No. 10

QUADRATIC EQUATIONS

- **Factorization Method Continued…**

Q) Solve the following quadratic equations by factorization method

vi] $y^2 + 2\sqrt{3}y + 3 = 0$

Sol: $1y^2 + 2\sqrt{3}y + 3 = 0$

$\therefore\ y^2 + \sqrt{3}y + \sqrt{3}y + 3 = 0$

$\therefore\ y(y + \sqrt{3}) + \sqrt{3}(y + \sqrt{3}) = 0$

$\therefore\ (y + \sqrt{3})(y + \sqrt{3}) = 0$

$\therefore\ (y + \sqrt{3}) = 0 \text{ or } (y + \sqrt{3}) = 0$

$\therefore\ y = -\sqrt{3} \text{ or } y = -\sqrt{3}$

Standard form

$\sqrt{3} \times \sqrt{3} \times 1 = 3$

$\sqrt{3} \times \sqrt{3} = 3$

To get $2\sqrt{3}$ both factors should have $\sqrt{3}$

'+' sign means adding

Place, $\sqrt{3}$ in both factors

$\sqrt{3} \times \sqrt{3}$

Find product of 3rd no. with 1st no.

...ors of 3 in ...hat by adding ...actors we get middle no.

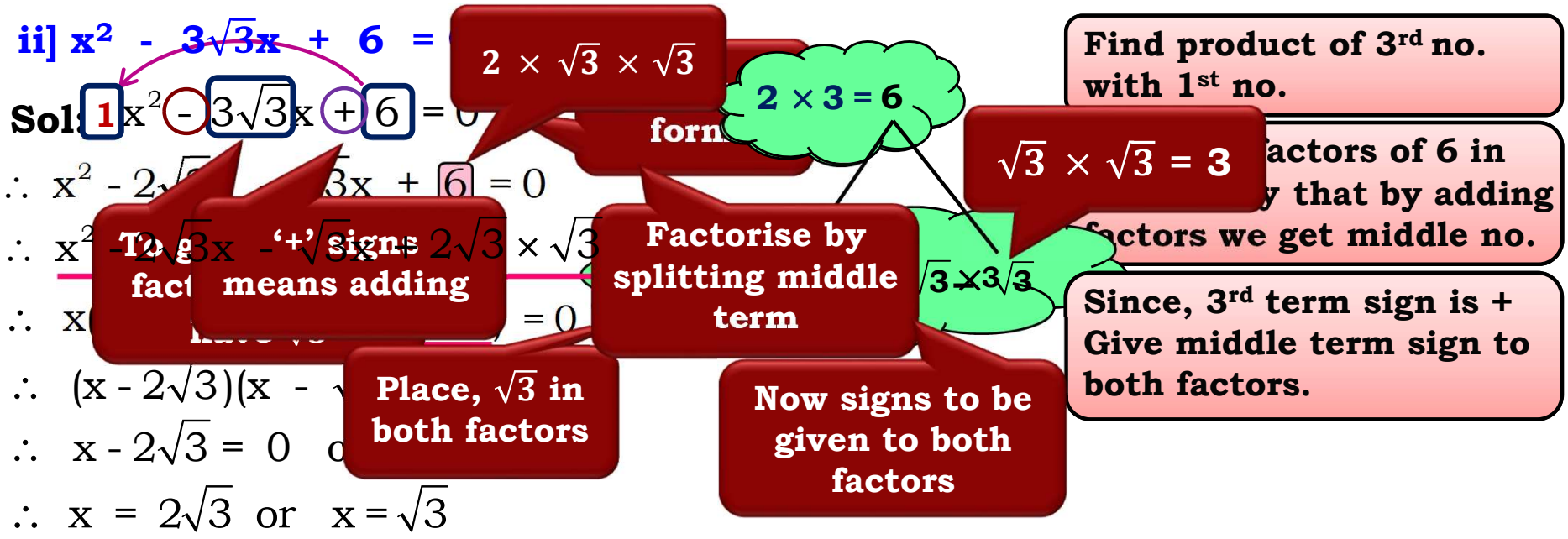
...since, 3rd term sign is + Give middle term sign to both factors.

