MODULE : 1

Lines and Angles

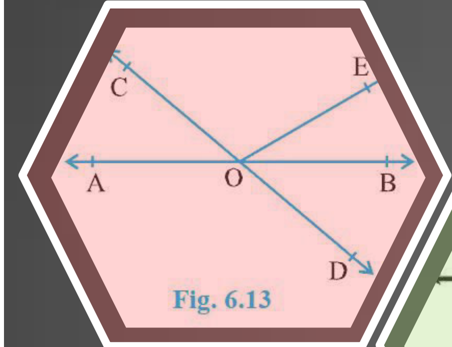
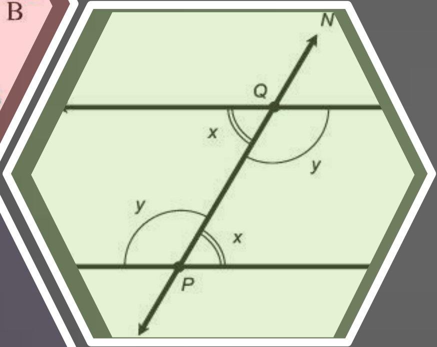
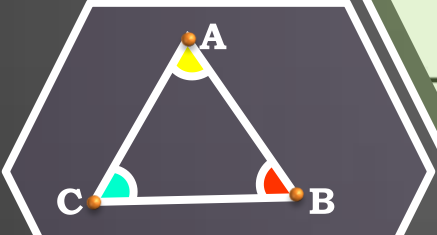


Fig. 6.13

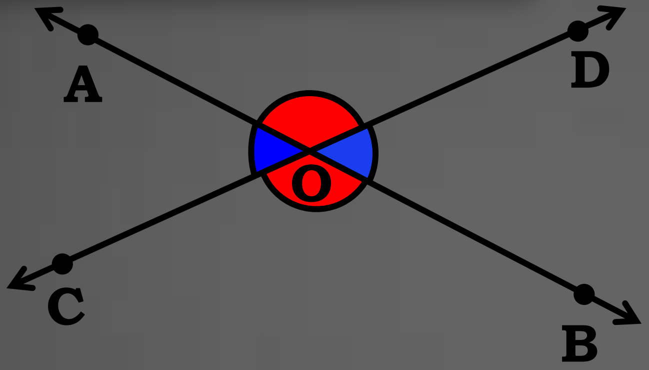




$\angle A + \angle B + \angle C = 180^\circ$

PAIRS OF ANGLES

➢ VERTICALLY OPPOSITE ANGLES



Note : Vertically Opposite angles are EQUAL

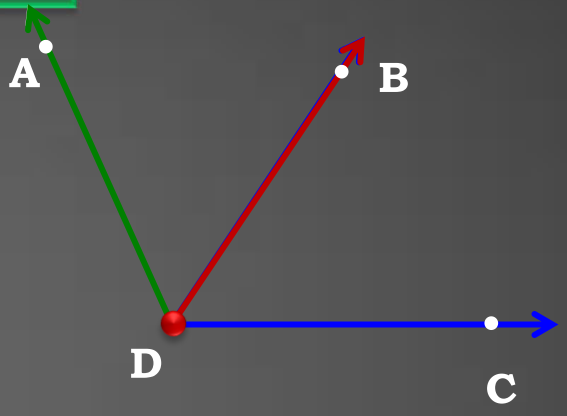
$\angle AOC = \angle BOD$, $\angle AOD = \angle COB$

MODULE : 2

Adjacent Angles

Two angles are said to be adjacent angles if

- They have common vertex.
- They have common ray.
- Non-common arms should lie on either side of common arm.



$\angle ADB$, $\angle BDC$

Have common vertex ‘D’.

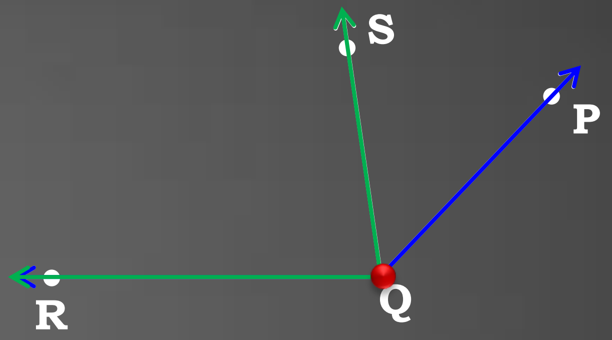
Have common arm ray BD.

ray AD and ray DC are non common arm lying on either side of common arm BD

Adjacent Angles

Two angles are said to be adjacent angles if

- ➢ They have common vertex.
- ➢ They have common ray.
- ➢ Non-common arms should lie on either side of common arm.



Consider ∠PQR and ∠SQR

Have common vertex Q.

QR is the common ray.

Non-common arms QS and QP lie on the same side of common arm QR

∠PQR and ∠SQR are not adjacent angles.

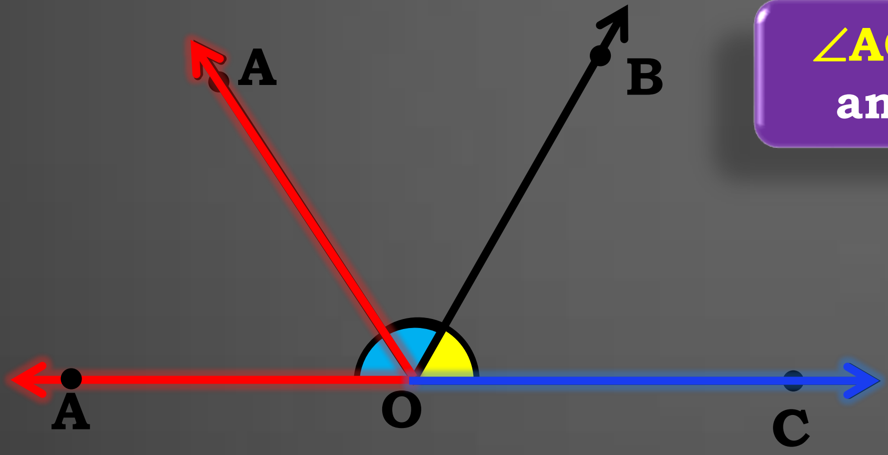
MODULE : 3

PAIRS OF ANGLES

➢ LINEAR PAIR ANGLES

Two adjacent angles are said to form a linear pair angle,

➢ ***If their non-common rays form a pair of opposite rays.***

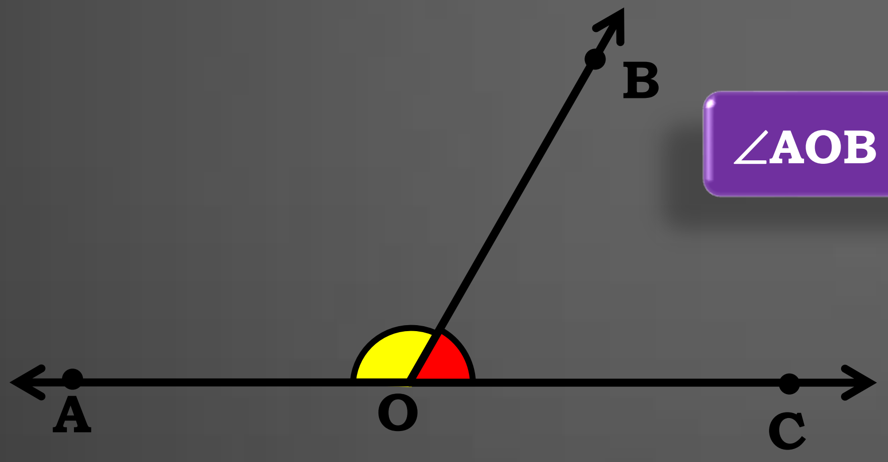


∠AOB and ∠BOC are the angles in a linear pair.

PAIRS OF ANGLES

➢ LINEAR PAIR ANGLES

The sum of the measure of a linear pair angle is 180°



$\angle AOB + \angle BOC = 180°$

MODULE : 4

Linear Pair Axiom

If a ray stands on a line, then the sum of the two adjacent angles so formed is 180^0 and vice–versa.

1. *If*, OC stands on line AB

   then, $\angle AOC + \angle COB = 180^0$

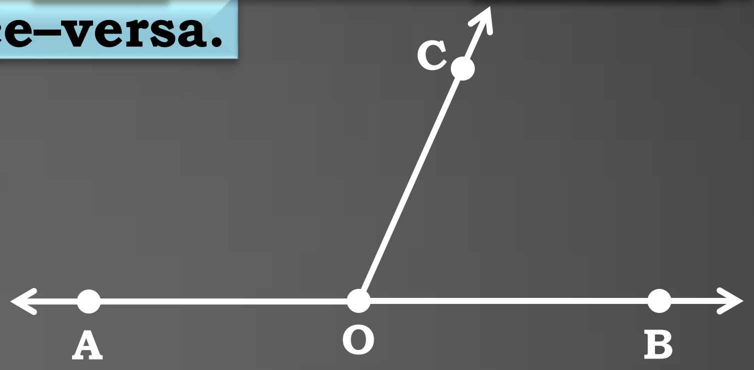
2. *If*

   $m\angle AOC + m\angle COB = 180^0$

   then,

   AB is a line and OC stands on the line.

   i.e. A , O , B are collinear



$\angle AOC + \angle COB = 180^0$

Lines which are parallel to a given line are parallel to each other.

If

AB || CD

and

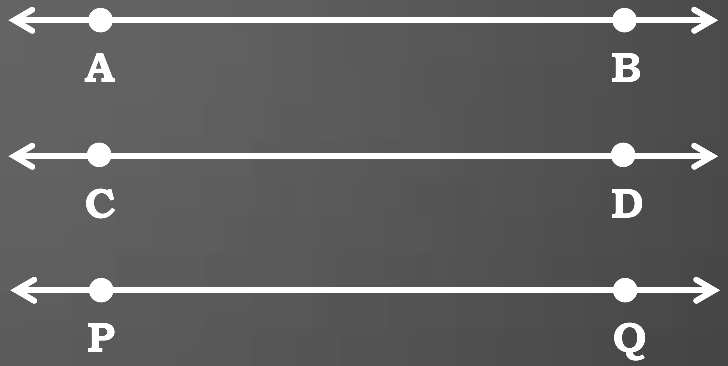
PQ || CD

then

AB || PQ

AB || CD

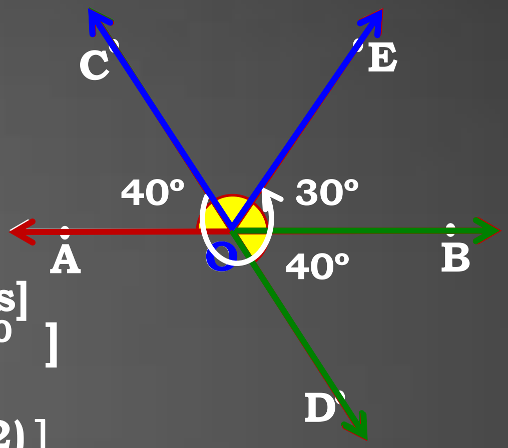
PQ || CD



MODULE : 5

Ex.: 6.1 - 1

In figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$, find $\angle BOE$ and reflex $\angle COE$



Sol: $\angle AOC + \angle BOE = 70^0$

$\therefore \angle BOE = 70^0 - \angle AOC$ ....(1)

But, $\angle AOC = \angle BOD$ [Vertically opposite angles]

$\therefore \angle AOC = 40^0$ ....(2) [$\because \angle BOD = 40^0$]

$\therefore \angle BOE = 70^0 - 40^0$ [From (1) and (2)]

$\therefore \angle BOE = 30^0$ ....(3)

