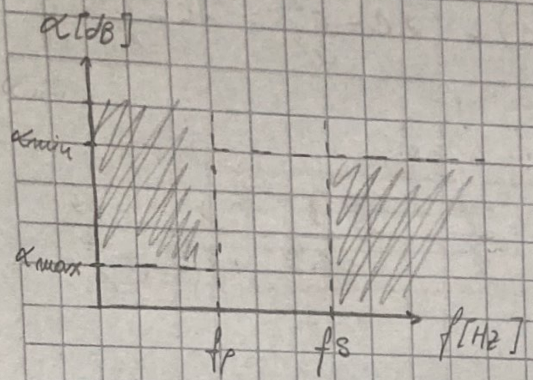
FECHA




$\alpha_{max}[dB] = 1\,dB$ $\qquad f_p[Hz] = 1500\,Hz$

$\alpha_{min}[dB] = 12\,dB$ $\qquad f_s[Hz] = 3000\,Hz$

$\alpha_{max}[dB] = 10\log(1+\xi^2\omega_p^{2n})$

$\alpha_{min}[dB] = 10\log(1+\xi^2\omega_s^{2n})$

Normalizo tal que $\omega_p = 1 \therefore \omega_s = 2$

$\boxed{\Omega\omega = 2\pi f_p}$

1) $|T(\omega)|^2 = \dfrac{1}{1+\xi^2\omega^{2n}}$

$\xi^2 = 10^{\alpha_{max}/10} - 1 = 0{,}259$

$\alpha_{min}[dB] = 10\log(1+0{,}259 \cdot 2^{2n}) \longrightarrow$ iterando $\Longrightarrow n = 3,\ \alpha = 12{,}45$

$$|T(\omega)|^2 = \frac{1}{1+\xi^2\omega^6} = \left[T(s)\cdot T(-s)\right]\Big|_{s=j\omega}$$

$$|T(\omega)|^2\Big|_{\omega=s/j} = \frac{1}{1-\xi^2 s^6} = \frac{-\xi^{-2}}{s^6-\xi^{-2}} = T(s)\cdot T(-s)$$

$$-\frac{1/\xi^2}{s^6-1/\xi^2} = \frac{k_I}{(s+a)(s^2+bs+c)} \cdot \frac{k_{II}}{(s-a)(s^2-bs+c)} = \frac{k}{s^3+(b+a)s^2+(c+ab)s+ac} \cdot \frac{k}{s^3+(a-b)s^2+(c+ab)s-ac}$$

$s^3+bs^2+cs+as^2+abs+ac \qquad s^3-bs^2+sc-as^2+abs-ac$

$$-\frac{1/\xi^2}{s^6-1/\xi^2} = \frac{k_I\cdot k_{II}}{s^6-(a+b)s^5+(c+ab)s^4-ac\,s^3+(b+a)s^5+(b+a)(a-b)s^4+(b+a)(c+ab)s^3 \ldots}$$

$\ldots -(b+a)ca\,s^2+(c+ab)s^4+(c+ab)(a-b)s^3+(c+ab)^2s^2-(c+ab)ac\,s+ac\,s^3-ac(a+b)s^2 \ldots$

$\ldots +ac(c+ab)s - a^2c^2$

$$\begin{cases} a-b+b-a=0 \\ c+ab-ab-b^2-a^2-ab+c+ab = 2c-a^2-b^2=0 \\ -ac+bc-ab^2-ac+a^2b+ac-bc-a^2b+ab^2-ac=0 \\ -abc-a^2c+c^2+2abc+a^2b^2-a^2c-abc = c^2+a^2b^2-2a^2c=0 \\ (c+ab)ac-(c+ab)ac=0 \end{cases}$$

- $2c = a^2+b^2$
- $a^2b^2+c^2 = 2a^2c$

$-a^2c^2 = -1/\xi^2 \longrightarrow ac = 1/\xi \quad \wedge \quad k_I\cdot k_{II} = -a^2c^2 = -1/\xi^2$

