Lecture 5

Module 17

Q.) Form the quadratic equation if its roots are -3 and -11

Sol: The roots of the quadratic equation are -3 and -11.

Let $\alpha = \boxed{-3}$ and $\beta = \boxed{-11}$

Sum of the roots :

$$\alpha + \beta = -3 + (-11) = -3 - 11 = \boxed{-14}$$

Product of the roots :

$$\alpha . \beta = -3 \times -11 = \boxed{33}$$

The required quadratic equation is

x^2 – (sum of the roots)x + Product of the roots = 0

$$x^2 - (\boxed{\alpha + \beta})x + \boxed{\alpha\beta} = 0$$

$$\therefore \quad x^2 - (-14)x + 33 = 0$$

$$\therefore \quad x^2 + 14x + 33 = 0$$

$\therefore$ The required quadratic equation is $x^2 + 14x + 33 = 0$

For forming a quadratic equation two things are required

Sum of the roots

Product of the roots

Q.) Form the quadratic equation if its roots are $\frac{1}{2}$ and $-\frac{3}{4}$

Sol: The roots of the quadratic equation are $\frac{1}{2}$ and $-\frac{3}{4}$

Let α and β be the roots of the quadratic equation

$$\alpha = \boxed{\frac{1}{2}} \text{ and } \beta = \boxed{-\frac{3}{4}}$$

Sum of the roots :

$$\alpha + \beta = \frac{1}{2} + \left(-\frac{3}{4}\right)$$

$$\therefore \alpha + \beta = \frac{1}{2} - \frac{3}{4}$$

$$\therefore \alpha + \beta = \frac{2 - 3}{4}$$

$$\therefore \alpha + \beta = \boxed{\frac{-1}{4}}$$

Product of the roots :

$$\alpha\beta = \frac{1}{2} \times \frac{-3}{4} = \boxed{-\frac{3}{8}}$$

Now, required quadratic equa

x^2 – (sum of the roots) x + Pr

$$x^2 - \boxed{(\alpha + \beta)}x + \boxed{\alpha\beta} = 0$$

$$\therefore x^2 - \left(-\frac{1}{4}\right)x + \left(-\frac{3}{8}\right) = 0$$

$$\therefore x^2 + \frac{1}{4}x - \frac{3}{8} = 0$$

Multiplying throughout by 8 we get,

$$\therefore 8x^2 + 2x - 3 = 0$$

$\therefore$ The required quadratic equation is $8x^2 + 2x - 3 = 0$

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$$8 \times x^2 + \cancel{8}^{2} \times \frac{1}{\cancel{4}}x - \cancel{8} \times \frac{3}{\cancel{8}} = 0$$

$$8x^2 + 2x - 3 = 0$$

Sum of the roots

Product of the roots

Q.) Form the quadratic equation if its roots are : (iv) -2 and $\frac{11}{2}$

Sol: The roots of the quadratic equation are -2 and $\frac{11}{2}$

Let α and β be the roots of the quadratic equation

$$\alpha = \boxed{-2} \text{ and } \beta = \boxed{\frac{11}{2}}$$

Sum of the roots :

$$\alpha + \beta = -2 + \frac{11}{2}$$

$$\therefore \alpha + \beta = \frac{-4 + 11}{2}$$

$$\therefore \alpha + \beta = \boxed{\frac{7}{2}}$$

Product of the roots :

$$\alpha\beta = -\cancel{2} \times \frac{11}{\cancel{2}}$$

$$\therefore \alpha\beta = \boxed{-11}$$

Now, required quadratic equation is

x^2 – (sum of the roots)x + Product o

$$x^2 - (\boxed{\alpha + \beta})x + \boxed{\alpha\beta} = 0$$

$$\therefore \quad x^2 - \frac{7}{2}x + (-11) = 0$$

$$\therefore \quad x^2 - \frac{7}{2}x - 11 = 0$$

Multiplying throughout by 2 we get,

$$\therefore \quad 2x^2 - 7x - 22 = 0$$

$\therefore$ The required quadratic equation is $2x^2 - 7x - 22 = 0$

$$2 \times x^2 - \cancel{2} \times \frac{7}{\cancel{2}}x - 2 \times 11 = 0$$

$$2x^2 - 7x - 22 = 0$$

Sum of the roots

Product of the roots

Module 18

Q.) Form the quadratic equation with real coefficients if its one of the root is $3 - 2\sqrt{5}$