...by ...iddle term

Now signs to be given to both factors

$\therefore$ **The roots of the given quadratic equations is** $-\sqrt{3}$

Q) Solve the following quadratic equations by factorization method



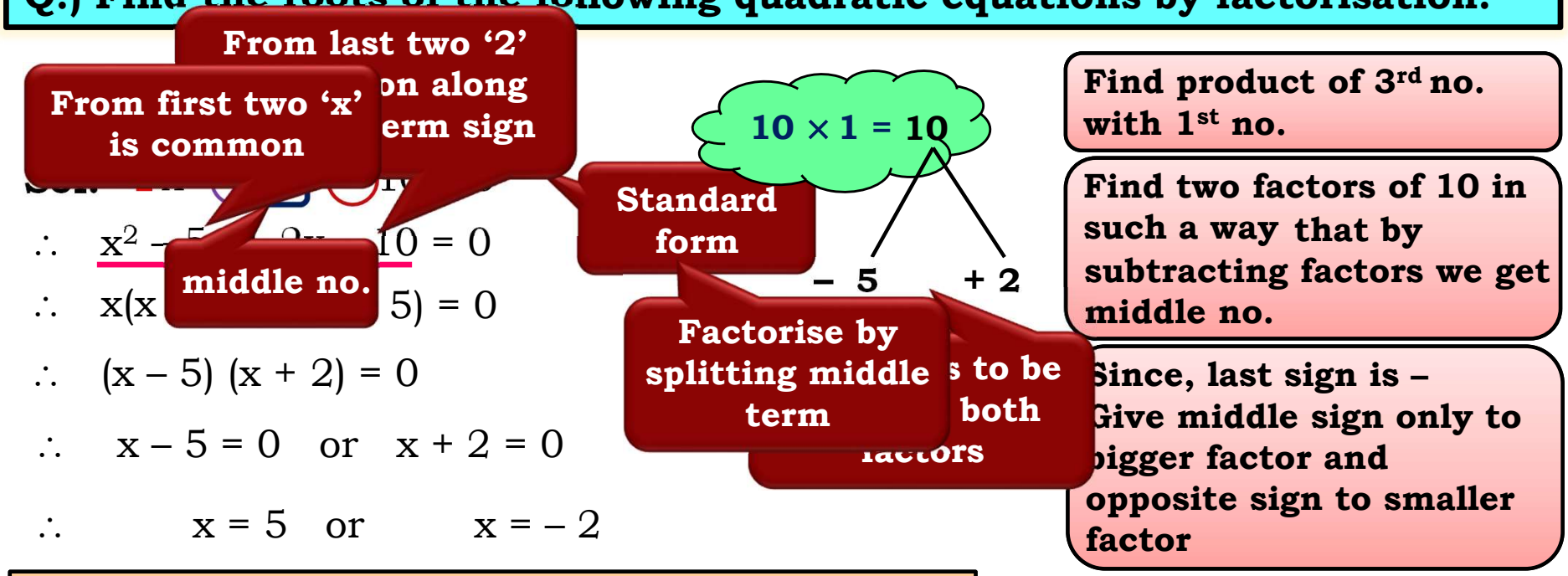
$\therefore$ **The roots of the given quadratic equations are $2\sqrt{3}$ and $\sqrt{3}$**

No. 11

QUADRATIC EQUATIONS

- **Sums based on Factorization Method**

Q.) Find the roots of the following quadratic equations by factorisation:



$\therefore \quad x^2 - \quad 10 = 0$

$\therefore \quad x(x \quad 5) = 0$

$\therefore \quad (x - 5)(x + 2) = 0$

$\therefore \quad x - 5 = 0 \quad \text{or} \quad x + 2 = 0$

$\therefore \quad x = 5 \quad \text{or} \quad x = -2$

$\therefore$ The roots of the given quadratic equations are 5 and – 2

Q.) Fin **ollowing quadratic equations by factorisation:**

From last two '3' on along erm sign

From first two '2x' is common

ndard form

6×2 12 $= 3 \times 4$

$+ 4$

$4 - 3 = 1$

o factorise by splitting iddle term

t of 3rd no.

Find two factors of 12 in such a way that by subtracting factors we get middle no.

