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Polynomials

1. A **polynomial** *p(x)* in one variable *x* is an algebraic expression in *x* of the form $p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots + a_2 x^2 + a_1 x + a_0$, where
   i. $a_0, a_1, a_2 \ldots\ldots a_n$ are constants
   ii. x is a variable
   iii. $a_0, a_1, a_2 \ldots\ldots a_n$ are respectively the **coefficients** of x^i
   iv. Each of $a_n x^n, a_{n-1} x^{n-1}, a_{n-2} x^{n-2}, \ldots\ldots a_2 x^2, a_1 x, a_0$, with $a_n \neq 0$, is called a **term** of a polynomial.

2. The highest power of the variable in a polynomial is called the **degree** of the polynomial.

3. A polynomial with one term is called a **monomial**.

4. A polynomial with two terms is called a **binomial**.

5. A polynomial with three terms is called a **trinomial**.

6. A polynomial with degree zero is called a **constant polynomial**. For example: 1, -3. The degree of non-zero constant polynomial is zero

7. A polynomial of degree one is called a **linear polynomial**. It is of the form *ax* + *b*. For example: *x* - 2, 4*y* + 89, 3*x* - *z*.

8. A polynomial of degree two is called a **quadratic polynomial**. It is of the form $ax^2 + bx + c$. where *a, b, c* are real numbers and $a \neq 0$ For example: $x^2 - 2x + 5$ etc.

9. A polynomial of degree three is called a **cubic polynomial** and has the general form $ax^3 + bx^2 + cx + d$. For example: $x^3 + 2x^2 - 2x + 5$ etc.

10. A **bi-quadratic polynomial** *p*(*x*) is a polynomial of degree four which can be reduced to quadratic polynomial in the variable $z = x^2$ by substitution.

11. The constant polynomial 0 is called the **zero polynomial**. Degree of zero polynomial is not defined.

12. The **value of a polynomial** f(x) at *x* = *p* is obtained by substituting *x* = *p* in the given polynomial and is denoted by *f*(*p*).

13. A real number '*a*' is a **zero** or root of a polynomial *p*(*x*) if *p* (*a*) = 0.

14. The number of real zeroes of *a* polynomial is less than or equal to the degree of polynomial.

15. Finding a zero or root of a polynomial f(x) means solving the polynomial equation *f*(*x*) = 0.

16. A non-zero constant polynomial has no zero.

17. Every real number is a zero of a zero polynomial.

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18. Division algorithm

If $p(x)$ and $g(x)$ are the two polynomials such that degree of $p(x) \geq$ degree of $g(x)$ and $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that:

$p(x) = g(x)\, q(x) + r(x)$

where, $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

19. Remainder theorem

Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If $p(x)$ is divided by the linear polynomial $(x - a)$, then remainder is $p(a)$.

i. If the polynomial $p(x)$ is divided by $(x + a)$, the remainder is given by the value of $p(-a)$.

ii. If $p(x)$ is divided by $ax + b = 0$; $a \neq 0$, the remainder is given by $p\left(\frac{-b}{a}\right)$; $a \neq 0$

iii. If $p(x)$ is divided by $ax - b = 0$, $a \neq 0$, the remainder is given by $p\left(\frac{b}{a}\right)$; $a \neq 0$

20. Factor theorem

Let $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number such that $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.

21. Converse of factor theorem

Let $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number. If $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.

i. $(x + a)$ is a factor of a polynomial $p(x)$ iff $p(-a) = 0$.

ii. $(ax - b)$ is a factor of a polynomial $p(x)$ iff $p(b/a) = 0$.

iii. $(ax + b)$ is a factor of a polynomial $p(x)$ iff $p(-b/a) = 0$.

iv. $(x - a)(x - b)$ is a factor of a polynomial $p(x)$ iff $p(a) = 0$ and $p(b) = 0$.

22. For applying factor theorem, the divisor should be either a linear polynomial of the form $(ax + b)$ or it should be reducible to a linear polynomial.

23. A quadratic polynomial $ax^2 + bx + c$ is **factorised by splitting the middle term** by writing b as $ps + qr$ such that $(ps)(qr) = ac$.

Then, $ax^2 + bx + c = (px + q)(rx + s)$

24. An **algebraic identity** is an algebraic equation which is true for all values of the variables occurring in it.

25. Some useful **quadratic identities**:

i. $(x+y)^2 = x^2 + 2xy + y^2$

ii. $(x-y)^2 = x^2 - 2xy + y^2$

iii. $(x-y)(x+y) = x^2 - y^2$

iv. $(x+a)(x+b) = x^2 + (a+b)x + ab$

v. $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here *x, y, z* are variables and *a, b* are constants.

26. Some useful **cubic identities**:

i. $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

ii. $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

iii. $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

iv. $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

v. $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

vi. If $x+y+z=0$ then $x^3 + y^3 + z^3 = 3xyz$

Here, *x, y* and *z* are variables.