Lecture 1

CHAPTER NO. 2

POLYNOMIAL

E X P R E S S I O N

P O L Y N O M I A L

Coefficient Coefficient

$2x + 5y$

$0, 1, 2, 3, 4, 5, \ldots$

Variable Variable

✓ $2x^{1} + 5y^{1}$

✓ $2y^{3} + 7x^{2} - y^{1}$

✓ $y^{2} - 17y^{0}$

2

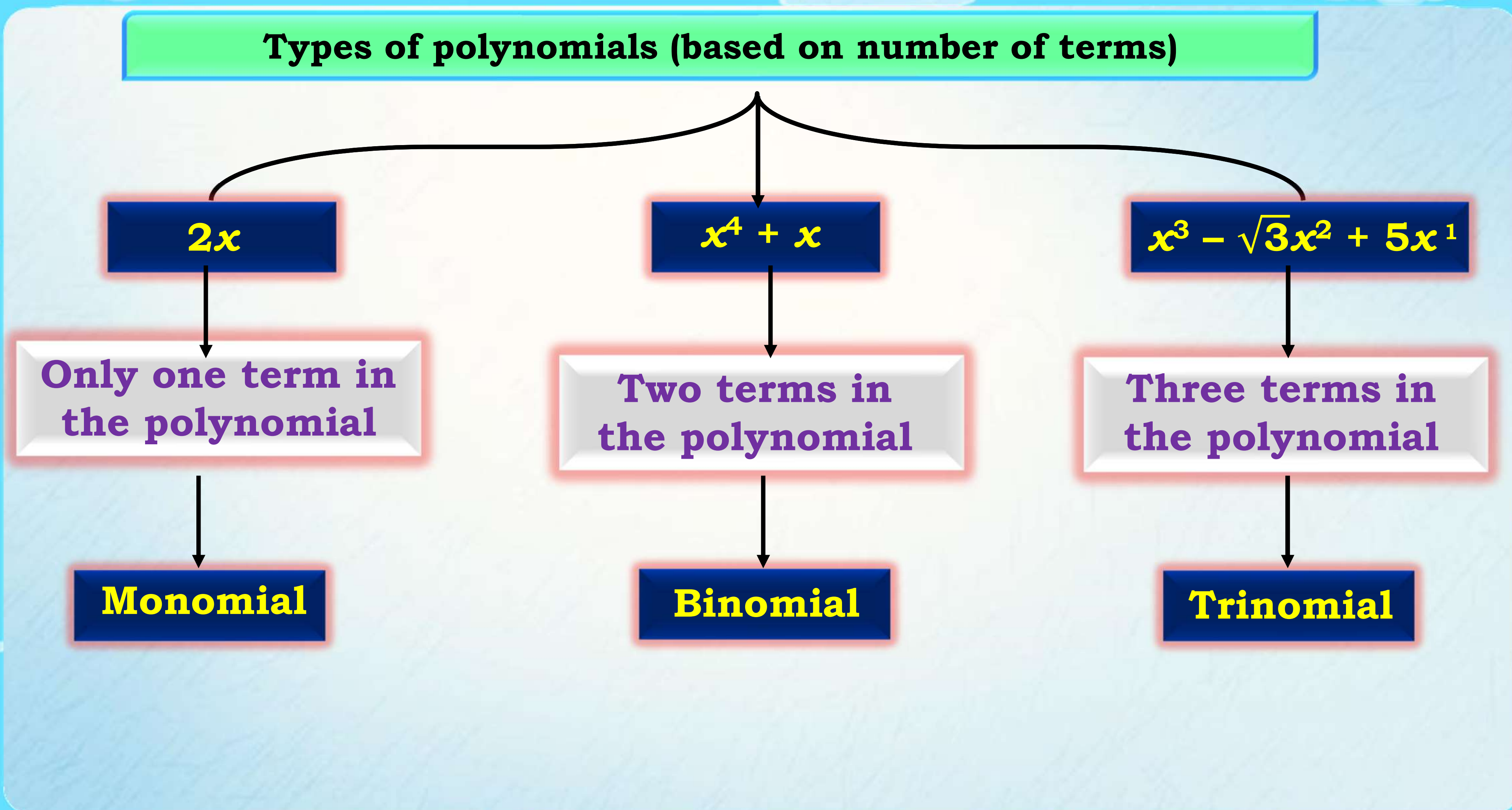
x

5

✗ $3x^{\frac{2}{3}} - x^{2}$

✗ $x^{-1} + 5x^{0}$

Polynomial is an expression in which all the powers of variables are whole numbers.



Types of polynomials (based on number of terms)
$2x$
$x^4 + x$
$x^3 - \sqrt{3}x^2 + 5x^1$
Only one term in the polynomial
Two terms in the polynomial
Three terms in the polynomial
Monomial
Binomial
Trinomial

Types of polynomial (based on degree)

$7y^{1}$ → Degree 1 → Linear polynomial

$2y^{2} + y + 1$ → Degree 2 → Quadratic polynomial

$x^{3} + x^{2} + 2x + 3$ → Degree 3 → Cubic polynomial

EXERCISE 2.1

Q. 1 **Which of the following expressions are polynomials in one variable and which are not ? State reasons for your answer.**

(i) $4x^{2} - 3x^{1} + 7x^{0}$

Sol. In $4x^2 - 3x + 7$, all the powers of x are whole numbers, so it is a polynomial in one variable x.

All the powers are whole numbers

(ii) $y^{2} + \sqrt{2}y^{0}$

Sol. In $y^2 + \sqrt{2}$, the powers of y is a whole number. So it is a polynomial in one variable y.

Since $\frac{1}{2}$ is not a whole number

Powers 2,1 and 0 are whole numbers.

$\sqrt{t}$ can be written as $t^{\frac{1}{2}}$

(iii) $3\sqrt{t} + t\sqrt{2}$

Sol. In $3\sqrt{t} + t\sqrt{2} = 3t^{1/2} + \sqrt{2}t$, here the power of the first term is $\frac{1}{2}$, which is not a whole number. Therefore, it is not a polynomial.

Powers 2 and 0 are whole numbers.

EXERCISE 2.1

Q. 1 Which of the following expressions are polynomials in one variable and which are not ? State reasons for your answer.

(iv) $y + \frac{2}{y}$

Sol. $y + \frac{2}{y}$

$= y + 2y^{-1}$ here the power of the second term is -1, which is not a whole number and so it is not a polynomial.

$\frac{2}{y}$ can be written as $2y^{-1}$

Since $2y^{-1}$ is not a whole number

(v) $x^{10} + y^{3} + t^{50}$

Sol. $x^{10} + y^3 + t^{50}$ is not a polynomial in one variable as three variables x, y, t occur in it.

Powers 10, 3 and 50 are whole numbers

but given polynomials are in three variables

Exercise 2.1

Q. 2 Write the coefficients of x^2 in each of the following :

(i) $2 + x^2 + x$

Sol. Coefficient of x^2 in

$2 + x^2 + x$ is 1

(ii) $2 - x^2 + x^3$

Sol. Coefficient of x^2 in

$2 - x^2 + x^3$ is -1

(iii) $\frac{\pi}{2}x^2 + x$

Coefficient means the number associated with variables

Sol. Coefficient of x^2 in

$\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$

(iv) $\sqrt{2}x - 1$

Sol. Coefficient of x^2 in

$\sqrt{2}x - 1$ is 0

EXERCISE 2.1

Q. 3 **Give one example each of a binomial of degree 35, and of a monomial of degree100.**

Sol. **Binomial of degree 35 may be taken as** $x^{35} - 8$

Monomial of degree 100 may be taken as $3x^{100}$

Monomial means an expression with one term

Binomial means an expression with two terms

EXERCISE 2.1

Q. 4 **Write the degree of each of the following polynomials.**

Highest power of variable t is 1

Highest power of variable y is 2

(i) $5x^3 + 4x^2 + 7x^1$

Sol. The highest power term is $5x^3$ and the power is 3. So, the degree is 3.

