Stat 142 MP Light 1

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Item 1

Write a general method function `taylor_swift` that would solve and return the answer computed from a specific Taylor Series expansion of a value x, given n, such that instead of summing up to infinity, you will only sum up to n. This function should accept at least three arguments, namely `x`, `n`, and a function named `func`. You may add more arguments as necessary.

```
taylor_swift <- function(x, n, func){

  # Description
  # Computes the Taylor series expansion of a function of interest evaluated at x

  # Parameters
  # x -- value at which to evaluate the approximation
  # n -- number of terms in the expansion
  # func -- a function that gives the expression being repeatedly summed in the expansion

  # Value
  # Returns the approximate numeric value of the function of interest evaluated at x

  sum <- 0 # sum will store the running total of the series
  j <- 0 # j will cycle from 0 to n (the index of the summation)
  while (j <= n) {
    sum <- sum + func(x,j)
    j <- j + 1
  }
  return(sum)
}
```

Item 2

Write a function named `geom` such that when you use it as an argument to `taylor_swift`, it will return the convergence of the Taylor Series expansion of $\frac{1}{1-x}$ up to n. Recall that

$$\frac{1}{1-x} = \sum_{j=0}^{\infty} x^j, \text{ for} - 1 < x < 1$$

```r
geom <- function(x, j){

  # Description
  # Computes the j^th term of the Taylor Series expansion of a geometric series

  # Parameters
  # x -- value at which to evaluate the series
  # j -- index of the term in the series

  # Value
  # Returns the numeric value the j^th term

   return(x^j)
}
```

Item 3

Write a function named `expo` such that when you use it as an argument to `taylor_swift`, it will return the convergence of the Taylor Series expansion of e^x up to n. Recall that

$$e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!}, \text{ for } -\infty < x < \infty$$

```r
expo <- function(x, j){

  # Description
  # Computes the j^th term of the Taylor Series expansion of e^x

  # Parameters
  # x -- value at which to evaluate the series
  # j -- index of the term in the series

  # Value
  # Returns the numeric value of the j^th term

  return(x^j / factorial(j))
}
```

Item 4

Create a control structure (not a function) that would perform the following:

a. Check at which n does `taylor_swift` approximate $\frac{1}{1-x}$, with error less than `1e-3`,if `x=0.5`.

```r
true_geom <- 1 / (1 - 0.5) # true numeric value of 1/(1-x) evaluated at x = 0.5
i = 0 # counter for the loop

repeat {
  if(abs(taylor_swift(x = 0.5, i, geom) - true_geom) < 1e-3){
    cat("n >=", i)
    break
  } else{
    i <- i + 1
  }
}
```

```
## n >= 10
```

This means that using any $n \geq 10$ in `taylor_swift` will produce an approximation of $\frac{1}{1-x}$ with error less than 10^{-3}.

b. Check at which n does `taylor_swift` approximate e^x , with error less than `1e-3`,if `x=1`.

```r
true_expo <- exp(1)  # true numeric value of e^x evaluated at x = 1
i = 0 # counter for the loop

repeat {
  if(abs(taylor_swift(x = 1, i, expo) - true_expo) < 1e-3){
    cat("n >=", i)
    break
  } else{
    i <- i + 1
  }
}
```

```
## n >= 6
```

This means that using any $n \geq 6$ in `taylor_swift` will produce an approximation of e^x with error less than 10^{-3}.

To verify 3b, let us look at the actual values.

```r
cat("e =", exp(1), "\n")
```

```
## e = 2.718282
```

```r
cat("Approximation of e =", taylor_swift(x=1, n=6, expo))
```

```
## Approximation of e = 2.718056
```

Clearly, the true value of e and the approximation of e produced by `taylor_swift` agree up to the third decimal, showing that the error is less than 10^{-3}.

References

GeeksforGeeks. (2025, July 23). *Program to Print a New Line in String.* https://www.geeksforgeeks.org/r-language/r-program-to-print-a-new-line-in-string/

– Used to fix presentation of results in Item 4b (for visual mode on R Markdown)

Hartman, G., et al. (n.d). *8.8: Taylor Series.* LibreTexts. https://math.libretexts.org/Bookshelves/Calculus/Calculus_3e_(Apex)/08%3A_Sequences_and_series/8.08%3A_Taylor_Series

– Used to verify formula for Taylor Series expansion of e^x

OpenAI. (2025, September 1). *ChatGPT (GPT-5) [Large language model].* https://chatgpt.com/share/68b52f95-5a6c-8004-81be-a283794f28b9

– Used to fix presentation of entire document in PDF output (font, divider, indentation)