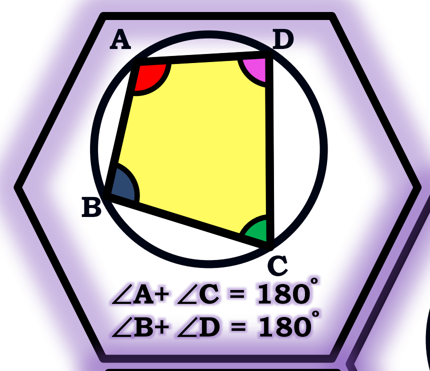
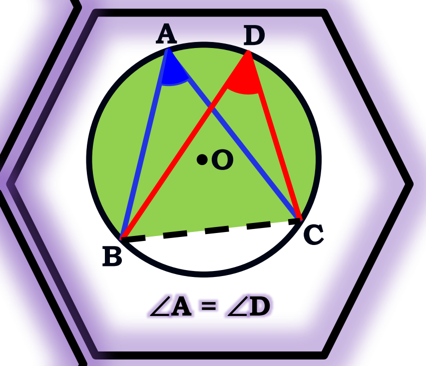
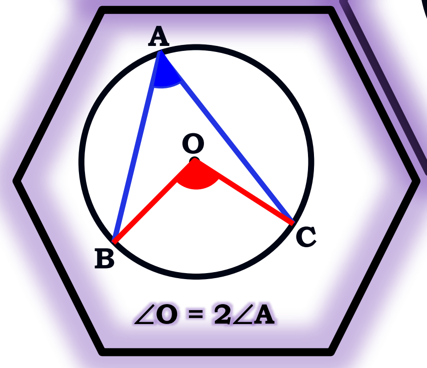
CIRCLE







MODULE 1

CIRCLE

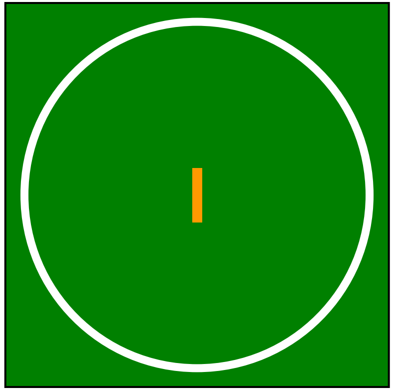
Let us see some examples of circle from day-to-day life.

Bangle

Ball

Coins

Why are these examples wrong ?

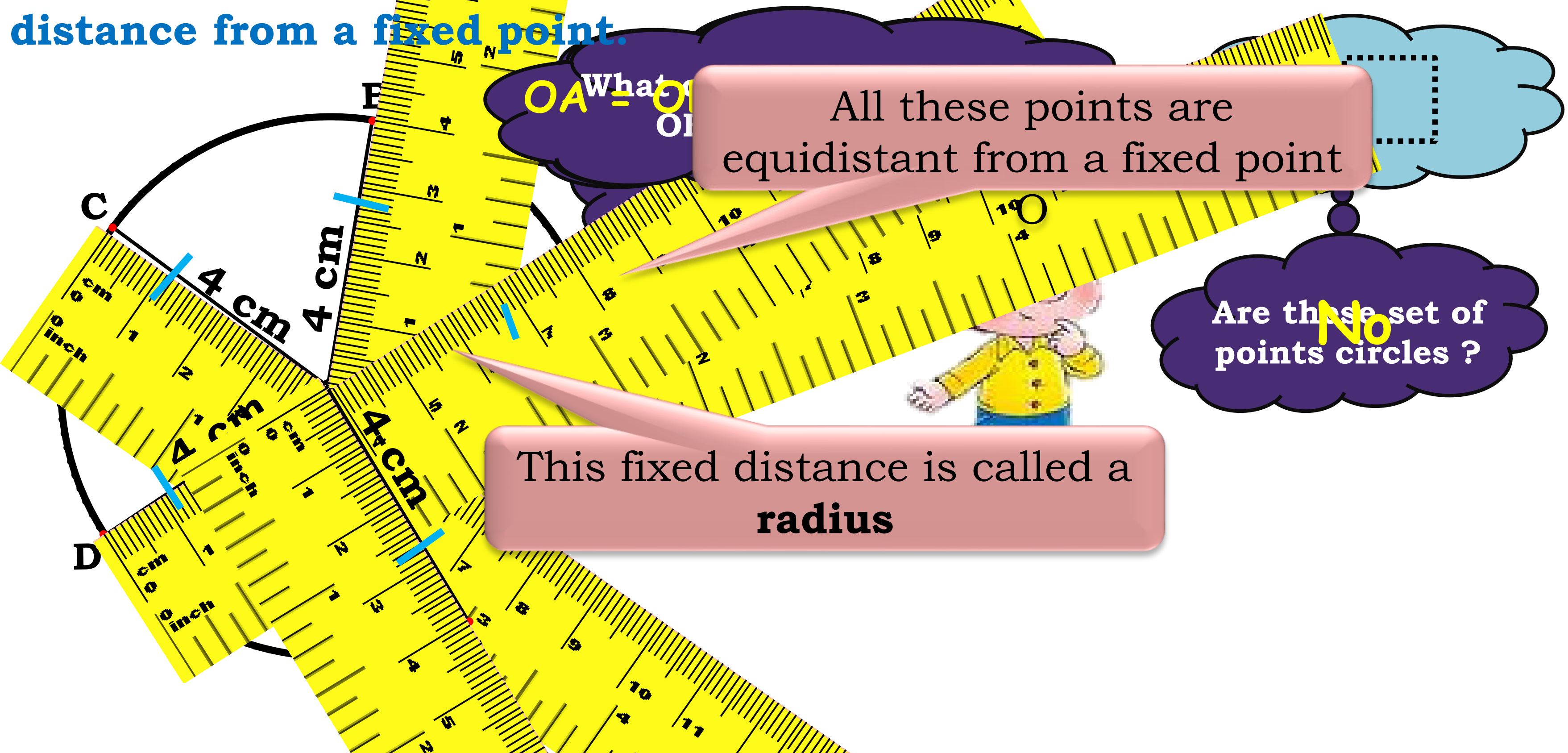


Boundary rope of a cricket ground

For this, let's understand the definition of circle

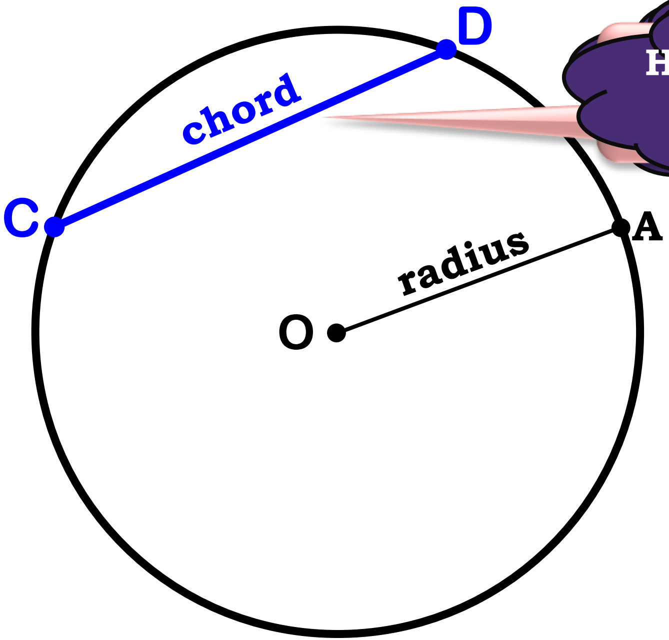
RCLE

Circle is the set o in a given plane which are at a fixed distance from a fixed point.



CIRCLE

A segment joining two distinct points on a circle is called a chord.



How many chords can we draw in a circle ?

Infinite

ed a

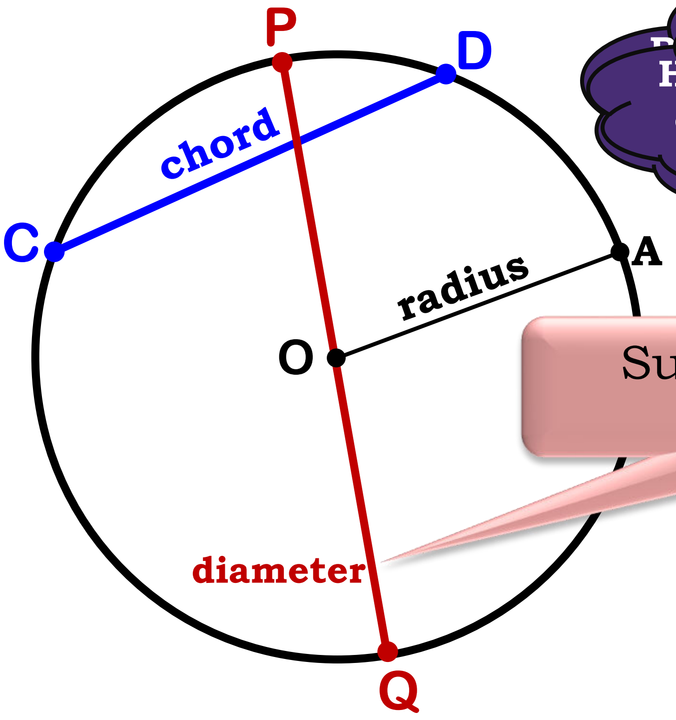
Remember :

Radii of the same

le are EQUAL

CIRCLE

A chord passing through the centre of the circle is called a diameter.



How many such diameters can we draw in a circle ?

Infinite

Such a chord is called a **diameter**

MODULE 2

THEOREM

Equal chords of congruent circles subtend equal angles at their centre

Given : AB and CD are two equal chords of congruent circles with centre at O and P resp.

To prove : $\angle AOB = \angle CPD$

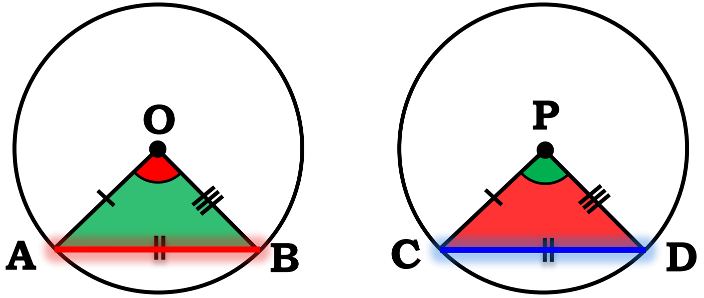
Proof :

In Δ AOB and Δ CPD,

AO = CP	[Radii of the congr
BO = DP	[Radii of the congr
AB = CD	[Given]

$\therefore$ $\Delta AOB \cong \Delta CPD$ [By SSS criterion]

$\therefore$ $\boldsymbol{\angle AOB = \angle CPD}$ [C.P.C.T.]

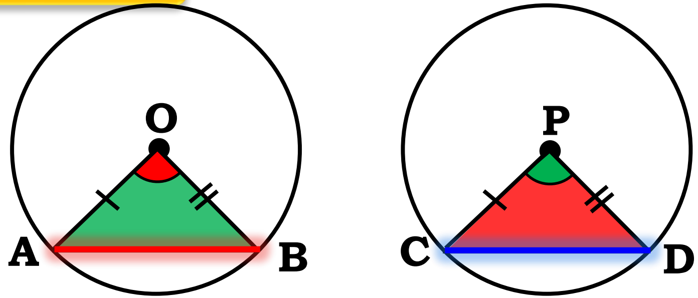


Angle subtended by chord CD at the centre is $\angle CPD$

MODULE 3

THEOREM

If chords of congruent circles subtend equal angles at their centre, then the chords are equal.



Given : $\angle AOB = \angle CPD$

To Prove : $AB = CD$

Proof :

In ΔAOB and ΔCPD,

$AO = CP$ [Rad

$\angle AOB = \angle CPD$ [

$BO = DP$ [Rad

$\therefore \Delta AOB \cong \Delta CPD$ [SAS criterion]

$\therefore$ **AB = CD** [C.P.C.T]

For proving chords equal, Prove triangles congruent

MODULE 4

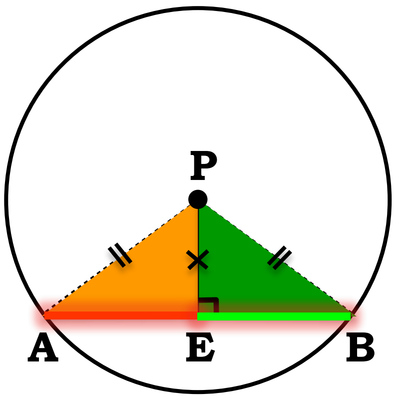
THEOREM

The perpendicular drawn from the centre of a circle to a chord bisects the chord

Given : In a circle with centre P,
PE ⊥ AB, A–E–B

To prove : AE = EB

Construction : Draw PA and PB



Proof :

By drawing PA and PB

Will ... to cr...

Fo... parts

... [Radii of same circle]

PE = PE [Common side]

ΔPEA ≅ ΔPEB [RHS rule]

∴ **AE = EB** [c.p.c.t.]

MODULE 5

Q. The radius of a circle with centre P is 25 cm and length of chord is 48 cm. Find the distance of the chord from centre P of the circle.

Given : In a circle with centre P,
PM ⊥ AB, PA = 25 cm,
AB = 48 cm

To find : PM

Sol : PM ⊥ AB,

$$\therefore AM = \frac{1}{2} AB$$

[Perpendicular drawn from centre of the circle to the chord bisects the chord]

$$\therefore AM = \frac{1}{2} \times 48$$

$\therefore$ AM = 24 cm

We know that, Perpendicular drawn from the centre to the chord, bisects the chord

[Pythagoras theorem]

$\therefore$ 25^2 = PM

Consider ΔPMA

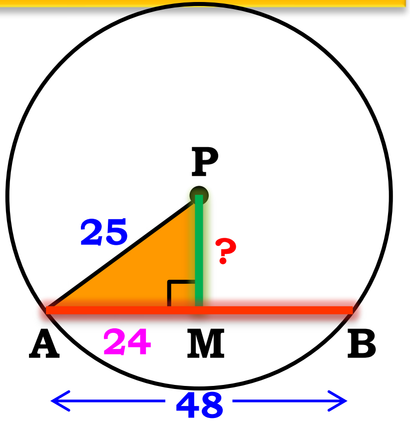
$\therefore$ 625 = P

$\therefore$ 625 − 576 = PM

$\therefore$ PM^2 = 49

$\therefore$ PM = 7 cm [Taking square roots]

Distance of chord AB from centre P is 7 cm.