Reflex of $\angle COE = 360 - \angle COE$

$\therefore \angle COE + \angle BOE + \angle BOD = 180^0$

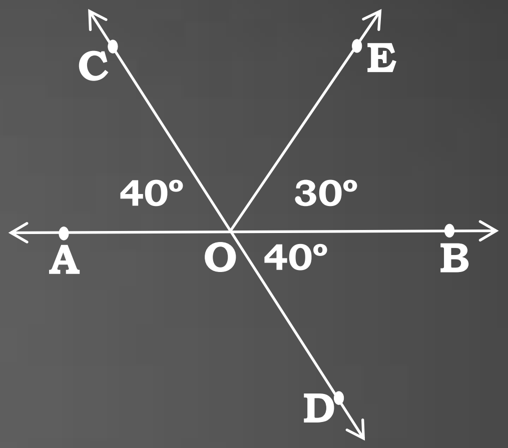
$\therefore \angle COE + 30^0 + 40^0 = 180^0$

$\therefore \angle COE = 180^0 - 40^0 - 30^0$

$\therefore \angle COE = 110^0$

Ex.: 6.1 - 1

In figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$



Sol:

$\therefore \angle COE = 110^0$

Reflex $\angle COE = 360° - 110-$

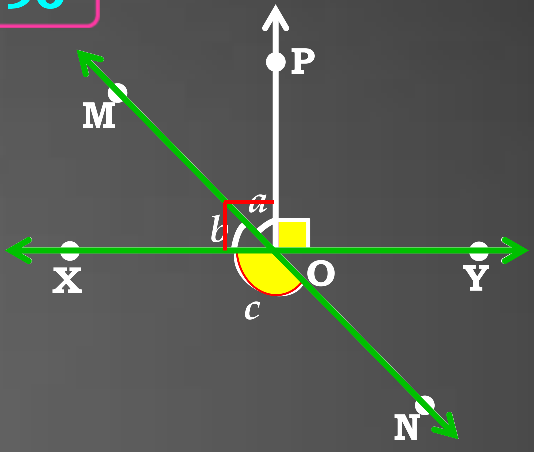
$\therefore$ $\boxed{\text{Reflex } \angle COE = 250°}$

Thank You

MODULE : 5

Ex. : 6.1 - 2

lines XY and MN intersect at O. If $\angle POY = 90^\circ$ and $a : b = 2 : 3$, find c.



Sol: $a : b = 2 : 3$

$\therefore a = 2x$ and $b = 3x$

$a + b = \angle POX$

$\angle POX = 90^\circ$ $[\because \angle POX + \angle POY = 180^0]$

$a + b = 90^0$

$2x + 3x = 90$

$5x = 90$

$x = \frac{\cancel{90}^{18}}{\cancel{5}} = 18^0$

$a = 2x$

$a = 2 \times 18$

$a = 36^0$

$b = 3x$

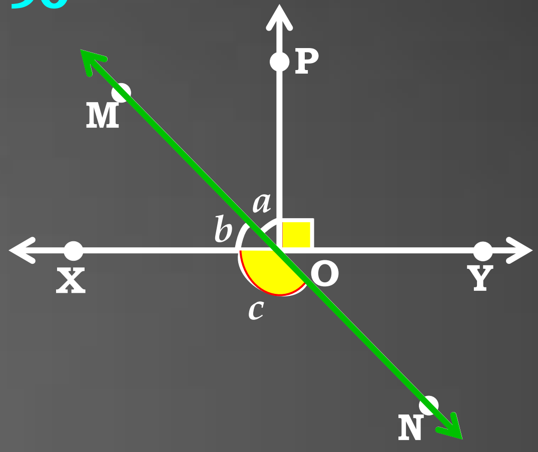
$b = 3 \times 18$

$\boxed{b = 54^0}$

Ex. : 6.1 - 2

lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$, find c.

$b = 54^0$



Sol:

$\angle MOX + \angle XON = 180°$

$\therefore \quad b + c = 180^0$

$\therefore \quad 54° + c = 180^0$

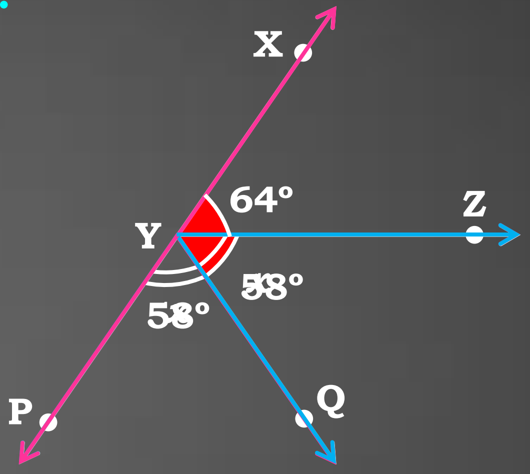
$\therefore \quad c = 180 - 54 = 126^0$

Hence, $\boxed{c = 126^0}$

MODULE : 6

Ex. : 6.1 - 6

It is given that $\boxed{\angle XYZ = 64^\circ}$ and $\boxed{\text{XY is produced to point P.}}$
If $\boxed{\text{ray YQ bisects } \angle ZYP,}$ $\boxed{\text{find } \angle XYQ}$ and reflex $\angle QYP$.



Sol. $\boxed{\angle XYZ = 64^\circ}$ [1]

ray YQ bisects $\angle ZYP$

let $\angle ZYQ = \angle QYP = x$ (1)

$\therefore \boxed{\angle ZYP = 2x}$ (2)

$\angle ZYX + \angle ZYP = 180$ [Linear Pair]

$64 + 2x = 180$ [from (1) and (2)]

$2x = 180 - 64$

$\boxed{x = 58^\circ}$

$\therefore \angle XYQ = \angle XYZ + \angle ZYQ$

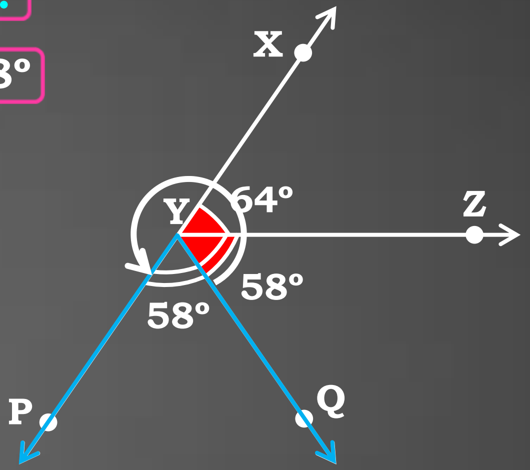
$= 64 + 58$

$= 122^\circ$

Ex. : 6.1 - 6

It is given that $\angle XYZ = 64°$ and XY is produced to point P. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and $\boxed{\text{reflex } \angle QYP.}$

$\boxed{x = 58°}$



Sol. Reflex $\angle QYP = 360° - \angle QYP$

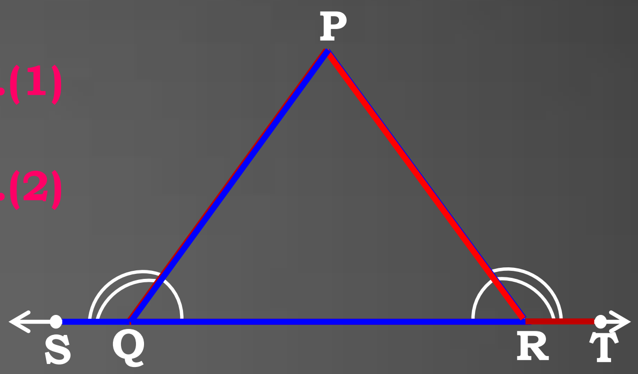
$= 360° - 58°$ From (3)

Reflex $\angle QYP = 302°$

MODULE : 7

Ex. : 6.1 - 3

In figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Sol:

$\angle PQS + \angle PQR = 180°$ [Linear Pairs] ....(1)

$\angle PRQ + \angle PRT = 180°$ [Linear Pairs] ....(2)

From (1) and (2), we have

$\angle PQS + \cancel{\angle PQR} = \cancel{\angle PRQ} + \angle PRT$

[$\because$ Each side = 180°] ....(3)

$\boxed{\angle PQS = \angle PRT}$

MODULE : 8

Ex. : 6.1 - 4

In figure, if $\boxed{x + y = w + z}$, then **prove that AOB is a line.**

Sol. Since the sum all angles round a point is equal to 360°

$\therefore \quad \angle BOC + \angle COA + \angle BOD + \angle AOD = 360°$

$\therefore \quad x + y + w + z = 360$

$\therefore \quad w + z + w + z = 360$

$\therefore \quad 2w + 2z = 360$

$\therefore \quad 2(w + z) = 360$

$\therefore \quad w + z = 180°$

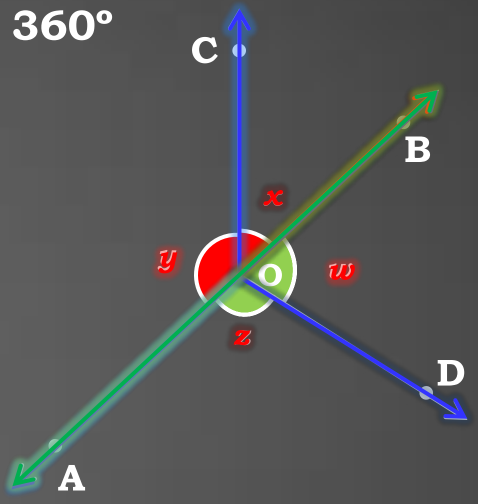
i.e. To prove $x + y = 180°$ or $w + z = 180°$

Thus, ∠BOC and ∠COA forms linear pairs.

Consequently OA and OB are two opposite rays.

Therefore,

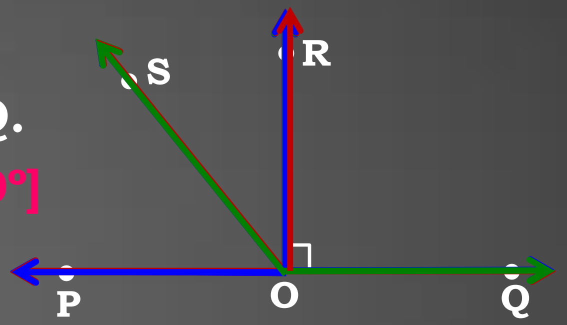
AOB is a straight line.



MODULE : 9

Ex. : 6.1 - 5

In figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.



Sol: Since OR is perpendicular to the line PQ.

