Assignment 2 -Probability and Random Variable

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I. Problem Statement-Problem 1.10

There are 5 % defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

II. Solutions

Let X be the number of defective items available in bulk of items.
let n =10 (total no. of samples=10)
therefore, q=1-p ,which is probability of getting a non defective item

$$q = 1 - \frac{1}{20} = \frac{19}{20} \tag{1}$$

From bernaulli's distribution, we know

$$Pr(X = r) = {}^{n}C_r p^r q^{n-r} \tag{2}$$

$$X \sim Bin(n = 10, p = 0.5) \tag{3}$$

We are required to find the probability that a sample of 10 items will not include more than 1 defective items

Therefore, the required probability is given by

$$\begin{aligned} Pr(X \leq 1) &= Pr(X = 0) + Pr(X = 1) \\ &= {}^{10}C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{10} + {}^{10}C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^9 \\ &= 1 \times 1 \times \left(\frac{19}{20}\right)^{10} + 10 \times \left(\frac{1}{20}\right) \times \left(\frac{19}{20}\right)^9 \\ &= \left(\frac{19}{20}\right)^{10} + \left(\frac{19}{20}\right)^9 \times \frac{1}{2} \\ &= \left(\frac{29}{20}\right) \times \left(\frac{19}{20}\right)^9 \end{aligned}$$

Hence the desired probability is $\frac{29}{20} \times \frac{19}{20}^9 = \frac{29}{20}\frac{19^9}{20^9}$ =0.9138