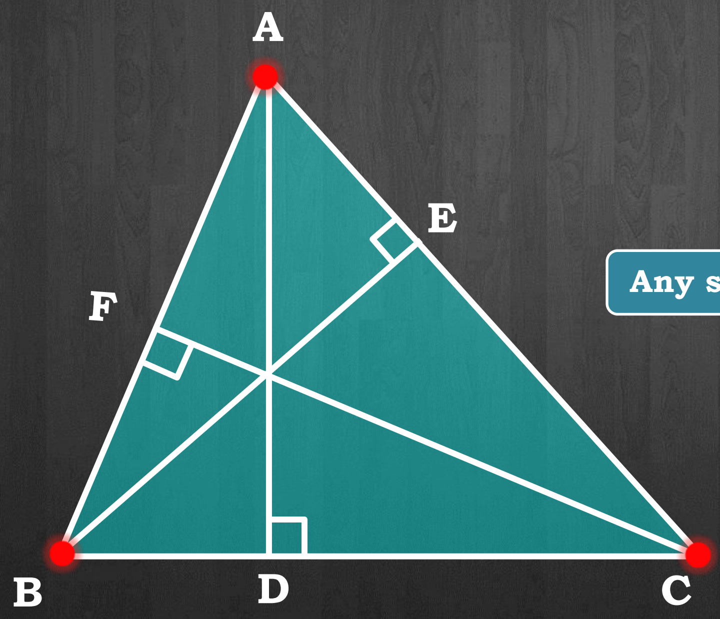
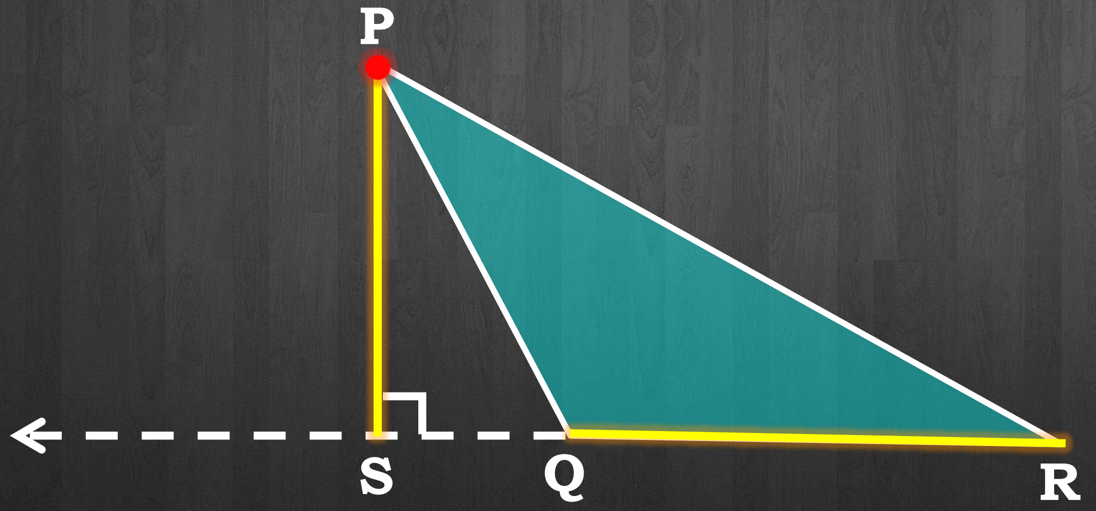
MODULE : 1

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

<u>BASE</u>	<u>HEIGHT</u>
BC	AD
AC	BE
AB	CF



Any side of a triangle can be the base.



Height	Base
PS	QR

Triangles betweeen same parallels

MODULE : 2

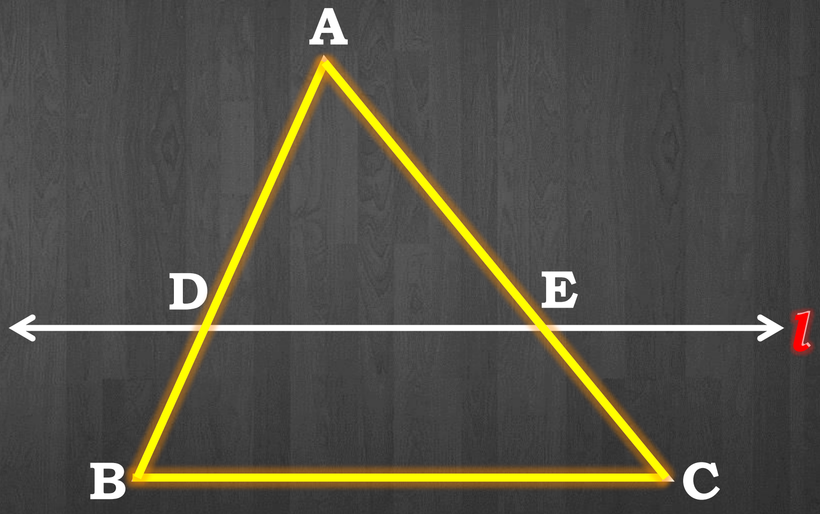
Basic Proportionality Theorem (B.P.T)

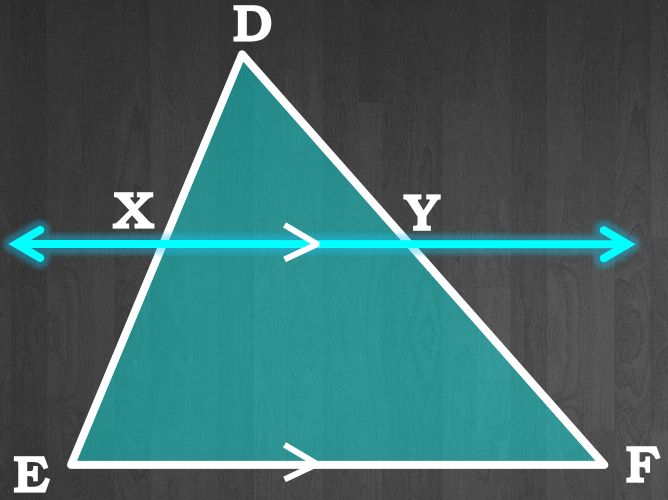
If a line parallel to a side of a triangle intersects other sides in two distinct points, then the other sides are divided in the same ratio by it.

In ΔABC,

line l || side BC

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad \text{[By B.P.T]}$$





In ΔDEF,

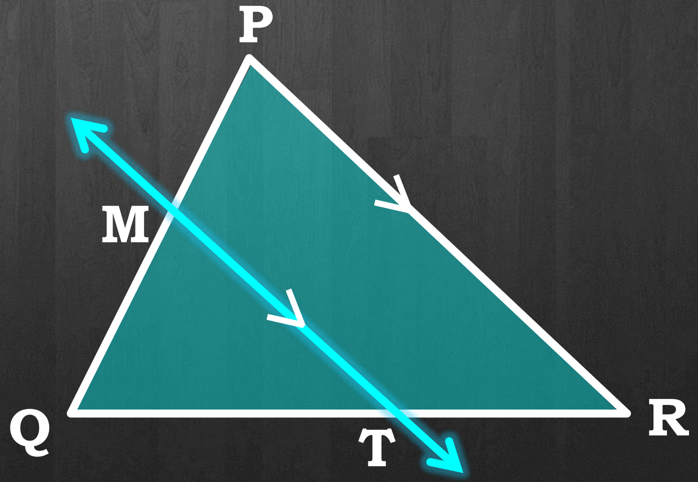
line XY || side EF

$$\therefore \quad \frac{DX}{XE} = \frac{DY}{YF} \quad \text{[By B.P.T]}$$

In ΔPQR,

line MT || side PR

$$\therefore \quad \frac{QM}{MP} = \frac{QT}{TR} \quad \textit{[By B.P.T]}$$



MODULE : 3

EXERCISE 6.2

Q.1. (i) Given : DE || BC To Find : (i) EC (ii) AD

Sol. In $\triangle ABC$,

$$DE \parallel BC$$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \textit{[By basic proportionality theorem]}$$

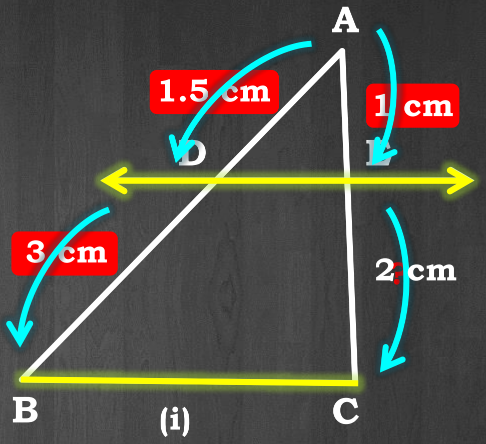
$$\therefore \quad \frac{1.5}{3} = \frac{1}{EC}$$

$$\therefore \quad 1.5 \times EC = 3 \times 1$$

$$\therefore \quad EC = \frac{3}{1.5}$$

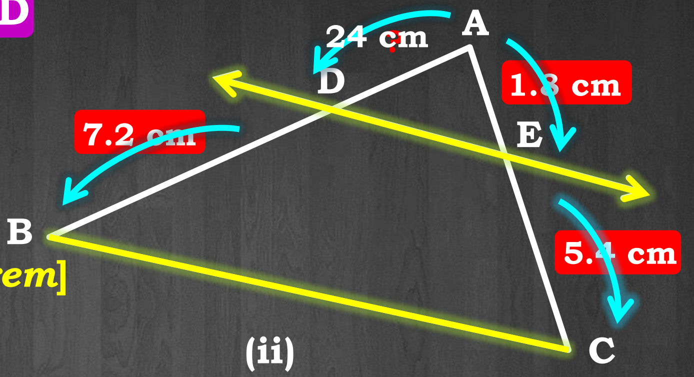
$$= \frac{\cancel{30}^{\,2}}{\cancel{15}}$$

$$\therefore \quad \boxed{EC = 2 \text{ cm}}$$



EXERCISE 6.2

Q.1. (ii) Given : DE || BC To Find : (i) EC (ii) AD



Sol. In ΔABC,

$DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{[By basic proportionality theorem]}$$

$$\therefore \frac{AD}{7.2} = \frac{1.8}{5.4}$$

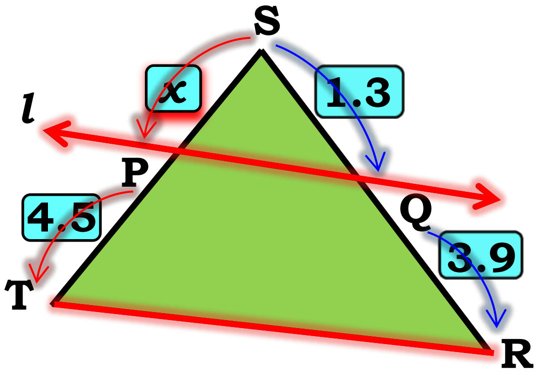
$$\therefore \frac{10 \times AD}{72} = \frac{18}{54} \times \frac{10}{10}$$

$$\therefore AD = \frac{\overset{1}{\cancel{18}} \times \overset{24}{\cancel{72}}}{\underset{3}{\cancel{54}} \times 10}$$

$$\therefore \boxed{AD = 2.4 \text{ cm}}$$

MODULE : 4

Find the value of 'x' in the figure, if line l is parallel to one of the sides of the given triangle :



Sol.

(ii) In ΔSTR,

line $l \parallel$ side TR [Given]

$$\therefore \frac{\boxed{SP}}{\boxed{PT}} = \frac{\boxed{SQ}}{\boxed{QR}} \quad \text{[By B.P.T]}$$

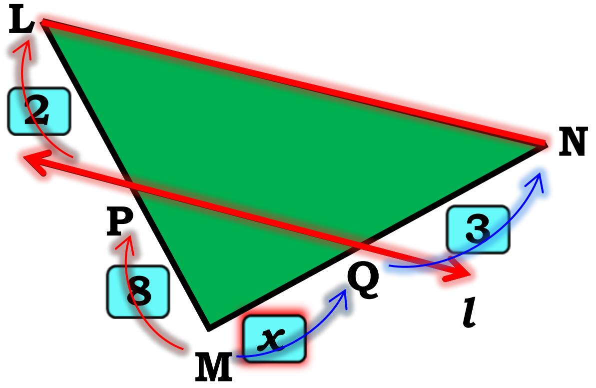
$$\therefore \frac{x}{4.5} = \frac{1.3}{3.9}$$

$$\therefore x = \frac{4.5 \times \cancel{1.3}}{\cancel{3.9}_{\,3}}$$

$$\therefore x = \frac{\overset{1.5}{\cancel{4.5}}}{\cancel{3}}$$

$$\therefore \boxed{x = 1.5}$$

Find the value of 'x' in the figure, if line l is parallel to one of the sides of the given triangle :



Sol.

(iii) In ΔLMN,

line $l \parallel$ side LN **[Given]**

$$\therefore \quad \frac{MP}{PL} = \frac{MQ}{QN} \quad \text{[By B.P.T]}$$

$$\therefore \quad \frac{8}{2} = \frac{x}{3}$$

$$\therefore \quad x = \frac{\overset{4}{\cancel{8}} \times 3}{\cancel{2}}$$

$$\therefore \quad \boxed{x = 12}$$

MODULE : 5

THEOREM : Basic Proportionality Theorem

If a line parallel to a side of a triangle intersects other sides in two distinct points, then the other sides are divided in the same ratio by it.

Given : line l ∥ side BC

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw seg EM ⊥ side AB and seg DN ⊥ side AC

Proof :

$\frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$

$\frac{A(\Delta ADE)}{A(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN}$ ...(ii) $= \frac{AE}{EC}$

A(ΔBDE) = A(ΔCDE) ...(iii) [If two triangles are in between two parallel lines with common base, then their areas are equal]

$\frac{AD}{DB} = \frac{AE}{EC}$

From (i), (ii) and (iii) we get,
L.H.S equal
So, the R.H.S will be equal

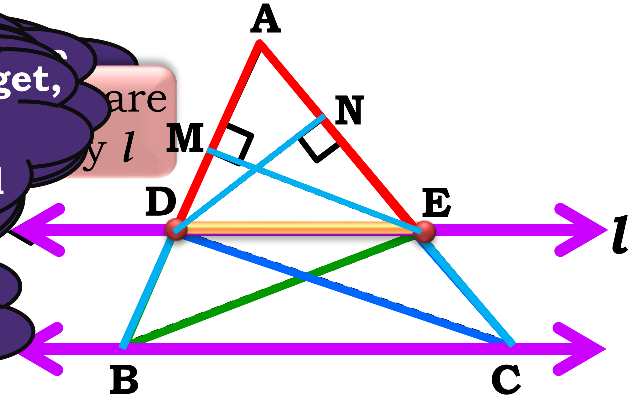
What can u say about A(ΔBDE) and A(ΔCDE) ?

A(ΔBDE) = A(ΔCDE)

Line l divides ... which 2 parts ?

DB

are ... by l

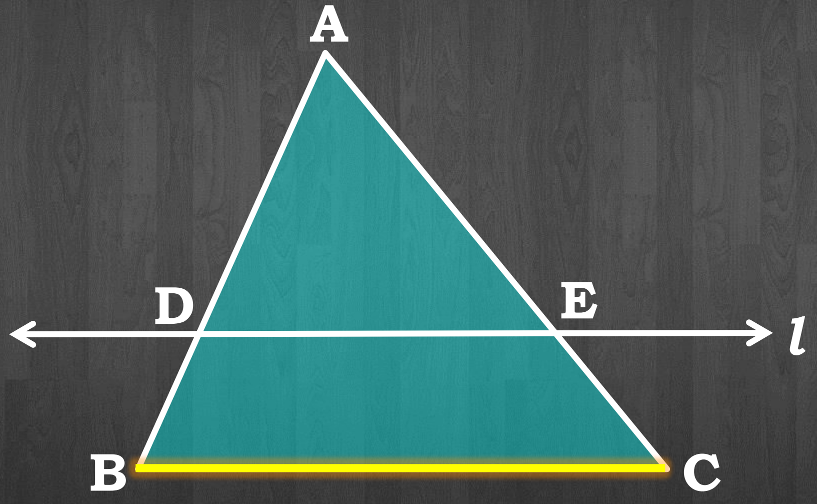


Thank You

MODULE : 6

Converse of Basic Proportionality Theorem

If a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.



In ΔABC,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\therefore$ line $l \parallel$ side BC   *[By converse of B.P.T]*

EXERCISE 6.2 - 2(i)

E and F are points on the sides PQ and PR respectively of a ΔPQR.
For each of the following cases, state whether EF || QR :
PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm.

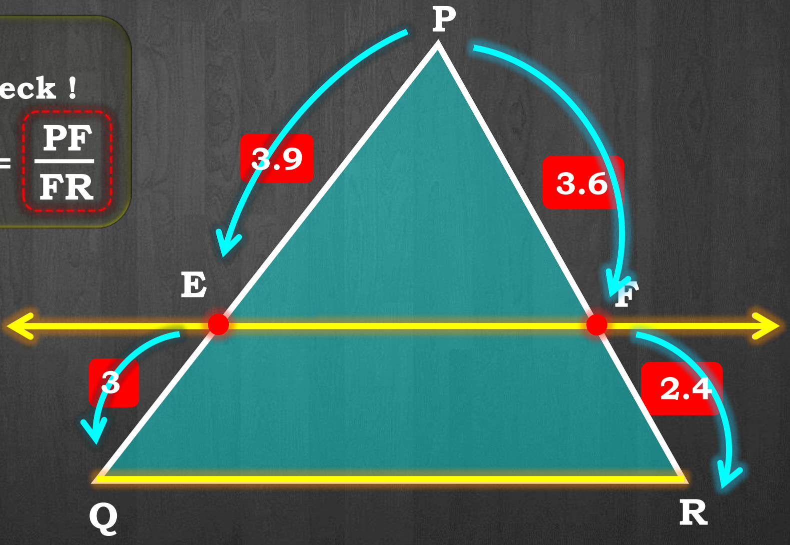
Sol. $$\frac{\boxed{PE}}{\boxed{EQ}} = \frac{3.9}{3}$$

$$\therefore \quad \frac{PE}{EQ} = \frac{\cancel{39}^{13}}{\cancel{30}_{10}}$$

$$\frac{PE}{EQ} = \frac{13}{10} \quad \ldots(i)$$

$$\therefore \quad \frac{PF}{FR} = \frac{3.6}{2.4}$$

Hint :
To Check !
$$\frac{PE}{EQ} = \frac{PF}{FR}$$



EXERCISE 6.2 - 2(i)

E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF ∥ QR :

PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm.

Sol.

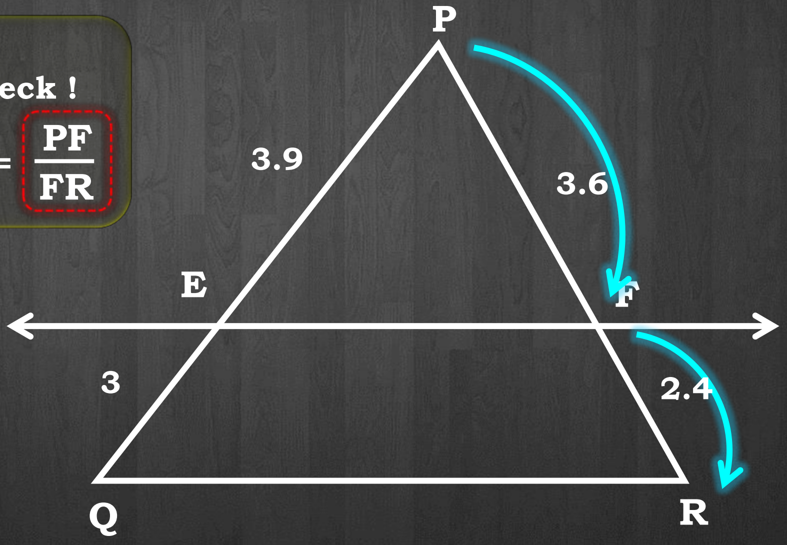
$$\frac{PF}{FR} = \frac{3.6}{2.4}$$

$$\therefore \quad \frac{PF}{FR} = \frac{\cancel{36}^{\,3}}{\cancel{24}_{\,2}}$$

$$\frac{PF}{FR} = \frac{3}{2} \quad \ldots\text{(ii)}$$

Hint :
To Check !

$$\frac{PE}{EQ} = \frac{PF}{FR}$$



In ΔPQR,

$$\frac{PE}{EQ} \neq \frac{PF}{FR} \quad \textit{[From (i) and (ii)]}$$

∴ Line EF is not parallel to side QR

MODULE : 7

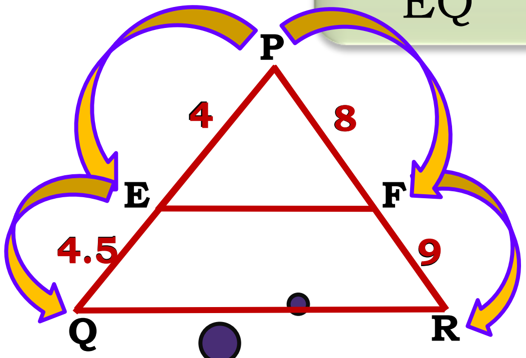
(ii) PE = 4cm, QE = 4.5cm, PF = 8cm & RF = 9cm

(ii) $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{\cancel{40}^{8}}{\cancel{45}_{9}} = \frac{8}{9}$

$\frac{PE}{EQ} = \frac{8}{9}$ ....(i)

$\frac{PF}{FR} = \frac{8}{9}$ ....(ii)

Hint : To check $\frac{PE}{EQ} = \frac{PF}{FR}$



$\therefore$ In ΔPQR,

From (i) and (ii)

$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore$ $\boxed{\textbf{EF} \parallel \textbf{QR}}$ ...ity theorem]

If we prove,

$\frac{PE}{EQ} = \frac{PF}{FR}$ **, can we say EF || QR ??**

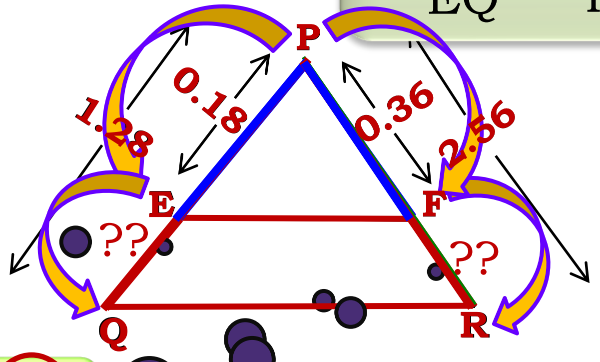
YES

(iii) PQ = 1.28cm, PR = 2.56cm, PE = 0.18cm & PF = 0.36cm

(iii) $\frac{PE}{EQ} = \frac{0.18}{EQ}$

Hint : To check

$\frac{PE}{EQ} = \frac{PF}{FR}$



EQ = PQ - PE

= 1.28 - 0.18

= 1.10

EQ = PQ - PE

$\therefore \frac{PE}{EQ} = \frac{\cancel{0.18}^{9}}{\cancel{1.10}_{55}}$

$\frac{PE}{EQ} = \frac{9}{55}$ ....(i)

$\frac{PF}{FR} = \frac{0.36}{FR}$

FR = PR - PF

FR = PR - PF

$\therefore \frac{PF}{FR} = \frac{\cancel{0.36}^{9}}{\cancel{2.20}_{55}}$

$\frac{PF}{FR} = \frac{9}{55}$ ....(ii)

If we pr

$\frac{PE}{EQ} = \frac{PF}{FR}$,

From (i) and (ii)

?? ??

YES

[by conve

proportionality theorem]

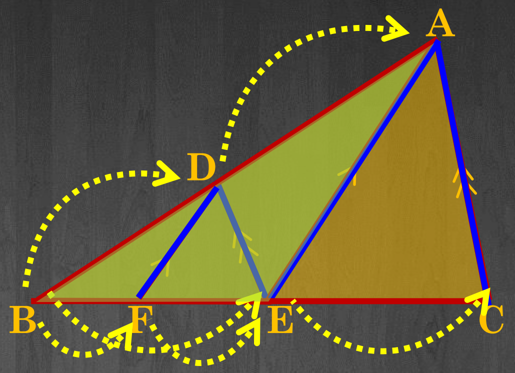
MODULE : 8

Ex 6.2 - 4

Given : DE || AC

DF || AE

Prove that : $\dfrac{BF}{FE} = \dfrac{BE}{EC}$



Proof : In ΔABC,

DE || AC ... [given]

$\dfrac{BE}{EC} = \dfrac{BD}{DA}$ ....(i) ... [by basic proportionality theorem]

In ΔABE,

DF || AE ... [given]

$\dfrac{BF}{FE} = \dfrac{BD}{DA}$ ....(ii) ... [by basic proportionality theorem.]

$\dfrac{BE}{EC} = \dfrac{BF}{FE}$ ... from (i) and (ii)

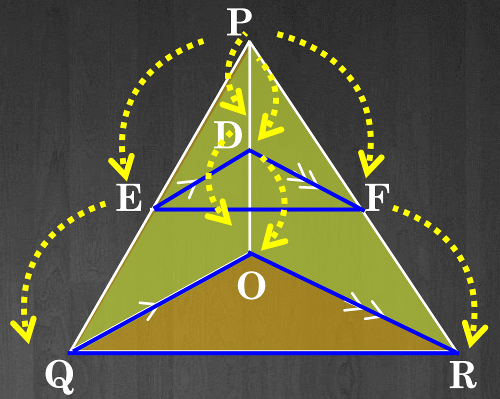
MODULE : 9

Ex 6.2 - 5

Given : $\boxed{DE \parallel OQ}$

$\boxed{DF \parallel OR}$

Show that : $EF \parallel QR$



Proof : In ΔPQO,

$DE \parallel OQ$

$$\frac{PE}{EQ} = \frac{PD}{DO} \quad \ldots(i)$$

... [by basic proportionality theorem.]

In ΔPRO,

$DF \parallel OR$

$$\frac{PF}{FR} = \frac{PD}{DO} \quad \ldots(ii)$$

... [by basic proportionality theorem.]

$$\therefore \boxed{\frac{PE}{EQ} = \frac{PF}{FR}} \quad \ldots(iii)$$

... from (i) and (ii)

In ΔPQR,

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

... from (iii)

$\therefore$ $EF \parallel QR$

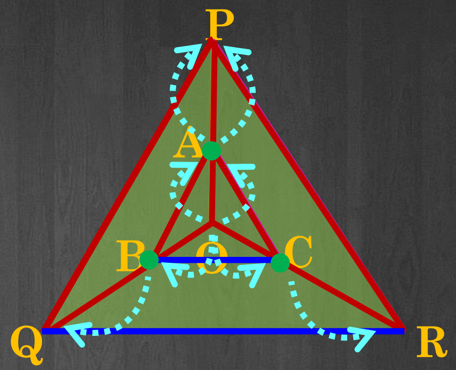
[by converse of basic proportionality theorem]

MODULE : 10

Ex 6.2 - 6

A, B & C are points on OP, OQ and OR respectively such that AB || PQ, AC || PR

Show that : BC || QR



Proof : In ΔPOQ,

AB || PQ ... [given]

$$\frac{OB}{BQ} = \frac{OA}{AP} \quad \text{...(i)}$$ ... [by basic proportionality theorem.]

In ΔPOR,

AC || PR ... [given]

$$\frac{OC}{CR} = \frac{OA}{AP} \quad \text{...(ii)}$$ ... [by basic proportionality theorem.]

$$\frac{OB}{BQ} = \frac{OC}{CR} \quad \text{...(iii)}$$ ... from (i) and (ii)

In ΔOQR,

$$\frac{OB}{BQ} = \frac{OC}{CR}$$ ... from (iii)

$\therefore$ **BC || QR** [by converse of basic proportionality theorem]

Thank You

MODULE : 11

Ex 6.2 - 7

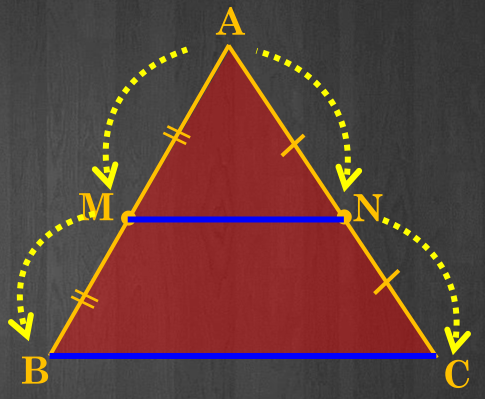
Prove that a line drawn through midpoint of one side of a triangle parallel to another side bisects the third side.

Given : In ΔABC,

$\boxed{\text{M is midpoint of side AB}}$

$\boxed{MN \parallel BC}$

Prove that : $AN = CN$



Proof : In ΔABC,

$MN \parallel BC$ ... [given]

$\boxed{\frac{AM}{MB} = \frac{AN}{NC}}$ ...(i) ... [by basic proportionality theorem]

$\boxed{\frac{AM}{MB} = 1}$ ...(ii) ... [M is midpoint of side AB]

$\frac{AN}{NC} = 1$ .... from (i) and (ii)

$AN = NC$

Ex 6.2 - 8

Prove that the line joining the midpoints of any two side of a triangle is parallel to the third side.

Given : In ΔABC,

M & N are midpoints of sides AB & AC resp.

To prove : $MN \parallel BC$

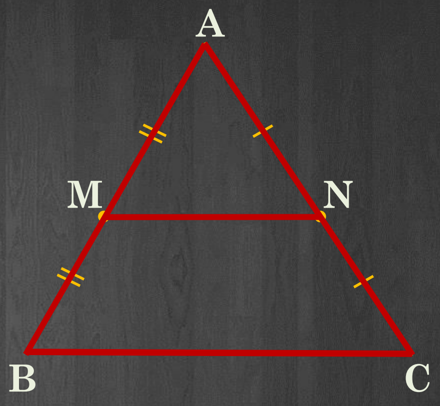
Proof : In ΔABC,

$$\frac{AM}{MB} = 1 \qquad \text{...(i)} \quad \text{[M is midpoint of side AB]}$$

$$\frac{AN}{NC} = 1 \qquad \text{...(ii)} \quad \text{[N is midpoint of side AC]}$$

$$\frac{AM}{MB} = \frac{AN}{NC} \qquad \text{... from (i) and (ii)}$$

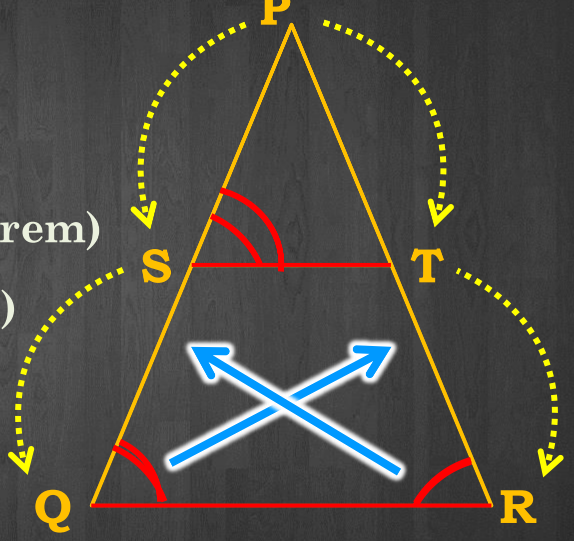
$MN \parallel BC$ ...[by converse of basic proportionality theorem]



MODULE : 12

Solved Example :

$\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.



