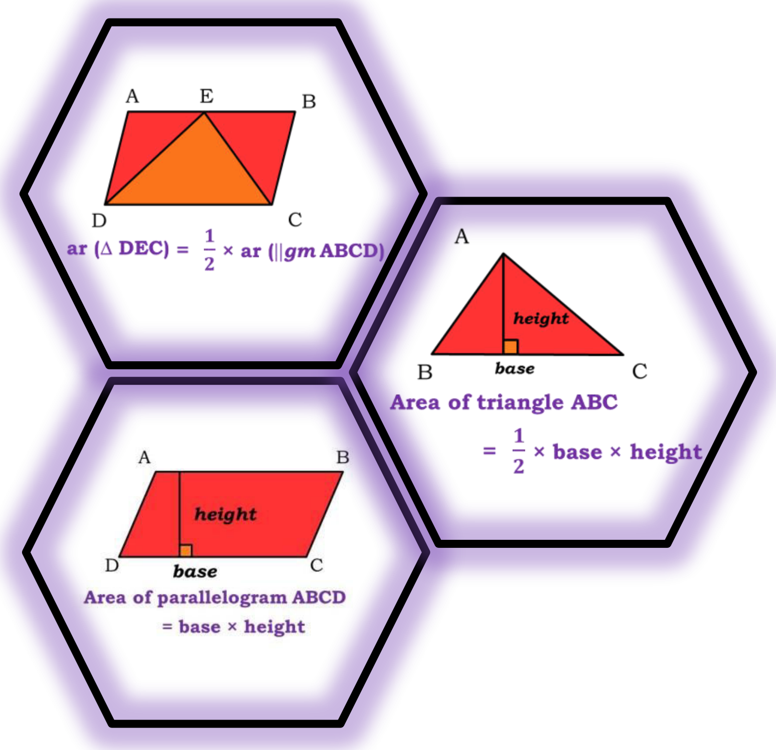
Module 1

AREAS OF PARALLELOGRAMS AND TRIANGLES



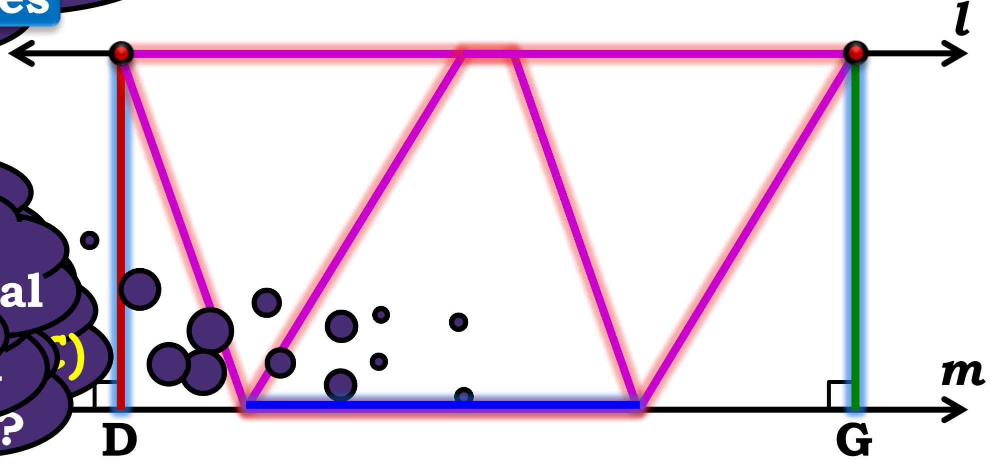
Let us consider two parallelograms on the same base & between the same parallel lines

ar($||^{gm}$BEFC) = BC × FG

What is the formula for area ...

Bases are equal

... on line m



Perpendicular distances between the same parallel lines are equal

Parallelograms on the same base and between the same parallels, are equal in area

Module 2

Let us consider a parallelogram and a triangle on the same base & between the same parallels

ar

ar(ΔBEC)

What is t

area o

height?

What can

abo

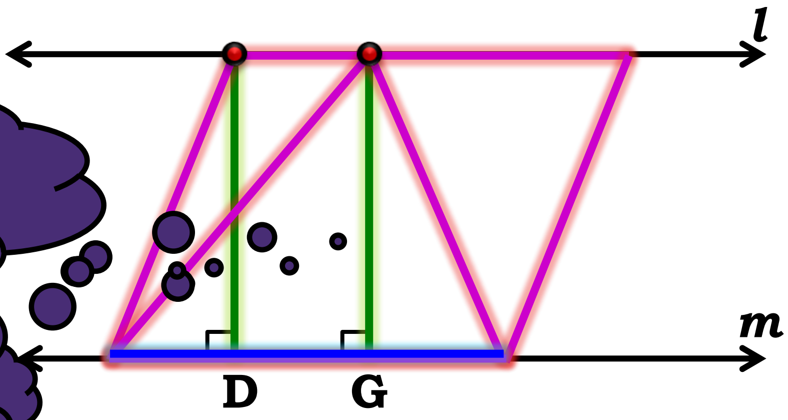
∴ ar(Δ

∴ **ar(ΔBEC)**

So, what can we conclude from this ?

Perpendicular distances between the same parallel lines are equal

B B C C



If a parallelogram & a triangle are on the same base and between the same parallels, then area of the triangle is half the area of a parallelogram

Module 3

Heights are equal

ar(ΔA...

Let us c...

ar(ΔACD)

$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$

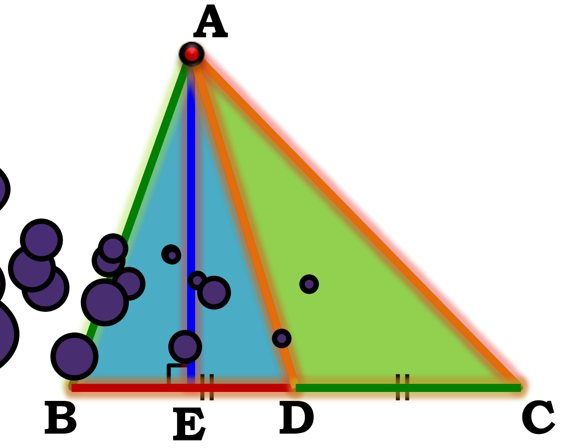
ar(ΔABD) = ...

