ПЗ 4 урок 5 Производная ф-ции 1й переменной

№1. $y = \frac{1}{x} + \frac{2}{x^2} - \frac{5}{x^3} + \sqrt{x} - \sqrt[3]{x} + \frac{3}{\sqrt{x}}$

$(x^\alpha)' = \alpha x^{\alpha-1}$, $(f(x)+g(x))' = f'(x) + g'(x)$

$y = x^{-1} + 2x^{-2} - 5x^{-3} + x^{1/2} - x^{1/3} + 3x^{-\frac{1}{2}}$

$y' = -x^{-2} - 4x^{-3} + 15x^{-4} + \frac{1}{2}x^{-1/2} - \frac{1}{3}x^{-2/3} - \frac{3}{2}x^{-\frac{3}{2}} =$

$= -\frac{1}{x^2} - \frac{4}{x^3} + \frac{15}{x^4} + \frac{1}{2\sqrt{x}} - \frac{1}{3\sqrt[3]{x^2}} - \frac{3}{2\sqrt{x^3}}$

№2. $y = x \cdot \sqrt{1+x^2}$

$y' = x' \cdot \sqrt{1+x^2} + x \cdot \sqrt{1+x^2}' = \sqrt{1+x^2} + x \cdot \left((1+x^2)^{\frac{1}{2}}\right)' =$

$= \sqrt{1+x^2} + x \cdot \frac{1}{2} \cdot (1+x^2)^{-\frac{1}{2}} = \sqrt{1+x^2} + \frac{1 \cdot x}{2\sqrt{1+x^2}} =$

$= \frac{2 + 2x^2 + x}{2\sqrt{1+x^2}} = \frac{2x^2 + x + 2}{2\sqrt{1+x^2}}$

№3. $y = \frac{2x}{1-x^2} = 2x \cdot (1-x^2)^{-1}$

$y' = (2x)' \cdot (1-x^2)^{-1} + 2x \cdot \left((1-x^2)^{-1}\right)' =$

$= \frac{2}{1-x^2} + 2x\left(-1 \cdot (1-x^2)^{-2}\right) =$

$= \frac{2}{1-x^2} + \frac{2x}{-(1-x^2)^2} = \frac{2}{1-x^2} - \frac{2x}{(1-x^2)^2} =$

$= \frac{2}{1-x^2}\left(1 - \frac{x}{1-x^2}\right)$

№5. $y = \ln\left(x + \sqrt{x^2+1}\right)$

$y' = \left(\ln\left(x + \sqrt{x^2+1}\right)\right)' = \left(\ln x + \ln\left(\sqrt{x^2+1}\right)\right)' =$

$= (\ln x)' + \left(\ln\sqrt{x^2+1}\right)' = \frac{1}{x} + \left(\frac{1}{2}\ln(x^2+1)\right)' =$

$= \frac{1}{x} + \left(\frac{1}{2} \cdot x \ln x\right)' + \left(\frac{1}{2}\ln 1\right)' = \frac{1}{x} + \frac{1}{x} + 0 = \frac{2}{x}$

№6. $y = x \cdot \ln\left(x + \sqrt{x^2+1}\right) - \sqrt{x^2+1}$

$y' = \left(x \cdot \ln\left(x + \sqrt{x^2+1}\right)\right)' - \left((x^2+1)^{\frac{1}{2}}\right)' =$

$= x' \cdot \ln\left(x + \sqrt{x^2+1}\right) + x \cdot \left(\ln\left(x + \sqrt{x^2+1}\right)\right)' - \frac{1}{2\sqrt{x^2+1}} =$

$= \ln x + \ln\sqrt{x^2+1} + x \cdot (\ln x)' + x \cdot \left(\ln\sqrt{x^2+1}\right)' - \frac{1}{2\sqrt{x^2+1}} =$

$= \ln x + \ln\sqrt{x^2+1} + x \cdot \frac{1}{x} + x \cdot \left(\frac{1}{2} \cdot 2\ln x + \frac{1}{2}\ln 1\right)' - \frac{1}{2\sqrt{x^2+1}} =$

$= \ln x + \ln\sqrt{x^2+1} + 1 + x \cdot \frac{1}{x} - \frac{1}{2\sqrt{x^2+1}} = 2 + \ln\left(x + \sqrt{x^2+1}\right) - \frac{1}{2\sqrt{x^2+1}}$

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№7* $y = \arcsin(\sin x)$

$y' = (\arcsin(\sin x))' = \frac{1}{\sqrt{1-\sin^2 x}} \cdot (\sin x)' = \frac{\cos x}{\sqrt{1-\sin^2 x}} =$

$= \frac{\cos x}{\sqrt{\cos^2 x}} = \frac{\cos x}{\cos x} = 1$

№4* $y = \sqrt{x + \sqrt{x + \sqrt{x}}} = \left(x + \left(x + x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$y' = \frac{1}{2} \cdot \left(x + \sqrt{x + \sqrt{x}}\right)^{-\frac{1}{2}} \cdot \frac{1}{2}\left(x + \sqrt{x}\right)^{-\frac{1}{2}} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} =$

$= \frac{1}{2 \cdot \sqrt{x + \sqrt{x + \sqrt{x}}}} \cdot \frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \frac{1}{2\sqrt{x}} =$

$= \frac{1}{8 \cdot \sqrt{x + \sqrt{x + \sqrt{x}}} \cdot \sqrt{x + \sqrt{x}} \cdot \sqrt{x}}$

№8* $\sin(1^\circ) = \sin\left(\frac{\pi}{180}\right)$

$f(x + \Delta x) \approx f(x) + f'(x) \cdot \Delta x$

$\sin 0^\circ = 0$

$x = 0^\circ \qquad \Delta x = 1^\circ$

$\sin 1^\circ = \sin\left(0^\circ + \frac{\pi}{180^\circ}\right) \approx \sin 0^\circ + \left(\sin\left(0^\circ + \frac{\pi}{180^\circ}\right)\right)' =$

$= \sin 0^\circ + \cos 0^\circ \cdot \frac{\pi}{180} = 0 + 1 \cdot \frac{\pi}{180} = \frac{\pi}{180}$