NOTA

$$\begin{cases} 2c = a^2 + b^2 \longrightarrow 2c = a^2 + 2c - \dfrac{c^2}{a^2} \longrightarrow \dfrac{c^2}{a^2} = a^2 \longrightarrow c = a^2 \quad (2) \\ a^2 b^2 + c^2 = 2a^2 c \longrightarrow b^2 = \dfrac{c(2a^2 - c)}{a^2} = 2c - \dfrac{c^2}{a^2} \overset{(2)}{=} 2c - \dfrac{c^2}{c} = c \\ a^2 c^2 = 1/\xi^2 \xrightarrow{(1)} c^3 = \dfrac{1}{\xi^2} \longrightarrow c = \dfrac{1}{\xi^{2/3}} \end{cases}$$

$$a^2 = \frac{1}{\xi^{2/3}} \longrightarrow a = \frac{1}{\xi^{1/3}} \qquad ac = \frac{1}{\xi} \longrightarrow \frac{1}{\xi^{1/3}} \cdot \frac{1}{\xi^{2/3}} = \frac{1}{\xi}$$

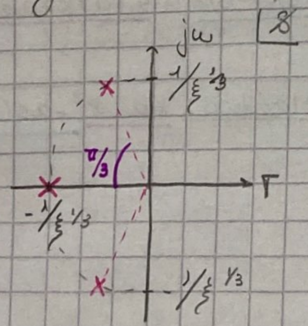
$$b = a = \frac{1}{\xi^{1/3}}$$

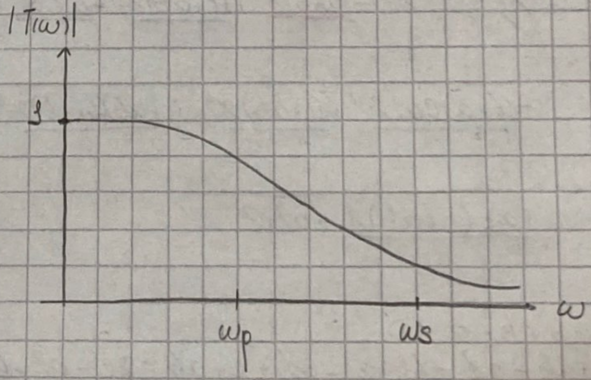
$$|T(s)| = \frac{ac}{(s+a)(s^2+bs+c)} = \frac{1/\xi}{(s + 1/\xi^{1/3})\left[s^2 + \frac{s}{\xi^{1/3}} + \left(\frac{1}{\xi^{1/3}}\right)^2\right]} = \frac{w_0^3}{(s+w_0)(s^2 + \frac{w_0}{Q}s + w_0^2)}$$

$$|T(s)| = \frac{ac}{s^3 + \underbrace{(b+a)}_{2a}s^2 + \underbrace{(c+ab)}_{c+a^2=2c}s + ac} = \frac{1/\xi}{s^3 + s^2 \cdot \frac{2}{\xi^{1/3}} + s \cdot \frac{2}{\xi^{2/3}} + \frac{1}{\xi}}$$

$$\begin{cases} w_0 = 1/\xi^{1/3} \\ Q = 1 \end{cases}$$

2) Diagrama de polos y ceros




apertura $= \dfrac{\pi}{n} = \dfrac{\pi}{3}$

$1/\xi^{1/3} \cong 1,25$

NOTA

$L = \Omega . s$
$c = s/\Omega$

HOJA Nº 5

FECHA

3) Por conocimiento previo de los circuitos $\qquad \xi \cong 0,51$

$$T_2(s) = \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \qquad \omega_0^2 = \frac{1}{LC} \qquad \frac{\omega_0}{Q} = \frac{R}{L}$$

$$T_1(s) = \frac{\omega_1}{s + \omega_1} \longrightarrow \text{para implementarlo uso un RC}$$

$$T_1(s) = \frac{1/sC}{1/sC + R} = \frac{1}{1 + sRC} = \frac{1/RC}{s + 1/RC}$$

$$\omega_0^2 = \frac{1}{\xi^{2/3}} = \frac{1}{LC} \longrightarrow C = \frac{\xi^{2/3}}{L} \longrightarrow C = \frac{\xi^{2/3}}{R \cdot \xi^{1/3}} = \frac{\xi^{1/3}}{R}$$