$\therefore \angle POR$ and $\angle ROQ$ [$\because$ Each = 90°]

$\therefore \angle POS$ and $\angle ROS = \angle ROQ$

$$\angle POS + \angle ROS = \angle QOS - \angle ROS$$

$$\therefore \angle ROS + \angle ROS = \angle QOS - \angle POS$$

$$\therefore 2\angle ROS = \angle QOS - \angle POS$$

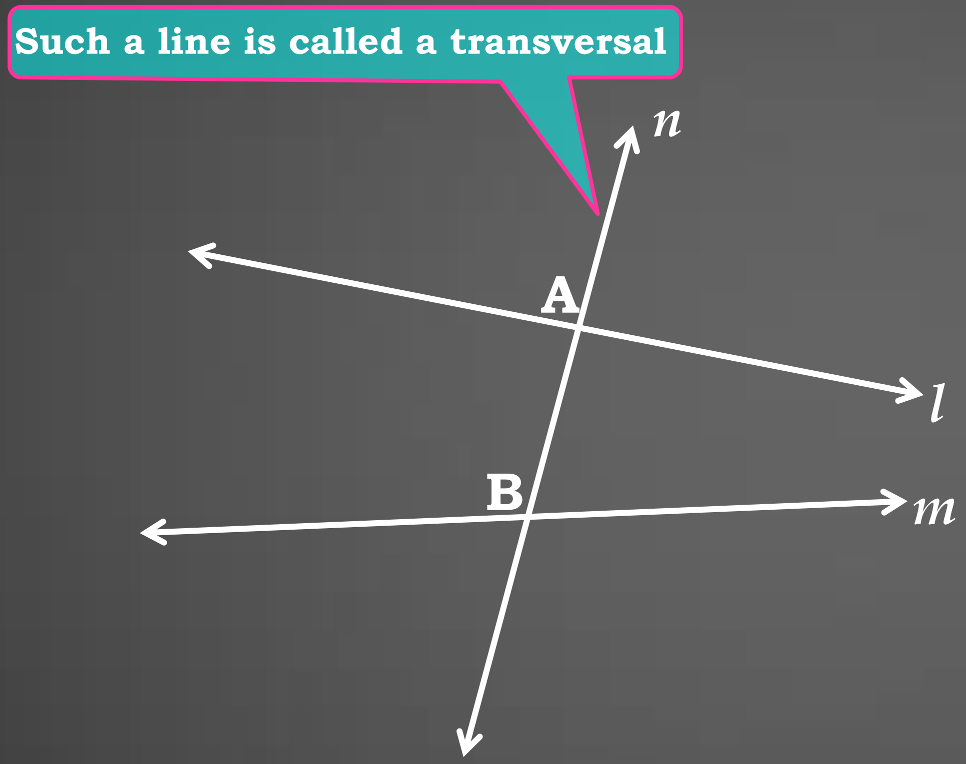
$$\therefore \boxed{\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)}$$

Thank You

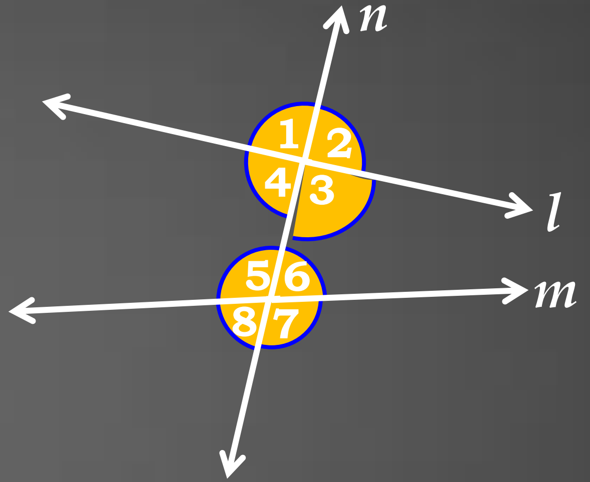
MODULE :

10

A line which intersects two or more lines at distinct points is called a transversal.



ANGLES MADE BY A TRANSVERSAL



Pairs of Corresponding angles :

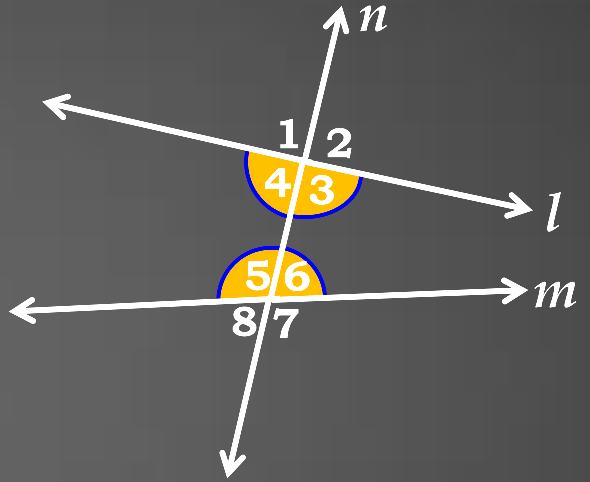
(i) $\angle 3, \angle 7$ (ii) $\angle 2, \angle 6$

(iii) $\angle 4, \angle 8$ (iv) $\angle 1, \angle 5$

Pairs of Alternate-Interior angles :

Pairs of Co-interior angles :

ANGLES MADE BY A TRANSVERSAL



Pairs of Corresponding angles :

(i) $\angle 3, \angle 7$ (ii) $\angle 2, \angle 6$

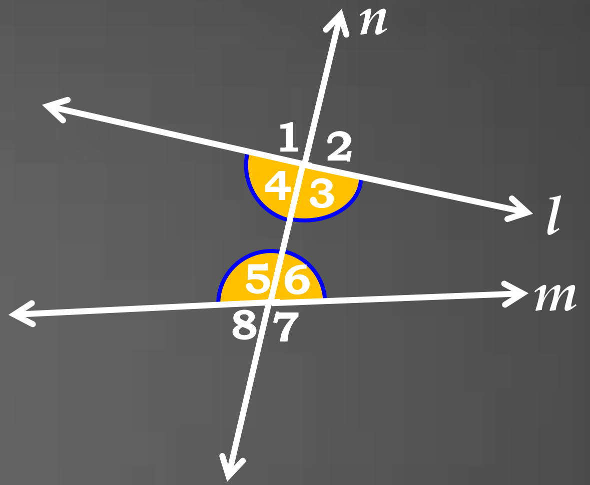
(iii) $\angle 4, \angle 8$ (iv) $\angle 1, \angle 5$

Pairs of Alternate-Interior angles :

(i) $\angle 4, \angle 6$ (ii) $\angle 3, \angle 5$

Pairs of Co-interior angles :

ANGLES MADE BY A TRANSVERSAL



Pairs of Corresponding angles :

(i) $\angle 3, \angle 7$ (ii) $\angle 2, \angle 6$

(iii) $\angle 4, \angle 8$ (iv) $\angle 1, \angle 5$

Pairs of Alternate-Interior angles :

(i) $\angle 4, \angle 6$ (ii) $\angle 3, \angle 5$

Pairs of Co-interior angles :

(i) $\angle 3, \angle 6$ (ii) $\angle 4, \angle 5$

MODULE : 11

$l \parallel m$ [Given]

Pairs of Corresponding angles :

(i) $\angle 3 = \angle 7$ (ii) $\angle 2 = \angle 6$

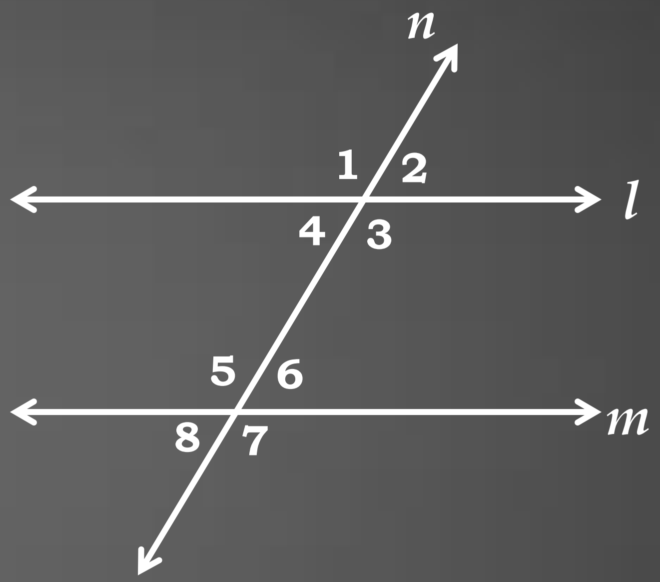
(iii) $\angle 4 = \angle 8$ (iv) $\angle 1 = \angle 5$

Pairs of Alternate-Interior angles :

(i) $\angle 4 = \angle 6$ (ii) $\angle 3 = \angle 5$

Pairs of Co-interior angles :

(i) $\angle 3 + \angle 6 = 180^\circ$ (ii) $\angle 4 + \angle 5 = 180^\circ$

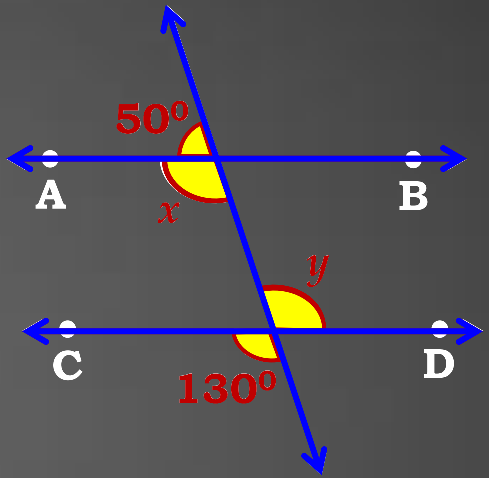


MODULE :

12

Ex. : 6.2 - 1

In figure,
find the values of x and y and
then show that AB || CD.



Sol: $50 + x = 180$ (Linear pair)

$\therefore$ $x = 180 - 50$

$\therefore$ $\boxed{x = 130}$

$\boxed{y = 130}$

$\therefore$ $\boxed{x = y}$

i.e. alternate interior angles are equal.

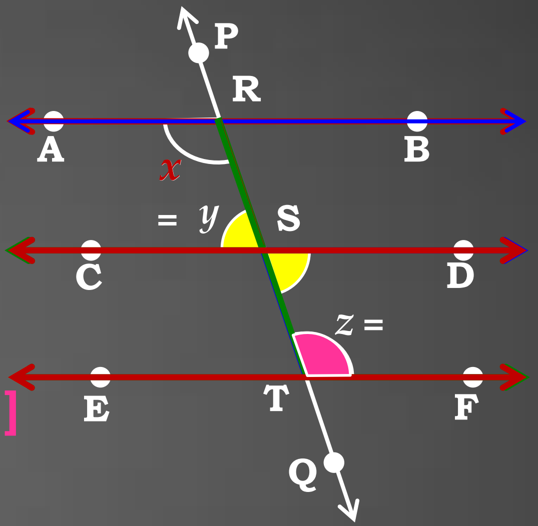
$\therefore$ **AB || CD.**

MODULE :

13

Ex. : 6.2 - 2

Q. In figure,
if AB || CD, CD || EF
and $y : z = 3 : 7$, find x.



Sol: $y : z = 3 : 7$ [given]

Let the common multiple be 'k'

$\therefore$ $\boxed{y = 3k}$, $\boxed{z = 7k}$,

Now,

$\angle CSR = \angle TSD$ [vertically opposite angle]

$\therefore$ $\angle TSD = y$ [$\because \angle CSR = y$]

CD || EF [given]

and ST is a transversal

$\therefore$ $\angle TSD + \angle STF = 180^0$

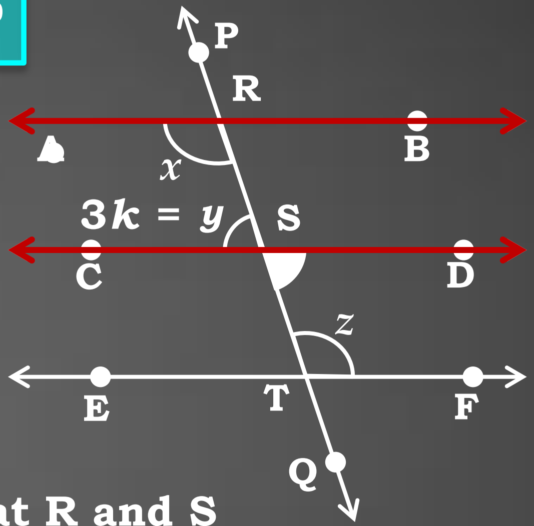
$\therefore$ $\boxed{y} + \boxed{z} = 180$

$\therefore$ $3k + 7k = 180^0$

Ex. : 6.2 - 2

$$\boxed{\underline{3k} + \underline{7k} = 180^0}$$

$$\boxed{x + \boxed{y} = 180}$$



$\therefore \quad 10k = 180$

$\therefore \quad k = \dfrac{\cancel{180}^{18}}{\cancel{10}}$

$\therefore \quad k = 18$

$\therefore \quad y = 3k = 3 \times 18$

$\therefore \quad \boxed{\boxed{y} = 54^0}$

Since AB || CD and transversal PQ intersects them at R and S respectively.