A line segm...
vertex of a Δ ...
of its op...

For base DC, the

So, what can we conclude from this ?

bases are equal



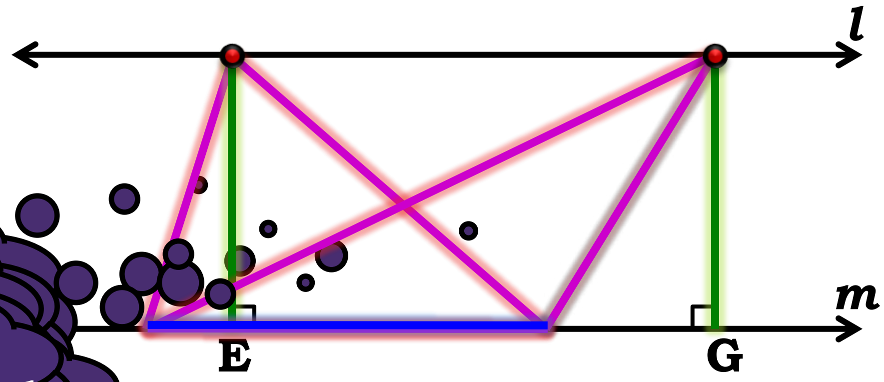
A median of a triangle divides it in to two triangles of equal areas

Module 4

Let us consider two triangles having the same base & between the same two parallels

$ar(\Delta ABC) = \frac{1}{2} \times BC \times AE$

$ar(\Delta DBC) = \frac{1}{2} \times BC \times DG$

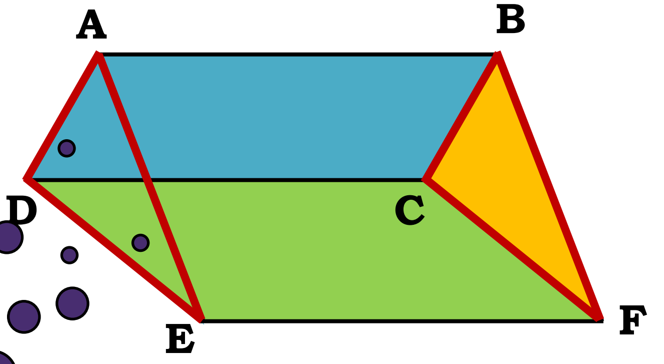


Bases are equal

If two triangles are on the same base (equal base) & between the same parallels, their areas are equal

Module 5

Q. In figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).



Soln. In parallelogram ABCD,

AD = BC ...(i) [opposite sides of parallelogram are equal]

In parallelogram DCFE,

DE = CF ...(ii) [opposite sides of parallelogram are equal]

In parallelogram ABFE,

AE = BF ...(iii) [opposite sides of parallelogram are equal]

In Δ ADE and Δ BCF,

AD = BC [Fr

DE = CF [Fr

AE = BF [From(

∴ By SSS criterion of congruence

ΔADE ≅ ΔBCF

∴ **ar(Δ ADE) = ar(Δ BCF)**

AE and BF are opposite sid parallelogram

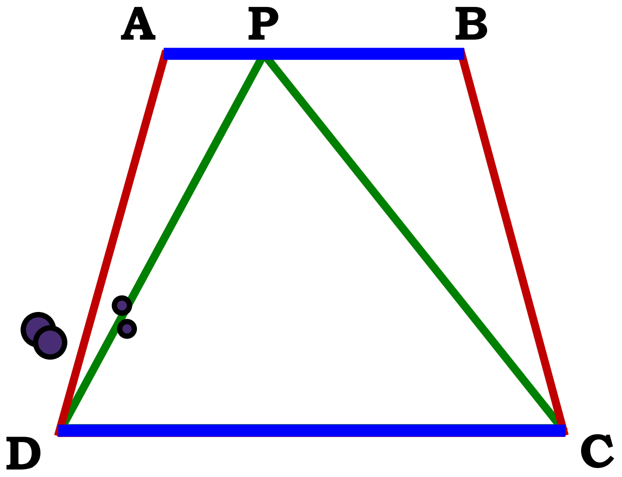
Triangles are congruent means they have same area

What do we know about the opposite sides of a parallelogram ?

They are equal

Module 6

Q. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.



Sol. (i):-

The two figures have a common base DC and lie between two parallel lines AB and DC

Does ΔPDC and □ABCD lie between two parallel lines ?

Yes, AB and DC

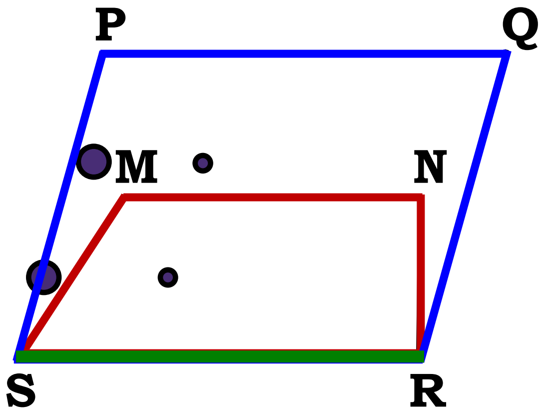
Sol. (ii):-

The two figure ... nd
does not lie

Does □MSRN and □PS ... mon

Does □MSRN and □PSRQ lie between two parallel lines ?