Degree means highest power of variable

Highest power of variable x is 3

(ii) $4 - y^2$

Sol. The highest power term is $-y^2$ and the power is 2. So, the degree is 2.

Highest power of variable x is 0

(iii) $5t^1 - \sqrt{7}t^0$

Sol. The highest power term is $5t$ and the power is 1. So, the degree is 1.

(iv) $3x^0$

Sol. The only term, here is 3 which can be written as $3x^0$
So , highest power of x is 0. Therefore, the degree is 0.

EXERCISE 2.1

Q. 5 Classify the following as linear, quadratic and cubic polynomials.

(i) $x^2 + x^1$

Soln. The highest power of $x^2 + x$ is 2, So, it is a quadratic polynomial.

Cubic polynomials means degree of polynomial is 3

Quadratic polynomials means degree of polynomial is 2

(ii) $x^1 - x^3$

Soln. The highest power of $x - x^3$ is 3, So, it is a cubic polynomial.

Linear polynomials means degree of polynomial is 1

Highest power of variable x is 2

So, the Degree of the polynomial is 2

(iii) $y^1 + y^2 + 4$

Soln. The highest power of $y + y^2 + 4$ is 2, So, it is a quadratic polynomial.

Highest power of variable y is 2

So, the Degree of the polynomial is 2

So, the Degree of the polynomial is 3

Highest power of variable x is 3

EXERCISE 2.1

Q. 5 **Classify the following as linear, quadratic and cubic polynomials.**

(iv) $1 + x^{1}$

Sol. The highest power of 'x' is 1 + x is 1,So, it is a linear polynomial.

Highest power of variable t is 1

So, the Degree of the polynomial is 1

Highest power of variable x is 1

(v) $3t^{1}$

Sol. The highest power of t in 3t is 1.So, it is a linear polynomial.

So, the Degree of the polynomial is 2

Highest power of variable r is 2

So, the Degree of the polynomial is 1

(vi) r^{2}

Sol. The highest power of r in r^2 is 2. So, it is a quadratic polynomial.

So, the Degree of the polynomial is 3

Highest power of variable r is 3

(vii) $7x^{3}$

Sol. The highest power of x in x^3 is 3. So, it is a cubic polynomial.

Lecture 2

Value of a polynomial

If $p(x)$ is a polynomial in x and if k is any real number, then the value obtained by putting $x = k$ in $p(x)$, is called the value of the polynomial $p(x)$ at $x = k$.

The value of $p(x)$ at $x = k$ is denoted by $p(k)$

e.g. If $p(x) = x^2 - 4x + 5$

then, $p(4) = (4)^2 - 4(4) + 5$

$= \cancel{16} - \cancel{16} + 5$

$= 5$

Replace x by 4

EXERCISE 2.2

Q. 1 **Find the value of the polynomial $5x - 4x^2 + 3$ at.**

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Sol. Let $p(x) = 5x - 4x^2 + 3$

(i) At $x = 0$, $p(0) = 5(0) - 4(0)^2 + 3$

$= 0 - 0 + 3$

$= 3$

(ii) At $x = -1$, $p(-1) = 5(-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3$

$= -6$

(iii) At $x = 2$, $p(2) = 5(2) - 4(2)^2 + 3$

$= 10 - 16 + 3$

$= -3$

EXERCISE 2.2

Q. 2 **Find p(0), p(1) and p(2) for each of the following polynomials :**

(i) $p(y) = y^2 - y + 1$ (ii) $p(t) = 2 + t + 2t^2 - t^3$

Sol. (i) Let $p(y) = y^2 - y + 1$

$p(0) = 0^2 - 0 + 1 = 1$

$p(1) = 1^2 - 1 + 1 = 1$

$p(2) = 2^2 - 2 + 1 = 3$

For p (2)
Substitute y = 2 in p(y)

For p (0)
Substitute y = 0 in p(y)

For p (1)
Substitute t = 1 in p(t)

(ii) Let $p(t) = 2 + t + 2t^2 - t^3$

$p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2$

$p(1) = 2 + 1 + 2(1)^2 - (1)^3 = 4$

$p(2) = 2 + 2 + 2(2)^2 - (2)^3$

$= 4 + 8 - 8 = 4$

For p (0)
Substitute t = 0 in p(t)

For p (2)
Substitute t = 2 in p(t)

For p (1)
Substitute y = 1 in p(y)

EXERCISE 2.2

Q. 2 **Find p(0), p(1) and p(2) for each of the following polynomials :**

(iii) $p(x) = x^3$ (iv) $p(x) = (x - 1)(x + 1)$

Sol. (iii) Let $p(x) = x^3$

$p(0) = (0)^3 = 0$

$p(1) = (1)^3 = 1$

$p(2) = (2)^3 = 8$

For p (0)
Substitute x = 0 in p(x)

For p (1)
Substitute x = 1 in p(x)

For p (2)
Substitute x = 2 in p(x)

(iv) Let $p(x) = (x - 1)(x + 1)$

$p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$

$p(1) = (1 - 1)(1 + 1) = (0)(2) = 0$

$p(2) = (2 - 1)(2 + 1) = (1)(3) = 3$

For p (0)
Substitute x = 0 in p(x)

For p (1)
Substitute x = 1 in p(x)

For p (2)
Substitute x = 2 in p(x)

ZEROES of the polynomial

$p(x) = x + 2$

for $x = -2$, $p(x) = 0$

ZERO

ZERO value of a polynomial is a number which when Substituted in the polynomial, the value is 0

$q(y) = y - 6$

$\therefore$ 6 is the zero of the polynomial q

y – 6 = 0

y = 6

If we

$r(x) = 2x - 10$

$\therefore$ 5 is the zero of the polynomial r

2x – 10 = 0

If we substitute x = 5 in the polynomial
it become 2(5) - 10
10 – 10
which is 0

EXERCISE 2.2

Q. 3 **Verify whether the following are zeroes of the polynomial, indicated against them.**

(i) $p(x) = 3x + 1, \quad x = -\frac{1}{3}$

Substitute $x = -\frac{1}{3}$ and check if the answer is zero

Sol. $p(x) = 3x + 1; \quad x = -\frac{1}{3}$

$$\therefore p\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{3}\right) + 1$$

