Challenging problem 8

Annu-EE21RESCH01010

Download latex code from here-

https://github.com/annu100/AI5002−Probability−and−Random−variables/tree/main.tex/challenging problems

I. Challenging problem 8

Let $X_1, X_2, ..., X_n$ be independent Poisson random variables with $E[X_i] = \mu_i$.Find the conditional distribution of $X_1, ..., X_n \Big| \sum_{i=1}^{n} X_i = y$

II. SOLUTIONS

The random variable X_i is distributed as Poisson if the density of X_i is given by
$E[X_i] = \mu_i$
Also, $X_i = \mu$
$f(x_i : \mu_i)= \begin{cases} \frac{e^{-\mu_i} \times \mu^{x_i}}{x_i!} & , x_i \geq 0 \\ 0 & , otherwise \end{cases}$
Since $X_1, X_2, ..., X_n$ be independent Poisson random variables-
Likelihood function L is required to be calculated and it is given by

$$L = \frac{e^{-\mu_1} \times \mu^{x_1}}{x_1!} \times \frac{e^{-\mu_2} \times \mu^{x_2}}{x_2!} \cdots \frac{e^{-\mu_n} \times \mu^{x_n}}{x_n!} \tag{1}$$

$$= \frac{e^{-\mu_n} \times \mu^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!} \tag{2}$$

$$\tag{3}$$

So,above case is only defined when
$\sum_{i=1}^{n} X_i = y$ for some y
otherwise L=0.
Here L is the conditional distribution of
$X_1, ..., X_n \Big| \sum_{i=1}^{n} X_i = y$
Also sum of two independent Possion R.V is also R,V with mean as sum of mean of individual random variables which provides for the proof of above expression for L.