Assignment 13-Probability and Random Variable

Annu-EE21RESCH01010

Download latex code from here-

https://github.com/annu100/AI5002−Probability−and−Random−variables/tree/main.tex/ASSIGNMENT_13

I. Gate-24 Solution

A binary symmetric channel (BSC) has a transition probability of $\frac{1}{8}$. If the binary transmit symbol X is such that $Pr(X = 0) = \frac{9}{10}$,then the probability of error for an optimum receiver will be-

II. SOLUTIONS

let crossover probability=p
q

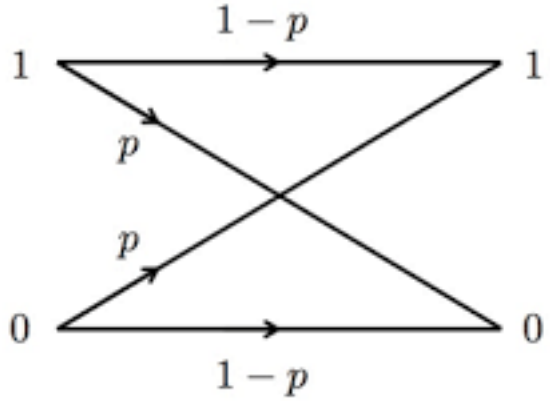

Figure 1: Channel transition diagram

x_0=0,x_1=1,y_0=0,y_1=1 for binary channel
Let x_0 and x_1 are two binary transmitted symbols.
y_0 and y_1 are recieved symbols.
transition probability=$Pr(y_1|x_0)$=$Pr(y_0|x_1)$ Given data-
$Pr(x_0) = \frac{9}{10}$
$Pr(x_1) = 1 - \frac{9}{10} = \frac{1}{10}$
$Pr(y_1|x_0) = p = \frac{1}{8}$
$Pr(y_0|x_1) = p = \frac{1}{8}$
$Pr(y_0|x_0) = 1 - p = 1 - \frac{1}{8} = \frac{9}{10}$
$Pr(y_1|x_1) = 1 - p = 1 - \frac{1}{8} = \frac{9}{10}$
Calculating probability values for MAP critera -

$$Pr(y_0|x_0) \times Pr(x_0) = \frac{7}{8} \times \frac{9}{10} \tag{1}$$
$$= \frac{63}{80} \tag{2}$$

$$Pr(y_0|x_1) \times Pr(x_1) = \frac{1}{8} \times \frac{1}{10} \tag{3}$$
$$= \frac{1}{80} \tag{4}$$

$$Pr(y_1|x_0) \times Pr(x_0) = \frac{1}{8} \times \frac{9}{10} \tag{5}$$
$$= \frac{9}{80} \tag{6}$$

$$Pr(y_1|x_1) \times Pr(x_1) = \frac{7}{8} \times \frac{1}{10} \tag{7}$$
$$= \frac{7}{80} \tag{8}$$

Now according to M.A.P criteria at reciever -

$$Pr(y_0|x_0) \times Pr(x_0) > Pr(y_0|x_1) \times Pr(x_1) \tag{9}$$

So,when a symbol is recieved is recieved as y_0 ,the decision can be made in favour of x_0 in an optimum way.

$$Pr(y_0|x_0) \times Pr(x_0) > Pr(y_0|x_1) \times Pr(x_1) \tag{10}$$

So,when a symbol is recieved is recieved as y_0 ,the decision can be made in favour of x_0 in an optimum way.
As

$$Pr(y_0|x_0) \times Pr(x_0) > Pr(y_0|x_1) \times Pr(x_1) \tag{11}$$
$$= (1 - p) \times \frac{9}{10} > p \times \frac{1}{10} \tag{12}$$

So,when a symbol is recieved is recieved as y_0 ,the decision can be made in favour of x_0 in an optimum way.

$$Pr(y_1|x_0) \times Pr(x_0) > Pr(y_1|x_1) \times Pr(x_1) \quad (13)$$

$$= p \times \frac{9}{10} > (1-p) \times \frac{1}{10} \quad (14)$$

So,when a symbol is recieved is recieved as y_1 ,the decision can be made in favour of x_0 in an optimum way.
So,for the given BSC channel ,with optimum reciver ,both the recieved symbols will be decoded as x_0.
Hence,the probability of error is equal to probability of transmitting x_1.
so,$Pr(error) = Pr(x_1) = \frac{1}{10}$