MODULE 6

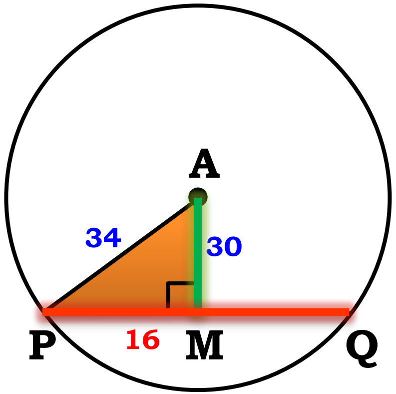
Q. AP = 34, AM = 30. If AM $\perp$ PQ, find the length of PQ.

Given : In a circle with centre A,
AM $\perp$ PQ,
AM = 30, AP = 34

To find : PQ

Sol :

Consider ΔAMP



In ΔAMP,

$\angle$AMP = 90°

$AP^2 = AM^2 + PM^2$

[Pythagoras theorem]

$\therefore 34^2 = 30^2 + PM^2$

$\therefore 1156 = 900 + PM^2$

$\therefore PM^2 = 1156 - 900$

$\therefore PM^2 = 256$

$\therefore PM = 16$ [Taking sq. roots]

AM $\perp$ PQ,

$\therefore PM = \frac{1}{2} PQ$

[Perpendicular d
centre of the c
chord bisec

We know that, Perpendicular drawn from the centre to the chord, bisects the chord

$\therefore 16 = \frac{1}{2} PQ$

$\therefore PQ = 32$

Length of PQ is 32 units.

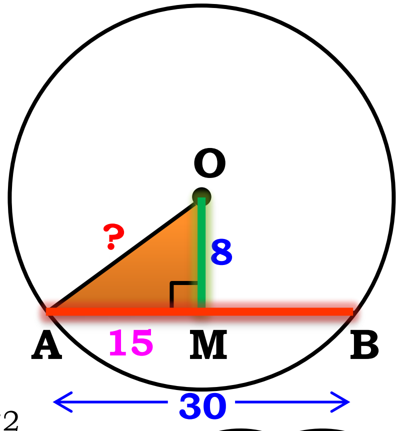
Thank You

MODULE 7

Q. A chord of length 30 cm is drawn at a distance of 8 cm from the centre of the circle. Find the radius of the circle.

Given : In a circle with centre O,
$OM \perp AB$,
$AB = 30$ cm, $OM = 8$ cm

To find : OA



Sol : $OM \perp AB$,

$$\therefore AM = \frac{1}{2} AB$$

[Perpendicular drawn fr... centre of the circle to the chord bisects the chord]

$$\therefore AM = \frac{1}{2} \times 30$$

$$\therefore AM = 15 \text{ cm}$$

We know that, Perpendicular drawn from the centre to the chord, bisects the chord

Consider ΔOMA

...M^2

...gora...

... $= 8^2 + \ldots$

$\therefore OA^2 = 64 + 225$

$\therefore OA^2 = 289$

$\therefore OA = 17$ cm [Taking square roots]

Radius of the circle is 17 cm.

MODULE 8

THEOREM

The segment joining midpoint of a chord and the centre of a circle is perpendicular to the chord.

Given : In a circle with cent

M is the midpoint

Draw PC and PD

To prove : $PM \perp CD$

Construction : Draw PC and PD

Proof :

In ΔPMC and ΔPMD,

Consider ΔPMC and ΔPMD

$PM = PM$ [Comm

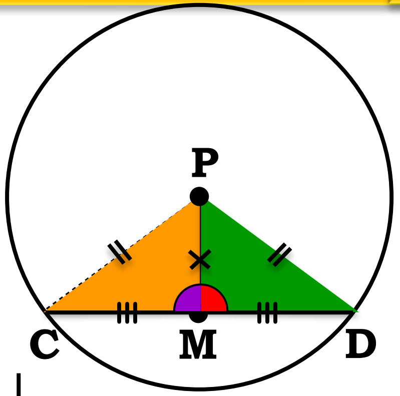
$CM = MD$ [M is midpoint of CD]

$PC = PD$ [Radii of the same circle]

$\therefore \Delta PMC \cong \Delta PMD$ [SSS criterion]

$\therefore \angle PMC = \angle PMD$ ...(i) [c.p.c.t.]

But, $\angle PMC + \angle PMD = 180^\circ$ [Angles in linear pair]



$\therefore \angle PMC + \angle PMC = 180^\circ$ [From (i)]

$\therefore 2\angle PMC = 180^\circ$

$\therefore \angle PMC = 90^\circ$

$\therefore$ **PM ⊥ CD**

MODULE 9

Q. **In a circle with centre P, chord AB is smaller than the sum of sides PA and PB by 4 cm. If the perimeter of ΔPAB is 144 cm, then find the length of PM.**

Sol.

$AB = PA + PB - 4$

$= x + x - 4$

$= (2x - 4)$ cm

Perimeter

= sum of all sides

Perimeter of ΔPAB = 144 cm

$\therefore$ $PA + PB + AB = 144$

$\therefore$ $x + x + 2x - 4 = 144$

$\therefore$ $4x - 4 = 144$

$\therefore$ $4x = 144 + 4$

$\therefore$ $4x = 148$

$\therefore$ $x = 37$

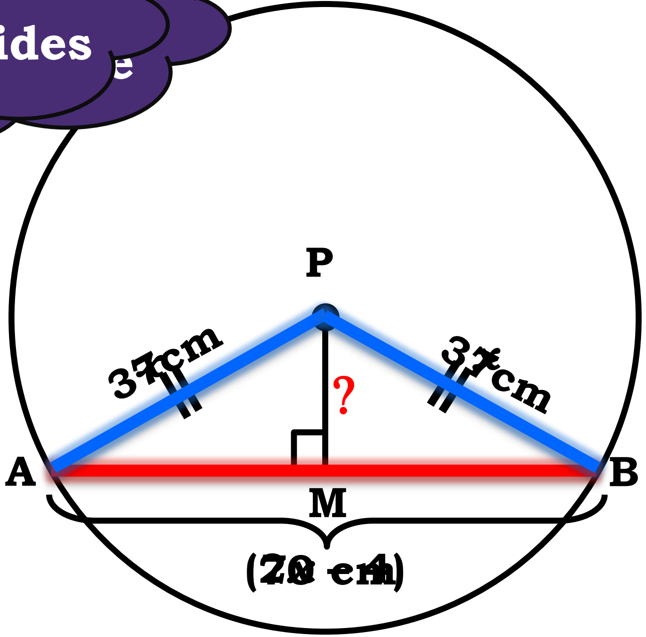
$\therefore$ $PA = PB = 37$ cm

$AB = (2x - 4)$

$= 2(37) - 4$

$= 74 - 4$

$\therefore$ $AB = 70$ cm



Q. In a circle with centre P, chord AB is smaller than the sum of sides PA and PB by 4 cm. If the perimeter of ΔPAB is 144 cm, then find the length of PM.

$PM \perp AB$

$AM = \frac{1}{2} AB$ [The perp. drawn from the centre of a circle to a chord ... chord]

$\therefore AM = \frac{1}{2} \times 70$

$\therefore AM = 35$ cm

In ΔPMA,

$PA^2 = PM^2$ ...

$\therefore 37^2 = PM^2$ ...

$\therefore 37^2 - 35^2 = PM^2$

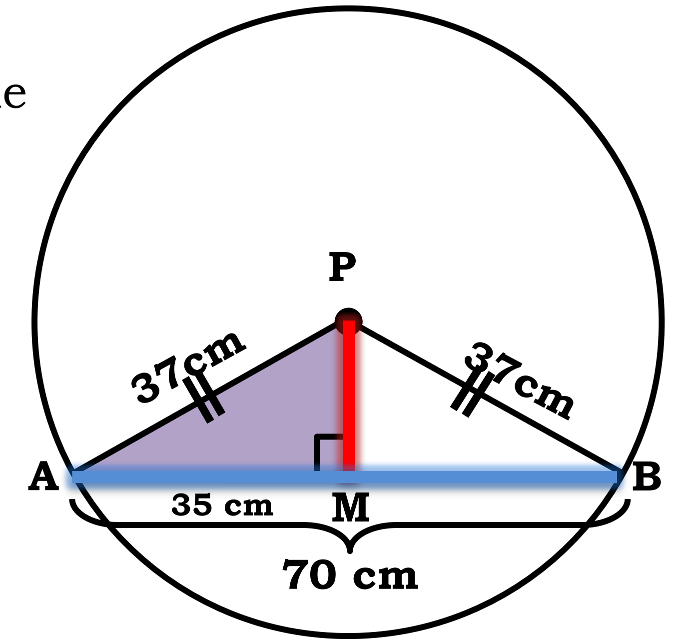
$\therefore PM^2 = (37 + 35)(37 - 35)$

$\therefore PM^2 = 72 \times 2$

$\therefore PM^2 = 144$

$\therefore \boxed{PM = 12 \text{ cm}}$ [Taking square-roots]

We know that, Perpendicular drawn from the centre to the chord, bisects the chord



MODULE 10

Converse Of Theorem Of Pythagoras

In a triangle, if the square of one side is equal to the sum of the squares of the remaining two sides, then the triangle is a right angled triangle.

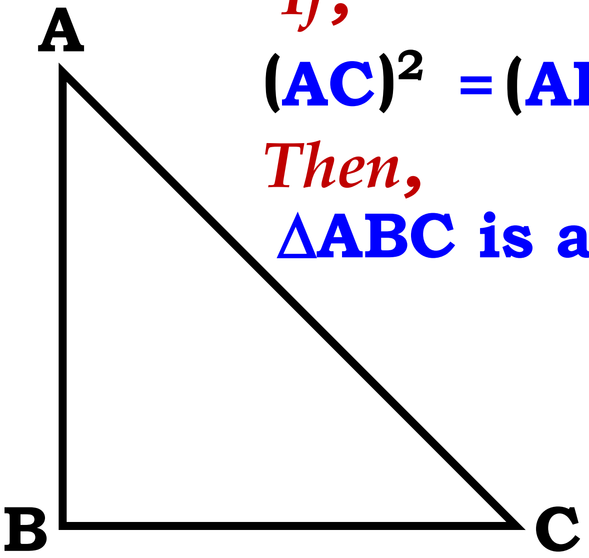
One side here refers to the largest side **i.e. side AC**

If,

$$(AC)^2 = (AB)^2 + (BC)^2$$

Then,

ΔABC is a right angled triangle



Q. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Solution :

Find : QR

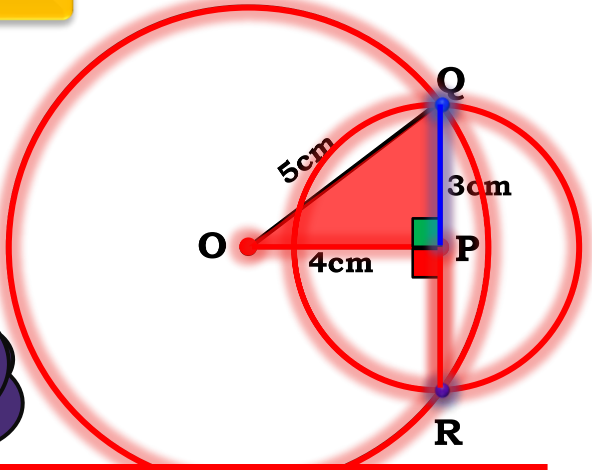
$OQ^2 = 5^2 = 25$

$PQ^2 + OP^2$

$OQ^2 = $ P...

∴

i.e. if $OQ^2 = PQ^2 + OP^2$, THEN angle opposite to OQ ($\angle OPQ$) is right angle



Similarly,

Adding (3) a...

$\angle QPO + \angle QPR = 90 + 9$...

WRONG SOLUTION

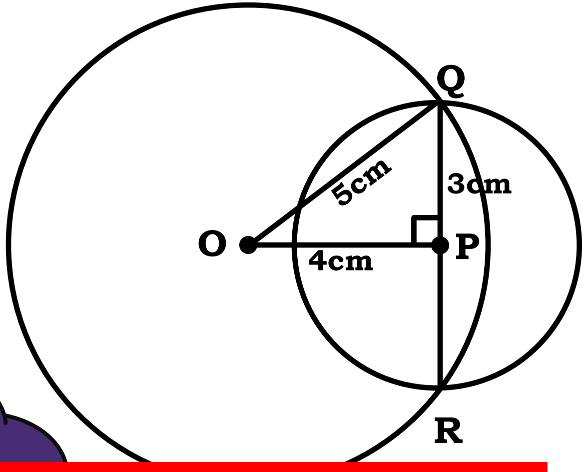
other two sides, then triangle is right triangle.

$\therefore \angle QPO + \angle QPR = 180^\circ$

$\therefore \angle QPO$ and $\angle QPR$ form linea...

∴ PQ and PR are opposite rays.

Q. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.



Solution :

Find : QR

$OP \perp QR$

$$PQ = \frac{1}{2} QR$$

[Perpendicular from the centre of the circle to the chord bisects the chord]

But, PQ = 3cm

We know that

the chord.