Sol: $\frac{PS}{SQ} = \frac{PT}{TR}$

$\therefore$ $ST \parallel QR$ (Basic proportionality theorem)

$\therefore$ $\boxed{\angle PST = \angle PQR}$ ...(1)(Corresponding angles)

But $\boxed{\angle PST = \angle PRQ}$ ...(2)

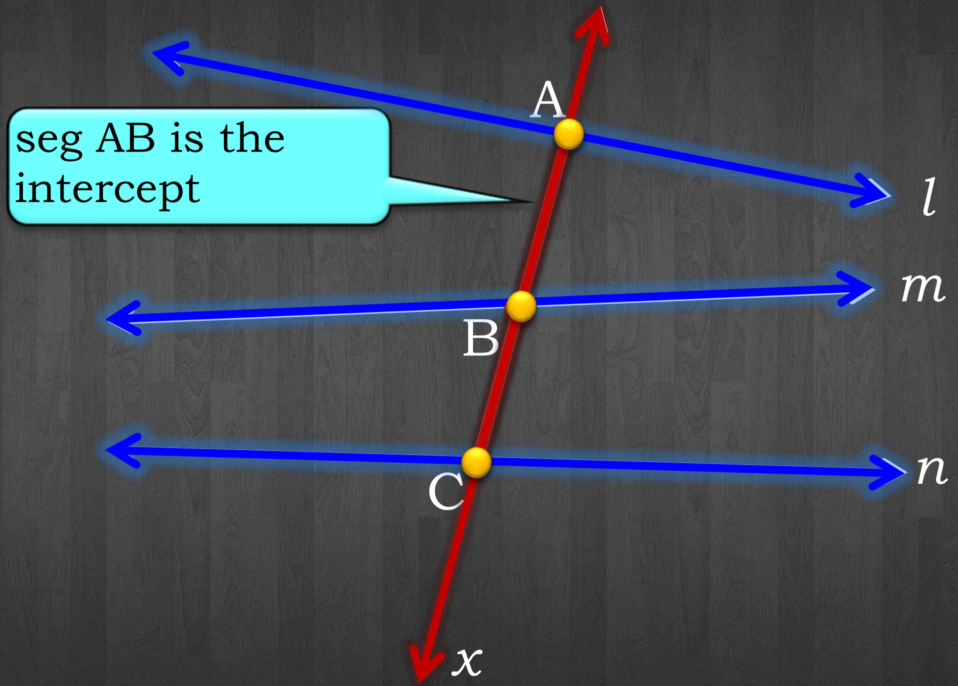
$\therefore$ $\angle PRQ = \angle PQR$ [From (1) and (2)]

and $PQ = PR$ (Sides opposite the equal angles)

i.e., PQR is an isosceles triangle.

MODULE : 13

INTERCEPT



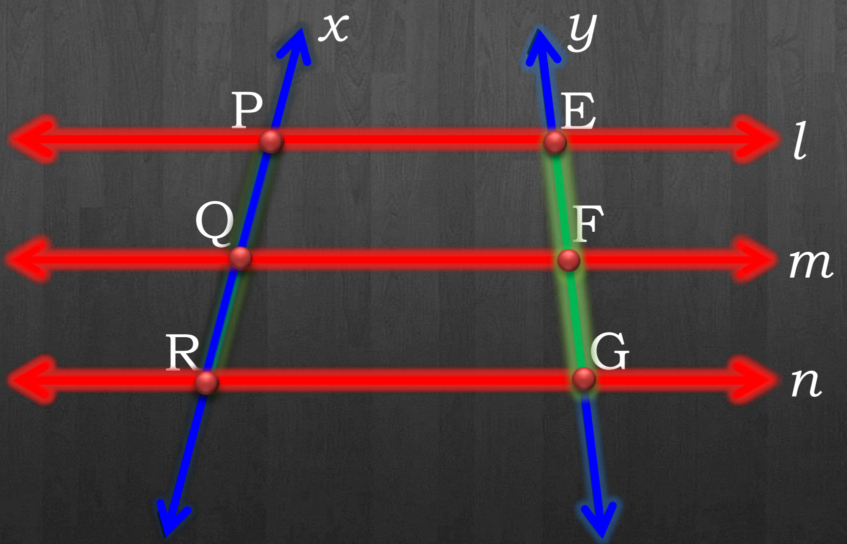
In the fig.,
seg AB is the intercept formed
on transversal x by lines l and m.

The ratio of the intercepts made on a transversal by **three parallel lines** is equal to the ratio of the **corresponding intercepts made on any other transversal** by the same parallel lines.

$$\frac{PQ}{QR} = \frac{EF}{FG}$$

$$\frac{QR}{PR} = \frac{FG}{EG}$$

$$\frac{PQ}{PR} = \frac{EF}{EG}$$



Intercepts on y

EF FG EG

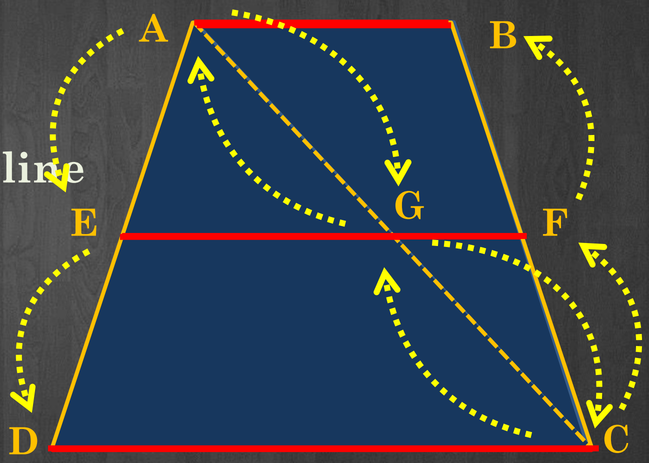
Intercepts on x

PQ QR PR

Solved Example :

ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB .

Show that $\dfrac{AE}{ED} = \dfrac{BF}{FC}$



Sol: AB || DC and EF || AB

$\therefore$ EF || DC (Lines parallel to the same line are parallel to each other)

Join AC intersecting EF at point G

In ΔADC,

EG || DC (As EF || DC)

$\therefore \quad \dfrac{AE}{ED} = \dfrac{AG}{GC}$ ...(1) [By Basic Proportionality Theorem]

In ΔCAB, FG || AB

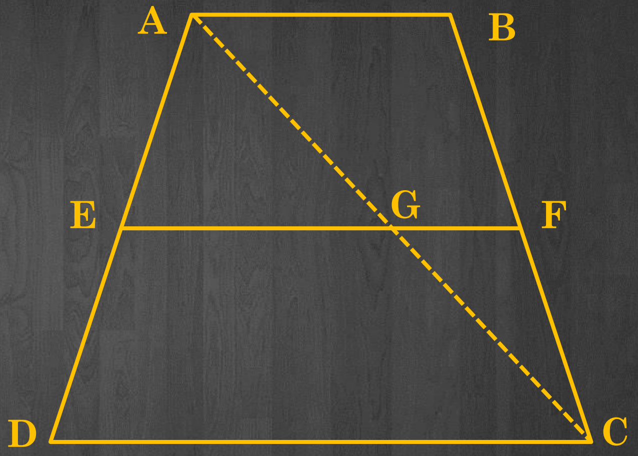
$\therefore \quad \dfrac{CG}{AG} = \dfrac{CF}{BF}$ ...(2)

Solved Example :

ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB .

Show that $\dfrac{AE}{ED} = \dfrac{BF}{FC}$

Sol:



$$\boxed{\frac{AE}{ED} = \frac{AG}{GC}} \quad \text{...(1)}$$

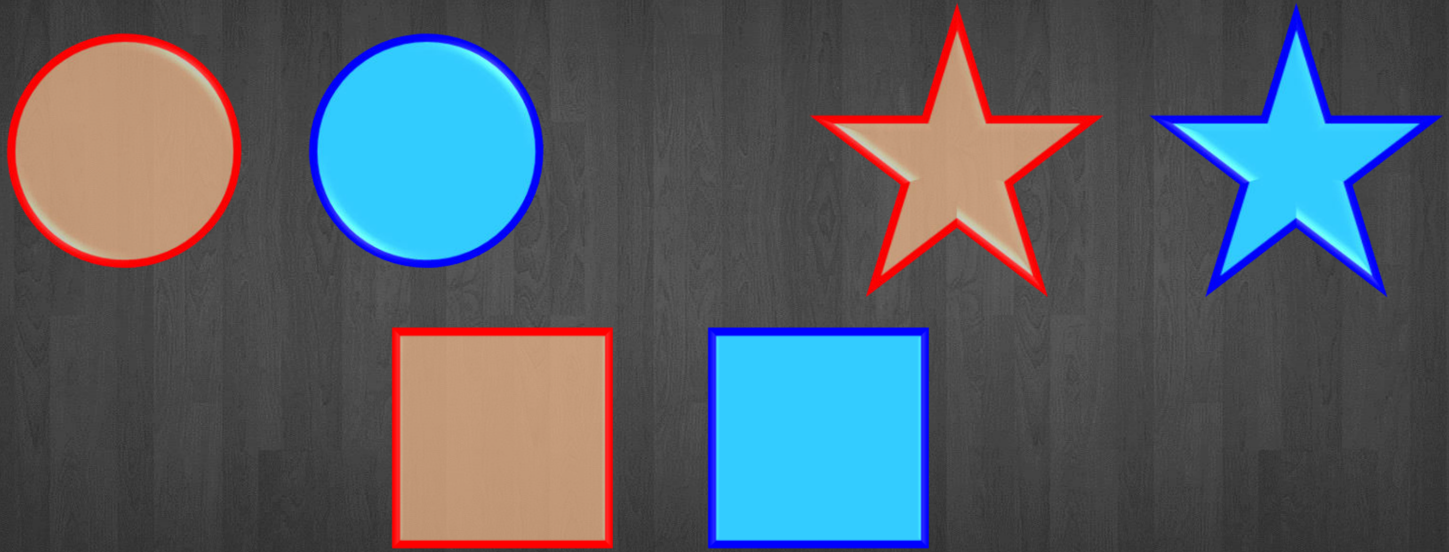
$$\boxed{\frac{CG}{AG} = \frac{CF}{BF}} \quad \text{...(2)}$$

i.e., $$\boxed{\frac{AG}{GC} = \frac{BF}{FC}} \quad \text{...(3)}$$

$$\frac{AE}{ED} = \frac{BF}{FC} \quad \text{[From (1) and (3)]}$$

MODULE : 14

CONGRUENT FIGURES



Figures having same shape and same size are called CONGRUENT FIGURES

SIMILAR OBJECTS

SIMILAR OBJECTS

SIMILAR OBJECTS

Figures whose SHAPE is SAME and size may or may not be same are similar.

SIMILAR OBJECTS

Figures whose shape is same and size may or may not be same are similar.

If

$\triangle ABC$ is similar to $\triangle PRQ$

then,

$\angle A \cong \angle P$

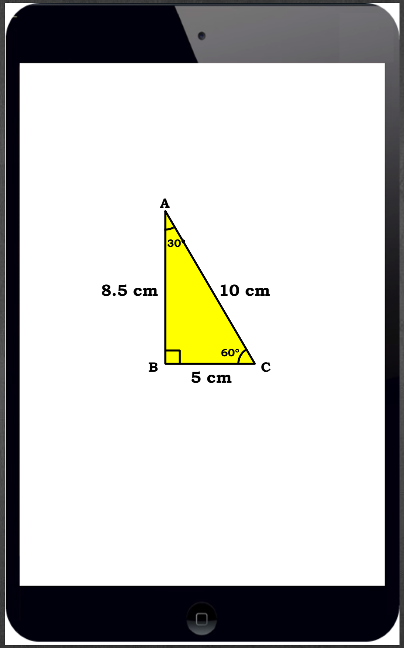
$\angle B \cong \angle R$

$\angle C \cong \angle Q$

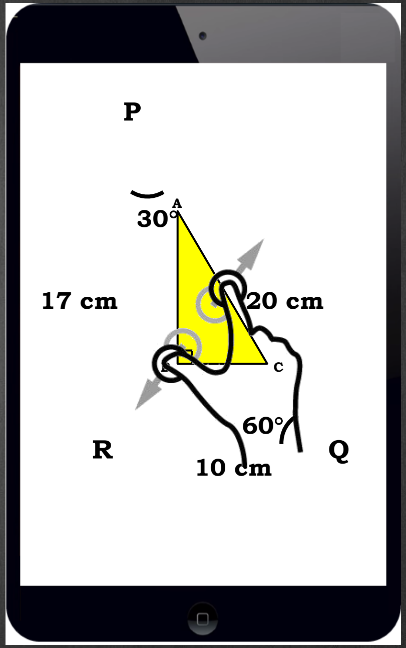
Corresponding angles of similar triangles are CONGRUENT.

$$\frac{AB}{PR} = \frac{BC}{QR} = \frac{AC}{PQ}$$

Corresponding sides of similar triangles are in PROPORTION



Zoom 2×

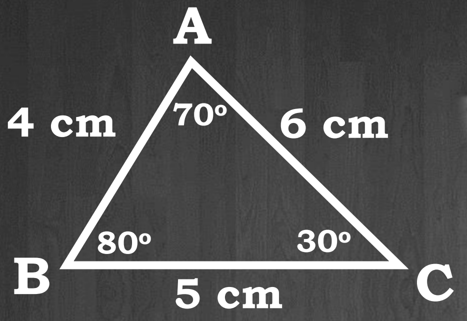


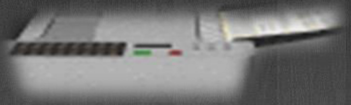
$$\frac{AB}{PR} = \frac{8.5}{17} = \boxed{\frac{1}{2}}$$

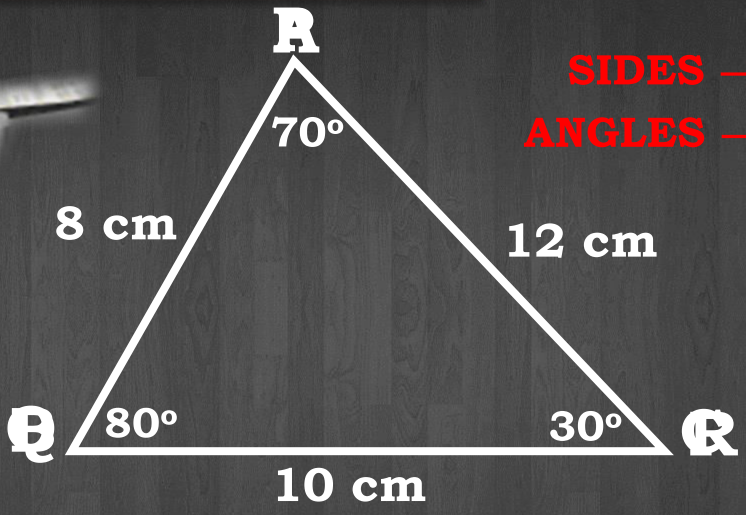
$$\frac{BC}{QR} = \frac{5}{10} = \boxed{\frac{1}{2}}$$

$$\frac{AC}{PQ} = \frac{10}{20} = \boxed{\frac{1}{2}}$$

∆ABC is similar to ∆PQR







SIDES → doubled

ANGLES → ?

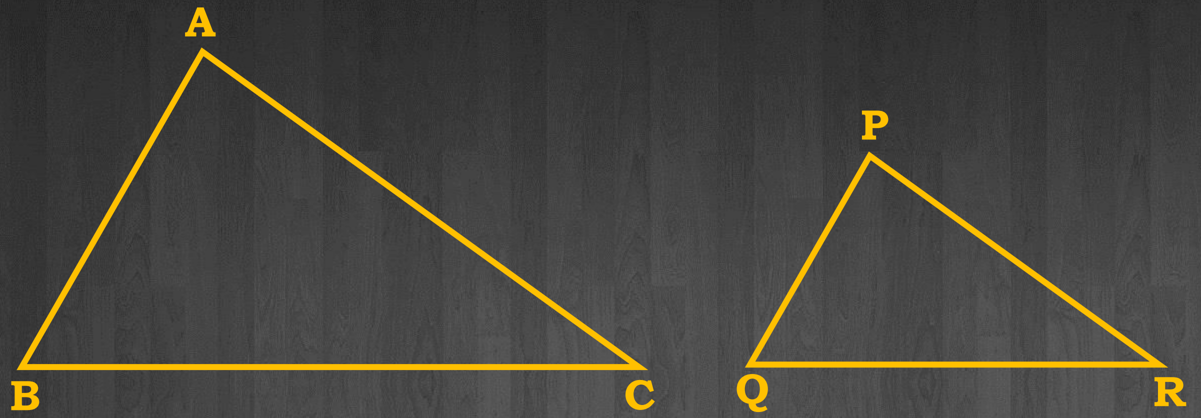
$$\frac{AB}{PQ} = \frac{1}{2} \qquad \frac{BC}{QR} = \frac{1}{2} \qquad \frac{AC}{PR} = \frac{1}{2}$$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

Corresponding pairs of similar triangles are in proportion.

$\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

Corresponding pairs of similar triangles are equal.



$$\Delta ABC \sim \Delta PQR$$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

Corresponding sides of similar triangles

$\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

Corresponding angles of similar triangles

SIMILAR TRIANGLES

$\Delta ABC \sim \Delta QRP$ [Given]

$\therefore \quad \dfrac{AB}{QR} = \dfrac{BC}{PR} = \dfrac{AC}{PQ}$ [c.s.s.t.] (Corresponding sides of similar triangles)

Also,

$$\left.\begin{aligned} \angle A &\cong \angle Q \\ \angle B &\cong \angle R \\ \angle C &\cong \angle P \end{aligned}\right\}$$

[c.a.s.t.] (Corresponding angles of similar triangles)

Thank You

MODULE : 15

EXERCISE 6.1

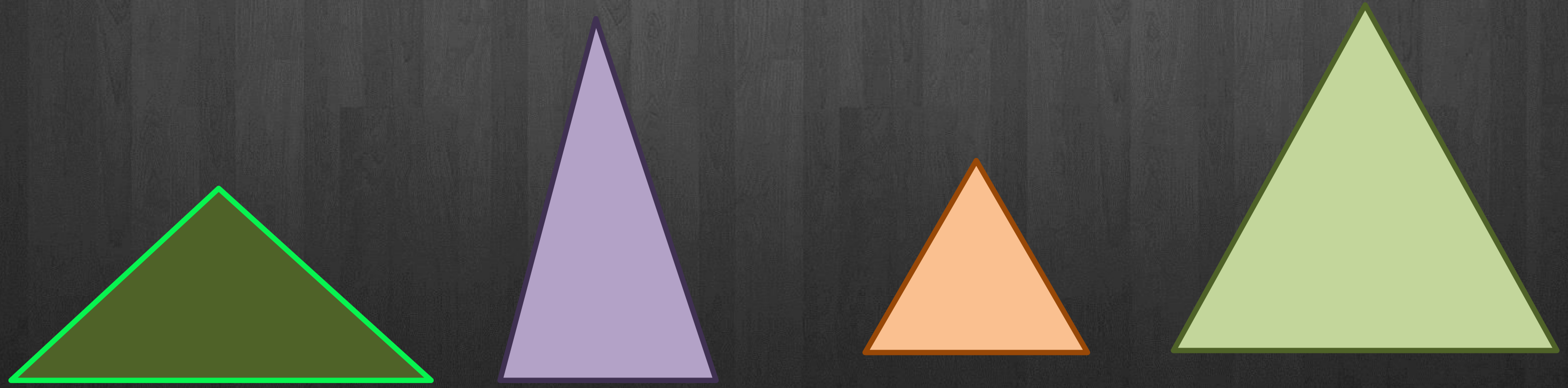
1. Fill in the blanks using the correct word given in brackets :

(i) All circles are similar. (congruent, similar)

(ii) All squares are similar. (similar, congruent)

(iii) All equilateral triangles are similar. (isosceles, equilateral)

Figures whose SHAPE is SAME and size may or may not be same are similar.



EXERCISE 6.1

1. Fill in the blanks using the correct word given in brackets :

(iv) Two polygons of the same number of sides are similar, if

(a) their corresponding angles are ____________ and

(b) their corresponding sides are ____________.

(equal, proportional)

Corresponding angles of similar polygons are CONGRUENT.

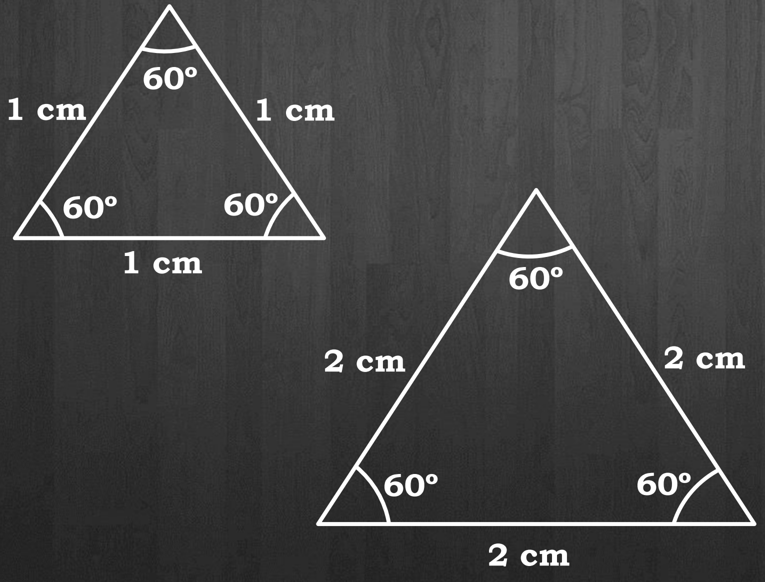
Corresponding sides of similar polygons are in PROPORTION

EXERCISE 6.1

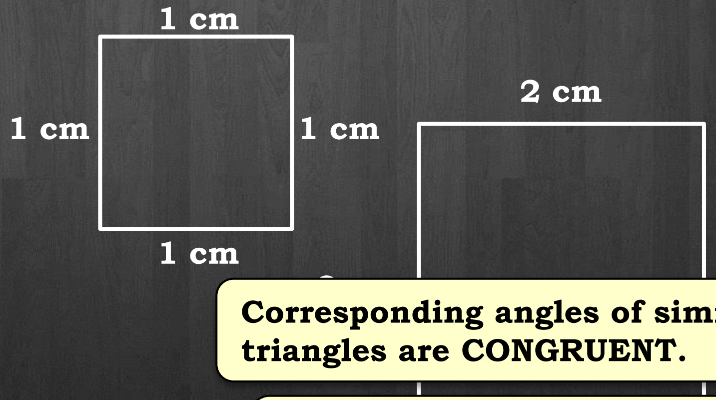
Q.2 Give two different examples of pair of :

(i) similar figures.

Two equilateral triangles with sides 1cm and 2cm



Two squares with sides 1cm and 2cm



Corresponding angles of similar triangles are CONGRUENT.

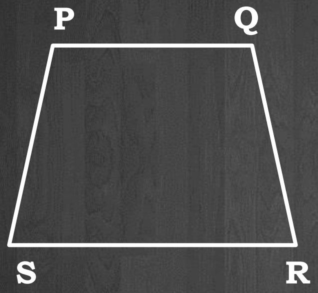
Corresponding sides of similar triangles are in PROPORTION

EXERCISE 6.1

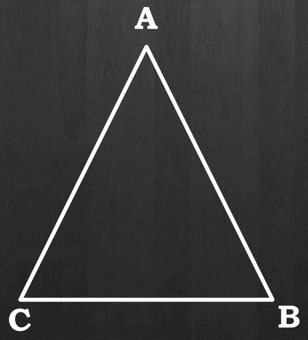
Q.2 Give two different examples of pair of :

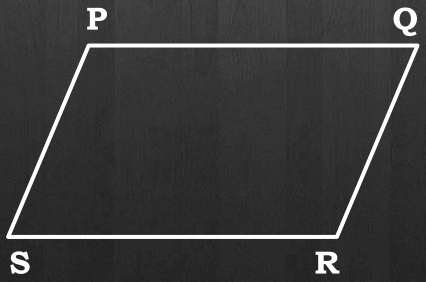
(ii) non-similar figures.

Trapezium and square



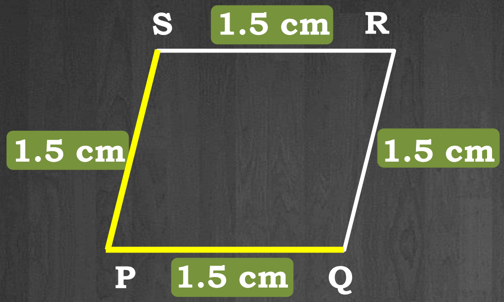
Triangle and parallelogram

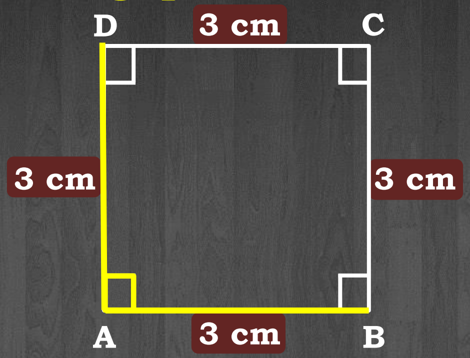




EXERCISE 6.1

Q.3 State whether the following quadrilaterals are similar or not:





Corresponding angles of similar triangles are CONGRUENT.

Corresponding sides of similar triangles are in PROPORTION

$$\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RS}{CD} = \frac{PS}{AD} = \frac{1.5}{3} = \frac{1}{2}$$

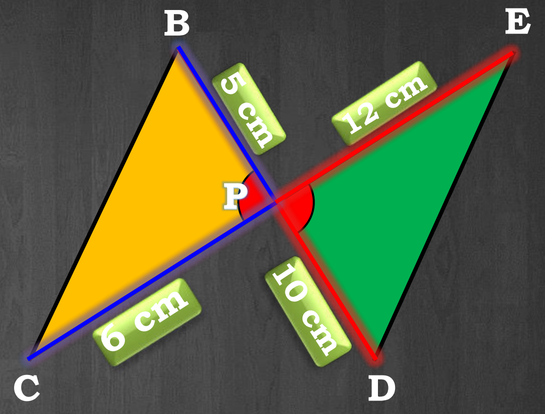
$\therefore$ corresponding sides of Quadrilaterals PQRS and ABCD are in proportion but corresponding angles of Quadrilaterals PQRS and ABCD are not congruent.

$\therefore$ □PQRS and □ABCD are not similar.

MODULE : 16

Example

BD and CE intersect each other at the point P
Is ΔPBC similar to ΔPDE ? Why ?



Proof : $\dfrac{BP}{DP} = \dfrac{5}{10}$

$\therefore \quad \dfrac{BP}{DP} = \dfrac{1}{2}$ ...(i)

$\dfrac{CP}{PE} = \dfrac{6}{12}$

$\therefore \quad \dfrac{CP}{PE} = \dfrac{1}{2}$ ...(ii)

In ΔPBC and ΔPDE

$\dfrac{BP}{DP} = \dfrac{CP}{PE}$ *[From (i) and (ii)]*

$\angle BPC = \angle DPE$ *[Vertically opposite angles]*

$\therefore \quad \Delta PBC \sim \Delta PDE$ *(SAS criterion)*

Example

Express x in term of a, b and c.