No



Q. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

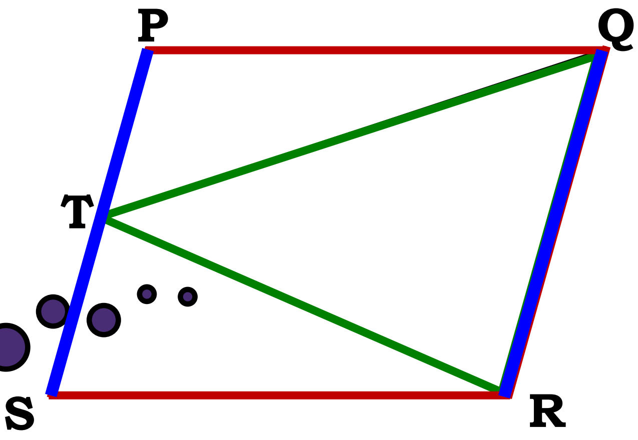
Sol. (iii):-

The two figures have a common base QR and lie between two parallel lines PS and QR

Does ∆RTQ

Does ∆RTQ and □RSPQ have a common base ?

Yes , RQ



Sol. (iv):-

The two figures lie between

lines AD and BC, bu

common bas

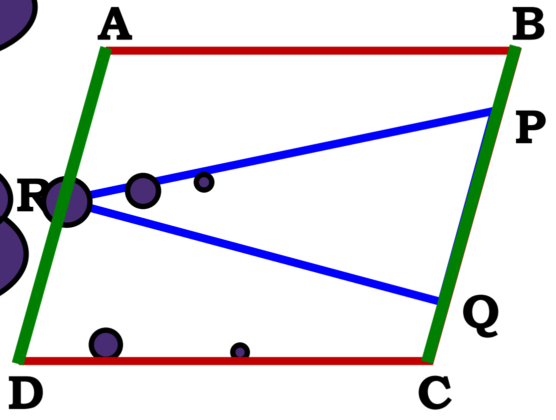
Q and

a

?

Does ∆PRQ and □ABCD lie between two parallel lines ?

Yes , BC and AD



Q. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

Sol. (v):-

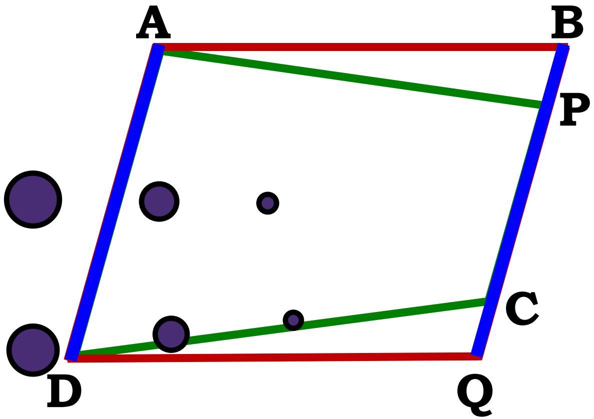
The two figures h
lie between two pa

Does □ADCP and □ADQB have a common base ?

Yes, AD

Does □ADCP and □ADQB lie between two parallel lines ?

Yes, AD and BQ

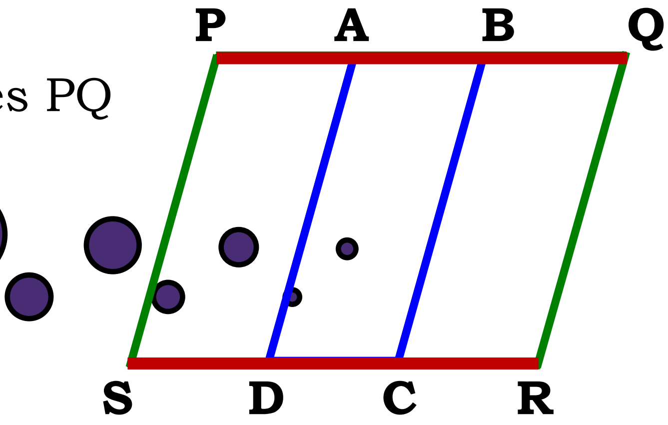


Sol. (vi):-

The two figures lie between two parallel lines PQ and SR but does not have a common base.

Does □DCBA and □SRQP lie between two parallel lines ?

Yes, PQ and SR



Thank You

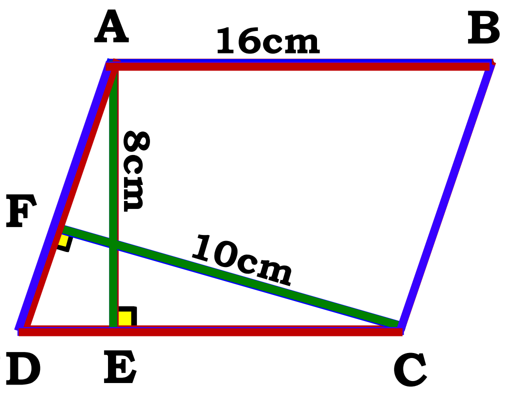
Module 7

Q. In figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD.
If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.

Soln.

Area of □ABCD = AD × CF .....(i)

Area of □ABCD = DC × AE ..... (ii)



From (1) and (2),

AD ×

What is the formula of area of a parallelogram ?

Base × height

∴

∴ AD

∴ $AD = \dfrac{16 \times \cancel{8}}{\cancel{10}\,5}$

∴ $AD = \dfrac{16 \times 4}{5}$

∴ $AD = \dfrac{\cancel{64}\ 12.8}{\cancel{5}}$

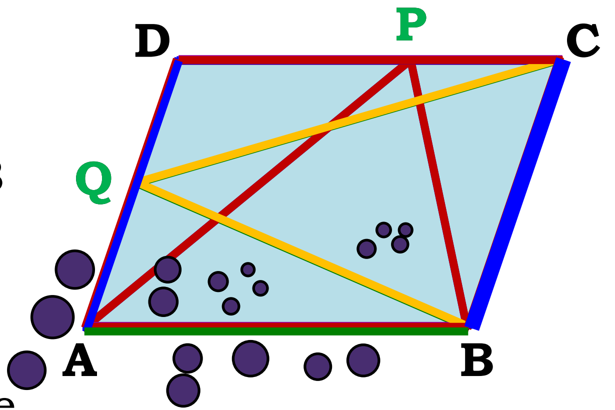
Any si

Since, opposite sides of parallelogram are equal.