Since, last sign is – Give middle sign only to bigger factor & opposite sign to smaller factor

$\therefore \quad 2x^2 + 4x - 3x - 6 = 0$

$\therefore \quad 2x(x + 2) - 3(x + 2) = 0$

Factorise by splitting middle

$\therefore \quad (2x - 3)(x + 2) = 0$

Either $(2x - 3) = 0$ or $(x + 2) = 0$ to be both factors

$\therefore \quad 2x - 3 = 0 \text{ or } x + 2 = 0$

$\therefore \quad 2x = 3 \text{ or } x = -2$

$\therefore \quad x = \frac{3}{2} \text{ or } x = -2$

$\therefore$ The roots of the given quadratic equations are $\frac{3}{2}$ and -2

No. 12

QUADRATIC EQUATIONS

- **Sums based on Factorization Method**

Q.) Find the roots of the following quadratic equations by factorisation:

(iv) $2x^2 - x + \frac{1}{8} = 0$

Sol: $2x^2 - x + \frac{1}{8} = 0$

$\therefore \quad 16x^2 - 8x + 1 = 0$

$\therefore \quad 16x^2 - 4x - 4x + 1 = 0$

$\therefore \quad (4x - 1)(4x - 1) = 0$

$\therefore \quad 4x - 1 = 0 \text{ or } 4x - 1 = 0$

$\therefore \quad x = \frac{1}{4} \text{ or } x = \frac{1}{4}$

From last two '1' ... along ... term sign

From first two '4x' is common

Standard form

1st ... middle no.

Factorise by ...

Either (4x – 1) = 0 or (4x – 1) = 0

adding ... to both ... factors

16 × 1 = 16 = 4 × 4

4 + 4 = 8

4

To factorise by splitting middle term

Find product of 3rd no. ... with 1st no.

... ctors of 16 in ... hat by adding ... get middle no.

Since, last sign is + Give middle sign to both ... the factors

∴ The roots of the given quadratic equations are $\frac{1}{4}$ and $\frac{1}{4}$

Q.) Fin **ollowing quadratic equations by factorisation:**

From last two '1' on along erm sign

From first two '10x' is common

middle no.

$\therefore$ $10x + 1 = 0$

$\therefore$ $(10x - 1)$

$\therefore$ $(10x - 1)(10x - 1) = 0$

$\therefore$ $10x - 1 = 0$ or $10x - 1 =$

$\therefore$ $x = \frac{1}{10}$ or $x = \frac{1}{10}$

Factorise by spli

Product of two brackets is zero

$1 \times 100 = 100 = 10 \times 10$

$10 + 10 = 20$

actorise by splitting dle term

of 3rd no.

Find two factors of 100 in such a way that by adding factors we get middle no.

Since, last sign is + Give middle sign to both the factors

$\therefore$ The roots of the given quadratic equations are $\frac{1}{10}$ and $\frac{1}{10}$

No. 13

QUADRATIC EQUATIONS

- **Factorization Method Continued …**
- **Factorization Method sums with 2 Terms**

Q) Solve the following quadratic equations by factorization method

vii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Sol: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\therefore \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\therefore \sqrt{2}x^2 + \sqrt{2} \times \sqrt{2}x + 5$

$\therefore \sqrt{2}x(x + \sqrt{2}) + 5$

$\therefore (x + \sqrt{2})($

$\therefore x + \sqrt{2} = 0$ or $\sqrt{2}x + 5 = 0$

$\therefore x = -\sqrt{2}$ or $x = \frac{-5}{\sqrt{2}}$

$\therefore$ **The roots of the given quadratic equations are** $-\sqrt{2}$ **and** $\frac{-5}{\sqrt{2}}$

$\sqrt{2} \times 5\sqrt{2} = 10$

$\times 10$

d product of 3rd no. ith 1st no.

factors of 10 in that by adding get middle no.

10

$1 + 10 = 7$

$2 + 5 = 7$

+5

+2

Factorise by splitting middle term

st sign is + Gi middle sign to both

Now signs to be given to both the factors

Q) Solve the following quadratic equations by factorization method

x) $9x^2 - 16 = 0$

Sol : $9x^2 - 16 = 0$

$\therefore (3x)^2 - (4)^2 = 0$

$\therefore (3x + 4)(3x - 4) = 0$

$\therefore 3x + 4 = 0$ or $3x - 4 = 0$

$\therefore x = -\frac{4}{3}$ or $x = \frac{4}{3}$

$\therefore$ **The roots of the given quadratic equations are** $-\frac{4}{3}$ **and** $\frac{4}{3}$

Only first & last terms are present

Factorise using $a^2 - b^2$

$a^2 - b^2 = (a + b)(a - b)$

xi) $3x - x^2 = 0$

$3x - x^2 = 0$

$\therefore 0 = x^2 - 3x$

$\therefore x^2 - 3x = 0$

$\therefore x(x - 3) = 0$

$\therefore x = 0$ or $(x - 3) = 0$

$\therefore x = 0$ or $x = 3$

roots of th... equations are 0 ar...

Arrange in a standard form

Only first and the middle terms are present.