WRONG SOLUTION

$$\therefore \quad 3 = \frac{1}{2} QR$$

$$\therefore \quad \boxed{QR = 6 \text{ cm}}$$

Q. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Find : QR

Sol:

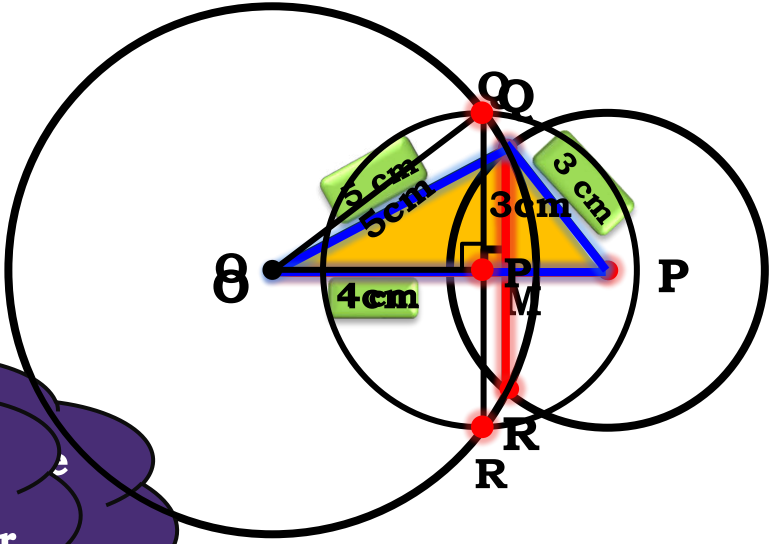
$OQ^2 = 5^2 = 25$ ... (i)

$PQ^2 + OP^2 = 3^2 + 4^2 = 25$ ... (ii)

$OQ^2 = PQ^2 + OP^2$ [From (i) and (ii)]

Chord QR is the diameter of circle with centre P

QR

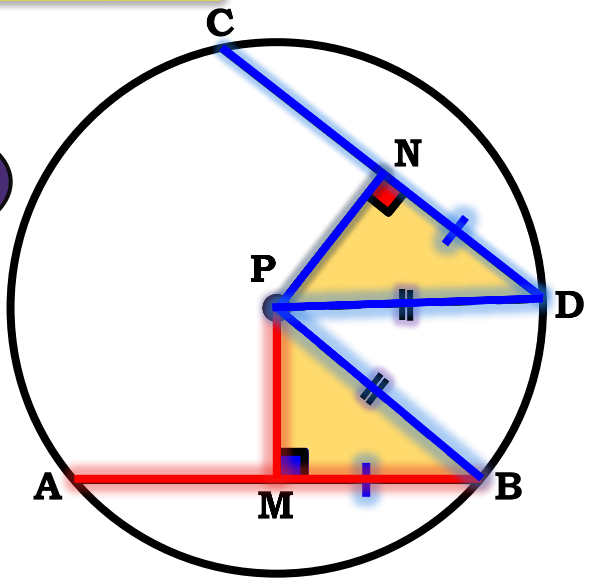


We know that, Segment joining centres of two intersecting circles is always the perpendicular bisector of a common chord

MODULE 11

THEOREM

Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).



Given: AB = CD

To prove : PM = PN

Const. : Draw PB and P

Proof :

AB = CD [Given]

We know that, Perpendicular drawn from the centre of the circle to the chord bisects the chord

$\therefore$ ½ AB = ½ CD

½ AB = MB

½ CD = ND

$\therefore$ MB = ND…(i) [Perpend … the centre of … chord bisects the c

In ΔPMB and ΔPND

$\angle$PMB = $\angle$PND = 90° [Given]

hypt. PB = hypt. PD [radii of same circle]

MB = ND [From (i)]

$\therefore$ ΔPMB $\cong$ ΔPND [RHS rule]

$\therefore$ **PM = PN** [c.p.c.t.]

Thank You

MODULE 12

THEOREM

Chords equidistant from the centre of a circle are equal in length.

Given: PM = PN

To prove : AB = CD

Const. : Draw PB &

Proof :

In ΔPMB and ΔPND

$\angle PMB = \angle PND = 90^o$ [Give

hypt. PB = hypt. PD [rad e]

PM = PN [Giv

$\therefore$ $\Delta PMB \cong \Delta PND$ [RHS r

$\therefore$ MB = ND [c.p.c.t.]

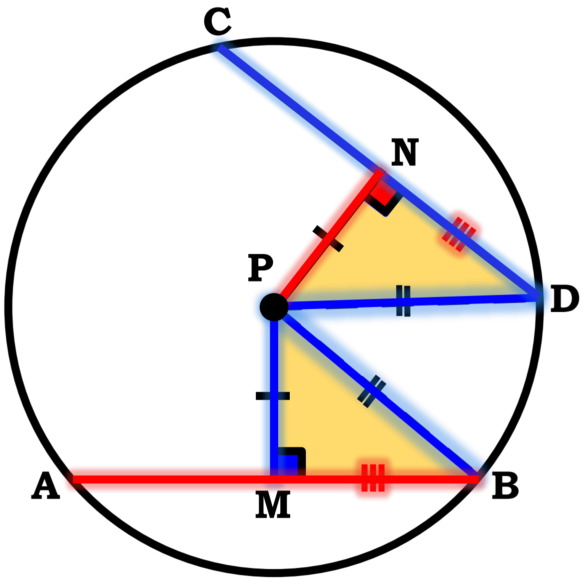
$\therefore$ 2MB = 2ND

$\therefore$ **AB = CD** [Perpendicular drawn from the centre to the chord, bisects the chord]

We know that, Perpendicular drawn from the centre of the circle to the chord bisects the chord

2 MB = AB

2 ND = CD



MODULE 13

Q. **If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corr. segments of the other chord.**

To prove : (i) AP = PD (ii) PB = CP

Construction : Draw OM ⊥ AB, ON ⊥ CD.

Proof :

$AM = MB = \frac{1}{2} AB$ .....(i)

$ND = NC = \frac{1}{2} CD$ .....(ii)

But, AB = CD .....(iii)

∴ AM = ND .....(iv)

and MB = NC .....(v) [fr

In ΔMPO and ΔNPO,

∠OMP = ∠ONP [Each 90°]

OP = OP [Common Side]

OM = ON [Equal chords are equidistant from centre]

∴ ΔOMP ≅ ΔONP [By RHS criterion]

∴ MP = NP ......(vi) [CPCT]

and (vi)

NP + ND

....(vii)

Subtracting (vii) from (iii)

AB – AP = CD – PD

∴ PB = CP

AB = CD

OM ⊥ AB

ON ⊥ CD

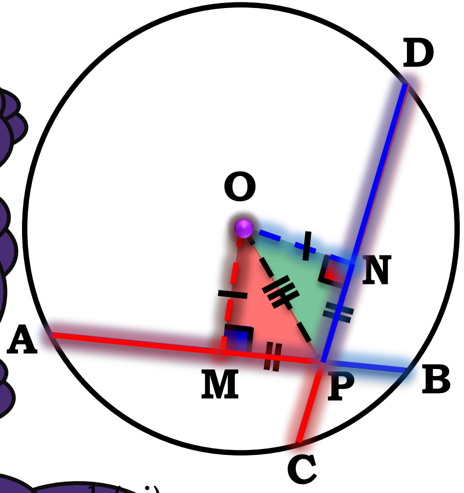
∴ OM = ON

Now,

ΔMPO

Equal chords are equidistant from the centre

perpendicular drawn from centre to the chord bisects the chord



MODULE 14

Q. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

To prove : ∠OPA = ∠OPD

Construction : Draw OE ⊥ CD

Proof :

Angle made by chord AB with OP is ∠OPA

OF = OE

we ... perpendicular ... to the ...re]

Equal chords are equidistant from the centre

In ΔO

∠OFP

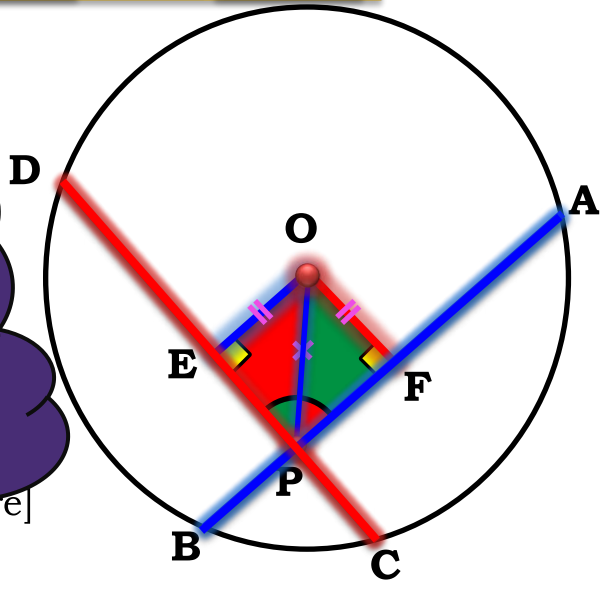
OP = OP

OF = OE

∴ ΔOPF ≅ ΔOPE

∴ ∠OPF = ∠OPE [C.P.C.T.]

∴ **∠OPA = ∠OPD** [P-F-A, P-E-D]



MODULE 15

Q. If a line intersects two concentric circles (circles with the same centre) with centre A at P, Q, R and S, then prove that PQ = RS.

To Prove : PQ = RS

Construction : Draw AM ⊥ PS.

Proof :

Consider l…

AM ⊥ c…

PM = M…

of th…

perpendicular from the centre to the chord

Now, for proving PQ = RS just prove QM = MR

PQ + … = MR + RS …(i) [P-Q-M, M-R-S]

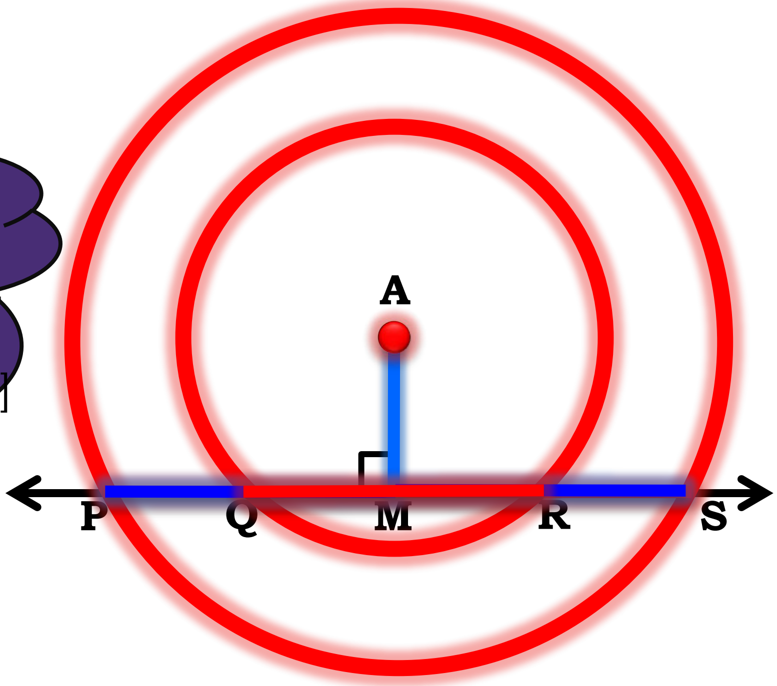
Cons…

AM ⊥ ch… …rd

QM = MR

PQ + $\cancel{QM}$ = $\cancel{QM}$ + RS [from (i) and (ii)]

PQ = RS



MODULE 16

CYCLIC QUADRILATERAL

A quadrilateral whose all the four vertices lie on a circle is called cyclic quadrilateral.

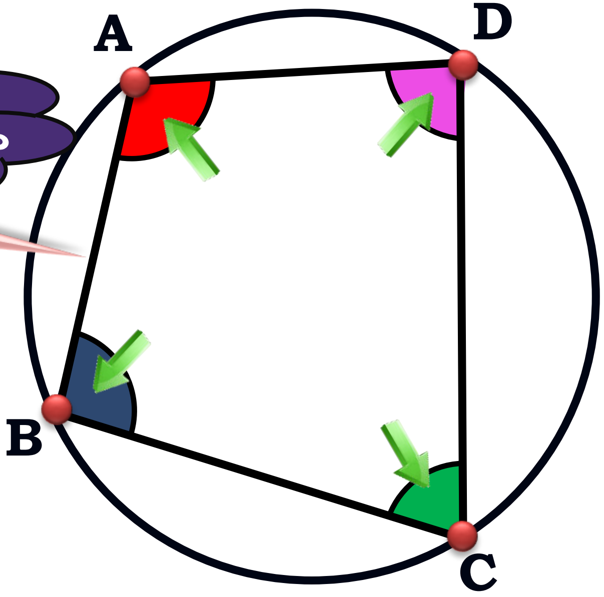
Property of a Cyclic quadrilateral

called a cyclic quadrilateral

The sum of either pair of opposite angles is 180°.

$\angle A$ & $\angle C$ = 180°

$\angle B$ & $\angle D$ = 180°



Q. If O is the centre of the circle and $\angle DAB = 50^\circ$, find the value of x and y in the following figure.

Soln.

In ΔBOA, $OA = OB$ [Radii of same circle]

$\therefore \angle OAB = \angle OBA = 50^0$ [Angles opposite to equal sides]

Now, $x = \angle OAB + \angle OBA$

$= 50 + 50$

$= 100$

$\therefore$ [illegible]

$\angle BC$ [illegible]

$\therefore \angle BC$ [illegible] $50 = 180$

$\therefore \angle BCD$ [illegible]

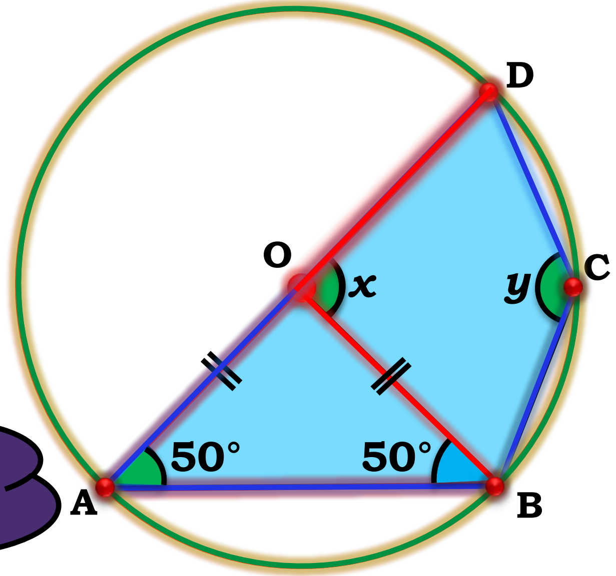
$\therefore \angle BCD = 130$

$\therefore \boxed{y = 130}$

What can we say about the opposite angles of a cyclic quadrilateral ?

Opposite angles of a cyclic quadrilateral are supplementary

opposite to the equal sides ?



$x = \angle OAB + \angle OBA$

Thank You

MODULE 17

Q. Prove that a cyclic parallelogram is a rectangle.

To prove : □ABCD is rectangle

Hint : prove $\angle A = 90°$

Proof : $\angle A = \angle C$ ... (1) [Opposite an... paral...

$\angle A + \angle C = 180°$ [Su... of a ... 0°]

$\angle A + \angle A = 180$ [F...

$\therefore \quad 2\angle A = 180$

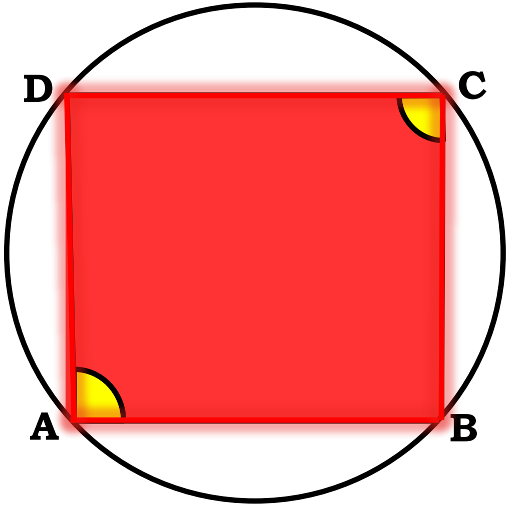
$\therefore \quad \angle A = 90°$

□ABCD is cyclic

A pa... rec...

