

Statistics

1. Three measures of central tendency are:

- i. Mean ii. Median iii. Mode

2. The **arithmetic mean**, also called the average, is the quantity obtained by adding all the observations and then dividing by the total number of observations.

3. Arithmetic mean may be computed by anyone of the following methods:

- i. Direct method
- ii. Short-cut method/ Assumed mean method
- iii. Step-deviation method

4. **Direct method** of finding mean:

If a variant X takes values $x_1, x_2, x_3, \dots, x_n$ with corresponding frequencies $f_1, f_2, f_3, \dots, f_n$ respectively, then arithmetic mean of these values is given by:

$$\bar{X} = \frac{\sum_{i=1}^n f_i x_i}{N}, \text{ where } N = \sum_{i=1}^n f_1 + f_1 + f_2 + \dots + f_n$$

5. **Class mark** = $\frac{1}{2}$ (Upper class limit + Lower class limit)

6. **Short-cut method/ assumed mean method** of finding mean:

Let x_1, x_2, \dots, x_n be values of a variable X with corresponding frequencies $f_1, f_2, f_3, \dots, f_n$ respectively. Let A be the assumed mean, $d_i = (x_i - A)$. Then:

$$\bar{X} = A + \frac{1}{N} \left(\sum_{i=1}^n f_i d_i \right)$$

Note that in case of continuous frequency distribution, the values of $x_1, x_2, x_3, \dots, x_n$ are taken as the mid-points or class-marks of the various classes.

7. **Step-deviation method** of finding mean:

Let x_1, x_2, \dots, x_n be values of a variable X with corresponding frequencies $f_1, f_2, f_3, \dots, f_n$ respectively. Let A be the assumed mean. Then:

$$\bar{X} = A + h \left\{ \frac{1}{N} \sum_{i=1}^n f_i u_i \right\}$$

Here, h is generally taken as common factor of the deviations, in case of ungrouped frequency distribution. And, in case of grouped frequency distribution, h is the class width, $u_i = \frac{x_i - A}{h} = \frac{d_i}{h}$

Note that in case of continuous frequency distribution, the values of $x_1, x_2, x_3, \dots, x_n$ are taken as the mid-points or class-marks of the various classes.

8. The step deviation method will be convenient to apply if all the deviations (d 's) have a common factor.

9. If class mark obtained, are in decimal form, then step deviation method is preferred to calculate mean.

10. **Median** is a measure of central tendency which gives the value of the middle observation in the data, arranged in order. It is that value such that the number of observations above it is equal to the number of observations below it.

11. For finding the median of a raw data, we arrange the given data in increasing or decreasing order.

If n is odd, then median is the value of $\left(\frac{n+1}{2}\right)^{th}$ observation.

If n is even, then median is the arithmetic mean of the values of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2} + 1\right)^{th}$ observations.

12. The **cumulative frequency** of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class to the frequency of the class.

13. In case of an **ungrouped frequency distribution**, we calculate the **median** by following the steps given below:

Step 1: Find the cumulative frequencies (c.f.) and obtain $N = \sum f_1$.

Step 2: Find $\frac{N}{2}$.

Step 3: Look for the cumulative frequency (c. f.) just greater than $\frac{N}{2}$ and determine the corresponding value of the variable. The value so obtained is the median.

14. In case of a **continuous frequency distribution**, we calculate the **median** by following the steps:

Step 1: Find the cumulative frequencies (c.f.) and obtain $N = \sum f_1$.

Step 2: Find $\frac{N}{2}$.

Step 3: Look for the cumulative frequency (c. f.) just greater than $\frac{N}{2}$ and determine the corresponding class. This class is known as the median class. (Note that the value of the median will lie in this class)

Step 4: Use the following formula to find median:

$$\text{Median} = l + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

Here, l = lower limit of the median class

f = frequency of the median class

h = width (size) of the median class

cf = cumulative frequency of the class preceding the median class

$$N = \sum f_1.$$

15. **Mode** is the value of the most frequently occurring observation in the data.

