Lecture_01

No. **1**

ARITHMETIC PROGRESSIONS

- **Introduction of sequence**

Sequence :

SEQUENCE IS A LIST OF NUMBERS WHICH FOLLOW A DEFINITE PATTERN

1ˢᵗ LIST pattern

2ⁿᵈ LIST

3ʳᵈ LIST

1, 3, 5, 7, 9, 11, 13

- 45, - 40, -

7, 11, 20, -1, ???

GUESS THE NEXT NUMBERS

GUESS THE NEXT NUMBERS

15, - 10, ...

NOT A SEQUENCE

In this chapter we will deal only with sequence

DOES NOT FOLLOW PATTERN

Sequence : List of Numbers

1st 2nd 3rd 4th term a_4

Each number in the list is called a term and is denoted by 'a'

1, 3, 5, 7, 9, 11, 13, 15, ...

- 45, - 40, - 35, - 30, - 25, - 20, - 15, - 10, ...

No. **2**

ARITHMETIC PROGRESSIONS

- **Introduction of Arithmetic Progressions (A.P.)**
- **Introduction of Terms 'a' and 'd'**

Arithmetic Progression

Difference b[etween] two [cons]ecutive terms **is constant**

$a_2 - a_1$ = 9 ... $a_3 - a_2$ = 11 - ... $a_4 - a_3$ = 13 – 11

Such type of sequence is known as

Terms which comes one after other [sequen]ce?

| a_1 | a_2 | a_3 | a_4 |

Sequence

7, 9, 11, 13, ...

For an A.P. 1st term a_1 is denoted as a

2   2   2

Yes

$a = 7, \quad d = 2$

AP IS A SEQUENCE IN WHICH DIFFERENCE BETWEEN ANY TWO CONSECUTIVE TERMS IS SAME

No. **3**

ARITHMETIC PROGRESSIONS

- Identifying whether a given list of numbers is A.P. or not

1) Which of the following are APs

In an AP difference between any two consecutive terms is constant

i) 2, 4, 8, 16, ...

Sol:

$a_1 = 2,$ $a_2 = 4,$ $a_3 = 8,$ $a_4 = 16$

Lets find difference between consecutive terms

$a_2 - a_1$ $a_3 - a_2$

$= 4 - 2$ $= 8 - 4$

$= 2$ $= 4$

$\therefore$ **As difference is not constant, the given list of numbers is not an AP.**

1) Which of the following are APs

In an AP difference between any two consecutive terms is constant

ii) **-10, -6, -2, 2, ...**

Sol:

Lets find difference between consecutive terms

$a_1 = -10$, $a_2 = -6$, $a_3 = -2$, $a_4 = 2$

$a_2 - a_1$	$a_3 - a_2$	$a_4 - a_3$
$= -6 - (-10)$	$= -2 - (-6)$	$= 2 - (-2)$
$= -6 + 10$	$= -2 + 6$	$= 2 + 2$
$= 4$	$= 4$	$= 4$

$\therefore$ **As difference is constant, the given list of numbers is an AP.**

1) Which of the following are APs

In an AP difference between any two conse~~cutive terms is constant~~

Lets find difference between consecutive terms

iii) **0.2, 0.22, 0.222, 0.2222, ...**

Sol:

$a_1 = 0.2, \ a_2 = 0.22, \ a_3 = 0.222, \ a_4 = 0.2222$

$a_2 - a_1$ $\qquad\qquad\quad$ $a_3 - a_2$

$= 0.22 - 0.2$ $\qquad\quad$ $= 0.222 - 0.22$

$= 0.02$ $\qquad\qquad\quad$ $= 0.002$

$\therefore$ **As difference is not constant, the given list of numbers is not an AP.**

1) Which of the following are APs

In an AP difference between any two conse~~cutive terms is constant~~

iv) **0, -4, -8, -12, ...**

Lets find difference between consecutive terms

Sol:

$a_1 = 0$, $a_2 = -4$, $a_3 = -8$, $a_4 = -12$

$a_2 - a_1$ $a_3 - a_2$ $a_4 - a_3$

$= -4 - 0$ $= -8 - (-4)$ $= -12 - (-8)$

$= -4$ $= -8 + 4$ $= -12 + 8$

                              $= -4$ $= -4$

∴ **As difference is constant, the given list of numbers is an AP.**

No.**4**

ARITHMETIC PROGRESSIONS

- Identifying whether a given list of numbers is A.P. or not

1) Which of the following are APs

Lets find difference between consecutive terms

v) 1, 3, 9, 27, ...

Sol:

$a_1 = 1$, $a_2 = 3$, $a_3 = 9$, $a_4 = 27$

$a_2 - a_1$ $a_3 - a_2$

$= 3 - 1$ $= 9 - 3$

$= 2$ $= 6$

$\therefore$ **As difference is not constant, the given list of numbers is not an AP.**

1) Which of the following are APs

In an AP difference between any two cons~~ecutive terms is constant~~

Lets find difference between consecutive terms

vi) a, 2a, 3a, 4a, ...

