Lecture 1

NUMBER SYSTEM

NUMBERS

Include negative of natural numbers

NATURAL NUMBERS : 1, 2, 3, 4, 5, ...

WHOLE NUMBERS : 0, 1, 2, 3, 4, 5, ...

INTEGERS : ... , -3, -2, -1, 0, 1, 2, 3, ...

NUMBERS

INTEGERS : … , -3, -2, -1, 0, 1, 2, 3, …

RATIONAL NUMBERS : If p is any integer and q any non-zero integer, then $\frac{p}{q}$ is a rational number.

$\frac{5}{9}$ $\frac{-3}{1}$? $\frac{4}{3}$ $\frac{1}{5}$

Ratio of integers

$$\text{Numerator} \rightarrow \frac{\text{Integer}}{\text{Non-zero Integer}} \leftarrow \text{Denominator}$$

RATIONAL NUMBERS

If p is any integer and q any non-zero integer, then $\frac{p}{q}$ is a rational number.

Decimal form of rational number is terminating or non-terminating & recurring.

$\frac{9}{4} = \boxed{2.25}$ $\rightarrow$ Terminating

$\frac{11}{3} = \boxed{3.666...} = 3.\bar{6}$ $\rightarrow$ Non-terminating and recurring

$4.914914... = 4.\overline{914}$

Identify Rational Numbers

$0.04\overline{8}$ → Rational number recurring

$4.\overline{914}$ → Rational number recurring

2.066 → Rational number

$0.0122547825...$ → Not a Rational number recurring

$3.1415926538...$ → Not a Rational number recurring

$2.61353029864...$ → Not a Rational number recurring

$1.90357415569...$ → Not a Rational number recurring

$270.\overline{253}$ → Rational number recurring

IRRATIONAL NUMBERS

Numbers whose decimal form is non-terminating and non-recurring are called Irrational numbers.

These decimal numbers are non-terminating and non-recurring

0.0122547825...

3.14159[illegible]6538...

2.6135[illegible]029864...

1.90357415569...

IRRATIONAL NUMBERS

Numbers whose decimal form is non-terminating and non-recurring are called Irrational numbers.

The square roots of numbers that are not perfect squares are Irrational numbers.

$\sqrt{2}$ = 1.414213562373...

$\sqrt{3}$ = 1.732050807568...

$\sqrt{5}$ = 2.236067977499...

$\sqrt{6}$ = 2.449489742783...

REAL NUMBERS

RATIONAL NUMBERS

$\dfrac{\text{Integer}}{\text{Non-zero integer}}$

Terminating or Non-terminating recurring

IRRATIONAL NUMBERS

Non-terminating Non-recurring

INTEGERS : ..., -2, -1, 0, 1, 2, ...

WHOLE NUMBERS : 0, 1, 2, 3, ...

NATURAL NUMBERS : 1, 2, 3, 4, ...

EUCLID'S DIVISION ALGORITHM

DIVIDEND = DIVISOR × QUOTIENT + REMAINDER

For two given positive integers *a* and *b* there exist unique integers *q* and *r* satisfying $a = bq + r\ ;\ 0 \le r < b$

DIVISOR

DIVIDEND

QUOTIENT

REMAINDER

Ex 8 ÷ 2

$$\begin{array}{r|l} 2 & 8 \;(4 \\ & -8 \\ \hline & 0 \end{array}$$

$8 = 2 \times 4 + 0$

$a = b \times q + r$

Example 2 : 15 ÷ 2

$$\begin{array}{r|l} 2 & 15 \;(7 \\ & -14 \\ \hline & 1 \end{array}$$

$15 = 2 \times 7 + 1$

$a = b \times q + r$

Exercise 1.1

Q.1 Is zero a rational number ? Can you write it in the form where p and q are $\frac{p}{q}$, integers and $q \neq 0$.

Sol: **Yes. Zero is a rational number.**

Zero can be written in any of the following ways.

$\frac{0}{1}, \frac{0}{-2}, \frac{0}{3}, \frac{0}{-7}$, **etc.**

Thus 0 can be written as $\frac{p}{q}$, where $p = 0$ and q is any non - zero integer .

$\therefore$ **Hence 0 is a rational number.**

Note: To find 'n' rational numbers between any two rational numbers, we will multiply both the numbers by $\frac{(n+1)}{(n+1)}$

Exercise 1.1

Q.2 Find six rational numbers between 3 and 4.

Sol: **Since we require 6 rational numbers between 3 and 4, So we write**

$\therefore \quad \frac{3}{1} = \frac{3}{1} \times \frac{7}{7} = \frac{21}{7}$ and $\frac{4}{1} = \frac{4}{1} \times \frac{7}{7} = \frac{28}{7}$

Also 21 < 22 < 23 < 24 < 25 < 26 < 27 < 28

$\therefore \quad \frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$

To find '6' rational numbers between any two rational numbers, we multiply both numbers by $\frac{(6+1)}{(6+1)} = \frac{7}{7}$

$\therefore$ **Hence six rational numbers between 3 and 4 are** $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$ **and** $\frac{27}{7}$

Exercise 1.1

Q.3 Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Soln: Since we require 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, So we write

$$\therefore \quad \frac{3}{5} = \frac{3}{5} \times \frac{6}{6} = \frac{18}{30} \text{ and } \frac{4}{5} = \frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$$

To find '5' rational numbers between any two rational numbers ,we multiply both numbers by $\frac{(5 + 1)}{(5 + 1)} = \frac{6}{6}$

Also 18 < 19 < 20 < 21 < 22 < 23 < 24

$$\therefore \quad \frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$$

$\therefore$ Hence 5 rational number between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{19}{30}$, $\frac{20}{30}$, $\frac{21}{30}$, $\frac{22}{30}$ and $\frac{23}{30}$.

Exercise 1.1

Q.4 State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

Integers: ...,-3,-2,-1,0,1,2,3,...
Whole numbers: 0,1,2,3,...

Sol: **TRUE** : Every natural number lies in the collection of whole numbers.

Natural numbers: 1,2,3,4,5,...
Whole numbers: 0,1,2,3,4,5

(ii) Every integer is a whole number.

whole numbers 0,1,2,3... tural

Sol: **FALSE** : – 3 , – 2... ar

...,-3,-2,-1 are not w

These can be also written as

(iii) Every rational number is a whol

But $\frac{1}{2}, \frac{3}{4}, \frac{5}{2}, \frac{7}{2}$ these are not whole number's

Sol: **FALSE** : $\frac{1}{2}, \frac{3}{4}, \frac{5}{2}, \frac{7}{2},$ are not a whole numbers.

Exercise 1.2

Q.1 State whether the following statements are true or false. Justify your answers.

