Assignment 14-Probability and Random Variable

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Download latex code from here-

https://github.com/annu100/AI5002−Probability−and−Random−variables/tree/main.tex/ASSIGNMENT_114

I. Gate-24 Solution

A fair coin is tossed till a head appears for the first time. The probability that the number of requried tosses is odd,is ....

II. SOLUTIONS

As we know
For odd no of tosses
We can get number of tosses like this
1, 3 ,5,7.........
Then the probability for getting head for the first time is-

$$Pr(\text{Head 1st time}) = (1/2)^1 + (1/2)^3 + (1/2)^5 + (1/2)^7 \dots \tag{1}$$

As we can see this is decreasing $G.P$ series
So sum ut upto infinity.
Sum is given by-
$S = \frac{a}{(1-r)}$ where. a is first term of the series=$\frac{1}{2}$
r is common. Ratio=$\frac{1}{4}$

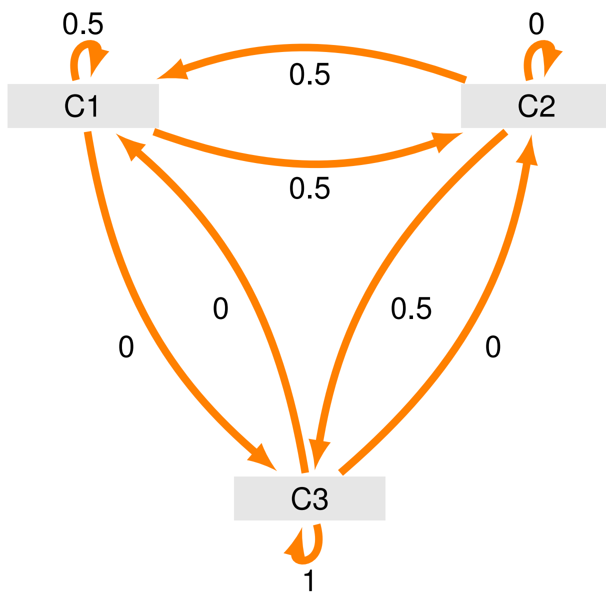
$$Pr(\text{Head 1st time}) = \frac{1/2}{(1-1/4)} \quad = \frac{2}{3} \tag{2}$$

So answer is 0.50/0.75
= 2/3

A. MARKOV CHAIN APPROACH

Probabilities at nth toss Probabilities at zero toss- $P_0 = (1, 0, 0)$
Probabilities after one toss-$P_0 = (0.5, 0.5, 0)$ Each state has the vector of probabilities for going to another state -
collecting it together to form matrix

$$M_3 = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$$



Note that:Above drawn markov chain is 3 states first order time homogeneous markov chain as transition probabilities are depending only on last one state.
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We can get P_1 by multiplying P_0 with M_3.
$P_1 = P_0 \times M_3$
Similarly to get P_2 ,we can multiply P_1 with M_3
$P_2 = P_1 \times M_3 = P_0 \times M_3^2$
So,in general $P_n = P_0 \times M_3^n$
here,n is number of tosses.
but n should be odd for our problem.
It is calculated for upto 100 tosses in python program.