Lecture 10

Module 34

Exercise 2.3

1. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Sol. Dividend = Divisor × Quotient + Remainder

$$x^3 - 3x^2 + x + 2 = g(x) \times (x-2) + (-2x+4)$$

$$\therefore \quad x^3 - 3x^2 + x + 2 = g(x) \times (x-2) - 2x + 4$$

$$\therefore \quad x^3 - 3x^2 + x + 2 + 2x - 4 = g(x) \times (x-2)$$

$$\therefore \quad g(x)(x-2) = x^3 - 3x^2 + x + 2 + 2x - 4$$

$$\therefore \quad g(x) = \frac{x^3 - 3x^2 + 3x - 2}{(x-2)}$$

Exercise 2.3

1. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Sol. Dividend = Divisor × Quotient + Remainder

$$g(x) = \frac{x^3 - 3x^2 + 3x - 2}{(x - 2)}$$

$$x^2 - x + 1$$

$$x - 2 \,\big)\, x^3 - 3x^2 + 3x - 2$$

$$\cancel{x^3} - 2x^2$$

$$(-) \quad (+)$$

$$- \cancel{x^2} + 3x - 2$$

$$- \cancel{x^2} + 2x$$

$$(+) \quad (-)$$

$$\cancel{x} - \cancel{2}$$

$$\cancel{x} - \cancel{2}$$

$$(-) \quad (+)$$

$$0$$

Dividend should be in index form

$\frac{x^3}{x} = x^2$ | $x^2(x - 2) = x^3 - 2x^2$

$\frac{-x^2}{x} = -x$ | $-x(x - 2) = -x^2 + 2x$

$\frac{x}{x} = 1$ | $1(x - 2) = x - 2$

$\therefore$ Quotient = $x^2 - x + 1$

Remainder = 0

Module 35

Exercise 2.3

1. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Sol. Since two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\therefore$ $x - \sqrt{\frac{5}{3}}$ and $x - \left(-\sqrt{\frac{5}{3}}\right)$

are the factors of the polynomials

$$\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \frac{x^2}{1} - \frac{5}{3}$$

$(a - b)(a + b) = a^2 - b^2$

$$= \frac{3x^2 - 5}{3} = \frac{1}{3}(3x^2 - 5)$$

i.e. $3x^2 - 5$ factor of the given polynomial. Now we divide the given polynomial by $3x^2 - 5$.

Exercise 2.3

1. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Sol. i.e. $3x^2 - 5$ factor of the given polynomial. Now we divide the given polynomial by $3x^2 - 5$.

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
3x^2 - 5 & \cancel{3x^4} + 6x^3 - 2x^2 - 10x - 5 \\
 & \underset{(-)}{\cancel{3x^4}} \quad \underset{(+)}{-}\ 5x^2 \\
\hline
 & + \cancel{6x^3} - 3x^2 - \cancel{10x} - 5 \\
 & \underset{(-)}{\cancel{6x^3}} \quad \underset{(+)}{-}\ \cancel{10x} \\
\hline
 & \cancel{3x^2} \quad \cancel{-5} \\
 & \underset{(-)}{\cancel{3x^2}} \quad \underset{(+)}{\cancel{-5}} \\
\hline
 & 0
\end{array}
$$

$\frac{\cancel{3x^4}^{\,2}}{\cancel{3x^2}} = x^2$	$x^2\,(3x^2 - 5) = 3x^4 - 5x^2$
$\frac{{}^{2}\cancel{6x^3}}{\cancel{3x^2}} = 2x$	$2x\,(3x^2 - 5) = 6x^3 - 10x$
$\frac{\cancel{3x^2}}{\cancel{3x^2}} = 1$	$1\,(3x^2 - 5) = 3x^2 - 5$

$\therefore$ Quotient = $x^2 + 2x + 1$

Remainder = 0

Exercise 2.3

1. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Sol.

$$
\begin{array}{r|l}
 & \boxed{x^2 + 2x + 1} \\
\hline
\boxed{3x^2 - 5} & \boxed{3x^4 + 6x^3 - 2x^2 - 10x - 5} \\
 & \underset{(-)}{3x^4} \qquad\quad \underset{(+)}{-} 5x^2 \\
\cline{2-2}
 & \qquad + 6x^3 - 3x^2 - 10x - 5 \\
 & \qquad \underset{(-)}{6x^3} \qquad\quad \underset{(+)}{-} 10x \\
\cline{2-2}
 & \qquad\qquad 3x^2 \qquad - 5 \\
 & \qquad\qquad \underset{(-)}{3x^2} \qquad \underset{(+)}{-} 5 \\
\cline{2-2}
 & \qquad\qquad\qquad\qquad \boxed{0}
\end{array}
$$

$\boxed{\text{Dividend}} = \boxed{\text{Divisor}} \times \boxed{\text{Quotient}} + \boxed{\text{Remainder}}$

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5$

$= (3x^2 - 5)\,(x^2 + 2x + 1) + 0$

Now, $x^2 + 2x + 1$ $\quad a^2 + 2ab + b^2 = (a + b)^2$

$= (x + 1)^2$

$= (x + 1)(x + 1)$

$\therefore \quad x + 1 = 0 \quad$ and $\quad x + 1 = 0$

$\therefore \quad x = -1 \quad$ and $\quad x = -1$

Its zeroes are -1 and -1

Therefore, the remaining zeroes of the given polynomial are -1 and -1.

Thank You