

Cubes and Cube Roots

1. Cube number

- Cube of a number is obtained when it is multiplied by itself thrice.
- If 'a' is a number, then the cube of 'a' is $a^3 = a \times a \times a$.
- Some of the cube numbers are 1, 8, 27, ...
- A natural number n is a perfect cube if n can be expressed as m^3 , for some natural number m. The numbers 1, 8, 27, ... are perfect cubes.

2. Properties of cubes

- Cubes of all even natural numbers are even.
- Cubes of all odd natural numbers are odd.
- Cubes of negative integers are negative.
- Cube of a number which ends in zero ends in three zeroes.
- Sum of the cubes of first n natural numbers is equal to the square of their sum.
- Unit digit of the cubes of the numbers ending in 1, 4, 5, 6 and 9 are 1, 4, 5, 6 and 9 respectively.
- Unit digit of the cubes of the numbers ending in 8 and 2 are 2 and 8 respectively.
- Unit digit of the cubes of the numbers ending in 3 and 7 are 7 and 3 respectively.
- For an integer a, a^3 is always greater than a^2 .
- If unit digit of a^2 is 9, then unit digit of a^3 is 7.
- Let x and y be any two integers, then $\sqrt[3]{xy} = \sqrt[3]{x} \times \sqrt[3]{y}$
- Let x and y be any two integers, then $\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}}, y \neq 0$

3. Adding consecutive odd numbers



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The cube a number natural 'n' is equal to the sum of 'n' consecutive odd natural numbers. Smallest among these 'n' consecutive number is $n^2 - (n - 1)$. The pattern is as follows:

$$1 = 1 = 1^3$$

$$3 + 5 = 8 = 2^3$$

$$7 + 9 + 11 = 27 = 3^3$$

$$13 + 15 + 17 + 19 = 64 = 4^3$$

And so on....

4. To find whether a number is a perfect cube or not:

- Express the number as the product of prime factors.
- If each factor appears three times in the prime factorization, the number is a perfect cube.
- If there are one or more factor(s) which do not appear 3 (or multiples of 3) times, the number is a non-perfect cube.
- $64 = 4 \times 4 \times 4$, is a perfect cube. $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$, is not a perfect cube.

5. Smallest multiple to convert a non-perfect cube into perfect cube

- Apply prime factorization to express the number as a product of prime factors.
- Arrange the same primes in a group of three.
- If a prime number can't be arranged in a group of 3, make it a group of 3 by multiplying with the smallest required number. Perform this operation with each of those prime numbers, which can't be grouped in 3.
- Product of all those numbers which have been multiplied will be the smallest multiple.
 Example: 1188 is not a perfect cube as $1188 = 2 \times 2 \times 3 \times 3 \times 3 \times 11$
 2 and 11 don't appear in groups of three.
 We need to multiply with $2 \times 11 \times 11 = 44$.
 Hence, the smallest multiple is 44.

Note: Same process can be applied for smallest divisor.

6. Cube root

- The cube root of a number x is that number whose cube gives x. It is denoted by $\sqrt[3]{x}$.
- For any positive integer x, we have $\sqrt[3]{-x} = -\sqrt[3]{x}$.

7. Finding cube root through prime factorization

- For finding the cube root of a perfect cube by prime factorization method, resolve it into prime factors; make triplets of similar factors and then take the product of prime factors, choosing one out of every triplet.
- Cube root of a perfect cube can also be evaluated through estimation.

8. Cube root of a cube number

Use the following steps to find the cube root of a cube number

- Take the number and start making group of three digits from the right.



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- The first group will give the unit's digit of the required cube root.
- Take the next group and find the greatest cube number less than this group whose unit's digit will become the ten's digit of the required cube root.

9. An interesting pattern about cubes and squares of natural numbers:

$$1^3 = 1$$

$$1^3 + 2^3 = (1 + 2)^2$$

$$1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2$$

$$1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2$$

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$$1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = (1 + 2 + 3 + 4 + \dots + n)^2$$

10. An interesting pattern to find the cube root of a natural number:

$$1^3 = 1$$

$$2^3 = 1 + 7 = 1 + \left(1 + \frac{2 \times 1}{2} \times 6 \right)$$

$$3^3 = 1 + 7 + 19 = 1 + \left(1 + \frac{2 \times 1}{2} \times 6 \right) + \left(1 + \frac{3 \times 2}{2} \times 6 \right)$$

$$4^3 = 1 + 7 + 19 + 37 = 1 + \left(1 + \frac{2 \times 1}{2} \times 6 \right) + \left(1 + \frac{3 \times 2}{2} \times 6 \right) + \left(1 + \frac{4 \times 3}{2} \times 6 \right)$$

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$$n^3 = 1 + 7 + 19 + 37 + \dots = 1 + \left(1 + \frac{2 \times 1}{2} \times 6 \right) + \left(1 + \frac{3 \times 2}{2} \times 6 \right) + \left(1 + \frac{4 \times 3}{2} \times 6 \right) + \dots + \left(1 + \frac{n \times (n-1)}{2} \times 6 \right)$$