We know that, opposite angles are supplementary

$\therefore$ □ABCD is a rectangle. [A parallelogram is a rectangle if one angle is 90°]



Q. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove it is a rectangle.

Proof :

OA = OC [radii of same circle]

OB = OD [radii

Diagonals bisect each oth

□ABCD is parallelogram

AC = BD [Diameters of the same circle]

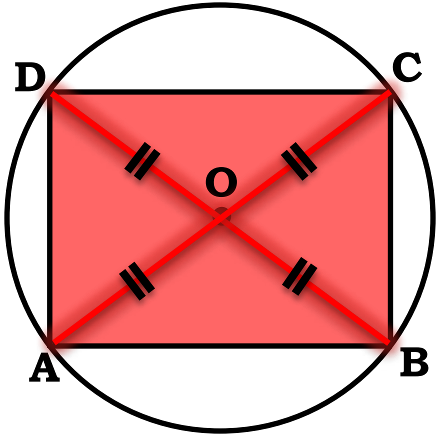
For a quad. t

(i) Diagonals sho

and (ii) they sh

OA, OB, OC and OD are radii of same circle

∴ OA = OB = OC = OD



□ ABCD is a rectangle.

[If diagonals of a parallelogram are equal, then it is rectangle]

<u>Hint : Prove</u>

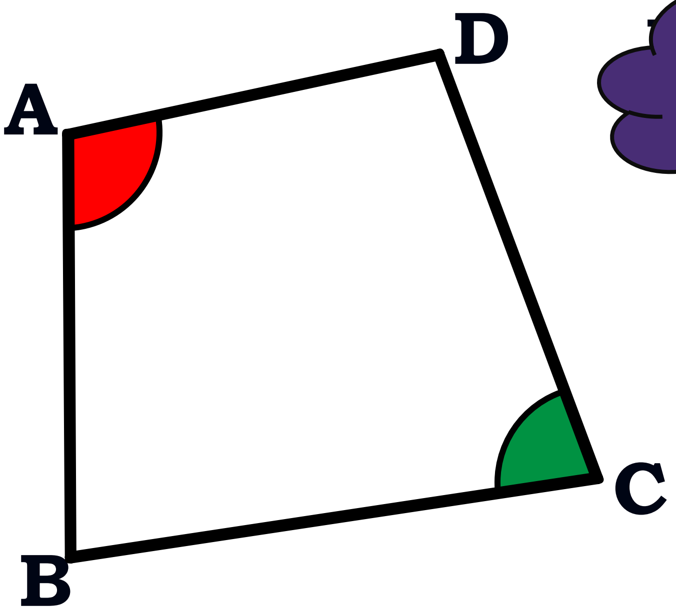
OA = OC

OB = OD

AC = BD

MODULE 18

If the sum of a pair of opposite angles of a quadrilateral is 180°, then quadrilateral is cyclic.



Let us consider □ABCD

180°

D is cyclic.

Q. If the non - parallel sides of a trapezium are equal, prove that it is cyclic.

Given : □ABCD is a trapezium , AD = BC

To prove : □ABCD is cyclic

Construction : D

Proof : $\angle A + \angle D$

In ΔDEA and Δ

$\angle DEA = \angle CFB$

AD = CB

DE = CF [Perpendicular distance between two parallel lines]

For proving angles equal, we need to prove triangles containing them to be congruent

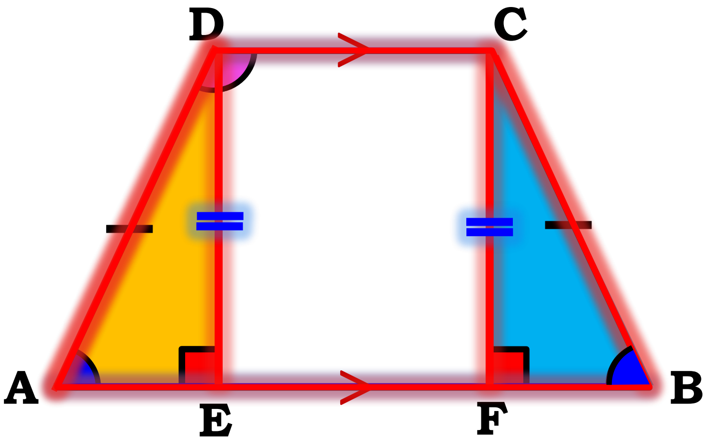
∴ $\angle A + \angle D = 180^\circ$

∴ ΔDEA ≅ ΔCFB [RHS criterion]

∴ $\angle A = \angle B$ .....(2) [CPCT]

$\angle B + \angle D = 180^\circ$ [From (1) and (2)]

∴ □ABCD is a cyclic trapezium [A quadrilateral is cyclic if a pair of opposite angles is supplementary]



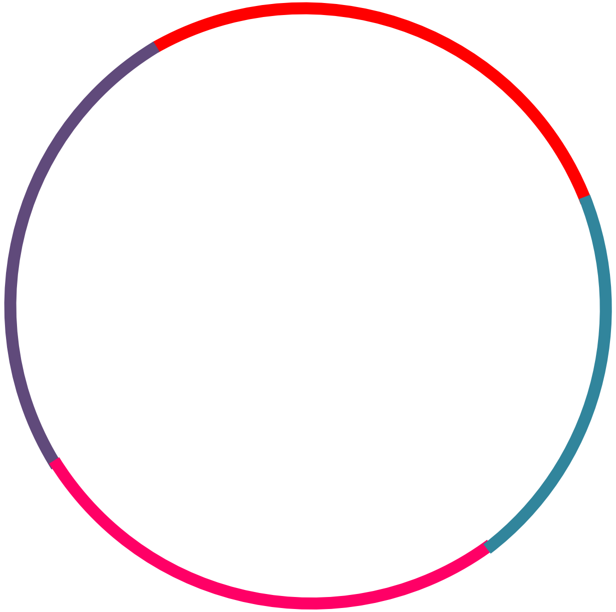
Hint :
To prove :
$\angle B + \angle D = 180^0$

MODULE 19

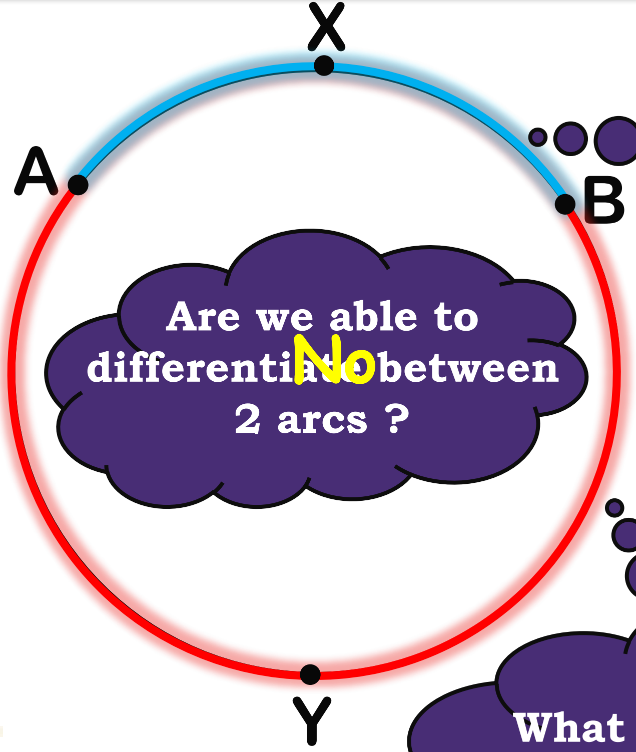
CIRCLE – ARC

An Arc is a part of a circle.

An arc is a part of a circle



NAMING OF AN ARC



Are we able to differentiate between 2 arcs ?

No

An arc is named by using 3 points

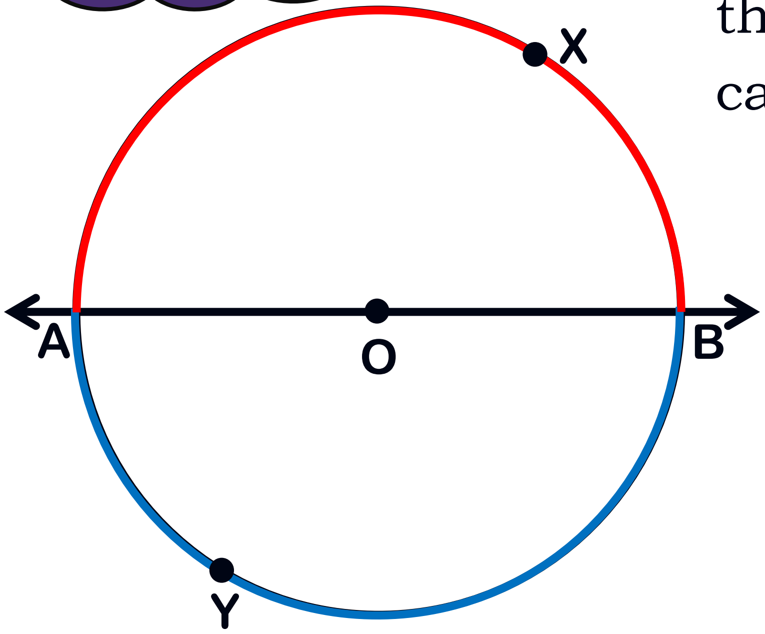
What is the name of this arc ?

arc AB

TERMS RELATED TO CIRCLE

Name the semicircles

arc AXB and arc AYB



<u>Semicircle</u> :

If a secant passes through the centre, then each arc so formed is called a Semicircle.

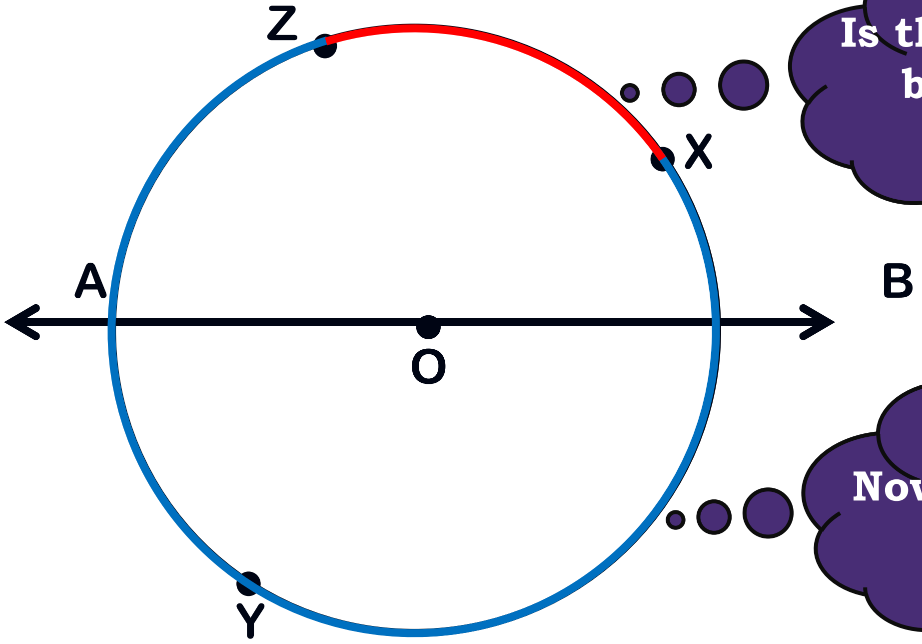
TERMS RELATED TO CIRCLE

Minor Arc :

An arc smaller than the semicircle is called a Minor arc.

Remember :

Only a minor arc can be named by 2 letters



Is this arc smaller or bigger than the semicircle ?

Smaller

...c :

...greater than the semicircle is called a Major arc.

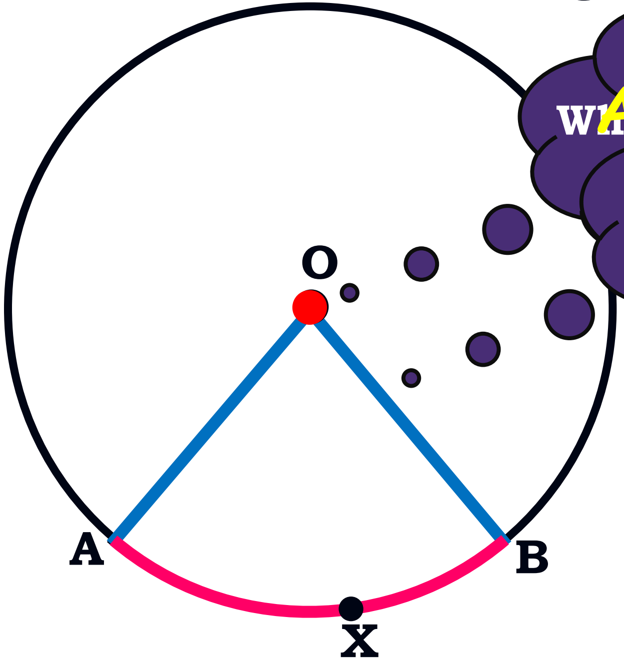
Now, what is this arc called ?

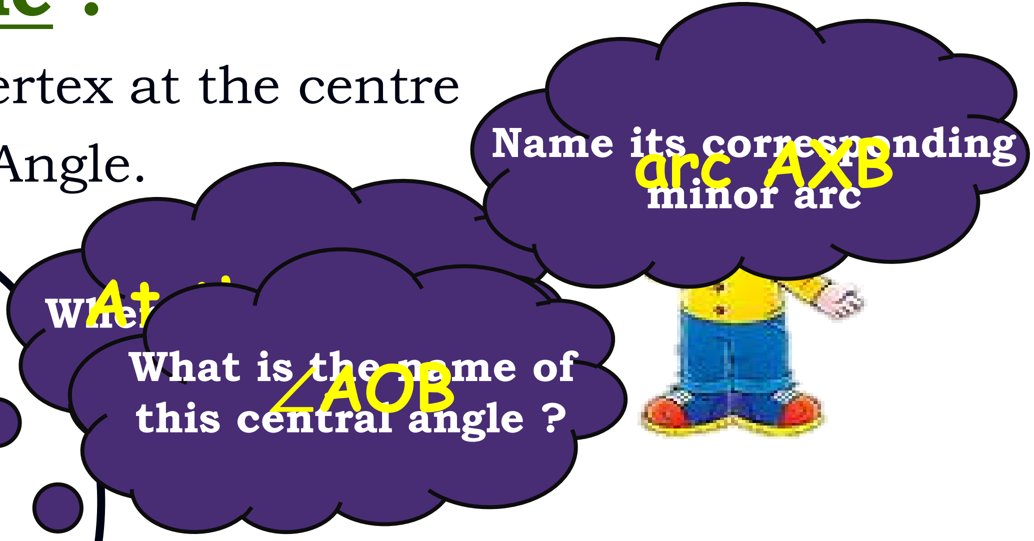
Major arc

TERMS RELATED TO CIRCLE

Central Angle :

An angle with its vertex at the centre is called a Central Angle.





