No. 38

QUADRATIC EQUATIONS

- **Word Problem Based on Numbers**

Q. Find two numbers whose sum is 27 and product is 182

Sol. Let one of the number be x

$\because$ Sum of two numbers is 27

$\therefore$ Other number is $(27 - x)$

According to the given condition,

$\therefore \quad x\,(27 - x) = 182$

$\therefore \quad 27x - x^2 = 182$

$\therefore \quad 0 = x^2 - 27x + 182$

$\therefore \quad 1x^2 - 27x + 182 = 0$

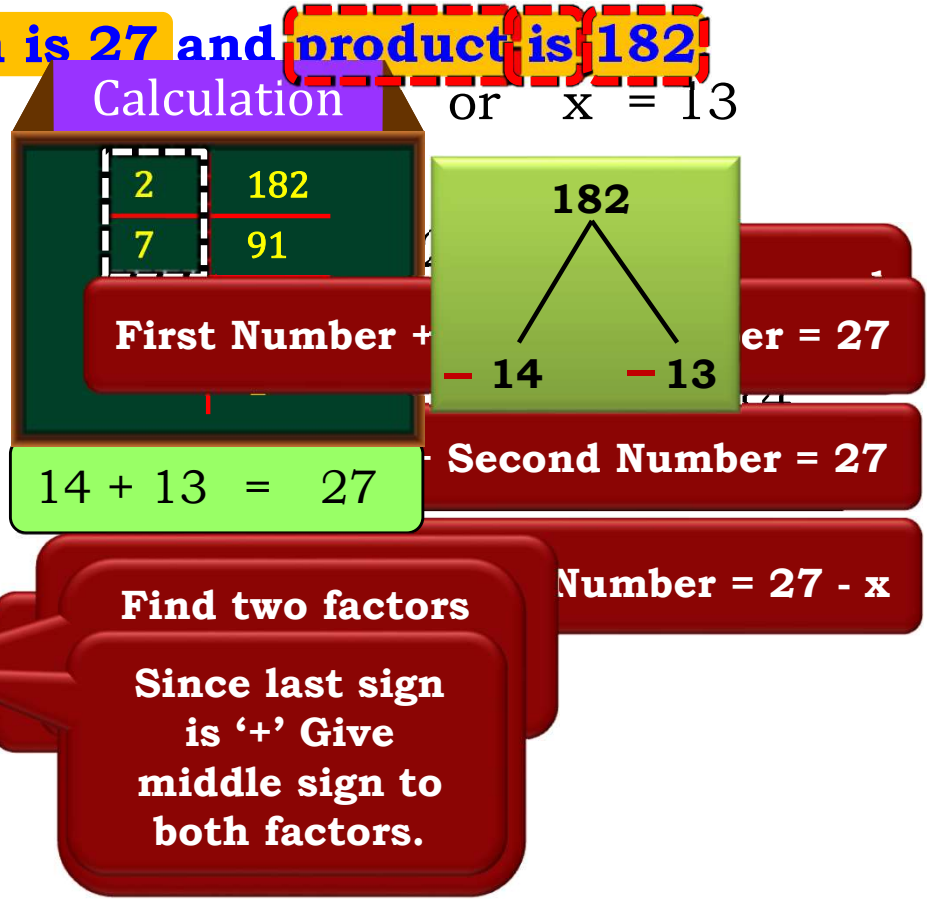
$\therefore \quad x^2 - 14x - 13x + 182 = 0$

$\therefore \quad x\,(x - 14) - 13\,(x - 14) = 0$

$\therefore \quad (x - 14)\,(x - 13) = 0$

$\therefore \quad x - 14 = 0 \quad \text{or} \quad x - 13 = 0$

$\therefore \quad x = 14 \quad \text{or} \quad x = 13$



No. 39

QUADRATIC EQUATIONS

- **Word Problem Based on Numbers**

Q. The sum of the squares of two consecutive positive numbers is 365. Find the numbers.

Sol. Let the two required consecutive p

Means one after the other $x + 1$

As per the given condition,

$$(x)^2 + (x+1)^2 = 365$$

$$\therefore \quad x^2 + x^2 + 2x + 1 = 365$$

$$\therefore \quad 2x^2 + 2x + 1 - 365 = 0$$

$$\therefore \quad 2x^2 + 2x - 364 = 0$$

Dividing throughout by 2

$$\therefore \quad 1x^2 + 1x - 182 = 0$$

$$\therefore \quad x^2 + 14x - 13x - 182 = 0$$

$$\therefore \quad x(x + 14) - 13(x + 14) = 0$$

$$\therefore \quad (x + 14)(x - 13) = 0$$

$$\therefore \quad x + 14 = 0 \text{ or } x - 13 = 0$$

$$\therefore \quad x = -14 \text{ or } x = 13$$

$\because$ The required numbers are natural numbers

$\therefore$ $x \neq -14$ Hence, $x = 13$

$\therefore$ $x + 1 = 13 + 1 = 14$

$\therefore$ The natu

What we have t in this sum

182

+ 14 − 13

cul

182 × 1 = 182

gn is '-' gn to the bigger factor & opposite sign to smaller factor.