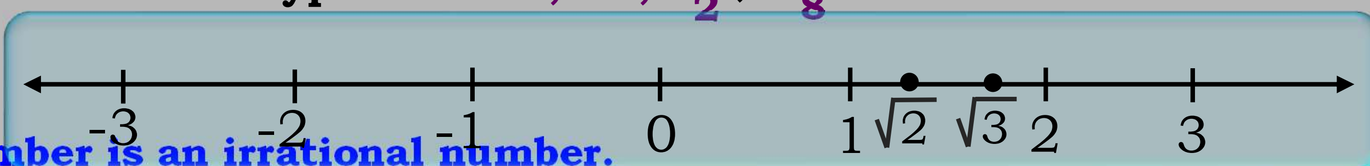
(i) Every irrational number is a real number.

Sol. **TRUE.** A real number is either rational or i

Every point on number line is a real number which is rational or irrational number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

Sol. **FALSE.** Number of other types like $-5, -4, \frac{1}{2}, -\frac{7}{8}$ also lie on the number line.



(iii) Every real number is an irrational number.

Sol. **FALSE.** Rational numbers are also real numbers.

Exercise 1.2

Q.2 **Are the square roots of all positive integers irrational ? If not, give an example of the square root of a number that is a rational number.**

Sol. **No.** **square roots of all positive integers are not irrational.**

e.g. 4, 9, 16, 25 .... etc. are positive integers but their square roots are rational numbers.

i.e. $\sqrt{4} = \frac{2}{1}$, $\sqrt{9} = \frac{3}{1}$, $\sqrt{16} = \frac{4}{1}$, $\sqrt{25} = \frac{5}{1}$, ...

Lecture 2

Exercise 1.3

Q.1 Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$

Sol. $\frac{36}{100}$ = 0.36, terminating decimal

(ii) $\frac{1}{11}$

Sol. $\frac{1}{11}$

$$
\begin{array}{r}
0.09090 \\
11 \overline{)\,1.0000} \\
-99 \\
\hline
100 \\
-99 \\
\hline
100 \\
-99 \\
\hline
1
\end{array}
$$

1 can be also written as 1.0000

$\therefore$ $\frac{1}{11} = 0.090909\ldots = 0.\overline{09}$, recurring decimal

Exercise 1.3

Q.1 Write the following in decimal form and say what kind of decimal expansion each has :

(iii) $4\frac{1}{8}$

Sol. $4\frac{1}{8} = \frac{32 + 1}{8} = \frac{33}{8} = 4.125$, terminating decimal

Mixed fraction

Is it terminating or recurring?

1.3
Q.1 (v) (vi)

Exercise 1.3

Q.1 **Write the following in decimal form and say what kind of decimal expansion each has :**

(iv) $\frac{2}{11}$

Sol. $\frac{2}{11}$

$\therefore \quad \frac{2}{11} = 0.1818... \qquad = 0.\overline{18}$, **recurring decimal**

$$
\begin{array}{r|l}
 & 0.1818 \\
11 & 2.0000 \\
 & -11 \\
 & \quad 90 \\
 & \; -88 \\
 & \quad\;\; 20 \\
 & \quad\;\; 90 \\
 & \quad -88 \\
 & \qquad\; 2
\end{array}
$$

2 can be also written as 2.0000

Exercise 1.3

Q.2 You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$, are, without actually doing the long division? If so, how?

Sol. decimals which are repetation of

1, 4, 2, 8, 5, 7

e.g. $2 \times \frac{1}{7}$

$\therefore 2 \times 0.142857$

$$\begin{array}{r} {\scriptstyle 1\,1\,1} \\ 0.142857 \\ \times\ 2 \\ \hline 0.285714 \end{array}$$

e.g. $3 \times \frac{1}{7}$

$\therefore 3 \times 0.142857$

$$\begin{array}{r} {\scriptstyle 1\quad 2\,1\,2} \\ 0.142857 \\ \times\ 3 \\ \hline 0.428571 \end{array}$$

Exercise 1.3

Q.2 You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$, are, without actually doing the long division? If so, how?

eg: $4 \times \left(\frac{1}{7}\right)$

$\therefore\ 4 \times 0.142857$

$\frac{1}{7} = 0.142857$

$$\begin{array}{r} {\scriptsize 1\ 1\ 3\ 2\ 2} \\ 0.142857 \\ \times\ 4 \\ \hline 0.571428 \end{array}$$

eg: $5 \times \left(\frac{1}{7}\right)$

$\therefore\ 5 \times 0.142857$

$\frac{1}{7} = 0.142857$

$$\begin{array}{r} {\scriptsize 2\ 1\ 4\ 2\ 3} \\ 0.142857 \\ \times\ 5 \\ \hline 0.714285 \end{array}$$

eg: $6 \times \left(\frac{1}{7}\right)$

$\therefore\ 6 \times 0.142857$

$\frac{1}{7} = 0.142857$

$$\begin{array}{r} {\scriptsize 2\ 1\ 5\ 3\ 4} \\ 0.142857 \\ \times\ 6 \\ \hline 0.857142 \end{array}$$

Exercise 1.3

Q.3 Express the following in the form $\frac{p}{q}$, where p and q are integers, $q \neq 0$

(i) $0.\overline{6}$

Hence multiply both sides by 10 which has one zero

0.66... × 10 will be 6.66....

$0.\overline{6}$ can be written as 0.66...

After the decimal point How many digits are repeating

Only one digit i.e 6

Sol: Let $x = 0.\overline{6}$

i.e $x = 0.66...$ ....(i)

Multiplying both sides by 10, we get

$10x = 6.66...$ ....(ii)

Subtracting (i) from (ii), we get

$10x - x = 6.66.... - 0.66...$

$$\begin{array}{r} 6.66 \\ -\ 0.66 \\ \hline 6.00 \end{array}$$

$\therefore\ 9x = 6.0$

$$x = \frac{6}{9}$$

$$\boxed{\therefore\ 0.\overline{6} = \frac{2}{3}}$$

Exercise 1.3

Q.3 Express the following in the form $\frac{p}{q}$, where p and q are integers, $q \neq 0$

(ii) $0.4\overline{7}$

Sol. Let $x = 0.4\overline{7} = 0.477....$

Multiplying both sides by 10, we get

$10x = 4.777....$ ------------- (1)

Multiplying both sides by 100, we get

$100x = 47.777....$ ------------- (2)

Subtracting (1) from (2), we get

$100x - 10x = (47.777....) - (4.777...)$

$90x = 43$

$\therefore \quad x = \frac{43}{90}$

$\therefore \quad \boxed{0.4\overline{7} = \frac{43}{90}}$

Exercise 1.3

Q.3 Express the following in the form $\frac{p}{q}$, where p and q

iii) $0.\overline{001}$

Sol: Let $x = 0.\overline{001}$

i.e $x = 0.001001...$ ....(i)

Multiplying both sides by 1000, we get

$1000x = 1.001001...$ ....(ii)

Subtracting (i) from (ii), we get

$$1000x - x = 1.001001... - 0.001001...$$

$$\therefore \quad 999x = 1$$

$$\therefore \quad x = \frac{1}{999}$$

$$\boxed{\therefore \quad 0.\overline{001} = \frac{1}{999}}$$

Hence multiply both sides by 1000 which has three zeros

he decimal point many digits are repeating

$0.001001 ..... \times 1000 = 1.001001....$

Three digit i.e 001

$$\begin{array}{r} 1.001001..... \\ -\ 0.001001.... \\ \hline 1.000000 \end{array}$$

Exercise 1.3

Q.4 **Express 0.99999 ..... in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and c... why the answer makes sense.**

Sol.