Sol:

$a_1 = a,$ $a_2 = 2a,$ $a_3 = 3a,$ $a_4 = 4a$

$a_2 - a_1$ $a_3 - a_2$ $a_4 - a_3$

$= 2a - a$ $= 3a - 2a$ $= 4a - 3a$

$= a$ $= a$ $= a$

$\therefore$ **As difference is constant, the given list of numbers is an AP.**

1) Which of the following are APs

In an AP difference between any two consecutive terms is constant

vii) $2, \dfrac{5}{2}, 3, \dfrac{7}{2}, \ldots.$

Lets find difference between consecutive terms

Sol:

$a_1 = 2, \quad a_2 = \dfrac{5}{2}, \quad a_3 = 3, \quad a_4 = \dfrac{7}{2},$

$a_2 - a_1$	$a_3 - a_2$	$a_4 - a_3$
$= \dfrac{5}{2} - 2$	$= 3 - \dfrac{5}{2}$	$= \dfrac{7}{2} - 3$
$= \dfrac{5-4}{2}$	$= \dfrac{6-5}{2}$	$= \dfrac{7-6}{2}$
$= \dfrac{1}{2}$	$= \dfrac{1}{2}$	$= \dfrac{1}{2}$

$\therefore$ **As difference is constant, the given list of numbers is an AP.**

1) Which of the following are APs

viii) $\dfrac{-1}{2}$, $\dfrac{-1}{2}$, $\dfrac{-1}{2}$, $\dfrac{-1}{2}$, ...

Sol:

$$a_1 = -\dfrac{1}{2}, \; a_2 = -\dfrac{1}{2}, \; a_3 = -\dfrac{1}{2}, \; a_4 = -\dfrac{1}{2},$$

$a_2 - a_1$	$a_3 - a_2$	$a_4 - a_3$
$= -\dfrac{1}{2} - \left(-\dfrac{1}{2}\right)$	$= -\dfrac{1}{2} - \left(-\dfrac{1}{2}\right)$	$= -\dfrac{1}{2} - \left(-\dfrac{1}{2}\right)$
$= -\dfrac{1}{2} + \dfrac{1}{2}$	$= -\dfrac{1}{2} + \dfrac{1}{2}$	$= -\dfrac{1}{2} + \dfrac{1}{2}$
$= 0$	$= 0$	$= 0$

$\therefore$ **As difference is constant, the given list of numbers is an AP.**

1) Which of the following are APs

In an AP difference between any two consecutive terms is constant

ix) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots \ldots$

Lets find difference between consecutive terms

Sol:

$a_1 = \sqrt{3},\ a_2 = \sqrt{6},\ a_3 = \sqrt{9},\ a_4 = \sqrt{12},$

$a_2 - a_1$ 　　　　　　 $a_3 - a_2$

$= \sqrt{6} - \sqrt{3}$ 　　　　 $= \sqrt{9} - \sqrt{6}$

　　　　　　　　　　　$= 3 - \sqrt{6}$

$\therefore$ **As difference is not constant, the given list of numbers is not an AP.**

No. **5**

ARITHMETIC PROGRESSIONS

- **Identifying whether the given situation / statement forms an A.P.**

2) In which of the following situations, does the list of numbers involved make an A.P. and why?

i) **The taxi fare after each km w**ere **re**... **or the first km and Rs.8 for each additional**...

> 23 – 15 = 8

> 31 – 23 = 8

> Lets make list of fare for successive kms

> Check difference between consecutive terms
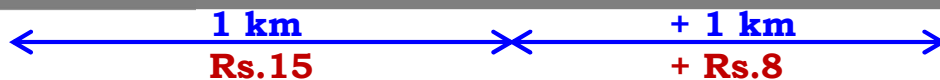
Sol: Fare for successive kms for taxi are:15, 23, 31,...

Since the difference between consecutive terms ... given sequence is an A.P.

1 km
Rs.15

+ 1 km
+ Rs.8

2) In which of the following situations, does the list of numbers involved make an A.P. and why?

ii) The amount of air pressure in a cylinder ... vacuum ... **removes of the air $\frac{1}{4}$ remaining in the cylinder at a time**

Sol: Amount of air remaining after removal of $\frac{1}{4}$ of air: V, $\frac{3}{4}V$, $\frac{9}{16}V$, ...

Since the difference between consecutive numbers is not an A.P.

