Pair of Linear Equations in Two Variables

1. A pair of Linear Equations in two variables:

- An equation of the form ax + by + c = 0, where a, b and c are real numbers, such that a and b are not both zero, is called a **linear equation in two variables**.
- Two linear equations in same two variables x and y are called **pair of linear equations in two variables**.

2. The **general form** of a pair of linear equations in two variables is

$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$

where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers, such that $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$

3. A system of linear equations in two variables represents two lines in a plane. For two given lines in a plane there could be three possible cases:

i. The two lines are intersecting, i. e., they **intersect at one point**.
ii. The two lines are **parallel**, i.e., they do not intersect at any real point
iii. The two lines are **coincident** lines, i.e., one line overlaps the other line.

4. A system of simultaneous linear equations is said to be

- **Consistent**, if it has **at least one solution.**
- **In-consistent**, if it has **no solution.**

5. If the lines

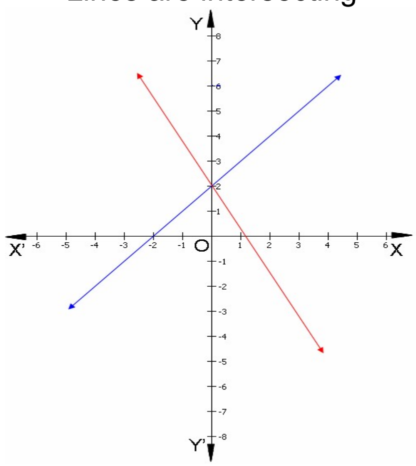
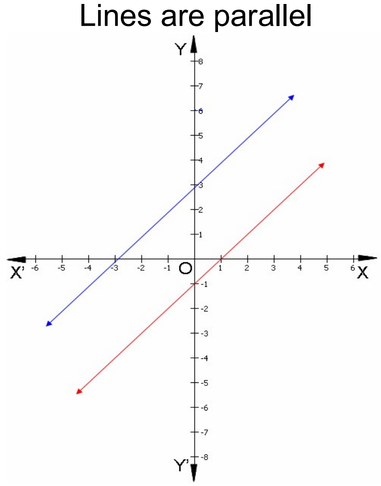
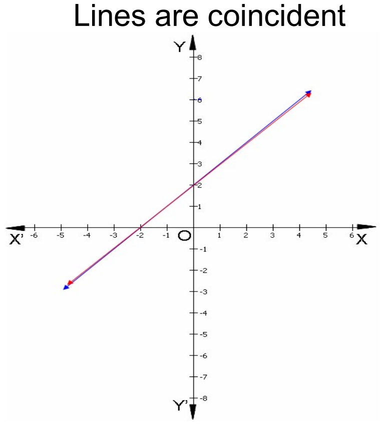
i. Intersect at a point, then that point gives the **unique solution** of the system of equations. In this case system of equations is said to be **consistent**.
ii. Coincide (overlap), then the pair of equations will have **infinitely many solutions**. System of equations is said to be **consistent.**
iii. are parallel, then the pair of equations has **no solution**. In this case pair of equations is said to be **inconsistent.**

6. Solution of a pair of Linear Equations in two variable:
System of equations can be solved using **Algebraic** and **Graphical Methods.**

7. Graphical Method:

- A linear equation in two variables is represented geometrically by a **straight line**.
- The graph of a pair of linear equations in two variables is represented by two lines.
  Steps:
  i. Draw the graphs of both the equations by finding two solutions for each.
  ii. Plot the points and draw the lines passing through them to represent the equations.
  iii. The behaviour of lines representing a pair of linear equations in two variables and the existence of solutions can be summarised as follows:

Anodiam
Education that enlightens!

Ratio of Coefficients	Graphical Representation	Nature of Solution	Defined as
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Lines are intersecting 	Unique solution	Consistent pair of equations
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Lines are parallel 	No solution	Inconsistent pair of equations
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Lines are coincident 	Infinitely many solutions	Dependent (consistent) pair of equations

8. Algebraic Method:

The most commonly used **Algebraic Methods** to solve a pair of linear equations in two variables are:

i. Substitution method
ii. Elimination method
iii. Cross-multiplication method

9. Substitution Method:

Steps followed for solving linear equations in two variables, using **substitution method**:

Step 1: Express the value of one variable, say y in terms of other variable x from either equation, whichever is convenient.

Step 2: Substitute the value of y in other equation and reduce it to an equation in one variable, i.e. in terms of x. There will be three possibilities:

a. If reduced equation is linear in x, then solve it for x to get **a unique solution.**
b. If reduced equation is a true statement without x, then system has **infinite solutions.**
c. If reduced equation is a false statement without x, then system has **no solution.**

Step 3: Substitute the value of x obtained in step 2, in the equation used in step 1, to obtain the value of y.

Step 4: The values of x and y so obtained is the coordinates of the solution of system of equations.

10. Elimination Method:

Steps followed for solving linear equations in two variables, by **elimination Method**:

Step 1: Multiply both the equations by some suitable non-zero constants to make the coefficients of variable x (or y) equal.

Step 2: Add or subtract both the equations to eliminate the variable whose coefficients are equal.

a. If an equation in one variable y (or x) is obtained, solve it for variable y (or x).
b. If a true statement involving no variable is obtained then the system has **infinite solutions**.
c. If a false statement involving no variable is obtained then the system has **no solution**.

Step 3: Substitute the value of variable y (or x) in either of the equation to get the value of other variable**.**

11. Cross Multiplication Method:

Steps followed for solving linear equations in two variables, by **cross multiplication method**:

Step 1: Write the equations in the general form.

$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$

Step 2: Arrange these in the following manner.

$$\frac{x}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \frac{y}{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$$

Here, the arrows between two numbers (coefficients) mean that they are to be multiplied and the second product is to be subtracted from the first product.

Step 3: Cross multiply:

$$\underset{(1)}{\frac{x}{b_1c_2 - b_2c_1}} = \underset{(2)}{\frac{y}{a_2c_1 - a_1c_2}} = \underset{(3)}{\frac{1}{a_1b_2 - a_2b_1}}$$

a. Comparing (1) and (3), we get the value of x

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

b. Comparing (2) and (3), we get the value of y

$$y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

From the above equations, obtain the value of x and y provided $a_1b_2 - a_2b_1 \neq 0$.

12. Equations which are not linear but can be reduced to linear form by some suitable substitutions are called equations reducible to linear form.

Reduced equation can be solved by any of the algebraic method (substitution, elimination or cross multiplication) of solving linear equation.

13. While solving problems based on time, distance and speed; following knowledge may be useful:
If speed of a boat in still water = u km/hr,
Speed of the current = v km/hr
Then,
Speed upstream = (u - v) km/hr
Speed downstream = (u + v) km/hr