Factorise by taking common

No. 14

QUADRATIC EQUATIONS

- **Completing The Squares Method**

2] Completing the Square Method

Lets us first understand what is a complete square ?

With regards to numbers

Are complete squares

100	$100 = 10^2$
25	$25 = 5^2$
4	$4 = 2^2$

With regards to expressions

	Are complete squares because
$x^2 + 4x + 4$	$x^2 + 4x + 4 = (x + 2)^2$
$x^2 - 6x + 9$	$x^2 - 6x + 9 = (x - 3)^2$
$x^2 - 10x + 25$	$x^2 - 10x + 25 = (x - 5)^2$

Solve the equation $z^2 + 6z = 0$ by completing the squares

Suppose the equation was $z^2 + 6z = 0$

Because it is $(z + 3)^2$

If we add 9 to both sides, we get

$z^2 + 6z + 9 = 0 + 9$

9 which is the third term is obtained by using the formula

The L.H.S of the equation is a complete square

$$\text{Third term} = \left(\frac{1}{2} \times \text{coefficient of } z\right)^2$$

Because it is $(z + 3)^2$

$$= \left[\frac{1}{2} \times 6\right]^2$$

9 which is the third term is obtained by using the formula

$$= 3^2$$

$$= 9$$

$z^2 + 6z + 9 = 0 + 9$

$z^2 + 6z + 9 - 8 = 0 - 8$

The L.H.S of the equation is not a complete square

The L.H.S of the equation is a complete square

No. 15

QUADRATIC EQUATIONS

- **Sums based on Completing The Squares Method Contd....**

Q) Solv… uadratic equations by completing square.

i) z^2 +…

Sol: $1z^2 + 6z - 8 = 0$

Coefficient of 1st term is 1

$\therefore$ $z^2 + 6z = 8$ ……(i)

To express LHS in square form

Third …nt of z $\Big)^2$

Take middle term … square root … 3rd term

Make it whole square

Ad… …we get

$z^2 + … 9 = 8 + 9$

$\therefore (z + 3)^2 = 17$

Taking square roots on both sides

$\therefore \quad z + 3 = \pm\sqrt{17}$

$\therefore \quad z = -3 \pm \sqrt{17}$

$\therefore z = -3 + \sqrt{17}$ or $z = -3 - \sqrt{17}$

STEPS

1) Coefficient of square term should be 1
2) Constant on the RHS
3) Find the third term
4) Add third term on both the sides
5) Express L.H.S. in square form.
6) Take square root & write the solution

$\therefore$ **The roots of the given quadratic equations are $-3 + \sqrt{17}$ and $-3 - \sqrt{17}$**

Q) Solve the following quadratic equations by completing square.

ii) $x(x-1) = 1$

Sol: $x(x-1) = 1$

$\therefore 1x^2 - 1x = 1$ .....(i)

Third term = $\left(\frac{1}{2} \text{ of coefficient of } x\right)^2$

$= \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

To express LHS in square form

Take middle term

Make it whole square

Add third term on both sides of (i) to get

$x^2 - x + \frac{1}{4} = 1 + \frac{1}{4}$

$\therefore \left(x - \frac{1}{2}\right)^2 = \frac{5}{4}$

Taking square roots on both sides

$\therefore x - \frac{1}{2} = \pm\frac{\sqrt{5}}{2}$

STEPS

1) Coefficient of first term should be 1
2) Constant on the RHS
3) Find the third term
4) Add third term on both the sides
5) Express L.H.S. in square form.
6) Take square root & write the solution

$\therefore x = \frac{1}{2} \pm \frac{\sqrt{5}}{2} = \frac{1 \pm \sqrt{5}}{2}$

$\therefore x = \frac{1+\sqrt{5}}{2}$ or $x = \frac{1-\sqrt{5}}{2}$

$\therefore$ **The roots of the given quadratic equations are $\frac{1+\sqrt{5}}{2}$ and $\frac{1-\sqrt{5}}{2}$**

No. 16

QUADRATIC EQUATIONS

- **Sums based on Completing The Squares Method Contd....**

Q. Solve ... adratic equations by completing square.

iii) $4p^2$

Sol : $4p^2 + 7 = 12p$

Dividing throughout by 4, we get

$\therefore p^2 + \frac{7}{4} = 3p$

$\therefore p^2 - 3p = -\frac{7}{4}$ .....(i)