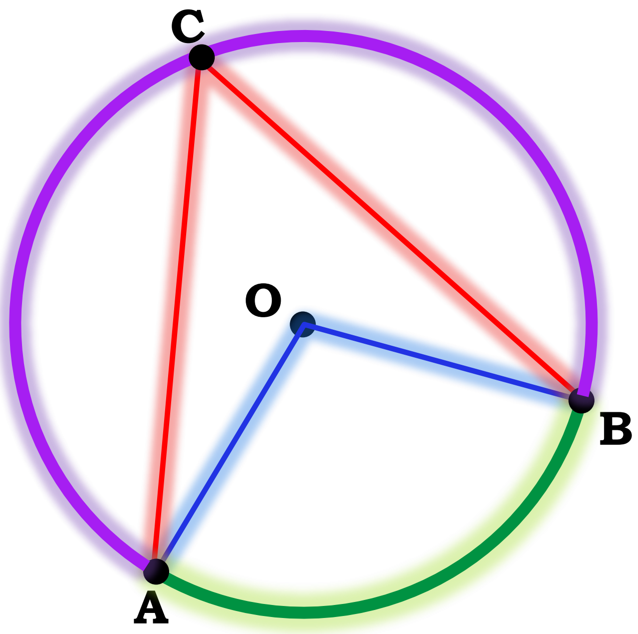
MODULE 20

THEOREM

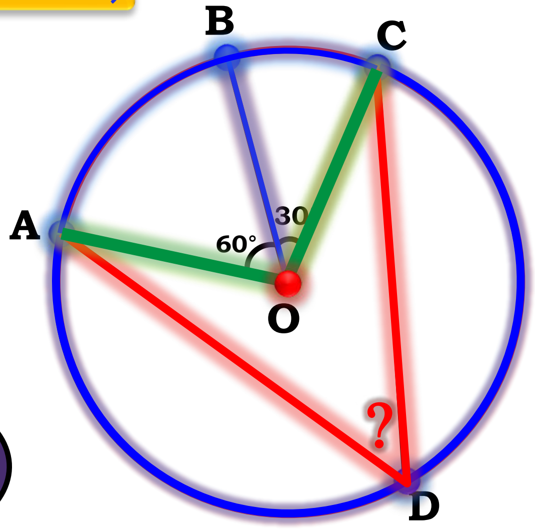
The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\angle AOB = 2\angle ACB$$

$\angle ACB$



Q. A, B and C are 3 points on a circle with centre O such that $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.



Sol.

$\angle AOC = 2\angle ADC$

∠ADC is subtended by which arc?

∠AOC is subtended by which arc?

arc ABC

Angle ... at the centre ... the angle sub... the remai... circle.

$\therefore \angle ADC = \frac{1}{2}\angle AOC$

But $\angle AOC = \angle A$...

$= 6$...

$\therefore \boxed{\angle AOC = 90^\circ}$

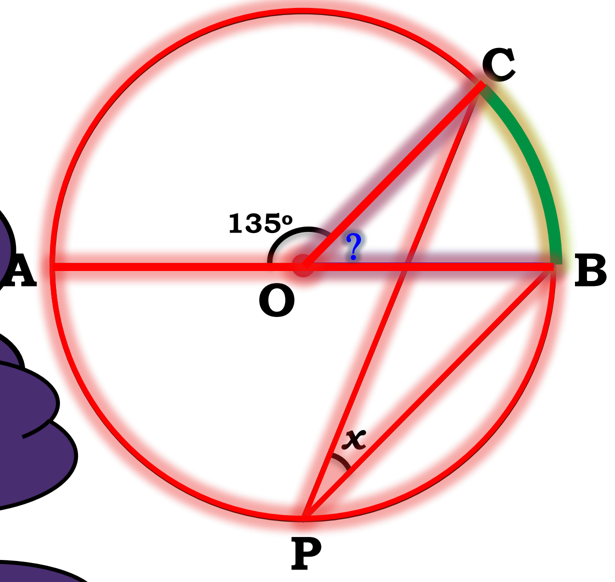
$\therefore \angle ADC = \frac{1}{2} \times 90^\circ =$

$\therefore \boxed{\angle ADC = 45^\circ}$

∠AOC = 2 ∠ADC

MODULE 21

Q. In the fig., O is the centre of the circle, find the value of x.



Sol.

$\angle COB = 2\angle CPB$

[The angle s...
double the...
remaini...

$\angle CPB$ ...

$\angle COB + \angle C$...

$\therefore \angle COB + 13$...

$\therefore \angle COB = 180 - 1$...

$\therefore \boxed{\angle COB = 45^\circ}$

$\therefore \angle CPB =$ ...

$\therefore \angle CPB = 22.5$...

i.e. $\boxed{x = 22.5^\circ}$

We...

Angle ...
at the ...
angle ...
the rem...

$\angle C$...

Now, let us find $\angle COB$

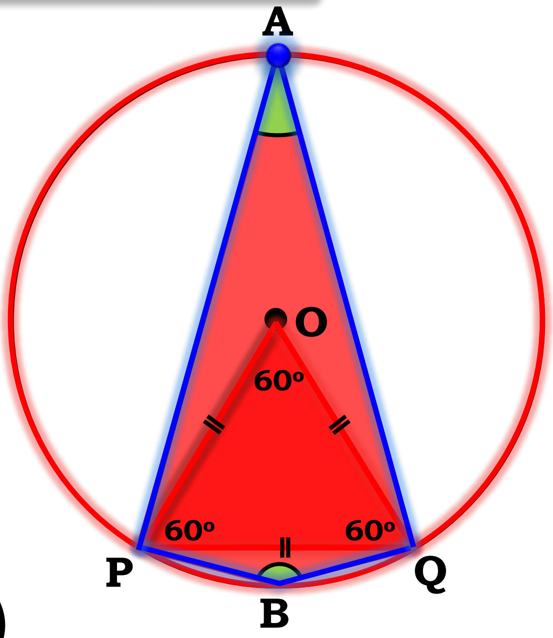
$\angle COB$ and $\angle COA$ are angles forming linear pair

$\therefore \angle COB + \angle COA = 180^\circ$

Thank You

MODULE 22

Q. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.



Sol.

To find :
(i) ∠PBQ
(ii) ∠PAQ

In ΔPOQ,

PQ = OP = OQ [Given]

∴ ΔPOQ is an equilateral triangle

∴ ∠POQ = ∠OPQ = ∠OQP = 60º

∠POQ = 2 ∠PAQ [Th

∴ 60 = 2 ∠PA

∴ $\boxed{∠PAQ = 30º}$

∠PBQ + ∠PAQ = 180º [Opposite angles of cyclic quadrilateral are supplementary]

∴ ∠PBQ + 30 = 180

∴ $\boxed{∠PBQ = 150º}$

□APBQ is cyclic

Acc.
Angle
at the
angle su
the remaining part of the circle.

Opposite angles are supplementary

the arc PBQ at the centre

MODULE 23

Q. Two congruent circles with centres O and O′ intersect at A and B. If ∠AO′B = 50°, then find ∠APB.

Sol.

∠AOB = 2

[The angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle]

In □A

OA = O

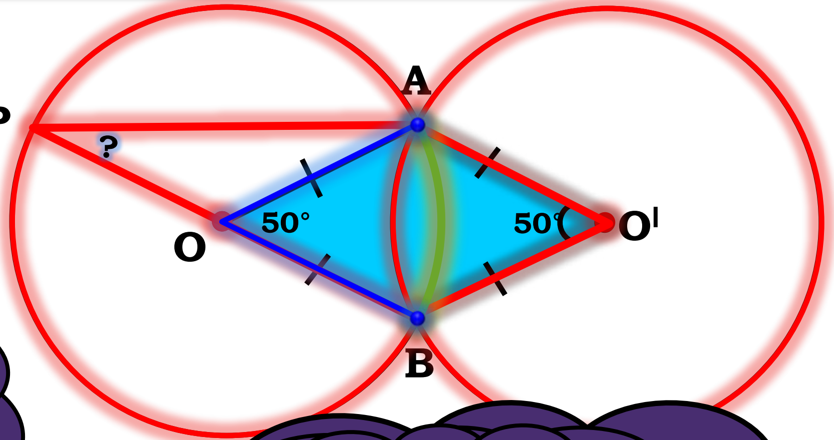
OB = O′A

∴ □AOBO′ is a rhombus

∠AOB = 50° [Opposite angles of a

∴ 50 = 2∠APB

∴ $\boxed{\angle APB = 25^\circ}$



Hint : Find ∠AOB

∠A subtended by arc AB

Also, ∠AOB is subtended by same arc AB at the centre.

What can we say about □AOBO′ ?

□AOBO′ is a rhombus [by definition]

MODULE 24

Q. If O is the centre of the circle, find the value of x

Sol. :

$\angle CBD + \angle ABC = 180^\circ$ ... s]

$\angle APC + \angle ABC =$ 1 ... a

$\angle CBD + \cancel{\angle ABC}$

$\angle CBD = \angle APC$

$\angle AOC = 2\angle A$...

[The angle subt...
the angle subt...
part of the...

$120 = 2\angle APC$

$\therefore \angle APC =$

$\therefore \angle CBD =$

i.e. $\boxed{x = 60^\circ}$

∠APC is

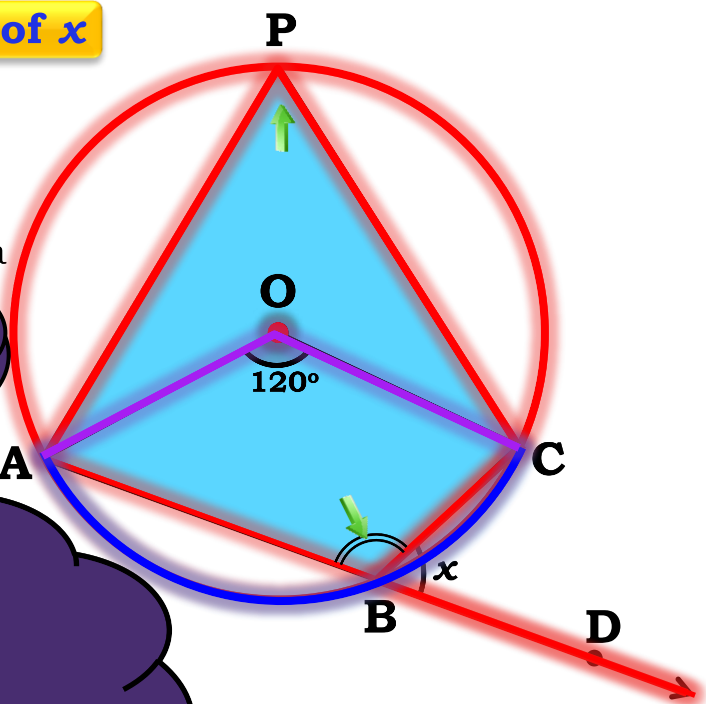
a

Now let us find ∠APC

subtended by same arc ABC at the centre.

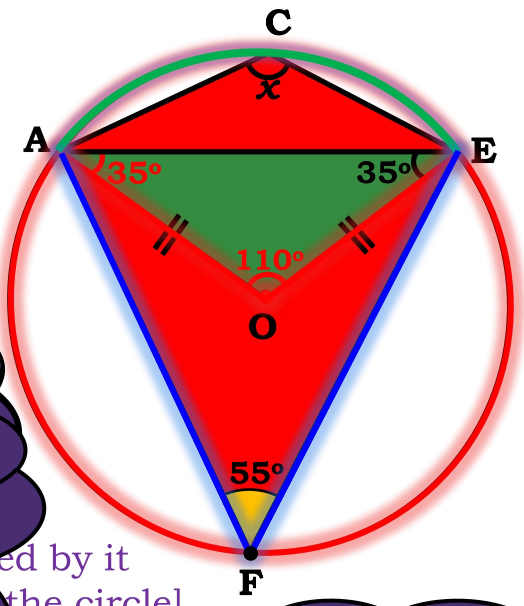
$\therefore \angle AOC = 2\angle APC$

...e.



MODULE 25

Q. If O is the centre of the circle, find the value of x.



Sol. :

In ΔAOE,

$\angle OEA + \angle OA$

$\therefore \quad 35 +$

$\therefore \quad 70$

$\therefore$

$\therefore$

Now, let us

Opposite angles are supplementary

$\angle AOE =$

ed by it

the circle]

$\therefore \quad 110^0$

$\square AFEC$ is cyclic.

$\angle AOE = 2 \angle AFE$

$\therefore \quad \angle AFE$

$\angle AFE + \angle ACE = 180^0$

$\therefore \quad 55 + x =$

$\therefore \quad x = 180 - 55^0$

$\therefore \quad \boxed{x = 125^o}$

MODULE 26

Q. In figure, $\angle PQR = 100^0$, where P,Q and R are points on a circle with center O. Find $\angle OPR$.

Construction: Join O and R , Draw $\angle PSR$

Sol. In ΔOPR, OP = OR [Radii of same circle]

$\angle OPR = \angle ORP$ [Angles opposite to equal sides]

Let $\angle OPR = \angle ORP = x$

In ΔPOR, $\angle OPR + \angle ORP + \angle POR = 180^0$

$x + x + \angle POR = 180$

$2x + \angle POR = 180$

$\angle POR = (180 - 2x)$

$\angle POR = 2\ \angle PSR$

$\angle PSR = \frac{(180 - 2x)}{2} = (90 -$

$\angle PQR + \angle PSR = 180°$

$\therefore\quad 100 + (90 - x) = 180$

$\therefore\quad 190 - x = 180$

$\therefore\quad x = 190 - 180$

$\therefore\quad x = 10$

$\therefore\quad \boxed{\angle OPR = 10}$

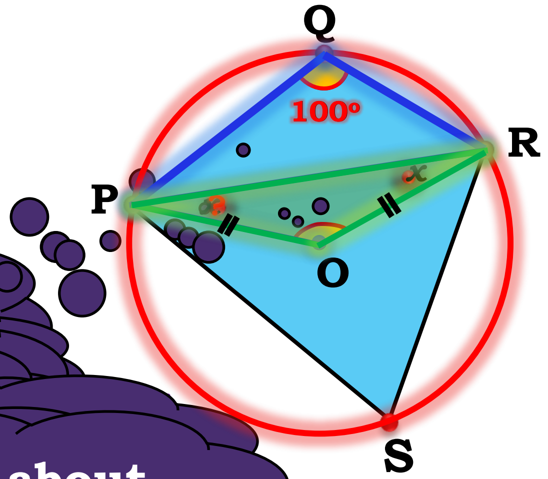
So

In ΔPOR

What can we say about opposite angles of a cyclic quadrilateral ?