$$= \cancel{-1} + \cancel{1}$$

$$= 0$$

$$\therefore \boxed{\frac{-1}{3} \text{ is a zero of } p(x)}$$

EXERCISE 2.2

Q. 3 **Verify whether the following are zeroes of the polynomial, indicated against them.**

(ii) $p(x) = 5x - \pi,\ x = \frac{4}{5}$

Sol. $p(x) = 5x - \pi,\ x = \frac{4}{5}$

$\therefore\ p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi$

$= 4 - \pi \neq 0$

$\therefore\ \boxed{\frac{4}{5} \text{ is not a zero of } p(x)}$

Substitute $x = \frac{4}{5}$ and check if the answer is zero

EXERCISE 2.2

Q. 3 Verify whether the following are zeroes of the polynomial, indicated against them.

(iii) $p(x) = x^2 - 1, \ x = 1, -1$

Substitute $x = -1$ and check if the answer is zero

Substitute $x = 1$ and check if the answer is zero

Sol. $p(x) = x^2 - 1, \ x = 1, -1$

$\therefore \ p(1) = 1^2 - 1 = 0$

$\therefore$ $\boxed{1 \text{ is a zero of } p(x)}$

Also, $p(-1) = (-1)^2 - 1$

$= 1 - 1$

$= 0$

$\therefore$ $\boxed{-1 \text{ is a zero of } p(x)}$

EXERCISE 2.2

Q. 3 **Verify whether the following are zeroes of the polynomial, indicated against them.**

(iv) $p(x) = (x + 1)(x - 2)$, $x = -1, 2$

Sol. $p(x) = (x + 1)(x - 2)$; $x = -1, 2$

Substitute $x = 2$ and check if the answer is zero

$\therefore$ $p(-1) = (-1 + 1)(-1 - 2)$

$= (0)(-3)$

$= 0$

$\therefore$ $\boxed{-1 \text{ is a zero of } p(x).}$

Also, $p(2) = (2 + 1)(2 - 2)$

$= 3(0)$

$= 0$

$\therefore$ $\boxed{2 \text{ is a zero of } p(x).}$

Substitute $x = -1$ and check if the answer is zero

EXERCISE 2.2

Q. 3 **Verify whether the following are zeroes of the polynomial, indicated against them.**

(v) $p(x) = x^2, x = 0$

Sol. $p(x) = x^2, x = 0$

$\therefore$ $p(0) = 0^2 = 0$

$\therefore$ $\boxed{0 \text{ is a zero of } p(x).}$

Substitute $x = 0$ and check if the answer is zero

EXERCISE 2.2

Q. 3 **Verify whether the following are zeroes of the polynomial, indicated against them.**

(vi) $p(x) = lx + m, \ x = -\frac{m}{l}$

Sol. $p(x) = lx + m; \ x = -\frac{m}{l}$

$\therefore \quad p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m$

Substitute $x = -\frac{m}{l}$ in $p(x)$

$= -\cancel{m} + \cancel{m}$

$= 0$

$\therefore \quad \boxed{-\frac{m}{l} \text{ is a zero of } p(x)}$

EXERCISE 2.2

Q. 3 **Verify whether the following are zeroes of the polynomial, indicated against them.**

(vii) $p(x) = 3x^2 - 1, \quad x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Sol. $p(x) = 3x^2 - 1, \quad x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Substitute $x = -\frac{1}{\sqrt{3}}$ in $p(x)$

$$\therefore \quad p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1$$

$$= \cancel{3}\left(\frac{1}{\cancel{3}}\right) - 1$$

$$= 1 - 1$$

$$= 0$$

$\therefore$ $\boxed{-\frac{1}{\sqrt{3}} \text{ is a zero of } p(x)}$

Substitute $x = \frac{2}{\sqrt{3}}$ in $p(x)$

$$\text{Also } p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1$$

$$= 4 - 1$$

$$= 3$$

$$\neq 0$$

$\therefore$ $\boxed{\frac{2}{\sqrt{3}} \text{ is not a zero of } p(x)}$

EXERCISE 2.2

Q. 3 **Verify whether the following are zeroes of the polynomial, indicated against them.**

(vii) $p(x) = 2x + 1, \ x = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ and check if the answer is zero

Sol. $p(x) = 2x + 1; \ x = \frac{1}{2}$

$\therefore \ p\left(\frac{1}{2}\right) = \cancel{2}\left(\frac{1}{\cancel{2}}\right) + 1$

$= 1 + 1$

$= 2$

$\neq 0$

$\therefore$ $\boxed{\frac{1}{2} \text{ is not a zero of } p(x).}$

Exercise 2.2

Q. 4 **Find the zero of the polynomial in each of the following cases.**

(i) $p(x) = x + 5$

Soln. We have to solve, $p(x) = 0$

$\therefore x + 5 = 0$

$\therefore x = -5$

$\therefore$ $\boxed{-5 \text{ is a zero of the polynomial } x + 5}$

EXERCISE 2.2

Q. 4 **Find the zero of the polynomial in each of the following cases.**

(ii) $p(x) = x - 5$

Sol. We have to solve, $p(x) = 0$

$\therefore \quad x - 5 = 0$

$\therefore \quad x = 5$

$\therefore$ $\boxed{\text{5 is a zero of the polynomial } x - 5}$

To find zero of polynomial we need to solve $p(x) = 0$

Exercise 2.2

Q. 4 **Find the zero of the polynomial in each of the following cases.**

(iii) $p(x) = 2x + 5$

Sol. We have to solve, $p(x) = 0$

To find zero of polynomial we need to solve $p(x) = 0$

$\therefore \quad 2x + 5 = 0$

$\therefore \quad x = \dfrac{-5}{2}$

$\therefore \quad \boxed{\dfrac{-5}{2} \text{ is a zero of the polynomial } x + 5}$

EXERCISE 2.2

Q. 4 **Find the zero of the polynomial in each of the following cases.**

(iv) $p(x) = 3x - 2$

To find zero of polynomial we need to solve $p(x) = 0$

Sol. We have to solve, $p(x) = 0$

$\therefore \quad 3x - 2 = 0$

$\therefore \quad 3x = 2$

$\therefore \quad x = \frac{2}{3}$

$\therefore$ $\frac{2}{3}$ is a zero of the polynomial $3x - 2$

EXERCISE 2.2

Q. 4 **Find the zero of the polynomial in each of the following cases.**

(v) $p(x) = 3x$

Sol. We have to solve, $p(x) = 0$

To find zero of polynomial we need to solve $p(x) = 0$

$\therefore \quad 3x = 0$

$\therefore \quad x = 0$

$\therefore$ $\boxed{0 \text{ is a zero of the polynomial } 3x}$

EXERCISE 2.2

Q. 4 **Find the zero of the polynomial in each of the following cases.**

(vi) $p(x) = ax,\ a \neq 0$

Sol. We have to solve, $p(x) = 0$

$$\therefore \quad ax = 0$$

$$\therefore \quad x = \frac{0}{a}$$

$$\therefore \quad x = 0$$

$\therefore$ **0 is a zero of the polynomial ax.**

EXERCISE 2.2

Q. 4 **Find the zero of the polynomial in each of the following cases.**

(vii) $p(x) = cx + d$, $c \neq 0$, c, d are real numbers.

Sol. We have to solve, $p(x) = 0$

$\therefore \quad cx + d = 0$

$\therefore \quad x = -\dfrac{d}{c}$

$\therefore$ $\boxed{-\dfrac{d}{c} \text{ is a zero of the polynomial } cx + d.}$

Lecture 3

Remainder Theorem: Let p(x) be any polynomial of degree greater than or equal to one and let a be any real number. If p(x) is divided by the linear polynomial x – a, then the remainder is p(a).