Third ... ient of p $\Big)^2$

$\left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

Add ... get

$p^2 - 3p + \frac{9}{4} = -\frac{7}{4} + \frac{9}{4}$

$\therefore \left(p - \frac{3}{2}\right)^2 = \frac{2}{4}$

Coefficient of 1st term is not 1

Divide 4 by 4 to make it 1

To express LHS

Take square root of 1st term

sign

... square root ... term

Make it whole square

STEPS

1) Coefficient of first term should be 1
2) Constant on the RHS
3) Find the third term
4) Add third term on both the sides
5) Express L.H.S. in square form.
6) Take square root & write the solution

Taking square roots on both sides

$\therefore p - \frac{3}{2} = \pm\frac{\sqrt{2}}{2}$

$\therefore p = \frac{3}{2} \pm \frac{\sqrt{2}}{2} = \frac{3 \pm \sqrt{2}}{2}$

$\therefore$ The roots of the given quadratic equations are $\frac{3+\sqrt{2}}{2}$ and $\frac{3-\sqrt{2}}{2}$

Q) Solve the following quadratic equations by completing square.

v) $4x^2 + 4\sqrt{3}x + 3 = 0$

Sol : $4x^2 + 4\sqrt{3}x + 3 = 0$

Dividing throughout by 4, we get

x^2

$\therefore$ ... (i)

Third term $= \left(\frac{1}{2} \times \text{coefficient of } x\right)^2$

$= \left(\frac{1}{2} \times \sqrt{3}\right)^2$

Coefficient of 1st term is not 1

Take middle term

square root

rd term

Make it whole square

Addi ... get

$\therefore$ $x^2 + \sqrt{\quad} + \frac{3}{4} - \frac{3}{4}$

$\therefore$ $\left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$

STEPS

1) Coefficient of square term should be 1
2) Constant on the RHS
3) Find the third term
4) Add third term on both the sides
5) Express L.H.S. in square form.
6) Take square root & write the solution

Taking square roots on both sides.

$x + \frac{\sqrt{3}}{\quad} = 0$

To express LHS in square form

$\frac{\sqrt{3}}{2}$

$\therefore$ **The root of the given quadratic equation is** $-\frac{\sqrt{3}}{2}$

No. 17

QUADRATIC EQUATIONS

- **Completing The Squares Method Contd...**

Q) Solve ... adratic equations by completing square.

iv) $3p^2 + 4 = -7p$

Sol : $3p^2 + 4 = -7p$

Coefficient of 1st term is not 1

Dividing throughout by 3, we get

$\therefore \frac{3p^2}{3} + \frac{4}{3} = \frac{-7p}{3}$

Divide 3 by 3 to make it 1

$\therefore p^2 + \frac{7}{3}p = -\frac{4}{3}$ .....(i)

Third term $= \left(\frac{1}{2} \times \text{coefficient of } p\right)^2$

To express LHS in square form

$= \left(\frac{1}{2} \times \frac{7}{3}\right)^2 = \frac{49}{36}$

Add ... square root ... get

Take middle term sign

... rd term

$\therefore p^2 + \frac{7}{3}p + \frac{49}{36} = -\frac{4}{3} + \frac{49}{36}$

STEPS

1) Coefficient of first term should be 1
2) Constant on the RHS
3) Find the third term
4) Add ... both the sides
5) Express ... square form.
6) Take square root & write the solution

$\therefore \left(p + \frac{7}{6}\right)^2 = \frac{-48 + 49}{36}$

$\therefore \left(p + \frac{7}{6}\right)^2 = \frac{1}{36}$

Taking square roots on both sides

Make it whole square

$\therefore p + \frac{7}{6} = \pm\frac{1}{6}$

$\therefore \left(p \pm \frac{7}{6}\right)^2 = \frac{1}{36}$

$\therefore p + \frac{7}{6} = \frac{-1}{6}$ or $p + \frac{7}{6} = \frac{1}{6}$

$\therefore p = \frac{-7-1}{6} = \frac{-8}{6}$ or $p = \frac{-7+1}{6} = \frac{-6}{6}$

Taking square roots on both sides

$\therefore p = -\frac{4}{3}$ or $p = -1$

$\therefore$ **The roots of the given quadratic equations are $-\frac{4}{3}$ and -1**

No. 18

QUADRATIC EQUATIONS

- **Completing The Squares Method Contd...**

Solve $ax^2 + bx + c = 0$ by completing square method.

