Assignment 4-Probability and Random Variable

Annu-EE21RESCH01010

Download latex code from here-

https://github.com/annu100/AI5002−Probability−and−Random−variables/tree/main/ASSIGNMENT_4

download python code from here

https://github.com/annu100/AI5002−Probability−and−Random−variables/blob/main/ASSIGNMENT_4/assignment_4.py

I. Problem Statement-Problem 3.10

How many times must a man toss a fair coin so that the probability of having at least one head is more than 90 %?

II. Solutions

Let r be the number for getting no. of heads.
let n =total no. of times a coin is tossed
therefore, q=1-p ,which is probability of getting a tail.Since it is the case of fair coin, therefore p=0.5 and q =0.5

$$p = \frac{1}{2} \tag{1}$$
$$q = 1 - \frac{1}{2} = \frac{1}{2} \tag{2}$$

From bernaulli's distribution, we know

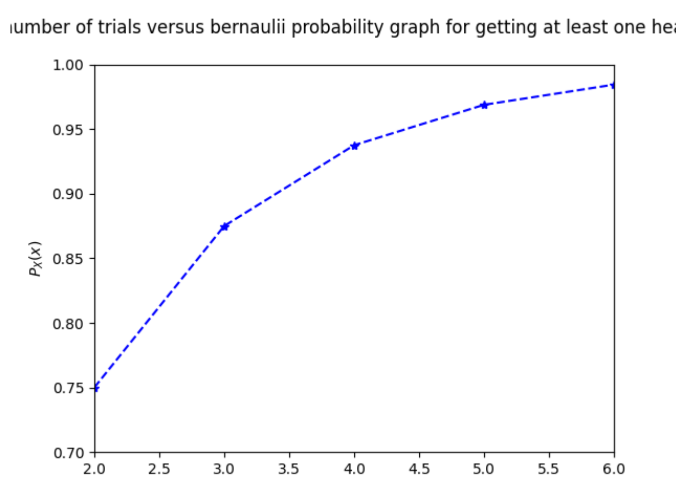
$$Pr(X = r) = {}^{n}C_r p^r q^{n-r} \tag{3}$$
$$X \sim Bin(n, p = 0.5) \tag{4}$$

We are required to find the number of trials such that the sample probability of having at least one head is more than 90%

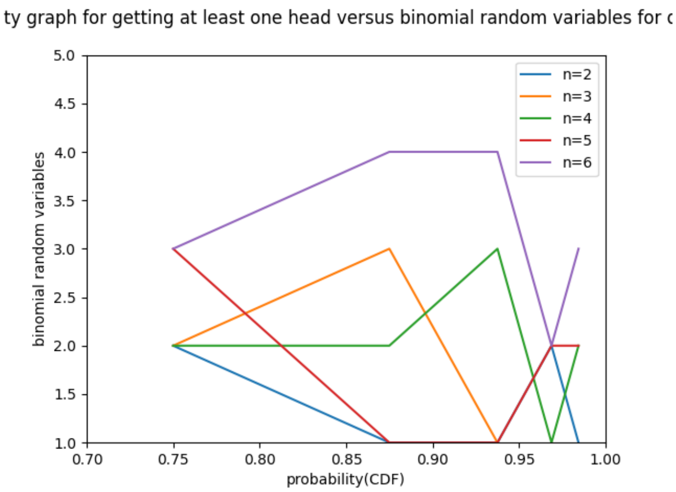
$$Pr(X \geq 1) = 1 - Pr(X = 0) \tag{5}$$
$$= 1 - {}^{n}C_0 0.5^0 0.5^{n-0} > 0.9 \tag{6}$$
$$= 1 - (\frac{1}{2})^n > 0.9 \tag{7}$$
$$= (\frac{1}{2})^n < 0.1 \tag{8}$$
$$= 2^n > 10 \tag{9}$$

This implies $n \geq 4$

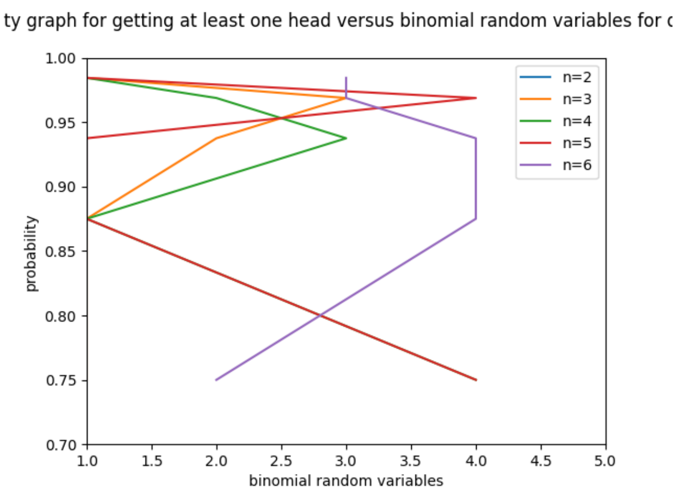
Therefore, the required number of trials must be greater than or equal to 4.From graph,we can also see