$\therefore \quad \angle ARS + \angle RSC = 180^\circ$ [Interior angles are supplementary]

$\therefore \quad x + y = 180$

$x = 180 - y = 180 - 54 = 126$

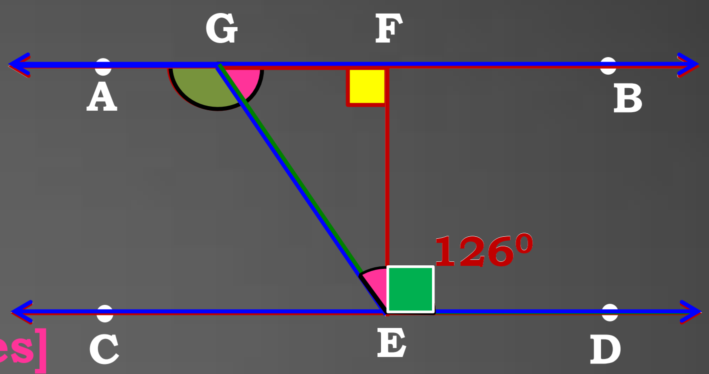
Hence, $\boxed{x = 126}$

MODULE :

14

Ex . : 6.2 - 3

Q. In figure, if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126^\circ$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.



Sol: Since $AB \parallel CD$ and transversal GE cuts them at G and E respectively.

$\angle AGE = \angle GED$ [Alternate angles]

$\therefore \angle AGE = 126^\circ$ [$\because \angle GED = 126^\circ$ (given)]

$\therefore \angle GEF = \angle GED - \angle FED$

$\therefore \angle GEF = 126^\circ - 90^\circ$

$\therefore \angle GEF = 36^\circ$

and, $\angle AGE + \angle FGE = 180^0$ [Linear pair angles]

$\therefore 126^0 + \angle FGE = 180^0$

$\therefore \angle FGE = 180^0 - 126^0$

$\therefore \boxed{\angle FGE = 54^\circ}$

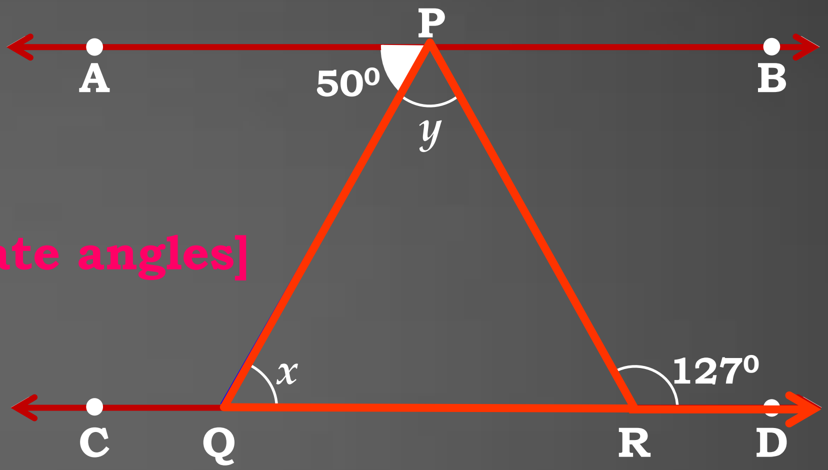
Thank You

MODULE :

15

Ex. : 6.2 - 5

In figure, if AB || CD, $\angle APQ = 50°$ and $\angle PRD = 127°$, find x and y.



Sol: AB || CD and transversal PQ intersects them at P and Q respectively.

$\therefore \quad \angle PQR = \angle APQ$ [Alternate angles]

$\therefore \quad \boxed{x = 50°}$

$\angle PRD$ is an exterior angle of ΔPQR

$\therefore \angle PRD = \angle QPR + \angle PQR$ [Exterior angle is equal to sum of the two opposite interior angles]

$\therefore 127° = y + 50°$

$\therefore 127° - 50° = y$

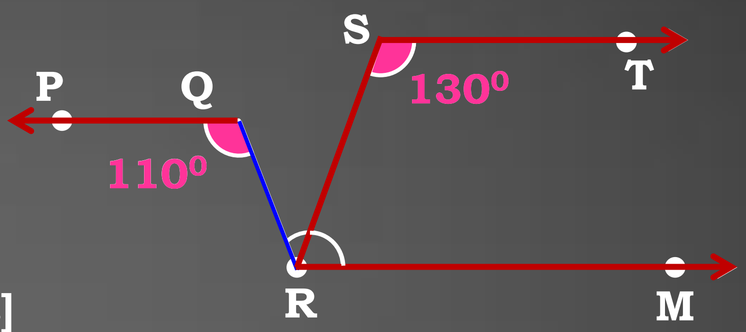
$\therefore \boxed{y = 77°}$

MODULE :

16

Ex.: 6.2 - 4

If PQ || ST, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, find $\angle QRS$.



Sol: Draw a line passing through point R and parallel to line PQ.

$\therefore$ PQ || RM .... (i)

On transversal QR.

$\angle PQR = \angle QRM$ [Alternate interior Angles]

$\therefore \angle PQR = \angle QRM = 110^\circ$ ... (ii)

PQ || ST ... (iii) (Given)

$\therefore$ ST || RM From (i) & (iii)

on transversal SR.

$\angle RST + \angle SRM = 180^\circ$ ... [co-interior angles]

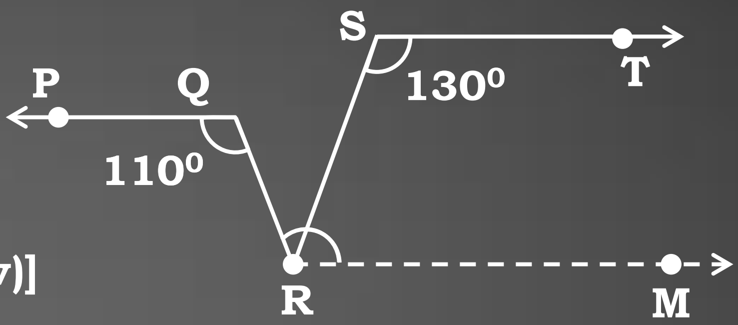
$\therefore 130^\circ + \angle SRM = 180^\circ$

$\therefore \angle SRM = 180^\circ - 130$

$\therefore \angle SRM = 50^\circ$ ... (iv)

Ex . : 6.2 - 4

If $PQ \parallel ST$, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, find $\angle QRS$.



Sol:

$$\angle QRM = \angle QRS + \angle SRM$$

$$110^\circ = \angle QRS + 50^\circ \quad \text{[From (ii) \& (iv)]}$$

$$\therefore \angle QRS = 110^\circ - 50^\circ$$

$$\therefore \angle QRS = 60^\circ$$

$$\angle PQR = \angle QRM = 110^\circ \quad \dots \text{(ii)}$$

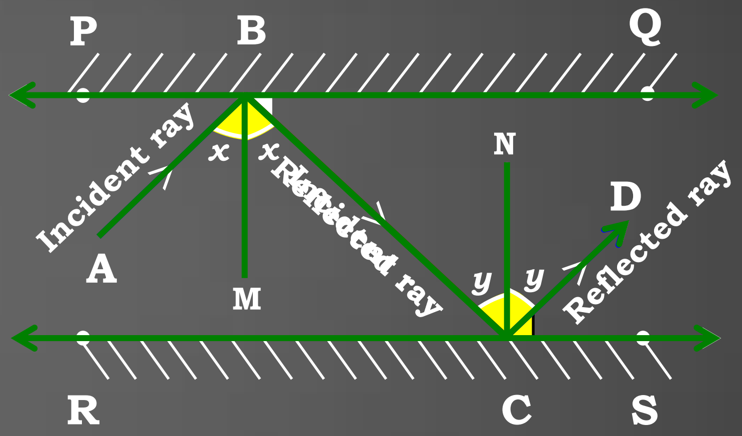
$$\angle SRM = 50^\circ \quad \dots \text{(iv)}$$

MODULE :

17

Ex . : 6.2 - 6

In figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back long CD. Prove that AB || CD.



Sol:

Draw normal MB and NC at B and C respectively.

Let $\angle ABM = \angle MBC = x$ (i)

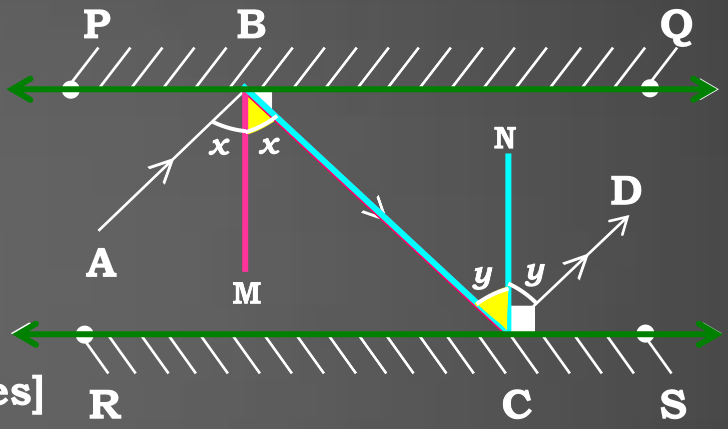
and $\angle BCN = \angle NCD = y$ (ii)

Now: $\angle ABC = \angle ABM + \angle MBC = x + x = 2x$ (iii)

$\therefore$ $\angle BCD = \angle BCN + \angle NCD = y + y = 2y$ (iv)

Ex . : 6.2 - 6

In figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back long CD. Prove that AB || CD.



Sol: PQ || RS

Taking BC as the transversal,

$\angle MBC = \angle BCN$ [Alternate interior angles]

$x = y$ (v) [From (i) and (ii)]

$\angle ABC = \angle BCD$ [From (iii), (iv) and (v)]

But, these are a pair of alternate interior angles on transversal BC.

$\therefore$ AB || CD

$\angle ABC = 2x$ ... (iii)

$\angle BCD = 2y$ ... (iv)

$x = y$ ... (v)

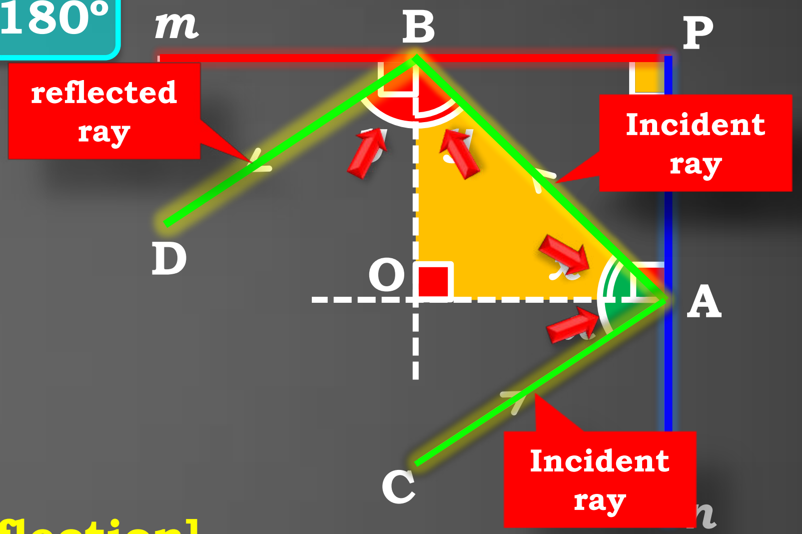
MODULE :

18

Q. *m* and *n* are two plane mirrors perpendicular to each other. Show that the incident ray CA is parallel to the reflected ray BD.