182
91
13
1

14 - 13 = 1

No. 40

QUADRATIC EQUATIONS

- **Word Problem Based on Numbers**

Q. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Sol :

Let larger number be x

$(\text{Smaller no.})^2 = 8x$

$\therefore$ Smaller no $= \sqrt{8x}$

As per the given condition

$x^2 - 8x =$

$\therefore \quad 1x^2 - 8x - 180 =$

$\therefore \quad x^2 - 18x + 10x - 180 =$

$\therefore \quad x(x - 18) + 10(x - 18) = 0$

$\therefore \quad (x - 18)(x + 10) = 0$

$\therefore \quad (x - 18) = 0 \text{ or } (x + 10) = 0$

$\therefore \quad x = 18 \text{ or } x = -10$

(Larger no.)^2 – (smaller no.)^2 = 180

x^2 – 8x = 180

In a comparative statement which comes later

18 – 10 = 8

180 × 1 = 180

'–' sign means subtracting

Since Give middle sign to the bigger factor & opposite sign to smaller factor.

180

– 18 **+ 10**

er no is 18 and ller no is 12

culation

	180
2	90
3	45
3	15
5	5
	1

18 - 10 = 8

No. 41

QUADRATIC EQUATIONS

- **Word problem based on marbles**

Q.] Solve the following problems.

(i) John and Jivanti together have 45 marbles. Both ... marbles each, and the p... is 124. We would like to fi... h.

Sol : Let the no. of marbles ...

Total no...

∴ No. of m...

After los...
John ha...

Jivanti ...

Accordi...

(x –

∴ $40x - 5x = 124$

∴ $-x \; 24 = 0$

∴ $x^2 - 45x + 324 = 0$

middle no.

Select the bigger number

From last two '9' ... mon along ... d term sign

first two 'x' ... is common

Try all combinations with the bigger number

Find product of 3rd no. with 1st no.

Find two factors of 324 in such a way that by adding factors we get middle no.

Since, last sign is + Give middle sign to both the factors

Now signs to b... given to both

... e by

$324 \times 1 = 324$

	162
3	81
3	27
3	9
3	3

324

(3×3) (2×2×3×3)

–9 –36

45

When $x = 36 \Rightarrow 45 - x = 45 - 36 = 9$

When $x = 9 \Rightarrow 45 - x = 45 - 9 = 36$

∴ **Either John had 36 marbles and Jivanti had 9 marbles or vice - versa.**

No. 42

QUADRATIC EQUATIONS

- **Word Problem based on consecutive numbers**

WORD PROBLEMS BASED ON CONSECUTIVE NUMBERS

How do we express 6 in terms of 4 ?	5, 6	5, 5+1	x, x+1
3 Consecutive Numbers	5, 6, 7	5, 5+1, 5+2	x, x+1, x+2
4 Consecutive Numbers	5, 6, 7, 8	5, 5+1, 5+2, 5+3	x, x+1, x+2, x+3
2 Consecutive ODD Numbers	3, 5	3, 3+2	x, x+2
How do we express 5 in terms of 3 ?	4, 6	4, 4+2	x , x+2

Q) The sum of the squares of two consecutive natural numbers is 113. Find the numbers.

Sol. Let the two consecutive

As per the given conditi

$(x)^2 + (x+1)^2 = 113$

$\therefore \quad x^2 + x^2 + 2x + 1 = 113$

$\therefore \quad 2x^2 + 2x + 1 - 113 = 0$

$\therefore \quad 2x^2 + 2x - 112 = 0$

Dividing throughout by 2

$\therefore \quad 1x^2 + x - 56 = 0$

$\therefore \quad x^2 + 8x - 7x - 56 = 0$

$\therefore \quad x(x + 8) - 7(x + 8) = 0$

$\therefore \quad (x + 8)(x - 7) = 0$

Since we are subtracting the factors give middle term sign to

What we need to find ?

opposite sign to the smaller factor

8 – 7 = 1

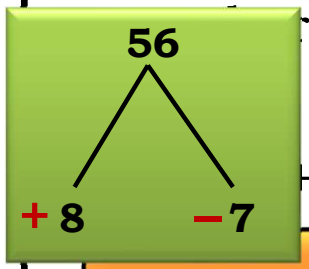
x and x + 1

rs are natural

rs

Hence, x = 7

+ 1 = 7 + 1 = 8



$\therefore$

The two required consecutive natural numbers are 7 and 8.