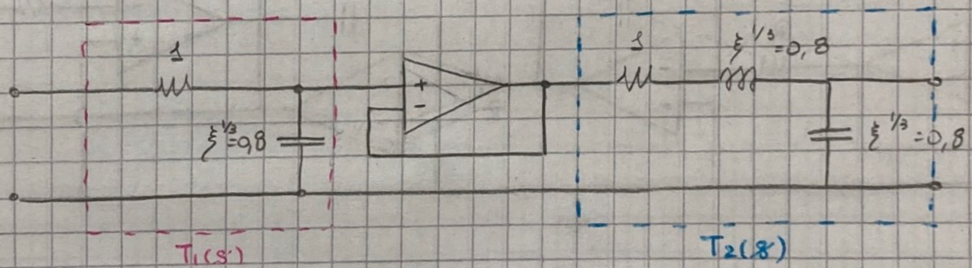
$$\frac{\omega_0}{Q} = \frac{R}{L} \longrightarrow \frac{1}{\xi^{1/3}} = \frac{R}{L} \longrightarrow L = R \cdot \xi^{1/3}$$

Normalizo para $\Omega_Z = R$

$R' = 1$
$L' = \xi^{1/3} \cong 0,8$
$C' = \xi^{1/3} \cong 0,8$

corroboro que $\omega_1 = \omega_0$

$$T_1(s) = \frac{1/C}{s + 1/C} = \frac{1/\xi^{1/3}}{s + 1/\xi^{1/3}}$$




4) $C = 100\,uF \qquad \Omega_\omega = 2\pi f_p = 2\pi \cdot 1500\,Hz \qquad \Omega_Z = R$

$$C = \frac{C'}{\Omega_\omega \Omega_Z} = \frac{\xi^{1/3}}{2\pi f_p R} \longrightarrow R = \frac{0,8}{2\pi f_p \cdot C} = 848,83\,\Omega$$

$$L = \frac{L' \Omega_Z}{\Omega_\omega} = \xi^{1/3} \cdot \frac{R}{2\pi f_p} = 72,05\,mHy$$

$R = R' \Omega_Z = \Omega_Z$

$$|T(s)| = \frac{\omega_1 \cdot \omega_0^2}{(s+\omega_1)(s^2 + \frac{\omega_0}{Q}s + \omega_0^2)} = \frac{\omega_1 \omega_0^2}{s^3 + \frac{\omega_0}{Q}s^2 + \omega_0^2 s + s^2\omega_1 + \frac{\omega_0\omega_1}{Q}s + \omega_0^2\omega_1} =$$

$$= \frac{1/LC\,RC}{s^3 + \left(\frac{R}{L} + \frac{1}{RC}\right)s^2 + s\left(\frac{1}{LC} + \frac{R}{L}\cdot\frac{1}{RC}\right) + \frac{1}{LC}\cdot\frac{1}{RC}} = \frac{1/C^2LR}{s^3 + \left(\frac{R}{L} + \frac{1}{RC}\right)s^2 + \frac{2}{LC}s + \frac{1}{C^2LR}}$$

NOTA

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5) Puedo reemplazar el RLC por el circuito Ackerberg-Mossberg que analizé en la TS2

$$|T_{A-M}(s)| = \frac{R_3}{R_1} \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$\omega_0 = \frac{1}{CR_3}$  $Q = \frac{R_2}{R_3}$

