Lecture_06

No. 39

Arithmetic Progressions

- **Sums based on S_n formula**

10) Show that a_1, ... rm an ... Also find the sum of t ... erms.

Lets find a_1, a_2 & a_3

Lets find the difference between the consecutive terms

Sol: $a_n = 3 + 4n$

$a_1 = 3 + 4(1)$ $\quad a_2 = 3 + 4(2$ ... (3) $\quad d = a_2 - a_1$ $\quad d = a_3 - a_2$

To find S_{15}

a_1 = ... $= 11 - 7$ $\quad = 15 - 11$

To find a_1 put n = 1

To find a_2 put n = 2

To find a_3 put n = 3

a_1 = ... $= 4$ $\quad = 4$

... sub. n = 15, a = 7 & d = 4

$\therefore$ **As 'd' is constant, th ... ers is an AP.**

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{15} = \frac{15}{2}[2(7) + (15 - 1)4]$$

$$= \frac{15}{2}[14 + 14 \times 4]$$

$$= \frac{15}{2}[14 + 56]$$

$$= \frac{15}{\cancel{2}} \times \cancel{70}^{35}$$

$$= 15 \times 35$$

$$\therefore S_{15} = 525$$

$\therefore$ $\boxed{\textbf{Sum of first 15 terms is 525}}$

No. 40

Arithmetic Progressions

- **Sums based on S_n formula**

12) Find the sum of first 40 positive integers divisible by 6.

Sol: The positive integers which are divisible by 6 are 6, 12, 18, 24, ...

$\therefore$ These numbers fo ... 6 and d = 6

For S_{40} substitute n = 40, a = 6 & d = 6

Difference between consecutive terms are same

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_{40} = \frac{40}{2}[2(6) + (40-1)(6)]$$

$$= 20[12 + 39(6)]$$

$$= 20[12 + 234]$$

$$\therefore \quad S_{40} = 20(246)$$

$$\therefore \quad \mathbf{S_{40} = 4920}$$

$\therefore$ Sum of first 40 positive integers divisible by 6 is 4920.

Q.13. Find the sum of the first 15 multiples of 8.

Sol: The multiples of 8 a

These numbers forr

For S_{15} substitute $n = 15$, $a = 8$ & $d = 8$

…ence between consecutive terms are same

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2(8) + (15-1)8]$$

$$= \frac{15}{2}[16 + (14)8]$$

$$= \frac{15}{2}[16 + 112]$$

$$= \frac{15}{2} \times 128$$

$$= 15 \times 64$$

$$\therefore \quad S_{15} = 960$$

∴ Sum of the first 15 multiples of 8 is 960

No. 41

Arithmetic Progressions

- **Sums based on S_n formula**

9) If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Sol: For given AP: $S_7 = 49$ & $S_{17} = 289$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_7 = \frac{7}{2}[2a + (7-1)d]$$

$$\therefore 49 = \frac{7}{2}[2a + 6d]$$

$$\therefore 49 = \frac{7}{\cancel{2}} \times \cancel{2}(a + 3d)$$

$$\therefore a + 3d = 7 \quad \ldots.(i)$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{17} = \frac{17}{2}[2a + (17-1)d]$$

$$\therefore 289 = \frac{17}{2}[2a + 16d]$$

$$\therefore a + 8d = 17 \quad \ldots.(ii)$$

Subtracting (i) from (ii)

$$\begin{array}{rcl} \cancel{a} + 8d & = & 17 \\ \cancel{a} + 3d & = & 7 \\ (-) \quad (-) & & (-) \\ \hline 5d & = & 10 \\ \hline \end{array}$$

$$\therefore d = 2$$

Substituting d = 2 in (i)

$$\therefore a + 3(2) = 7$$

$$\therefore a + 6 = 7$$

$$\therefore a = 1$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2(1) + (n-1)2]$$

$$= \frac{n}{2}[2 + 2n - 2]$$

$$= \frac{n}{\cancel{2}} \times \cancel{2}n$$

$$\therefore \boxed{S_n = n^2}$$

Take 2 common

Substitute, n = 7

For given value of S_{17}, ...the formula

Substitute, a = 1 & d = 2

Take 2 common

Lets solve it by elimination method

Substitute, n = 17

Lets find S_n

No. 42

Arithmetic Progressions

- **Sums based on a_n and S_n formula**

2) Find the sum of the following AP.

(i) 34 + 32 + 30 + … + 10

Sol: For given AP: 34, 32, 30, …, 10.