They are supplementary

any point on the circle



Thank You

MODULE 27

THEOREM

Angles in the same segment of a circle are equal.

Given : ∠ACB an... ne

se...

To prove...

Construc...

Proof :

∠AOB

∠AOB =

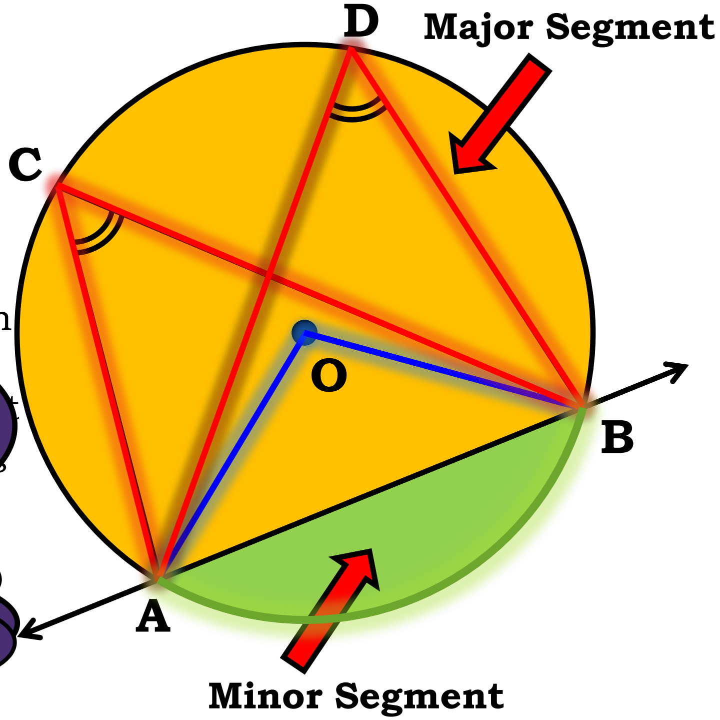
∠ADB is subtended by arc AB

Also, ∠AOB is subtended by same arc AB at the centre.

∴∠AOB = 2∠ADB

$\therefore$ $\cancel{2}\angle ACB = \cancel{2}\angle A$...

$\therefore$ $\angle ACB = \angle AD$...



MODULE 28

Q. In figure, $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$.

Soln. $\angle BDC = \angle BAC$ [Angles in the same ....(1)

∠BDC lies in segment BDC

In ΔABC,

[Sum of all angles of a triangle]

Consider ΔABC

∴

Even ∠BAC lies in the same segment

∴ $\angle B$

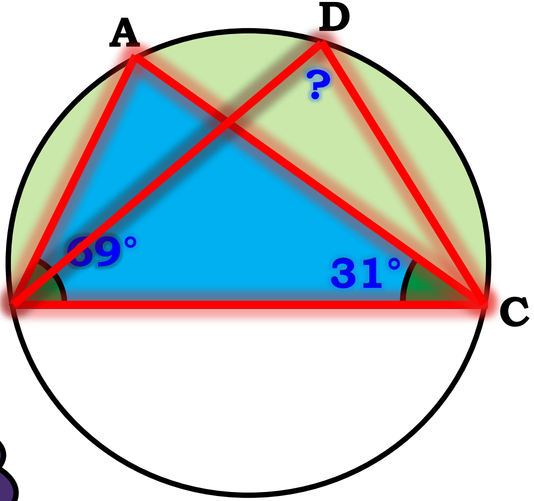
=

∴ $\angle BAC =$

∴ ∠BDC = ∠BAC

From (1) and (2),

$\boxed{\angle BDC = 80^\circ}$



MODULE 29

Q. In figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.

Soln. ∠BAC = ∠BDC

[Angles in the sa ...(1)

∠BAC lies in segment BAC

∠BEC + ∠CED = 180°

∴ 130 + ∠CED = 180

∴ ∠CED = 180 – 130

Even ∠BDC lies in the same segment

∴ ∠CED = 50°

In ΔEDC,

∠ECD + ∠CED

20° + 5

Consider ΔEDC

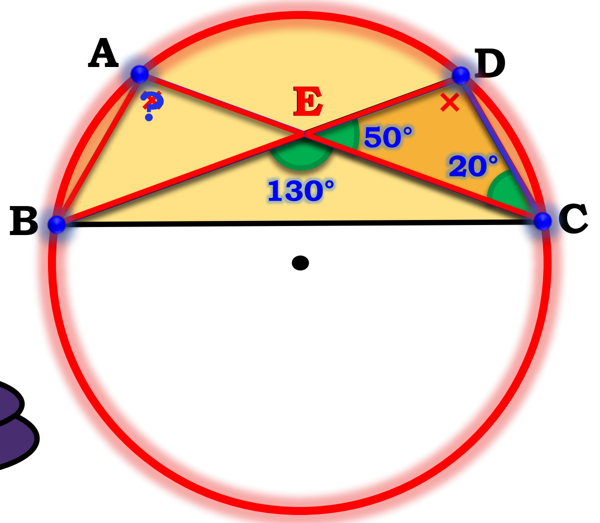
7

∠EDC = – 70

∠EDC = 110°

i.e. ∠BDC = 110° ....(2)

From (1) and (2) $\boxed{∠BAC = 110°}$



MODULE 30

Q. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC$ is 30°, find $\angle BCD$. Further, if $AB = BC$, find $\angle ECD$.

Sol. $\angle BDC = \angle BAC = 30^\circ$ [An… …me …ment

In ΔDBC

$\angle BDC + \angle DBC + \angle BCD = 180^\circ$

$30^\circ + 70^\circ +$ …

$\therefore \angle BCD = 180 - 100$

$\therefore \boxed{\angle BCD = 80^\circ}$

In ΔACB

$AB = BC$ [Given]

$\angle BCA = \angle$ …

i.e . $\angle BCE = 30^\circ$

…triangle are equal]

$\angle ECD = \angle BCD - \angle BCE$

$= 80 - 30$

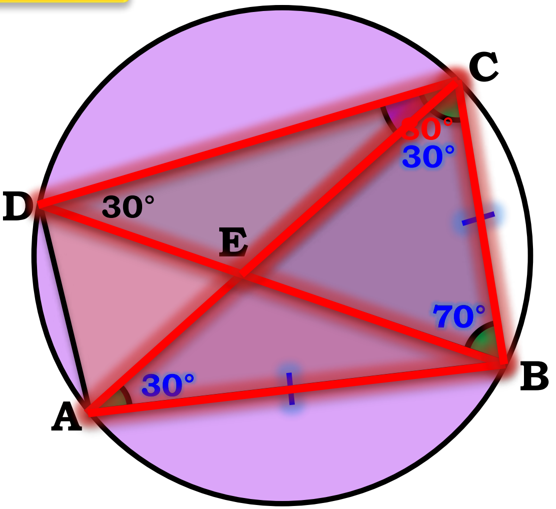
$\boxed{\angle ECD = 50^\circ}$

Consider ΔDBC

Even $\angle BDC$ lies in the same segment

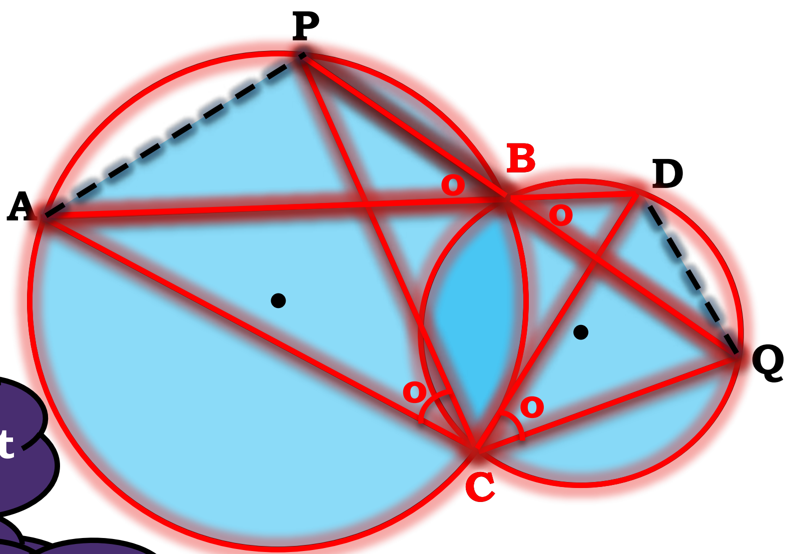
Consider ΔACB

…DC = $\angle BAC$



MODULE 31

Q. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see figure). Prove that $\angle ACP = \angle QCD$.



Sol.

$\angle ACP = \angle ABP$ [∵ $\angle$AC
se

$\angle QCD$ lies in segment QCD

$\angle QCD = \angle QBD$ [∵ A
se

$\angle ABP = \angle QBD$

Even

Ev

$\angle ABP = \angle QBD$

Vertically opposite angles

∴ From (1), (2) and (3), we h

∴ $\angle$

∴ $\angle QCD = \angle QBD$

$\boxed{\angle ACP = \angle QCD}$

MODULE 32

Q. ΔABC and ΔADC are two right triangles with common hypt. AC

Prove : $\angle CAD = \angle CBD$.

Proof :

$\angle ADC + \angle ABC$ = 90 …

=

□ABCD is cyclic.

[A quadrilateral is cyclic i… angles is supplementar…

$\angle CAD = \angle CBD$

[Angles in the same segment]

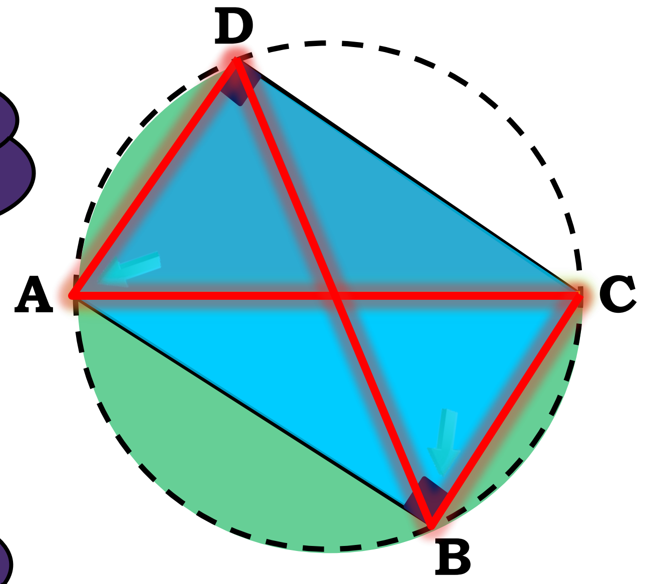
$\angle ADC$ + …

Therefore, vertices A,B,C,D lie on a circle

opp… is supplement…

$\angle CAD$ lie in the major segment CAD

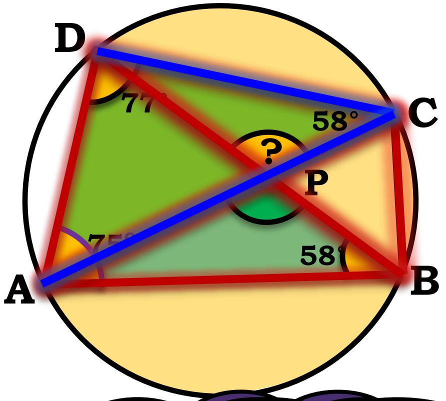
$\therefore \angle CAD = \angle CBD$



Thank You

MODULE 33

Q. ABCD is a cyclic quadrilateral in which $\angle BAD = 75^0$, $\angle ABD = 58^0$ and $\angle ADC = 77^0$, AC and BD intersect at P, then find $\angle DPC$.



Sol.

$\angle DPC = \angle APB$ [Vertically opposite angles]

$\angle DBA = \angle DCA = 58^0$ [Angles in the same segment]

In ΔADC

$\therefore \angle ADC + \angle DCA + \angle DAC = 180^0$

$\therefore 77 + 58 + \angle DAC = 180^0$

$\therefore 135 + \angle DAC = 180^0$

$\therefore \angle DAC = 180 - 135$

$\therefore \angle DAC = 45^o$

$\therefore \angle DAC + \angle CAB = \angle DAB$

$\therefore 45 + \angle CAB = 75$

$\therefore \angle CAB = 75 - 45$

$\therefore \angle CAB = 30^0$

i.e $\angle PAB = 30^0$

In ΔAPB

$\angle DAB$ is ... of

Consider ΔAPB

$\angle DAC$ and $\angle CAB$

Sum of all angles of a triangle is 180^0

$\therefore \angle APB = \angle DPC$ [vertically opposite angles]

$\therefore \angle DPC = 92^0$

MODULE 34

THEOREM

Angle in a semicircle is a right angle.