It is an irrational root

Then other root will be its conjugate

Sol: If one of the root of the quadratic equation is $3 - 2\sqrt{5}$, then the other root is $3 + 2\sqrt{5}$,

Let $\alpha = \boxed{3 - 2\sqrt{5}}$ and $\beta = \boxed{3 + 2\sqrt{5}}$

Sum of the roots :

Now, $\alpha + \beta = 3 - \cancel{2\sqrt{5}} + 3 + \cancel{2\sqrt{5}}$

$\therefore \alpha + \beta = \boxed{6}$

Product of the roots :

and $\alpha . \beta = \boxed{(3 - 2\sqrt{5})}\boxed{(3 + 2\sqrt{5})}$

$(a - b)(a + b) = a^2 - b^2$

$\therefore \alpha\beta = (3)^2 - (2\sqrt{5})^2$

$\therefore \alpha\beta = 9 - (4 \times 5)$

$\therefore \alpha\beta = 9 - 20$

$\therefore \alpha\beta = \boxed{-11}$

Now, required quadratic equation is

For forming a quadratic equation there are two things required

$x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0$

$x^2 - \boxed{(\alpha + \beta)}x + \boxed{\alpha\beta} = 0$

Sum of the roots

Product of the roots

$\therefore x^2 - 6x + (-11) = 0$

$\therefore x^2 - 6x - 11 = 0$

$\therefore$ The required quadratic equation is $x^2 - 6x - 11 = 0$

Q.) Form the quadratic equa... if it ... of the equation $2\sqrt{3}$ ...

It is an irrational root

Then other root will be its conjugate

Sol: If one root of the quadratic equation is $2\sqrt{3} - 4$, then the other r...

For forming a quadratic equation there are two things required

Let $\alpha = 2\sqrt{3} - 4$ and $\beta = 2\sqrt{3} + 4$

Sum of the roots :

$\alpha + \beta = 2\sqrt{3} - \cancel{4} + 2\sqrt{3} + \cancel{4}$

$\therefore \alpha + \beta = \boxed{4\sqrt{3}}$

Product of the roots :

$\alpha\beta = (2\sqrt{3} - 4)(2\sqrt{3} + 4)$

$(a - b)(a + b) = a^2 - b^2$

$\therefore \alpha\beta = (2\sqrt{3})^2 - (4)^2$

$\therefore \alpha\beta = (4 \times 3) - 16$

$\therefore \alpha\beta = 12 - 16$

$\therefore \alpha\beta = \boxed{-4}$

Now, required quadratic equatio...

x^2 – (sum of the roots) x + Produ...

Sum of the roots

Product of the roots

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - 4\sqrt{3}x + (-4) = 0$

$x^2 - 4\sqrt{3}x - 4 = 0$

$\therefore$ The required quadratic equation is
$x^2 - 4\sqrt{3}x - 4 = 0$

Q.) Form the quadratic equation if its one of the root is $\sqrt{5} - \sqrt{3}$

It is an irrational root

Then other root will be its conjugate

Sol: If one root of the quadratic equation is $\boxed{\sqrt{5} - \sqrt{3}}$, then the other root is $\sqrt{5} + \sqrt{3}$

Let $\alpha = \boxed{\sqrt{5} - \sqrt{3}}$ and $\beta = \boxed{\sqrt{5} + \sqrt{3}}$

Now required quadratic equation is

For forming a quadratic equation there are two required

x – the roots = 0

$$x^2 - \boxed{(\alpha + \beta)}x + \boxed{\alpha\beta} = 0$$

Sum of the roots

$$\therefore x^2 - 2\sqrt{5}x + 2 = 0$$

Product of the roots

equation

0

Sum of the roots :

$$\alpha + \beta = \sqrt{5} - \cancel{\sqrt{3}} + \sqrt{5} + \cancel{\sqrt{3}}$$

$$\therefore \alpha + \beta = \boxed{2\sqrt{5}}$$

Product of the roots :

$$\alpha\beta = \boxed{(\sqrt{5} - \sqrt{3})}\boxed{(\sqrt{5} + \sqrt{3})}$$

$$(a - b)(a + b) = a^2 - b^2$$

$$\therefore \alpha\beta = (\sqrt{5})^2 - (\sqrt{3})^2$$

$$\therefore \alpha\beta = 5 - 3$$

$$\therefore \alpha\beta = \boxed{2}$$

Module 19

Formulae we need to know

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

Q.) If the sum of the roots of the quadratic equation is 3 and sum of their cubes is 63, find the quadratic equation.