Sol: In ΔPKN and ΔLKM

$\angle K = \angle K$ (Common angle)

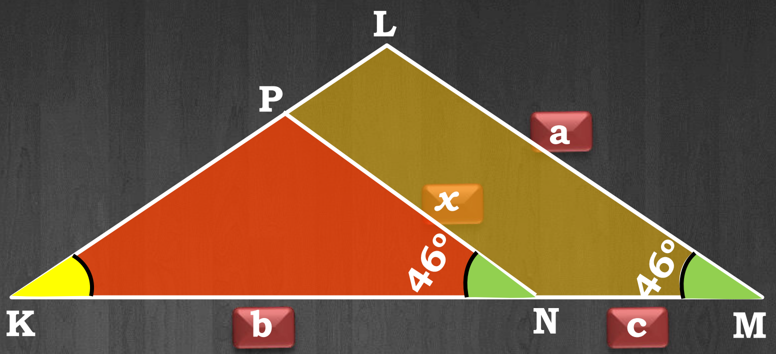
$\angle KNP = \angle KML$ (Given)

$\therefore \Delta PKN \sim \Delta LKM$ (AA criterion)

$$\frac{KN}{KM} = \frac{PN}{LM}$$ (Corresponding sides of similar triangles)

$$\frac{b}{(b + c)} = \frac{x}{a}$$

$$\boxed{x = \left(\frac{ab}{b + c}\right)}$$



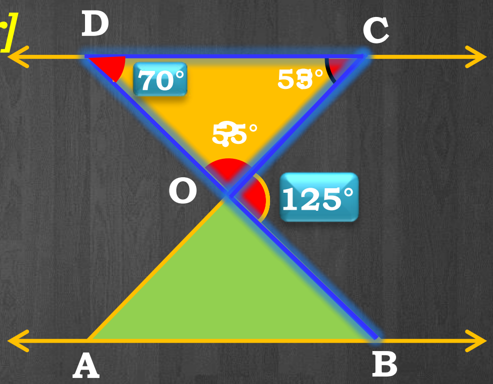
MODULE : 17

EX.6.3 (Q.2)

$\Delta ODC \sim \Delta OBA$, $\angle BOC = 125^\circ$, $\angle CDO = 70^\circ$

Find $\angle DOC$, $\angle DCO$ and $\angle OAB$

Soln. $\angle DOC + \angle BOC = 180^\circ$ *[Angles in a linear pair]*



$\therefore \quad \angle DOC + 125 = 180$

$\therefore \quad \angle DOC = 180 - 125$

$\therefore \quad \boxed{\angle DOC = 55^\circ}$ ...(i)

$\angle BOC = \angle CDO + \angle DCO$

[Exterior angle is equal to sum of its two interior opposite angles]

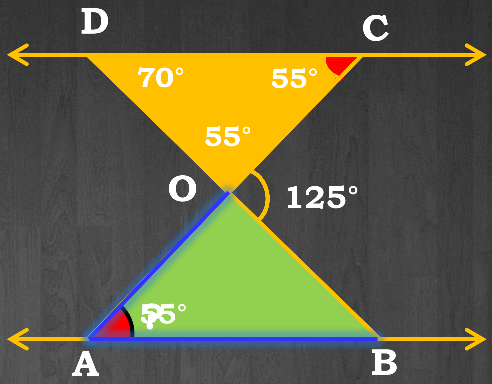
$\therefore \quad 125 = 70 + \angle DCO$

$\therefore \quad \angle DCO = 125 - 70$

$\therefore \quad \boxed{\angle DCO = 55^\circ}$ ...(ii)

Q. Given : $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$, $\angle CDO = 70°$

Find : $\angle DOC$, $\angle DCO$ and $\angle OAB$



Soln. $\angle DCO = 55°$ ...(ii)

$\Delta ODC \sim \Delta OBA$ *[Given]*

$\angle DCO = \angle OAB$ *[corresponding angles of similar triangles]*

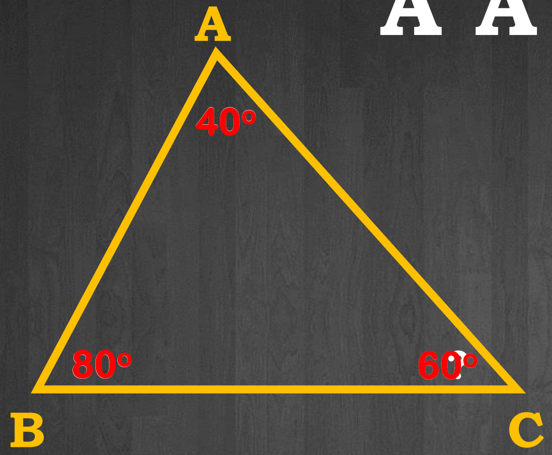
$\therefore$ $\angle OAB = 55°$

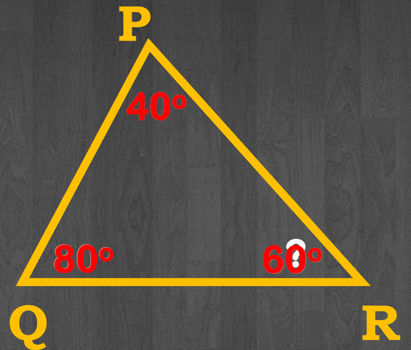
MODULE : 18

CRITERIA OF SIMILARITY OF TWO TRIANGLES

- **AAA** Criterion Of Similarity
- **SSS** Criterion Of Similarity
- **SAS** Criterion Of Similarity

A A A Criterion Of Similarity





In ΔABC and ΔPQR,

$\angle A = \angle P$

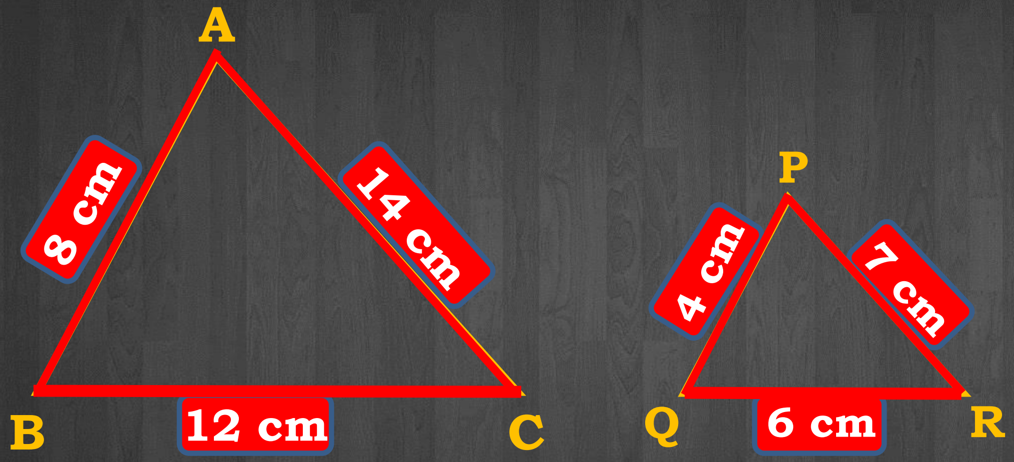
$\angle B = \angle Q$

$\angle C = \angle R$

$\therefore \quad \Delta ABC \simeq \Delta PQR$

$\therefore \quad \Delta ABC \sim \Delta PQR$

SSS Criterion Of Similarity



In ΔABC and ΔPQR,

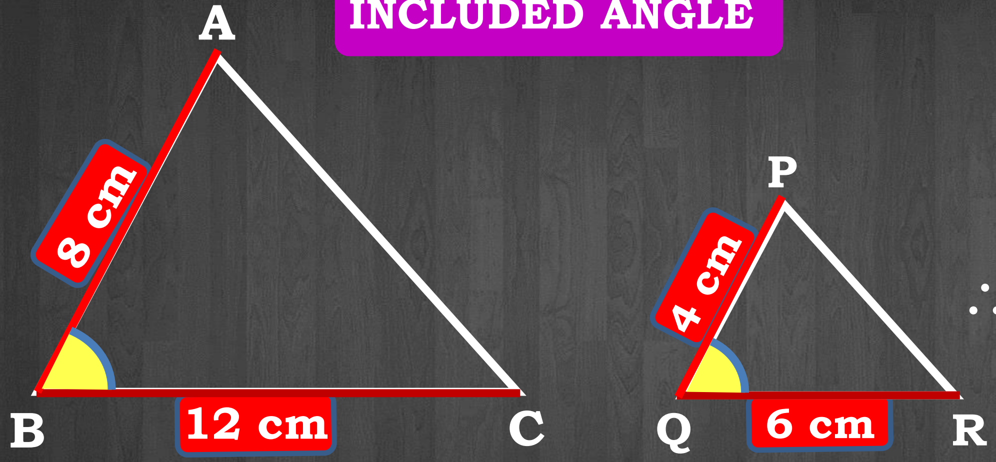
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$\therefore$ $\Delta ABC \sim \Delta PQR$

$$\boxed{\frac{AB}{PQ}} = \frac{8}{4} = \frac{2}{1} \qquad \boxed{\frac{BC}{QR}} = \frac{12}{6} = \frac{2}{1} \qquad \boxed{\frac{AC}{PR}} = \frac{14}{7} = \frac{2}{1}$$

S A S Criterion Of Similarity

INCLUDED ANGLE



$$\boxed{\frac{AB}{PQ}} = \frac{8}{4} = \frac{2}{1} \qquad \boxed{\frac{BC}{QR}} = \frac{12}{6} = \frac{2}{1}$$

In ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

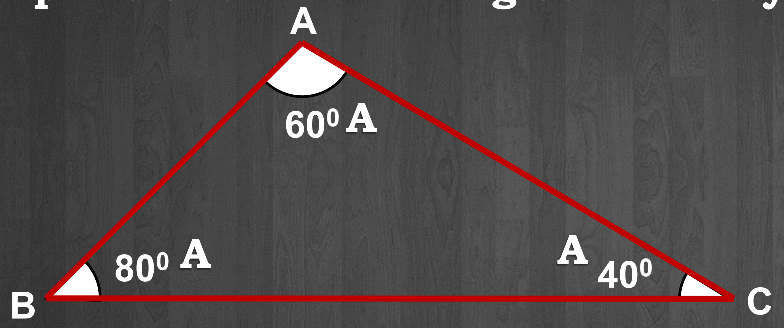
$$\angle B = \angle Q$$

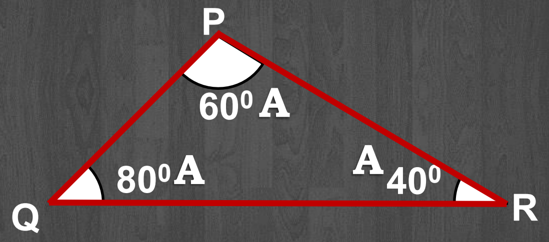
$$\therefore \quad \Delta ABC \sim \Delta PQR$$

MODULE : 19

EX.6.3 Q.1 (i)

State which pairs of triangles are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :





Soln. In ΔABC & ΔPQR

$\angle A = \angle P = 60^0$

$\angle B = \angle Q = 80^0$

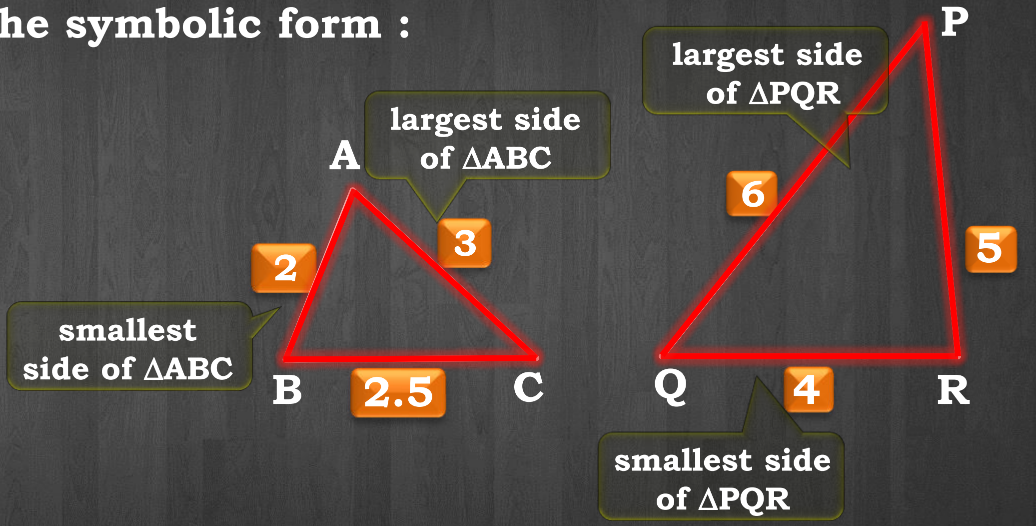
$\angle C = \angle R = 40^0$

$\therefore$ ΔABC & Δ PQR are similar

Δ ABC ~ Δ PQR by AAA similarity

EX.6.3 Q.1 (ii)

State which pairs of triangles are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :



Sol. $\dfrac{AB}{QR} = \dfrac{\cancel{2}}{\cancel{4}_{2}} = \dfrac{1}{2}$ ...(i)

$\dfrac{AC}{PQ} = \dfrac{\cancel{3}}{\cancel{6}_{2}} = \dfrac{1}{2}$ ...(ii)

$\dfrac{BC}{PR} = \dfrac{\cancel{2}.5}{\cancel{5}_{2}} = \dfrac{1}{2}$ ...(iii)

In ΔABC and ΔPQR

$$\frac{AB}{QR} = \frac{AC}{PQ} = \frac{BC}{PR}$$

$\therefore$ $\Delta ABC \sim \Delta QRP$ ***[By SSS criterion]***

EX.6.3 Q.1 (iii)

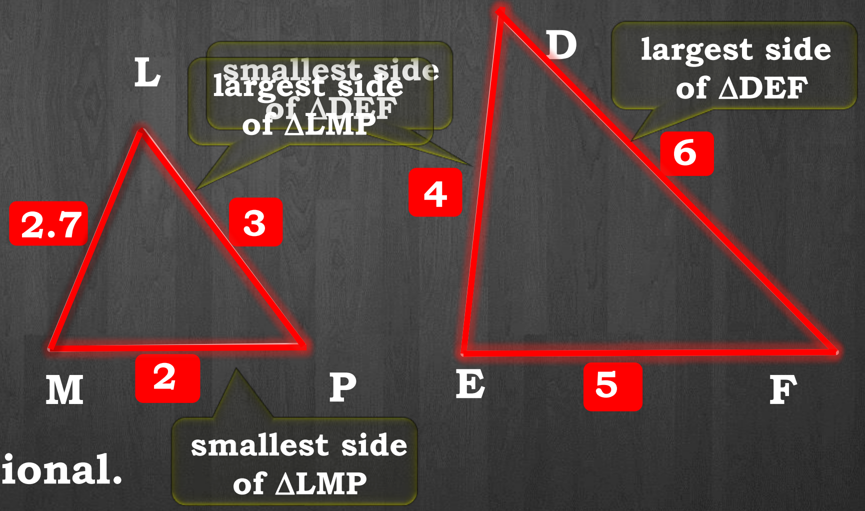
State which pairs of triangles are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :

Sol.

$$\frac{MP}{DE} = \frac{\cancel{2}}{\cancel{4}_{2}} = \frac{1}{2} \quad \text{...(i)}$$

$$\frac{LP}{DF} = \frac{\cancel{3}}{\cancel{6}_{2}} = \frac{1}{2} \quad \text{...(ii)}$$

$$\frac{LM}{EF} = \frac{2.7}{5} \quad \text{...(iii)}$$



Corresponding sides are not proportional.

$\therefore$ **ΔLMP and ΔDEF are not similar.**

Thank You

MODULE : 20

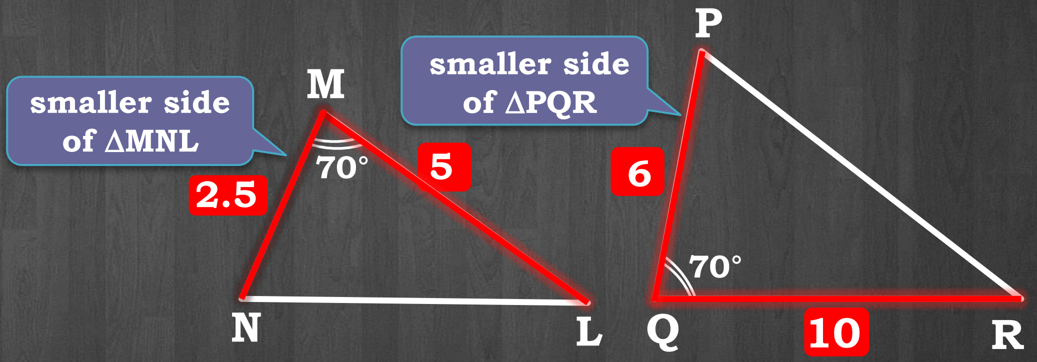
EX.6.3 Q.1 (iv)

State which pairs of triangles are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :

Sol.

$$\frac{\boxed{MN}}{\boxed{PQ}} = \frac{2.5}{6} = \boxed{\frac{5}{12}} \quad \ldots\text{(i)}$$

$$\frac{\boxed{ML}}{\boxed{QR}} = \frac{\cancel{5}}{\cancel{10}_{2}} = \boxed{\frac{1}{2}} \quad \ldots\text{(ii)}$$

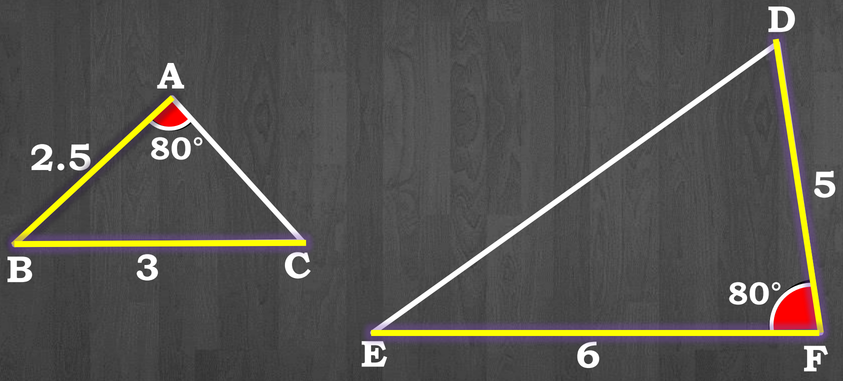


Corresponding sides are not proportional.

$\therefore$ **ΔMNL and ΔPQR are not similar.**

EX.6.3 Q.1 (v)

State which pairs of triangles are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :

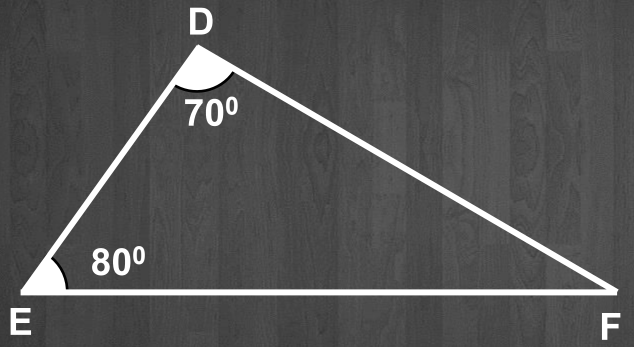


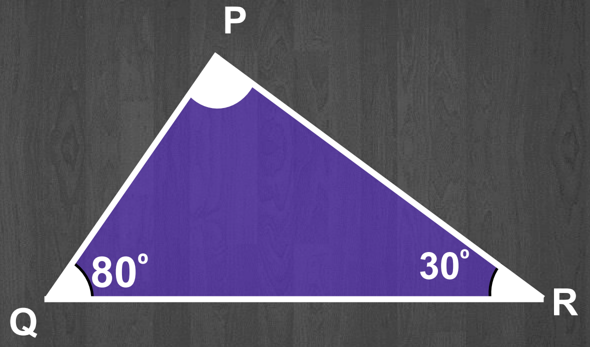
Sol :

Two triangles are not similar as a given pair of equal angles is not included between given sides.

EX.6.3 Q.1 (vi)

State which pairs of triangles are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :





Soln. In ΔPQR

$$\angle P + \angle Q + \angle R = 180^0$$

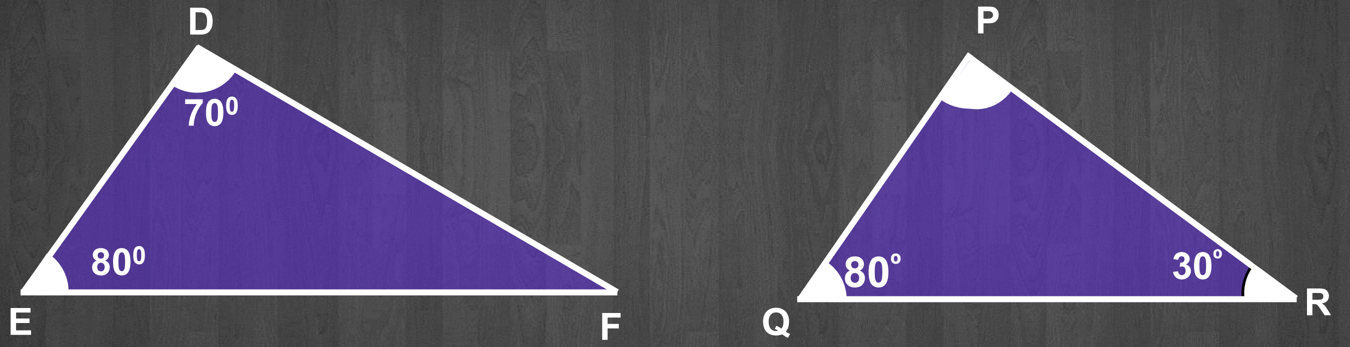
$$\angle P + 80 + 30 = 180^0$$

$$\angle P = 180^0 - 110^0$$

$$\angle P = 70^0$$

EX.6.3 Q.1 (vi)

State which pairs of triangles are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :



Soln.

In Δ DEF and ΔPQR,

$\angle D = \angle P$ .... [each is 70^0]

$\angle E = \angle Q$ .... [each is 80^0]

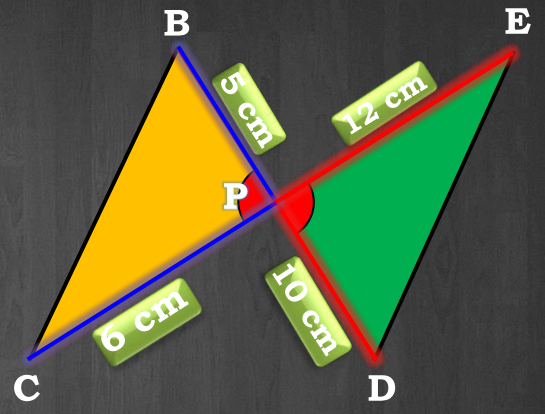
$\therefore$ $\boxed{\Delta\ DEF \sim \Delta\ PQR}$ .... [by AA similarity criterion]

MODULE : 21

Example

BD and CE intersect each other at the point P

Is ΔPBC similar to ΔPDE ? Why ?



Proof : $\dfrac{BP}{DP} = \dfrac{5}{10}$

$\therefore \quad \dfrac{BP}{DP} = \dfrac{1}{2} \quad \text{...(i)}$

$\dfrac{CP}{PE} = \dfrac{6}{12}$

$\therefore \quad \dfrac{CP}{PE} = \dfrac{1}{2} \quad \text{...(ii)}$

In ΔPBC and ΔPDE

$\dfrac{BP}{DP} = \dfrac{CP}{PE}$ *[From (i) and (ii)]*

$\angle BPC = \angle DPE$ *[Vertically opposite angles]*

$\therefore \; \Delta PBC \sim \Delta PDE$ *(SAS criterion)*

Example

Express x in term of a, b and c.

Sol: In ΔPKN and ΔLKM

$\angle K = \angle K$ (Common angle)

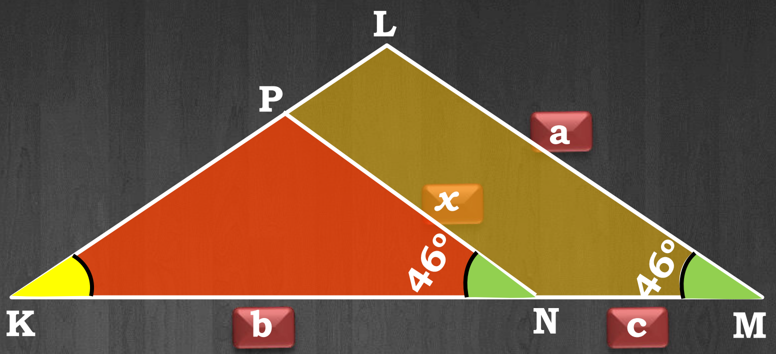
$\angle KNP = \angle KML$ (Given)

$\therefore \Delta PKN \sim \Delta LKM$ (AA criterion)

$$\frac{KN}{KM} = \frac{PN}{LM}$$ (Corresponding sides of similar triangles)

$$\frac{b}{(b + c)} = \frac{x}{a}$$

$$\boxed{x = \left(\frac{ab}{b + c}\right)}$$

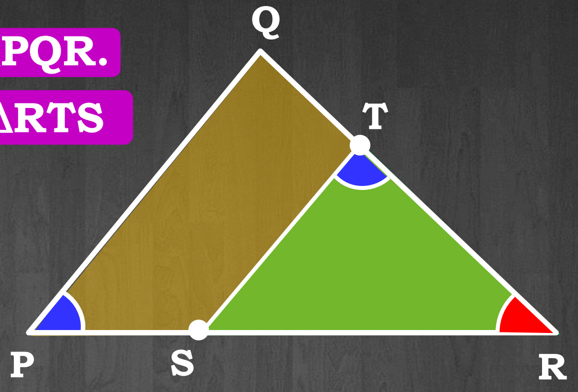


MODULE : 22

EX.6.3 Q.5

S and T are points on sides PR and QR of ΔPQR.

such that $\angle P = \angle RTS$ Show that : $\Delta RPQ \sim \Delta RTS$



Proof :

In ΔRPQ and ΔRTS,

$\angle RPQ = \angle RTS$ *[Given]*

$\angle R = \angle R$ *[common angle]*

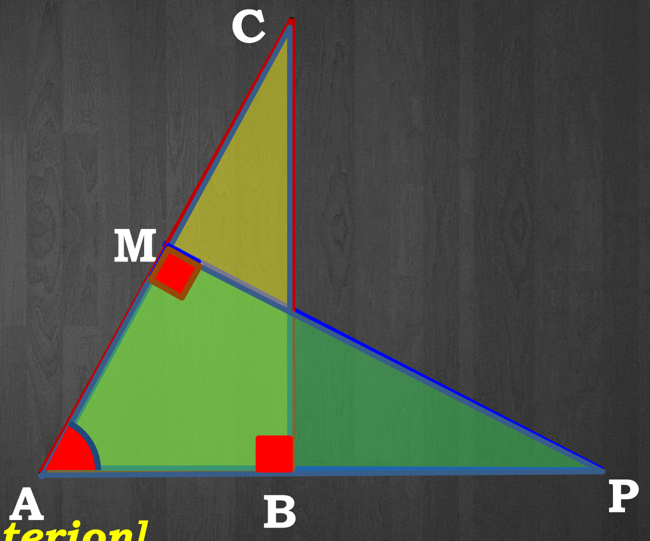
$\therefore$ $\boxed{\Delta RPQ \sim \Delta RTS}$ *[by AA similarity criterion]*

EX.6.3 Q.9

ΔABC & ΔAMP are two right triangles, right angled at B & M respectively.

Prove that : i) $\Delta ABC \sim \Delta AMP$

$$\text{ii) } \frac{CA}{PA} = \frac{BC}{MP}$$



Soln.

In ΔABC & ΔAMP

$\angle BAC = \angle MAP$ ..... *[common angle]*

$\angle ABC = \angle AMP$ ..... *[each is 90°]*

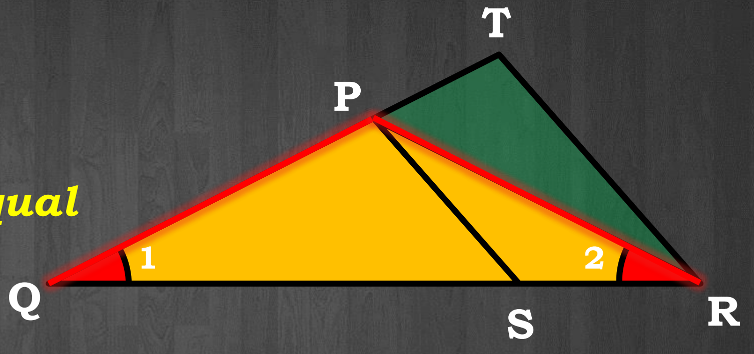
$\therefore$ $\boxed{\Delta ABC \sim \Delta AMP}$ ..... *[by AA similarity criterion]*

$\therefore$ $\boxed{\frac{CA}{PA} = \frac{BC}{MP}}$ ..... *[corresponding sides of similar triangles]*

MODULE : 23

EX.6.3 Q.4

$\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$, Show that : $\Delta PQS \sim \Delta TQR$



Proof : In ΔPQR,

$\angle PQR = \angle PRQ$ *[Given]*

$PQ = PR$ *...(i) [sides opposite to equal angles are equal]*

$\frac{QR}{QS} = \frac{QT}{PR}$ *...(ii) [Given]*

$\therefore \frac{QR}{QS} = \frac{QT}{PQ}$ *...(iii) [from (i) & (ii)]*

In ΔTQR and ΔPQS,

$\frac{QR}{QS} = \frac{QT}{PQ}$ *[From (iii)]*

$\angle Q = \angle Q$ *[common angle]*

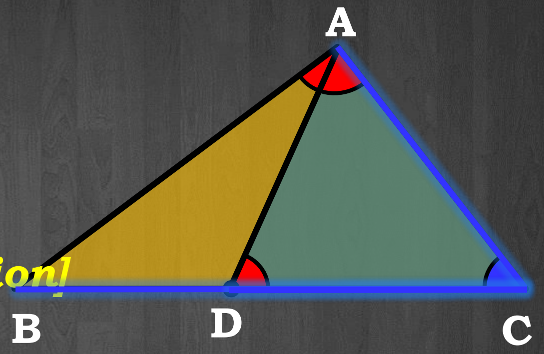
$\therefore \Delta TQR \sim \Delta PQS$ *[by SAS similarity criterion]*

i.e. $\Delta PQS \sim \Delta TQR$

EX.6.3 Q.13

D is a point on side BC of ΔABC such that $\angle ADC = \angle BAC$

Show that : $CA^2 = CB \times CD$



Proof : In ΔCAB and ΔCDA,

$\angle BAC = \angle ADC$ *[Given]*

$\angle C = \angle C$ *[Common angle]*

$\Delta CAB \sim \Delta CDA$ *[By AA similarity criterion]*

$\therefore \dfrac{CA}{CD} = \dfrac{CB}{CA}$ *[corresponding sides of similar triangles]*

$\therefore \boxed{CA^2 = CB \times CD}$

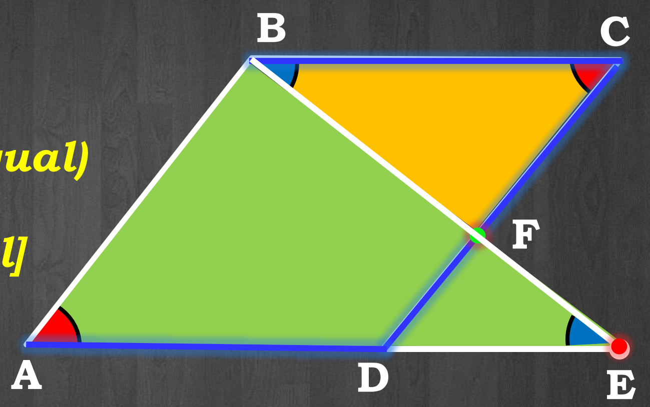
i.e. $CA \times CA = CB \times CD$

i.e. $\dfrac{CA}{CD} = \dfrac{CB}{CA}$

MODULE : 24

EX.6.3 Q.8

E is a point on side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that : $\Delta ABE \sim \Delta CFB$



Proof : ABCD is a parallelogram *[Given]*

$\therefore$ $\angle A = \angle C$ *...(i) [Opposite angles of parallelogram are equal)*

