

Assignment 3 -Probability and Random Variable

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I. PROBLEM STATEMENT-PROBLEM 2.10

Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

II. SOLUTIONS

Bag 1 contains 3 red and 4 black balls.
Bag 2 contains 4 red and 5 black balls.

let C1: Event of transferring black ball from bag 1 to 2

let C2: Event of transferring red ball from bag 1 to 2

let A : Event that the ball drawn from 2 is red after the transfer of a ball from bag 1 to bag 2.

$$Pr(C1) = \frac{4}{7}$$

$$Pr(C2) = \frac{3}{7}$$

$$Pr(A/C1) = \frac{4}{10} = \frac{2}{5}$$

$$Pr(A/C2) = \frac{5}{10} = \frac{1}{2}$$

From Baye's theoram

$$Pr(\text{Drawn ball is red}) = P(A)$$

$$\begin{aligned} &= Pr\left(\frac{A}{C1}\right) \times Pr(C1) + Pr(C2) \times Pr\left(\frac{A}{C2}\right) \\ &= \frac{4}{10} \times \frac{4}{7} + \frac{5}{10} \times \frac{3}{7} \\ &= \frac{16 + 15}{70} \\ &= \frac{31}{70} \end{aligned}$$

The probability that the transferred ball is black
It is equal to conditional probability of C1 when
event A has already happened

The desired probability is given by

$$\begin{aligned} Pr\left(\frac{C1}{A}\right) &= \frac{Pr(C1 \cap A)}{Pr(A)} \\ &= \frac{Pr\left(\frac{A}{C1}\right)Pr(C1)}{Pr(A)} \\ &= \frac{\frac{4}{10} \times \frac{4}{7}}{\frac{31}{70}} \\ &= \frac{16}{31} \end{aligned}$$

Hence the desired probability is

$$\frac{16}{31} = 0.516$$