MATHS

Real Numbers

1. Euclid's Division Lemma:

Given positive integers a and *b,* there exists unique integers q and r satisfying
$a = bq + r$, where $0 \leq r < b$

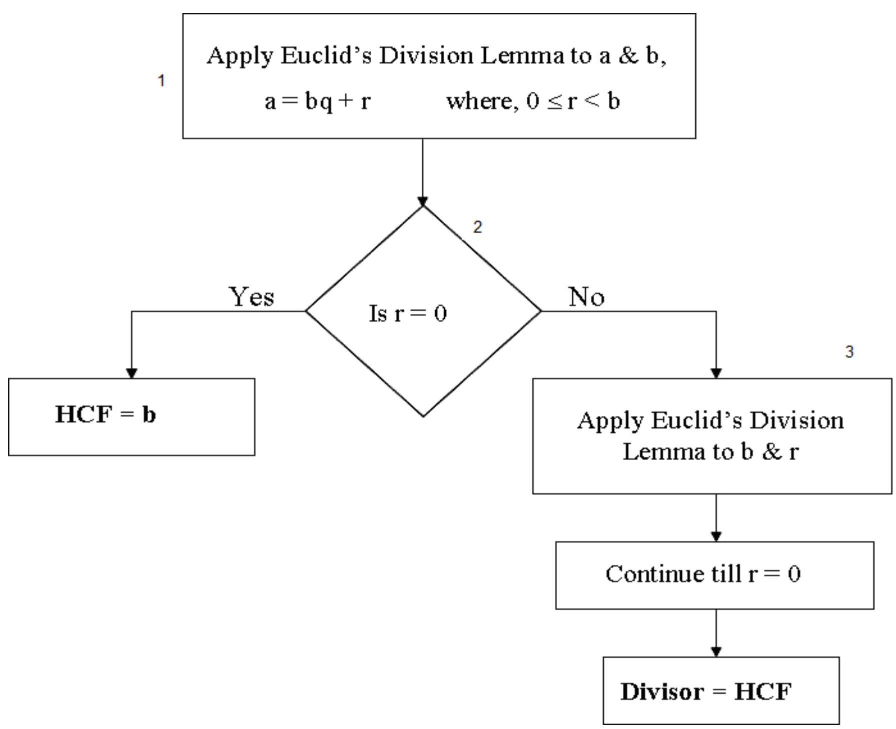
- **Lemma** is a proven statement used for proving another statement.

2. Euclid's Division Algorithm:

- An **algorithm** is a series of well defined steps which gives a procedure for solving a type of problem.
- This algorithm is a technique to compute the **H.C.F** of two given positive integers.
- According to this algorithm, the **HCF** of any two positive integers 'a' and 'b', with a > b, is obtained by following the steps given below:

Step 1: Apply Euclid's division lemma, to 'a' and 'b', to find q and r, such that $a = bq + r$, $0 \leq r < b$.
Step 2: If $r = 0$, the HCF is b. If $r \neq 0$, apply Euclid's division lemma to b and r.
Step 3: Continue the process till the remainder is zero. The divisor at this stage will be HCF (a, b). Also, note that HCF (a, b) = HCF (b, r).

Euclid's Division Algorithm can be summarized as follows:



- Euclid's Division Algorithm is stated for only positive integers but it can be extended for all integers except zero, i.e., $b \neq 0$.

3. **Real Numbers:**

- The numbers which can be represented in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called **Rational numbers**.
- Any number that cannot be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called **Irrational numbers**.
- There are more irrational numbers than rational numbers between two consecutive numbers.
- Rational and Irrational numbers together constitute **Real numbers**.

4. **Properties of Irrational numbers**:

i. The **Sum, Difference, Product** and **Division** of two irrational numbers need not always be an irrational number.

ii. **Negative** of an irrational number is an irrational number.

iii. **Sum** of a **rational** and an **irrational** number is irrational.

iv. **Product** and **Division** of a non-zero rational and irrational number is always irrational.

5. **Fractions:**

- **Terminating fractions** are the fractions which leaves remainder 0 on normal division.
- **Recurring fractions** are the fractions which never leave a remainder 0 on normal division.

6. **Properties related to prime numbers:**

- If p is a prime and divides a^2, then p divides a, where 'a' is a positive integer.
- If p is a prime, then $\sqrt{p}$ is an irrational number.
- A number ends with the digit zero if and only if it has 2 and 5 as two of its prime factors.

7. **Decimal Expansion:**

- The decimal expansion of rational number is either **terminating** or **non-terminating recurring (repeating).**
- If the decimal expansion of rational number **terminates**, then we can express the number in the form of $\frac{p}{q}$, where p and q are co prime, and the prime factorization of **q is of the form 2^n5^m**, where n and m are non negative integers.
- If x = $\frac{p}{q}$ is a rational number, such that the prime factorization of q is of the form 2^n5^m, where n, m are non-negative integers. Then, x has a decimal expansion which **terminates.**

- If the denominator of a rational number is of the form 2^n5^m, then it will terminate after n places if n > m or after m places if m > n.

- The decimal expansion of an irrational number is **non-terminating, non-recurring.**

8. **Fundamental Theorem of Arithmetic**:

   Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

- The procedure of finding **HCF(Highest Common Factor)** and **LCM(Lowest Common Multiple)** of given two positive integers a and b:

i. Find the prime factorization of given numbers.

ii. HCF(a, b) = **Product of the smallest power of each common prime factors in the numbers.**

iii. LCM(a, b) = **Product of the greatest power of each prime factors, involved in the numbers.**

9. **Relationship between HCF and LCM** of two numbers:

   If a and b are two positive integers, then HCF (a, b) $\times$ LCM (a, b) = a $\times$ b

10. **Relationship between HCF and LCM of three numbers:**

$$LCM\ (p, q, r) = \frac{p.\ q.\ r\ .\ \text{HCF}\ (p, q, r)}{\text{HCF}\ (p, q).\text{HCF}\ (q, r).\ \text{HCF}\ (p, r)}$$

$$HCF\ (p, q, r) = \frac{p.\ q.\ r\ .\ \text{LCM}\ (p, q, r)}{\text{LCM}\ (p, q).\ \text{LCM}\ (q, r).\ \text{LCM}\ (p, r)}$$