BC ∥ AD *[Opposite sides of parallelogram are parallel]*

$\therefore$ BC ∥ AE

On transversal BE

$\angle AEB = \angle CBF$ *...(ii) [alternate interior angles]*

In ΔABE and ΔCFB,

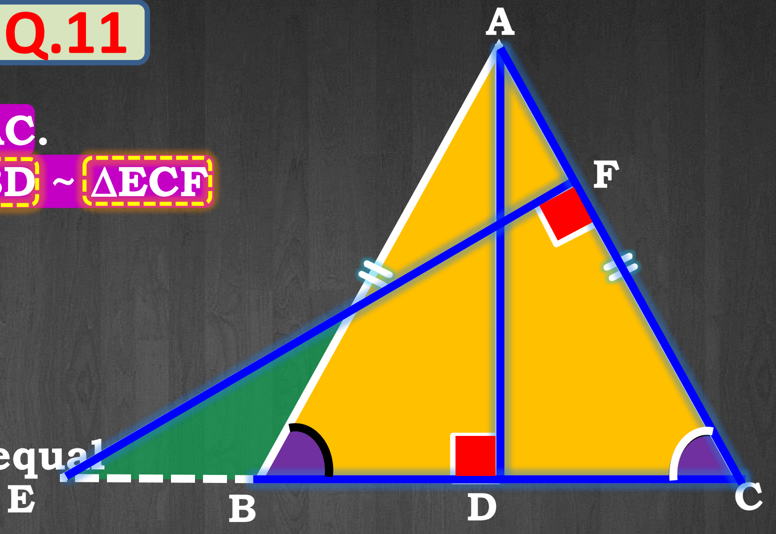
$\angle A = \angle C$ *[From (i)]*

$\angle AEB = \angle CBF$ *[From (ii)]*

$\therefore$ $\Delta ABE \sim \Delta CFB$ *[By AA similarity criterion]*

EX.6.3 Q.11

E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC, EF ⊥ AC Prove that : ΔABD ~ ΔECF



Proof :

In ΔABC,

AB = AC [Given]

∴ ∠ABC = ∠ACB [Angles opposite to equal sides are equal]

∴ ∠ABD = ∠ECF ...(i)

In ΔABD and ΔECF,

∠ABD = ∠FCE [From (i)]

∠ADB = ∠EFC [Each is 90°]

∴ ΔABD ~ ΔECF [By AA similarity criterion]

Thank You

MODULE : 25

EX.6.3 Q.7

Altitudes AD and CE of ΔABC intersect each other at the point P

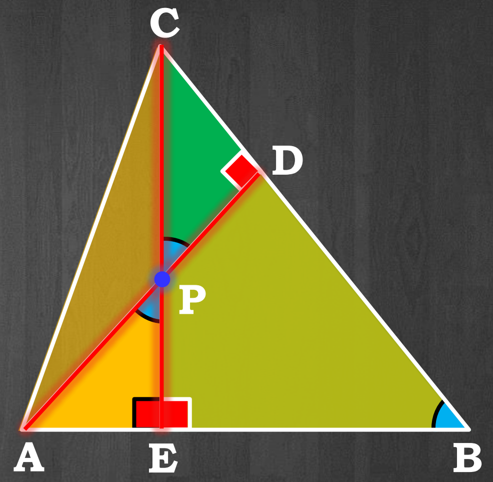
Show that :

(i) $\Delta AEP \sim \Delta CDP$

(ii) $\Delta ABD \sim \Delta CBE$

(iii) $\Delta AEP \sim \Delta ADB$

(iv) $\Delta PDC \sim \Delta BEC$



Proof : In ΔAEP and ΔCDP,

$\angle AEP = \angle CDP$ *[each is 90°]*

$\angle APE = \angle CPD$ *[vertically opposite angles]*

$\therefore \quad \Delta AEP \sim \Delta CDP$ *[by AA similarity criterion]*

In ΔABD and ΔCBE,

$\angle ADB = \angle CEB$ *[each is 90°]*

$\angle B = \angle B$ *[common angle]*

$\therefore \quad \Delta ABD \sim \Delta CBE$ *[by AA similarity criterion]*

EX.6.3 Q.7

Q. Given : Altitudes AD and CE of ΔABC intersect each other at the point P

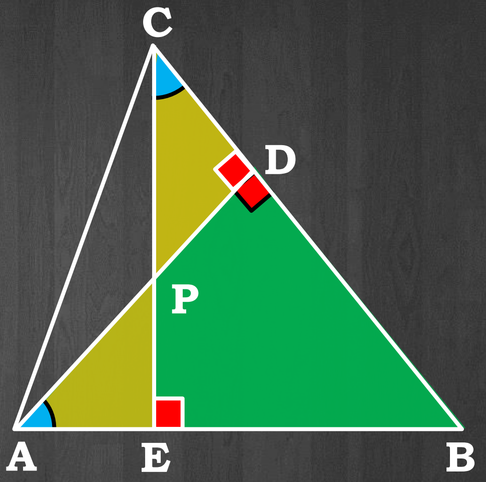
Show that :

✓(i) $\Delta AEP \sim \Delta CDP$

✓(ii) $\Delta ABD \sim \Delta CBE$

✓(iii) $\Delta AEP \sim \Delta ADB$

✓(iv) $\Delta PDC \sim \Delta BEC$



Proof : In ΔAEP and ΔADB,

$\angle AEP = \angle ADB$ *[each is 90°]*

$\angle PAE = \angle BAD$ *[common angle]*

$\therefore$ $\Delta AEP \sim \Delta ADB$ *[by AA similarity criterion]*

In ΔPDC and ΔBEC,

$\angle PDC = \angle BEC$ *[each is 90°]*

$\angle PCD = \angle BCE$ *[common angle]*

$\therefore$ $\Delta PDC \sim \Delta BEC$ *[by AA similarity criterion]*

MODULE : 26

Solved Example

If a line intersects sides AB and AC of a ΔABC at D and E respectively and is parallel to BC, prove that $\dfrac{AD}{AB} = \dfrac{AE}{AC}$

Sol:

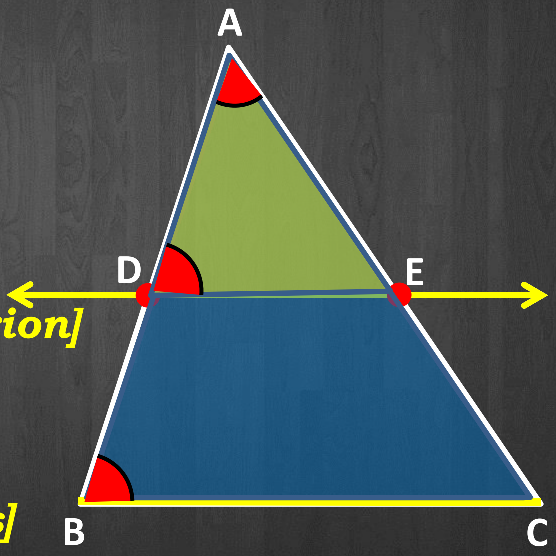
In ΔADE & ΔABC

$\angle ADE = \angle ABC$ ..... *[corresponding angles]*

$\angle DAE = \angle BAC$ ..... *[common angle]*

$\Delta ADE \sim \Delta ABC$ ..... *[by AA similarity criterion]*

$$\boxed{\frac{AD}{AB} = \frac{AE}{AC}}$$ ....(i) *[corresponding sides of similar triangles]*



EX.6.2 Q.3

If **LM || CB** and **LN || CD,** prove that $\frac{AM}{AB} = \frac{AN}{AD}$

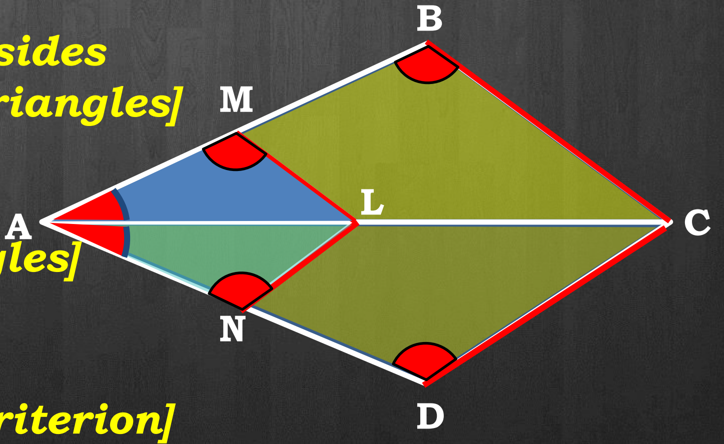
Sol: In ΔAML & ΔABC

$\angle AML = \angle ABC$ ..... *[corresponding angles]*

$\angle MAL = \angle BAC$ ..... *[common angle]*

$\Delta AML \sim \Delta ABC$ ..... *[by AA similarity criterion]*

$\frac{AM}{AB} = \frac{AL}{AC}$ ....(i) *[corresponding sides of similar triangles]*



In ΔANL & ΔADC

$\angle ANL = \angle ADC$ ..... *[corresponding angles]*

$\angle NAL = \angle DAC$ ..... *[common angle]*

$\Delta ANL \sim \Delta ADC$ ..... *[by AA similarity criterion]*

EX.6.2 Q.3

If LM ∥ CB and LN ∥ CD, prove that $\dfrac{AM}{AB} = \dfrac{AN}{AD}$

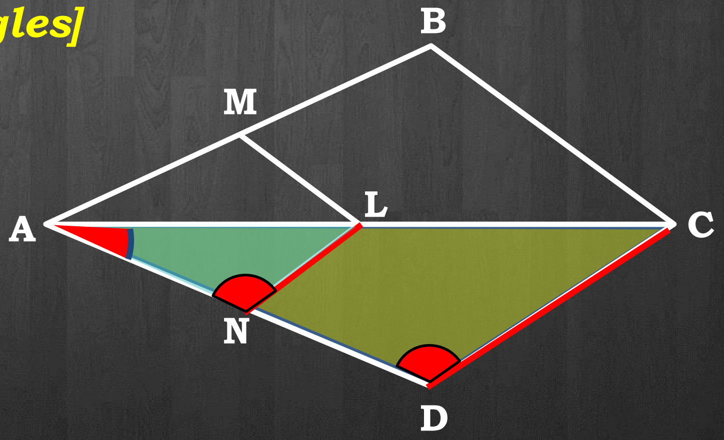
$$\boxed{\dfrac{AM}{AB} = \dfrac{AL}{AC}} \quad ....(i)$$

Sol:

$\Delta ANL \sim \Delta ADC$ ..... *[by AA similarity criterion]*

$$\boxed{\dfrac{AN}{AD} = \dfrac{AL}{AC}}$$ .....(ii) *[corresponding sides of similar triangles]*

$$\boxed{\dfrac{AM}{AB} = \dfrac{AN}{AD}}$$ ..... *[from (i) and (ii)]*



MODULE : 27

EX.6.3 Q.3

Diagonals AC and BD of a trapezium ABCD with AB || CD intersect each other at the point O. Using a similarity criterion for two triangles, Show that $\frac{OA}{OC} = \frac{OB}{OD}$

Proof : AB || DC

$\angle DCA = \angle CAB$ *[Alternate angles]*

$\angle DCO = \angle OAB$ *... (i)*

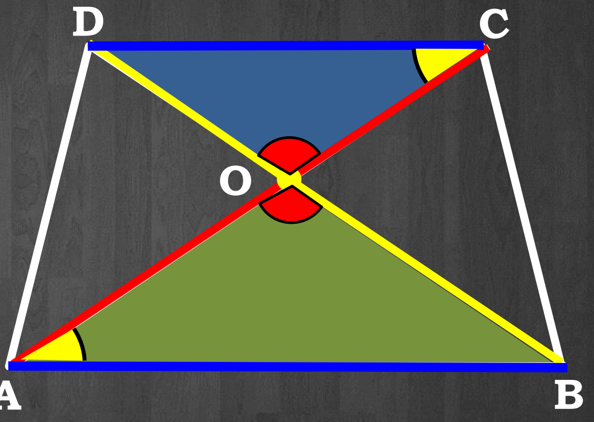
In ΔOAB and ΔOCD

$\angle AOB = \angle COD$ *[Vertically opposite angles]*

$\angle BAO = \angle OCD$ *[From i]*

$\therefore \quad \Delta OAB \sim \Delta OCD$ *[By AA criterion]*

$\therefore \quad \frac{OA}{OC} = \frac{OB}{OD}$ *[Corresponding sides of similar triangles are proportional]*

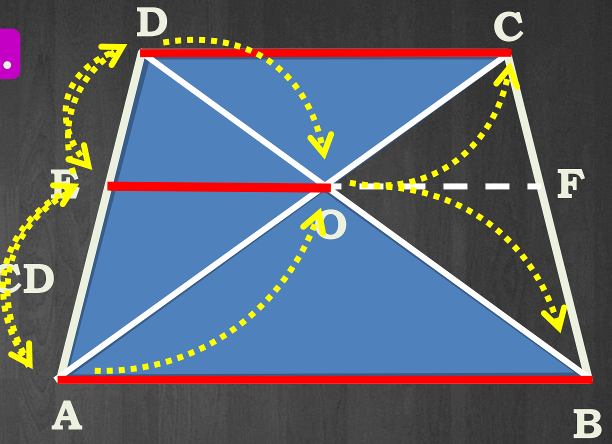


EX.6.2 Q.9

ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O.

Show that $\frac{AO}{BO} = \frac{CO}{DO}$

Construction :



Draw a line EF through point O, such that EF || CD

In ΔADC, EO || CD

$\therefore \frac{AE}{ED} = \frac{AO}{OC}$ ...(1)

In ΔABD, OE || AB

$\therefore \frac{ED}{AE} = \frac{OD}{BO}$

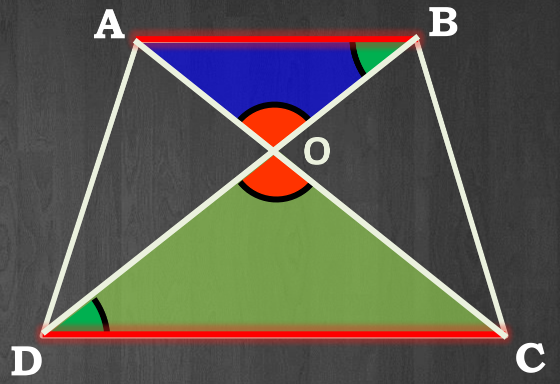
i.e $\frac{AE}{ED} = \frac{BO}{OD}$ ...(2)

$\therefore \frac{AO}{OC} = \frac{BO}{OD}$ ...[From (i) and (ii)]

MODULE : 28

EX.6.2 Q.10

The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$ show that ABCD is a trapezium



Proof : $\frac{AO}{BO} = \frac{CO}{DO}$

Hint : To prove AB ∥ BC

$\therefore \boxed{\frac{AO}{CO} = \frac{BO}{DO}}$ ...(i)

In Δ AOB and ΔCOD,

$\frac{AO}{CO} = \frac{BO}{DO}$ *[From (i)]*

$\angle AOB = \angle COD$ *[Vertically Opposite angles]*

$\therefore \Delta AOB \sim \Delta COD$ *[By SAS criterion]*

$\boxed{\angle ABO} = \boxed{\angle CDO}$ *[c.a.s.t.]*

$\therefore \angle ABD = \angle CDB$

But, these are a pair of alternate interior angles on transversal BD.

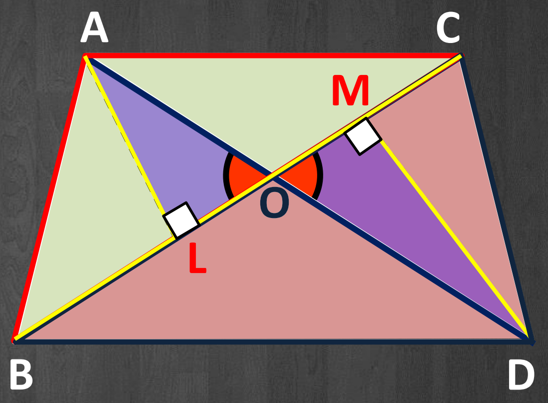
$\therefore$ AB ∥ DC

$\therefore$ ABCD is a trapezium.

MODULE : 29

EX.6.4 (Q.3)

ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that : $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AO}{DO}$.



Construction : Draw AL ⊥ BC and DM ⊥ BC.

Proof.

$$\frac{ar\ (\Delta ABC)}{ar\ (\Delta DBC)} = \frac{\cancel{\frac{1}{2}} \times \cancel{BC} \times AL}{\cancel{\frac{1}{2}} \times \cancel{BC} \times DM}$$

$$\frac{ar\ (\Delta ABC)}{ar\ (\Delta DBC)} = \frac{AL}{DM} \quad \text{.... (i)}$$

In ΔALO and ΔDMO

$\angle ALO = \angle DMO$ .... *[each 90°, construction]*

$\angle AOL = \angle DOM$ .... *[vertically opposite angles]*

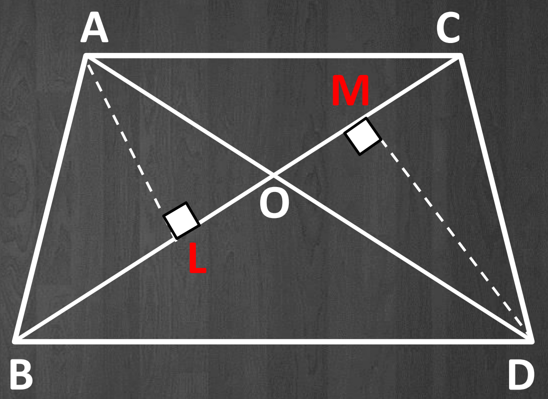
$\therefore\ \Delta ALO \sim \Delta DMO$ .... *[by AA similarity criterion]*

EX.6.4 (Q.3)

ABC and DBC are two triangles on the same base BC. If AD intersects BC at O,

show that : $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AO}{DO}$.

Proof.



$\Delta ALO \sim \Delta DMO$

$\therefore \dfrac{AO}{DO} = \dfrac{AL}{DM}$ .... *(ii)*

.... [corresponding sides of similar triangles.]

$\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AL}{DM}$ .... (i)

D

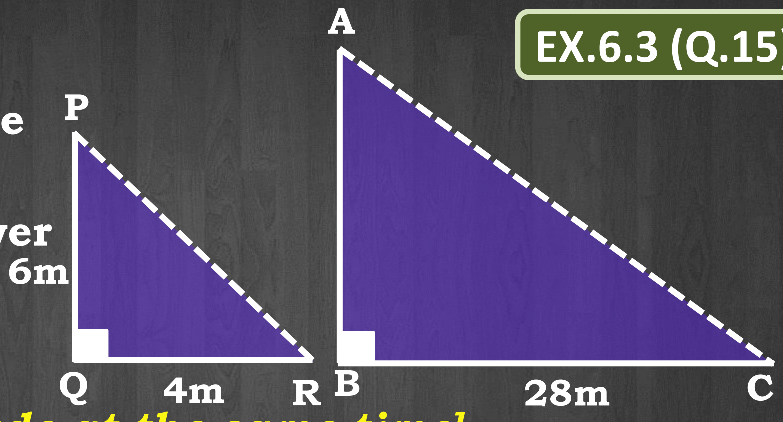
$\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AO}{DO}$ .... *from (i) & (ii)*

Thank You

MODULE : 30

Q. A vertical pole of length 6m casts a shadow 4m long on the ground and at the, same time a tower casts a shadow 28m Find the height of the tower.

EX.6.3 (Q.15)



Sol: PQ represents the pole

QR represents the shadow of the pole

AB represents the tower

BC represents the shadow of the tower

In ΔPQR and ΔABC,

$\angle PQR = \angle ABC$ .....*[each is 90°]*

$\angle PRQ = \angle ACB$ .....*[Sun's altitude at the same time]*

$\therefore \Delta PQR \sim \Delta ABC$ .....*[by AA similarity]*

$$\frac{PQ}{AB} = \frac{QR}{BC}$$ .....*[corresponding sides of similar triangles]*

$$\frac{6}{AB} = \frac{\cancel{4}}{\cancel{28}_{7}}$$

$$\frac{6}{AB} = \frac{1}{7}$$

$AB = 7 \times 6$

$\therefore AB = 42m$

$\therefore$ **Height of the tower is 42m**

MODULE : 31

A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Sol: Let AB denote the lamp-post and CD the girl after walking for 4 seconds away from the lamp-post. From the figure, you can see that DE is the shadow of the girl. Let DE be x metres.

Now, BD = 1.2 m × 4 = 4.8 m.

Note that in ΔABE and ΔCDE

$\angle B = \angle D$ (Each is of 90° because lamp-post as well as the girl are standing vertical to the ground)

and $\angle E = \angle E$ (Same angle)

So, $\Delta ABE \sim \Delta CDE$ (AA similarity criterion)

Therefore, $$\frac{BE}{DE} = \frac{AB}{CD}$$

i.e., $$\frac{4.8 + x}{x} = \frac{3.6}{0.9}$$ $\left(90 \text{ cm} = \frac{90}{100} \text{ m} = 0.9\text{m}\right)$

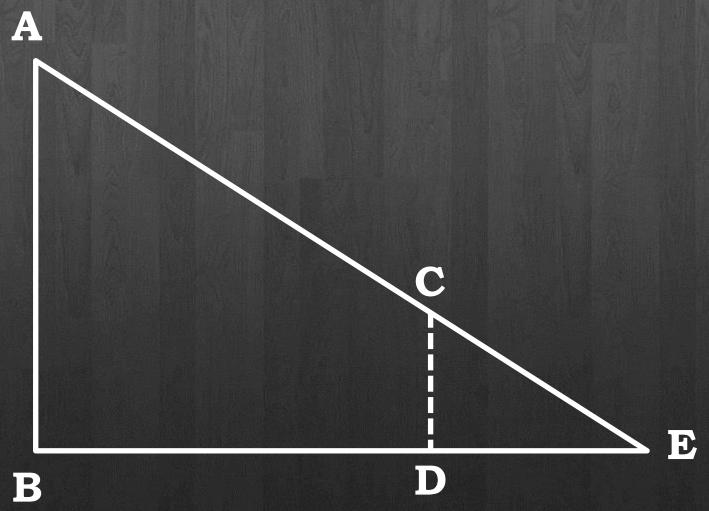
A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Sol: i.e., $4.8 + x = 4x$

i.e., $3x = 4.8$

i.e., $x = 1.6$

So, the shadow of the girl after walking for 4 seconds is 1.6 m long.



MODULE : 32

Q. Δ ABE $\cong$ Δ ACD

Show that : Δ ADE ~ Δ ABC

Δ ABE $\cong$ ΔACD

EX.6.3 (Q.6)

Proof : .... (given)

AB = AC .... (i)

AE = AD

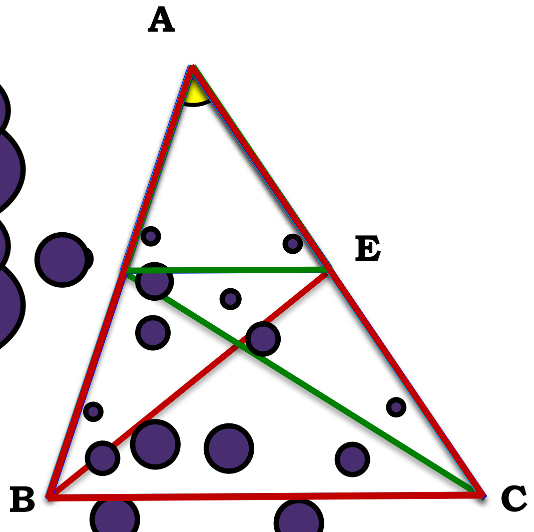
AD = AE .... (ii)

$\frac{AD}{AB} = \frac{AE}{AC}$ .... (iii)

If we have the ratio of corresponding sides equal,

Triangles are similar by which criterion ??

SAS



In Δ ADE & Δ ABC

S .... from (iii)

S

A

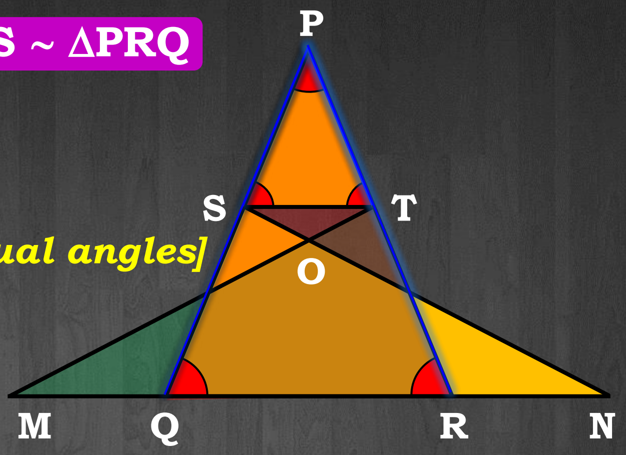
$\angle$A = $\angle$A .... (common angle)

$\therefore$ **Δ ADE ~ Δ ABC** .... (by SAS similarity crite

We took AD and AE because they belong to ΔADE

MODULE : 33

Q. $\angle PST = \angle PTS$, $\Delta NSQ \cong \Delta MTR$, Prove : $\Delta PTS \sim \Delta PRQ$



Proof : In ΔPST

$\angle PST = \angle PTS$ *[Given]*

$\therefore$ $PS = PT$ ...(i) *[Side opposite to equal angles]*

$\Delta NSQ \cong \Delta MTR$ *[Given]*

$\therefore$ $\angle SQN = \angle TRM$ ...(ii) *[c.p.c.t]*

In ΔPQR

$\angle PQR = \angle PRQ$ *[From (ii)]*

$\therefore$ $PQ = PR$ ...(iii) *[Side opposite to equal angles]*

In ΔPTS and ΔPRQ

$\frac{PS}{PQ} = \frac{PT}{PR}$ *[Dividing (i) and (iii)]*

$\angle P = \angle P$ *[common angle]*

$\therefore$ $\Delta PTS \sim \Delta PRQ$ *[SAS criterion]*

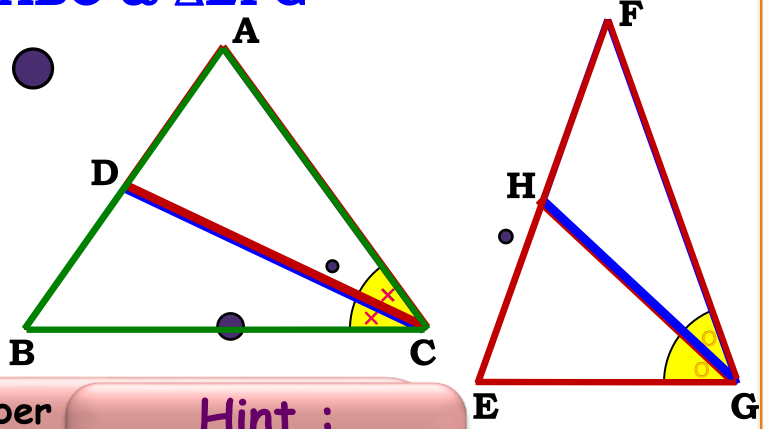
MODULE : 34

EX.6.3 (Q.10)

Q. CD & GH are respectively the bisectors of $\angle ACB$ & $\angle EGF$ such that D & H lie on sides AB & FE of ΔABC & ΔEFG respectively. $\Delta ABC \sim \Delta FEG$.

Prove that : i) $\frac{CD}{GH} = \frac{AC}{FG}$ ii) $\Delta DCB \sim \Delta HGE$

iii) Δ



Soln.

$\Delta ABC \sim$

$\angle ACB$

$\therefore \frac{1}{2}\angle A$

To prove:

$\frac{CD}{GH} = \frac{AC}{FG}$

$\angle EGH = \angle FGH = \frac{1}{2}\angle FGE$

Does GH & FG form one

Which triangle ??

ΔHGF

Multiplying both sides by $\frac{1}{2}$

Remember

Whenever we

products in th

➢ Numerator

triangle and

denominator should form

other triangle.

Hint :

To prove:

$\sim$

$\angle BCD = \angle ACD = \frac{1}{2}\angle ACB$

B]

.. [GH bis

$\angle ACB]$

Does CD & AC form one

Which triangle ??

