# ПЗ 5 урок 6

№1. $\operatorname{arctg}\left(\frac{y}{x}\right) = \ln\sqrt{x^2+y^2}$

$$F(x, y(x)) = \operatorname{arctg}\left(\frac{y}{x}\right) - \ln\sqrt{x^2+y^2} = 0$$

$$F'_x = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{y}{x}\right)'_x - \frac{1}{\sqrt{x^2+y^2}}\left(\sqrt{x^2+y^2}\right)'_x =$$

$$= \frac{1}{1+\frac{y^2}{\cancel{x^2}}}\cdot\frac{-y}{\cancel{x^2}} - \frac{1}{\sqrt{x^2+y^2}}\cdot\frac{1}{2\sqrt{x^2+y^2}} =$$

$$= -\frac{y}{x^2+y^2} - \frac{1}{2(x^2+y^2)} = \frac{-\cancel{2}y-\cancel{2}x}{\cancel{2}(x^2+y^2)} = \frac{-y-x}{x^2+y^2}$$

$$F'_y = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{y}{x}\right)'_y - \frac{1}{\sqrt{x^2+y^2}}\left(\sqrt{x^2+y^2}\right)'_y =$$

$$= \frac{1}{1+\frac{y^2}{x^2}}\cdot\frac{1}{x} - \frac{1}{\sqrt{x^2+y^2}}\cdot\frac{1\cdot 2y}{2\sqrt{x^2+y^2}} =$$

$$= \frac{1}{x+\frac{y^2}{x}} - \frac{1\cdot 2y}{2(x^2+y^2)} = \frac{x}{x^2+y^2} - \frac{1\cdot 2y}{2(x^2+y^2)} =$$

$$= \frac{\cancel{2}x-\cancel{2}y}{\cancel{2}(x^2+y^2)} = \frac{x-y}{x^2+y^2}$$

$$y' = -\frac{F'_x}{F'_y} = -\frac{-x-y}{x^2+y^2}\cdot\frac{x^2+y^2}{x-y} = \frac{x+y}{x-y}$$

# ПЗ 5 урок 6

№1. $\operatorname{arctg}\frac{y}{x} = \ln\sqrt{x^2+y^2}$

<u>Проверка кода</u>

$$-\frac{F'_x}{F'_y} = \frac{\frac{x}{x^2+y^2} + \frac{y}{x^2\left(1+\frac{y^2}{x^2}\right)}}{-\frac{y}{x^2+y^2} + \frac{1}{x\left(1+\frac{y^2}{x^2}\right)}} = \frac{\frac{x+y}{x^2+y^2}}{-\frac{y}{x^2+y^2}+\frac{x}{x^2+y^2}} =$$

$$= \frac{x+y}{\cancel{x^2+y^2}}\cdot\frac{\cancel{x^2+y^2}}{x-y} = \frac{x+y}{x-y}$$

N2. $\begin{cases} y = \frac{t^2}{t-1} \\ x = \frac{t}{t^2-1} \end{cases}$ $\quad y'_x = \frac{y'_t}{x'_t}$

$$y'_t = \left(t^2\cdot(t-1)^{-1}\right)' = 2t\cdot(t-1)^{-1} + t^2\cdot(-1)(t-1)^{-2} =$$

$$= \frac{2t}{t-1} - \frac{t^2}{(t-1)^2} = \frac{2t(t-1)-t^2}{(t-1)^2} = \frac{t(2t-2-t)}{(t-1)^2} = \frac{t(t-2)}{(t-1)^2}$$

$$x'_t = \left(t\cdot(t^2-1)^{-1}\right)' = \frac{1}{t^2-1} + \frac{-t}{(t^2-1)^2}\cdot 2t = \frac{t^2-1-2t^2}{(t^2-1)^2} =$$

$$= -\frac{t^2+1}{(t^2-1)^2}$$

$$y'_x = -\frac{t(t-2)}{(t-1)^2}\cdot\frac{(t^2-1)^2}{(t^2+1)} = -\frac{t(t-2)\cancel{(t-1)^2}(t+1)^2}{\cancel{(t-1)^2}(t^2+1)} =$$

$$= \frac{(2t-t^2)(t+1)^2}{t^2+1} = \frac{(2t^2+2t-t^3-t^2)(t+1)}{t^2+1} =$$

$$= \frac{\cancel{t^3}+2t^2-t^4+2t^2-t^2+2t-\cancel{t^3}}{t^2+1} = \frac{-t^4+3t^2+2t}{t^2+1}$$

## Логарифмирование

$$y' = f(x)\cdot(\ln f(x))'$$

N3. $y = (x^2+2)^5\cdot(3x-x^3)^3$

$$y' = (x^2+2)^5(3x-x^3)^3\cdot\left(\frac{5}{x^2+2}\cdot 2x + \frac{3}{3x-x^3}\cdot(3-3x^2)\right) =$$

$$= (x^2+2)^5(3x-x^3)^3\cdot\left(\frac{10x}{x^2+2} + \frac{9(1-x^2)}{3x-x^3}\right) =$$

$$= (x^2+2)^{\cancel{5}\,4}(3x-x^3)^{\cancel{3}\,2}\cdot\frac{10x(3x-x^3)+(9-9x^2)(x^2+2)}{\cancel{(x^2+2)(3x-x^3)}} =$$