EXERCISE 2.3

Q. 1 **Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by**

(i) $x + 1$

Sol: $p(x) = x^3 + 3x^2 + 3x + 1$

$x + 1 = 0$

$x = -1$

(i) Divisor $= x + 1$

Substitute $x = -1$ in $p(x)$

$\therefore$ **By Remainder theorem,**

Remainder $= p(-1)$

$= (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$= \cancel{-1} + \cancel{3} - \cancel{3} + \cancel{1}$

$= \boxed{0}$

EXERCISE 2.3

Q. 1 **Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by**

(iii) $x - \frac{1}{2}$

$x - \frac{1}{2} = 0$

$x = \frac{1}{2}$

$x + \pi = 0$

$\therefore \quad x = -\pi$

Sol: $p(x) = x^3 + 3x^2 + 3x + 1$

Divisor $= x - \frac{1}{2}$

$\therefore$ By Remainder theorem,

$$\text{Remainder} = p\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1$$

$$= \frac{1 + 6 + 12 + 8}{8}$$

$$= \boxed{\frac{27}{8}}$$

EXERCISE 2.3

Q. 1 **Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by**

(iv) $x + \pi$

$x + \pi = 0$

$\therefore \quad x = -\pi$

Sol: $p(x) = x^3 + 3x^2 + 3x + 1$

Divisor $= x + \pi$

Substitute $x = -\pi$ in $p(x)$

$\therefore$ **By Remainder theorem,**

Remainder $= p(-\pi)$

$= (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$= \boxed{-\pi^3 + 3\pi^2 - 3\pi + 1}$

EXERCISE 2.3

Q. 1 Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(v) $5 + 2x$

Sol: $p(x) = x^3 + 3x^2 + 3x + 1$

Divisor $= 5 + 2x$

$\therefore$ By Remainder theorem,

$$\text{Remainder} = p\left(\frac{-5}{2}\right)$$

$$= \left(\frac{-5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 3$$

$$= \frac{-125}{8} + \frac{75}{2} - \frac{15}{2} + 1$$

$$= \frac{-125 + 125 + 60 + 8}{8}$$

$$= \boxed{\frac{-27}{8}}$$

$5 + 2x = 0$

$\therefore \quad 5 = -2$

$\therefore \quad \frac{5}{-2} = x$

Substitute $x = \frac{5}{-2}$ in $p(x)$

EXERCISE 2.3

Q. 2 **Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.**

Sol: $p(x) = x^3 - ax^2 + 6x - a$

Divisor $= x - a$

If $x - a = 0$
$x = a$

Substitute $x = a$ in $p(x)$

$\therefore$ By Remainder theorem,

$$p(\mathrm{a}) = (a)^3 + a(a)^2 + 6(\mathrm{a}) - a$$

$$= a^3 - a^3 + 6a - a$$

$$= \boxed{5a}$$

EXERCISE 2.3

Q. 3 **Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.**

Sol: $p(x) = 3x^3 + 7x$

Divisor $= 7 + 3x$

$\therefore$ By Remainder theorem,

$$\text{Remainder} = p\left(\frac{-7}{3}\right)$$

$$= 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right)$$

$$= 3\left(\frac{-343}{27}\right) - \frac{49}{3}$$

$$= \frac{-343}{9} - \frac{49}{3} \times \frac{3}{3}$$

$$= \frac{-343 - 147}{4}$$

$$= \frac{-490}{9} \neq 0$$

Since Remainder $\neq 0$

$\therefore$ $\boxed{7 + 3x \text{ is not a factor of } 3x^3 + 7x.}$

Lecture 4

Factor Theorem : If $p(x)$ is a polynomial of degree $n > 1$ and a is any real number, then

(i) $x - a$ is a factor of $p(x)$, if $p(a) = 0$,

and

(ii) $p(a) = 0$, if $x - a$ is a factor of $p(x)$.

Exercise 2.4

Q. 1 **Determine which of the following polynomials has $(x + 1)$ a factor :**

(i) $x^3 + x^2 + x + 1$

$x + 1 = 0$

$x = -1$

Substitute $x = -1$ in $p(x)$

Sol. $p(x) = x^3 + x^2 + x + 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$p(-1) = \cancel{-1} + \cancel{1} \cancel{-1} + \cancel{1}$

$p(-1) = 0$

If the value of $p(-1)$ is zero , then we say that $(x + 1)$ is a factor of $p(x)$

$\therefore$ $\boxed{(x + 1) \text{ is a factor of } p(x).}$

Exercise 2.4

Q. 1 **Determine which of the following polynomials has $(x + 1)$ a factor :**

(ii) $x^4 + x^3 + x^2 + x + 1$

$x + 1 = 0$
$x = -1$

Sol. $p(x) = x^4 + x^3 + x^2 + x + 1$

Substitute $x = -1$ in $p(x)$

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

If the value of p(–1) is zero , then we say that $(x + 1)$ is a factor of $p(x)$

$p(-1) = \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + 1$

$p(-1) = 1$

$p(-1) \neq 0$

$\therefore$ $\boxed{(x + 1) \text{ is not a factor of } p(x).}$

Exercise 2.4

Q. 1 **Determine which of the following polynomials has $(x + 1)$ a factor :**

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Sol. $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$p(-1) = \boxed{(-1)^4} + \boxed{3(-1)^3} + \boxed{3(-1)^2} + (\cancel{-1}) + \cancel{1}$

$p(-1) = 1 \cancel{-3} + \cancel{3} + 0$

$p(-1) = 1 \neq 0$

$\therefore$ $\boxed{(x + 1) \text{ is not a factor of } p(x).}$

If the value of p(–1) is zero, then we say that $(x + 1)$ is a factor of $p(x)$

Substitute $x = -1$ in $p(x)$

$x + 1 = 0$

$x = -1$

EXERCISE 2.4

Q. 1 **Determine which of the following polynomials has $(x + 1)$ a factor :**

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Sol. $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

$p(-1) = (-1)^3 - (-1)^2 - (-1)(2 + \sqrt{2}) + \sqrt{2}$

$p(-1) = -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$

$p(-1) = 2\sqrt{2} \neq 0$

$\therefore$ $\boxed{(x + 1) \text{ is not a factor of } p(x).}$

Substitute $x = -1$ in p(x)

If the value of p(−1) is zero , then we say that $(x + 1)$ is a factor of $p(x)$

$x + 1 = 0$

$x = -1$

EXERCISE 2.4

Q. 2 Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = 2x^3 + x^2 - 2x - 1,\ g(x) = x + 1$

If $p(-1) = 0$ by factor theorem we conclude $g(x)$ factor is $p(x)$

Sol. $p(x) = 2x^3 + x^2 - 2x - 1,\ g(x) = x + 1$

To find $p(-1)$
Substitute $x = -1$ is $p(x)$

$$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$$

$$= -2 + 1 + 2 - 1$$

$$= 0$$

$\therefore$ $\boxed{g(x) \text{ is a factor of } p(x).}$

To determine weather $g(x)$ is a factor of $p(x)$

Solve $g(x) = 0$

i.e $x + 1 = 0$

$\therefore$ $x = -1$

EXERCISE 2.4

Q. 2 **Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases :**

(ii) $p(x) = x^3 + 3x^2 + 3x + 1,\ g(x) = x + 2$

Sol. $p(x) = x^3 + 3x^2 + 3x + 1,\ g(x) = x + 2$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -1$

$\neq 0$

$\therefore$ g(x) is not a factor of p(x).