ΔDCA

$\angle EGH = \angle FGH = \frac{1}{2}\angle FGE$

$\frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$ (i)

$\angle BCD = \angle ACD = \frac{1}{2}\angle ACB$ (ii)

$\angle EGH = \angle FGH = \frac{1}{2}\angle FGE$ (iii)

EX.6.3 (Q.10)

$=$ ... (iv) ... from (i), (ii), (iii)

$\angle B = \angle E$ ... (v) ...[corresponding angles of similar triangles]

$\angle A = \angle F$ ... (vi)

In ΔADC & ΔFHG,

$\angle ACD = \angle FGH$ ... from(iv)

... from (vi)

$\Delta DCA \sim \Delta HGF$ ... [by AA similarity criterion]

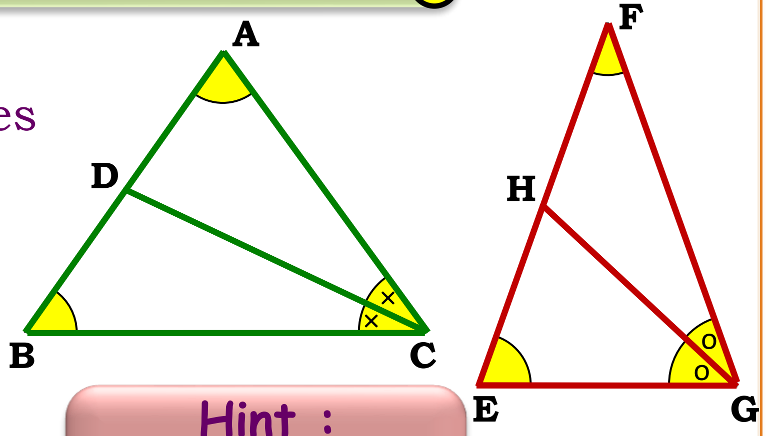
$\frac{CD}{GH} = \frac{AC}{FG}$ ... [corresponding sides of similar triangles]

In ΔDBC & ΔHEG,

$=$ ... from (i), (ii), (iii)

... from (v)

$\Delta DCB \sim \Delta HGE$ ... [by AA similarity criterion]



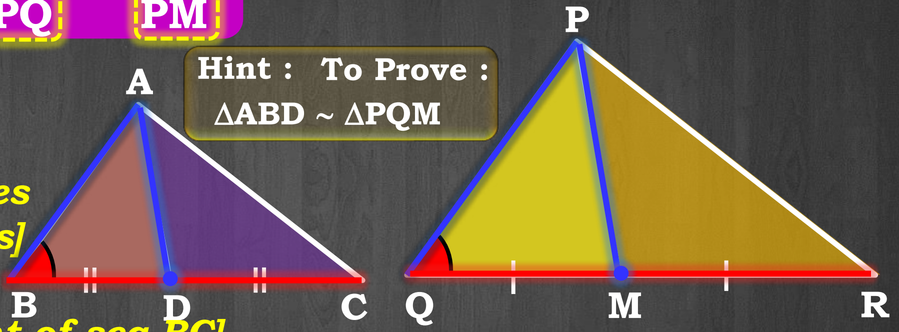
Hint :
To prove:
Δ DCA ~ Δ HGF

Thank You

MODULE : 35

EX.6.3 (Q.16)

If AD and PM are medians of triangles ABC and PQR respectively where $\Delta ABC \sim \Delta PQR$, prove that $\dfrac{AB}{PQ} = \dfrac{AD}{PM}$



Hint : To Prove : $\Delta ABD \sim \Delta PQM$

Proof : $\Delta ABC \sim \Delta PQR$ [Given]

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR}$ *...(i) [corresponding sides of similar triangles]*

$BC = 2BD$ *...(ii) [D is the midpoint of seg BC]*

$QR = 2QM$ *...(iii) [M is the midpoint of seg QR]*

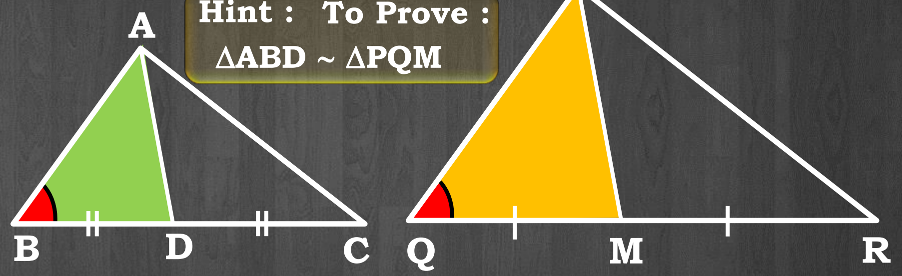
$\therefore \dfrac{AB}{PQ} = \dfrac{\cancel{2}BD}{\cancel{2}QM}$ *[From (i), (ii), (iii)]*

$\therefore \dfrac{AB}{PQ} = \dfrac{BD}{QM}$ *...(iv)*

$\angle B = \angle Q$ *...(v) [corresponding angles of similar triangles]*

EX.6.3 (Q.16)

Q. If AD and PM are medians of triangles ABC and PQR respectively where $\Delta ABC \sim \Delta PQR$, prove that $\dfrac{AB}{PQ} = \dfrac{AD}{PM}$



Hint : To Prove : $\Delta ABD \sim \Delta PQM$

Proof : $\therefore \dfrac{AB}{PQ} = \dfrac{BD}{QM}$ ...(iv)

$\angle B = \angle Q$ ...(v)

In ΔABD and ΔPQM,

$\dfrac{AB}{PQ} = \dfrac{BD}{QM}$ [From (iv)]

$\angle B = \angle Q$ [From (v)]

$\therefore \Delta ABD \sim \Delta PQM$ [by SAS similarity criterion]

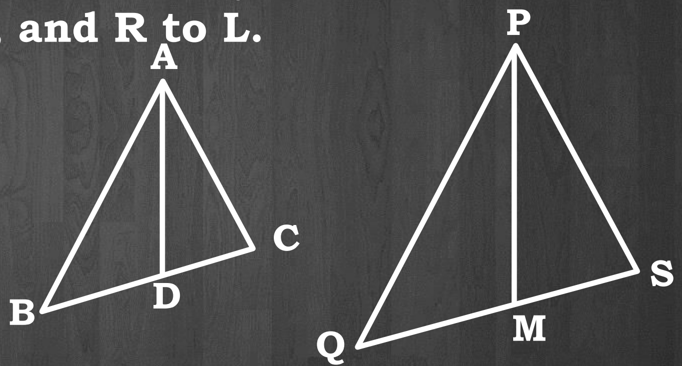
$\therefore \dfrac{AB}{PQ} = \dfrac{AD}{PM}$ [corresponding sides of similar triangles]

MODULE : 36

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

Sol: Given that, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

Let us extent AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

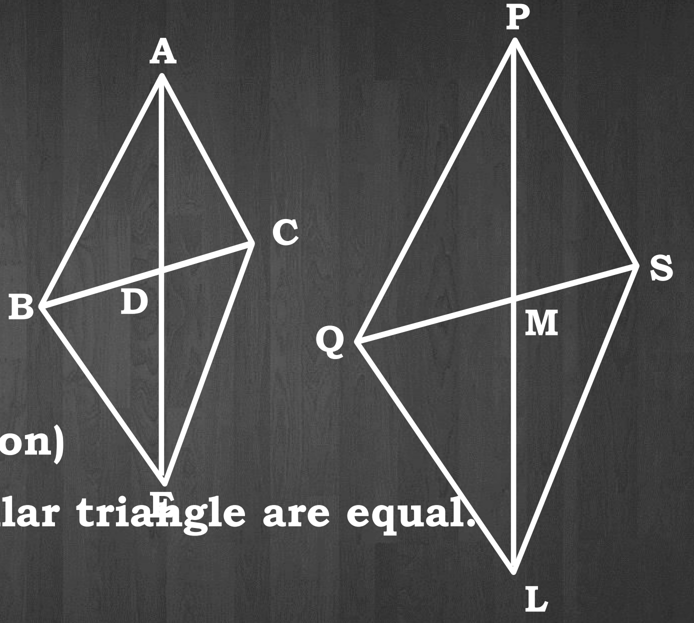
Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

Sol:

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$



$\therefore \quad \Delta ABE \sim \Delta PQL$ (by SSS similarity criterion)

We know that corresponding angles of similar triangle are equal.

$\therefore \quad \angle BAE = \angle QPL \quad \ldots (1)$

Similarly, it can be proved that $\Delta AEC \sim \Delta PLR$ and

$\therefore \quad \angle CAE = \angle RPL \quad \ldots (2)$

Adding equation (1) and (2), we obtain

$\therefore \quad \angle BAE + \angle CAE = \angle QPL + \angle RPL$

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Sol: $\Rightarrow \angle CAB = \angle RPQ$ ... (3)

In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ Given

$\angle CAB = \angle RPQ$ [using equation (3)]

$\Delta ABC \sim \Delta PQR$ (By SAS similarity criterion)

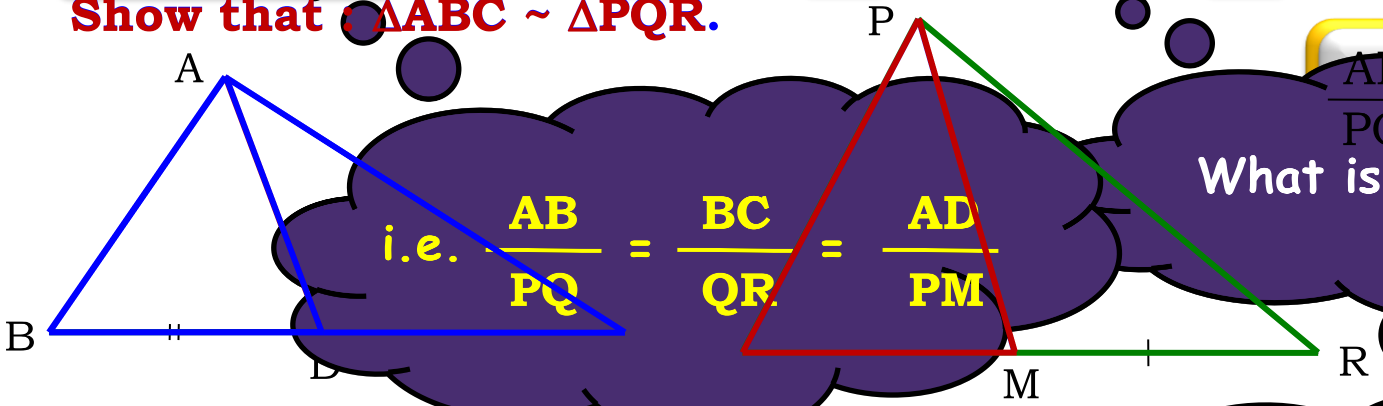
MODULE : 37

Q. Given :

Sides AB and BC and median AD of ΔABC are respectively proportional to sides PQ and QR and median PM of ΔPQR.

Show that : ΔABC ~ ΔPQR.

EX.6.3 (Q.14)



i.e. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

What is a median ??

Do PQ, QM and PM form one triangle ??

ΔPQM

Soln.

BC = 2BD

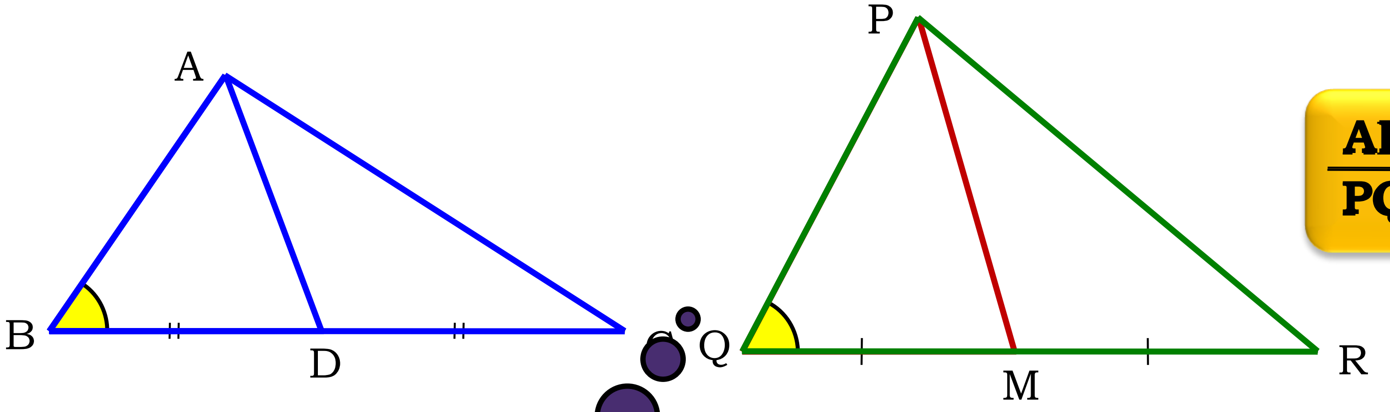
QR = 2QM

$\therefore \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$

...(i)

...(iv)

.... from ... (iii)



EX.6.3 (Q.14)

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

In ΔABD & ΔPQM

S

Triangles are similar
by which criterion ??

SAS criterion

.... [on]

$\angle B = \angle Q$ ...(v)

.... [corresponding angles of similar triangles]

In ΔABC & ΔPQR

S A S

n (i)

om (v)

Triangles are similar
by which criterion ??

SSS criterion

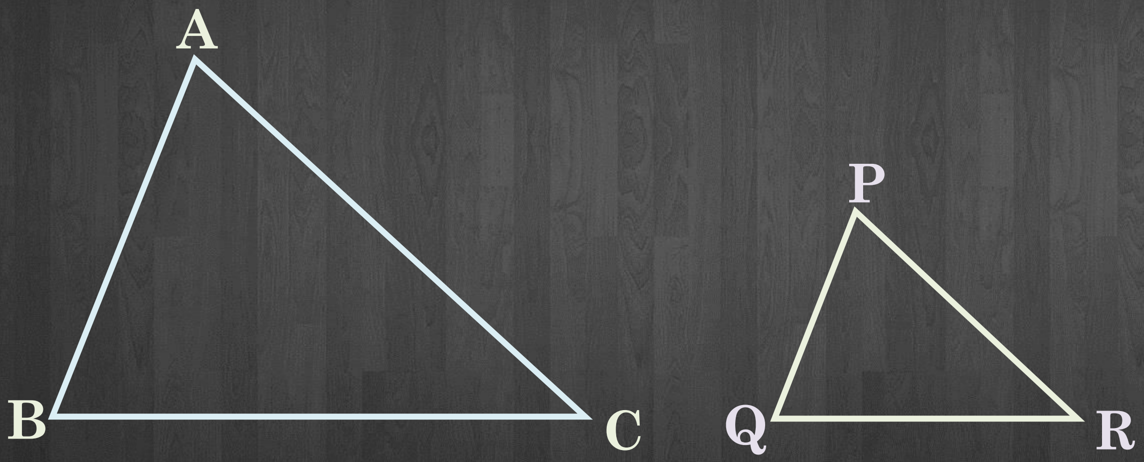
∴

.... [by SAS similarity criterion]

MODULE : 38

AREAS OF SIMILAR TRIANGLES

The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.



$\Delta ABC \sim \Delta PQR$

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

EX.6.4 (Q.1)

Q. Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, $64 cm^2$ and $121 cm^2$. If EF = 15.4cm, find BC.

Soln. $\Delta ABC \sim \Delta DEF$ ... (given)

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$ ... The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides

$$\therefore \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\therefore \frac{8}{11} = \frac{BC}{15.4}$$ [Taking square root]

$$\therefore BC = \frac{8 \times 15.4}{11}$$

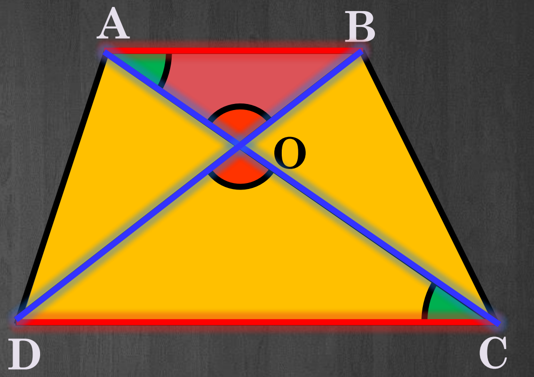
$$\therefore BC = 8 \times 1.4$$

$$\therefore \boxed{BC = 11.2 \text{ cm}}$$

MODULE : 39

EX.6.4 (Q.2)

Q. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.



To Find - ar (AOB) : ar (COD)

Proof : In ΔAOB and ΔCOD,

$\angle BAO = \angle DCO$ [Alternate angles]

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\therefore$ $\Delta AOB \sim \Delta COD$ [by AA similarity criterion]

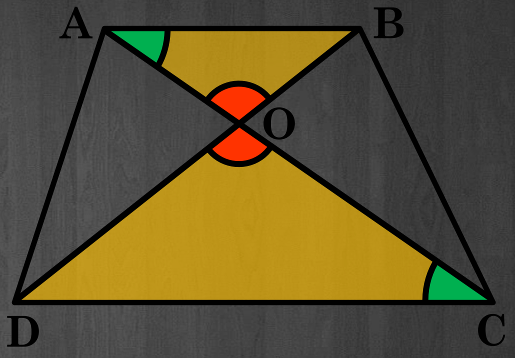
$$\therefore \frac{ar\,(AOB)}{ar\,(COD)} = \frac{AB^2}{CD^2} \quad \ldots(i)$$

[The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides]

$$\therefore \frac{ar\,(AOB)}{ar\,(COD)} = \left(\frac{AB}{CD}\right)^2$$

EX.6.4 (Q.2)

Q. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.



To Find - ar (AOB) : ar (COD)

Proof :

$$\frac{ar\,(AOB)}{ar\,(COD)} = \left(\frac{AB}{CD}\right)^2$$

$$\therefore \frac{ar\,(AOB)}{ar\,(COD)} = \left(\frac{2\,\cancel{CD}}{\cancel{CD}}\right)^2 \quad [\because AB = 2CD]$$

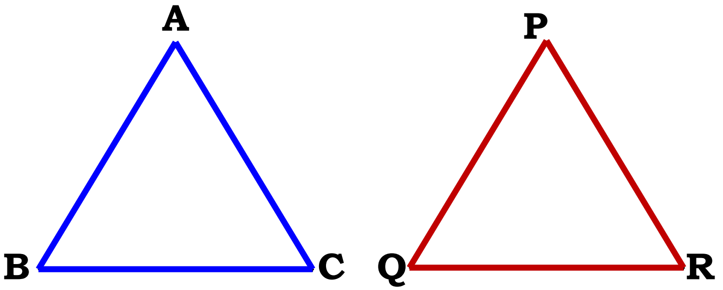
$$\therefore \frac{ar\,(AOB)}{ar\,(COD)} = \frac{4}{1}$$

$$\therefore \boxed{ar\,(AOB) : ar\,(COD) = 4 : 1}$$

Thank You

MODULE : 40

Q. If the areas of two similar triangles are equal then prove that they are congruent. EX.6.4 (Q.4)



Given :

$\Delta ABC \sim \Delta PQR$

$ar(\Delta ABC) = ar(\Delta PQR)$

To Prove : $\Delta ABC \cong \Delta PQR$

Proof.

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} \quad [i]$$ … [given] …

The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides

But … [given]

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = 1 \quad [ii]$$

$$\therefore \frac{1}{1} =$$ … from [i] & [ii]

EX.6.4 (Q.4)

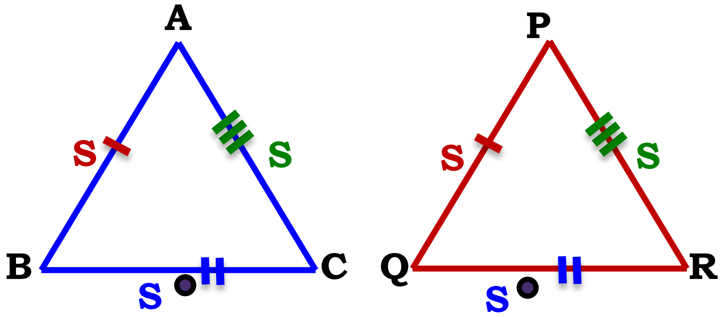
$$1 = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

$\therefore$ = , = , =

$\therefore$ $AB^2 = PQ^2$, $BC^2 = QR^2$, $AC^2 = PR^2$

$\therefore$ $AB = PQ$, $BC = QR$, $AC = PR$

$\therefore$ $\boxed{\Delta ABC \cong \Delta PQR}$ ... [by SSS congruency]



Taking square roots

Triangles are congruent by which test ??

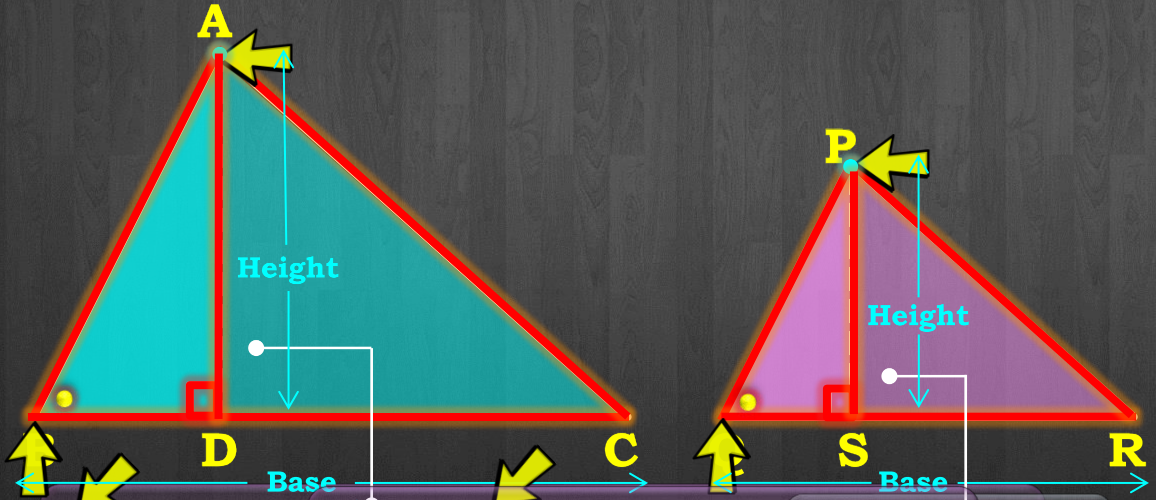
SSS

MODULE : 41

THEOREM : Areas of Similar Triangles

Given : $\Delta ABC \sim \Delta PQR$

To prove : $\dfrac{ar\,(ABC)}{ar\,(PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{AC^2}{PR^2}$



The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides

Hint : (i) Prove $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$ (ii) Draw AD $\perp$ BC, PS $\perp$ QR $\dfrac{AB}{PQ} = \dfrac{AD}{PS}$

Proof :

$$\frac{ar\,(ABC)}{ar\,(PQR)} = \frac{\tfrac{1}{2} \times BC \times AD}{\tfrac{1}{2} \times QR \times PS}$$

$$\frac{ar\,(ABC)}{ar\,(PQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$$

$\Delta ABC \sim \Delta PQR$

$$\boxed{\frac{AB}{PQ} = \frac{BC}{QR}}$$

$\angle B = \angle Q$

$\Delta ADB \sim \Delta PSQ$

$$\boxed{\frac{AD}{PS} = \frac{AB}{PQ}}$$

$$\frac{ar\,(ABC)}{ar\,(PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$$

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$$

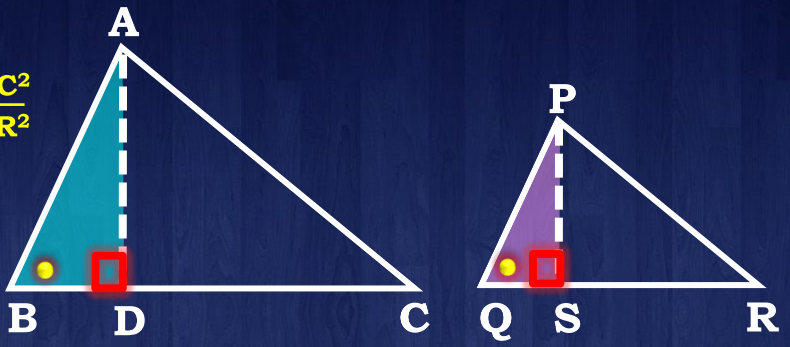
THEOREM : Areas of Similar Triangles

Given : $\Delta ABC \sim \Delta PQR$

To prove : $\dfrac{A(\Delta ABC)}{A(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{AC^2}{PR^2}$

Construction : Draw $AD \perp BC$

and $PS \perp QR$



Hint : Prove :

(i) $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$ (ii) $\dfrac{AB}{PQ} = \dfrac{AD}{PS}$

Proof :

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$$

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC}{QR} \times \frac{AD}{PS} \quad \text{...(i)}$$

$\Delta ABC \sim \Delta PQR$ [Given]

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad \text{...(ii) [c.s.s.t]}$$

$\therefore \angle B = \angle Q$ ...(iii) [c.a.s.t]

In ΔADB and ΔPSQ

$\angle ADB = \angle PSQ$

$\angle B = \angle Q$ [From (iii)]

$\therefore \Delta ADB \sim \Delta PSQ$ [By AA similarity criterion]

$$\therefore \frac{AD}{PS} = \frac{AB}{PQ} \quad \text{...(iv)}$$

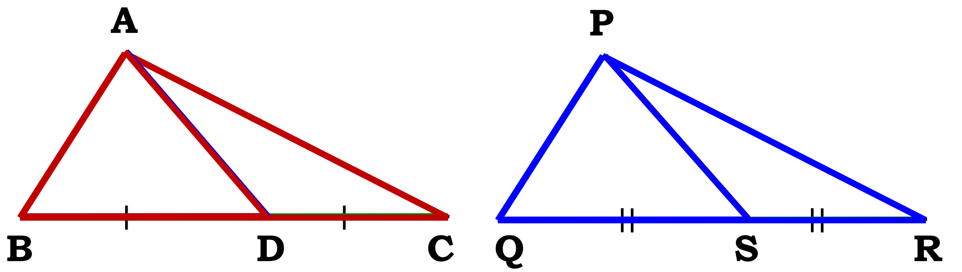
$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ} \quad \text{[From (i), (ii) \& (iv)]}$$

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} \quad \text{...(v)}$$

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} \quad \text{[From (ii) \& (v)]}$$

MODULE : 42

Q. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians. **EX.6.4 (Q.6)**



Given : $\Delta ABC \sim \Delta PQR$, AD & PS are their corresponding medians.

Prove that : $\dfrac{ar\ (\Delta ABC)}{ar\ (\Delta PQR)} = \dfrac{AD^2}{PS^2}$

Proof. $\Delta ABC \sim \Delta PQR$ ....[given]

$\therefore \quad = \dfrac{AB^2}{PQ^2}$ ...[i] .... (The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides)

$\dfrac{AB}{PQ} = \dfrac{BC}{QR}$ ... [ii] .... [corresponding sides of similar triangles]

$BC = 2BD$ ... [iii]

.... [$\because$ D is mid point of BC]

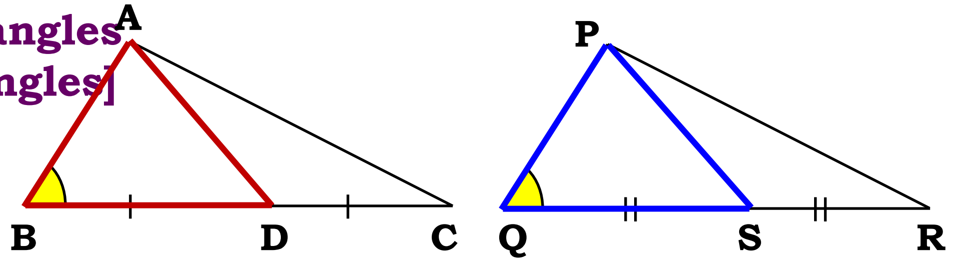
$QR = 2QS$ ... [iv]

.... [$\because$ S is mid point of QR]

EX.6.4 (Q.6)

$$\therefore \frac{AB}{PQ} = \frac{\cancel{2}BD}{\cancel{2}QS}$$ ....[v] ....from [ii] [iii] and [iv]

$\angle B = \angle Q$ ....[vi] ....[corresponding angles of similar triangles]



In ΔABD and ΔPQS

....from[v]

....from[vi]

$\Delta ABD \sim \Delta PQS$ ....[by SAS similarity criterion]

$$\frac{AB}{PQ} = \frac{AD}{PS}$$ ....[corresponding sides of similar triangles]

$$\therefore \frac{AB^2}{PQ^2} = \frac{AD^2}{PS^2}$$ ....[vii]

$$\therefore \boxed{\frac{ar\ (\Delta ABC)}{ar\ (\Delta PQR)} = \frac{AD^2}{PS^2}}$$ ....from [i] and [vii]

MODULE : 43

RESULTS RELATED TO THE RATIO OF THE AREAS OF TWO SIMILAR TRIANGLES

RESULT – 1

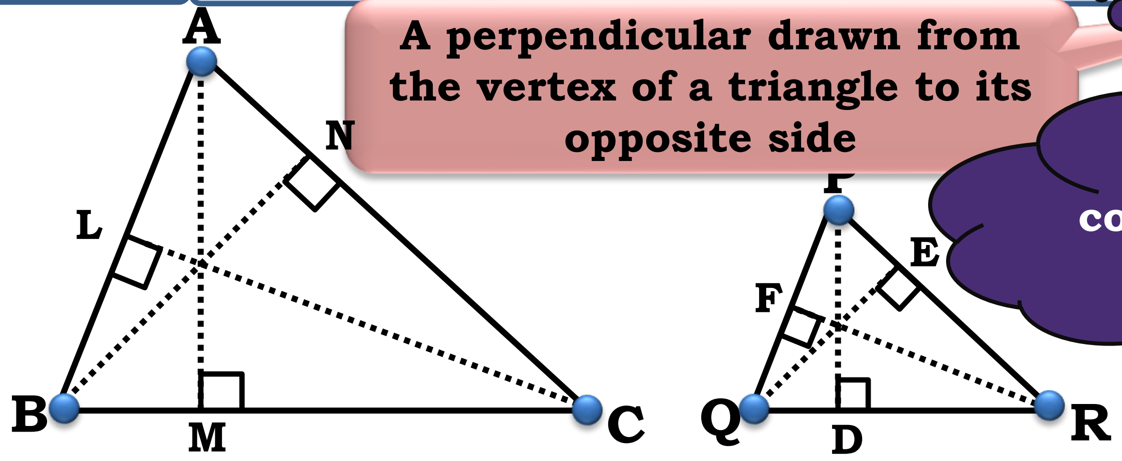
The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes.

A perpendicular drawn from the vertex of a triangle to its opposite side

AM ↔ PD

BN ↔ QE

CL ↔ RF



$\Delta ABC \sim \Delta PQR$

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(AM)^2}{(PD)^2} = \frac{(BN)^2}{(QE)^2} = \frac{(CL)^2}{(RF)^2}$$

RESULTS RELATED TO THE RATIO OF THE AREAS OF TWO SIMILAR TRIANGLES

RESULT – 2

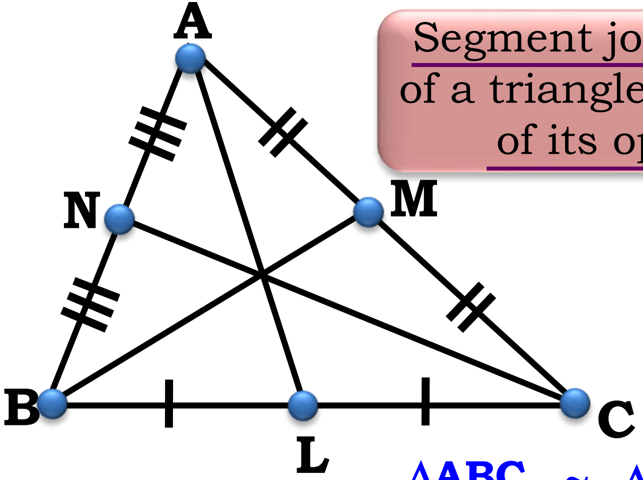
The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding medians.

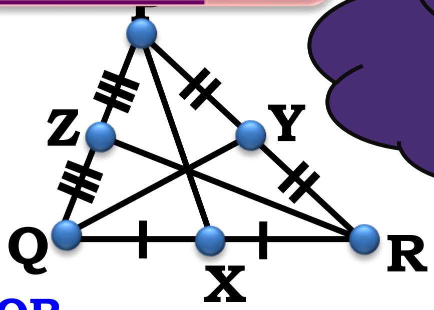
Segment joining the vertex of a triangle to the midpoint of its opposite side

$AL \leftrightarrow PX$

$BM \leftrightarrow QY$

$CN \leftrightarrow RZ$



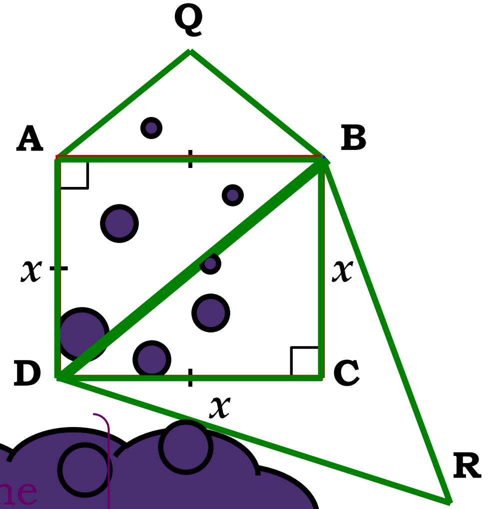


$\Delta ABC \sim \Delta PQR$

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(AL)^2}{(PX)^2} = \frac{(BM)^2}{(QY)^2} = \frac{(CN)^2}{(RZ)^2}$$

MODULE : 44

Q. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.