∴ $\boxed{AD = 12.8cm}$

Module 8

Q. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).



Soln. ΔAPB and □ABCD stand on the same base AB and lie between the sam... AB and DC.

∴ ar (ΔAPB) = $\frac{1}{2}$ ar (□ABCD)

Similarly ΔBQC ar... ame base AB and lie betwe... ...lels AB and DC.

∴ ar (ΔBQC ... (ii)

From (i...

∴ **ar (APB) =** ...

Consider AB as ...

Also, parallelogram ABCD lie on the same base AB

Also, ΔAPB and □ABCD li... same ... AB

Also, pa... ABC... sam...

Also, ΔQCB a... □ABCD lie b... same parall... BC and B...

So, what can we say about areas of ΔBQC and □ABCD ?

ar (ΔBQC) = $\frac{1}{2}$ ar (□ABCD)

Module 9

Q. In figure, E is any point on median AD of a ΔABC.
Show that ar (ABE) = ar (ACE).

Given : AD is a median of ΔABC and E is any point on AD.

To prove : ar (ΔABE) = ar (ΔACE)

Proof : AD is the median of ΔABC

ar (ΔABD) = ar (ΔACD) … (i)

Also, ED is the median of Δ…

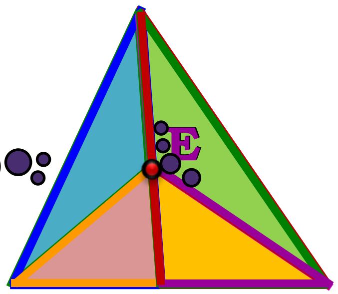
∴ ar (ΔBED) = ar (ΔC…

Subtracting (ii) from (i), we g…

ar (ΔABD) – ar (ΔBED) = ar (ΔACD) – ar (ΔCED)

∴ **ar (ΔABE) = ar (ΔACE)**

ΔABD and ΔACD are the two parts of ΔABC



ar (ΔABE) = ar (ΔABD) – ar (ΔBED)

ar (ΔACE) = ar (ΔACD) – ar (ΔCED)

In ΔEBC, ED the median

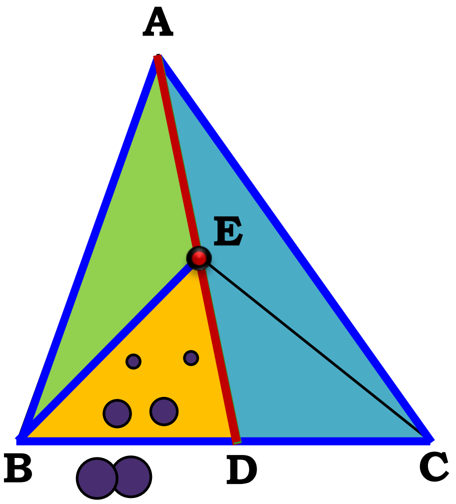
So, what can we say about areas of ΔBED and ΔCED ?

ar (ΔBED) = ar (ΔCED)

Module 10

Q. In a triangle ABC, E is the mid-point of median AD.

Show that ar (ΔBED) = $\frac{1}{4}$ ar(ΔABC).



Given : In ΔABC, E is the mid-point of the median AD

To prove : ar (ΔBED) = $\frac{1}{4}$ ar (ΔABC)

Proof : AD is a median of ΔABC

$\therefore$ ar (ΔABD) = $\frac{1}{2}$ ar (ΔABC) [Median divides a triangle into two triangles of equal area]

BE is a median of ΔABD

$\therefore$ ar (ΔBED) = $\frac{1}{2}$ ar (ΔABD) [Median divides into two triangles

What can we say about the areas of ΔBED and ΔABD ?

ar (ΔBED) = $\frac{1}{2}$ ar (ΔABD)

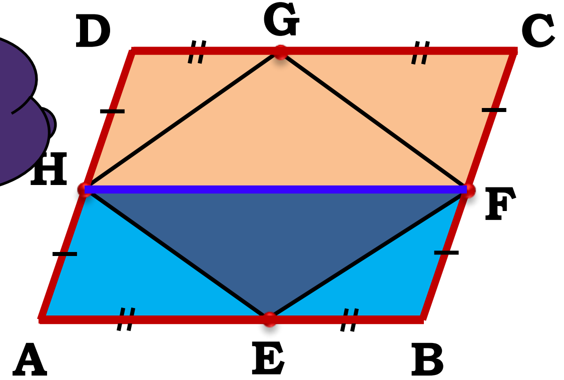
ar (ΔBED) = $\frac{1}{2} \times \frac{1}{2}$ ar (ΔABC)

$\therefore$ **ar (ΔBED) = $\frac{1}{4}$ ar (ΔABC)**

Thank You

Module 11

Q. If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD. Show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD).



Soln. ΔHGF ar...

HF ar...

HF a...

ar(HGF) = $\frac{1}{2}$ ...

ΔHGF and □HDLF have a common base HF

Yes,

Also, they lie between the same two parallel lines DC & HF

□ EFGH is made up of which two triangles

Yes, ar(ΔHGF) and ...

ΔHEF ... same b... paralle...

ΔHEF and □HABF have a common base HF

Also, they lie between the same two parallel lines AB & HF

ar(ΔHEF) = $\frac{1}{2}$ ar(□ABFH) ... (II)

... (□ EFGH) = ar(ΔHGF) + ar(ΔHEF)

... DCF) + ar(□ ABFH)

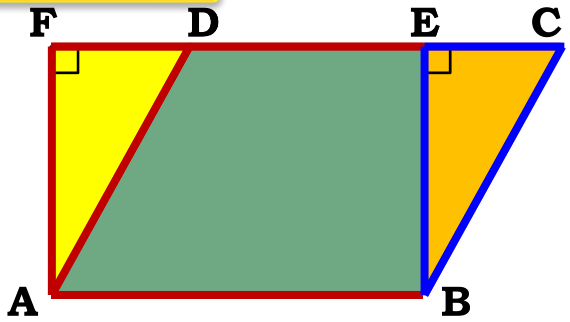
∴ Adding (1) and (2), we get

ar (ΔHGF) + ar (ΔHEF) = $\frac{1}{2}$ [ar ...