No. 43

QUADRATIC EQUATIONS

- **Word Problem based on two consecutive even natural numbers**

Q The sum of the squares of two consecutive even natural numbers is 100. Find the numbers.

Sol. Let the two consecutive e… d $x+2$.

As per the given condition,

$(x)^2 + (x+2)^2 = 100$

$\therefore x^2 + x^2 + 4x + 4 = 100$

$\therefore 2x^2 + 4x + 4 - 100 = 0$

$\therefore 2x^2 + 4x - 96 = 0$

(Dividing throughout by 2)

$\therefore 1x^2 + 2x - 48 = 0$

$\therefore x^2 + 8x - 6x - 48 = 0$

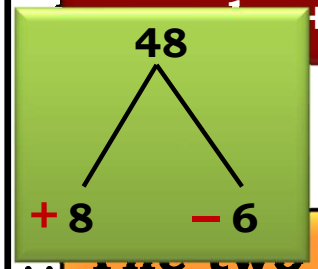
$\therefore x(x + 8) - 6(x + 8) = 0$

$\therefore (x + 8)(x - 6) = 0$

Since we are subtracting the factors give middle term sign to the bigger factor and the opposite sign to the smaller factor

$\because$ The required numbers are natural

… + 2



Hence, x = 6

… 2 = 6 + 2 = 8

… required consecutive even natural numbers are 6 and 8.

No. 44

QUADRATIC EQUATIONS

- **Word Problem based on three consecutive odd natural numbers**

(Q) Three consecutive odd natural numbers are such that the product of the first and third is greater than four times the middle by 1. Find the numbers.

What we need to find ?

How to assume three consecutive odd natural numbers ?

Sol.

Let the three required consecutive odd natural numbers be x, $x + 2$ and $x + 4$.

x, x + 2 and x + 4

As per the given condition,

$x \times (x + 4) = 4 \times (x + 2) + 1$

$\therefore \quad x^2 + 4x = 4x + 8 + 1$

$\therefore \quad x^2 = 9$

Taking square root on both the sides we get,

$\therefore \quad x = \pm 3$

$\because$ The required numbers are natural numbers.

$\therefore \quad x \neq -3$ Hence $x = 3$

$\therefore \quad x + 2 = 3 + 2 = 5$

$\therefore \quad x + 4 = 3 + 4 = 7$

$\therefore$ **The three consecutive odd natural numbers are 3, 5 and 7 respectively.**

No. 45

QUADRATIC EQUATIONS

- **Word Problem based on five consecutive numbers**

(Q) **The sum of squares of five natural consecutive numbers is 1455. Find the numbers.**

Sol. Let the five required consecutive natural numbers be x, $x + 1$, $x + 2$, $x + 3$ and $x + 4$.

As per the given condition,

$(x)^2 + (x + 1)^2 + (x + 2)^2 + (x + 3)^2$

$\therefore\ x^2 + x^2 + 2x + 1 + x^2 + 4x +$

$\therefore\ x^2 + x^2 + x^2 + x^2 + x^2 + 2x +$

$\therefore\ 5x^2 + 20x - 1425 = 0$

(Dividing throughout by 5)

$\therefore\ 1x^2 + 4x - 285 = 0$

$\therefore\ x^2 - 15x + 19x - 285 = 0$

$\therefore\ x(x - 15) + 19(x - 15) = 0$

$\therefore\ (x - 15)(x + 19) = 0$

$\therefore\ x - 15 = 0$ or $x + 19 = 0$

$\therefore\ x = 15$ or $x = -19$

$(a+b)^2 = a^2 + 2ab + b^2$

consecutive natural numbers ?

sign to the smaller factor

285

+19 −15

Calculation

5	285
3	57
19	19
	1

nce, $x = 15$

$1 = 16$

$2 = 17$

$3 = 18$

$\therefore\ x + 4 = 15 + 4 = 19$

The required five consecutive natural numbers are 15, 16, 17, 18 and 19.

No. 46

QUADRATIC EQUATIONS

- **Word Problem based on four consecutive numbers**

(Q) The product of four consecutive positive integers is 840. Find these numbers.

Sol. Let the four consecutive positive integers be x, $x + 1$, $x + 2$, and $x + 3$.