Proof :

$\angle CAB + \angle ABD = 180^\circ$



Draw AO and BO normals to the mirrors 'm' and 'n' at A and B respectively

AO $\perp$ *n* and BO $\perp$ *m*

Also, line *n* $\perp$ line *m*

$\therefore$ OA $\perp$ OB

$\angle OAB = \angle OAC = x$

$\angle OBA = \angle OBD = y$

[$\because$ Angle of incidence = Angle of reflection]

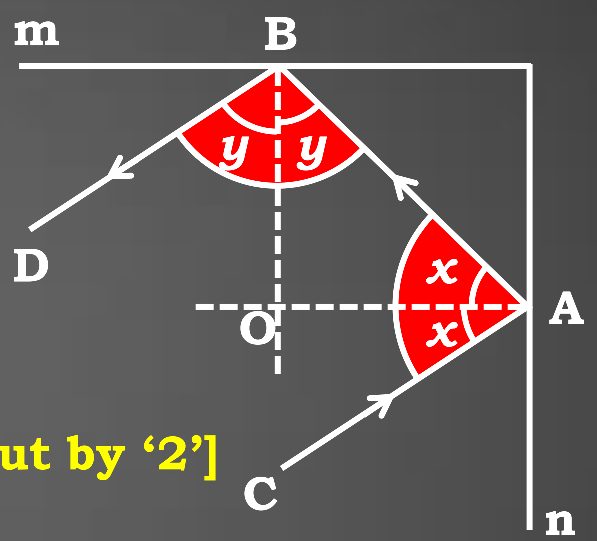
In ΔAOB,

$\angle AOB + \angle OAB + \angle OBA = 180^\circ$ [Angle sum property of a triangle]

$\therefore 90 + x + y = 180$

Q. *m* and *n* are two plane mirrors perpendicular to each other. Show that the incident ray CA is parallel to the reflected ray BD.

Proof : $\boxed{\overset{2x}{\angle CAB} + \overset{2y}{\angle ABD} = 180^\circ}$



$\therefore \quad 90 + x + y = 180$

$\therefore \quad x + y = 180 - 90$

$\therefore \quad \boxed{x + y = 90}$

$\therefore \quad \boxed{2x} + \boxed{2y} = 180$ [Multiplying throughout by ‘2’]

$\therefore \angle CAB + \angle ABD = 180^\circ$

A pair of interior angles are supplementary

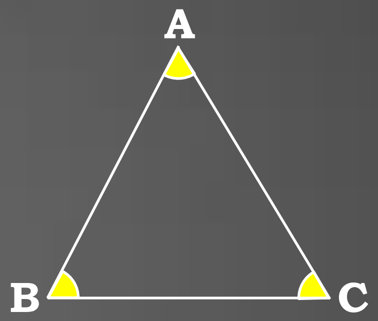
$\therefore \quad CA \parallel BD$

MODULE :

19

Angle sum property of a triangle

Sum of all angles of a triangle is 180^0



In ΔABC,

$$\angle ABC + \angle BAC + \angle ACB = 180^0$$

In ΔPQR, $\angle PQR = 60^0$, $\angle PRQ = 50^0$

Find measure of angle $\angle QPR$

In ΔPQR,

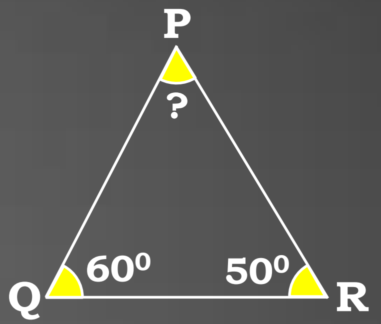
$$\angle PQR + \angle PRQ + \angle QPR = 180^0$$

$$\therefore \quad 60^0 + 50^0 + \angle QPR = 180^0$$

$$\therefore \quad 110^0 + \angle QPR = 180^0$$

$$\therefore \quad \angle QPR = \underline{180^0} - \underline{110^0}$$

$$\therefore \quad \boxed{\angle QPR = 70^0}$$



Exterior Angle Of A Triangle

A ... a linear pair with ... e is said to be ... of a triangle.

$\angle A$ & $\angle B$ are interior angles; Also, they are opposite to $\angle ACB$

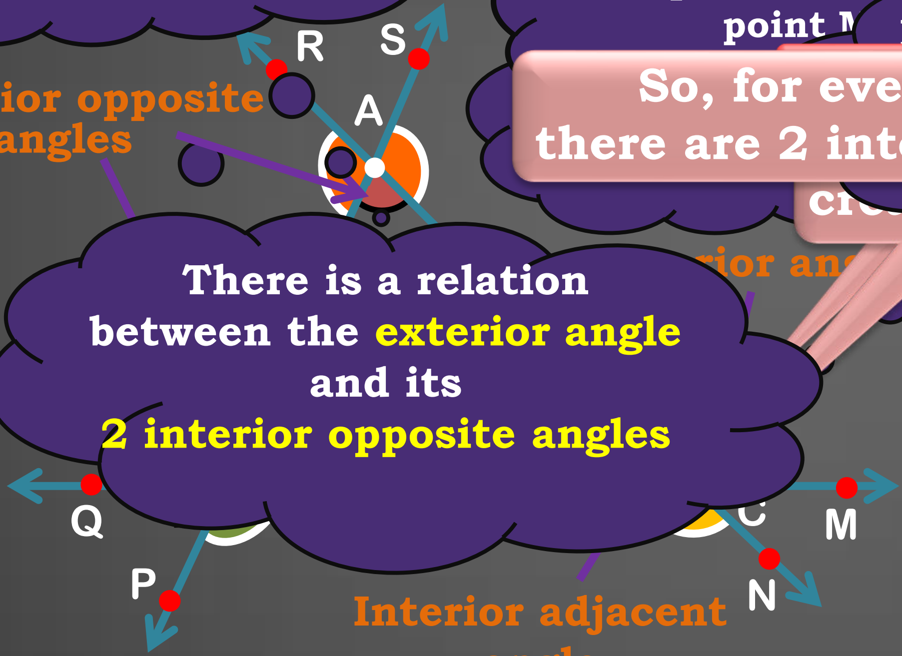
Let us produce side BC ... point M

Now, let us create 2

So, for every exterior angle, there are 2 interior opposite angles

... exterior angles

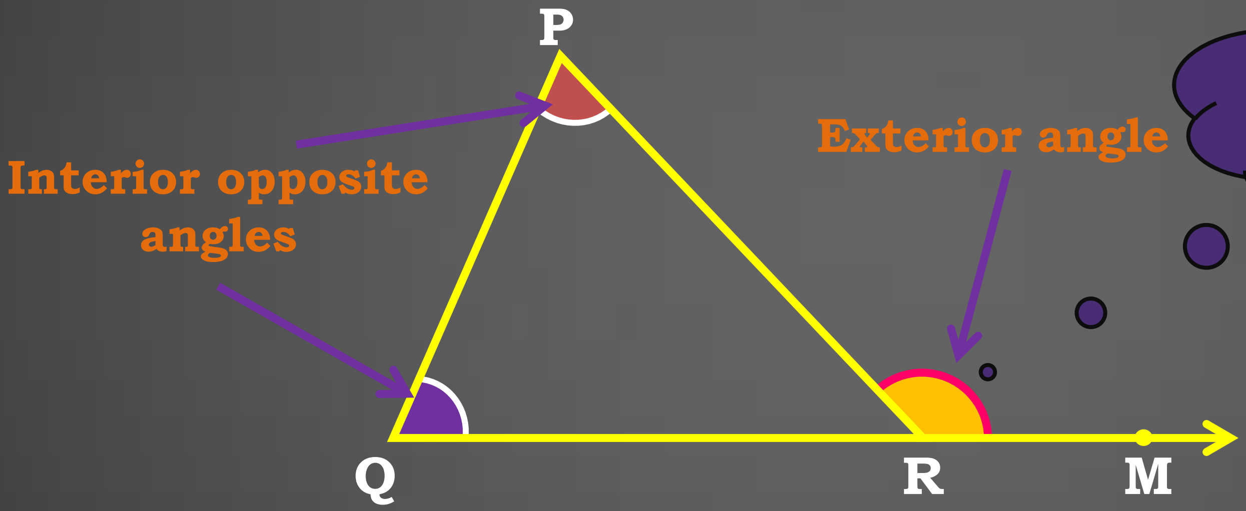
Exterior angle	Interior opposite angles
	& $\angle B$
	$\angle C$
	$\angle C$



There is a relation between the **exterior angle** and its **2 interior opposite angles**

Theorem

If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.



Which are the 2 interior opposite angles?

$\angle PQR$ & $\angle QPR$

$$\angle PRM = \angle PQR + \angle QPR$$

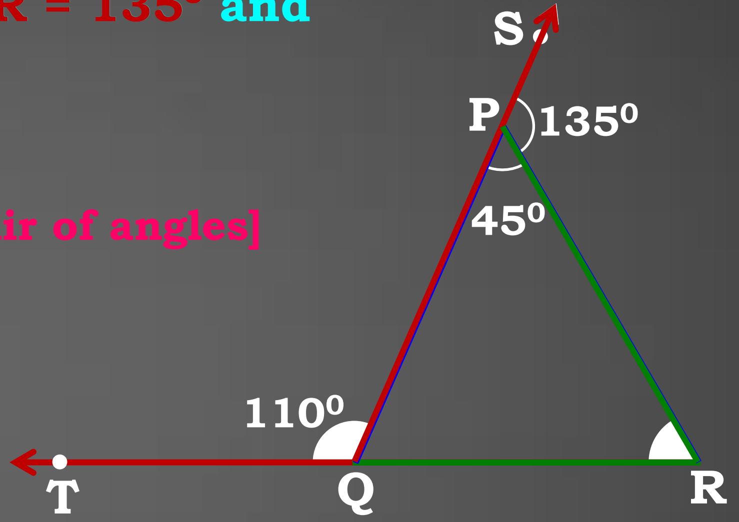
Thank You

MODULE :

20

Ex . : 6.3 - 1

Q. In figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR = 135^\circ$ and $\angle TQP = 110^\circ$, find $\angle PRQ$.