∴ ar (EFGH) = $\frac{1}{2}$ ar (ABCD)

Module 12

Q. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.



ar (□ABCD) = ar (□ABEF)

Proof :

AB = CD ...(i) [Opposite sides of a parallelogram are equal]

EF = AB ...(ii) [Opposite sides of a rectangle are equal]

∴ AB + EF = AB + CD

DA > FA

BC > BE

∴ DA + BC > FA + BE ...(vi)

∴ AB + BC + CD + DA > AB + BE + EF + FA [Adding (iii) and (vi)]

∴ Perimeter of parallelogram ABCD > Perimeter of rectangle ABEF

Hint:
To prove:
(AB + BC + CD + DA)
(AB + BE + EF + FA)

What do we know about AB and CD?

AB and CD are opposite

Adding (i) and (ii)

Adding (iii) and (vi)

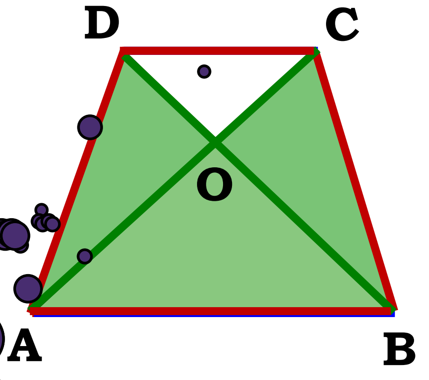
triangle BEC

sides

Module 13

Q. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.

Prove that ar (Δ AOD) = ar (Δ BOC).



ar(ΔAOD) = ar(ΔABD) – ar(ΔAOB)

ar(ΔBOC) = ar(ΔABC) – ar(ΔAOB)

Proof. ΔABC an

betwe

∴ ar (ΔAI

∴ ar (ΔABD) – ar (ΔAOB) = ar

tracting ar (ΔAOB) from both sides]

∴ **ar (ΔAOD) = ar (ΔBOC)**

ΔABD and ΔABC

common base

between same pa

lines AB and

ΔABC is made up of ΔBOC & ΔAOB

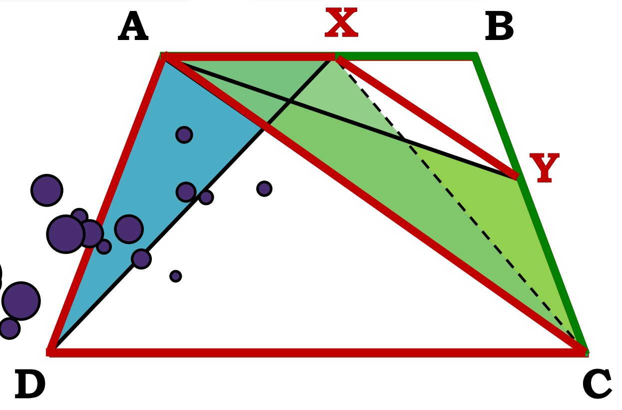
ar(ΔABD) = ar(ΔABC)

areas of ΔABD and ΔABC ?

Module 14

Q. ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (Δ ADX) = ar (Δ ACY).

Construction. Draw XC



Proof. ΔACX and ΔADX have same base AX and are between same parallels AB and DC

ar (ΔADX) = ar (ΔACX) ....(i)

ΔACX and ΔACY h... d

are betw...

ar (... ...i)

Fro... o

Do we have one more triangle having the same base AC and between the same two parallel lines AC and XY ?

Yes, ΔACX

... AX and DC ?

So, wh...

ar (Δ...

of ΔACX and ...

∵ AB || DC

∴ AX || DC

Thank You

Module 15

Q. In figure, AP || BQ || CR. Prove that ar(Δ AQC) = ar(Δ PBR).

Proof. ar (AQC) = ar (AQB) + ar (BQC) … (i)

ar (PBR) = ar (PB … (ii)

ΔAQB and ΔPBQ li … en
same parallel lines

… (iii)

ΔBQC and ΔQBR … en
same parallel line

Adding (iii) & (iv),

ar (AQB) + ar (

ar (ΔAQC) = ar (ΔPBR) [From (i), (ii) & (v)]

ΔAQC is made up of which

ΔPBR is made up of which

ΔPBQ and ΔQBR

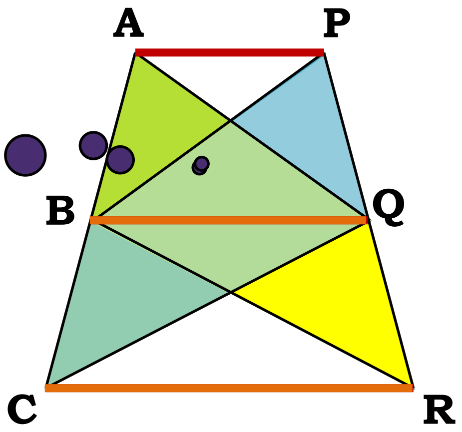
They lie on same base BQ

What can we say about ΔBQC
and ΔQBR ?

They lie on same base BQ
and between same parallel
lines BQ and CR

So, what can we say about
areas of ΔBQC and ΔQBR ?