Solve $g(x) = 0$
i.e $x + 2 = 0$
$\therefore\ x = -2$

To find $p(-2)$
Substitute $x = -2$ is p(x)

To determine weather g(x) is a factor of p(x)

If $p(-2) = 0$ by factor theorem we conclude g(x) factor is p(x)

EXERCISE 2.4

Q. 2 Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases :

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

Sol. $p(x) = x^3 - 4x^2 + x + 6 \quad g(x) = x - 3$

$p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 9$

$= 0$

$\therefore$ $\boxed{g(x) \text{ is a factor of } p(x).}$

To find p(3)
Substitute $x = 3$ is $p(x)$

To determine weather $g(x)$ is a factor of $p(x)$

If $p(-3) = 0$ by factor theorem we conclude $g(x)$ factor is $p(x)$

Solve $g(x) = 0$

i.e $x - 3 = 0$

$\therefore \quad x = 3$

Lecture 5

EXERCISE 2.4

Q. 3 **Find the value of k, if x – 1 is a factor of p(x) in each of the following cases :**

(i) $p(x) = x^2 + x + k$

Sol. Since, $(x - 1)$ is a factor of $p(x)$ by factor theorem

$$p(x) = x^2 + x + k$$

For finding p(1) substitute $x = 1$ in $p(x)$

$$\therefore \quad p(1) = 0$$

$$\therefore \quad 1^2 + 1 + k = 0$$

$$\therefore \quad \boxed{k = -2}$$

EXERCISE 2.4

Q. 3 **Find the value of k, if x – 1 is a factor of p(x) in each of the following cases :**

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

For finding p(1) substitute $x = 1$ in $p(x)$

Sol. Since, $(x - 1)$ is a factor of p(x) by factor theorem

$$p(x) = 2x^2 + kx + \sqrt{2}$$

$$(1) = 0$$

$$\therefore \quad 2(1)^2 + k(1) + \sqrt{2} = 0$$

$$\therefore \quad 2 + k + \sqrt{2} = 0$$

$$\therefore \quad \boxed{k = -(2 + \sqrt{2})}$$

EXERCISE 2.4

Q. 3 **Find the value of k, if x – 1 is a factor of p(x) in each of the following cases :**

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

Sol. Since, $(x - 1)$ is a factor of $p(x)$ by factor theorem

$\therefore \quad p(1) = 0$

$p(x) = kx^2 - \sqrt{2}x + 1$

For finding p(1) substitute $x = 1$ in $p(x)$

$\therefore \quad k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\therefore \quad k - \sqrt{2} + 1 = 0$

$\therefore \quad \boxed{k = \sqrt{2} - 1}$

Exercise 2.4

Q. 3 **Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases :**

(iv) $p(x) = kx^2 - 3x + k$

Sol. Since, $(x - 1)$ is a factor of $p(x)$ by factor theorem

For finding $p(1)$ substitute $x = 1$ in $p(x)$

$\therefore \quad p(1) = 0$

$p(x) = kx^2 - 3x + k$

$\therefore \quad k(1)^2 - 3(1) + k = 0$

$\therefore \quad k - 3 + k = 0$

$\therefore \quad 2k - 3 = 0$

$\therefore \quad 2k = 3$

$\therefore \quad \boxed{k = \frac{3}{2}}$

EXERCISE 2.4

Q. 4 **Factories :**

(i) $12x^2 - 7x + 1$

Sol. $12x^2 - 7x + 1$

$12 \times 1 = 12 = 4 \times 3$

$1 + 12 \neq 7$

$-4 + -3 = 7$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= \boxed{(3x - 1)(4x - 1)}$

EXERCISE 2.4

Q. 4 **Factories :**

(ii) $2x^2 + 7x + 3$

Sol. $2x^2 + 7x + 3$

$= 2x^2 + 6x + 1x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= \boxed{(x + 3)(2x + 1)}$

$3 \times 2 = 6 \quad = 6 \times 1$

$3 + 2 \neq 7$

$+6 + +1 = 7$

EXERCISE 2.4

Q. 4 **Factories :**

(iii) $6x^2 + 5x - 6$

Sol. $\boxed{6}x^2 + \boxed{5x} - \boxed{6}$

$= 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= \boxed{(2x + 3)(3x - 2)}$

$6 \times 6 = 36 = 9 \times 4$

$6 \quad - \quad 6 \neq 5$

$+9 \quad - -4 = 5$

EXERCISE 2.4

Q. 4 **Factories :**

(iv) $3x^2 - x - 4$

Sol. $\boxed{3}x^2 - \boxed{x} - \boxed{4}$

$4 \times 3 = 12$

$-4 - +3 = 1$

$= \underline{3x^2 - 4x} + \underline{3x - 4}$

$= x\underline{(3x - 4)} + 1\underline{(3x - 4)}$

$= \boxed{(3x - 4)(x + 1)}$

Lecture 6

EXERCISE 2.4

Q. 5 **Factorise :**

(i) $x^3 - 2x^2 - x + 2$

Sol. $\underline{x^3 - 2x^2} - \underline{x + 2}$

$= x^2\boxed{(x - 2)} - 1\boxed{(x - 2)}$

$= (x - 2)(x^2 - 1)$

$= \boxed{(x - 2)(x + 1)(x - 1)}$

Since there are 4 terms and the coefficient of alternate terms are same let us remove common from first two terms and then next two terms

EXERCISE 2.4

Q. 5 **Factorise :**

(iv) $2y^3 + y^2 - 2y - 1$

Since are 4 terms let us remove common from first two terms and then next two terms

Sol. Let $p(y) = \underline{2y^3 + y^2} - \underline{2y - 1}$

$= y^2\boxed{(2y + 1)} - 1\boxed{(2y + 1)}$

$= (2y + 1)(y^2 - 1)$

$\therefore \quad \boxed{2y^3 + y^2 - 2y - 1 = (2y + 1)(y + 1)(y - 1)}$

EXERCISE 2.4

Q. 5 **Factorise :**

(ii) $x^3 - 3x^2 - 9x - 5$

Sol. Let $p(x) =$

The consta

$\therefore$ The possibl

Now, $p(-1)$

Rough work :

$\frac{\cancel{x^3}^{\,2}}{\cancel{x}} = x^2$ (Divisor) $= x^3 + x^2$

$\frac{-4\cancel{x^2}}{\cancel{x}} = -4x$ (Divisor) $= -4x^2 - 4x$

$\frac{-5\cancel{x}}{\cancel{x}} = -5$ (Divisor) $= -5x - 5$

For x +
P – 1 should be a zero.

$= 0$

In this case first we check if x + 1 is a factor of p(x), if not so Then we check weather x – 1 is a factor p(x)

$\therefore$ $(x + 1)$ is a factor of $p(x)$.

To obtain the second factor, divide $p(x)$ by $(x + 1)$

$$
\begin{array}{r|l}
 & x^2 - 4x - 5 \\
x + 1 & x^3 - 3x^2 - 9x - 5 \\
 & \underline{x^3 + x^2} \\
 & -4x^2 - 9x \\
 & \underline{-4x^2 - 4x} \\
 & -5x - 5 \\
 & \underline{-5x - 5} \\
 & 0
\end{array}
$$

$\therefore$ Second factor $= x^2 - 4x - 5$

$= x^2 + x - 5x - 5$

$= x(x + 1) - 5(x + 1)$

$= (x + 1)(x - 5)$

$\therefore$ $p(x) = (x + 1)(x + 1)(x - 5)$

$$\boxed{x^3 - 9x^2 - 9x - 5 = (x + 1)(x + 1)(x - 5)}$$

EXERCISE 2.4

Q. 5 Factorise :

(iii) $x^3 + 13x^2 + 32x + 20$

Sol. Let $p(x) = x$

The constant

$\therefore$ The possible f

Now, $p(-1) = ($

For $x + 1$ to be a factor of $p(x)$
P – 1 should be a zero.

