# Quadratic Equations

**1. Introduction to Quadratic equation**

If $p(x)$ is a quadratic polynomial, then $p(x) = 0$ is called a **quadratic equation**.
The general or standard form of a quadratic equation, in the variable $x$, is given by $ax^2 + bx + c = 0$, where $a, b, c$ are real numbers and $a \neq 0$.

**2. Roots of the quadratic equation**

- The value of $x$ that satisfies an equation is called the **zeroes** or **roots** of the equation.
- A real number $\alpha$ is said to be a solution/root of the quadratic equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$.
- A quadratic equation has **at most two roots**.

3. A quadratic equation can be solved by following algebraic methods:
   i. Splitting the middle term (factorization)
   ii. Completing squares
   iii. Quadratic formula

**4. Splitting the middle term (or factorization) method**

- If $ax^2 + bx + c$, $a \neq 0$, can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- Steps involved in solving quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ by **splitting the middle term** (or factorization) method:
  **Step 1:** Find the product $ac$.
  **Step 2:** Find the factors of 'ac' that add to up to $b$, using the following criteria:
  i. If $ac>0$ and $b>0$, then both the factors are positive.
  ii. If $ac>0$ and $b<0$, then both the factors are negative.
  iii. If $ac<0$ and $b>0$, then larger factor is positive and smaller factor is negative.
  iv. If $ac<0$ and $b<0$, then larger factor is negative and smaller factor is positive.
  **Step 3:** Split the middle term into two parts using the factors obtained in the above step.
  **Step 4:** Factorize the quadratic equation obtained in the above step by grouping method. Two factors will be obtained.
  **Step 5:** Equate each of the linear factors to zero to get the value of x.

**5. Completing the square method**

- Any quadratic equation can be converted to the form $(x + a)^2 – b^2 = 0$ or $(x – a)^2 + b^2 = 0$ by adding and subtracting the constant term. This method of finding the roots of quadratic equation is called the method of completing the square.
- The steps involved in solving a quadratic equation by **completing the square**, are as follows:
  **Step 1:** Make the coefficient of $x^2$ unity.
  **Step 2:** Express the coefficient of x in the form $2 \times x \times p$.
  **Step 3:** Add and subtract the square of $p$.
  **Step 4:** Use the square identity $(a + b)^2$ or $(a – b)^2$ to obtain the quadratic equation in the required form $(x + a)^2 - b^2 = 0$ or $(x – a)^2 + b^2 = 0$.
  **Step 5:** Take the constant term to the other side of the equation.
  **Step 6:** Take the square root on both the sides of the obtained equation to get the roots of the given quadratic equation.

**6. Quadratic formula**

The roots of a quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ can be calculated by using the **quadratic formula:**

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}, \text{ where } b^2-4ac \geq 0$$

If $b^2 - 4ac < 0$, then equation does not have real roots.

**7. Discriminant of a quadratic equation**

For the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, the expression $b^2 - 4ac$ is known as **discriminant**.

8. **Nature of the roots** of a quadratic equation:
   i. If $b^2 - 4ac > 0$, the quadratic equation has **two distinct real roots**.
   ii. If $b^2 - 4ac = 0$, the quadratic equation has **two equal real roots**.
   iii. If $b^2 - 4ac < 0$, the quadratic equation has **no real roots**.

9. There are many equations which are not in the quadratic form but can be reduced to the quadratic form by simplifications.

**10. Application of quadratic equations**

- The applications of quadratic equation can be utilized in solving real life problems.
- Following points can be helpful in solving word problems:
  i. Every two digit number '*xy*' where $x$ is a ten's place and $y$ is a unit's place can be expressed as $xy = 10x + y$.
  ii. Downstream: It means that the boat is running in the direction of the stream
  Upstream: It means that the boat is running in the opposite direction of the stream
  Thus, if
  Speed of boat in still water is $x$ km/h
  And the speed of stream is $y$ km/h
  Then the speed of boat downstream will be $(x + y)$ km/h and in upstream it will be $(x - y)$ km/h.
  iii. If a person takes x days to finish a work, then his one day's work = $\frac{1}{x}$