

Module

1

AREAS RELATED TO CIRCLE

- **Introduction**
- **Formula : Area and circumference
of circle and semicircle**

CIRCLE

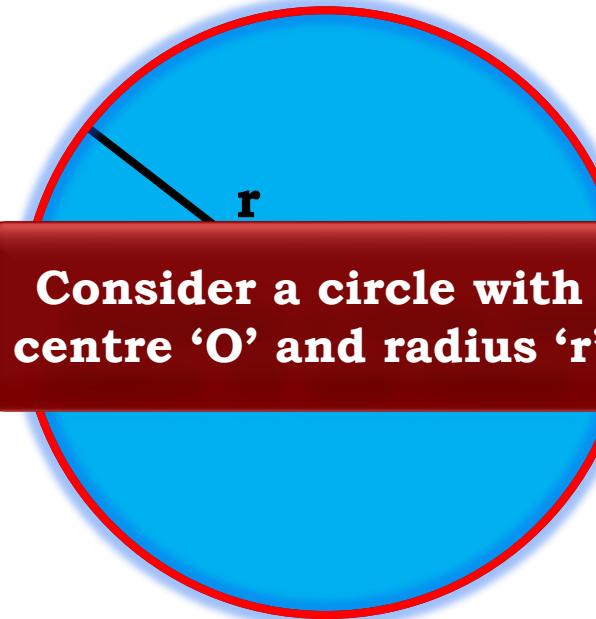
$$\text{Area of circle} = \pi r^2$$

$$\text{Circumference of the circle} = 2\pi r$$

or

$$= \pi d$$

Consider a circle with centre 'O' and radius 'r'

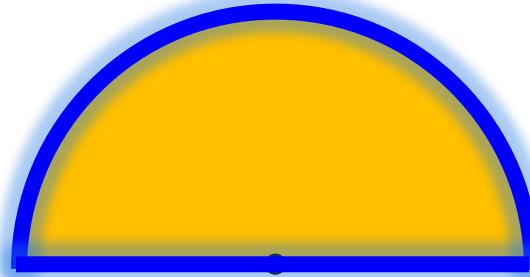


We know that,
Diameter = $2r$

Half

SEMI -CIRCLE

We know that,
Diameter = $2r$



$$\text{Area of semicircle} = \frac{1}{2} \times \text{Area of circle}$$

$$\text{Area of semicircle} = \frac{1}{2} \pi r^2$$

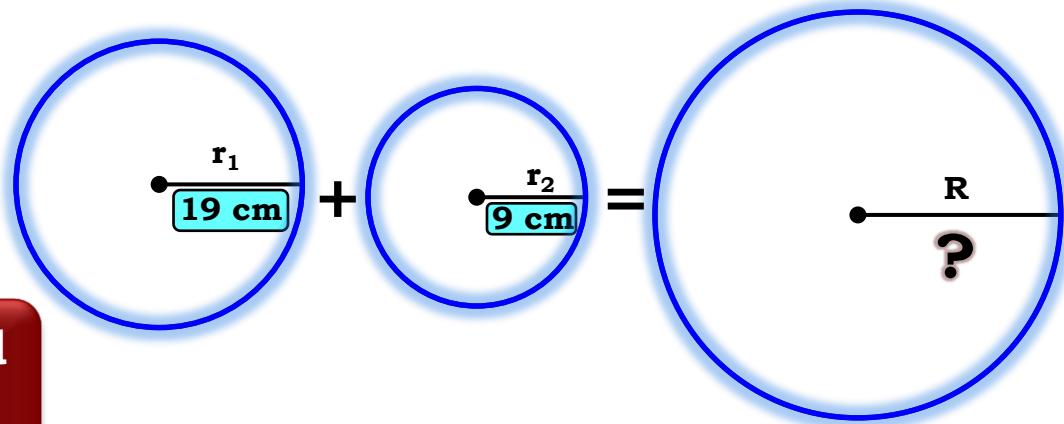
$$\begin{aligned}\text{Perimeter of semicircle} &= \pi r + d \\ &= \pi r + 2r\end{aligned}$$

Q. The radii of two circles are 19 cm and 9 cm respectively.

Find the radius of the circle which has circumference equal to the sum of the circumference of the two circles.

Sol. Let the radius of the bigger circle be R .

$$\therefore 2\pi R = 2\pi r_1 + 2\pi r_2$$



What is the formula to find circumference $2\pi r$ of a circle?

\therefore $2\pi R = 2\pi r_1 + 2\pi r_2$

$$\therefore R = 19 + 9$$

$$\therefore R = 28 \text{ cm}$$

\therefore Radius of the bigger circle is 28 cm.

Q. The radii of two circles are 8 cm and 6 cm respectively.

Find the radius of the circle having area equal to the sum of the areas of the two circles.