$ax^2 + bx + c = 0$

Sol: $ax^2 + bx + c = 0$

Dividing throughout by a, we get

Divide 'a' by 'a' to make it 1

$\therefore x^2 + \frac{b}{a}x = -\frac{c}{a}$ .....(i)

Third term $= \left(\frac{1}{2} \times \text{coefficient of } x\right)^2$

$= \left(\frac{1}{2} \times \frac{b}{a}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

Adding '$\frac{b^2}{4a^2}$' on both sides of (i)

$\therefore x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a} \times \frac{4a}{4a}$

6) Take square root & write the solution

$\therefore \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$

$\therefore \left(x + \frac{b}{2a}\right)^2 = \frac{(b^2 - 4ac)}{4a^2}$

Same denominator, take it common

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

This is the formula to find value of 'x'

$\therefore$ **$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$**

No. 19

QUADRATIC EQUATIONS

- **Sum based on Completing The Squares Method**

Q.) Find the roots of the following quadratic equations, if they exist by the method of completing the square :

i) $2x$

Sol: $2x^2 - 7x + 3 = 0$

Coefficient of 1st term is not 1

Dividing throughout by 2, we get

Divide 2 by 2 to make it 1

x^2

$\therefore$ ..... (1)

Third term $= \left(\frac{1}{2} \times \text{co-efficient of } x\right)^2$

Take middle term

uare root term

$\frac{49}{16}$

Adding tion (1), we get

Make it whole square

$x^2 - \frac{7}{2}x$ $\frac{49}{16}$

$\therefore \quad \left(x - \frac{7}{4}\right)^2 = \frac{-3 \times 8}{2 \times 8} + \frac{49}{16}$

$= \frac{-24 + 49}{16}$

$\therefore \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

To express LHS in square form

both sides,

$\therefore$ 4

$x = \frac{7}{4} \pm \frac{5}{4}$

STEPS

1) Coefficient of first term should be 1
2) Constant on the RHS
3) Find the third term
4) Add third term on both the sides
5) Express L.H.S. in square form.
6) Take square root & write the solution

$\therefore$ **The roots of the given quadratic equation are 3 and $\frac{1}{2}$.**

No. 20

QUADRATIC EQUATIONS

- **Sum based on Completing The Squares Method**

Q.) Find the roots of the following quadratic equations, if they exist by the method of completing the square :

Coefficient of 1st term is not 1

ii) 2

Sol: $2x^2 + x - 4 = 0$

Dividing throughout by 2, we get

Divide 2 by 2 to make it 1

$\therefore$ ..... (1)

Third term $= \left(\frac{1}{2} \times \text{co-efficient of } x\right)^2$

Take middle term

uare root term

Adding tion (1), we get

Make it whole square

$x^2 + \frac{1}{2}x$

$\therefore \quad \left(x + \frac{1}{4}\right)^2 = \frac{32+1}{16}$

$\therefore \quad \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$

Taking square root on both sides,

$\therefore \quad x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

$\therefore \quad x = \frac{-1}{4} \pm \frac{\sqrt{33}}{4}$

$\quad\quad\quad -1 \pm \sqrt{33}$

STEPS

1) Coefficient of first term should be 1
2) Constant on the RHS
3) Find the third term
4) Add third term on both the sides
5) Express L.H.S. in square form.
6) Take square root & write the solution

No. 21

QUADRATIC EQUATIONS

- **Sum based on Completing The Squares Method**

Q.) Find the roots of the following quadratic equations, if they exist by the method of completing the square :

Coefficient of 1st term is not 1

(iv) $2x^2 + x + 4 = 0$

Sol: $2x^2 + x + 4 = 0$

Dividing throughout by 2, we get

Divide 2 by 2 to make it 1

$\therefore \quad x^2 + \frac{1}{2}x = -2 \quad \ldots\ldots (1)$

$$\text{Third term} = \left(\frac{1}{2} \times \text{co-efficient of } x\right)^2$$

Take middle term sign … square root … term

Make it whole square

Adding … to both sides of (1), we get

$$x^2 + \frac{1}{2}x + \frac{1}{16} = -2 + \frac{1}{16}$$

$$\therefore \quad \left(x + \frac{1}{4}\right)^2 = \frac{-32 + 1}{16}$$

STEPS

1) Coefficient of first term should be 1
2) Constant on the RHS
3) Find the third term
4) Add third term on both the sides
5) Express L.H.S. in square form.

To express LHS in square form

$$\left(x + \frac{1}{4}\right)^2 = \frac{-31}{16} < 0$$

But square of a real number cannot be negative,

$\therefore$ **The given quadratic equation has no real roots OR roots do not exist.**

Thank You