ДЗ 3 урок 4

ж*) $\lim\limits_{x\to+0} \frac{5^x-1}{x} = \lim\limits_{t\to0} \frac{t}{\log_5(1+t)}$

$t = 5^x - 1$, $x = \log_5(1+t)$

$5^x \to 5^0 = 1$, при $x\to0$, но $5^x \neq 1$, т.к. $x\neq0$, а $x\to0$

$\lim\limits_{x\to+0} \frac{5^{\cancel{x}}-1}{x} = \ln(5) \approx 1{,}61$

$$\boxed{\lim_{x\to0} \frac{a^x-1}{x} = \ln(a)}$$

з*) $\lim\limits_{x\to+\infty} \frac{\ln(x^2-x+1)}{\ln(x^{10}+x+1)} = \left(\frac{\infty^2-\infty+1}{\infty+1}\right) = \left(\frac{+\infty}{+\infty}\right)$

$\lim\limits_{x\to+\infty} \frac{\ln(x^2(1-\frac{1}{x}+\frac{1}{x^2}))}{\ln(x^{10}(1+\frac{1}{x^9}+\frac{1}{x^{10}}))} = \lim\limits_{x\to+\infty} \frac{\ln x^2 + \ln(1-\frac{1}{x}+\frac{1}{x^2})}{\ln x^{10} + \ln(1+\frac{1}{x^9}+\frac{1}{x^{10}})} =$

$= \lim\limits_{x\to+\infty} \frac{2\ln x + \ln(1-\frac{1}{x}+\frac{1}{x^2})}{10\ln x + \ln(1+\frac{1}{x^9}+\frac{1}{x^{10}})} = \lim\limits_{x\to+\infty} \frac{\cancel{2\ln x}\left(1+\frac{1}{2\ln x}\ln(1-\frac{1}{x}+\frac{1}{x^2})\right)}{\cancel{2\ln x}\left(5+\frac{1}{2\ln x}\ln(1+\frac{1}{x^9}+\frac{1}{x^{10}})\right)} = \frac{1}{5}$

$\to 0$, $\to 1$, $\to 0$, $\to 1$, $\to 0$, 0

g**) $\lim\limits_{x\to0} \frac{\sqrt{2}\,x^2\sin4x}{(1-\cos2x)^{3/2}} = \lim\limits_{x\to0} \frac{\sqrt{2}\,x^2\sin4x}{(1-1+2\sin^2x)^{3/2}} = \lim\limits_{x\to0} \frac{\sqrt{2}\,x^2\sin4x}{\sqrt{2}^3\sin^3x} =$

$$\boxed{\cos2x = 1-2\sin^2x}$$

$= \lim\limits_{x\to0} \frac{x^2\,\cancel{2}\sin2x\cdot\cos2x}{\cancel{2}\sin^3x} = \lim\limits_{x\to0} \frac{2x^2\sin x\cos x\cos2x}{\sin^3x} =$

$= \lim\limits_{x\to0} \frac{2\cdot\overbrace{x}^{\to1}\cdot\overbrace{x}^{\to1}}{\sin x\,\sin x}\cdot\overbrace{\cos x}^{\to1}\cdot\overbrace{\cos2x}^{\to1} = 2$