Sol: We have,

$\angle QPR + \angle SPR = 180^0$ [Linear pair of angles]

$\therefore \angle QPR + 135^0 = 180$

$\therefore \angle QPR = 180 - 135 = 45^0$

Now, $\angle TQP = \angle QPR + \angle PRQ$

[Exterior angle is equal to sum of the two opposite interior angles]

$\therefore 110^0 = 45^0 + \angle PRQ$

$\therefore \angle PRQ = 110^0 - 45^0 = 65^0$

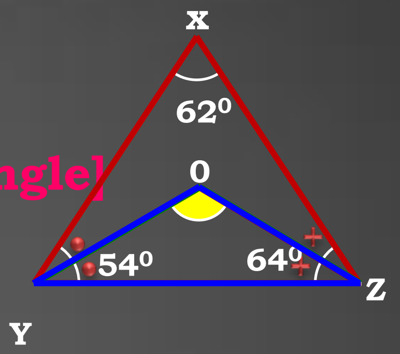
$\therefore \boxed{\angle PRQ = 65^0}$

MODULE :

21

Ex. : 6.3 - 2

Q. In figure, $\angle X = 62^\circ$, $\angle XYZ = 54^\circ$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$.



Sol: Consider ΔXYZ,

$\angle YXZ + \angle XYZ + \angle XZY = 180^0$ [Angle sum property of a triangle]

$\therefore\ 62^0 + 54^0 + \angle XZY = 180^0$ [$\because$ YXZ = 62°, $\angle XYZ$ = 54°]

$\therefore\ \angle XZY = 180^0 - 62^0 - 54^0 = 64^0$

Since YO and ZO are bisectors of $\angle XYZ$ and $\angle XZY$.

$$\angle OYZ = \frac{1}{2} \times \angle XYZ = \frac{1}{\cancel{2}} \times \cancel{54}^0 = 27^0$$

and $$\angle OZY = \frac{1}{2} \times \angle XZY = \frac{1}{\cancel{2}} \times \cancel{64}^0 = 32^0$$

$\boxed{\angle OYZ = 27^0}$

$\boxed{\angle OZY = 32^0}$

In ΔOYZ, we have

$\angle YOZ + \angle OYZ + \angle OZY = 180^0$ [Angle sum property]

$\therefore\ \angle YOZ + 27^0 + 32^0 = 180^0$

$\therefore\ \angle YOZ = 180^0 - 27^0 - 32^0 = 121^0$

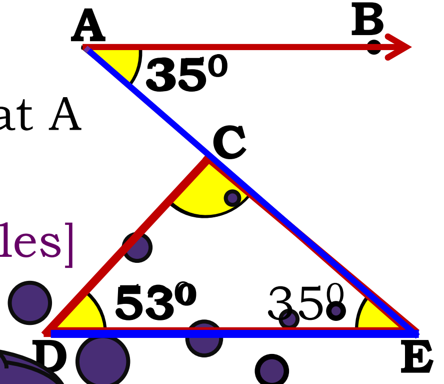
$\boxed{\text{Hence, } \angle OZY = 32^0 \text{ and } \angle YOZ = 121^0}$

MODULE :

22

Ex.: 6.3 - 3

Q. In figure, If AB || DE, ∠BAC = 35° and ∠CDE = 53°, find ∠DCE.



Soln. Since AB || DE and transversal AE intersects them at A and E respectively.

∴ ∠DEA = ∠BAE [Alternate interior angles]

∴ ∠DEC = 35°

∴ i.e. ∠DEA = ∠BAE

In **ΔDEC**, we have

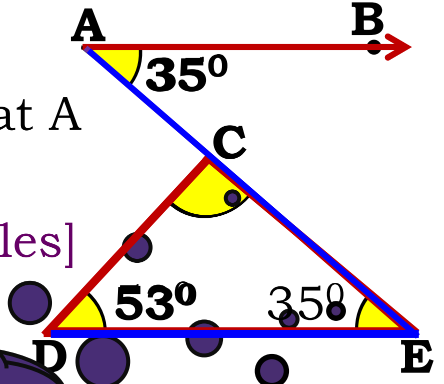
∠DCE + ∠DEC + ∠CDE = 180°

∠DCE + …

Hence, ∠DCE = 92°

Ex.: 6.3 - 3

Q. In figure, If AB || DE, ∠BAC = 35° and ∠CDE = 53°, find ∠DCE.



Soln. Since AB || DE and transversal AE intersects them at A and E respectively.

∴ ∠DEA = ∠BAE [Alternate interior angles]

∴ ∠DEC = 35°

∴

In **ΔDEC**, we have

∠DCE+∠DEC+

∠DCE+

is 180°

35°

i.e. ∠DEA = ∠BAE

If we find ∠DEC we will get the requi

∠DEA = ∠DEC [A - C - E]

Hence, ∠DCE = 92^0

MODULE :

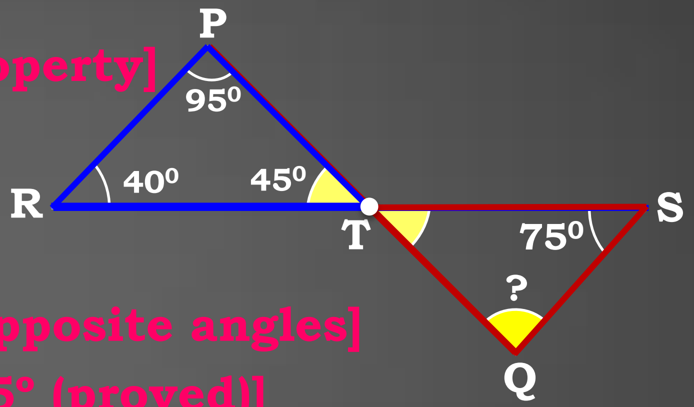
23

Ex.: 6.3 - 4

In figure, if lines PQ and RS intersect at point T, such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$, find $\angle SQT$.

Sol:

In ΔPRT, we have



$\angle PRT + \angle RTP + \angle TPR = 180^0$ [Angle sum property]

$\therefore\ 40^0 + \angle RTP + 95^0 = 180^0$

$\therefore\ \angle RTP = 180^0 - 40^0 - 95^0$

$\boxed{\angle RTP = 45^0}$

$\angle STQ = \angle RTP$ [Vertically opposite angles]

$\therefore\ \boxed{\angle STQ = 45^0}$ [$\because \angle RTP = 45°$ (proved)]

In ΔTQS, we have

$\angle SQT + \angle STQ + \angle TSQ = 180^0$ [Angle sum property]

$\therefore\ \angle SQT + 45^0 + 75^0 = 180^0$ [$\because \angle STQ = 45°$ (proved)]

$\therefore\ \angle SQT = 180^0 - 45^0 - 75^0$

$\angle SQT = 60^0$

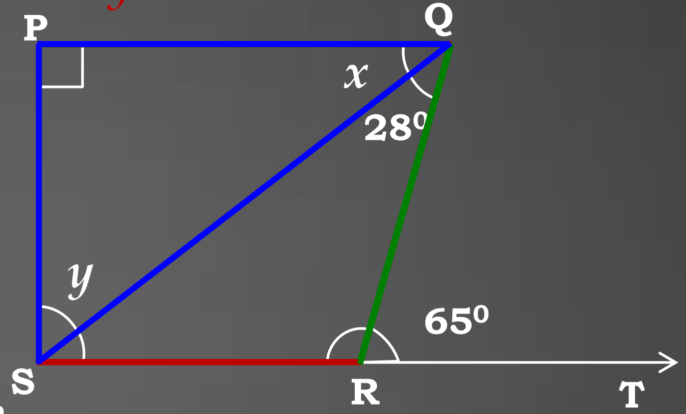
Hence, $\angle SQT = 60^0$

MODULE :

24

Ex.: 6.3 - 5

Q. In figure, If $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y.



Sol: PQ || SR and QR is a transversal line.

$\angle PQR = \angle QRT$ [Alternate interior angles]

$x + 28 = 65$

$x = 65 - 28$

$\boxed{x = 37}$

Using angle sum property for ΔSPQ, we obtain

$\angle SPQ + x + y = 180$

$90 + 37 + y = 180$

$y = 180 - 90 - 37$

$y = 53$

$\therefore$ $\boxed{x = 37 \text{ and } y = 53}$

MODULE :

25

Extra Example

If QT ⊥ PR, ∠ TQR = 40° and ∠ SPR = 30°, find x and y.

Sol.

In △ TQR, $90° + 40° + x = 180°$

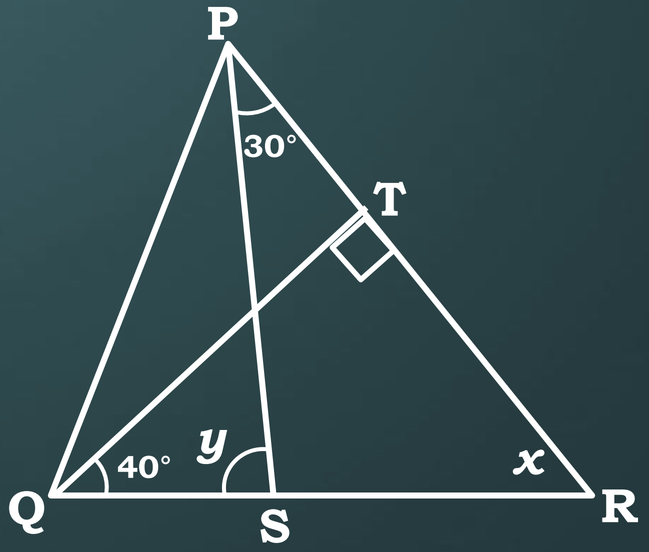
(Angle sum property of a triangle)

∴ $\boxed{x = 50°}$

$y = \angle SPR + x$

$y = 30° + 50°$

$\boxed{y = 80°}$



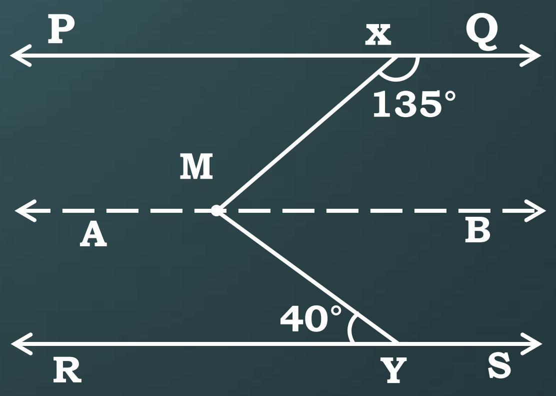
MODULE :

26

Extra Example

In Fig., if PQ || RS, $\angle$ MXQ = 135° and $\angle$ MYR = 40°, find $\angle$ XMY

Sol. Here, we need to draw a line AB parallel to line PQ, through point M as shown in Fig.



Now, AB || PQ and PQ || RS.

$\therefore$ AB || RS

$\therefore$ $\angle QXM + \angle XMB = 180°$

(AB || PQ, Interior angles on the same side of the transversal XM)

$\angle QXM = 135°$

$\therefore$ $135° + \angle XMB = 180°$

$\therefore$ $\angle XMB = 45°$ (1)

$\angle BMY = \angle MYR$ (AB || RS, Alternate angles)

$\therefore$ $\angle BMY = 40°$ (2)

Adding (1) and (2), you get

$\angle XMB + \angle BMY = 45° + 40°$

$\therefore$ $\boxed{\angle XMY = 85°}$

Thank You

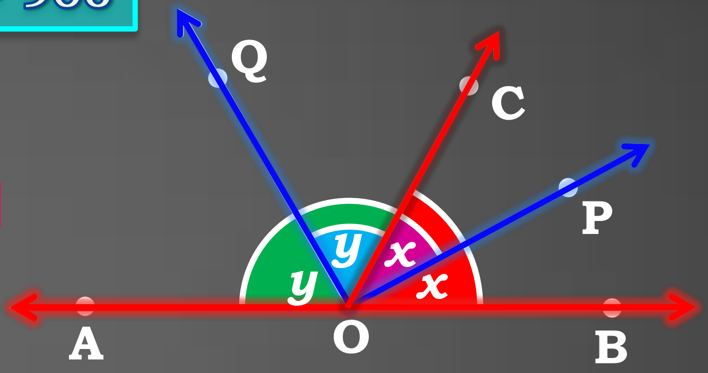
MODULE :

27

Q. OP and OQ bisects $\angle BOC$ and $\angle AOC$ resp. Show that $\angle POQ = 90°$.

Proof :

Just prove : $(x + y) = 90o$



OA and OB are on the same line

$\angle AOC + \angle COB = 180°$ [Linear pair]

$\therefore\ 2y + 2x = 180$

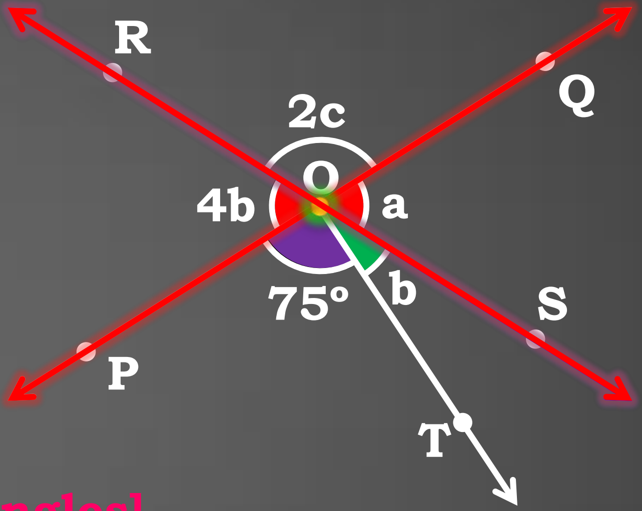
$\therefore\ 2(y + x) = 180$

$\therefore\ x + y = 90$

$\angle COP + \angle QOC = 90$

$\therefore\ \angle POQ = 90°$

Q. Two straight lines PQ and RS intersect each other at O. If $\angle POT = 75°$, find the values of a, b and c.



Sol. Since OR and OS are in the same line.