Sol. Let the radius of the bigger circle be R

$$\therefore \pi R^2 = \pi r_1^2 + \pi r_2^2$$

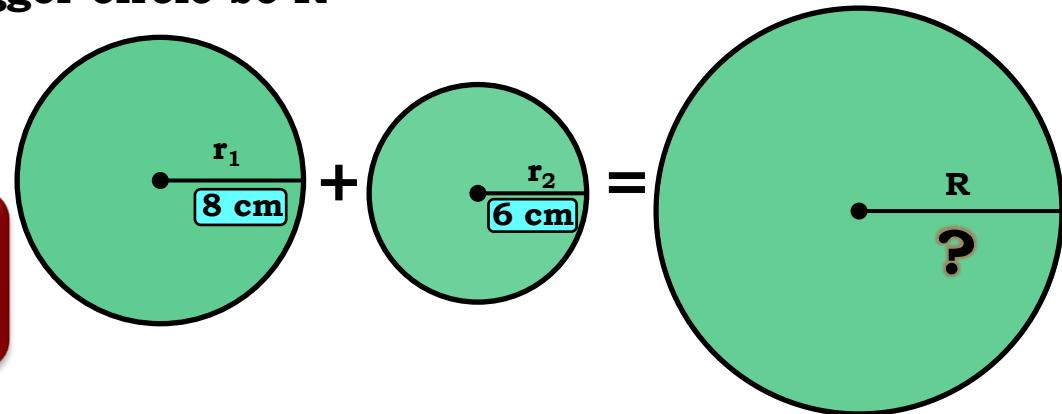
$$\therefore \cancel{\pi} R^2 = \cancel{\pi} (r_1^2 + r_2^2)$$

**What is the formula
to find πr^2 area of a
circle?**

$$\therefore R^2 = 100$$

$$\therefore R = 10 \text{ cm}$$

∴ Radius of the bigger circle is 10 cm



**Q. If the perimeter and the area of a circle are numerically equal,
then find the radius of circle.**

Sol. Perimeter of a circle = Area of the circle

$$\therefore 2\pi r = \pi r^2$$

$$\therefore 2r = r^2$$

$$\therefore 2 = \frac{r^2}{r} \quad [\because r \neq 0]$$

$$\therefore r = 2 \text{ units.}$$

Module 2

AREAS RELATED TO CIRCLE

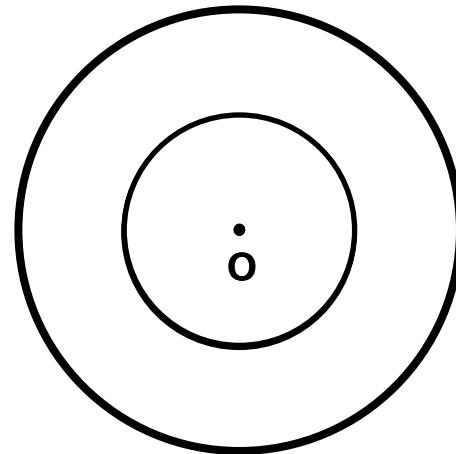
- Sum based on Area of circle

UNDERSTAND!

$\text{Ar (shaded portion)} = \text{Ar (bigger circle)} - \text{Ar (smaller circle)}$



Let us understand
how to find area of
shaded region ?



Q. The following figure depicts an archery target marked with its five scoring regions. Let us find area of Gold circular region Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and the width of all the bands is 10.5 cm wide. Area of circle = πr^2 . Find the area of each of the five scoring regions.

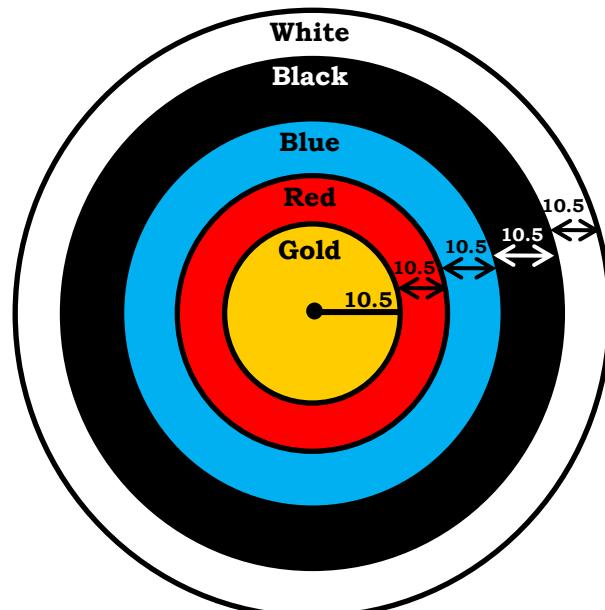
Sol.

$$\text{Radius of gold circular region } (r_1) = \frac{21}{2} = 10.5 \text{ cm}$$

$$\begin{aligned}
 \text{Area of gold circular region} &= \pi r_1^2 \\
 &= \frac{22}{7} \times 10.5 \times 10.5 \\
 &= 22 \times 1.5 \times 10.5 \\
 &= 346.5 \text{ cm}^2
 \end{aligned}$$

∴

Area of gold circular region is 346.5 cm^2



Q. The following figure depicts an archery target marked with its five scoring rings. Let us find radius of red circular region Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and the width of the Red ring is 1.5 cm wide. Find the area of Red ring.

