№3* Решить систему линейных ур-ний методом LU-разложения

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 1 \\ 11x_1 + 7x_2 + 5x_3 = -6 \\ 9x_1 + 8x_2 + 4x_3 = -5 \end{cases}$$

$A = LU$

$Ly = b$

$Ux = y$

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 11 & 7 & 5 \\ 9 & 8 & 4 \end{pmatrix} \xrightarrow{\text{II} - \frac{11}{2}\text{I}} \begin{pmatrix} 2 & 1 & 3 \\ 0 & 1{,}5 & -11{,}5 \\ 0 & 3{,}5 & -9{,}5 \end{pmatrix} \xrightarrow{\text{III} - \frac{7}{3}\text{II}} \underbrace{\begin{pmatrix} 2 & 1 & 3 \\ 0 & 1{,}5 & -11{,}5 \\ 0 & 0 & 52/3 \end{pmatrix}}_{U} = U$$

$Ly = b$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 11/2 & 1 & 0 \\ 9/2 & 7/3 & 1 \end{pmatrix}$$

$$\begin{cases} y_1 = 1 \\ 11/2\, y_1 + y_2 = -6 \\ 9/2\, y_1 + 7/3\, y_2 + y_3 = -5 \end{cases} \qquad \begin{cases} y_1 = 1 \\ y_2 = -6 - 11/2 = -11{,}5 \\ y_3 = -5 - 9/2 - 7/3 \cdot (-11{,}5) = \frac{161}{6} - \frac{9}{2} - 5 = \frac{67}{3} - \frac{15}{3} = \frac{52}{3} \end{cases}$$

$$\begin{cases} y_1 = 1 \\ y_2 = -11{,}5 \\ y_3 = 52/3 \end{cases}$$

$Ux = y$

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 1 \\ 1{,}5x_2 - 11{,}5x_3 = -11{,}5 \\ 52/3\, x_3 = 52/3 \end{cases} \qquad \begin{cases} x_1 = -2/2 = -1 \\ x_2 = 0 \\ x_3 = 1 \end{cases}$$

Проверка:

$$\begin{cases} 2 \cdot (-1) + 0 + 3 \cdot 1 = 1 \\ 11 \cdot (-1) + 7 \cdot 0 + 5 \cdot 1 = -6 \\ 9 \cdot (-1) + 8 \cdot 0 + 4 \cdot 1 = -5 \end{cases}$$

Верно!