

Real Numbers

1. Euclid's Division Lemma:

Given positive integers a and b , there exists unique integers q and r satisfying
 $a = bq + r$, where $0 \leq r < b$

➤ **Lemma** is a proven statement used for proving another statement.

2. Euclid's Division Algorithm:

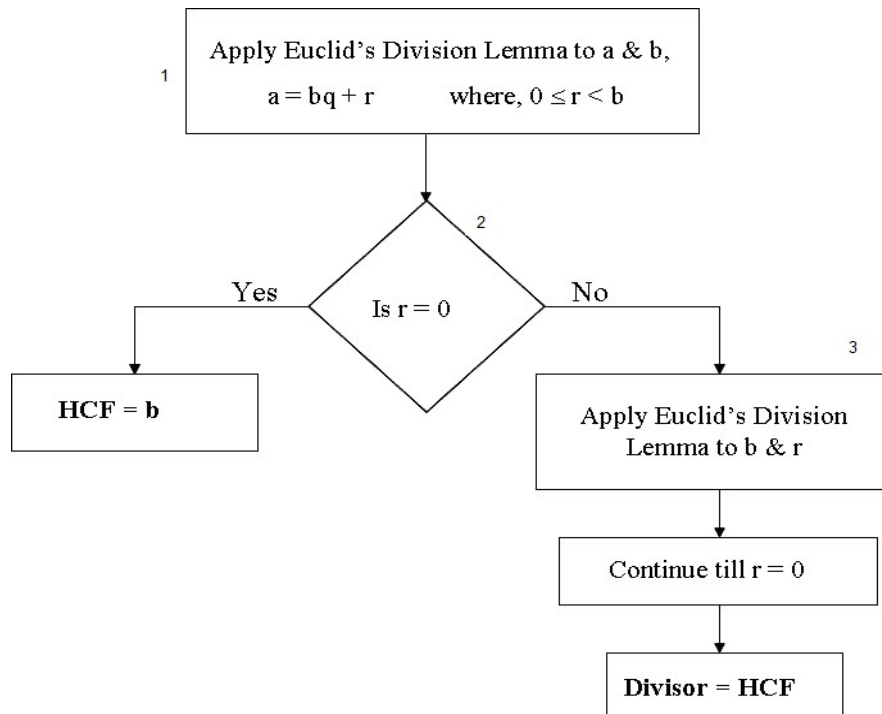
- An **algorithm** is a series of well defined steps which gives a procedure for solving a type of problem.
- This algorithm is a technique to compute the **H.C.F** of two given positive integers.
- According to this algorithm, the **HCF** of any two positive integers ' a ' and ' b ', with $a > b$, is obtained by following the steps given below:

Step 1: Apply Euclid's division lemma, to ' a ' and ' b ', to find q and r , such that $a = bq + r$, $0 \leq r < b$.

Step 2: If $r = 0$, the HCF is b . If $r \neq 0$, apply Euclid's division lemma to b and r .

Step 3: Continue the process till the remainder is zero. The divisor at this stage will be HCF (a , b).
Also, note that $\text{HCF}(a, b) = \text{HCF}(b, r)$.

Euclid's Division Algorithm can be summarized as follows:



- Euclid's Division Algorithm is stated for only positive integers but it can be extended for all integers except zero, i.e., $b \neq 0$.

3. Real Numbers:

- The numbers which can be represented in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called **Rational numbers**.
- Any number that cannot be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called **Irrational numbers**.
- There are more irrational numbers than rational numbers between two consecutive numbers.
- Rational and Irrational numbers together constitute **Real numbers**.

4. Properties of Irrational numbers:

- The **Sum, Difference, Product** and **Division** of two irrational numbers need not always be an irrational number.
- Negative** of an irrational number is an irrational number.
- Sum** of a **rational** and an **irrational** number is irrational.
- Product** and **Division** of a non-zero rational and irrational number is always irrational.

5. Fractions:

- **Terminating fractions** are the fractions which leaves remainder 0 on normal division.
- **Recurring fractions** are the fractions which never leave a remainder 0 on normal division.

6. Properties related to prime numbers:

- If p is a prime and divides a^2 , then p divides a , where ' a ' is a positive integer.
- If p is a prime, then \sqrt{p} is an irrational number.
- A number ends with the digit zero if and only if it has 2 and 5 as two of its prime factors.

7. Decimal Expansion:

- The decimal expansion of rational number is either **terminating** or **non-terminating recurring (repeating)**.

- If the decimal expansion of rational number **terminates**, then we can express the number in the form of $\frac{p}{q}$, where p and q are co prime, and the prime factorization of **q is of the form $2^n 5^m$** , where n and m are non negative integers.
- If $x = \frac{p}{q}$ is a rational number, such that the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which **terminates**.
- If the denominator of a rational number is of the form $2^n 5^m$, then it will terminate after n places if $n > m$ or after m places if $m > n$.
- The decimal expansion of an irrational number is **non-terminating, non-recurring**.

8. Fundamental Theorem of Arithmetic:

Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

- The procedure of finding **HCF(Highest Common Factor)** and **LCM(Lowest Common Multiple)** of given two positive integers a and b:
 - i. Find the prime factorization of given numbers.
 - ii. $HCF(a, b) = \text{Product of the smallest power of each common prime factors in the numbers.}$
 - iii. $LCM(a, b) = \text{Product of the greatest power of each prime factors, involved in the numbers.}$

9. Relationship between HCF and LCM of two numbers:

If a and b are two positive integers, then $HCF(a, b) \times LCM(a, b) = a \times b$

10. Relationship between HCF and LCM of three numbers:

$$LCM(p, q, r) = \frac{p \cdot q \cdot r \cdot HCF(p, q, r)}{HCF(p, q) \cdot HCF(q, r) \cdot HCF(p, r)}$$

$$HCF(p, q, r) = \frac{p \cdot q \cdot r \cdot LCM(p, q, r)}{LCM(p, q) \cdot LCM(q, r) \cdot LCM(p, r)}$$