Sol.

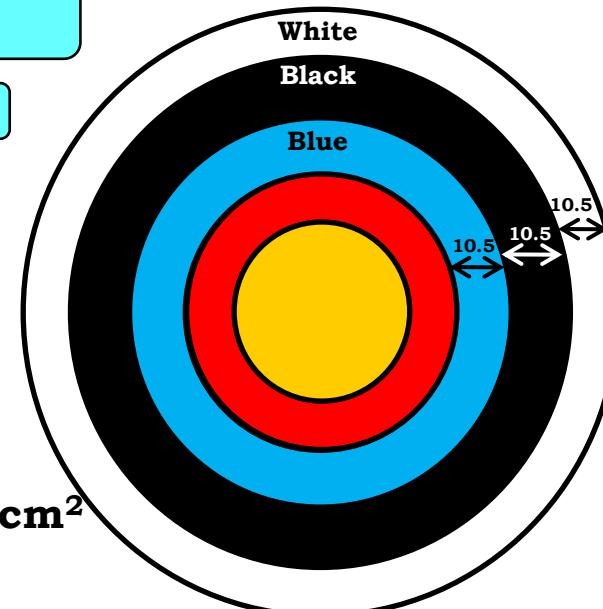
$$A(\text{Red ring}) = A(\text{Red circular region}) - A(\text{Gold circular region})$$

$$\text{Radius of gold circular region } (r_1) = \frac{21}{2} = 10.5 \text{ cm}$$

$$\text{Radius of red circular region } (r_2) = r_1 + 10.5 = 21 \text{ cm}$$

$$\begin{aligned}\text{Area of red ring} &= \pi r_2^2 - \pi r_1^2 = \pi(r_2^2 - r_1^2) \\&= \frac{22}{7} \times (21^2 - 10.5^2) \\&= \frac{22}{7} \times (21 + 10.5)(21 - 10.5) \\&= \frac{22}{7} \times 31.5 \times 10.5 = 1039.5 \text{ cm}^2\end{aligned}$$

\therefore Area of red ring is 1039.5 cm^2



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Module

AREAS RELATED TO CIRCLE

- Sum based on Area of circle (part-II)

Q. The following figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. Let us find area of Blue ring presenting Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the radius of blue circular region.

Sol.

$$A(\text{Blue ring}) = A(\text{Blue circular region}) - A(\text{Red circular region})$$

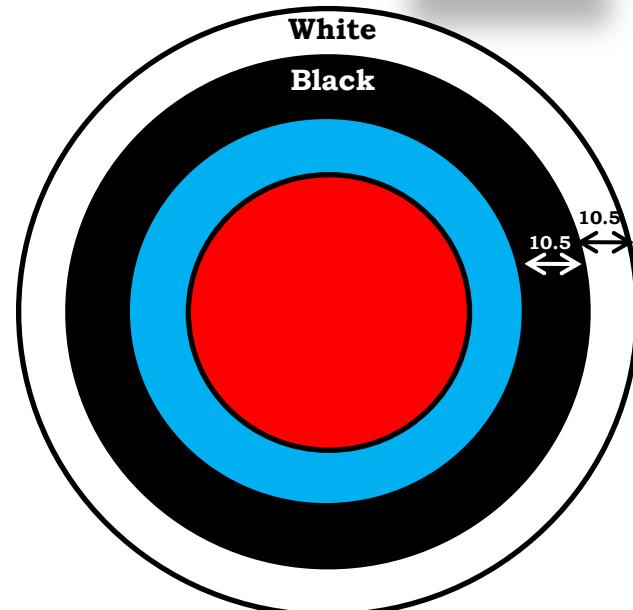
$$r_2 = 21$$

$$\text{Radius of blue circular region } (r_3) = r_2 + 10.5$$

$$\therefore r_3 = 31.5 \text{ cm}$$

$$\begin{aligned} A(\text{Blue ring}) &= \pi r_3^2 - \pi r_2^2 = \pi(r_3^2 - r_2^2) \\ &= \frac{22}{7} \times (31.5^2 - 21^2) \\ &= \frac{22}{7} \times (31.5 + 21)(31.5 - 21) \\ &= \frac{22}{7} \times 52.5 \times 10.5 = 1732.5 \end{aligned}$$

\therefore Area of blue ring is 1732.5 cm^2



Q. The following figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. Let us find area of Black ring representing Gold score is 21 cm and each of the other bands is 10.5 cm wide.