Rough work :

$\frac{\cancel{x^3}^{2}}{\cancel{x}} = x^2$ (Divisor) $= x^3 + x^2$

$\frac{12x^{\cancel{2}}}{\cancel{x}} = 12x$ (Divisor) $= 12x^2 + 12x$

$\frac{20\cancel{x}}{\cancel{x}} = 20$ (Divisor) $= 20x + 20$

$\therefore$ In this case first we check if $x + 1$ is a factor of $p(x)$, if not so Then we check weather $x - 1$ is a factor $p(x)$

To obtain the second factor, divide $p(x)$ by $(x + 1)$

$\therefore$ Second factor $= x^2 + 12x + 20$

$= \underline{x^2 + 10x} + \underline{2x + 20}$

$= x\,\underline{(x + 10)} + 2\,\underline{(x + 10)}$

$= (x + 10)(x + 2)$

$\therefore$ $p(x) = (x + 1)(x + 10)(x + 2)$

$$\boxed{x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 10)(x + 2)}$$

$$\begin{array}{r|l} & x^2 + 12x + 20 \\ \hline x + 1 & \cancel{x^3} + 13x^2 + 32x + 20 \\ & \underset{-}{\cancel{x^3}} \underset{-}{+} x^2 \\ \hline & \cancel{12x^2} + 32x \\ & \underset{-}{\cancel{12x^2}} \underset{-}{+} 12x \\ \hline & \cancel{20x} + 20 \\ & \underset{-}{\cancel{20x}} \underset{-}{+} 20 \\ \hline & 0 \end{array}$$

Lecture 7

EXERCISE 2.5

Q. 1 **Use suitable identities to find the following products :**

> **Use identity** $(x + a)(x + b) = x^2 + (a + b)x + ab$

(i) $(x + 4)(x + 10)$

Sol. $(\underset{x}{x} + \underset{a}{4})(\underset{x}{x} + \underset{b}{10})$

$= x^2 + (4 + 10)\,x + 4 \times 10$

$= \boxed{x^2 + 14x + 10}$

> **Use identity** $(x + a)(x + b) = x^2 + (a + b)x + ab$

(ii) $(x + 8)(x - 10)$

Sol. $(\underset{x}{x} + \underset{a}{8})(\underset{x}{x} - \underset{b}{10})$

$= x^2 + (8 + (-10))\,x + 8 \times (-10)$

$= \boxed{x^2 - 2x - 80}$

> **Use identity** $(x + a)(x + b) = x^2 + (a + b)x + ab$

(iii) $(3x + 4)(3x - 5)$

Sol. $(\underset{x}{3x} + \underset{a}{4})(\underset{x}{3x} - \underset{b}{5})$

$= 3x^2 + (4 + (-5))\,3x + 4 \times (-5)$

$= \boxed{3x^2 - 3x - 80}$

EXERCISE 2.5

Q. 1 **Use suitable identities to find the following products :**

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

Use identity $(a + b)(a - b) = a^2 - b^2$

Sol. $\left(\underbrace{y^2}_{a} + \underbrace{\frac{3}{2}}_{b}\right)\left(\underbrace{y^2}_{a} - \underbrace{\frac{3}{2}}_{b}\right) = (y^2)^2 - \left(\frac{3}{2}\right)^2$

$= \boxed{y^4 - \frac{9}{4}}$

(v) $(3 - 2x)(3 + 2x)$

Sol. $(\underbrace{3}_{a} - \underbrace{2x}_{b})(\underbrace{3}_{a} + \underbrace{2x}_{b}) = \overset{(3)^2}{a^2} - \overset{(2x)^2}{b^2}$

$= \boxed{9 - 4x^2}$

Use identity $(a + b)(a - b) = a^2 - b^2$

EXERCISE 2.5

Q. 2 **Evaluate the following products without multiplying directly :**

(i) 103 × 107

Soln. $103 \times 107 = (\underset{x}{100} + \underset{a}{3})(\underset{x}{100} + \underset{b}{7})$

$= 100^2 + (3 + 7)100 + 3 \times 7$

$= 10000 + 1000 + 21$

$= \boxed{\mathbf{11021}}$

(ii) 95 × 96

Soln. $95 \times 96 = (\underset{x}{100} - \underset{a}{5})(\underset{x}{100} - \underset{b}{4})$

$= 100^2 + (a + b)100 + 5 \times 4$

$= 10000 - 900 + 20$

$= \boxed{\mathbf{9120}}$

EXERCISE 2.5

Q. 2 **Evaluate the following products without multiplying directly :**

(iii) 104×96

Soln. $104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - (4)^2$

$= 10000 - 16$

$= \boxed{9984}$

EXERCISE 2.5

Q. 3 Factorise the following using appropriate identities :

(i) $9x^2 + 6xy + y^2$

$$a^2 + 2ab + b^2 = (a + b)^2$$

Soln. $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x + y)^2$

$= \boxed{(3x + y)\ (3x + y)}$

(ii) $4y^2 - 4y + 1$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Soln. $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)^2$

$= \boxed{(2y - 1)\ (2y - 1)}$

Exercise 2.5

Q. 3 **Factorise the following using appropriate identities :**

(iii) $x^2 - \dfrac{y^2}{100}$

Soln. $x^2 - \dfrac{y^2}{100} = \underbrace{(x)^2}_{a} - \underbrace{\left(\dfrac{y}{10}\right)^2}_{b}$

$$= \boxed{\left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)}$$

Lecture 8

EXERCISE 2.5

Q. 4 Expand each of the following using suitable identities :

(i) $(x + 2y + 4z)^2$

Soln. $(x + 2y + 4z)^2$

We know,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

$$(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(x)(4z)$$

$$= \boxed{x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx}$$

(iii) $(-2x + 3y + 2z)^2$

Soln. $(-2x + 3y + 2z)^2 = [(-2x) + 3y + 2z]^2$

We know,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

$$[(-2x) + 3y + 2z]^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(-2x)(2z)$$

$$= \boxed{4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx}$$

EXERCISE 2.5

Q. 4 **Expand each of the following using suitable identities :**

(ii) $(2x - y + z)^2$

Soln. $(2x - y + z)^2$

We know,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$[2x + (-y) + z]^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$$

$$= \boxed{4x^2 + y^2 + z^2 - 4xy + 2yz - 4zx}$$

EXERCISE 2.5

Q. 4 Expand each of the following using suitable identities :

(iv) $(3a - 7b - c)^2$

Soln. $(3a - 7b - c)^2$

We know,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$[3a + (-7b) + (-c)]^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$$

$$= \boxed{9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ab}$$

EXERCISE 2.5

Q. 4 **Expand each of the following using suitable identities :**

(v) $(-2x + 5y - 3z)^2$

Soln. $(-2x + 5y - 3z)^2 = [(-2x) + 5y + (-3z)]^2$

We know,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$[(-2x) + 5y + (-3z)]^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2 \quad (\quad) + 2 (\quad) + 2$

$= \boxed{4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx}$

EXERCISE 2.5

Q. 4 **Expand each of the following using suitable identities :**

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Soln. $$\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2 = \left[\frac{1}{4}a + \left(-\frac{1}{2}b\right) + 1\right]^2$$

We know,

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$$

$$\left[\frac{1}{4}a + \left(-\frac{1}{2}b\right) + 1\right]^2 = \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\quad\right) \quad + 2 \quad (\quad) + 2(\quad)\left(\quad\right)$$

$$= \boxed{\frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a}$$

EXERCISE 2.5

Q. 5 **Factorise :**

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

Soln. $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= [2x + 3y + (-4z)]^2$

$= (2x + 3y - 4z)^2$

$= \mathbf{(2x + 3y - 4z)(2x + 3y - 4z)}$

The

It is in form of a^2, b^2 and c^2

i.e. $(a + b + c)^2$

Lets arrange these terms as

d 2ca

It is in the form of

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

EXERCISE 2.5

Q. 5 **Factorise :**

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

Soln. $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

$= (\sqrt{2}x)^2 + (-y)^2 + (-2\sqrt{2}z)^2 + 2(\sqrt{2}x)(-y) + 2(-y)(-2\sqrt{2}z) + 2(-2\sqrt{2}z)(\sqrt{2}x)$

$= [\sqrt{2}x + (-y)\,(-2\sqrt{2}z)]^2$

$= (\sqrt{2}x - y - 2\sqrt{2}z)^2$

Lets arrange these terms as 2ab, 2bc and 2ca

i.e. $(a + b + c)^2$

Lecture 9

EXERCISE 2.5

Q. 6 **Write the following cubes in expanded form :**

(i) $(2x + 1)^3$

It is in the form of $(a + b)^3$

a = 2x and b = 1

Soln.