Let $x = 0.9999 .....$ $= 0.\overline{9}$ --------- (1)

Multiplying both sides by 10, we get

$10x = 9.9999 .....$ --------- (2)

Subtracting (1) from (2), we get

$10x - x = (9.9999....) - (0.9999....)$

$9x = 9$

$x = 1$

$\therefore$ $\boxed{0.9999 .. = 1}$

Yes, at a glance we are surprised at our answer.

when we observe that 0.9999.... goes on forever.

1 and 0.9999.......and hence they are equal.

After the decimal point How many digits are repeating

$0.9999 \times 10 = 9.9999$

0.9999 can be written as $0.\overline{9}$

Hence multiply both sides by 10 which has one zero

Only one digit i.e 9

$-\quad 0.9999....$

9.0000

But the answer makes sense

So there is no gap between

Lecture 3

Exercise 1.3

1 can be also written as 1.0000......

Q.5 What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Sol.

$$
\begin{array}{r|l}
 & 0.0588235294117647 \\
\hline
17 & 1.0000000000000000 \\
 & -85 \\
 & 150 \\
 & -136 \\
 & 140 \\
 & -136 \\
 & 40 \\
 & -34 \\
 & 60 \\
 & -51 \\
 & 90 \\
 & -85 \\
 & 50 \\
 & -34 \\
 & 160 \\
 & -153 \\
 & 70 \\
 & -68 \\
 & 20 \\
 & -17 \\
 & 30 \\
 & -17 \\
 & 130 \\
 & -119 \\
 & 110 \\
 & -102 \\
 & 80 \\
 & -68 \\
 & 120 \\
 & -119 \\
 & 1
\end{array}
$$

$\because$ 1 < 17

$\because$ 10 < 17

But 100 > 17

Hence

In t... er decimal ... 6 digits

$\therefore$ The maximum number of digits in the quotient while computing $\frac{1}{17}$ are 16.

$\therefore \frac{1}{17} = \boxed{0.\overline{0588235294117647}}$

Exercise 1.3

Q.6 Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Sol. The denominator of the given rational number has either 2 or 5 or both of them as the only prime factors.

$$\frac{1}{8} = \frac{1}{2 \times 2 \times 2}$$

$$\begin{array}{r|l} 8 & 10 \\ & \underline{8} \\ & 20 \\ & \underline{16} \\ & 40 \\ & \underline{40} \\ & 0 \end{array} \quad = 0.125$$

Let us divide

$$\frac{1}{25} = \frac{1}{5 \times 5}$$

$$\begin{array}{r|l} 25 & 100 \\ & \underline{100} \\ & 0 \end{array} \quad = 0.04$$

$\because$ 10 < 25

$$\frac{1}{50} = \frac{1}{2 \times 5 \times 5}$$

$$\begin{array}{r|l} 50 & 100 \\ & \underline{100} \\ & 0 \end{array} \quad = 0.02$$

$\because$ 10 < 50

$$\begin{array}{r|r} 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Let us divide

If the denominator of a rational number in standard form has no prime factors other than 2 or 5, then and only then is can be represented as a terminating decimal.

Exercise 1.3

Q.7 **Write three numbers whose decimal expansions are non - terminating non - recurring.**

Sol. **Three numbers whose decimal representations are non - terminating** and non - recurring are $\sqrt{2}, \sqrt{3}, \sqrt{5}$, ... etc.

OR

0.1010010001 ...., −1.2323423452....... and 0.4040040004

For the number to have non-terminating, non-recurring decimal expansions. It should be irrational number

Exercise 1.3

Q.8 Find three different irrational numbers between the rational $\frac{5}{7}$ and $\frac{9}{11}$

Soln.

$$\begin{array}{r|l} & 0.7142\ldots \\ 7 & \boxed{5}\boxed{.0}000 \\ & -49 \\ & \quad 10 \\ & \quad -7 \\ & \quad\ 30 \\ & \quad -28 \\ & \qquad 20 \\ & \qquad -14 \\ & \qquad\ \ 6 \\ & \qquad\ \ \vdots \end{array}$$

$$\begin{array}{r|l} & 0.81.. \\ 11 & 9.00 \\ & -88 \\ & \quad 20 \\ & \quad -11 \\ & \qquad 9 \end{array}$$

Write any three different Irrational numbers between 0.714285... and 0.81...

$\therefore$ $\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

Thus three different irrational numbers between $\frac{5}{7}$ & $\frac{9}{11}$ are 0.727207200....., 0.7676676667........ and 0.8080080008.......