Prove that : $\text{ar}(\Delta AQB) = \dfrac{1}{2}\,\text{ar}(\Delta BRD)$

Let AB = x

Proof : ... [two equilateral triangles are always similar]

$$\therefore \frac{\text{ar}(\Delta AQB)}{\text{ar}(\Delta BRD)} = \frac{AB^2}{BD^2} \quad \ldots\text{(i)}$$

[The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides]

□ABCD is a square ...

Let side of square be x units

$\therefore$ AB = BC = CD = AD = x units

ΔAQB is an

Ratio of the areas of two similar triangles = Ratio of the squares of their corresponding sides

We considered AB and ... find the relation between them

$$\frac{\text{ar }(\Delta AQB)}{\text{ar }(\Delta BRD)} = \frac{AB^2}{BD^2} \quad \ldots\text{(i)}$$

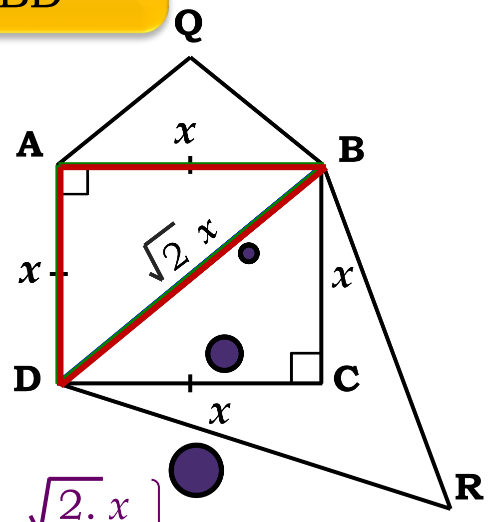
In ΔBAD

$\therefore \angle BAD = 90^o$ ...[angle of a square]

$\therefore BD^2 = AB^2 + AD^2$ ...[Pythagoras theorem]

$\therefore BD^2 = \underline{\quad}^2 + \underline{\quad}^2$

$\therefore BD^2 = 2x^2$ ...(ii)



$[\because AB = x,\ BD = \sqrt{2}.\,x]$

$$\therefore \frac{\text{ar }(\Delta AQB)}{\text{ar }(\Delta BRD)} = \frac{\cancel{x^2}}{2.\cancel{x^2}}$$

$$\therefore \frac{\text{ar }(\Delta AQB)}{\text{ar }(\Delta BRD)} = \frac{1}{2}$$

$$\therefore \boxed{\textbf{ar }(\Delta AQB) = \frac{1}{2}\ \textbf{ar }(\Delta BRD)}$$

Lets apply Pythagoras theorem to any one triangle

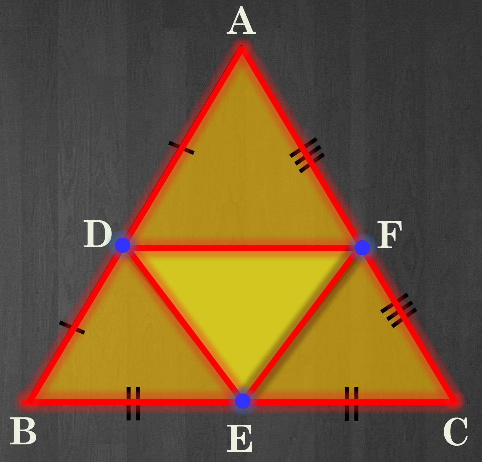
By Pythagoras theorem

Thank You

MODULE : 45

EX.6.4 (Q.5)

Q. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.



To Find - ar (DEF) : ar(ABC)

Proof : In ΔABC

D, E and F are midpoints of sides AB, BC and CA respectively [Given]

By Midpoint theorem,

$$DF = \frac{1}{2} BC$$

$$\therefore \boxed{\frac{DF}{BC} = \frac{1}{2}} \quad ...(i)$$

$$DE = \frac{1}{2} CA$$

$$\therefore \boxed{\frac{DE}{CA} = \frac{1}{2}} \quad ...(ii)$$

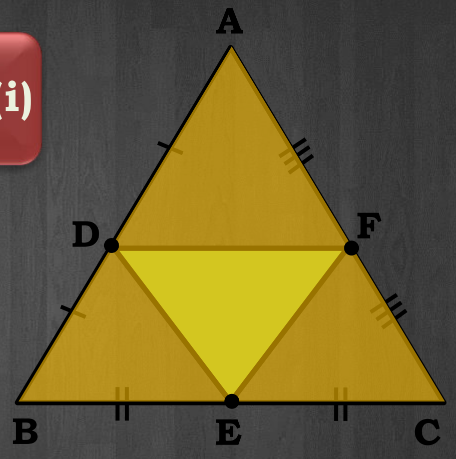
$$EF = \frac{1}{2} AB$$

$$\therefore \boxed{\frac{EF}{AB} = \frac{1}{2}} \quad ...(iii)$$

$$\therefore \frac{DF}{BC} = \frac{DE}{CA} = \frac{EF}{AB} \quad ...(iv)$$

[From (i), (ii) and (iii)]

Q. D, E and F are respectively the mid-points of sides AB, BC and CA of DABC. Find the ratio of the areas of ΔDEF and ΔABC.



To Find - ar (DEF) : ar(ABC)

$$\frac{DF}{BC} = \frac{1}{2} \quad \ldots(i)$$

Proof : In ΔDEF and ΔCAB

$$\frac{DF}{BC} = \frac{DE}{CA} = \frac{EF}{AB} \quad \text{[From (iv)]}$$

$\therefore$ ΔDEF ~ ΔCAB [by SSS similarity criterion]

$$\therefore \frac{ar(DEF)}{ar(ABC)} = \frac{DF^2}{BC^2} \quad \ldots(i)$$

The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides

$$\therefore \frac{ar(DEF)}{ar(ABC)} = \left(\frac{1}{2}\right)^2 \quad \text{[From (i)]}$$

$$\therefore \frac{ar(DEF)}{ar(ABC)} = \frac{1}{4}$$

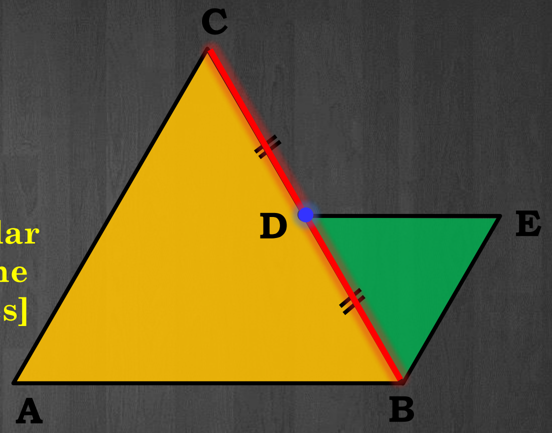
$\therefore$ **ar (DEF) : ar(ABC) = 1 : 4**

MODULE : 46

EX.6.4 (Q.8)

Q. ΔABC and ΔBDE are two equilateral triangles such that D is the midpoint of BC. Ratio of the areas of triangles ABC and BDE is

(A) 2 : 1 (B) 1 : 2 (C) 4 : 1 (D) 1 : 4.



To find : - ar (ABC) : (BDE)

Sol. Equilateral triangles are similar

$\Delta ABC \sim \Delta BDE$

$$\therefore \frac{ar\,(ABC)}{ar\,(BDE)} = \frac{BC^2}{BD^2}$$ [The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides]

$$\therefore \frac{ar\,(ABC)}{ar\,(BDE)} = \left(\frac{\boxed{BC}}{BD}\right)^2 \quad \text{...(i)}$$

$$\boxed{BC = 2BD} \quad \text{...(ii) [D is mid point of BC]}$$

$$\therefore \frac{ar\,(ABC)}{ar\,(BDE)} = \left(\frac{2\cancel{BD}}{\cancel{BD}}\right)^2 \quad \text{[From (i) and (ii)]}$$

$$\therefore \frac{ar\,(ABC)}{ar\,(BDE)} = \frac{4}{1}$$

$\therefore$ **ar (ABC) : (BDE) = 4 : 1**

EX.6.4 (Q.9)

Q. If sides of two similar triangles are in the ratio 4 : 9 then areas of these triangles are in the ratio ....

(A) 2 : 3 (B) 4 : 9 (C) 81 : 16 (D) 16 : 81

Sol. Let A_1 and A_2 be the areas of two similar triangles and S_1 and S_2 be their corresponding sides.

The two triangles are similar [Given]

$$\therefore \quad \boxed{\frac{S_1}{S_2} = \frac{4}{9}} \quad \text{...(i) [Given]}$$

$$\therefore \quad \frac{A_1}{A_2} = \boxed{\left(\frac{S_1}{S_2}\right)^2}$$

[The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides]

$$\therefore \quad \frac{A_1}{A_2} = \left(\frac{4}{9}\right)^2$$

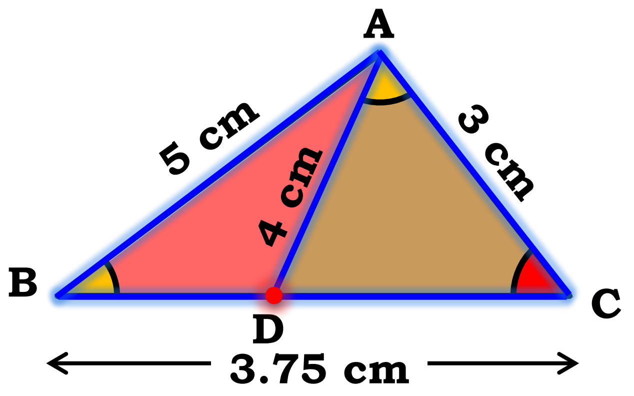
[From (i)]

$$\therefore \quad \frac{A_1}{A_2} = \frac{16}{81}$$

$$\therefore \quad \boxed{A_1 : A_2 = 16 : 81}$$

MODULE : 47

In the adjoining figure, D is a point on BC such that $\angle ABD = \angle CAD$. If $AB = 5$ cm, $AD = 4$ cm and $AC = 3$ cm. Find (i) BC, (ii) DC, (iii) $A(\Delta ACD) : A(\Delta BCA)$



Proof : In ΔABC and ΔDAC,

$\angle ABC = \angle DAC$ [Given]

$\angle ACB = \angle DCA$ [Common angle]

$\therefore \Delta ABC \sim \Delta DAC$ ...(i) [By AA criterion]

$\therefore \dfrac{AB}{DA} = \dfrac{BC}{AC} = \dfrac{AC}{DC}$

$\therefore \boxed{\dfrac{5}{4} = \dfrac{BC}{3}} = \dfrac{3}{DC}$ ...(ii)

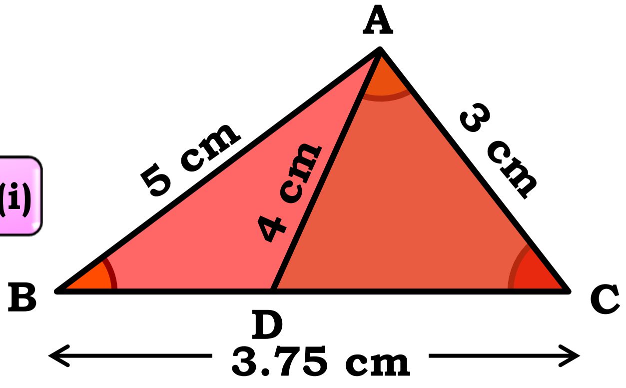
$\therefore \dfrac{5}{4} = \dfrac{BC}{3}$ [From (ii)]

$\therefore 5 \times 3 = BC \times 4$

$\therefore BC = \dfrac{15}{4}$

$\therefore \boxed{BC = 3.75 \text{ cm}}$

In the adjoining figure, D is a point on BC such that ∠ABD = ∠CAD. If AB = 5 cm, AD = 4 cm and AC = 3 cm. Find (i) BC, (ii) DC, (iii) A(ΔACD) : A(ΔBCA)



Proof : $\boxed{\dfrac{5}{4}} = \dfrac{BC}{3} = \boxed{\dfrac{3}{DC}}$ ...(ii)

$\Delta ABC \sim \Delta DAC$ ...(i)

$\therefore \quad \dfrac{5}{4} = \dfrac{3}{DC}$ [From (ii)]

$\therefore \quad 5 \times DC = 3 \times 4$

$\therefore \quad DC = \dfrac{12}{5}$

$\therefore \quad \boxed{DC = 2.4 \text{ cm}}$

(iii) $\Delta DAC \sim \Delta ABC$ [From (i)]

$\therefore \quad \dfrac{ar(\Delta DAC)}{ar(\Delta ABC)} = \dfrac{\boxed{AD^2}}{\boxed{AB^2}}$

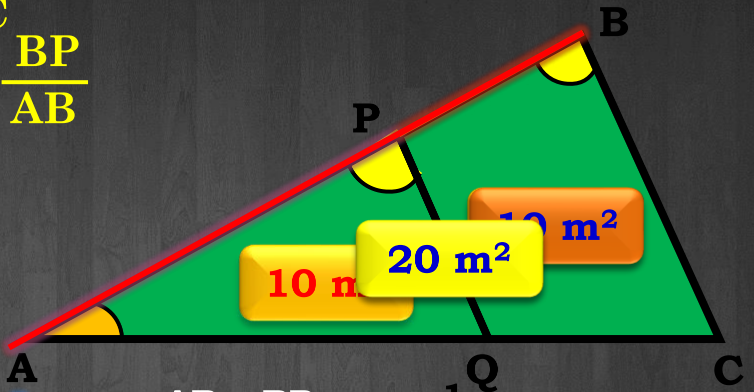
$\therefore \quad \dfrac{A(\Delta ACD)}{A(\Delta BCA)} = \dfrac{(4)^2}{(5)^2}$

$\therefore \quad \dfrac{A(\Delta ACD)}{A(\Delta BCA)} = \dfrac{16}{25}$

$\therefore \quad \boxed{A(\Delta ACD) : A(\Delta BCA) = 16 : 25}$

MODULE : 48

Q. In ΔABC, PQ is a line segment intersecting AB at P and AC at Q such that $PQ \parallel BC$. If PQ divides ΔABC into two equal parts equal in area. Find : $\frac{BP}{AB}$



Sol.

$$ar(APQ) = \frac{1}{2}\, ar(ABC)$$

$$\therefore \quad \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{2} \quad .....(i)$$

In ΔAPQ and ΔABC

$$\angle A = \angle A$$

$$\angle APQ = \angle ABC$$

$$\therefore \quad \Delta APQ \sim \Delta ABC$$

$$\therefore \quad \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2}$$

$$\therefore \quad \frac{1}{2} = \frac{AP^2}{AB^2} \quad \text{[from (i)]}$$

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{AP}{AB}$$

$$\therefore \quad \frac{AP}{AB} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad \frac{AB - BP}{AB} = \frac{1}{\sqrt{2}}$$

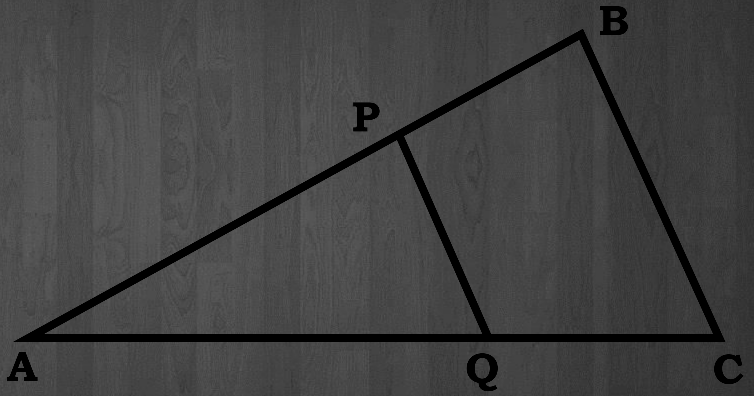
$$\therefore \quad \cancel{\frac{AB}{AB}} - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad 1 - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad 1 - \frac{1}{\sqrt{2}} = \frac{BP}{AB}$$

$$\therefore \quad \frac{BP}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

Q. In ΔABC, PQ is a line segment intersecting AB at P and AC at Q such that $PQ \parallel BC$. If PQ divides ΔABC into two equal parts equal in area. Find : $\dfrac{BP}{AB}$



Sol.

$$\therefore \quad \frac{AP}{AB} = \frac{1}{\sqrt{2}}$$

Let the common multiple be x

$$\therefore \quad AP = x \quad \text{and } AB = \sqrt{2}\,x$$

$$\text{But, } AB = AP + PB \quad (A - B - P)$$

$$\therefore \quad PB = AB - AP$$

$$\therefore \quad PB = \sqrt{2}\,x - x$$

$$\therefore \quad PB = (\sqrt{2} - 1)\,x$$

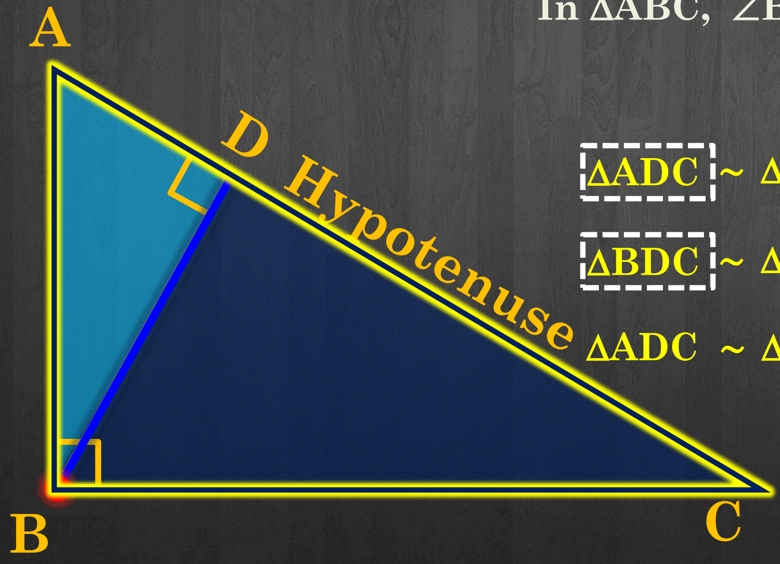
$$\therefore \quad \boxed{\frac{BP}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}}$$

MODULE : 49

THEOREM ON SIMILARITY OF RIGHT ANGLED TRIANGLES

In a right angled triangle, if the perpendicular is drawn from the vertex of the right angle to the hypotenuse, then the triangles on either side of the perpendicular are similar to the original triangle and to each other.

In ΔABC, $\angle B = 90^{o}$



$\boxed{\Delta ADC} \sim \Delta ABC$

$\boxed{\Delta BDC} \sim \Delta ABC$

$\Delta ADC \sim \Delta BDC$

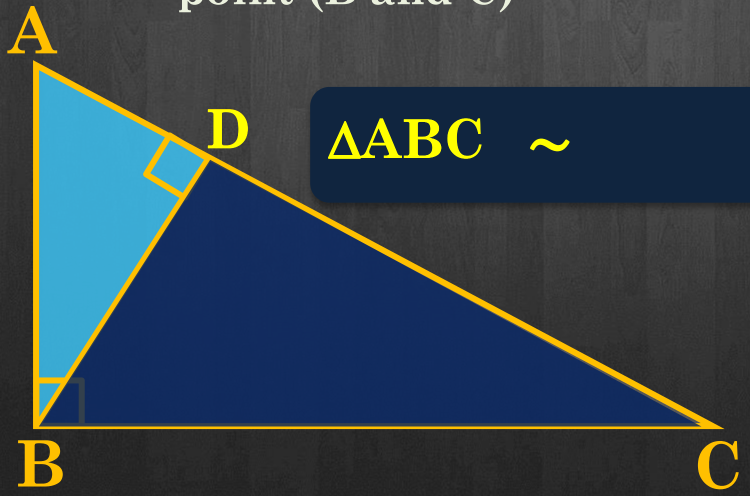
$\Delta ABC \sim \Delta ADB \sim \Delta BDC$

TRICK FOR NAMING THE THREE TRIANGLES (with respect to the figure)

Step 1 : Write the name of the bigger right angled triangle (ΔABC) with the right angle ($\angle B$) in between.

Step 2 : Pick up the fourth point (D) and place it between the first two points (A and B)

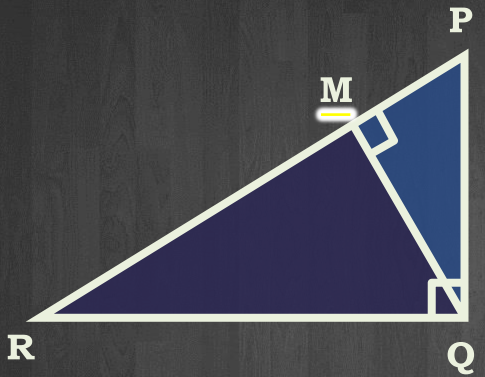
Step 3 : Now, place the fourth point (D) between the second and the third point (B and C)



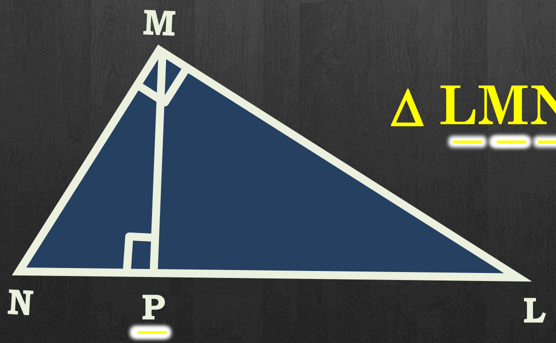
$\Delta ABC \sim \quad \sim$

This will give us the name of the second triangle

EXAMPLE



$\Delta\, PQR \sim \Delta\, PMQ \sim \Delta\, QMR$

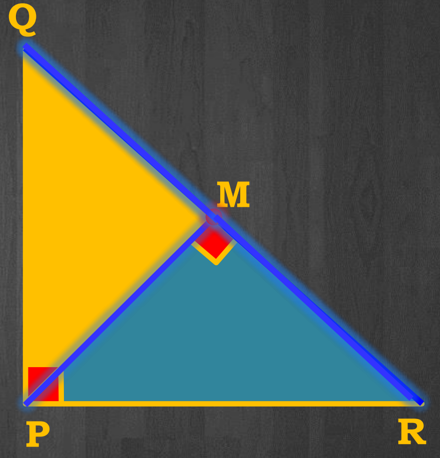


$\Delta\, LMN \sim \Delta\, LPM \sim \Delta\, MPN$

MODULE : 50

Ex.6.5 (Q.2)

PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM \times MR$.



Proof : In ΔPQR,

$\angle QPR = 90°$

$PM \perp QR$

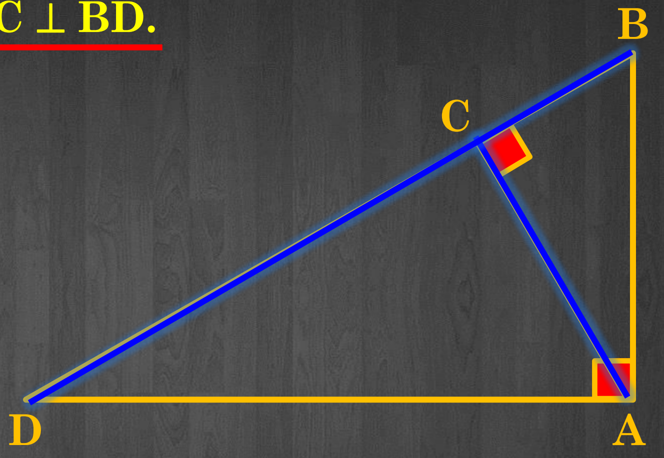
$\therefore \quad \Delta QMP \sim \Delta PMR$ [If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other]

$\frac{QM}{PM} = \frac{PM}{MR}$ [corresponding sides of similar triangles]

$\therefore \quad \boxed{PM^2 = QM \times MR}$

Ex.6.5 (Q.3)

ABD is a triangle right angled at A and $AC \perp BD$.
Show that :
(i) $AB^2 = BC \times BD$ ✓
(ii) $AC^2 = BC \times CD$
(iii) $AD^2 = BD \times CD$



Proof : In ΔBAD, $m\angle BAD = 90^{\circ}$

$AC \perp BD$

$\therefore \Delta BCA \sim \Delta BAD \sim \Delta ACD$ ...(i)

[If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular a are similar to the whole triangle and to each other]

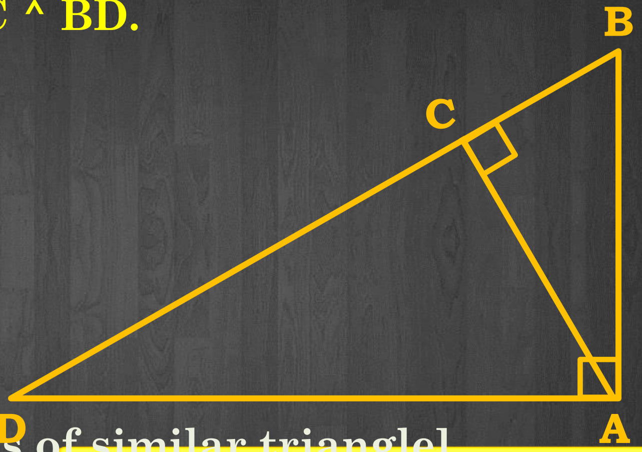
$\Delta BCA \sim \Delta BAD$ [From (i)]

$\therefore \dfrac{BC}{BA} = \dfrac{BA}{BD}$ [corresponding sides of similar triangles]

$\therefore \boxed{AB^2 = BC \times BD}$

Ex.6.5 (Q.3)

ABD is a triangle right angled at A and AC ^ BD.
Show that :
(i) $AB^2 = BC \times BD$
(ii) $AC^2 = BC \times CD$
(iii) $AD^2 = BD \times CD$



$\Delta BAD \sim \Delta BCA \sim \Delta ACD$ ...(i)

Proof : $\Delta BCA \sim \Delta ACD$ [From (i)]

$$\therefore \quad \frac{BC}{AC} = \frac{AC}{CD} \qquad \text{[corresponding sides of similar triangle]}$$

$$\therefore \quad \boxed{AC^2 = BC \times CD}$$

$\Delta BAD \sim \Delta ACD$ [From (i)]

$$\therefore \quad \frac{BD}{AD} = \frac{AD}{CD} \qquad \text{[corresponding sides of similar triangle]}$$

$$\therefore \quad \boxed{AD^2 = BD \times CD}$$

Thank You

MODULE : 51

$\angle ACB = 90^\circ$ and $CD \perp AB$. Prove that $\dfrac{BC^2}{AC^2} = \dfrac{BD}{AD}$.

Sol: $\boxed{\Delta ACB} \sim \Delta ADC \sim \boxed{\Delta CDB}$ ....(i) [If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other]

$$\therefore \quad \frac{AC}{AD} = \frac{AB}{AC}$$

$$\therefore \quad AC^2 = AB \,.\, AD \quad \text{... (ii)}$$

$$\Delta ACB \sim \Delta CDB$$

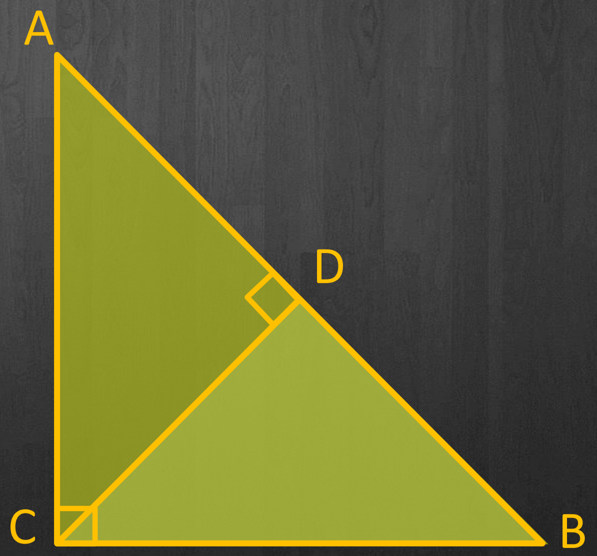
$$\therefore \quad \frac{AB}{CB} = \frac{BC}{DB}$$

$$\therefore \quad BC^2 = AB \,.\, DB \quad \text{... (iii)}$$

$$\therefore \quad \frac{BC^2}{AC^2} = \frac{\cancel{AB} \,.\, BD}{\cancel{AB} \,.\, AD}$$

From (ii) and (iii)

$$\text{i.e} \quad \frac{BC^2}{AC^2} = \frac{BD}{AD}$$



MODULE : 52

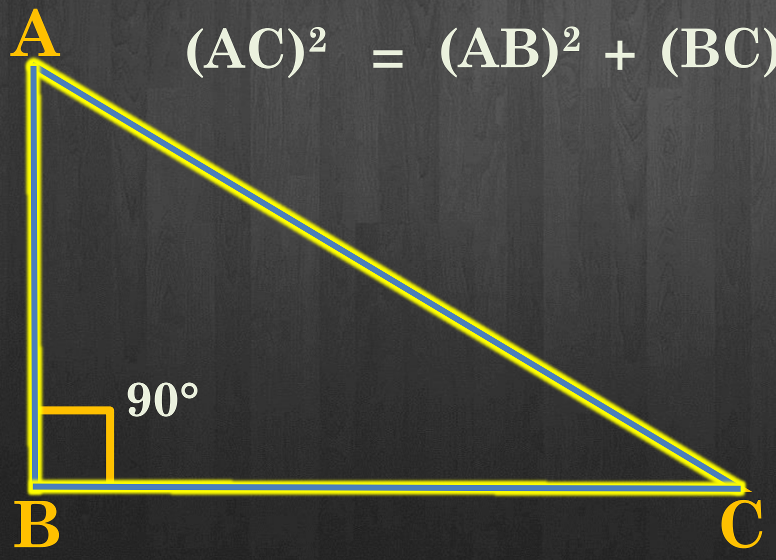
THEOREM OF PYTHAGORAS

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the remaining two sides.

