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Algebraic Expressions and Identities

1. What are constant and variable?

- Constant is a symbol that takes fixed numerical value which is represented by letters.
- Variable is a symbol that takes various numerical values which is represented by letters.

2. Algebraic Expression

A combination of constants and variables connected by the operators +, -, $\times$ and $\div$ is known as an algebraic expression.
Example: $15x^2 - 9x + 4xy$ is an algebraic expression.

3. Introduction to Term

Terms are different parts of the algebraic expression which are separated by '+' or '-'
Example: In the expression, $15x^2 - 9x + 4xy$, there are three terms, namely, $15x^2$, 9x and 4xy.

4. Monomial, Binomial and Trinomials

- An algebraic expression having only one term is called a **monomial**.
  Example: –5, 2z, $\frac{1}{2}x^2, 5xyz, -\frac{7}{3}a^3b^2c$ are monomials.
- An algebraic expression having two terms is called a **binomial**.
  Example: x – y, 3x + 5y, $a = \frac{7b}{3}$ are binomials.
- An algebraic expression having three terms is called a **trinomial**.
  Example: a + b + c, x – 2y + 3c, $x^2y + \frac{xy^3}{3} - \frac{5}{7}x^2$ are trinomials.

5. Factors

- An algebraic expression is further divided in to a product of one or more numbers and/or literals.
- These numbers and literals are known as factors of that particular term.
- Consider the example: $\frac{1}{3}xy^3$
  Here, $\frac{1}{3}$, x and y are the factors of the term $\frac{1}{3}xy^3$

6. Numerical factor and Literal factor

- Constant factor of the expression is called **numerical factor**, whereas the variable is called the **literal factor**.
- Example: The expression, $2x - 6xy^3 + 12x^2z^2$, can be rewritten as $2x\left(1 - 3xy^3 + 6xz^2\right)$
  Here, 2 is called the numerical factor and 'x' is called the literal factor.

7. **Coefficient**
A factor of the term is called the coefficient of the term.
Example: In the term '– 5xy', coefficient of x is – 5y, coefficient of y is –5x, Coefficient of xy is –5.

8. **Like Terms**
- Terms whose variables and their exponents are same and may be with different coefficients are called **like terms**.
- Example: Consider the expression, $x^2y + 3xy^2 - \frac{1}{3}xy + \frac{7}{3}xy^2 - 5x^2y$.

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In the above expression, the terms, x^2y and $-5x^2y, 3xy^2$ and $\frac{7}{3}xy^2$ are like terms.

9. Unlike Terms

- Terms which are not like terms are called Unlike Terms.
- Example: Consider the expression, $x^2y + 3xy^2 - \frac{1}{3}xy + \frac{7}{3}xy^2 - 5x^2y$.

  In the above expression, the terms, x^2y and $\frac{7}{3}xy^2, 3xy^2$ and $-5x^2y$ are Unlike terms.

10. Polynomial

An algebraic expression having one or more terms in which the variables involved have only non-negative integral powers is called a polynomial.

Example: $2 - 3x + 5x^2y - \frac{1}{3}xy^3$ is a polynomial.

11. Degree of a Polynomial

- In case of a polynomial in one variable, the highest power of the variable is called the degree of the polynomial.

  Eg: $5x^3 - 7x + \frac{3}{2}$ is a polynomial in x of degree 3.
- In case of a polynomial having more than one variable, the sum of the power of the variables in each term is taken up and the highest sum so obtained is called the degree of the polynomial.
  Eg: $5x^3 - 2x^2y^2 - 3x^2y + 9y$ is a polynomial of degree 4 in x and y.

12. Addition of algebraic expressions

- While adding algebraic expressions, we add the like terms and keep the unlike terms as they are.
- The sum of several like terms is a like term whose coefficient is the sum of the coefficients of those like terms.

13. Subtraction of algebraic expressions

- While subtracting algebraic expressions, change the sign of each term to be subtracted in the second expression and then add.
- The sum of several like terms is a like term whose coefficient is the sum of the coefficients of those like terms.

14. Multiplication of algebraic expressions

- Rule of signs: The product of two factors with the same signs is positive and the product of two factors with different sign is negative.
- Product of two monomials = (product of their coefficients) $\times$ (product of their variables).
- While multiplying a polynomial by a monomial, multiply every term of the polynomial by the monomial.

- While multiplying a polynomial by a binomial (or trinomial, or polynomial), multiply term by term. That is, every term of the polynomial is multiplied by every term in the binomial (or trinomial, or polynomial).

15. Horizontal method of multiplication

We are able to carry out the multiplication term by term using the distributive law.

Consider two binomials, say, $(x + y)$ and $(u + v)$.

Thus, we have, $(x + y) \times (u + v) = x \times (u + v) + y \times (u + v)$.

That is, $(x + y) \times (u + v) = (x \times u + x \times v) + (y \times u + y \times v)$

That is, $(x + y) \times (u + v) = xu + xv + yu + yv$

This method is also known as horizontal method for multiplication.

16. Column method of multiplication

Column method of multiplication of polynomials is very similar to the multiplication of two whole numbers. This method is used when the binomials being multiplied contain terms with like terms.

Example:

$$\begin{array}{rl} 4x + 3y & \\ \times \quad 5x + 2y & \\ \hline 20x^2 + 15xy \quad & \text{multiplying 4x+3y by 5x} \\ +8xy+6y^2 & \text{multipllying 4x+3y by 2y} \\ \hline 20x^2 + 23xy + 6y^2 & \text{adding the like terms} \\ \hline \end{array}$$

17. Dividing a polynomial by a monomial

- Quotient of two monomials = (quotient of their coefficients) $\times$ (quotient of their variables).
- While dividing a polynomial by a monomial, divide every term of the polynomial by the monomial.

18. What is an identity?

- An identity is an equality, which is true for all values of variables in it.
- On the other hand, an equation is true only for certain values of the variables in it. An equation is not an identity.

19. Some standard and important identities:

i. $(a + b)^2 = a^2+2ab+b^2$
ii. $(a – b)^2 = a^2 – 2ab + b^2$
iii. $(a + b)(a – b) = a^2 – b^2$
iv. $(x + a)(x + b) = x^2 + (a + b)x + ab$

20. Use of algebraic identities

- The algebraic identities are useful in carrying out the square and products of algebraic expressions.
- They can also be used as an alternative method to calculate product of numbers.