$a = 34, \quad d = 32 - 34 = -2, \quad a_n$

We know that,

$$a_n = a + (n - 1)\,d$$

$$\therefore\ 10 = 34 + (n - 1)(-2)$$

$$\therefore\ 10 - 34 = (n - 1)(-2)$$

$$\therefore\ -24 = (n$$

$$\therefore\ 12 = n -$$

$$\therefore\ n = 13$$

$$\text{Now,}\quad S_n = \frac{n}{2}[a + a_n]$$

$$\therefore\ S_{13} = \frac{13}{2}[34 + 10]$$

$\therefore\ S$

6

We need to find sum of … P

Substitute,
$a_n = 10$, $a = 34$ & $d = -2$

We need value of 'n' to use S_n formula

To find number of terms check which term is 10. Because, it is the last term.

That means, We need to find S_{13}

For S_{13} substitute, $a = 34$, $a_n = 10$ & $n = 13$

2) Find the sum of the following AP.

(ii) -5 + (-8) + (-11) +...+ (-230)

Sol: For given AP: -5, -8, -11, ..., -230.

$a = -5, \quad d = -8 - (-5) = -3, \quad a_n$

We know that,

$$a_n = a + (n - 1)\, d$$

$$\therefore \; -230 = -5 + (n - 1)(-3)$$

$$\therefore \; -230 + 5 = (n - 1)(-3)$$

$$\therefore \; -225 = (n - 1$$

$$\therefore \; 75 = n - 1$$

$$\therefore \; n = 76$$

$$\text{Now,} \quad S_n = \frac{n}{2}[a + a_n]$$

$$\therefore \; S_{76} = \frac{\cancel{76}^{38}}{\cancel{2}}[-5 + (-230)]$$

$\therefore \; S_{76}$

- 8930

We need to find sum of ... P

Substitute, a_n = -230, a = -5 & d = -3

We need value of 'n' to use S_n formula

To find number of terms check which term is -230 Because, it is the last term.

That means, We need to find S_{76}

For S_{76} substitute, a = -5, a_n = -230 & n = 76

No. 43

Arithmetic Progressions

- **Sums based on a_n and S_n formula**

3) In an AP.

(viii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.

Sol:

$$a_{12} = a + 11d$$

$$\therefore \quad 37 = a + 11\,(3$$

$$\therefore \quad 37 = a +$$

$$\therefore \quad 37 - 33 = a$$

$$\therefore \quad a = 4$$

$$S_n = \frac{n}{2}\,[a + a_n]$$

$$S_{12} = \frac{12}{2}\,[4 + 37]$$

$$= 6 \times 41$$

$$= 246$$

$$\therefore \quad \boxed{a = 4,\ S_{12} = 246}$$

For given value of a_{12} Let's use the formula

Lets fi

Substitute $n = 12$, $a = 4$ & a_n i.e $a_{12} = 37$

3) In an AP.

i) Given $a = 5$, $d = 3$, $a_n = 50$

Sol: For given AP:

$a = 5$, $d = 3$, $a_n = 50$

We know that,

$$a_n = a + (n - 1)\,d$$

$$\therefore\ 50 = 5 + (n - 1)(3)$$

$$\therefore\ 50 - 5 = (n - 1)(3)$$

$$\therefore\ 45 = (n - 1)(3)$$

$$\therefore\ 15 = n - 1$$

$$\therefore\ n = 16$$

$$\text{Now,}\ S_n = \frac{n}{2}[a + a_n]$$

$$= \frac{\cancel{16}^{\,8}}{\cancel{2}}[5 + 50]$$

$$= 8\,[55]$$

$$\therefore\ S_n = 440$$

$$\therefore\ \boxed{n = 16,\ S_n = 440}$$

For given value of a_n, Lets use the formula

Substitute, $a_n = 50$, $a = 5$ & $d = 3$

Now lets find S_n

For S_n substitute, $a = 5$, $a_n = 50$ & $n = 16$

3) In an AP.

ii) Given $a = 7$, $a_{13} = 3$

For given value of a
Lets use the formu

For S_{13} substitute,
$n = 13$, $a = 7$ & $a_{13} = 35$

Sol: For given AP:

$a = 7, \quad a_{13} = 35$

We know that,

$$a_{13} = a + 12d$$

For a_{13} substitute,
$a = 7$ & $a_{13} = 35$

$$\therefore \; 35 = 7 + 12d$$

$$\therefore \; 35 - 7 = 12d$$

$$\therefore \; 28 = 12d$$

$$\therefore \; d = \frac{\cancel{28}^{\,7}}{\cancel{12}_{\,3}}$$

$$\therefore \; d = \frac{7}{3}$$

Now lets find S_{13}

$$\text{Now,} \quad S_n = \frac{n}{2}[a + a_n]$$

$$= \frac{13}{\cancel{2}}[\cancel{42}^{\,21}]$$

$$= 13[21]$$

$$\therefore \; S_{13} = 273$$

$$\therefore \; \boxed{d = \frac{7}{3}, \quad S_{13} = 273}$$

Thank You