Sol: Let α and β be the roots of the required quadratic equation

$\boxed{\alpha + \beta} = \boxed{3}$

$\boxed{\alpha^3 + \beta^3} = \boxed{63}$

$\boxed{\alpha^3 + \beta^3} = (\boxed{\alpha + \beta})^3 - 3\alpha\beta\,(\boxed{\alpha + \beta})$

$\therefore\ 63 = (3)^3 - 3\alpha\beta\ (3)$

$\therefore\ 63 = 27 - 9\alpha\beta$

$\therefore\ 9\alpha\beta = 27 - 6$

$\therefore\ 9\alpha\beta = -36$

$\therefore\ \alpha\beta = \boxed{-4}$

We know that the re ion is

$x^2 - (\boxed{\alpha + \beta})x$

$\therefore\ x^2 - 3x + (-$

$\therefore\ x^2 - 3x$

$\therefore$ The required quadratic equation is $x^2 - 3x - 4 = 0$

Since there is one unknown we use one formula

alue

No

of the roots

Product of the roots

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

Q.) If the difference of the roots of the quadratic equati... their cubes is 215, find the quadratic equation.

Sol: Let α and β be the roots of a quadratic equation.

$\boxed{\alpha - \beta} = \boxed{5}$

$\boxed{\alpha^3 - \beta^3} = \boxed{215}$

Also, $\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$

$\therefore\ 215 = (5)^3 + 3\,\alpha.\beta(5)$

$\therefore\ 215 = 125 + 15\,\alpha.\beta$

$\therefore\ 215 - 125 = 15\,\alpha.\beta$

$\therefore\ 90 = 15\,\alpha.\beta$

$\therefore\ \alpha.\beta = \frac{90}{15}$

$\therefore\ \boxed{\alpha.\beta} = \boxed{6}$

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\,\alpha.\beta$

$\therefore\ (5)^2 = (\alpha + \beta)^2 - 4\,(6)$

$\therefore\ 25 = (\alpha + \beta)^2 - 24$

For forming a quadratic equation there are two things required.

Sum of the roots

Product of the roots

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

Since both are unknown we need to use two formulae

No

$x^2 - 7x + 6 = 0$ or $x^2 + 7x + 6 = 0$

$\therefore$ The required quadratic equation is $x^2 - 7x + 6 = 0$ or $x^2 + 7x + 6 = 0$.

Module 20

Q. If α and β are the zeros of the quadratic polynomial $\boxed{f(x) = x^2 - 5x + 4}$, find the value of $\boxed{\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta}$.

Sol. $f(x) = 1x^2 - 5x + 4$

Here $a = 1,\ b = -5,\ c = 4$

$\because$ α and β are the zeros of $f(x)$

$$\therefore \quad \alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{1} = 5$$

$$\text{and} \quad \alpha\beta = \frac{c}{a} = \frac{4}{1} = 4$$

$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta + \alpha}{\alpha\beta} - 2\alpha\beta$$

$$= \frac{\beta + \beta}{\alpha\beta} - 2\alpha\beta$$

$$= \frac{5}{4} - \underline{2 \times 4} = \frac{5}{4} - \frac{8}{1}$$

$$= \frac{5 - 32}{4} = \frac{-27}{4}$$

$$\therefore \quad \boxed{\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{-27}{4}}$$

Q. If α and β are the zeros of the quadratic polynomial $p(y) = 5y^2 - 7y + 1$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

Sol. $p(y) = 5y^2 - 7y + 1$

Here $a = 5$, $b = -7$, $c = 1$

$\because$ α and β are the zeros of $p(y)$

$$\therefore \quad \alpha + \beta = \frac{-b}{a} = \frac{-(-7)}{5} = \frac{7}{5}$$

$$\text{and} \quad \alpha\beta = \frac{c}{a} = \frac{1}{5}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$= \frac{\frac{7}{5}}{\frac{1}{5}} = \frac{7}{5} \div \frac{1}{5} = \frac{7}{\cancel{5}} \times \frac{\cancel{5}}{1} = 7$$

$$\therefore \quad \boxed{\frac{1}{\alpha} + \frac{1}{\beta} = 7}$$

Thank You