$(2x + 1)^3 = (2x)^3 + 3(2x)(1)(2x + 1) + (1)^3$

$[\because (a + b)^3 = (a)^3 + 3ab(a + b) + b^3]$

$= 8x^3 + 6x(2x + 1) + 1$

$= \mathbf{8x^3 + 12x^2 + 6x + 1}$

EXERCISE 2.5

Q. 6 **Write the following cubes in expanded form :**

(ii) $(2a - 3b)^3$

x = 2a and y = 3b

It is in the form of
$(x - y)^3$

Soln.

$$(2a - 3b)^3 = (2a)^3 - 3(2a)(3b)(2a - 3b) - (3b)^3$$

$$[\because (x - y)^3 = (x)^3 - 3xy(x - y) - y^3]$$

$$= 8a^3 - 18ab(2a - 3b) - 27b^3$$

$$= \mathbf{8a^3 - 36a^2b + 54ab^2 - 27b^3}$$

EXERCISE 2.5

Q. 6 **Write the following cubes in expanded form :**

(iii) $\left[\frac{3}{2}x + 1\right]^3$

$a = \left(\frac{3}{2}\right)x$ and $b = 1$

It is in the form of $(a + b)^3$

Soln.

$$\left[\frac{3}{2}x + 1\right]^3 = \left(\frac{3}{2}x\right)^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x + 1\right) + (1)^3$$

$[\because (a + b)^3 = (a)^3 + 3ab(a + b) + b^3]$

$$= \frac{27}{8}x^3 + \frac{9x}{2}\left(\frac{3}{2}x + 1\right) + 1$$

$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$$

EXERCISE 2.5

Q. 6 **Write the following cubes in expanded form :**

(iv) $\left[x - \frac{2}{3}y\right]^3$

a = x and b = $\left(\frac{2}{3}\right)$y

It is in the form of $(a - b)^3$

Soln.

$$\left[x - \frac{2}{3}y\right]^3 = x^3 - 3\,(x)\left(\frac{2}{3}y\right)\left(x - \frac{2}{3}y\right) - \left(\frac{2}{3}y\right)^3$$

$[\because (a-b)^3 = (a)^3 - 3ab(a - b) - b^3]$

$$= x^3 - 2xy\left(x - \frac{2}{3}y\right) - \frac{8}{27}y^3$$

$$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$$

EXERCISE 2.5

Q. 7 **Evaluate the following using suitable identities :**

(i) $(99)^3$

$(a - b)^3 = a^3 - 3ab(a - b) - b^3$

a = 100 and b = 1

Soln. $(99)^3 = (100 - 1)^3$

$= (100)^3 - 3(100)(1)(100 - 1) - (1)^3$

$= 1000000 - 300(99) - 1$

$= 1000000 - 29700 - 1$

$= \boxed{\mathbf{970299}}$

EXERCISE 2.5

Q. 7 **Evaluate the following using suitable identities :**

(ii) $(102)^3$

Soln. $(102)^3 = (100 + 2)^3$

It is in the form of $(a + b)^3$

a = 100 and b = 2

$(a + b)^3 = a^3 + 3ab(a + b) + b^3$

$= (100)^3 + 3(100)(2)(100 + 2) + (2)^3$

$= 1000000 + 600(102) + 8$

$= 1000000 + 61200 + 8$

$= \boxed{1061208}$

EXERCISE 2.5

Q. 7 **Evaluate the following using suitable identities :**

Lets

a = 1000 and b = 2

00

(iii) $(998)^3$

Soln. $(998)^3 = (1000 - 2)^3$

It is in the form of (a –

$(a - b)^3 = a^3 - 3ab(a - b) - b^3$

$= (1000)^3 - 3(1000)(2)(1000 - 2) - (2)^3$

$= 1000000000 - 6000(998) - 8$

$= 1000000000 - 5988000 - 8$

$= \boxed{\mathbf{994011992}}$

EXERCISE 2.5

Q. 8 **Factorise each of the following :**

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

Soln: $8a^3 + b^3 + 12a^2b + 6ab^2$

$= x^3 + 3x^2y + 3xy^2 + y^3$

$= (2a)^3 + 3(4a^2)(b) + 3(2a)(b^2) + (b)^3$

$= (2a)^3 + 3(2a)^2 b + 3(2a)(b)^2 + (b)^3$

$= (2a + b)^3$

$= \boxed{(2a + b)(2a + b)(2a + b)}$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

Soln: $8a^3 - b^3 - 12a^2b + 6ab^2$

$= x^3 - 3x^2y + 3xy^2 - y^3$

$= (2a)^3 - 3(4a^2)(b) - 3(2a)(b^2) - (b)^3$

$= (2a)^3 - 3(2a)^2 b - 3(2a)(b)^2 - (b)^3$

$= (2a - b)^3$

$= \boxed{(2a - b)(2a - b)(2a - b)}$

Polynomial has 4 terms and highest power of 3

(i) $x^3 + 3x^2y + 3xy^2 + y^3 = (x + y)^3$

(ii) $x^3 - 3x^2y + 3xy^2 - y^3 = (x - y)^3$

All positive → 1st

Some negative → 2nd

EXERCISE 2.5

Q. 8 **Factorise each of the following :**

(iii) $27 - 125a^3 - 135ab^2 + 225a^2b$

Soln: $27 - 125a^3 - 135ab^2 + 225a^2$

$= x^3 - 3x^2y + 3xy^2 - y^3$

$= (3)^3 - 3\,(9)\,(5a) + 3\,(3)\,(25a^2) - (5a)^3$

$= (3)^3 - 3\,(3)^2\,(5a) + 3\,(3)\,(5a)^2 - (5a)^3$

$= (3 - 5a)^3$

$= \boxed{(3 - 5a)(3 - 5a)(3 - 5a)}$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

Soln: $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= x^3 - 3x^2y + 3xy^2 - y^3$

$= (4a)^3 - 3\,(16a^2)\,(3b) + 3\,(4a)\,(9b^2) - (3b)^3$

$= (4a)^3 - 3\,(4a)^2\,(3b) + 3\,(4a)\,(3b)^2 - (3b)^3$

$= (4a - 3b)^3$

$= \boxed{(4a - 3b)(4a - 3b)(4a - 3b)}$

Polynomial has 4 terms and highest power of 3

$x^3 + 3x^2y + 3xy^2 + y^3 = (x + y)^3$

$x^3 - 3x^2y + 3xy^2 - y^3 = (x - y)^3$

Some negative → 2nd

EXERCISE 2.5

Q. 8 **Factorise each of the following :**

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Soln: $27p^3 - \frac{1}{216} - \frac{9}{2}qp^2 + \frac{1}{4}pq^2$

$= x^3 - 3x^2y + 3xy^2 - y^3$

$= (3p)^3 - 3(3p)^2\left(\frac{1}{6}\right) + 3(3p)\left(\frac{1}{6}\right)^2 - \left(\frac{1}{6}\right)^3$

