Lecture_11

No. 69

1. ARITHMETIC PROGRESSIONS

- **Sum based on ‘t_n’ and ‘S_n’**

Q.6) Obtain the sum of the 56 terms of an A. P. whose 19th and 38th terms are 52 and 148 respectively.

Given : $t_{19} = 52$

To find : $S_{56} = ?$

We know

$t_n = a + (n - 1)\,d$

Now, $t_{19} = a +$

$52 = a +$

i.e. $a + 18d = 52$ .... (1)

Also $t_{38} = a + (38 - 1)d$

∴ $148 = a + 37d$

i.e. $a + 37d = 148$ .... (2)

Adding (1) and (2)

$$\begin{aligned} a + 18\,d &= 52 \\ a + 37\,d &= 148 \\ \hline \boxed{2a + 55d = 200} \end{aligned}$$ .... (3)

$S_n = \frac{n}{2}[2a + (n - 1)d]$

∴ $S_{56} = \frac{56}{2}[2a + (56 - 1)d]$

$+ 55d]$

00 [from (3)]

500

n of first 56 terms of A.P. is 5600.

To find, S_{56} replace n by 56 in S_n

given

e of t_{38}

replace n by 38

By subtracting we get $d = \frac{96}{19}$

$d = 5.05...$

Same co–efficient & Same sign

Lets add the 2 equations

No. 70

1. ARITHMETIC PROGRESSIONS

- **Sums based on 't_n' and 'S_n'**

6. Second and fourth term of on A.P. is 12 and 20 respectively. Find the sum of first 25 terms of that A.P.

Sol. $t_2 = 12, \quad t_4 = 20$

t_n 1) d

t_2

12

For giv

o

For gi value

replace n by 4

$\therefore \quad a + d = 12 \quad .....(i)$

$t_4 = a + (4 - 1) d$

$20 = a + 3d$

$\therefore \quad a + 3d = 20 \quad .....(ii)$

Subtracting (ii) from, (i),

$$\begin{array}{rcl} a + d & = & 12 \\ a + 3d & = & 20 \\ (-) \quad (-) & & (-) \\ \hline -2d & = & -8 \end{array}$$

$\therefore \quad d = 4$

Substituting d = 4 in (i),

$a + 4 = 12$

$a = 12 - 4$

$\therefore \quad a = 8$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{25} = \frac{25}{2}[2a + (25 - 1)d]$

$= \frac{25}{2}[2(8) + 24 (4)]$

$= \frac{25}{2}[16 + 96]$

$= \frac{25}{2}[112]$

$\therefore \quad S_{25} = 1400$

$\therefore$ **Sum of 25 terms of the A.P is 1400.**

No. 71

Q. The ratio of sum of n term of two A.P 's is $(7n + 1) : (4n + 27)$. Find the ratio of their m^{th} terms.

Sol. Let a_1 , a_2 be the first terms of two A.P's and d_1, d_2 be the common different of two A.P's.

Then, the sums of their n terms are given by

$$S_n = \frac{n}{2}[2a_1 + (n-1)d_1] \quad \text{and,}$$

$$S_n' = \frac{n}{2}[2a_2 + (n-1)d_2]$$

$$\therefore \quad \frac{S_n}{S_n'} = \frac{\boxed{\cancel{\frac{n}{2}}}[2a_1 + (n-1)d_1]}{\boxed{\cancel{\frac{n}{2}}}[2a_2 + (n-1)d_2]}$$

$$\therefore \quad \boxed{\frac{S_n}{S_n'} = \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2}}$$

It is given that $\boxed{\frac{S_n}{S_n'}} = \frac{7n + 1}{4n + 27}$

$$\therefore \quad \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n + 1}{4n + 27} \qquad \text{...(i)}$$

Q. The ratio of sum of n term of two A.P 's is $(7n + 1) : (4n + 27)$. Find the ratio of their m^{th} terms.

Sol.

$$\therefore \quad \frac{2a_1 + (n - 1)\, d_1}{2a_2 + (n - 1)\, d_2} = \frac{7n + 1}{4n + 27} \qquad \ldots(i)$$

To find the ratio of the m^{th} of the two A.P' s, we replace n by $(2m - 1)$ in equation (i)

$$\therefore \quad \frac{2a_1 + [2m - 1 - 1]\, d_1}{2a_2 + [2m - 1 - 1]\, d_2} = \frac{7\,(2m - 1) + 1}{4\,(2m - 1) + 27}$$

$$\therefore \quad \frac{2a_1 + (2m - 2)\, d_1}{2a_2 + (2m - 2)\, d_2} = \frac{14m - 7 + 1}{8m - 4 + 27}$$

$$\therefore \quad \frac{2a_1 + 2(m - 1)d_1}{2a_2 + 2(m - 1)d_2} = \frac{14m - 6}{8m + 23}$$

$$\therefore \quad \frac{2\,[a_1 + (m - 1)\, d_1]}{2\,[a_1 + (m - 1)\, d_1]} = \frac{14m - 6}{8m + 23}$$

Hence, the ratio of the m^{th} terms of the two A.P's is $(14m - 6) : (8m + 23)$.

No. 72

Q.4) In winter, the temperature at a hill station from Monday to Friday is in A.P. The sum of the temperatures of Monday, Tuesday and Wednesday is zero and the sum of the temperatures of Thursday and Friday is 15. Find the temperature of each of the five days.

Sol: Let the temperatures of hill station

Tem **T** **T** **Tem** **Temperature on Friday**

$a - 2d, \quad a - d, \quad a, \quad a + d, \quad a + 2d$

respectively

From Monday to Friday means 5 consecutive days

As per the 1

$a - 2d + a$

$\therefore \quad 3a - 3d =$

So, sum is based on 5 consecutive terms of A.P.

$\therefore \quad 3a =$

$\therefore \quad \boxed{a =}$

As per the 2nd condition,

$a + d + a + 2d = 15$

$\therefore \quad 2a + 3d = 15$

$\therefore \quad 2d + 3d = 15$

$\therefore \quad 5d = 15$

$\therefore \quad d = 3$

$\therefore \quad a = 3$

With the value of a and d lets find temperature of all days

When $a = 3$ & $d = 3$

$\therefore \quad a - 2d = 3 - 2(3) = 3 - 6 = -3$

$\therefore \quad a - d = 3 - 3 = 0$

$a = 3$

$\therefore \quad a + d = 3 + 3 = 6$

$\therefore \quad a + 2d = 3 + 2(3) = 3 + 6 = 9$

$\therefore$ **The temperatures from Monday to Friday are – 3, 0, 3, 6 and 9 respectively**

Thank You