(A) \\ &= Pr(\frac{A}{C1}) \times Pr(C1) \\ &+ Pr(C2) \times Pr(\frac{A}{C2}) \\ &= \frac{4}{10} \times \frac{4}{7} + \frac{5}{10} \times \frac{3}{7} \\ &= \frac{16+15}{70} \\ &= \frac{31}{70} \end{aligned}$$

The probability that the transferred ball is black It is equal to conditional probability of C1 when event A has already happened
Calculating probabilities for markov chain.

$$\begin{aligned} Pr(\frac{C1}{A}) &= \frac{Pr(C1 \cap A)}{Pr(A)} \\ &= \frac{Pr(\frac{A}{C1})Pr(C1)}{Pr(A)} \\ &= \frac{\frac{4}{10} \times \frac{4}{7}}{\frac{31}{70}} \\ &= \frac{16}{31} \end{aligned}$$

$$\begin{aligned} Pr(\frac{C2}{A}) &= \frac{Pr(C2 \cap A)}{Pr(A)} \\ &= \frac{Pr(\frac{A}{C2})Pr(C2)}{Pr(A)} \\ &= \frac{\frac{5}{10} \times \frac{3}{7}}{\frac{31}{70}} \\ &= \frac{15}{31} \end{aligned}$$

We have been asked to find out $Pr(C|A)$ Hence the desired probability is

$$\frac{16}{31} = 0.516$$

III. SIMULATION PART

Using random variable simulation,bernaulli random variables are generated for the two cases:- P(X=0)=probability for drawnball P(X=1)=probability for transffered ball

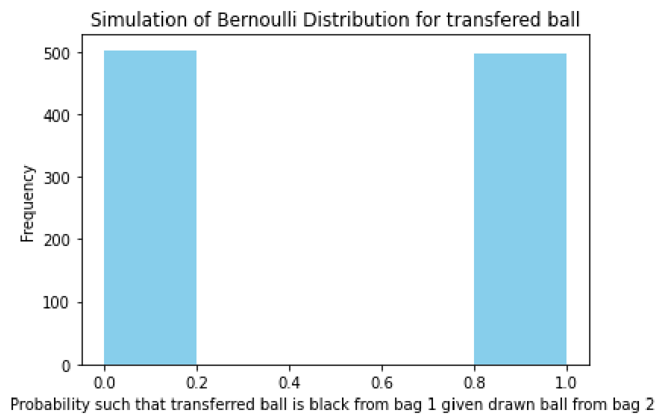


Figure 1: simulation of bernaulli distribution for transferred ball