$= \left(3p - \frac{1}{6}\right)^3$

$= \boxed{\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)}$

Polynomial has 4 terms and highest power of 3

(i) $x^3 + 3x^2y + 3xy^2 + y^3 = (x + y)^3$

✓(ii) $x^3 - 3x^2y + 3xy^2 - y^3 = (x - y)^3$

Some negative → 2^{nd}

Lecture 10

EXERCISE 2.5

Q. 9 **Verify :**

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Soln: RHS $= (x + y)(x^2 - xy + y^2)$

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - \cancel{x^2y} + \cancel{xy^2} + \cancel{x^2y} - \cancel{xy^2} + y^3$

$= x^3 + y^3$

$=$ LHS

$\therefore$ **LHS = RHS ...... Hence verified**

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Soln: RHS $= (x - y)(x^2 + xy + y^2)$

$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + \cancel{x^2y} + \cancel{xy^2} - \cancel{x^2y} - \cancel{xy^2} - y^3$

$= x^3 - y^3$

$=$ LHS

$\therefore$ **LHS = RHS ...... Hence verified**

EXERCISE 2.5

Q. 10 **Factorise each of the following :**

$$a^3 + b^3 = (a + b)\,[(a)^2 - (ab) + (b)^2]$$

(i) $27y^3 + 125z^3$

Soln: $27y^3 + 125z^3 = \underbrace{(3y)}_{a}{}^3 + \underbrace{(5z)}_{b}{}^3$

$= (3y + 5z)\,[(3y)^2 - (3y)(5z) + (5z)^2]$

$\boxed{= (3y + 5z)\,(9y^2 - 15yz + 25z^2)}$

$$a^3 - b^3 = (a - b)\,[(a)^2 + (ab) + (b)^2]$$

(ii) $64m^3 - 343n^3 = \underbrace{(4m)}_{a}{}^3 - \underbrace{(7n)}_{b}{}^3$

$= (4m + 7n)\,[(4m)^2 + (4m)(7n) + (7n)^2]$

$\boxed{= (4m - 7n)\,(16m^2 + 28mn + 49n^2)}$

EXERCISE 2.5

Q. 11 **Factorise : $27x^3 + y^3 + z^3 - 9xyz$.**

Soln: $27x^3 + y^3 + z^3 - 9xyz$

a b c

$= (3x)^3 + (y)^3 + (z)^3 - 3\,(3x)\,(y)\,(z)$

$= (3x + y + z)\,[(3x)^2 + (y)^2 + (z)^2 - (3x)(y) - (y)(z) - (z)(3x)]$

$= \boxed{(3x + y + z)\,(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)}$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)\,[a^2 + b^2 + c^2 - ab - bc - ca]$

Exercise 2.5

Q. 12 Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Soln: RHS $= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$a^2 - 2ab + b^2 = (a - b)^2$

$= \frac{1}{2}(x + y + z)\ [x^2 - 2xy + y^2 + y^2 - 2yz + z^2 + z^2 - 2zx + x^2]$

$= \frac{1}{2}(x + y + z)\ [2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx]$

$a^2 - 2ab + b^2 = (a - b)^2$

$= \frac{1}{\cancel{2}}(x + y + z)\ \cancel{2}[x^2 + y^2 + z^2 - xy - yz - zx]$

$a^2 - 2ab + b^2 = (a - b)^2$

$= (x + y + z)\ [x^2 + y^2 + z^2 - xy - yz - zx]$

$= x^3 + y^3 + z^3 - 3xyz =$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$\therefore$ LHS = RHS .... Hence verified

Lecture 11

EXERCISE 2.5

Q. 13 **If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.**

Soln: $x + y + z = 0$

We know,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$$

$$\therefore\ x^3 + y^3 + z^3 - 3xyz = (0)\ (x^2 + y^2 + z^2 - xy - yz - xz)$$

$$\therefore\ x^3 + y^3 + z^3 - 3xyz = (0)$$

$$\therefore\ x^3 + y^3 + z^3 = 3xyz$$

$\therefore$ $\boldsymbol{x^3 + y^3 + z^3 = 3xyz}$ **....... Hence proved.**

EXERCISE 2.5

Q. 14 **Without actually calculating the cubes, find the value of each of the following :**

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Soln. **(i) $(-12)^3 + (7)^3 + (5)^3$**

Let $x = -12, \quad y = 7, \quad z = 5$

$\therefore \quad x + y + z = -12 + 7 + 5$

$x + y + z = 0$

We know

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)\ (x^2 + y^2 + z^2)$

$\because \quad x + y + z = 0$

We get,

$x^3 + y^3 + z^3 - 3xyz = 0$

$\therefore \quad x^3 + y^3 + z^3 = 3xyz$

Resubstituting the values of x, y, z

$\therefore \quad (-12)^3 + (7)^3 + (5)^3 = 3(-12)\ (7)(5)$

$= \mathbf{-1260}$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Let $x = 28, \quad y = -15, \quad z = -13$

$\therefore \quad x + y + z = 28 - 15 - 13$

$x + y + z = 0$

We know

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)\ (x^2 + y^2 + z^2)$

$\because \quad x + y + z = 0$

We get,

$x^3 + y^3 + z^3 - 3xyz = 0$

$\therefore \quad x^3 + y^3 + z^3 = 3xyz$

Resubstituting the values of x, y, z

$\therefore \quad (28)^3 + (-15)^3 + (-13)^3 = 3\ (28)\ (-15)\ (-13)$

$= \mathbf{16380}$

EXERCISE 2.5

Q. 15 Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given :

(i) **Area : $25a^2 - 35a + 12$**

Soln. Area of a rectangle = $l \times b$

$25a^2 - 35a + 12 = 25a^2 - 15a - 2$

$12 \times 25 = 300 = 15 \times 20$

$12 + 25 \neq 35$

$-15 + -20 = 35$

$= 7y\ (5a - 3) - 4\ (5a - 3)$

$= \boxed{(5a - 3)\ (5a - 4)}$

$\therefore$ **Possible length and breadth are (5a – 3) & (5a – 4) units**

EXERCISE 2.5

Q. 15 Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given :

(i) **Area : $35y^2 + 13y - 12$**

Soln. Area of a rectangle = $l \times b$

$35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$

$12 \times 35 = 420 = 15 \times 28$

$12 - 35 \neq 13$

$+28 - -15 = 13$

$= 7y\,(5y + 4) - 3\,(5y + 4)$

$= \boxed{(5y + 4)\,(7y - 3)}$

$\therefore$ Possible length and breadth are $(5y + 4)$ & $(7y - 3)$ units

EXERCISE 2.5

Q. 16 **What are the possible expressions for the dimensions of the cuboids whose volumes are given below ?**

(i) Volume : $3x^2 - 12x$ **(ii) Volume : $12ky^2 + 8ky - 20k$**

Soln. (i) Volume of cuboid $= 3x^2 - 12x$

$l \times b \times h = 3x(x - 4)$

$= 3 \times x \times (x - 4)$

$\therefore$ **Possible dimensions of cuboid are 3, x and (x – 4) units.**

(ii) Volume of cuboid $= 12ky^2 + 8ky - 20k$

$l \times b \times h = 4k(\boxed{3}y^2 + \boxed{2y} - \boxed{5})$

$= 4k(3y^2 + 5y - 3y - 5)$

$= 4k[y(3y + 5) - 1(3y + 5)]$

$= 4k(3y + 5)(y - 1)$

$3 \times 5 = 15$ $= 1 \times 15$

$1 - 15 \neq 15$

$-3 + 5 = 15$

$\therefore$ **Possible dimensions of cuboid are 4k, 3y + 5 and y – 1 units.**

Thank You