As per the given condition,

$x \times (x + 1) \times (x + 2) \times (x + 3) = 840$

$\therefore \quad x(x + 3) \times (x + 1)(x + 2) = 840$

$\therefore \quad (x^2 + 3x) \times (x^2 + 2x + x + 2) = 840$

$\therefore \quad (x^2 + 3x)(x^2 + 3x + 2) = 840$

Let, $x^2 + 3x = m$; We get, $m(m + 2) = 840$

$\therefore \quad m^2 + 2m - 840 = 0$

$\therefore \quad m^2 + 30m - 28m - 840 = 0$

$\therefore \quad m(m + 30) - 28(m + 30) = 0$

$\therefore \quad (m + 30)(m - 28) = 0$

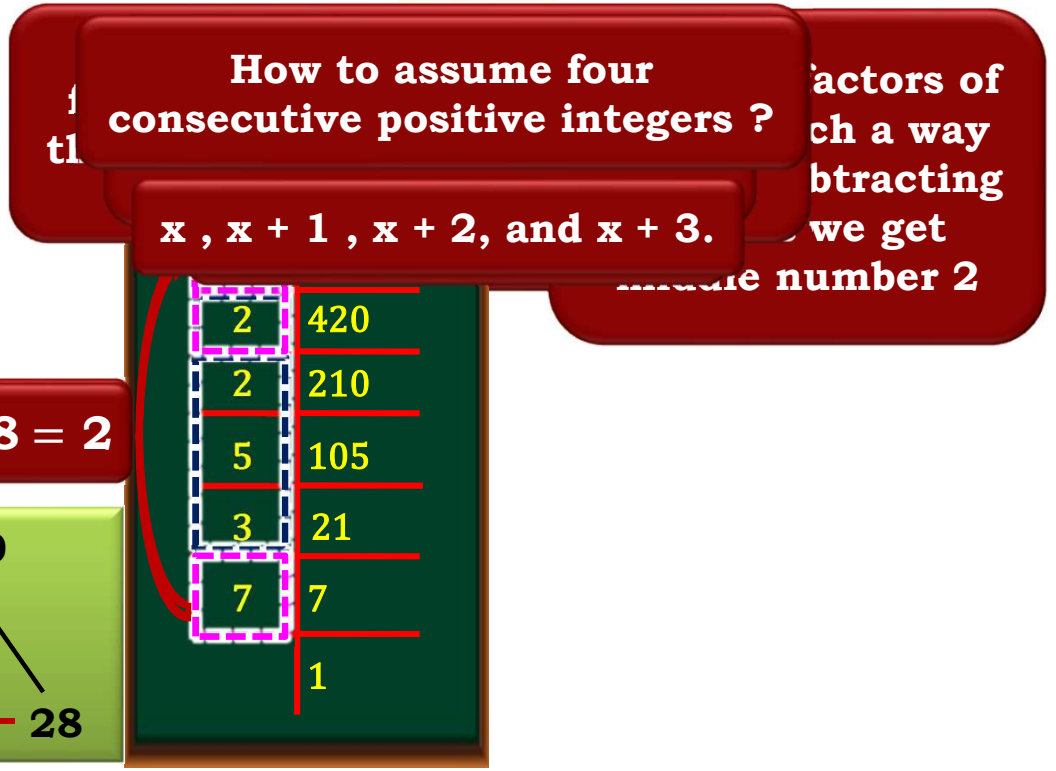
$\therefore \quad m + 30 = 0$ or $m - 28 = 0$

$\therefore \quad m = -30$ or $m = 28$

Resubstituting the value of m we ge

$x^2 + 3x = -30$ .....(i)

or $x^2 + 3x = 28$ .....(ii)



(Q) The product of four consecutive positive integers is 840. Find these numbers.

Sol. Let the four consecutive positive integer be x , x + 1 , x + 2, and x + 3.

As per the given condition,

x(x + 1)(x + 2)(x + 3) = 840

∴ (x² + 3x)(x² + 3x + 2) = 840

Let's write it in standard form

Find two factors of ... middle number 3

Let's write it in standard form

∴ m(m + 2) = 840

∴ m² + 2m = 840

∴ m² + 2m – 840 = 0

∴ m² + 30m – 28m – 840 = 0

∴ m(m + 30) – 28(m + 30) = 0

∴ m + 30 = 0 or m – 28 = 0

∴ m = – 30 or m = 28

Resubstituting the value of m we get,

$x^2 + 3x = -30$ .....(i)

or $x^2 + 3x = 28$ .....(ii)

From (i) $1x^2 + 3x + 30 = 0$

28 → – 4, + 7

7 – 4 = 3

4(1)(30)

20

∵ $b^2 - 4ac < 0$

∴ The roots of the above quadratic equation are not real. Hence not considered.

From (ii), $1x^2 + 3x - 28 = 0$

∴ $x^2 + 7x - 4x - 28 = 0$

∴ $x(x + 7) - 4(x + 7) = 0$

∴ $(x + 7)(x - 4) = 0$

∴ $x + 7 = 0$ or $x - 4 = 0$

∴ $x = -7$ or $x = 4$

The required number is positive,

∴ $x \neq -7$ Hence, $x = 4$

∴ $x + 1 = 4 + 1 = 5$

∴ $x + 2 = 4 + 2 = 6$

∴ $x + 3 = 4 + 3 = 7$

∴ **The four consecutive positive integers are 4, 5, 6 and 7**

Thank You