Exercise 1.3

Q.8 Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$

Sol. $\sqrt{23}$ **is an irrational number as 23 is not a perfect square.**

(ii) $\sqrt{225}$

Sol. $\sqrt{225} = \sqrt{15 \times 15} = 15$ **which is rational.**

$\therefore$ $\sqrt{225}$ **is a rational number.**

(iii) **0.3796**

Sol. **0.3796 is a rational number as it is terminating decimal.**

(iv) **7.478478.....**

Sol. **7.478478 .... is non-terminating but repeating so it is a rational**

(v) **1.101001000100001....**

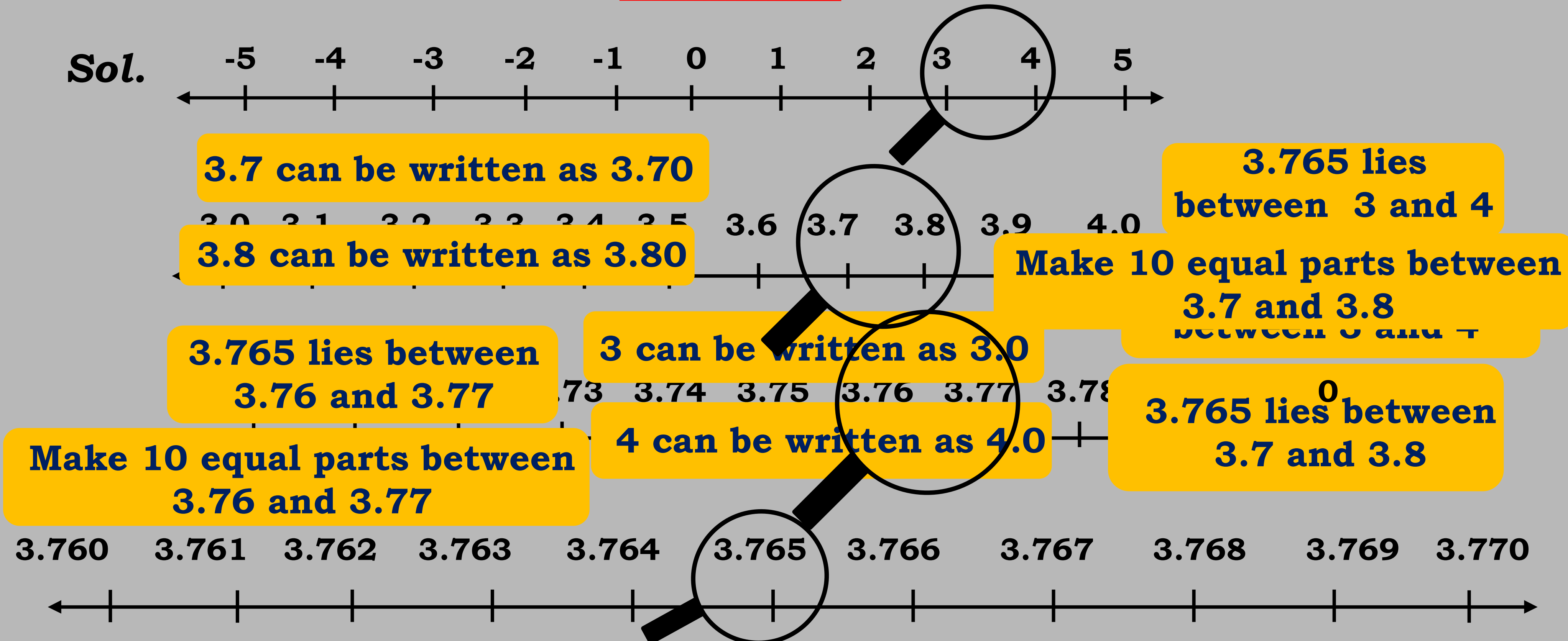
Sol. **1.1010010001..... is non - terminating and non - repeating, so it is an irrational number.**

Lecture 4

Exercise 1.4

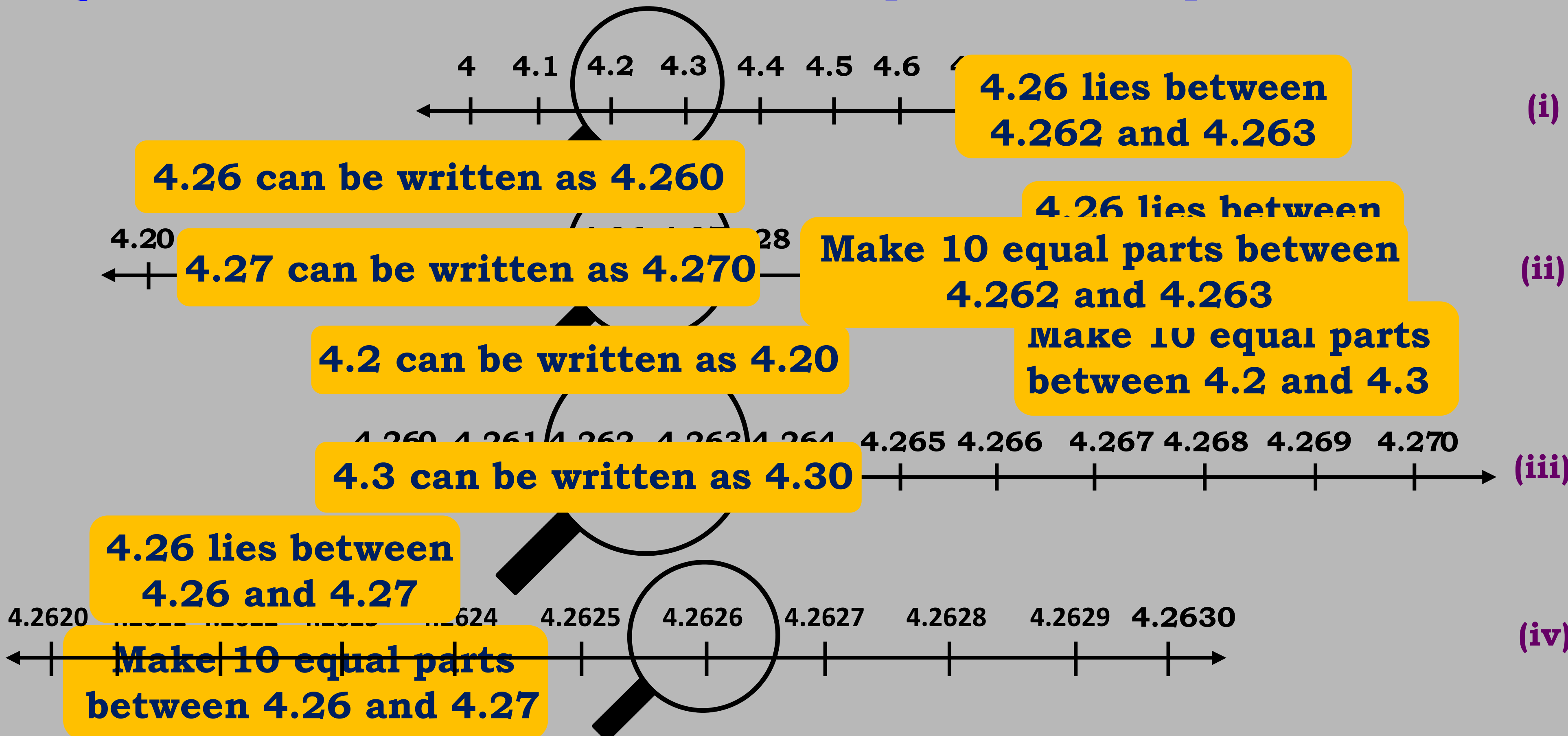
Q. 1 **Visualize 3.765 on the number line using successive magnification.**

Sol.