$\omega_0 = 1/\xi^{1/3}$  $Q = 1$

$C = \frac{\xi^{1/3}}{R_3}$  $R_2 = R_3$

Propongo $\Omega_z = R_3$ y $\Omega_\omega = 2\pi f_p$

$C' = \xi^{1/3}$

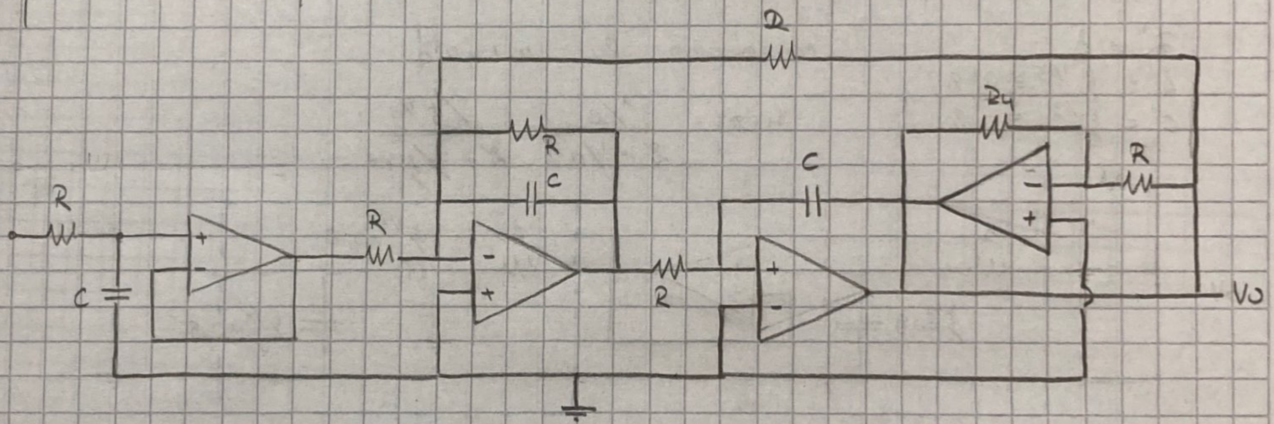
$R_2' = 1$

$R_3' = 1$

$|T(0)|\Big|_{MAX\,plan} = 1 \longrightarrow |T_{A-M}(0)| = \frac{R_3}{R_1} \therefore R_3 = R_1 \longrightarrow R_1' = 1$

$\begin{cases} C = 100\,nF = \frac{C'}{\Omega_z \Omega_\omega} \longrightarrow R_3 = 848{,}83\,\Omega \\ R_1 = R_2 = R_3 = R_4 = R \end{cases}$

Dado que $R_4$ no afecta $T(s)$, $R_4 = R_3$




BONUS

Red renormalizada $\longrightarrow \omega_B = \Omega_\omega \xi^{-1/n} = \Omega_\omega \frac{1}{\xi^{1/n}}$  $\frac{\omega}{\omega_B} = \frac{\omega}{\Omega_\omega \xi^{-1/n}}$

Solo normalizando con $\Omega_\omega$, $|T(\omega)|^2 = \frac{1}{1+\xi^2\omega^{2n}} = \frac{1}{1+(\omega.\xi^{1/n})^{2n}} = \frac{1}{1+\left(\frac{\omega}{\omega_B}\right)^{2n}}$

Normalizando con $\omega_B$, $|T(\omega)|^2 = \frac{1}{1+\omega^{2n}}$

Ahora trabajo como si fuera un Butter de tercer orden

$$|T_{B_3}(s)| = \frac{\omega_0}{s+\omega_0} \cdot \frac{\omega_0^2}{s+2\cos\varphi.\omega_0.s+\omega_0^2}$$

como es de tercer órden $\varphi = \frac{\pi}{3}$ (apertura $= \pi/n$)

Dado que está renormalizado, $\omega_0 = 1$

$$|T_{B_3}(s)| = \frac{1}{(s+1)(s+1.s+1)}$$

NOTA

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