$\frac{1}{4}\left(\frac{3}{4}V\right)$

$\frac{3}{4}V - V = \frac{-V}{4}$

$\frac{9}{16}V - \frac{3}{4}V = \frac{-3V}{16}$

Lets make list of air pressure remaining after every successive removal

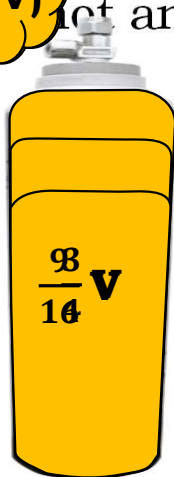Check difference between consecutive terms

Lets initial amount air pressure in a cylinder be 'V'

Air pressure remaining after

Air pressure remaining after 2nd removal ' $\frac{3}{4}V - \frac{1}{4}\left(\frac{3}{4}V\right)$ '

$\frac{9}{16}V$

No. 6

ARITHMETIC PROGRESSIONS

- **Identifying whether the given situation / statement forms an A.P.**

2) In which of the following situations, does the list of numbers involved make an A.P. and why?

iii) **The cost of digging a well after every** 200 − 150 = 50 **digging** 250 − 200 = 50 **t costs Rs.150 for the first meter and rises by** **or each** **equent meter.**

Sol: Cost for digging each subsequent meters are: 150, 200, 250,...

Since the difference between consecutive terms is constant, given sequence is an A.P.

Lets make list of cost of digging a well each subsequent meters
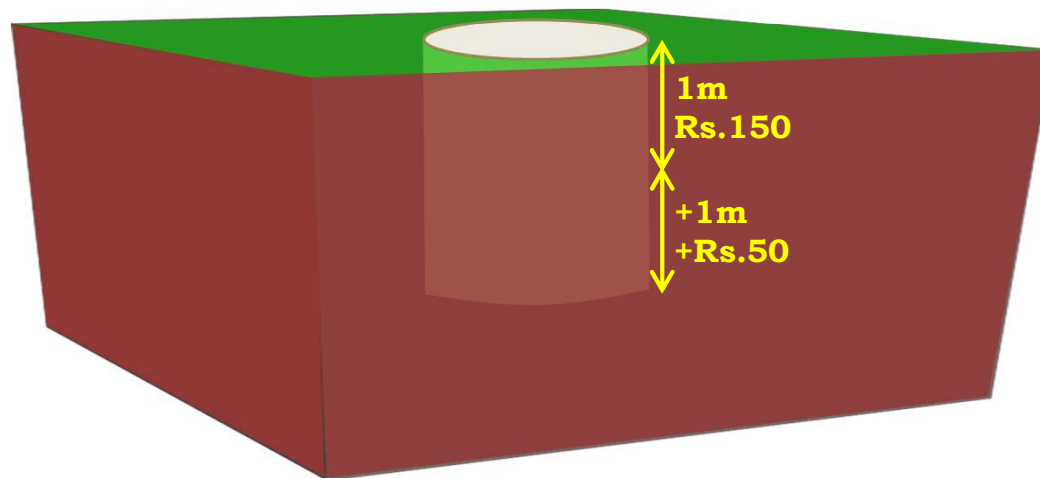
1m
Rs.150

+1m
+Rs.50

2) In which of the following situations, does the list of numbers involved make an A.P. and why?

iv) The amount of money in the account every year, when **Rs.10000 is** deposited at compound interest at 8% per annum.

Sol: Amount of money in the account for successive years 10000, 10800, 11664, ...

The difference between consecutive terms is not same, so the given list of numbers is not an A.P.

10800 − 10000 = 800

11664 − 10800 = 864

Rs.10800

After 1 year

Lets make list of amount for successive years

Amount = Principle + Interest

Check difference between consecutive terms

8% of Rs.10800
8% of Rs.10000

Rs.10000

$\dfrac{8}{100} \times 10800$ **+** **=** Rs.10800

Principle **Interest** **Amount**

No. **7**

ARITHMETIC PROGRESSIONS

- **Finding the value of first term (a) and common difference (d)**

3) For following APs, write first term and the common difference:

i) 3, 1, -1, -3, ...

Sol: For given AP: 3, 1, -1, -3, ...

First term(a) = 3

Common difference(d) = 1

> **Calculate Common difference as a term minus previous term**

ii) -5, -1, 3, 7, ...

Sol: For given AP: -5, -1, 3, 7, ...

First term(a) = -5

Common difference(d) = -1

> **Calculate Common difference as a term minus previous term**

3) For following APs, write first term and the common difference:

iii) $\dfrac{1}{3}, \dfrac{5}{3}, \dfrac{9}{3}, \dfrac{13}{3}, ...$

Sol: For given AP : $\dfrac{1}{3}, \dfrac{5}{3}, \dfrac{9}{3}, \dfrac{13}{3}, ...$

First term(a) = $\dfrac{1}{3}$

Common difference(d) = $\dfrac{5}{3} - \dfrac{1}{3} = \dfrac{1}{3}$

> Calculate Common difference as a term minus previous term

iv) 0.6, 1.7, 2.8, 3.9, ...

Sol: For given AP: 0.6, 1.7, 2.8, 3.9, ...

First term(a) = 0.6

Common difference(d) = 1.7 – 0.6

> Calculate Common difference as a term minus previous term

Thank You