$\therefore \quad \angle ROP + \angle POT + \angle TOS = 180°$

$\therefore \quad 4b + 75 + b = 180$

$\therefore \quad 5b + 75 = 180$

$\therefore \quad 5b = 105$

$\therefore \quad \boxed{b = 21}$

Since PQ and RS intersect at O. Therefore,

$\angle QOS = \angle POR$ [Vertically opp. Angles]

$\therefore \quad a = 4b$

$\therefore \quad a = 4 \times 21$

$\therefore \quad a = 84$ [$\because$ b = 21]

Q. Two straight lines PQ and RS intersect each other at O. If $\angle POT = 75°$, find the values of a, b and c.

$a = 84$

Sol. Now, OR and OS are in the same line.
Therefore.

$\angle ROQ + \angle QOS = 180°$ [Linear pair]

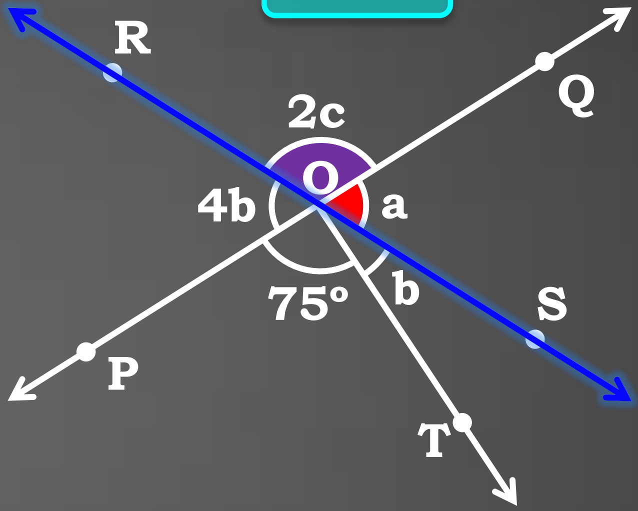
$\therefore \quad 2c + \boxed{a} = 180$

$\therefore \quad 2c + 84 = 180$

$\therefore \quad 2c = 96$

$\therefore \quad c = 48$

Hence, a = 84, b = 21 and c = 48



MODULE :

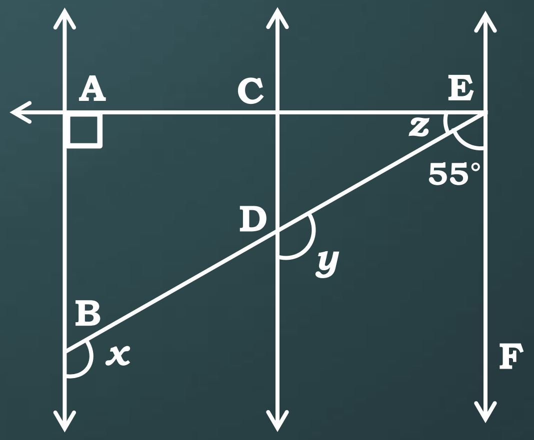
28

Extra Example.6

In Fig., AB || CD and CD || EF. Also EA ⊥ AB. If ∠ BEF = 55°, find the values of x, y and z

Sol. $y + 55^\circ = 180^\circ$

(Interior angles on the same side of the of the transversal ED)



∴ $y = 180^\circ - 55^\circ = 125^\circ$

$x = y$ (AB || CD, Corresponding angles axiom)

∴ $x = 125^\circ$

Now, since AB || CD and CD || EF

∴ AB || EF.

∴ ∠ EAB + ∠ FEA = 180° (Interior angles on the same side of the transversal EA)

∴ $90^\circ + z + 55^\circ = 180^\circ$

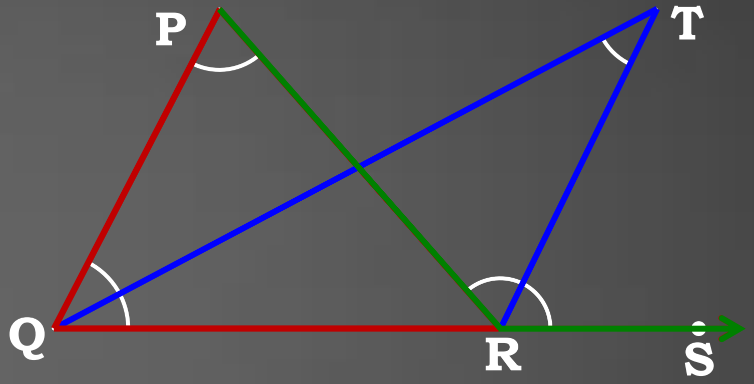
∴ $z = 35^\circ$

MODULE :

29

Ex . : 6.3 - 6

In figure, QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove $\angle QTR = \frac{1}{2}\angle QPR$.



Sol: $\angle TRS$ is an exterior angle to $\angle QTR$

$$\angle TRS = \angle QTR + \angle TQR$$

$$\therefore \angle QTR = \boxed{\angle TRS} - \boxed{\angle TQR} \quad \ldots \text{(i)}$$

$$\therefore \angle QTR = \frac{1}{2}\angle PRS - \frac{1}{2}\angle PQR$$

$$\therefore \angle QTR = \frac{1}{2}\boxed{(\angle PRS - \angle PQR)} \quad \ldots \text{(ii)}$$

$\angle PRS$ is an exterior angle to $\angle PQR$

$$\angle PRS = \angle PQR + \angle QPR$$

$$\therefore \boxed{\angle PRS - \angle PQR = \angle QPR} \quad \ldots \text{(iii)}$$

$$\therefore \boxed{\angle QTR = \frac{1}{2}\angle QPR}$$

MODULE :

30

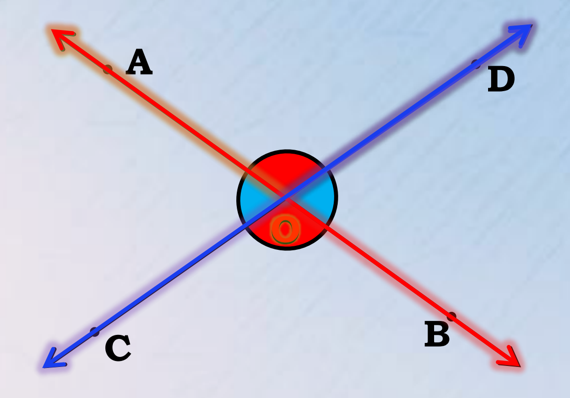
VERTICALLY OPPOSITE ANGLES THEOREM

If two lines intersect each other, then vertically opposite angles are equal.

Given: Line AB and line CD intersect each other in point O.

To prove : (i) $\boxed{\angle AOC = \angle BOD}$ ✓

(ii) $\boxed{\angle AOD = \angle BOC}$ ✓



Proof :

Line AB and line CD intersect in point O.

$\angle AOC + \angle AOD = \boxed{180^\circ}$ ...(i) *[Angles in a linear pair]*

$\angle AOD + \angle BOD = \boxed{180^\circ}$ ...(ii) *[Angles in a linear pair]*

$\angle AOC + \cancel{\angle AOD} = \cancel{\angle AOD} + \angle BOD$ *[From (i) and (ii)]*

$\therefore \quad \angle AOC = \angle BOD$

Similarly, it can be proved that $\angle AOD = \angle BOC$.

Thank You

MODULE :

31

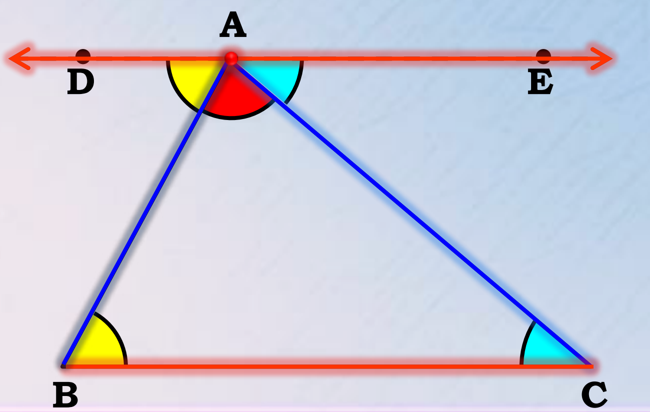
ANGLE SUM PROPERTY OF A TRIANGLE

Sum of all angles of a triangle is 180°.

Given : ΔABC is any triangle

To prove : $\angle ABC + \angle ACB + \angle BAC = 180°$

Construction : Through the point A, draw a DE || BC



Proof :

DE || BC

On transversal AB,

$\therefore \angle ABC = \angle DAB$ ...(i) *[Alternate-interior angles]*

On transversal AC,

$\therefore \angle ACB = \angle EAC$ ...(ii) *[Alternate-interior angles]*

$\angle ABC + \angle ACB = \angle DAB + \angle EAC$

[Adding (i) and (ii)]

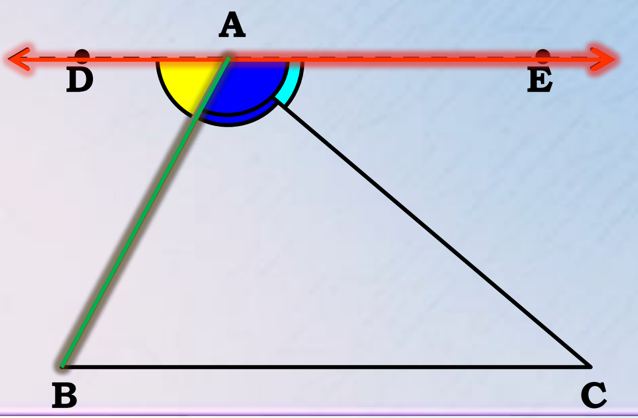
ANGLE SUM PROPERTY OF A TRIANGLE

Sum of all angles of a triangle is 180°.

Given : ΔABC is any triangle

To prove : $\angle ABC + \angle ACB + \angle BAC = 180°$

Construction : Through the point A, draw a DE || BC



Proof :

$\angle ABC + \angle ACB = \angle DAB + \angle EAC$

Adding $\angle BAC$ on both sides,

$\angle ABC + \angle ACB + \angle BAC = \angle DAB + \angle EAC + \angle BAC$

$\therefore \angle ABC + \angle ACB + \angle BAC = \angle DAB + \angle EAB$

$\therefore \angle ABC + \angle ACB + \angle BAC = 180°$

[Angles in a linear pair]

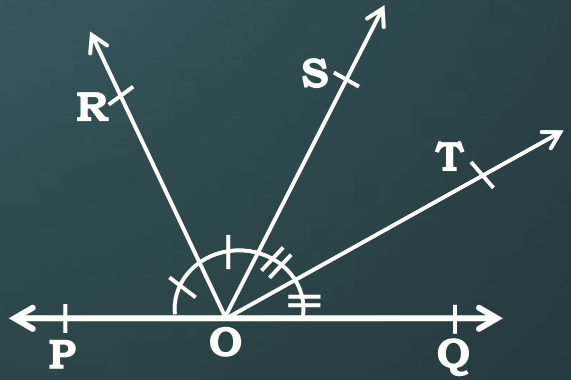
MODULE :

32

Extra Example

Ray OS stands on a line POQ. Ray OR and ray OT are angle bisectors of $\angle POS$ and $\angle SOQ$, respectively. If $\angle POS = x$, find $\angle ROT$.