1 вар. $= (x^2+2)^{\cancel{5}\,4}(3x-x^3)^2\cdot(30x^2-10x^4+9x^2+18-9x^4-18x^2) =$

$= (x^2+2)^4(3x-x^3)^2(-19x^4+21x^2+18)$

2 вар. $= (x^2+2)^4(3x-x^3)^3\cdot 10x + 9(1-x^2)(x^2+2)^5(3x-x^3)^2$

что соответствует рассчетам в Python

N4. $y = x^x$

$y' = x^x\cdot(\ln x^x)' =$

$= (e^{\ln x^x})' = (e^{x\ln x})' = e^{x\ln x}\cdot(x\ln x)' = e^{x\ln x}\cdot\left(1\cdot\ln x + x\cdot\frac{1}{x}\right) =$

$= x^x\cdot(\ln x+1)$

N 4 второй способ

$y = x^x \to$ пролог-ем $\ln y = \ln x^x \to \ln y = x \ln x$

$(\ln y)'_x = (x \ln x)'_x$

$\frac{1}{y} \cdot y' = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$\frac{y'}{y} = \ln x + 1$

$y' = y(\ln x + 1)$ , $y = x^x$

$y' = x^x(\ln x + 1)$

N 5 $y = \frac{(2-x^2)^3 \cdot (x-1)^2}{(2x^3-3x)(e^x)}$

$y' = \frac{(2-x^2)^3 \cdot (x-1)^2}{(2x^3-3x) \cdot e^x} \cdot \left(\frac{3}{2-x^2} \cdot (-2x) + \frac{2}{x-1} - \frac{1}{2x^3-3x} \cdot (6x^2-3) - 1\right) =$

$\left(\ln \frac{1}{e^x}\right)' = (\ln e^{-x})' = (-x)' \cancel{\ln e} = -1$

$= \frac{(2-x^2)^3 \cdot (x-1)^2}{(2x^3-3x) \cdot e^x} \cdot \left(\frac{-6x}{2-x^2} + \frac{2}{x-1} - \frac{6x^2-3}{2x^3-3x} - 1\right)$

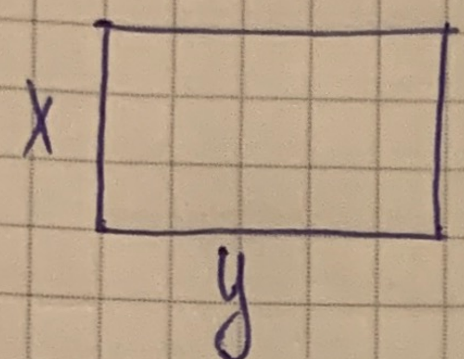
№6* $y = \operatorname{arctg} x \rightarrow x = \operatorname{tg} y$ $\qquad (\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$y'_x = \frac{1}{x'_y}$

$x'_y = (\operatorname{tg} y)' = \frac{1}{\cos^2 y} = 1 + \operatorname{tg}^2 y = 1 + \operatorname{tg}^2(\operatorname{arctg}(x)) = 1 + x^2$

$y'_x = \frac{1}{1+x^2}$

№7.




$S_{max}$ при $P = 144$ см

$x, y$ - ?

$S = x * y$

$P = 2x + 2y$

$2x + 2y = 144$

$x + y = 72$

$x = 72 - y$ $\qquad x = 72 - 36 = 36$

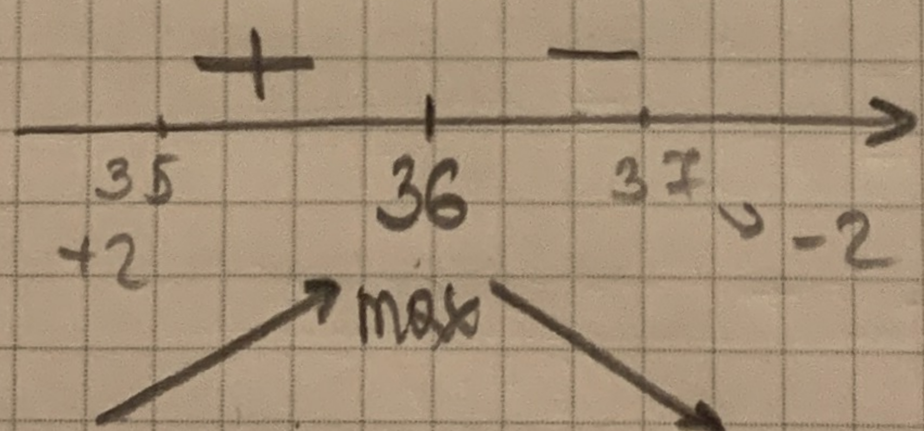
$S = (72 - y)y = 72y - y^2$

$S' = 72 - 2y$

$72 - 2y = 0$

$72 = 2y$

$y = 36$




Максимальное значение ~~сторон~~ площади будет достигнуто при $x = y = 36$ см, $S_{max} = 1296$ см$^2$