

Assignment 1 -Probability and Random Variable

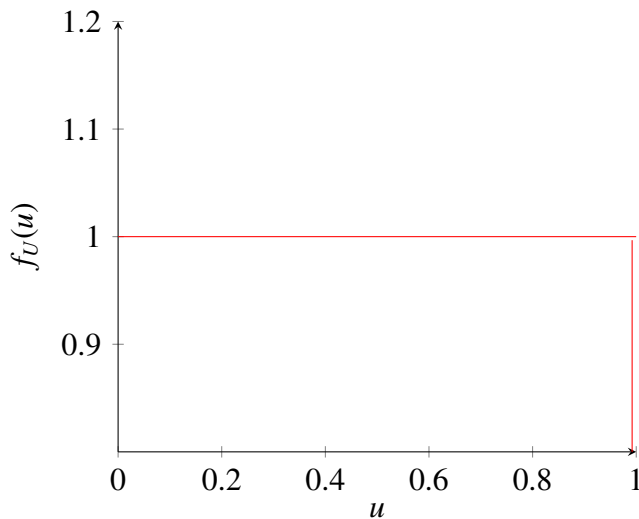
Annu-EE21RESCH01010

Problem Statement- Generate samples of $V = -2 \ln(1 - U)$ and plot its CDF and comment over it.

Solutions:

Given U is uniformly distributed random variable over $\in (0, 1)$.

V is also a random variable which is the function of uniformly distributed random variable U . Below is probability density function of U .



$$V = -2 \ln(1 - U)$$

Cummulative distribution function of random variable V is defined as

$$\begin{aligned} F_V(v) &= \Pr(V \leq v) \\ &= \Pr(-2 \ln(1 - U) \leq v) \\ &= \Pr(\ln(1 - U) \geq (-v)/2) \\ &= \Pr((1 - U) \geq \exp(-v/2)) \\ &= \Pr(U \leq (1 - \exp(-v/2))) \\ &= (1 - (\exp(-v/2))) \end{aligned}$$

$$F_V(v) = 1 - \exp(-v/2)$$

$$v = F_V(\text{inv}(U))$$

Now, the Generation of random samples can be easily done using above equation. Some samples are

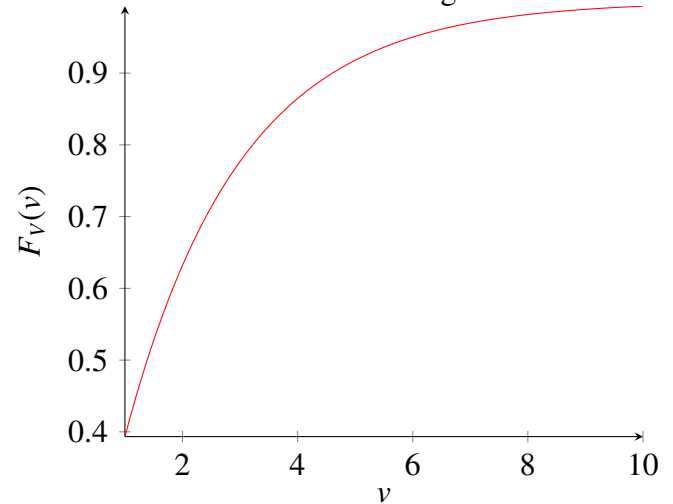
given below-

Since $U \in [0, 1]$, therefore V is always positive and thus $V \geq 0$.

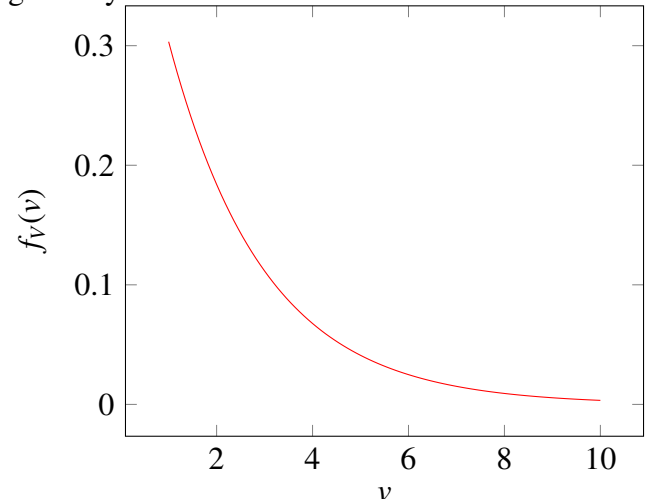
- $u=0, v=0$
- $u=0.9, v=4.605$
- $u=0.5, v=1.38$
- $u=0.75, v=2.77$
- $u=0.25, v=0.5735$
- $u=0.6, v=1.8325$

PDF of V is the differentiation of CDF $F_V(v)$ with respect to v . $f_V(v) = \frac{d(F_V(v))}{dv} = \exp((-v)/2)/2$

Plot CDF of random Variable V is given below



Probability density function of random variable V is given by-



Comment: Obtaining the PDF for the function of random variable involves deriving a distribution. $V = \text{function}(U)$.

The random variable is the function of another random variable. From the uniformly distributed random variable U , We get another random variable V which is exponential in nature. From any known probability distribution, we can derive it's function probability distribution. Once we have CDF of random variable, we can easily get probability density function by just differentiating CDF. From graph we can see CDF is always increasing(monotonically increasing function) Note: Function of random variable is itself a random variable .