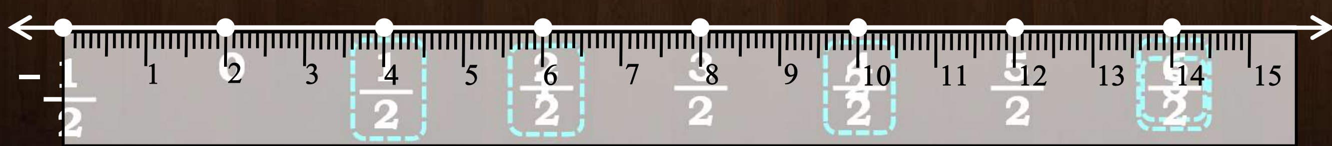
Exercise 1.4

Q. 2 Visualise 4.26 on the number line up to 4 decimal places.



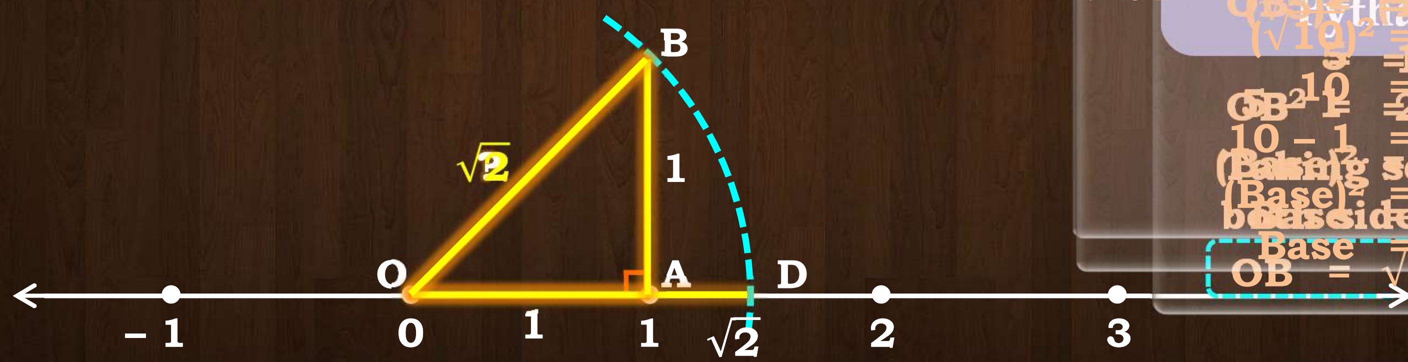
Represent $\frac{1}{2}$, 3, $\sqrt{2}$ on the number line.

$\sqrt{2}$ is not seen on the number line



Represent $\frac{1}{2}$, 3, $\sqrt{2}$ on the number line.

To plot $\sqrt{10}$ on the number line



In ΔOAB

By Pythagoras Theorem

$(Hypotenuse)^2 = (Base)^2 + (Height)^2$

$OB^2 = (1)^2 + (1)^2$

$OB^2 = 1 + 1$

$OB^2 = 2$

Taking square root on both sides we get,

$OB = \sqrt{2}$ units

To plot $\sqrt{10}$ on the number line consider,

Hypotenuse = $\sqrt{10}$

Height = 1 unit

Base = ?

Note : To plot an irrational number on the number line consider,

Hypotenuse = irrational number

Height = 1 unit

Base = Obtained by Pythagoras theorem

Practice set 1.2

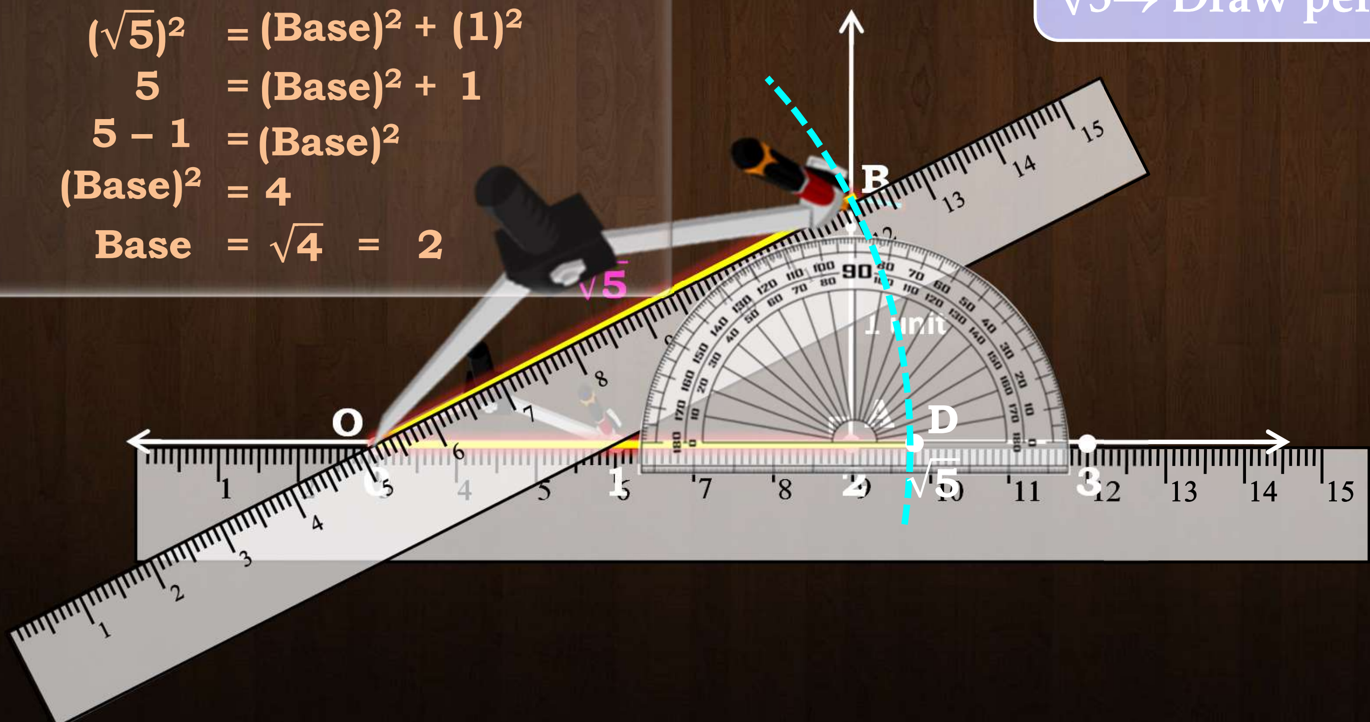
Q.2 Represent the number $\sqrt{5}$ on the number line.

Scale :
3 cm = 1 unit

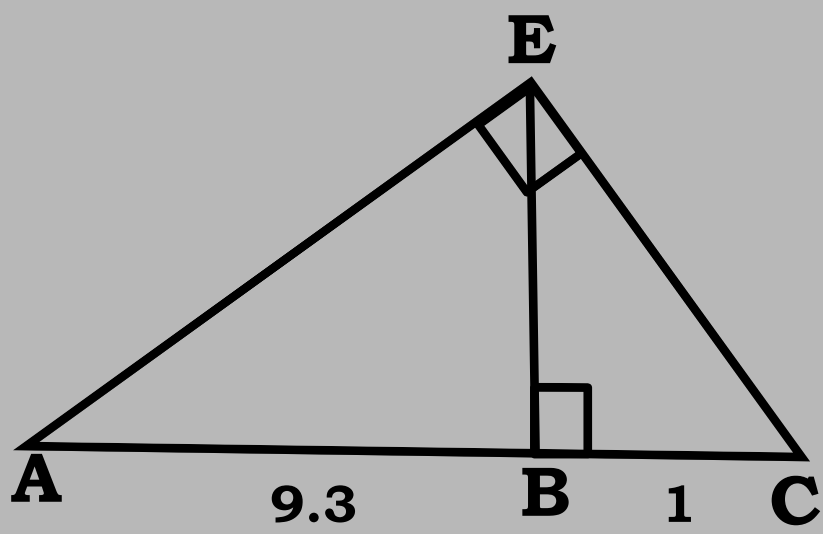
To draw
$\sqrt{5} \rightarrow$ Draw perpendicular at 2