In ΔABC,

$\angle ABC = 90^{o}$

$(AC)^2 = (AB)^2 + (BC)^2$ [By Pythagoras theorem]



THEOREM : Theorem of Pythagoras

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the remaining two sides.

Given : In ΔABC, m∠ABC =]

To prove : $AC^2 = AB^2 + BC^2$

Constr

Proof :

No

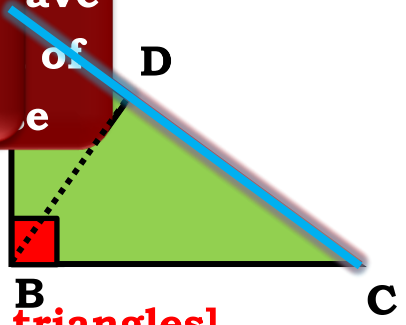
Can we drop perpendiculars from A and C ?

We already have it

ave

of

e



W

For similar

So, we will drop a perpendicular from B to side AC

[Given]

triangle

[Construction]

a

vertex

to the opposite side

e

pply?

we require should be a right angled triangle

[c.s.s.t]

[c.s.s.t]

iii),

angled triangles]

A

∴

∴ A

ΔABC ~ ΔI

$\therefore \dfrac{BC}{DC} = \dfrac{A}{BC}$

$\therefore \boxed{BC^2} = DC \times AC$ ...(iii)

(AD × AC) + (DC × AC)

$C^2 = AC\ (AD + DC)$

$C^2 = AC \times AC$

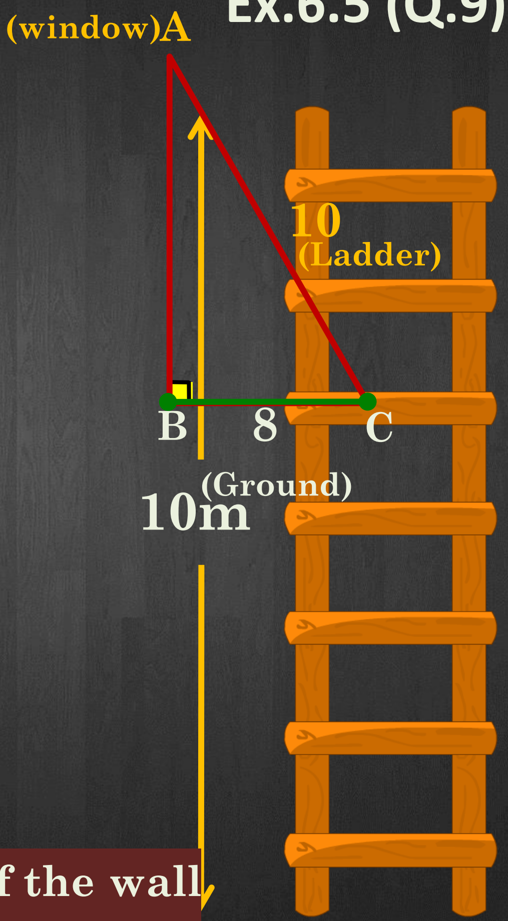
$C^2 = AC^2$

$\therefore \quad AC^2 = AB^2 + BC^2$

MODULE : 53

Ex.6.5 (Q.9)

Q. A ladder 10m long reaches a window 8m above the ground. Find the distance of the foot of the ladder from base of the wall.



Soln. AB represents the height at which ladder reaches the window above the ground. AC represents length of the ladder.

$AB = 8m$, $AC = 10m$,

In ΔABC, $\angle ABC = 90^0$

$AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\therefore (\quad)^2 = (\quad)^2 + BC^2$

$\therefore 100 = 64 + BC^2$

$\therefore BC^2 = 100 - 64$

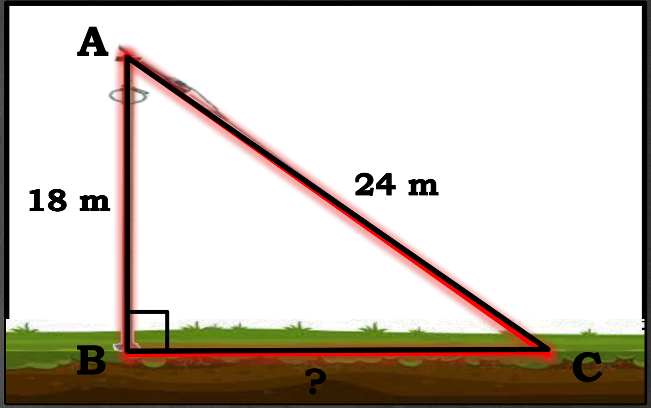
$\therefore BC^2 = 36$

$\therefore BC = 6m$

$\therefore$ Distance of the foot of the ladder from the base of the wall is 6m.

Ex.6.5 (Q.10)

Q. A guy wire is attached to a vertical pole of height 18m is 24m long & has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire be will be taut?



Sol. AB represents the length of vertical pole.

AC represents the length of wire

BC is the distance of stake from the base of the pole.

In ΔABC, $\angle ABC = 90^\circ$

$AC^2 = AB^2 + BC^2$ [by Pythagoras theorem]

$\therefore \quad 24^2 = 18^2 + BC^2$

$\therefore \quad 576 = 324 + BC^2$

$\therefore \quad BC^2 = 576 - 324 = 252$

$\therefore \quad BC = \sqrt{252} = \sqrt{36 \times 7}$

$\therefore \quad BC = 6\sqrt{7}$

Stake should be at a distance of $6\sqrt{7}$ m from the base of the pole.

MODULE : 54

Ex.6.5 (Q.11)

Q. An aeroplane leaves an airport and flies due north at the speed of 1000km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will the two planes be after $1\frac{1}{2}$ hours.

Soln. AB represents the distance covered by 1st areoplane.
AC represents the distance covered by 2nd areoplane.

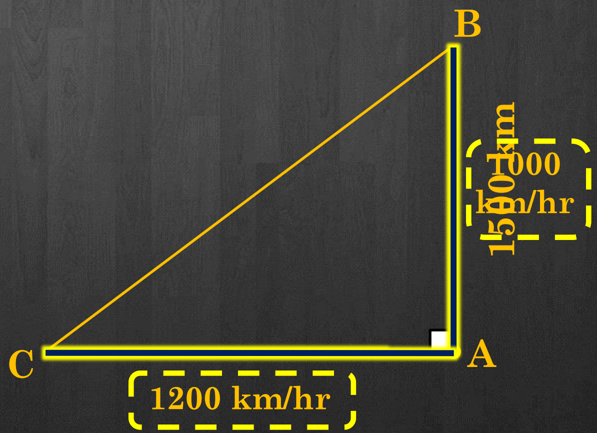
$$1\frac{1}{2} \text{ hours} = \frac{3}{2} \text{ hours.}$$

Distance = speed × time

$$AB = \overset{500}{\cancel{1000}} \times \frac{3}{\cancel{2}} = 1500 \text{ km}$$

Distance = speed × time

$$AC = \overset{600}{\cancel{1200}} \times \frac{3}{\cancel{2}} = 1800 \text{ km}$$



Ex.6.5 (Q.11)

Q. An aeroplane leaves an airport and flies due north at the speed of 1000km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will the two planes be after 1

Soln. In ΔBAC, $\angle BAC = 90^0$

$(BC)^2 = (AB)^2 + (AC)^2$ ... [by Pythagoras theorem]

$\therefore (BC)^2 = \underline{(1500)^2} + \underline{(1800)^2}$

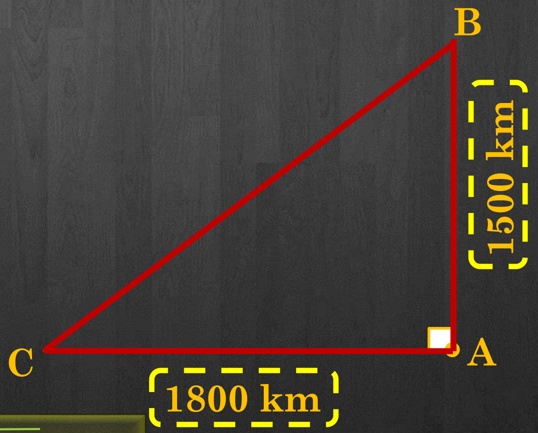
$\therefore (BC)^2 = 2250000 + 3240000$

$\therefore (BC)^2 = 5490000$

$\therefore BC = \sqrt{5490000} = \sqrt{\underline{9} \times 61 \times \underline{100} \times \underline{100}}$

$\therefore BC = 300\sqrt{61}$ km

$\therefore$ **Distance between the two aeroplanes = $300\sqrt{61}$ km**



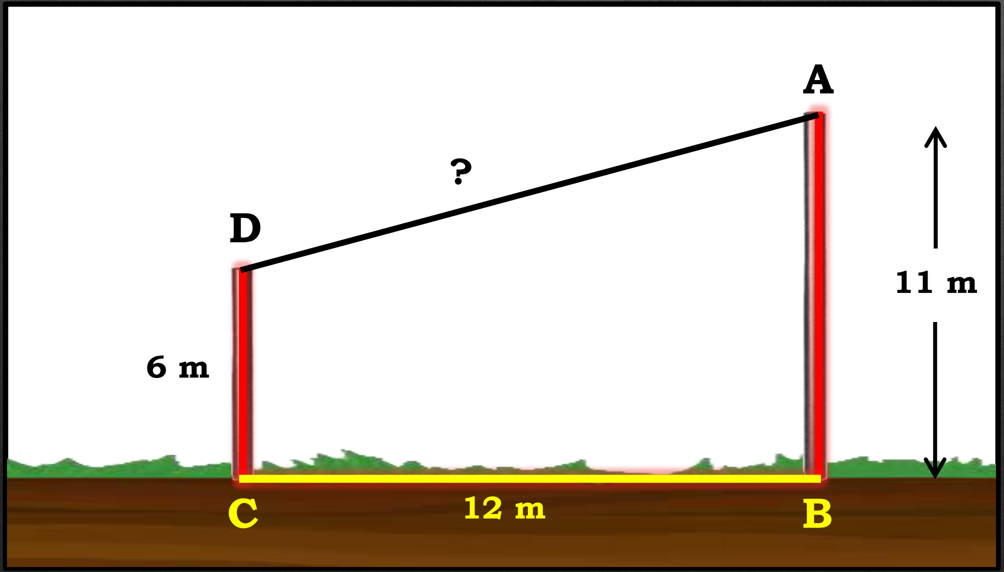
MODULE : 55

Ex.6.5 (Q.12)

Q. Two poles of heights 6m and 11m stand on a plane ground. If the distance between the feet of the poles is 12m, find the distance between their tops.

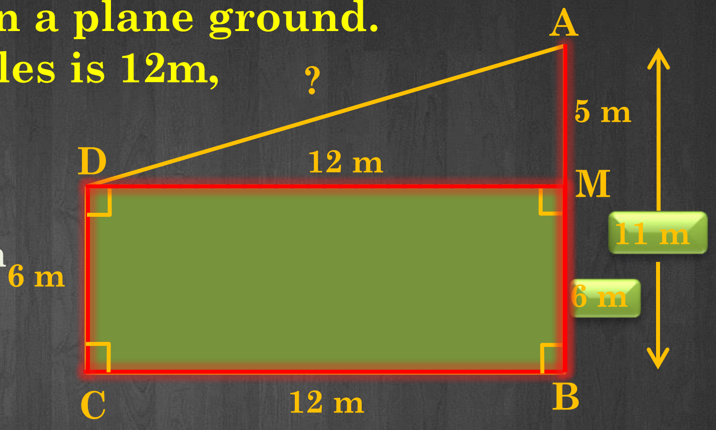
Sol. AB and DC represent heights of two poles.

BC and AD represents the distance between their feet and tops respectively.



Ex.6.5 (Q.12)

Q. Two poles of heights 6m and 11m stand on a plane ground. If the distance between the feet of the poles is 12m, find the distance between their tops.



Sol. AB and DC represent heights of two poles.

BC and AD represents the distance between their feet and tops respectively.

Draw DM ⊥ AB

MBCD is a Rectangle

DM = BC = 12m

DC = MB = 6m

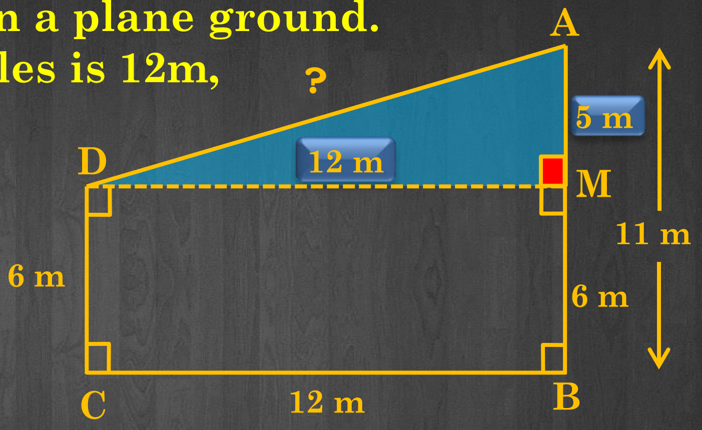
AM = AB – MB

∴ AM = 11 – 6

∴ AM = 5m

Ex.6.5 (Q.12)

Q. Two poles of heights 6m and 11m stand on a plane ground. If the distance between the feet of the poles is 12m, find the distance between their tops.



Sol. In ΔAMD, $\angle AMD = 90°$

$AD^2 = AM^2 + DM^2$ [by Pythagoras theorem]

$\therefore AD^2 = 5^2 + 12^2$

$\therefore AD^2 = 25 + 144$

$\therefore AD^2 = 169$

$\therefore AD = 13m$

Distance between the tops is 13 m.

Thank You

MODULE : 56

Ex.6.5 (Q.4)

Q. ABC is an isosceles triangle right angled at C.
Prove that $AB^2 = 2AC^2$.

Given : In ΔABC, $\boxed{\angle C = 90^\circ}$
$\boxed{AC = BC}$

To Prove : $AB^2 = 2AC^2$

Proof :

In ΔACB, $\angle C = 90^0$

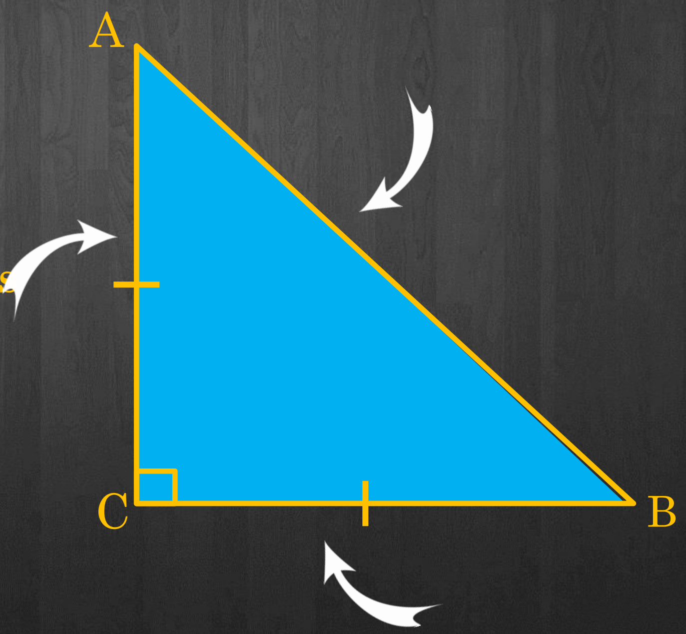
$AB^2 = AC^2 + BC^2$ ... (i) ... [by Pythagoras theorem]

$BC = AC$

$\therefore$ $BC^2 = AC^2$ ... (ii)

$\therefore$ $AB^2 = \underline{AC^2 + AC^2}$ ... [From (i) and (ii)]

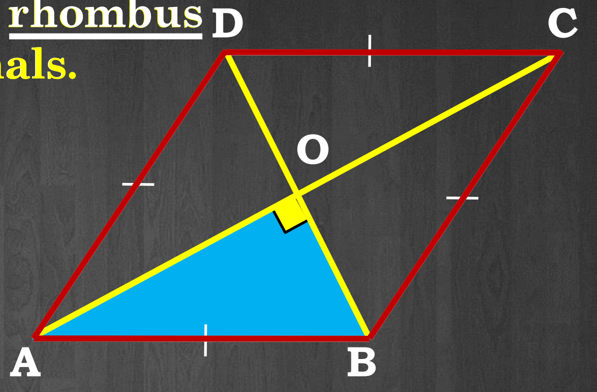
$\therefore$ $\boxed{AB^2 = 2AC^2}$



MODULE : 57

Ex.6.5 (Q.7)

Q. Prove that sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.



Given : ABCD is a rhombus

AC & BD intersect at 'O'.

To Prove : $AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

Proof : ABCD is a rhombus

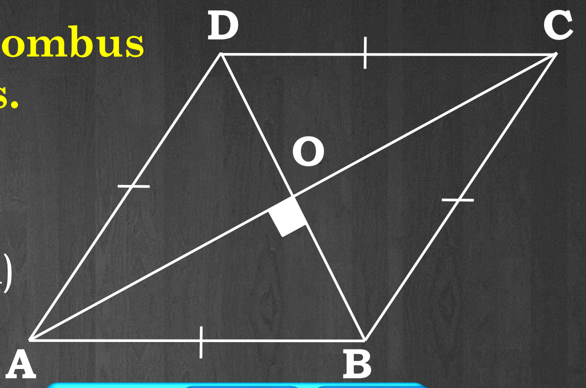
In AOB, AOB = 90^0 ... [diagonals of rhombus are perpendicular to each other]

$AB^2 = AO^2 + BO^2$ ... (i) ... [by Pythagoras theorem]

$$\left.\begin{aligned} AO &= \frac{1}{2} AC \\ BO &= \frac{1}{2} BD \end{aligned}\right\}$$ ... (ii) ...[diagonals of a rhombus bisect each other]

Ex.6.5 (Q.7)

Q. Prove that sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.



$AB^2 = AO^2 + BO^2$

$AO = \frac{1}{2} AC$

$BO = \frac{1}{2} BD$

To Prove : $AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

$$AB^2 = \left(\frac{1}{2} AC\right)^2 + \left(\frac{1}{2} BD\right)^2 \quad \text{... from (i) and (ii)}$$

$$\therefore \quad AB^2 = \frac{1}{4} AC^2 + \frac{1}{4} BD^2$$

$$\therefore \quad 4AB^2 = AC^2 + BD^2$$

$$\therefore \quad AB^2 + AB^2 + AB^2 + AB^2 = AC^2 + BD^2 \quad \text{... (iii)}$$

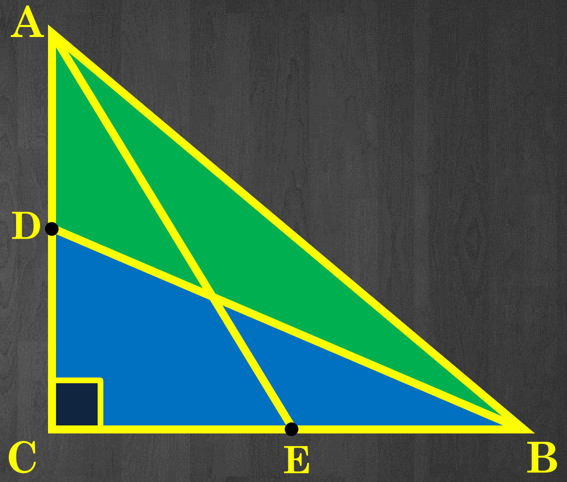
$$AB = BC = CD = AD \quad \text{... (iv) .... [sides of a rhombus]}$$

$$\therefore \quad AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2 \quad \text{... from (iii) and (iv)}$$

MODULE : 58

Ex.6.5 (Q.13)

Q. D & E are points on the sides CA & CB respectively of ΔABC right angled at C.



Prove that : $AE^2 + BD^2 = AB^2 + DE^2$

Proof : In $\Delta ACE, \angle ACE = 90^0$

In $\Delta DCE, \angle DCE = 90^0$

$AE^2 = AC^2 + CE^2$ ... (i) ... [by Pythagoras theorem]

$DE^2 = DC^2 + CE^2$ ... (v) ... [by Pythagoras theorem]

In $\Delta DCB, \angle DCB = 90^0$

$BD^2 = BC^2 + CD^2$ ... (ii) ... [by Pythagoras theorem]

$AE^2 + BD^2 = AB^2 + DE^2$ ... [From (iii), (iv) & (v)]

$AE^2 + BD^2 = AC^2 + CE^2 + BC^2 + CD^2$ ... (iii) ... [by Adding (i) and (ii)]

$AE^2 + BD^2 = AC^2 + CE^2 + BC^2 + CD^2$ ... (iii)

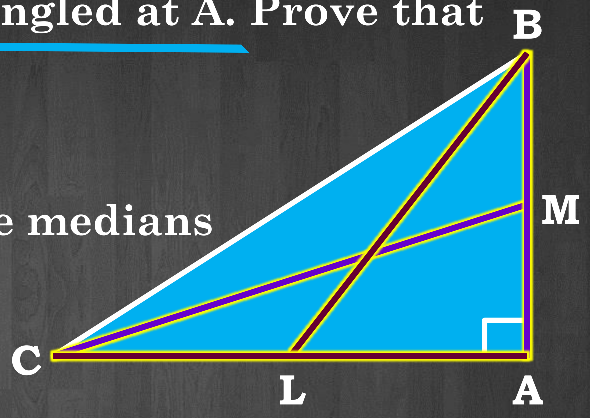
$AB^2 = AC^2 + BC^2$ ... (iv)

...y Pythagoras theorem]

MODULE : 59

Solved Example

BL and CM are medians of a triangle ABC right angled at A. Prove that $4\,(BL^2 + CM^2) = 5\,BC^2$.



Sol: $\left.\begin{aligned} \boxed{CL = LA = \frac{1}{2}CA} \\ AM = BM = \frac{1}{2}AB \end{aligned}\right\}$ ......(1) [BL and CM are medians of a ΔABC]

In ΔABC, $\angle A = 90^0$

$\therefore\ BC^2 = AB^2 + AC^2$ (Pythagoras Theorem) ......(2)

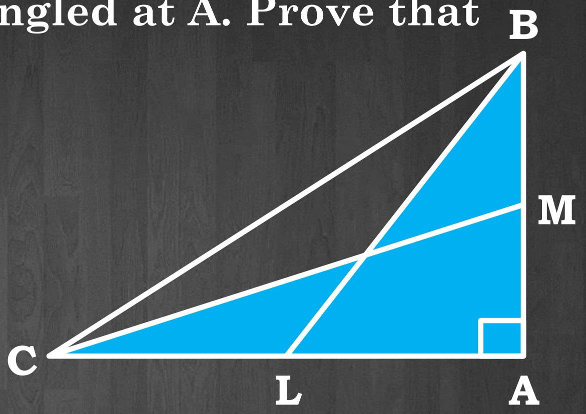
In ΔABL, $\angle A = 90^0$

$\therefore\ BL^2 = \underline{AL^2} + AB^2$ (Pythagoras Theorem)

$BL^2 = \left(\frac{AC}{2}\right)^2 + AB^2$ ....[From (1)]

Solved Example

BL and CM are medians of a triangle ABC right angled at A. Prove that $4\,(BL^2 + CM^2) = 5\,BC^2$.



Sol: $\therefore \quad BL^2 = \dfrac{AC^2}{4} + AB^2$

$\therefore \quad 4\,BL^2 = AC^2 + 4\,AB^2 \quad \ldots\ldots(3)$

In ΔCMA, $\angle A = 90^0$

$\therefore \quad CM^2 = AC^2 + AM^2$

$\therefore \quad CM^2 = AC^2 + \left(\dfrac{AB}{2}\right)^2 \quad$ [From……(1)]

$\therefore \quad CM^2 = AC^2 + \dfrac{AB^2}{4}$

$4\,CM^2 = 4\,AC^2 + AB^2 \quad \ldots\ldots(4)$

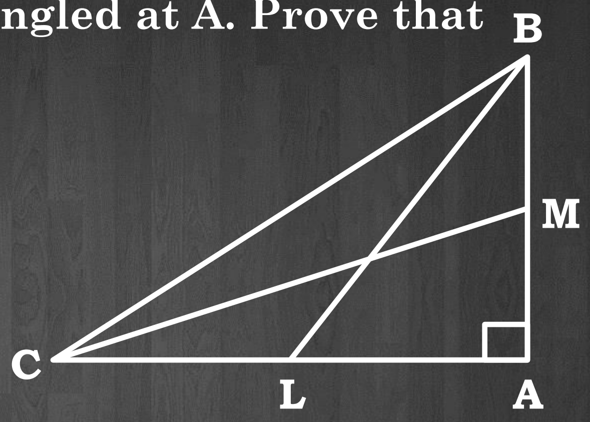
$CL = LA = \dfrac{1}{2}\,CA$

$AM = BM = \dfrac{1}{2}\,AB$

Solved Example

BL and CM are medians of a triangle ABC right angled at A. Prove that

$\boxed{4\,(BL^2 + CM^2) = 5\,BC^2.}$



Sol:

$$\therefore\quad 4BL^2 = AC^2 + 4\,AB^2 \quad ......(3)$$

$$\therefore\quad 4CM^2 = 4\,AC^2 + AB^2 \quad ......(4)$$

$$\therefore\quad 4\,BL^2 + 4CM^2 = AC^2 + 4AB^2 + 4AB^2 + 4AC^2 + 4AB^2$$

(Adding (3) and (4), we have)

$$\therefore\quad 4\,(BL^2 + CM^2) = 5\,(AC^2 + AB^2)$$

$$\boxed{\therefore\quad 4\,(BL^2 + CM^2) = 5\,BC^2}$$

[From (1)]

$$CL = LA = \frac{1}{2}CA$$

$$AM = BM = \frac{1}{2}AB$$

MODULE : 60

Solved Example

If $AD \perp BC$, prove that $AB^2 + CD^2 = BD^2 + AC^2$.

Sol: From ΔADC, we have

$$\boxed{AC^2 = AD^2 + CD^2} \quad \text{(Pythagoras Theorem)} \quad (1)$$

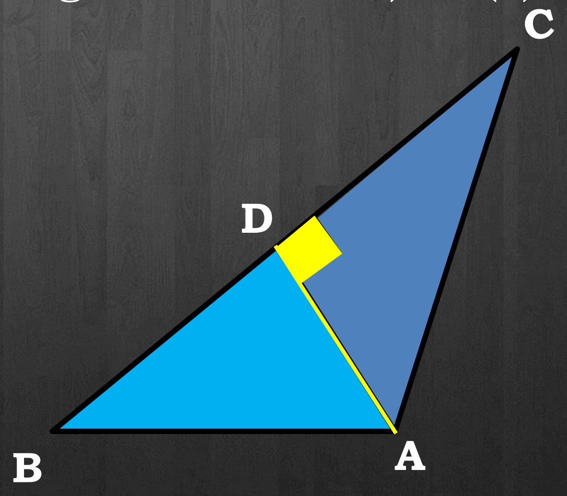
From ΔADB, we have

$$\boxed{AB^2 = AD^2 + BD^2} \quad \text{(Pythagoras Theorem)} \quad (2)$$

Subtracting (1) from (2), we have

$$AB^2 - AC^2 = BD^2 - CD^2$$

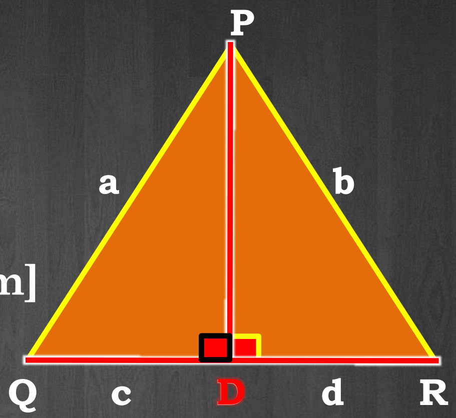
$$AB^2 + CD^2 = BD^2 + AC^2$$



In DPQR, PD ^ QR, such that D lies on QR.

If PQ = a, PR = b, QD = c and DR = d

Prove : $(a + b)(a - b) = (c + d)(c - d)$



Proof : In DPDQ, $\angle PDQ = 90^o$

$PD^2 + QD^2 = PQ^2$ [By Pythagoras theorem]

$\therefore \quad PD^2 + c^2 = a^2$

$\therefore \quad PD^2 = a^2 - c^2$ ...(i)

In DPDR, $\angle PDR = 90^o$

$PD^2 + DR^2 = PR^2$ [By Pythagoras theorem]

$\therefore \quad PD^2 + d^2 = b^2$

$\therefore \quad PD^2 = b^2 - d^2$ ...(ii)

$\therefore \quad a^2 - c^2 = b^2 - d^2$ [From (i) and (ii)]

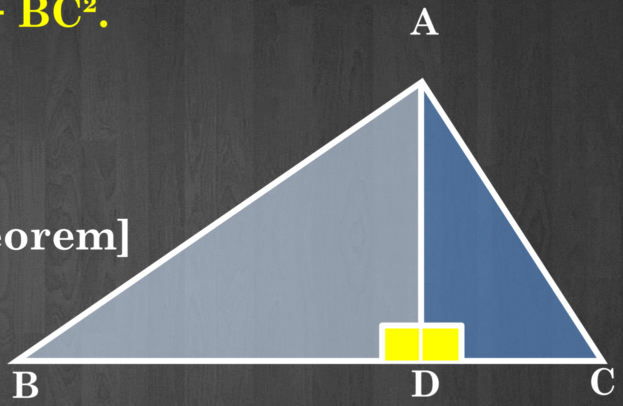
$\therefore \quad a^2 - b^2 = c^2 - d^2$

$\therefore \quad (a + b)(a - b) = (c + d)(c - d)$

MODULE : 61

Ex.6.5(Q.14)

Q. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$.