16. In an ungrouped frequency distribution, mode is the value of the variable having maximum frequency.

17. In a **grouped frequency distribution**, the modal class is the one with highest frequency and the **mode** can be calculated by the following formula

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

l = lower limit of the modal class

h = size of the class interval

f_1 = frequency of the modal class

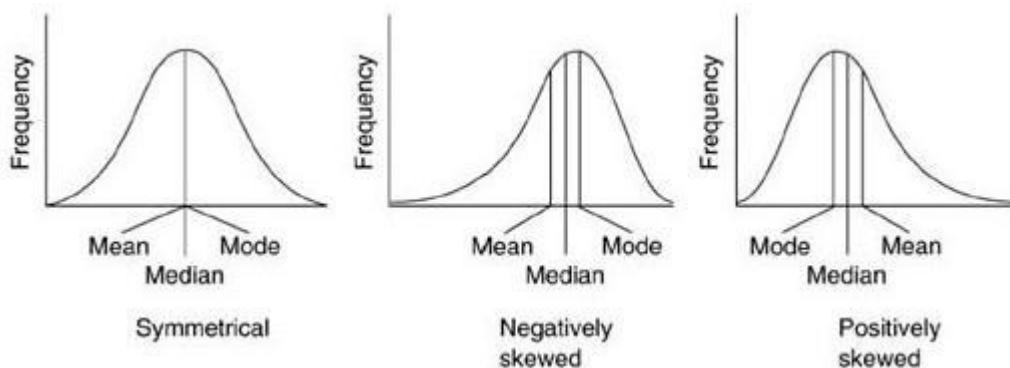
f_0 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class

18. The most frequently used measure of central tendency is the mean, because the mean is calculated by taking into account all the observations of a given data. And it lies between the smallest and the largest value of the data.
19. The biggest drawback in considering mean is that it is affected by the extreme values. One large or small number can distort the average. In that case, median is a better measure of central tendency. While, when the most repeated value or the most wanted one is required, then mode is used.
20. When all three measures of central tendency are equal, the distribution is called **symmetrical distribution**.
21. When the values of mean, median and mode are not equal, then the distribution is known as **asymmetrical or skewed**. In this case, the distribution can be positively skewed or negatively skewed.

Negatively skewed distributions have a few extremely low scores, while positively skewed distributions have a few extremely high scores.

- When the data is negatively skewed, then $\text{Mean} < \text{Median} < \text{Mode}$
- When the positively skewed, then $\text{Mean} > \text{Median} > \text{Mode}$



22. Three measure of central values are connected by the following relation:

$$3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$$

23. The **cumulative frequency** is the accumulated or sum of frequencies up to a particular point. A table showing the cumulative frequencies is called a **cumulative frequency distribution**.

24. There are two types of cumulative frequencies:

- i. **Less than type cumulative frequency distribution:** It is found by adding sequentially the frequencies of all the earlier classes including the class adjacent to which it is written. The cumulate is started from the lowest to the highest size.
- ii. **More than type cumulative frequency distribution:** It is obtained by finding the cumulate of frequencies starting from the highest to the lowest class.

25. A cumulative frequency distribution can be represented graphically by means of an **ogive**.

26. There are two types of ogives:

- i. **'Less than' ogive:** In a less than ogive the upper limit of a class (x axis) is plotted against its cumulative frequency (y axis) as a point on the ogive. The 'less than ogive' is a rising curve.
- ii. **'More than' ogive:** In a 'more than ogive' the lower limit of a class (x axis) is plotted against its cumulative frequency (y axis) as a point on the ogive. The 'more than ogive' is a falling curve.

27. The ogives can be drawn only when the given class intervals are continuous and if this is not the case then first the class intervals are made continuous.

28. In order to determine the **median from less than ogive or more than ogive**, we follow the steps given below:

Step 1: Draw more than or less than ogive as asked in question. Find $\frac{N}{2}$, where N is the total number of observations.

Step 2: Locate the $\frac{N}{2}$ cumulative frequency on the y-axis.

Step 3: Draw a line parallel to x-axis through the point obtained in step 2, cutting the cumulative frequency curve at a point *P* (say).

Step 4: Draw perpendicular *PM* from *P* on the x-axis. The x-coordinate of point M is the median value.

29. If we draw **less than ogive and more than ogive on the same graph**, then **median** can be obtained by following the steps given below:

Step 1: Draw both ogives on the same graph.

Step 2: Identify the point of intersection of both ogives and mark it as Q (say).

Step 3: Draw perpendicular from Q on x-axis.

Step 4: The point of perpendicular on x-axis is the median.