To prove : $\angle ACB = 90^\circ$

Proof :

$\angle AOB = 2$

F

Take point C

$\therefore$

We know that

Angle sub

at the c

angle s

the rem

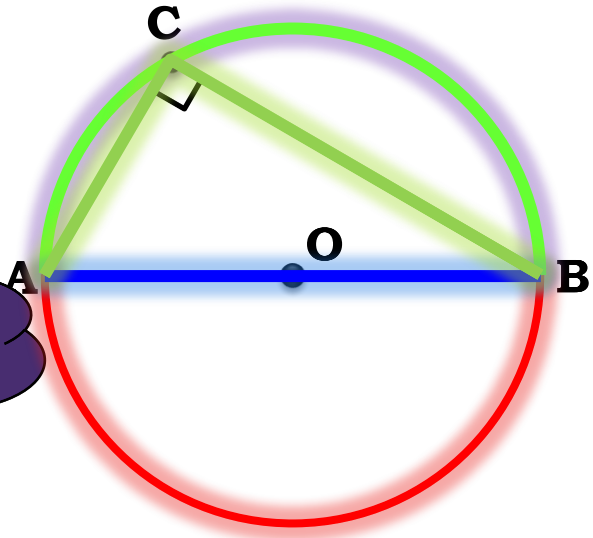
circle

$\angle AOB = 180^\circ$

[straight angle]

$\therefore$

$\angle AOB = 2\angle ACB$



Q. AB is a diameter of the circle such that ∠A = 35° and ∠Q = 25°, find ∠PBR.

Sol. :

∠PBR = ∠PBA + ∠ABR

In ΔAPB,

∠PBA + ∠PAB + ∠APB =

∴ ∠PBA + 35 + 90

∴ ∠PBA + 125 = 180

∴ ∠PBA = 180 - 125

∴ ∠PBA = 55°

∠ABR = ∠BAQ + ∠BQA

= 35 + 25

∴ ∠ABR = 60°

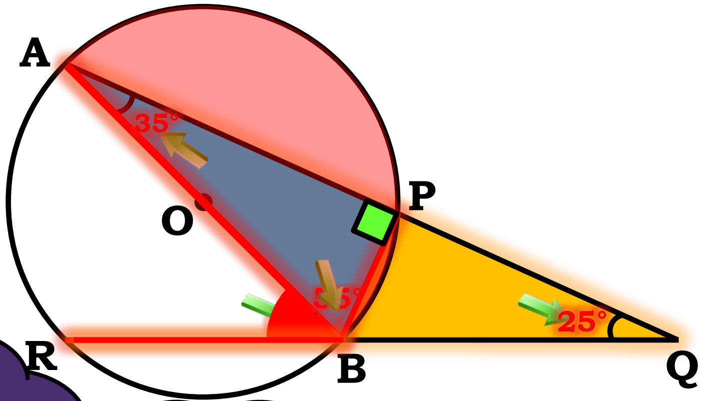
Now, find :
(i) ∠PBA and
(ii) ∠ABR

∠PBR is
made up of

We know that,
sum of all
angles of
triangle

opposite angles

∠APB = 90°
[Angle in a semicircle]



MODULE 35

Q. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Construction : Draw AD

Proof :

$\angle ADB = 90^\circ \quad \ldots (1)$

$\angle ADC = 90^\circ \quad \ldots (2)$

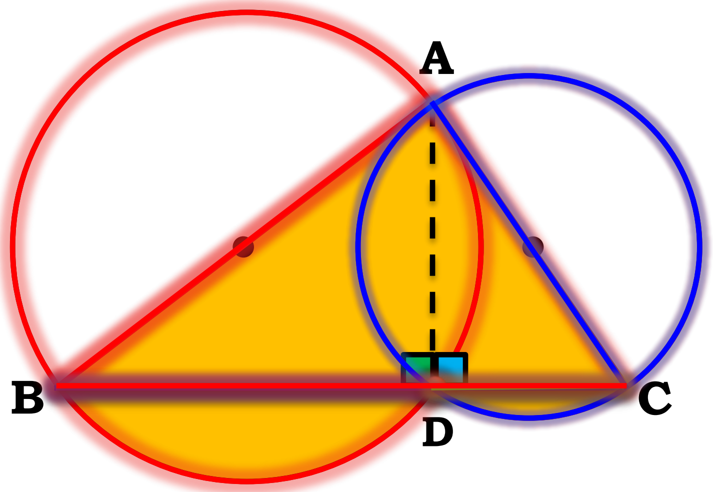
Adding (1) and (2),

$\angle ADB + \angle ADC = 90^\circ + 90^\circ$

$\angle ADB + \angle ADC = 180^\circ$

i.e. BD and DC are opposite rays
BDC is a straight line

Hence, $\boxed{\text{D lies on BC.}}$



$\angle ADC = 90^\circ$

MODULE 36

THEOREM

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

To Prove : $\angle AOB = 2\angle ACB$

Construction : Draw ray OC, take point M on it such that C-O-M

Proof : In ΔOAC

$OA = OC$ [ra

$\angle OAC = \angle OCA$ es]

Let, $\angle OAC = \angle OCA$

$\angle AOM$ is an exteri

$\angle AOM = \angle OAC + \angle$

$\angle AOM = x + x$

$\angle AOM = 2x$

In ΔOBC

$OB = OC$

$\angle OBC = \angle OCB$

Let, $\angle OBC = \angle OCB$

$\angle BOM$ is an exterior angle of ΔBOC

$\angle BOM = \angle OBC + \angle OCB$ [exterior angle is equal to sum of interior opposite angles]

$\angle BOM = y + y$

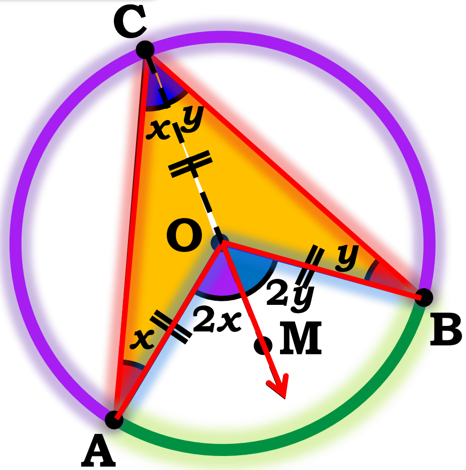
$\angle BOM = 2y$ .....(4)

We know that, exterior angle is equal to sum of two interior opposite angles.

ual to sum interior

$\therefore \angle BOM = \angle OBC + \angle OCB$

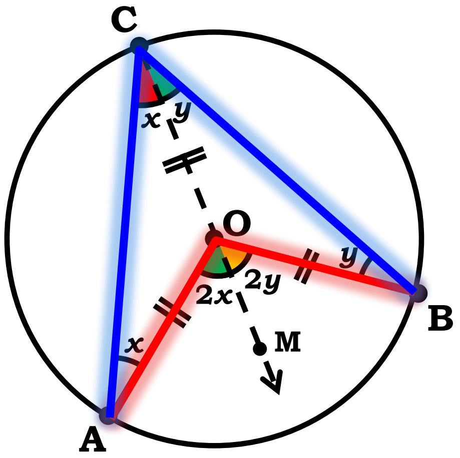
$\therefore \angle AOM = \ldots + \angle OCA$



THEOREM

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

To Prove : $\angle AOB = 2 \angle ACB$



Proof :

$\angle AOB = \angle AOM + \angle BOM$ [A

$= 2x + 2y$

$= 2(x + y)$

$= 2(\angle OCA + \angle OCB)$ [From (1) and (3)]

$\angle AOB = 2 \angle ACB$

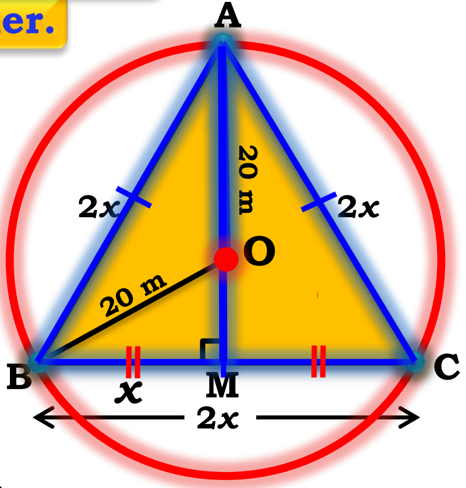
$\angle AOB$ is made up of two angles i.e. $\angle AOM$ and $\angle BOM$

MODULE 37

Q. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Construction : OM ⊥ BC

AB = BC = AC = ?



Sol. Let AB = BC = AC

Now, BM ... from the ... cle to a ... d.]

∴ BM = ...

∴ BM ...

Point ...

In eq ... nd

the centroi...

AM is a median.

In equilateral tr... e

are one and sam...

Height of equilatera... 2 side

$AM = \frac{\sqrt{3}}{2} \times AB$

We know that centroid trisects each median

...entre ... a

i.e it divides each median in the ratio 2:1

$\mathbf{OA = \frac{2}{3}\ AM}$ **and**

$\mathbf{OM = \frac{1}{3}\ AM}$

$AM = \frac{\sqrt{3}}{2} \times 2x$

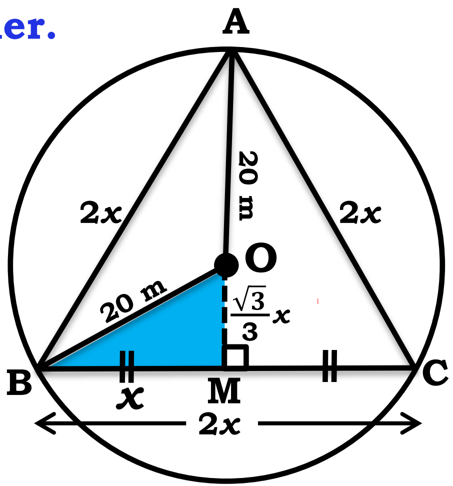
$AM = \sqrt{3}\,x$

$OM = \frac{1}{3}\ AM$ [centroid trisects each median]

$= \frac{1}{3} \times \sqrt{3}\,x$

$OM = \frac{\sqrt{3}}{3}\,x$

Q. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.



Now, in right angled ΔOMB,

$$OM^2 + BM^2 = OB^2$$

$$\left(\frac{\sqrt{3}}{3}x\right)^2 + x^2 = 20^2$$

$$\therefore \frac{3x^2}{9} + x^2 = 400$$

$$\therefore 3x^2 + 9x^2 = 400 \times 9$$

$$\therefore 12x^2 = 400 \times 9$$

$$\therefore x^2 = \frac{\overset{100}{\cancel{400}} \times \overset{3}{\cancel{9}}}{\underset{\cancel{3}\,1}{\cancel{12}}}$$

$$\therefore x^2 = 300$$

$$\therefore x = \sqrt{300}$$

$$\therefore x = \sqrt{100 \times 3}$$

$$\therefore x = 10\sqrt{3}$$

$$\therefore BM = 10\sqrt{3}$$

$$\therefore BC = 2BM$$

$$= 2 \times 10\sqrt{3}$$

$$BC = 20\sqrt{3}\text{ m}$$

The length of each string is $20\sqrt{3}$ m

THEOREM

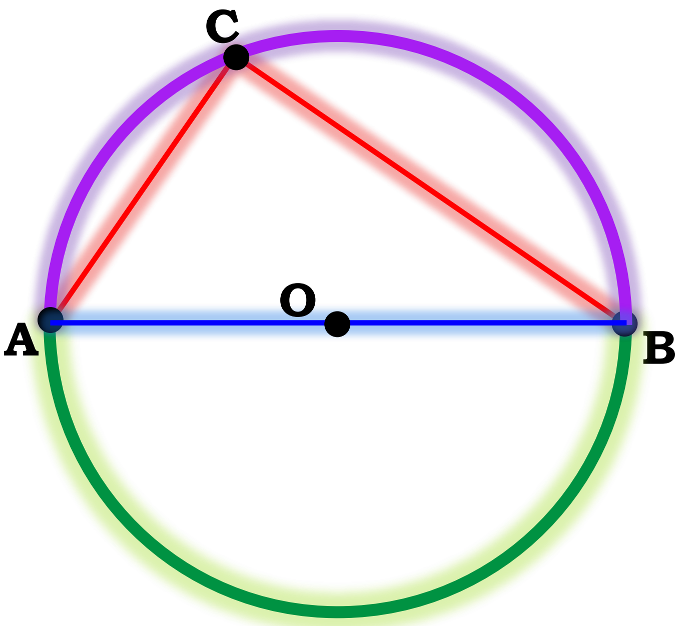
The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

CASE - II

- Arc AB is a semicircle.

$$\angle AOB = 2\angle ACB$$

$\angle ACB$



THEOREM

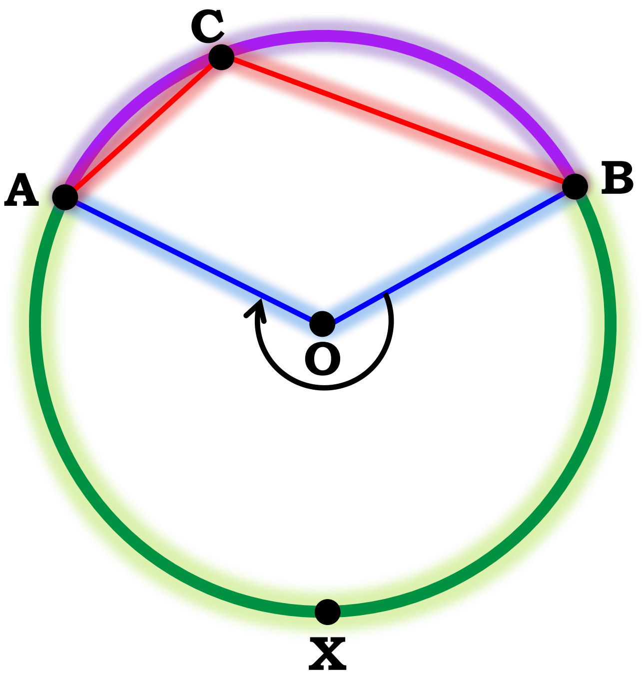
The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

CASE - III

- Arc AXB is major.

$\angle AOB =$

$\angle ACB$



Thank You

MODULE 38

Q. The circles with centres O and P intersect each other in the points Q and R. Their radii are 10 cm and 8 cm resp. The length of the common chord QR is 12 cm. Find the distance between their centres.

Sol. :

Find : OP

seg OM $\perp$ chord QR

$QM = \frac{1}{2} QR$

$\therefore \quad QM = \frac{1}{2} \times 12$

$\therefore \quad QM = 6$ cm.