By Pythagoras Theorem,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$$
$$(\sqrt{5})^2 = (\text{Base})^2 + (1)^2$$
$$5 = (\text{Base})^2 + 1$$
$$5 - 1 = (\text{Base})^2$$
$$(\text{Base})^2 = 4$$
$$\text{Base} = \sqrt{4} = 2$$



PROPERTY OF GEOMETRIC MEAN



This property is applicable only to right angled triangles

In $\triangle AEC$,

$m\ \angle AEC = 90^\circ$

seg EB $\perp$ hypotenuse AC

$\therefore\quad EB^2 = AB \times BC$

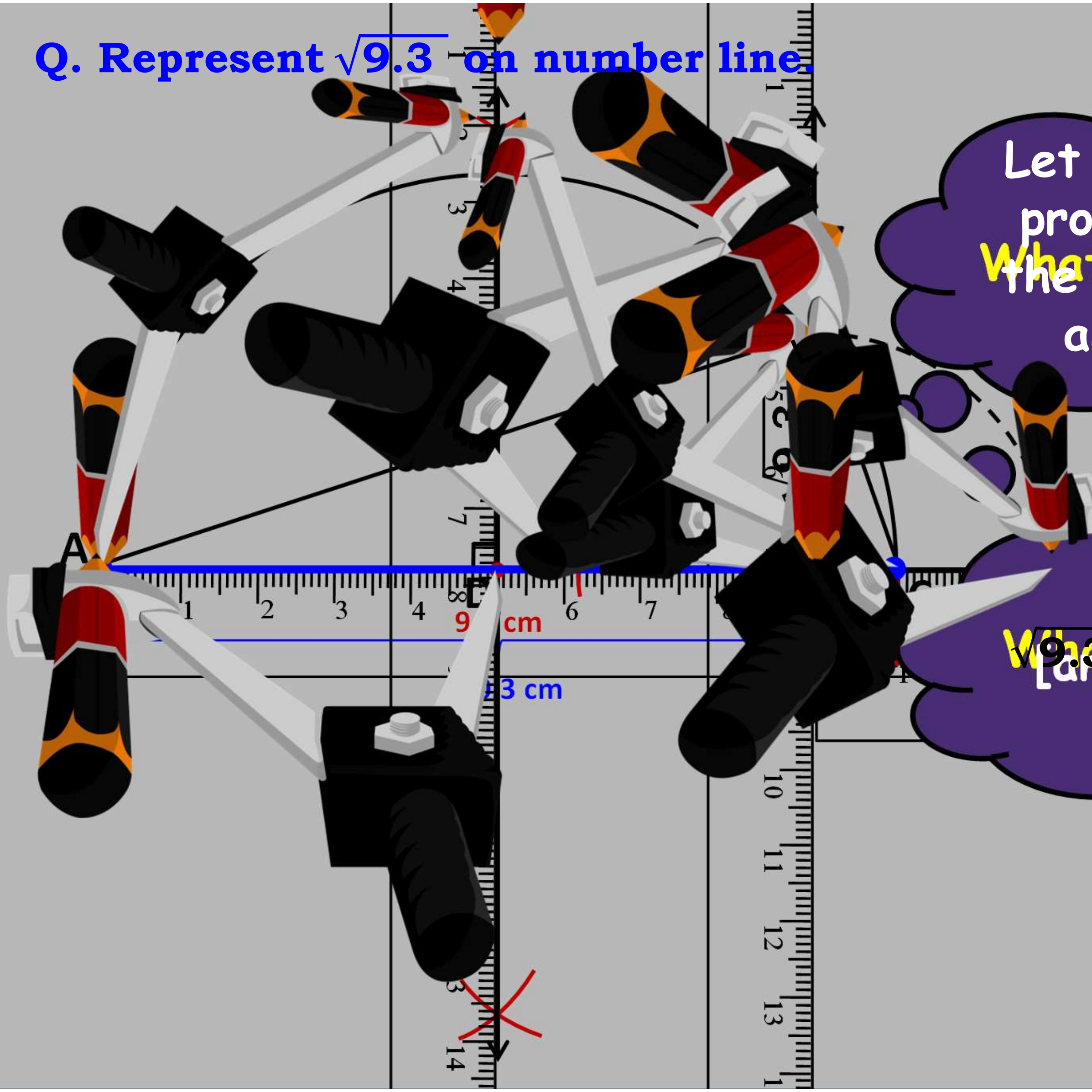
EB is the geometric mean of AB and BC

$$EB^2 = AB \times BC$$

$$= 9.3 \times 1$$

$$EB = \sqrt{9.3}$$

Q. Represent $\sqrt{9.3}$ on number line.



Let us use the same property by joining the sides EA and EC and form ΔAEC

What? How?

In ΔAEC,

$m\angle AEC = 90^\circ$

seg EB $\perp$ hypotenuse AC

$$\therefore \quad EB^2 = AB \times BC$$

$$= 9.3 \times 1$$

$$EB = \sqrt{9.3}$$

What is $\angle AEC$?

[angle subtended by semicircle]

...compass rotate BE so that it intersect seg AC at point F

Lecture 5

Exercise 1.5

Q.1 Classify the following numbers as rational or irrational :

(i) $2 - \sqrt{5}$

Sol. $2 - \sqrt{5}$ **is an irrational number being a difference between a rational and an irrational.**

Rational number – Irrational number = Irrational number

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

Sol. $(3 + \sqrt{23}) - \sqrt{23} = 3 + \cancel{\sqrt{23}} - \cancel{\sqrt{23}}$

$= 3$, **which is a rational number.**

(iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}}$

Ratio of two integers

Sol. $\dfrac{2\cancel{\sqrt{7}}}{7\cancel{\sqrt{7}}} = \dfrac{2}{7}$, **which is a rational number.**

Exercise 1.5

Q.1 Classify the following numbers as rational or irrational :

(iv) $\frac{1}{\sqrt{2}}$

Sol. $\frac{1}{\sqrt{2}}$ **is irrational being the quotient of a rational and an irrational.**

(v) 2π

Sol. 2π **is irrational being the product of rational and irrational.**

Exercise 1.5

Use the identity $(a+b)(a-b)=a^2-b^2$

Replace a by 3 and b by $\sqrt{3}$

Q. 2 **Simplify each of the following expressions**

(i) $(3+\sqrt{3})(2+\sqrt{2})$

Sol. $(3+\sqrt{3})(2+\sqrt{2}) = 3(2+\sqrt{2}) + \sqrt{3}(2+\sqrt{2})$

$= \boxed{6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

Sol. $(3+\sqrt{3})(3-\sqrt{3})$

$= (3)^2 - (\sqrt{3})^2$

Expand using $(a+b)^2 = a^2 + 2ab + b^2$

$= 9 - 3$

$= \boxed{6}$

(iii) $(\sqrt{5}+\sqrt{2})^2$

Sol. $(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2 + 2(\sqrt{5})(\sqrt{2}) + (\sqrt{2})^2$

$= 5 + 2\sqrt{10} + 2$

$= \boxed{7 + 2\sqrt{10}}$

Exercise 1.5

Q. 2 Simplify each of the following expressions :

(iv) $(5 - \sqrt{2})\ (5 + \sqrt{2})$

Use the identity $(a+b)(a-b)=a^2-b^2$

Replace a by 5 and b by $\sqrt{2}$

Sol. $(5 - \sqrt{2})\ (5 + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2$

$= 5 - 2$

$= \boxed{3}$

Q. 3 Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction ?

Sol. There is no contradiction as either c or d are irrational and hence π is an irrational number.