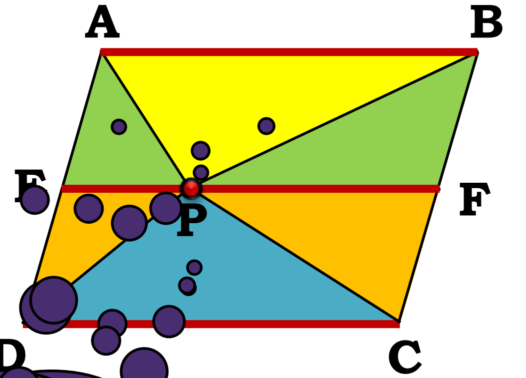
ar(ΔBQC) = ar(ΔQBR)



MODULE 16

Q. In figure, P is a point in the interior of a parallelogram ABCD. Show that

(1) ar (ΔAPB) + ar (ΔPCD) = $\frac{1}{2}$ ar (□ABCD)



Construction : Draw EF such that EF || AB

Proof : ΔAPB and □ABFE stand on the same base AB and lie between th... B and EF.

∴ ar (ΔAPB) = $\frac{1}{2}$ ar ...

ΔPCD and □EFCD stand... same base DC and lie betw... same parallels DC and EF.

∴ ar (ΔPCD) = $\frac{1}{2}$ ar (□EFCD) ..... (ii)

...d (ii), we get

...EFCD)]

∴ ar (ΔAPB) + ar (ΔPCD)

= $\frac{1}{2}$ ar (□ABCD) ... (iii)

Consider AB as the

Also, ΔAPB & □A...

Also, ΔPCD & □EFCD

So, what can we say

Also, D... of

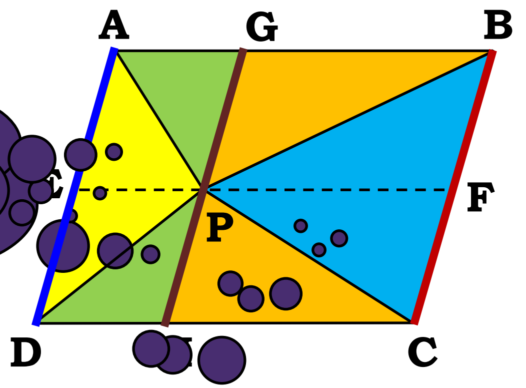
ar (ΔPCD) = $\frac{1}{...}$

Adding (i) and (ii)

MODULE 17

(2) **ar (ΔAPD) + ar (ΔPBC) = ar (ΔAPB) + ar (ΔPCD)**

Construction : Draw GH such that GH || AD || BC



ΔAPD and □AGHD are on the same ba...

lie between the same paralle...

Also, ΔAPD and □AGDH lie between the same two parallels AD and GH

So what can we say about their areas ?

ar (ΔAPD) = $\frac{1}{2}$ ar (□AGHD)

∴ ar (ΔAPD) = $\frac{1}{2}$ ar (...

ΔPCB and □GHCB are on the ...

lie between the same paral...

Adding (iv) and (v)

∴ ar (ΔPCB) = $\frac{1}{2}$ ar (□GH...

Also, ΔPCB and □GHCB lie between the same two parallels GH and BC

So what can we say about their areas ?

ar (ΔPCB) = $\frac{1}{2}$ ar (□GHCB)

...CD) ... (iii)

Adding (iv) and (v),

ar (ΔAPD) + ar (ΔPCB) = $\frac{1}{2}$ [ar (□AGHD) + ar (□GBCH)]

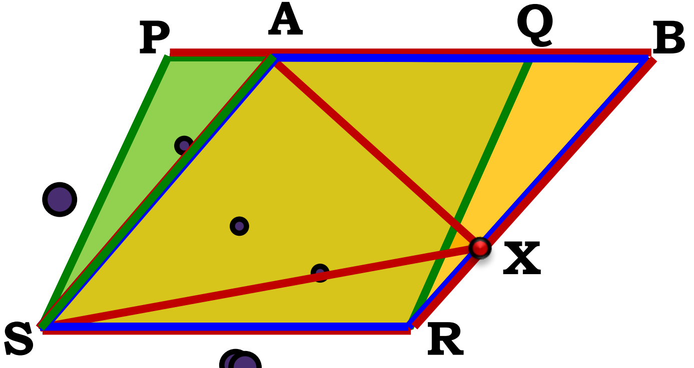
∴ ar (ΔAPD) + ar (ΔPCB) = $\frac{1}{2}$ ar (□ABCD) ... (vi)

∴ **ar (ΔAPD) + ar (ΔPBC) = ar (ΔAPB) + ar (ΔPCD)** [From (iii) and (vi)]

Thank You

MODULE 18

Q. In figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that :



(i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ ar (PQRS)

Proof. (i) … e base RS and lie … nd PB.

Does, □PQRS and □ABRS lie between same parallels?

Yes, AS and BR

ar (ΔAXS) = $\frac{1}{2}$ ar (□ABRS)

$\therefore$ ar (□P…) = ar (□ABRS) …(i)

(ii) ΔAXS and □ABRS stan… ie between the same parallels …

Does, □PQRS and □ABRS lie between … have a com… par…

Yes, PB and SR

So what can we say about areas of □PQRS and □ABRS ?

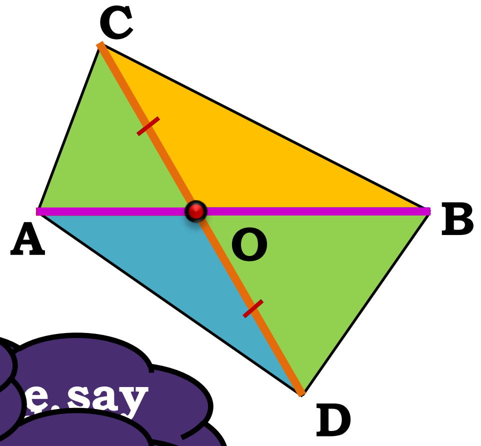
ar(□PQRS) = ar(□ABRS)

$\therefore$ ar (ΔAXS) = $\frac{1}{2}$ …

$\therefore$ ar (ΔAXS) = $\frac{1}{2}$ ar (□PQRS)

MODULE 19

Q. In figure, ABC and ABD are two triangles on the same base AB. If line segment CD is bisected by AB at O show that ar (ΔABC) = ar (ΔABD).