Sol. Ray OS stands on the line POQ.



$$\therefore \angle POS + \angle SOQ = 180^\circ$$

$$\angle POS = x$$

$$x + \angle SOQ = 180^\circ$$

$$\angle SOQ = 180^\circ - x$$

Now, ray OR bisects $\angle POS$, therefore,

$$\angle ROS = \frac{1}{2} \times \angle POS$$

$$= \frac{1}{2} \times x = \frac{x}{2}$$

$$\angle SOT = \frac{1}{2} \times \angle SOQ$$

$$= \frac{1}{2} \times (180^\circ - x)$$

$$= 90^\circ - \frac{x}{2}$$

Extra Example.2

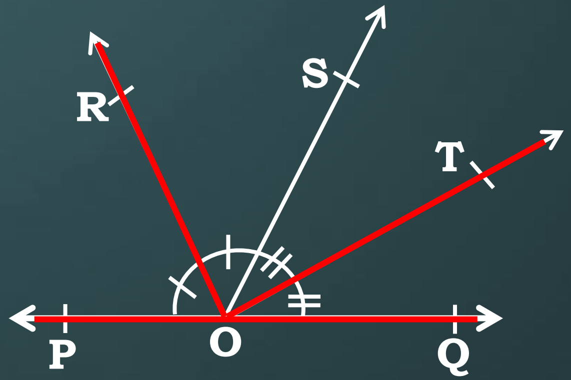
Ray OS stands on a line POQ. Ray OR and ray OT are angle bisectors of $\angle$ POS and $\angle$ SOQ, respectively. If $\angle$ POS = x, find $\angle$ ROT.

Sol.

$$\angle ROT = \angle ROS + \angle SOT$$

$$= \frac{x}{2} + 90° - \frac{x}{2}$$

$$\therefore \quad \boxed{\angle ROT = 90°}$$

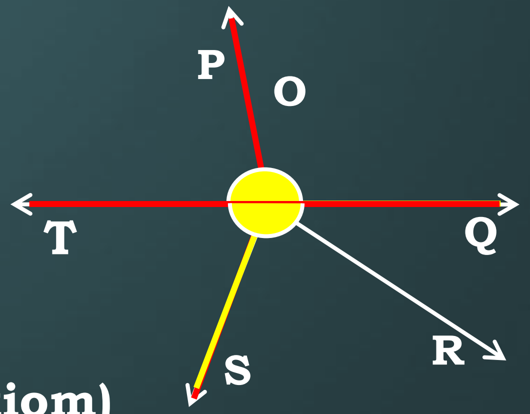


MODULE :

33

Extra Example.3

In the given Fig. OP, OQ, OR and OS are four rays. Prove that $\angle POQ + \angle QOR + \angle SOR + \angle POS = 360°$.



Sol.

In the given Fig., you need to produce any of the rays OP, OQ, OR or OS backwards to a point. Let us produce ray OQ backwards to a point T so that TOQ is a line.

Now, ray OP stands on line TOQ.

$\therefore \quad \angle TOP + \angle POQ = 180° \quad (1)$ (Linear pair axiom)

Similarly, ray OS stands on line TOQ

$\therefore \quad \angle TOS + \angle SOQ = 180° \quad (2)$

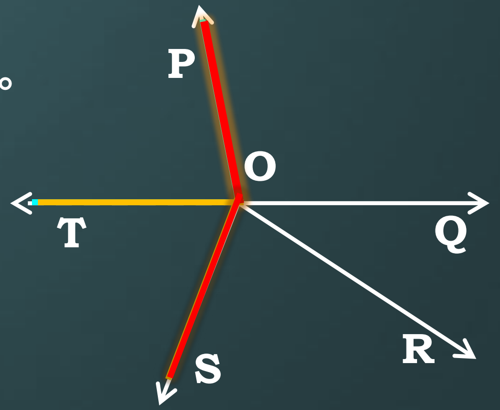
$\angle SOQ = \angle SOR + \angle QOR \quad (3)$

$\therefore \quad \angle TOS + \angle SOR + \angle QOR = 180° \quad (4)$ [From (2)and(3)]

Now, adding (1) and (4), you get

Extra Example.3

In the given Fig. OP, OQ, OR and OS are four rays. Prove that $\angle POQ + \angle QOR + \angle SOR + \angle POS = 360°$.



Sol.

$\boxed{\angle TOP} + \angle POQ + \boxed{\angle TOS} + \angle SOR + \angle QOR = 360°$

But $\angle TOP + \angle TOS = \angle POS$

$\therefore \quad \boxed{\angle POQ + \angle QOR + \angle SOR + \angle POS = 360°}$

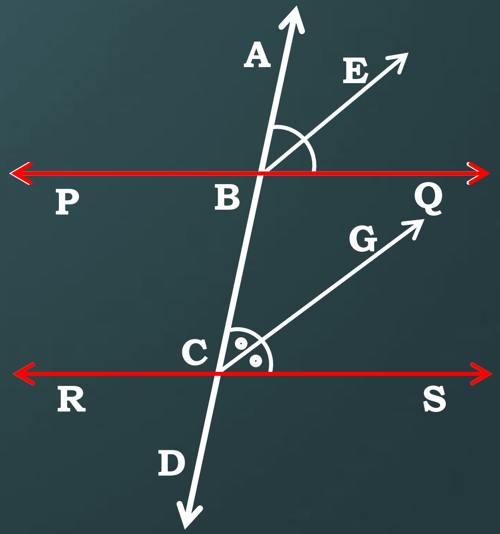
MODULE :

34

Extra Example.5

If a transversal intersects two lines such that the bisectors of a pair of corresponding angles are parallel, then prove that the two lines are parallel.

Sol. In Fig. a transversal AD intersects two lines PQ and RS at points B and C respectively. Ray BE is the bisector of $\angle ABQ$ and ray CG is the bisector of $\angle BCS$; and $BE \parallel CG$.



It is given that ray BE is the bisector of $\angle ABQ$.

$\therefore \quad \angle ABE = \frac{1}{2} \angle ABQ \qquad (1)$

Similarly, ray CG is the bisector of $\angle BCS$.

$\therefore \quad \angle BCG = \frac{1}{2} \angle BCS \qquad (2)$

But $BE \parallel CG$ and AD is the transversal

$\therefore \quad \angle ABE = \angle BCG$ (Corresponding angles axiom) (3)

Substituting (1) and (2) in (3), you get

$$\frac{1}{2} \angle ABQ = \frac{1}{2} \angle BCS$$

$\therefore \quad \angle ABQ = \angle BCS$

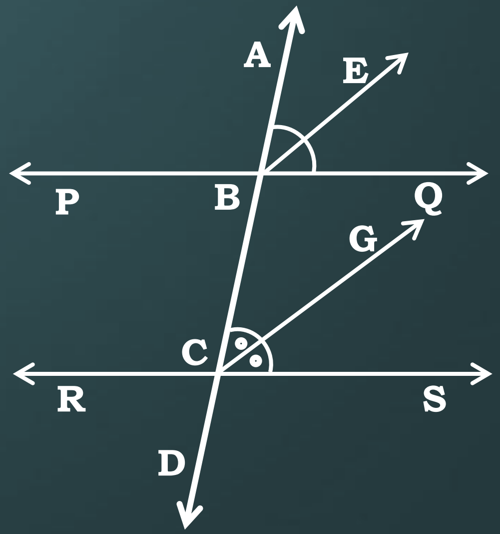
Extra Example.5

If a transversal intersects two lines such that the bisectors of a pair of corresponding angles are parallel, then prove that the two lines are parallel.

Sol.

But, they are the corresponding angles formed by transversal AD with PQ and RS; and are equal.

$\therefore$ PQ || RS (Converse of corresponding angles axiom)



Extra Example

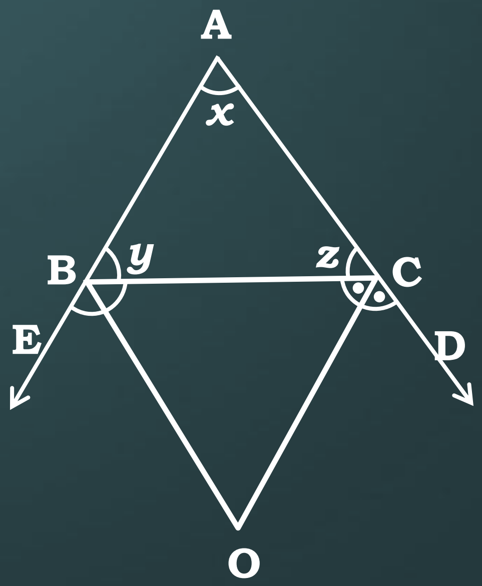
The sides AB and AC of $\triangle ABC$ are produced to points E and D respectively. If bisectors BO and CO of $\angle CBE$ and $\angle BCD$ respectively meet at point O, then prove that $\angle BOC = 90° - \frac{1}{2}\angle BAC$.

Sol. Ray BO is the bisector of $\angle CBE$.

$\therefore \quad \angle CBO = \frac{1}{2}\angle CBE$

$= \frac{1}{2}(180° - y)$

$= 90° - \frac{y}{2} \qquad (1)$



Similarly, ray CO is the bisector of $\angle BCD$.

$\angle BCO = \frac{1}{2}\angle BCD$

$= \frac{1}{2}(180° - z)$

$= 90° - \frac{z}{2} \qquad (2)$

In $\triangle BOC$, $\angle BOC + \angle BCO + \angle CBO = 180° \qquad (3)$

Extra Example

The sides AB and AC of $\triangle ABC$ are produced to points E and D respectively. If bisectors BO and CO of $\angle CBE$ and $\angle BCD$ respectively meet at point O, then prove that $\angle BOC = 90° - \frac{1}{2}\angle BAC$.

Sol. Substituting (1) and (2) in (3), you get

$$\angle BOC + 90° - \frac{z}{2} + 90° - \frac{y}{2} = 180°$$

$$\angle BOC = \frac{z}{2} + \frac{y}{2}$$

$$\angle BOC = \frac{1}{2}(y + z)$$

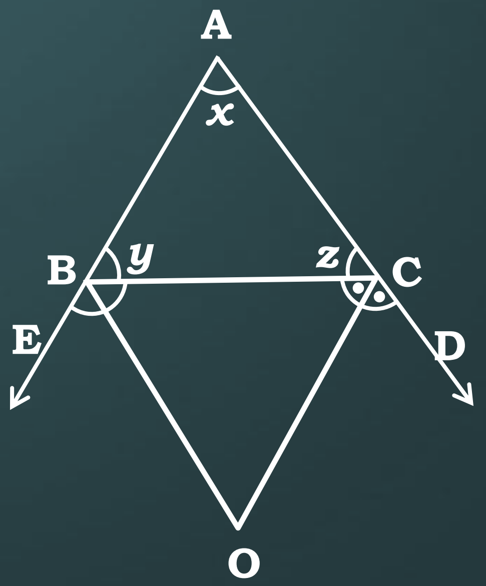
$x + y + z = 180°$ (Angle sum property of a triangle)

$$y + z = 180° - x$$

$$\angle BOC = \frac{1}{2}(180° - x)$$

$$= 90° - \frac{x}{2}$$

$$\boxed{\angle BOC = 90° - \frac{1}{2}\angle BAC}$$



Thank You