Lecture 6

Write the conjugate of the following surds and find the product of each pair

Surds	Conjugate	Product
$\sqrt{3} - \sqrt{2}$	$\sqrt{3} + \sqrt{2}$	$\left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{3} + \sqrt{2}\right) = \left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2 = 3 - 2 = 1$
$\sqrt{5} + 1$	$\sqrt{5} - 1$	$\left(\sqrt{5} + 1\right)\left(\sqrt{5} - 1\right) = \left(\sqrt{5}\right)^2 - (1)^2 = 5 - 1 = 4$
$4\sqrt{3} + \sqrt{27}$	$4\sqrt{3} - \sqrt{27}$	$\left(4\sqrt{3} + \sqrt{27}\right)\left(4\sqrt{3} - \sqrt{27}\right) = \left(4\sqrt{3}\right)^2 - \left(\sqrt{27}\right)^2$ $= 16 \times 3 - 27 = 21$
$\sqrt{xy^2} - \sqrt{x^2y}$	$\sqrt{xy^2} + \sqrt{x^2y}$	$\left(\sqrt{xy^2} - \sqrt{x^2y}\right)\left(\sqrt{xy^2} + \sqrt{x^2y}\right) = \left(\sqrt{xy^2}\right)^2 - \left(\sqrt{x^2y}\right)^2$ $= xy^2 - x^2y$

Q.5 Rationalize the denominator :

i) $\dfrac{4}{\sqrt{5}}$

Sol. $\dfrac{4}{\sqrt{5}} = \dfrac{4}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$

$= \boxed{\dfrac{4\sqrt{5}}{5}}$

ii) $\dfrac{1}{\sqrt{12}}$

Sol. $\dfrac{1}{\sqrt{12}} = \dfrac{1}{\sqrt{4 \times 3}}$

$= \dfrac{1}{2\sqrt{3}}$

$= \dfrac{1}{2\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \boxed{\dfrac{\sqrt{3}}{6}}$

Exercise 1.5

Q.5 Rationalise the denominators of the following :

(i) $\dfrac{1}{\sqrt{7}}$

Sol. $\dfrac{1}{\sqrt{7}} = \dfrac{1}{\sqrt{7}} \times \dfrac{\sqrt{7}}{\sqrt{7}}$

$= \boxed{\dfrac{\sqrt{7}}{7}}$

Exercise 1.5

Q.5 Rationalise the denominators of the following :

(ii) $\dfrac{1}{\sqrt{7} - \sqrt{6}}$

$(a + b)(a - b) = a^2 - b^2$

Sol. $\dfrac{1}{\sqrt{7} - \sqrt{6}}$

$= \dfrac{1}{\sqrt{7} - \sqrt{6}} \times \dfrac{\sqrt{7} + \sqrt{6}}{\sqrt{7} + \sqrt{6}}$

$= \dfrac{\sqrt{7} + \sqrt{6}}{(\sqrt{7})^2 - (\sqrt{6})^2}$

$= \dfrac{\sqrt{7} + \sqrt{6}}{7 - 6}$

$= \sqrt{7} + \sqrt{6}$

Conjugate of $\sqrt{7} - \sqrt{6}$ is $\sqrt{7} + \sqrt{6}$

Multiply $\sqrt{7} + \sqrt{6}$ to numerator and denominator

Exercise 1.5

Q.5 Rationalise the denominators of the following :

$(a + b)(a - b) = a^2 - b^2$

(iii) $\dfrac{1}{\sqrt{5} + \sqrt{2}}$

Sol. $\dfrac{1}{\sqrt{5} + \sqrt{2}}$

$$= \frac{1}{\sqrt{5} + \sqrt{2}} \times \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

$$= \frac{\sqrt{5} - \sqrt{2}}{(\sqrt{5})^2 - (\sqrt{2})^2}$$

$$= \frac{\sqrt{5} - \sqrt{2}}{5 - 2} = \frac{\sqrt{5} - \sqrt{2}}{3}$$

Conjugate of $\sqrt{5} + \sqrt{2}$ is $\sqrt{5} - \sqrt{2}$

Multiply $\sqrt{5} - \sqrt{2}$ to numerator and denominator

Exercise 1.5

Q.5 Rationalise the denominators of the following :

(iv) $\dfrac{1}{\sqrt{7}-2}$

Sol. $\dfrac{1}{\sqrt{7}-2}$

$= \dfrac{1}{\sqrt{7}-2} \times \dfrac{\sqrt{7}+2}{\sqrt{7}+2}$

$= \dfrac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$

$= \dfrac{\sqrt{7}+2}{7-4}$

$= \dfrac{\sqrt{7}+2}{3}$

$(a + b)(a - b) = a^2 - b^2$

Conjugate of $\sqrt{7}-2$ is $\sqrt{7}+2$

Multiply $\sqrt{7}+2$ to numerator and denominator

i). If $\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a - b\sqrt{6}$, find the values of a and b.

Sol. $\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a - b\sqrt{6}$

$$\therefore \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = a - b\sqrt{6}$$

$$\therefore \frac{\left(\sqrt{2}+\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2} = a - b\sqrt{6}$$

$(a + b)(a - b) = a^2 - b^2$

$$\therefore \frac{\sqrt{2}\left(3\sqrt{2}+2\sqrt{3}\right)+\sqrt{3}\left(3\sqrt{2}+2\sqrt{3}\right)}{18-12} = a - b\sqrt{6}$$

$$\therefore \frac{6 + 2\sqrt{6} + 3\sqrt{6} + 6}{6} = a - b\sqrt{6}$$

$\dfrac{12 + 5\sqrt{6}}{}$ … $a \quad b\sqrt{6}$

Conjugate of

$3\sqrt{2} - 2\sqrt{3}$ **is**

$3\sqrt{2} + 2\sqrt{3}$

$a - b\sqrt{6}$

$\therefore \quad 2 + \frac{5}{6}\sqrt{6} = a + (-b\sqrt{6})$

Multiply

$3\sqrt{2} + 2\sqrt{3}$ **to**

$\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)$

$= \left(3\sqrt{2}\right)^2 - \left(2\sqrt{3}\right)^2$

$b = \dfrac{5}{6}$

Q. Show that : $\dfrac{3\sqrt{2}}{3+\sqrt{6}} + \dfrac{6}{2\sqrt{3}+\sqrt{6}} - \dfrac{4\sqrt{3}}{\sqrt{2}+\sqrt{6}} = 0$