To prove : ar (ΔABC) = ar (ΔABD)

Proof : ar (ΔABC) = ar (ΔAOC) + ar (ΔBOC) … (i)

ar (ΔABD) = ar (ΔAOD) + ar (ΔBOD) … (ii)

ar (ΔAOC) = ar (ΔAOD) [∵ AO is t

ar (ΔBOC) = ar (ΔBOD) [∵ BO

ar (ΔAOC) + ar (ΔBOC) = ar (ΔAOD) + ar (ΔBOD) [Adding (iii) and (iv)] … (v)

∴ ar (ΔABC) = ar (ΔABD) [From (i), (ii), (v)]

Adding (iii) and (iv)

we say

BO is the median of ΔBCD

ΔBOC and ΔBOD ?

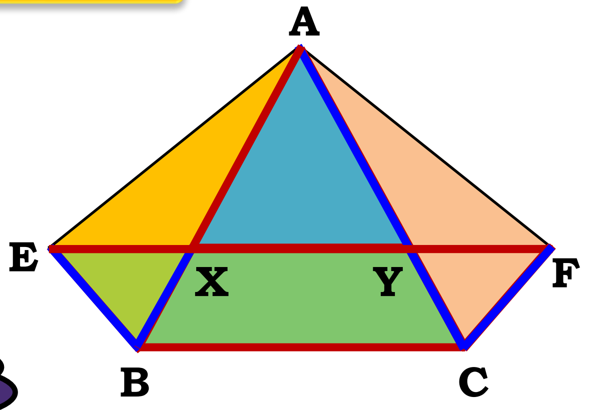
ar (ΔBOC) = ar (ΔBOD)

ar (ΔABC) = ar (ΔAOC) + ar (ΔB

ar (ΔAOD) + ar (ΔBOD)

MODULE 20

Q. XY is a line parallel to side BC of triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that ar (ΔABE) = ar (ΔACF).



Proof. □BCYE is a parallelogram [EY || BC and BE || CY]

Since ΔABE and □BCY... ...se BE and between th...

So what can we say about their areas?

ar(ΔACF) = $\frac{1}{2}$ ar(□BCFX)

ar(ΔABE...

□BCFX is a parallelogram

L.H.S. are equal

R.H.S. are equal

...commo... base BC and lie between same pair of parallel lines

...and between the... sam...

ar... (i...

Is there any relation between area...

Yes, ar (□BCFX) ... (□BCYE)

Also, □BCFX lie on the same base CF

lie b... parallel line CF and BX

But □BCFX and □BCYE are on the sa... parallels BC and EF.

∴ ar (BCFX) = ar (BCYE) …… (iii)

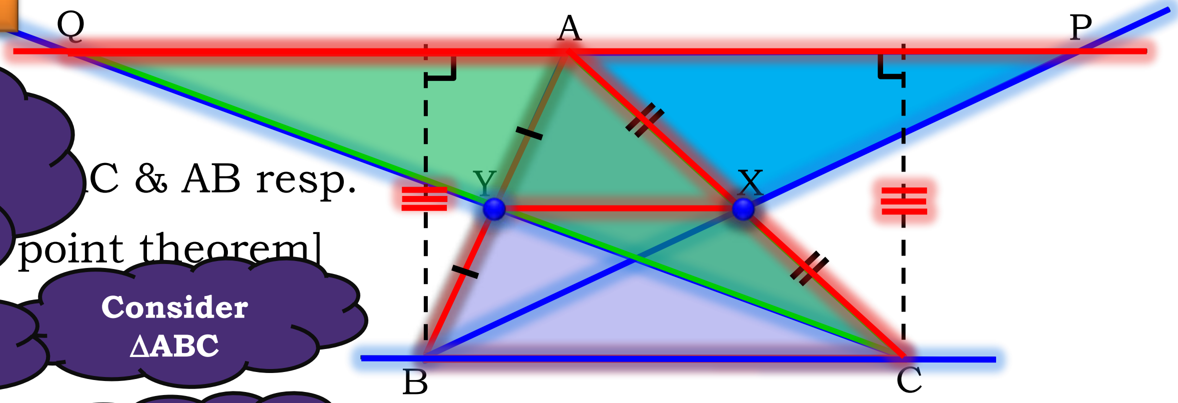
ar (ΔABE) = ar (ΔACF)

Thank You

MODULE 21

Q. X & Y are midpoints of AC & AB resp. of ΔABC. QP || BC & CYQ and BXP are straight lines prove that ar (ΔABP) = ar (ΔACQ)

Hint : prove AQ = AP



Proof :

In ... C & AB resp.

∴ ... point theorem]

But, BC ...

XY ...

[Given]

XY || QA [From ...

∴ Y is midpoint of QC [Converse of midpoint theorem]

ΔABP and ΔACQ lie between same two parallel lines PQ and BC

∴ Their heights are equal.

Consider ΔABC

Now, for proving areas equal we just need to prove their bases equal.

Consider ΔACQ

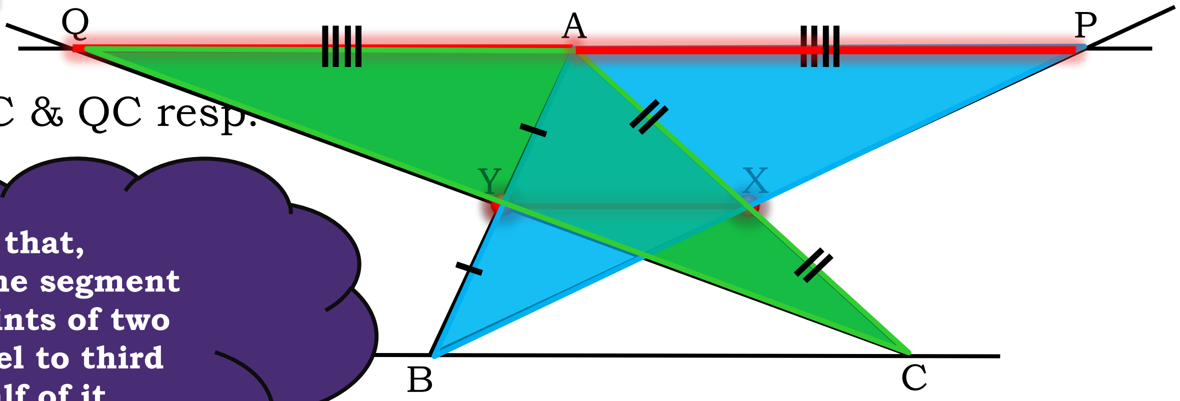
...orem ...n we use ?