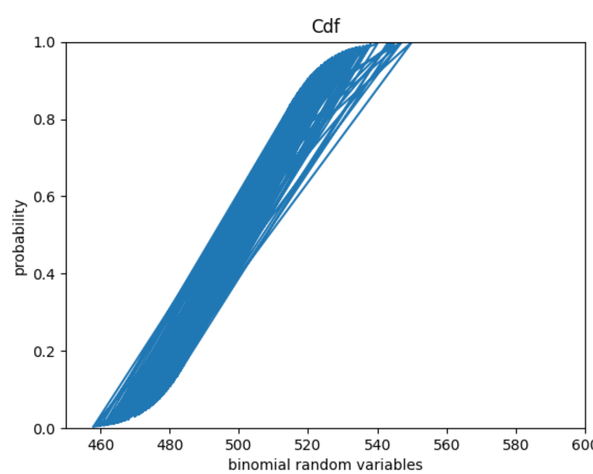
Above is is the graph of no. of trials versus probability



Above is is the graph of binomial random number generated versus probability(cdf)

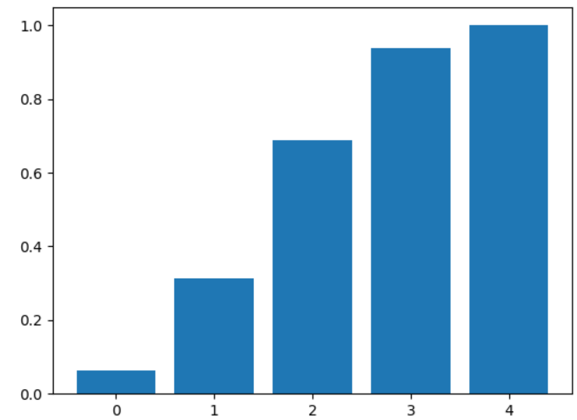


Above is is the graph of binomial random number generated versus probability
$(c1 - binom.cdf(1, n, p) + binom.pmf(1, n, p))$



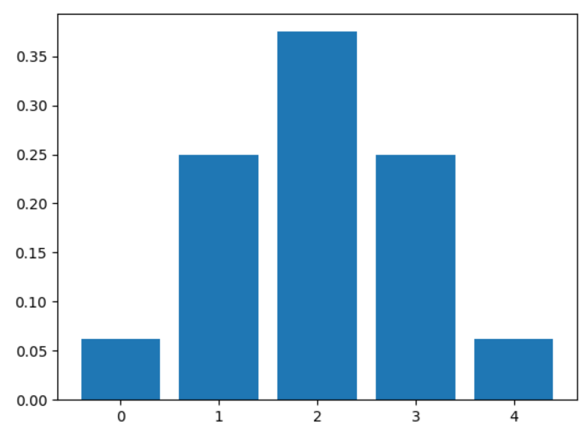
Above is is the graph of binomial random number generated versus probability for large data set and large number of trials

Figure 1: cdf



Above is the graph of number of success versus probability(cdf) for n=4

Figure 2: pmf



Above is is the graph of number of success versus probability(pmf) for n=4

The above result for $n \geq 4$ is verified many times.

1) Using mathematical calculation
2) By seeing the grapgh of no. of trial versus probability
3) Using binomial random numner generation in the python program by generating 1000 random numbers and then calculating probability.

4) At last, a graph is drawn between random variables generated versus required probability,which is often calculated using CDF,probability=1 $- binom.cdf(1, n, p) + binom.pmf(1, n, p)$ This grapgh is random,since every simulation gives different random variables generation
5) A graph of CDF is also drawn in order to observe the properties associated with cdf of binomial R.V
6) Further, the graph of CDF and PMF is obtained (for n=4) for different number of success. So for atmost 1 head our probability must be 1-CDF(1)+PMF(1) which is grearer than 0.9 .Hence our results again verified.