Proof : $\boxed{BD = 3CD}$

In ΔADB, $m\angle ADB = 90^\circ$

$\therefore\ AB^2 = AD^2 + \boxed{BD^2}$ [By Pythagoras theorem]

$\therefore\ AB^2 = AD^2 + (3CD)^2$ [From (i)]

$\therefore\ AB^2 = \boxed{AD^2} + 9CD^2$

In ΔADC, $m\angle ADC = 90^\circ$

$AD^2 + CD^2 = AC^2$ [By Pythagoras theorem]

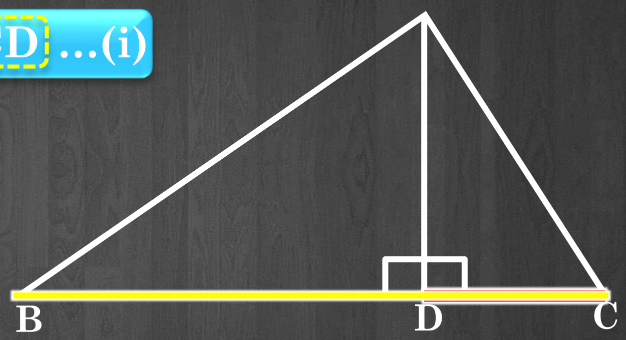
$\therefore\ \boxed{AD^2 = AC^2 - CD^2}$ ...(iii)

$\therefore\ AB^2 = AC^2 - CD^2 + 9CD^2$ [From (ii) and (iii)]

$\therefore\ AB^2 = AC^2 + 8\,CD^2$ ...(iv)

Ex.6.5(Q.14)

Q. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$.



$BD = 3CD$ ...(i)

Proof : $AB^2 = AC^2 + 8CD^2$ ...(iv)

$BC = BD + CD$ [B-D-C]

$\therefore BC = 3CD + CD$ [From (i)]

$\therefore BC = 4CD$

$\therefore \frac{BC}{4} = CD$ ...(v)

$\therefore AB^2 = AC^2 + 8 \times \left(\frac{BC}{4}\right)^2$ [From (iv) and (v)]

$\therefore AB^2 = AC^2 + \cancel{8} \times \frac{BC^2}{\cancel{16}_{2}}$

$\therefore AB^2 = AC^2 + \frac{BC^2}{2}$

Multiplying throughout by 2,

$\therefore 2AB^2 = 2AC^2 + BC^2$

Thank You

MODULE : 62

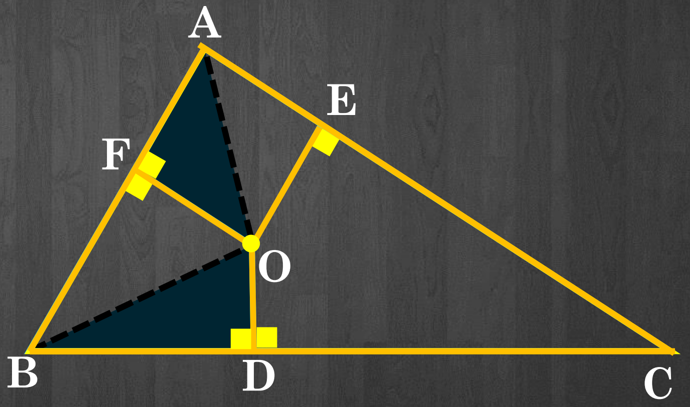
Ex.6.5(Q.8)

O is a point in the interior of ΔABC. $OD \perp BC$, $OE \perp AC$ & $OF \perp AB$.

Show that :

i) $\boxed{OA^2} + \boxed{OB^2} + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$



Proof :

(i) In ΔAFO, $\angle AFO = 90^0$

$\therefore \quad OA^2 = AF^2 + OF^2 \quad \ldots$ (i)

... [by Pythagoras theorem]

(ii) In ΔBDO, $\angle BDO = 90^0$

$\therefore \quad OB^2 = BD^2 + OD^2 \quad \ldots$ (ii)

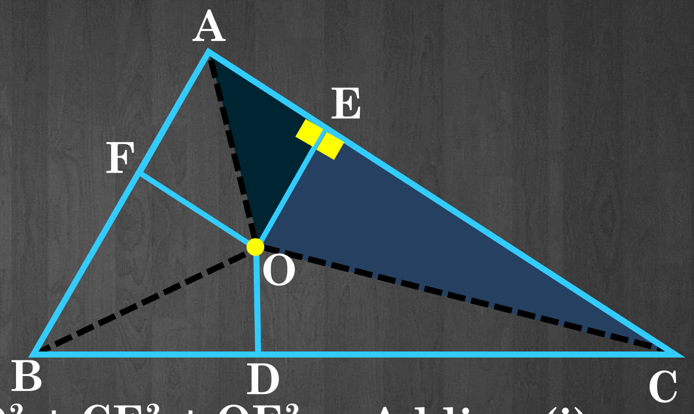
... [by Pythagoras theorem]

Ex.6.5(Q.8)

Show that :

i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$



In ΔCEO, $\angle CEO = 90^0$

$\therefore \quad OC^2 = CE^2 + OE^2 \quad \dots \text{(iii)}$

... [by Pythagoras theorem]

$\therefore \; OA^2 + OB^2 + OC^2 = AF^2 + OF^2 + BD^2 + OD^2 + CE^2 + OE^2$ ... Adding (i), (ii) and (iii)

$\therefore \; OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

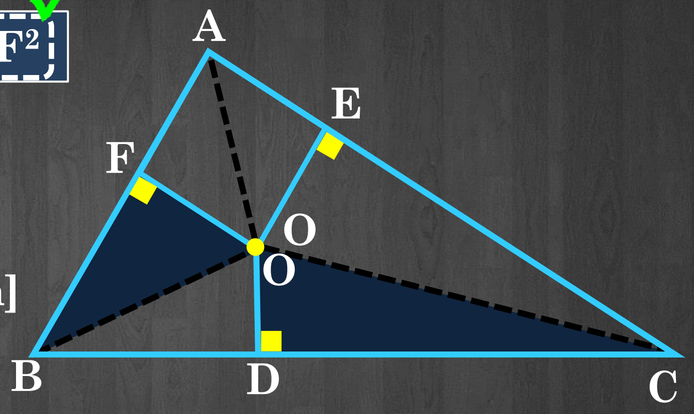
In ΔAEO, $\angle AEO = 90^0$

$\therefore \quad OA^2 = OE^2 + AE^2$ ... (iv) [by Pythagoras theorem]

$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

$OA^2 = OE^2 + AE^2 \ldots \text{(iv)}$

Show that : $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$



In ΔCDO, $\angle CDO = 90^0$

$\therefore$ $OC^2 = OD^2 + CD^2 \ldots \text{(v)}$

[by Pythagoras theorem]

In ΔBFO, $\angle BFO = 90^0$

$\therefore$ $OB^2 = BF^2 + OF^2 \ldots \text{(vi)}$ [by Pythagoras theorem]

$OA^2 + OC^2 + OB^2 = OE^2 + AE^2 + OD^2 = CD^2 + BF^2 + OF^2$

$OA^2 + OC^2 + OB^2 - OD^2 - OE^2 - OF^2 = AE^2 + CD^2 + BF^2$

$\therefore$ $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

MODULE : 63

Ex.6.5(Q.8)

O is any point inside a rectangle ABCD.

Prove that $OB^2 + OD^2 = OA^2 + OC^2$

Construction : through Q, draw $\boxed{PQ \parallel BC}$ intersecting AB and CD at P and Q respectively.

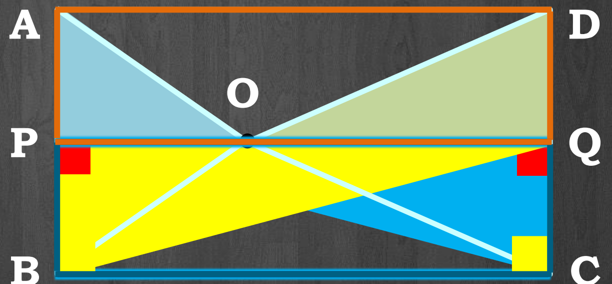
Sol: $PQ \parallel BC$

$\therefore$ $PQ \perp AB$ and $PQ \perp DC$

($\angle B = 90°$ and $\angle C = 90°$)

$\angle BPQ = 90°$ and $\angle CQP = 90°$

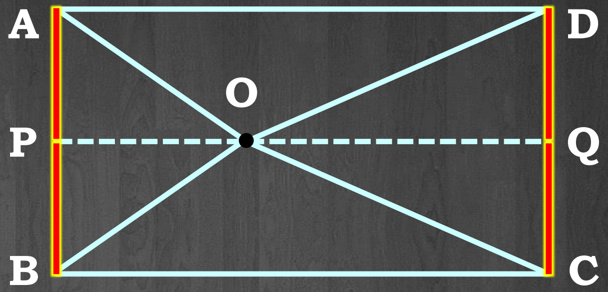
$\therefore$ BPQC and APQD are both rectangles.



$$\left.\begin{array}{lll} \text{In } \Delta OPB, & OB^2 = BP^2 + OP^2 & (1) \\ \text{In } \Delta OQD, & OD^2 = OQ^2 + DQ^2 & (2) \\ \text{In } \Delta OQC, & OC^2 = OQ^2 + CQ^2 & (3) \\ \text{In } \Delta OAP, & OA^2 = AP^2 + OP^2 & (4) \end{array}\right\} \text{[by Pythagoras theorem]}$$

O is any point inside a rectangle ABCD.

Prove that $\boxed{OB^2 + OD^2 = OA^2 + OC^2}$



Sol: Adding (1) and (2),

$$OB^2 + OD^2 = BP^2 + OP^2 + OQ^2 + DQ^2$$

$$= CQ^2 + OP^2 + OQ^2 + AP^2$$

(As BP = CQ and DQ = AP)

$$= CQ^2 + OQ^2 + OP^2 + AP^2$$

$$\boxed{OB^2 + OD^2 = OC^2 + OA^2}$$

[From (3) and (4)]

$OB^2 = BP^2 + OP^2$ ...(1)

$OD^2 = OQ^2 + DQ^2$ ...(2)

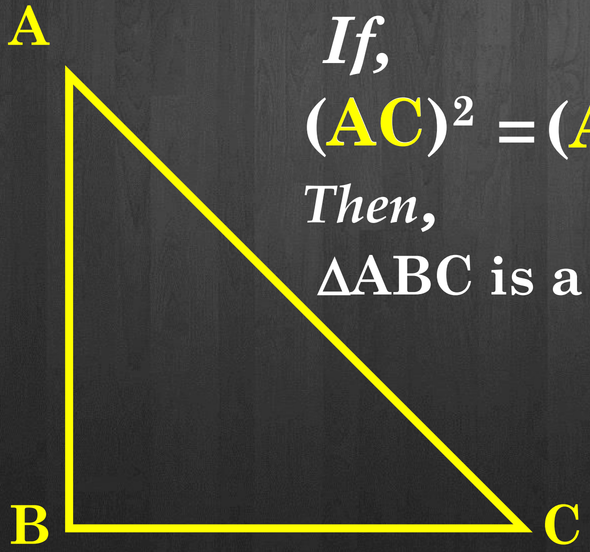
$OC^2 = OQ^2 + CQ^2$ ...(3)

$OA^2 = AP^2 + OP^2$ ...(4)

MODULE : 64

Converse Of Theorem Of Pythagoras

In a triangle, if the $\boxed{\text{square}}$ of $\boxed{\text{one side}}$ is equal to the sum of the squares of the remaining two sides, then the triangle is a right angled triangle.



If,

$$(AC)^2 = (AB)^2 + (BC)^2$$

Then,

ΔABC is a right angled triangle

Ex.6.5 (Q.1 (i, ii))

Sides of some triangles are given below. Determine which of them are right triangles.

(i) 7cm, 24cm, 25cm

Soln. $\boxed{25^2 = 625}$ ...(i)

$\therefore \quad 7^2 + 24^2 = 49 + 576$

$\therefore \quad \boxed{7^2 + 24^2 = 625}$ ...(ii)

$\therefore \quad 25^2 = 7^2 + 24^2$ [From (i) and (ii)]

$\therefore$ The given triangle is a right angled triangle.

[By Converse of Pythagoras theorem]

Ex.6.5 (Q.1 (i, ii))

Sides of some triangles are given below. Determine which of them are right triangles.

(ii) $\underbrace{3cm,}\underbrace{8cm,}\underbrace{6cm}$

Soln. $\boxed{8^2 = 64}$ ...(i)

$\therefore \quad 3^2 + 6^2 = 9 + 36$

$\therefore \quad \boxed{3^2 + 6^2 = 45}$ ...(ii)

$\therefore \quad 8^2 \neq 3^2 + 6^2$ [From (i) and (ii)]

$\therefore$ The given triangle is not a right angled triangle.

Ex.6.5 (Q.1 (iii))

Sides of some triangles are given below. Determine which of them are right triangles.

(iii) 50cm, 80cm, 100cm

Sol. $\boxed{(100)^2 = 10000}$ ...(i)

$\therefore \quad (50)^2 + (80)^2 = 2500 + 6400$

$\therefore \quad \boxed{(50)^2 + (80)^2 = 8900}$ ...(ii)

$\therefore \quad (100)^2 \neq (50)^2 + (80)^2$ [From (i) and (ii)]

$\therefore$ The given triangle is not a right angled triangle.

Ex.6.5 (Q.1 (iv))

Sides of some triangles are given below. Determine which of them are right triangles.

(iv) 13cm, 12cm, 5cm

Sol. $\boxed{(13)^2 = 169}$ ...(i)

$\therefore$ $(12)^2 + (5)^2 = 144 + 25$

$\therefore$ $\boxed{(12)^2 + (5)^2 = 169}$ ...(ii)

$\therefore$ $(13)^2 = (12)^2 + (5)^2$ [From (i) and (ii)]

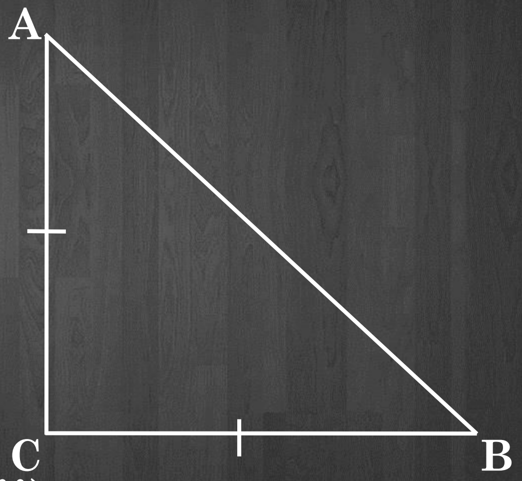
$\therefore$ The given triangle is a right angled triangle.

[By Converse of Pythagoras theorem]

MODULE : 65

Ex.6.5 (Q.5)

Q. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.



Proof : $AB^2 = 2AC^2$

$\therefore$ $AB^2 = AC^2 + AC^2$ ... (i)

$\therefore$ $AC = BC$ ... (ii)

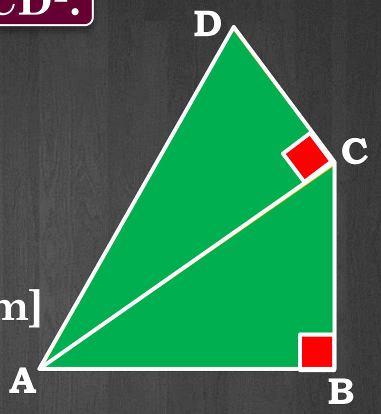
$\therefore$ $AB^2 = AC^2 + BC^2$ ... From (i) and (ii)

$\therefore$ Δ ABC is a right angled triangle ...[by converse of Pythagoras theorem]

Ex.6.5 (Q.5)

In a quadrilateral ABCD. $\angle B = 90^o$, $AD^2 = AB^2 + BC^2 + CD^2$.

Prove : $\angle ACD = 90^o$



Proof : In right triangle ABC,

$\angle B = 90^o$

$\therefore$ $AC^2 = AB^2 + BC^2$ ...(i) [Pythagoras theorem]

$AD^2 = AB^2 + BC^2 + CD^2$ ...(ii)[Given]

$\therefore$ $AD^2 = AC^2 + CD^2$ ...(iii)[From (i) and (ii)]

In ΔADC,

$AD^2 = AC^2 + CD^2$ [From (iii)]

$\therefore$ $\angle ACD = 90^o$ [By Converse of Pythagoras theorem]

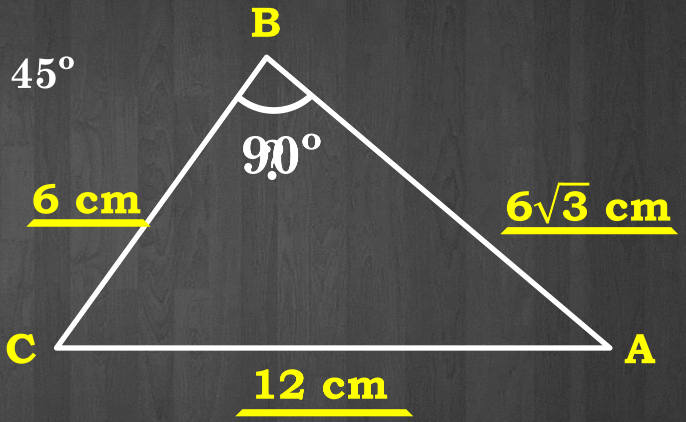
Ex.6.5 (Q.17)

Tick the correct answer and justify :

In Δ ABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm,

The angle B is :

(A) 120° (B) 60° (C) 90° (D) 45°



Sol: $AB^2 = 108$

$AC^2 = 144$

$BC^2 = 36$

$AB^2 + BC^2 = 108 + 36 = 144$

$AB^2 + BC^2 = AC^2$

$\therefore$ The given triangle is a right angled triangle at B.

$\therefore \quad \angle B = 90^\circ$ [By Converse of Pythagoras Theorem]

Hence, the correct answer is (C).

Thank You

MODULE : 66

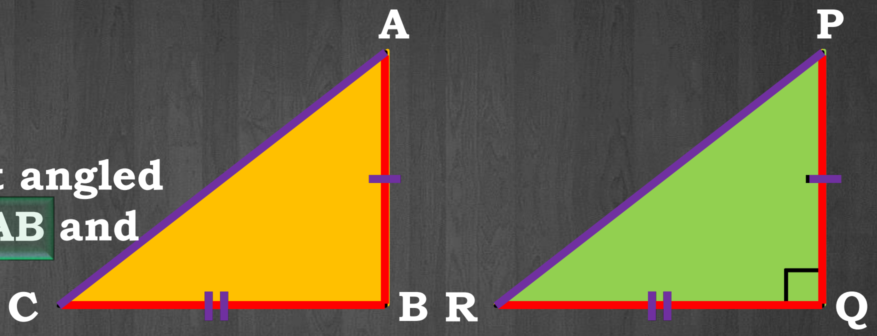
In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Given: In ΔABC,

$$AC^2 = AB^2 + BC^2$$

To prove $\angle B = 90^\circ$

Construction: construct ΔPQR right angled at Q such that $PQ = AB$ and $QR = BC$



Proof: In ΔPQR, $\angle PQR = 90^\circ$

$\therefore$ $PR^2 = PQ^2 + QR^2$ (By Pythagoras Theorem)

$\therefore$ $PR^2 = AB^2 + BC^2$ (construction) ...(i)

But $AC^2 = AB^2 + BC^2$ (Given) ...(ii)

$\therefore$ $AC = PR$ [From (1) and (2)] ...(iii)

In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Given:

In ΔABC and ΔPQR,

$AB = PQ$ (construction)

$BC = QR$ (construction)

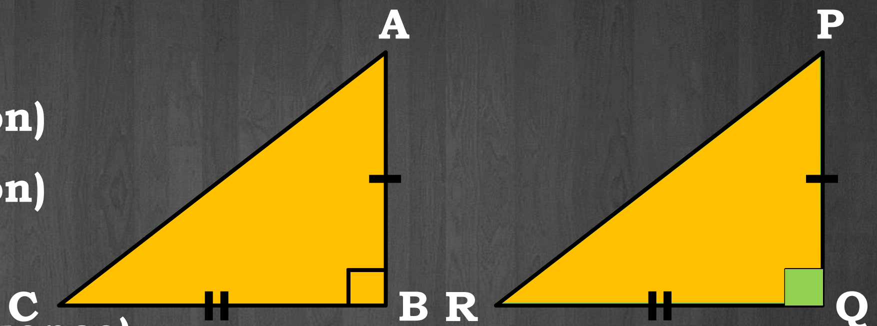
$AC = PR$ [From (iii)]

$\therefore \Delta ABC \cong \Delta PQR$ (SSS congruence)

$\therefore \angle \; \cong \; \angle$ (CPCT)

But $\angle Q \cong 90^\circ$ (construction)

$\therefore \angle B \cong 90^\circ$

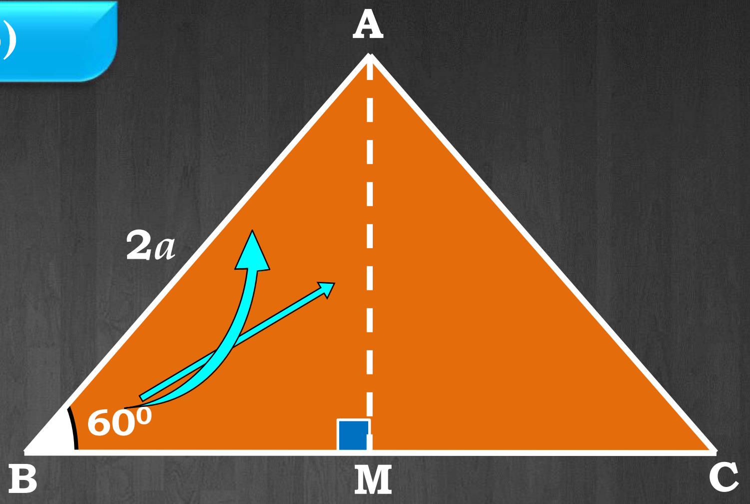


$AC = PR$ ...(iii)

MODULE : 67

Ex.6.5 (Q.6)

ΔABC is an equilateral triangle of side $2a$. Find each of its altitudes.



Construction : Draw $AM \perp BC$

Soln :

$$AB = BC = AC = 2a$$

In ΔABC,

$AM \perp BC$ ... [construction]

$\angle B = 60^0$ ... [angle of an equilateral triangle]

In ΔAMB, $\sin 60^\circ = \dfrac{AM}{AB}$

$$\therefore \frac{\sqrt{3}}{2} = \frac{AM}{2a}$$

$$\therefore AM = \frac{\sqrt{3}}{\cancel{2}} \times \cancel{2}a$$

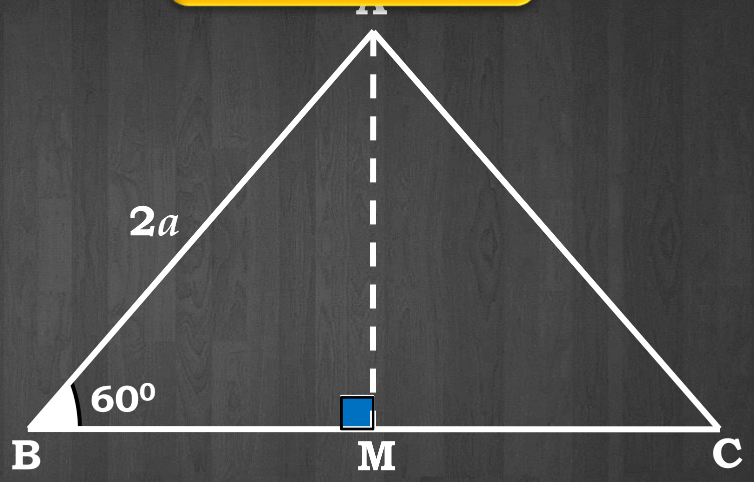
Ex.6.5 (Q.6)

ΔABC is an equilateral triangle of side $2a$.
Find each of its altitudes.

Construction : Draw $AM \perp BC$

Soln :

$$\sin 60^\circ = \frac{AM}{AB}$$



$\therefore \quad AM = \sqrt{3}\, a$

Length of altitude AM $= \sqrt{3}\, a$ unit

All the altitudes of an equilateral triangle are equal

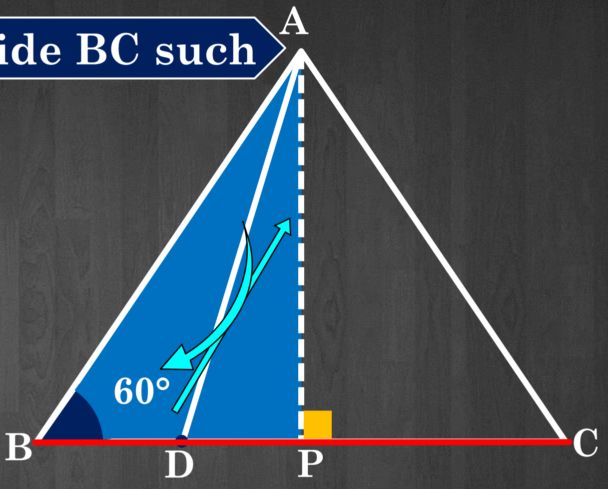
$\therefore$ $\boxed{\text{Length of all the altitudes } = \sqrt{3}\, a \text{ unit}}$

MODULE : 68

Ex.6.5 (Q.15)

In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$.

Prove that $9AD^2 = 7AB^2$.



Construction : Draw $AP \perp BC$

Proof : In right ΔAPD,

$AD^2 = AP^2 + PD^2$ ...(i) (by Pythagoras theorem)

$\angle B = 60^0$ ... [angle of an equilateral triangle]

$AP \perp BC$ ... [construction]

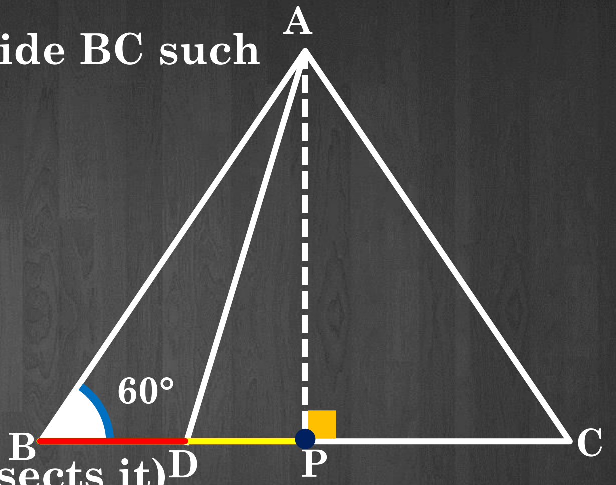
In right ΔAPB,

$\sin 60^0 = \frac{AP}{AB}$ $\qquad \frac{\sqrt{3}}{2} = \frac{AP}{AB}$ $\qquad \therefore \quad AP = \frac{\sqrt{3}}{2}AB$ ... (ii)

Ex.6.5 (Q.15)

In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3} BC$.

Prove that $9AD^2 = 7AB^2$.



Construction : Draw $AP \perp BC$

Proof : $PD = BP - BD$ ... (iii)

$BP = CP$ (altitude of an equilateral triangle drawn to the base bisects it)

$\therefore$ P is midpoint of BC

$\therefore$ $BP = \frac{1}{2} BC$ ... (iv)

$\therefore$ $BD = \frac{1}{3} BC$ ... (v)

$\therefore$ $PD = \frac{1}{2} BC - \frac{1}{3} BC$ .... from (iii) (iv) & (v)

Ex.6.5 (Q.15)

In an equilateral triangle ABC, D ... that $BD = \frac{1}{3}BC$.

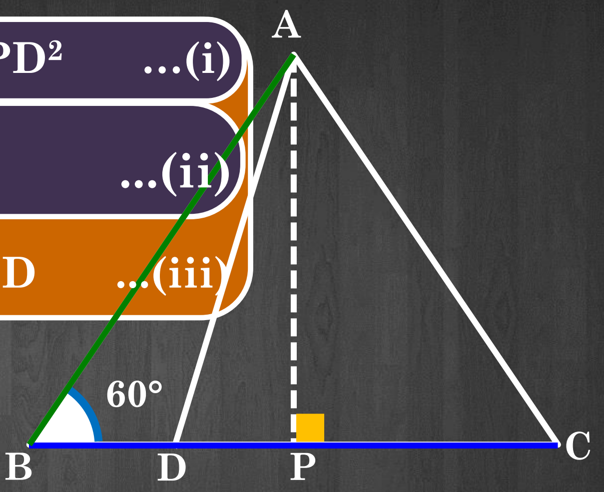
Prove that $9AD^2 = 7AB^2$.

Construction : Draw $AP \perp BC$

$$AD^2 = AP^2 + PD^2 \quad \text{...(i)}$$

$$AP = \frac{\sqrt{3}}{2}AB \quad \text{...(ii)}$$

$$PD = BP - BD \quad \text{...(iii)}$$



Proof :

$$\therefore \quad PD = BC\left(\frac{1}{2} - \frac{1}{3}\right)$$

$$\therefore \quad PD = BC\left(\frac{3-2}{6}\right)$$

$$\therefore \quad PD = BC\left(\frac{1}{6}\right)$$

$$\therefore \quad PD = \frac{1}{6}AB \quad \text{... (vi)}$$

$$AD^2 = \left(\frac{\sqrt{3}}{2}AB\right)^2 + \left(\frac{1}{6}AB\right)^2 \text{... [from (i), (ii) and (vi)]}$$

Ex.6.5 (Q.15)

In an equilateral triangle A

that BD $= \frac{1}{3}$ BC.

Prove that $9AD^2 = 7AB^2$.

$$AD^2 = \left(\frac{\sqrt{3}}{2}AB\right)^2 + \left(\frac{1}{6}AB\right)^2 \quad \text{... [from (i), (ii) and (vi)]}$$

Construction : Draw AP ⊥ BC

Proof :

$$= \frac{3}{4}AB^2 + \frac{1}{36}AB^2$$

$$= AB^2\left(\frac{3}{4} + \frac{1}{36}\right)$$

9

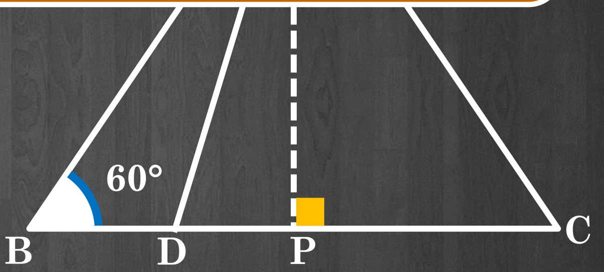
$$= AB^2\left(\frac{3 \times 9 + 1}{36}\right)$$

$$= AB^2\left(\frac{27 + 1}{36}\right)$$

$$= \left(\frac{\cancel{28}^{\,7}}{\cancel{36}_{\,9}}\right)AB^2$$

$$\therefore \; AD^2 = \frac{7}{9}AB^2$$

$$\therefore \; \boxed{9AD^2 = 7AB^2}$$



Thank You