Midpoint Theorem

Q. X & Y are mid point of AC & AB resp. of ΔABC. QP||BC & CYQ and BXP are straight lines prove that ar (ΔABP) = ar (ΔACQ)

Hint : prove AQ = AP

Proof :



In ΔAQC

X and Y are midpoints of AC & QC resp.

$XY = \frac{1}{2} AQ$ ... (ii)

Similarly

$XY = \frac{1}{2}$ ...

$\cancel{\frac{1}{2}} AQ = \cancel{\frac{1}{2}}$ ...

We know that,
In a triangle line segment joining midpoints of two sides is parallel to third side and half of it.

AQ = AP

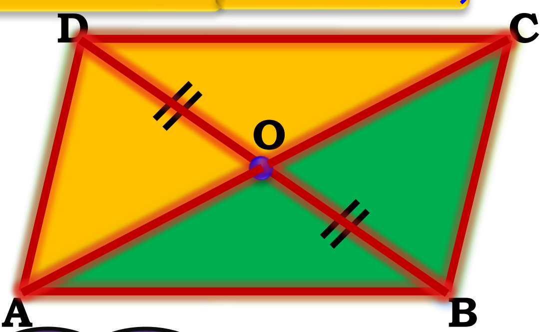
ΔABP and ΔACQ have equal heights and bases.

∴ Their areas are equal.

∴ ar (ΔABP) = ar (ΔACQ)

MODULE 22

Q. The diagonals of □ABCD, AC & BD intersect in O prove that if BO = OD, The triangles ABC and ΔADC are equal in area.



Proof :

Prove : ar (ΔABC) = ar (ΔADC)

In ΔADB

DO = OB [Given]

∴ O is mid point of DB

∴ AO is median for ΔABC

∴ ar (ΔAOB) = ar (ΔAOD) ...(i)

In ΔCBD

O is mid point of BD

∴ CO is median for ΔCBD

∴ ar (ΔBOC) = ar (ΔDOC) ...(ii)

Consider ΔADB

Consider ΔCBD

...now that ...edian divides triangle in two equal areas.

ar (ΔAOB) = ar (ΔAOD)

<u>Adding (i) & (ii)</u>

ar (ΔAOB) + ar (ΔBOC) = ar (ΔAOD) + ar (ΔDOC)

∴ **ar (ΔABC) = ar (ΔADC)**

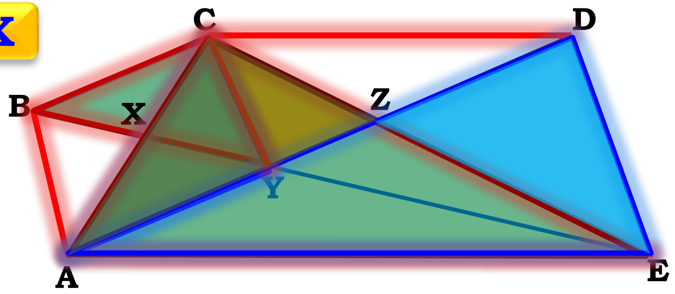
MODULE 23

Q. In fig. CD || AE & CY || BA

1. Name a triangle equal in area of ΔCBX

2. Prove that ar (ΔZDE) = ar (ΔCZA)

3. Prove that ar (□BCZY) = ar (ΔEDZ)



Proof .

ar (ΔCYB) = ar (ΔCYA)

[If two triangles are on the same base a

same parallel lines, then their areas

Also, they have common base CY

∴ Their areas are equal

A (ΔCBX) + A (~~ΔCYX~~) = A (ΔAXY) +

A (ΔCBX) = A (Δ AXY)

A (ΔACE) = A (ΔADE) [If two tri e and between the reas are equal]

ΔCYA is made up of two triangles ΔAXY and ΔCYX

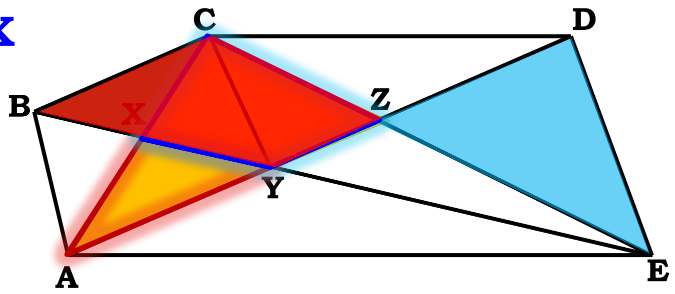
of two triangles ΔZDE and ΔAZE

A (ΔCZA) + A (~~ΔAZE~~) = A (ΔZDE) + A (~~ΔAZE~~)

A (ΔCZA) = A (ΔZDE)

Q. In fig. CD | | AE & CY | | BA

1. **Name a triangle equal in area of ΔCBX**
2. **Prove that ar (ΔZDE) = ar (ΔCZA)**
3. **Prove that ar (□BCZY) = ar (ΔEDZ)**



Proof .

ar (ΔCZA) = ar (ΔEDZ) [Proved]

ar (□CXYZ) + ar (ΔAXY) = ar (ΔEDZ)

ar (ΔAXY) = ar (ΔCBX) [Proved]

ar (□CXYZ) + ar (ΔCBX) = ar (ΔEDZ)

ar (□BCZY) = ar (ΔEDZ) [Area addition property]

Thank You