In ΔOMQ, $\angle$OM

$OQ^2 = $ O

$\therefore \quad 10^2 = $

$\therefore \quad 100 = $

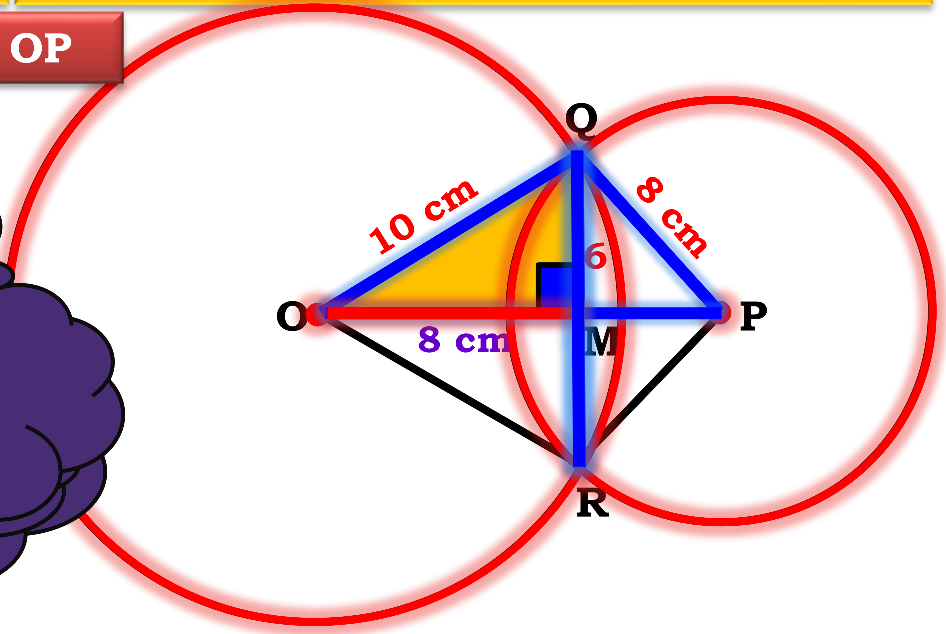
$\therefore \quad OM^2 = $ 10

$\therefore \quad OM^2 = 64$

$\therefore \quad OM = 8$ cm

Consider ΔOMQ

Let us find OM using Pythagoras theorem



Find : OP

In ΔQMP, $\angle QMP = 90^{o}$

$PQ^2 = Q$

Consider ΔQMP

$\therefore \quad 8^2 = 6^2$

$\therefore \quad 64 = 36 +$

$\therefore \quad 64 - 36 = MP^2$

$\therefore \quad MP^2$

Let us use Pythagoras theorem to find MP

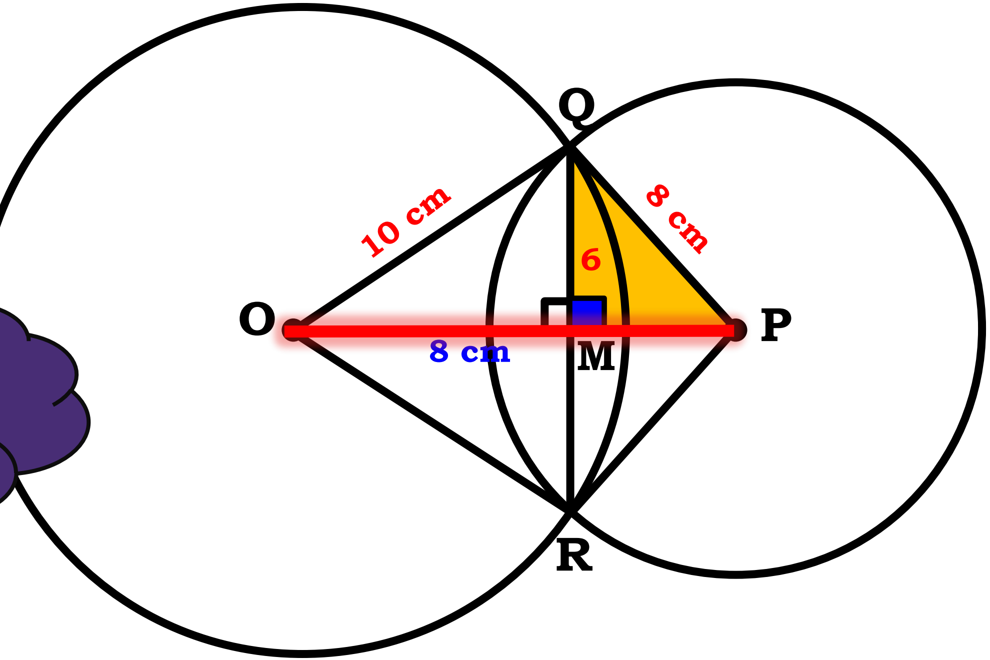
$\therefore \quad MP$

$\therefore \quad MP = \sqrt{4 \times 7}$

$\therefore \quad MP = 2\sqrt{7}$

$OP = OM + MP \quad [O – M – P]$

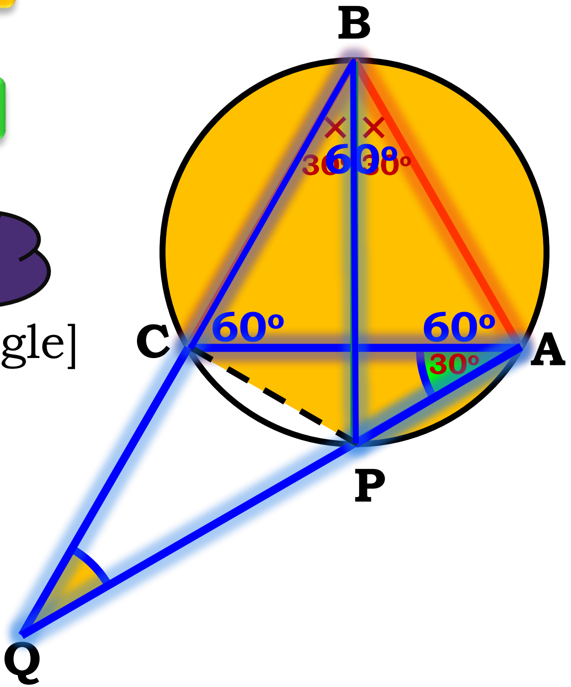
$\therefore \quad OP = 8 + 2\sqrt{7}$



The distance between their centres is $\left(8 + 2\sqrt{7}\right)$ cm

MODULE 39

Q. ΔABC is an equilateral triangle. Bisector of $\angle B$ intersects circumcircle of ABC in point P. Prove that CQ = CA



HINT : Prove $\angle CQA \cong \angle CAQ$

Proof :

ΔABC is an equi

$\angle BCA = \angle CAB = \angle$

Are CQ an

one

We k

angles i

segment are equal

Draw CP

$\angle CBP = \angle CAP$

and CA equal ?

$\angle CAQ$

angle]

$\angle CBP = \frac{1}{2} \angle CBA$ C]

$\therefore \quad \angle CBP = \frac{1}{2} \times 60$

$\therefore \quad \angle CBP = 30^\circ$ ...(ii)

$\angle CBP = \angle CAP$ ...(iii) [Angles in the same segment]

$\therefore \quad \angle CAP = 30^\circ$ [From (ii) and (iii)]

$\therefore \quad \angle CAQ = 30^\circ$ ...(iv) [A–P–Q]

Q. ΔABC is an equilateral triangle. Bisector of $\angle B$ intersects circumcircle of ABC in point P. Prove that CQ = CA

Prove that CQ = CA

HINT : Prove $\angle CQA \cong \angle CAQ$

Proof : $\angle BCA = \angle CAB = \angle CBA = 60^\circ$ ...(i)

$\angle CAQ = 30^\circ$ ...(iv)

$\angle BCA$ is an exterior angle of ΔCQA

$\therefore \angle BCA = \angle CQA + \angle CAQ$

$\therefore 60^\circ = \angle CQA$

$\therefore 60^\circ - 30^\circ = \angle CQA$

$\angle CQA = 30^\circ$ ...(v)

In ΔCQA,

$\angle CAQ = \angle CQA$ [From (iv) and (v)]

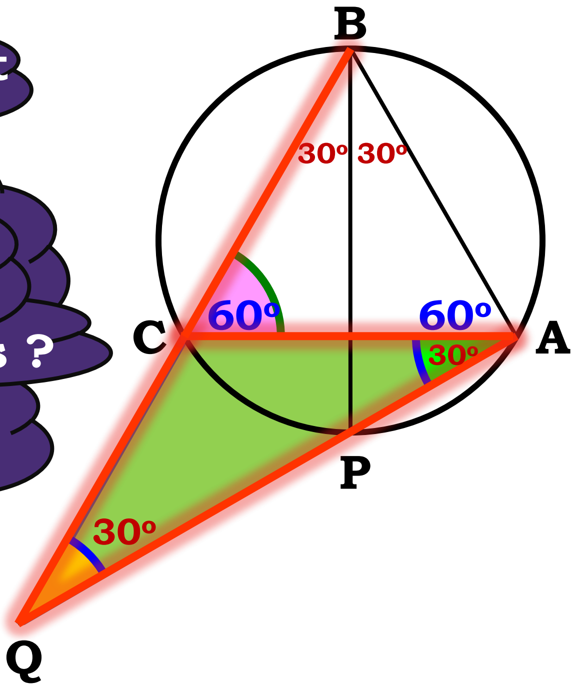
$\therefore CQ = CA$ [sides opposite to equal angles]

Now we need to get $\angle CQA = 30^\circ$

Now, consider ΔACQ

Can we relate ... gles ?

we know that exterior angle of a triangle is equal to sum of interior opposite angles



MODULE 40

Q. Bisector AD of ∠ BAC of ΔABC passes through the centre of the circumcircle of ABC as shown in figure. Prove that AB = AC.

Proof. In ΔAOB,

OA = OB

∴ ∠OAB = ∠OBA ...

In ΔAOC,

OA = OC

∴ ∠OAC = ∠OCA ...

But ∠OAB = ∠OAC ...

∴ ∠OBA = ∠OCA ....

Angles opposite to equal si

An

equa

Consider ΔAOC

angles to

equal sides are equal

In ΔOBC,

OB = OC [Radii of same circle]

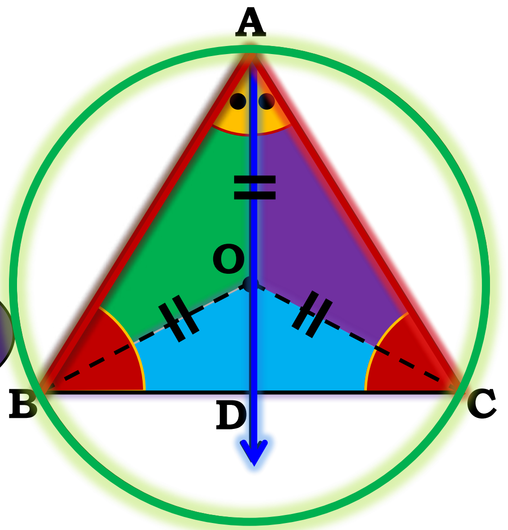
∴ ∠OBC = ∠OCB ....(v) [Angles opposite to equal sides]

Adding (iv) and (v)

∠OBA + ∠OBC = ∠OCA + ∠OCB

∴ ∠ABC = ∠ACB [Angle addition property]

∴ **AB = AC** [Sides opposite to equal angles]



MODULE 41

Q. ΔABC is an equilateral triangle. Find $\angle BDC$ and $\angle BEC$.

Sol .

In ΔABC,

$\angle BAC = 60^\circ$ ...(i)

$\angle BAC = \angle BDC$ ...(ii) [Angl... ...gle]

$\boxed{\angle BDC = 60^\circ}$ [E...

In cyclic □ABEC,

$\angle BAC + \angle BEC = 180^\circ$

$\therefore \quad 60 + \angle BEC = 180$

$\therefore \quad \angle BEC = 180 - 60$

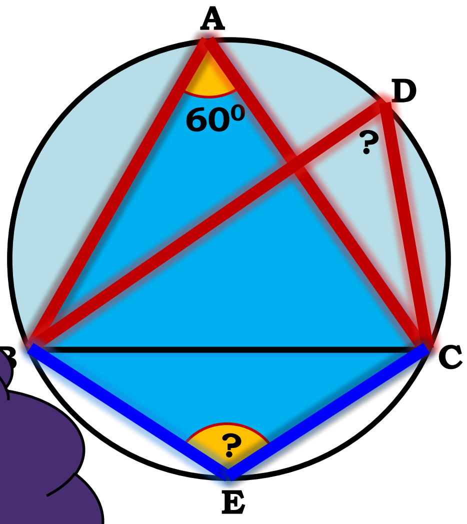
$\therefore \quad \boxed{\angle BEC = 120^\circ}$

∠BAC lies in
se...

□ABEC is
cyclic

in ...

We know that
Opposite angles of a
cyclic quadrilateral
are supplementary



MODULE 42

Q. **Prove** that **the quadrilateral formed by the angle bisectors** of a **cyclic quadrilateral** is also **cyclic.**

Proof :

To Prove : □PQRS is cyclic

$\angle Q + \angle QAB + \angle ABQ = 180^{\circ}$

$\therefore \angle Q + a + b = 180^{\circ}$

$\therefore \angle Q = 180$ ...

$\angle S + \angle SCD + \angle CD$ ...

$\therefore \angle S + c + d =$ ...

$\therefore \angle S =$ ...

Hint: $\angle Q + \angle S = 180^{\circ}$

We know that sum of all angles of quadrilateral is 360°

sum of ... angles of triangle is 180°

$\angle Q + \angle S = 180$ ... $c - d$

$\angle Q + \angle S = 360 - a - b - c - d$

$\angle Q + \angle S = 360 - (a + b + c + d)$

$\angle DAB + \angle ABC + \angle BCD +$ ...

$\therefore 2a + 2b$ ...

$\therefore 2(a +$ ...

$\therefore a + b + c + d = 180$ ...

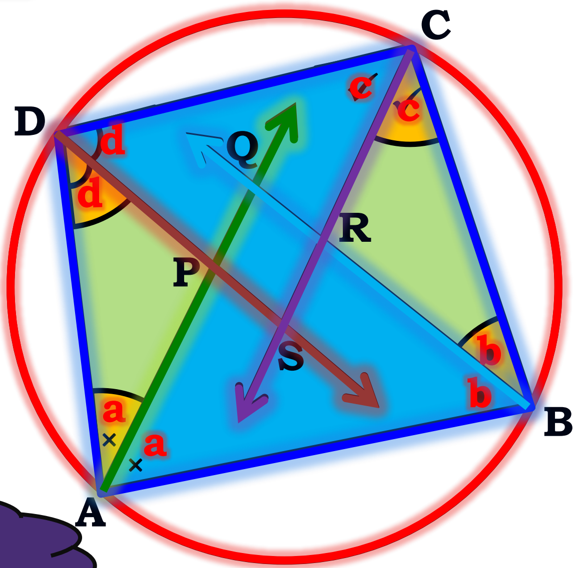
Now let us add Result no. (i) and (ii)

what should be subtracted from 360°?

$\therefore \angle Q + \angle S = 360 - 180$

$\therefore \angle Q + \angle S = 180^{\circ}$

∴ □PQRS is cyclic [If opposite angles of a quadrilateral are supplementary then that quadrilateral is cyclic.]



Thank You