(a + b)(a - b) = a² − b² in denominators and simplify the numerator

Sol. L.H.S. $= \dfrac{3\sqrt{2}}{3+\sqrt{6}} + \dfrac{6}{2\sqrt{3}+\sqrt{6}} - \dfrac{4\sqrt{3}}{\sqrt{2}+\sqrt{6}}$

Multiply $2\sqrt{3} - \sqrt{6}$ to the numerator and denominator to rationalize it

Multiply ... to the numerator and denominator to rationalize it

$\dfrac{4\sqrt{3}}{\left(\sqrt{2}+\sqrt{6}\right)} \times \dfrac{\left(\sqrt{2}-\sqrt{6}\right)}{\left(\sqrt{2}-\sqrt{6}\right)}$

$= \dfrac{\ldots}{(3)^2 - \left(\sqrt{6}\right)^2} + \dfrac{\ldots}{\left(2\sqrt{3}\right)^2 - \left(\sqrt{6}\right)^2} - \dfrac{\ldots\sqrt{18}}{\left(\sqrt{2}\right)^2 - \left(\sqrt{6}\right)^2}$

$= \dfrac{9\sqrt{2} - 3\sqrt{4}\ldots}{9-6} + \dfrac{\ldots}{12-6} - \dfrac{\ldots}{2-6}$

3 is common

6 is common

4 is common

$= \dfrac{9\sqrt{2} - 6\sqrt{3}}{3} + \dfrac{12\sqrt{3} - 6\sqrt{6}}{6} - \dfrac{4\sqrt{6} - 12\sqrt{2}}{-4}$

$= \dfrac{\cancel{3}\left(3\sqrt{2} - 2\sqrt{3}\right)}{\cancel{3}} + \dfrac{\cancel{6}\left(2\sqrt{3} - \sqrt{6}\right)}{\cancel{6}} + \dfrac{\cancel{4}\left(\sqrt{6} - 3\sqrt{2}\right)}{\cancel{4}}$

$= \cancel{3\sqrt{2}} - \cancel{2\sqrt{3}} + \cancel{2\sqrt{3}} - \cancel{\sqrt{6}} + \cancel{\sqrt{6}} - \cancel{3\sqrt{2}} = 0$

$\therefore \dfrac{3\sqrt{2}}{3+\sqrt{6}} + \dfrac{6}{2\sqrt{3}+\sqrt{6}} - \dfrac{4\sqrt{3}}{\sqrt{2}+\sqrt{6}} = 0$

Q. Simplify : $\dfrac{1}{\left(3+\sqrt{5}\right)^2}+\dfrac{1}{\left(3-\sqrt{5}\right)^2}$

Sol. $\dfrac{1}{\boxed{\left(3+\sqrt{5}\right)^2}}+\dfrac{1}{\boxed{\left(3-\sqrt{5}\right)^2}}$

$(a + b)(a - b) = a^2 - b^2$

$$= \frac{1}{3^2+2\times3\times\sqrt{5}+\left(\sqrt{5}\right)^2}+\frac{1}{3^2-2\times3\times\sqrt{5}+\left(\sqrt{5}\right)^2}$$

$$= \frac{1}{\underline{9}+6\sqrt{5}+\underline{5}}+\frac{1}{\underline{9}-6\sqrt{5}+\underline{5}}$$

$$= \frac{1}{14+6\sqrt{5}}+\frac{1}{14-6\sqrt{5}}$$

So lets cross multiply

$$= \frac{14-\cancel{6\sqrt{5}}+14+\cancel{6\sqrt{5}}}{\boxed{\left(14+6\sqrt{5}\right)\left(14-6\sqrt{5}\right)}}$$

$$= \frac{28}{(14)^2-\left(6\sqrt{5}\right)^2}$$

$$= \frac{28}{196-180}$$

$$= \frac{28}{16}$$

$$= \frac{7}{4}$$

Lecture 7

LAWS OF EXPONENTS FOR REAL NUMBERS

Product Law

$$a^m \times a^n = a^{m+n}$$

Quotient Law

$$a^m \div a^n = a^{m-n}$$

(Double Power Law)

$$(a^m)^n = a^{m \times n}$$

(Same Power Law)

$$a^m b^m = (ab)^m$$

Exercise 1.6

Q.1 Find.

(i) $64^{\frac{1}{2}}$

Sol. $64^{\frac{1}{2}} = 8^{2 \times \frac{1}{2}}$

$= 8$

(ii) $32^{\frac{1}{5}}$

Sol. $32^{\frac{1}{5}} = 2^{5 \times \frac{1}{5}}$

$= 2$

(iii) $125^{\frac{1}{3}}$

Sol. $125^{\frac{1}{3}} = 5^{3 \times \frac{1}{3}}$

$= 5$

Exercise 1.6

Q. 2 Find.

(i) $9^{\frac{3}{2}}$

Sol. $9^{\frac{3}{2}} = (3^2)^{\times \frac{3}{2}}$

$= 3^{2 \times \frac{3}{2}}$

$= 3^3$

$= 27$

(ii) $32^{\frac{2}{5}}$

Sol. $32^{\frac{2}{5}} = (2^5)^{\times \frac{2}{5}}$

$= 2^{5 \times \frac{2}{5}}$

$= 2^2$

$= 4$

Exercise 1.6

Q. 2 Find.

(iii) $16^{\frac{3}{4}}$

Sol. $16^{\frac{3}{4}} = (2^4)^{\times \frac{3}{4}}$

$= 2^{4 \times \frac{3}{4}}$

$= 2^3$

$= 8$

(iv) $125^{-\frac{1}{3}}$

Sol. $125^{-\frac{1}{3}} = (5^3)^{\times -\frac{1}{3}}$

$= 5^{3 \times -\frac{1}{3}}$

$= 5^{-1}$

Exercise 1.6

Q. 3 Simplify :

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

Sol. $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

$= 2^{\frac{2}{3} + \frac{1}{5}}$

$= 2^{\frac{10 + 3}{15}}$

$= 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3^3}\right)^7$

Sol. $\left(\frac{1}{3^3}\right)^7$

$= (3^{-3})^7$

$= 3^{-21}$

Exercise 1.6

Q. 3 Simplify :

(iii) $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

Sol. $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

$= 11^{\frac{1}{2}} \times 11^{-\frac{1}{4}}$

$= 11^{\frac{1}{2}-\frac{1}{4}}$

$= 11^{\frac{2-1}{4}}$

$= 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}} . 8^{\frac{1}{2}}$

Sol. $7^{\frac{1}{2}} . 8^{\frac{1}{2}}$

$= (7 \times 8)^{\frac{1}{2}}$

$= 56^{\frac{1}{2}}$

Thank You