Find the radius of black circular region

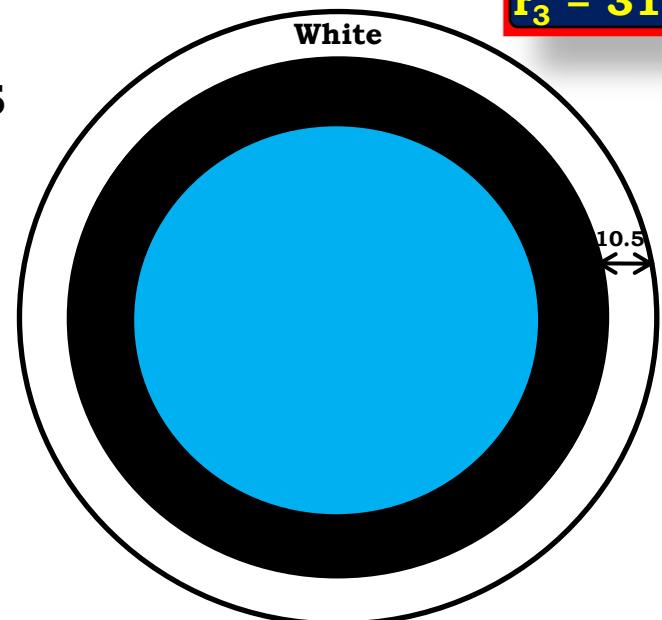
$$A(\text{Black ring}) = A(\text{Black circular region}) - A(\text{Blue circular region})$$

$$\begin{aligned}\text{Sol. Radius of black circular region } (r_4) &= r_3 + 10.5 \\ &= 31.5 + 10.5 \\ \therefore r_4 &= 42 \text{ cm}\end{aligned}$$

$$\begin{aligned}\therefore A(\text{Black ring}) &= \pi r_4^2 - \pi r_3^2 = \pi(r_4^2 - r_3^2) \\ &= \frac{22}{7} \times (42^2 - 31.5^2) \\ &= \frac{22}{7} \times (42 + 31.5)(42 - 31.5) \\ &= \frac{22}{7} \times 73.5 \times 10.5 = 2425.5 \text{ cm}^2\end{aligned}$$

∴ Area of black ring is 2425.5 cm²

$$r_3 = 31.5$$



Q. The following figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm. Let us find area of White ring. It is 10.5 cm wide. Find the radius of White circular region.

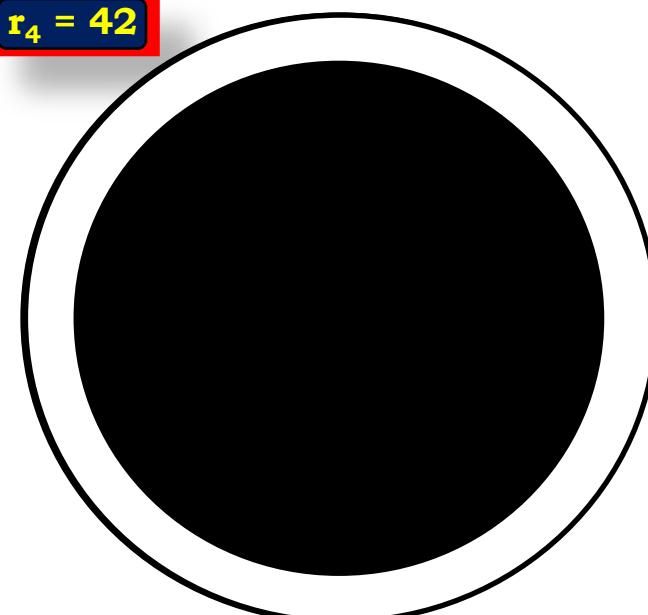
Sol. A (White ring) = A (White circular region) – A (Black circular region)

$$\begin{aligned}\text{Radius of white circular region } (r_5) &= r_4 + 10.5 \\ &= 42 + 10.5\end{aligned}$$

$$\therefore r_5 = 52.5 \text{ cm}$$

$$\begin{aligned}\therefore A (\text{White ring}) &= \pi r_5^2 - \pi r_4^2 = \pi (r_5^2 - r_4^2) \\ &= \frac{22}{7} \times (52.5^2 - 42^2) \\ &= \frac{22}{7} \times (52.5 + 42)(52.5 - 42) \\ &= \frac{22}{7} \times 94.5 \times 10.5 = 3118.5 \text{ cm}^2\end{aligned}$$

∴ Area of white ring is 3118.5 cm^2



4

Module

AREAS RELATED TO CIRCLE

- **Sum based on Area and
Perimeter of semicircle**

Q. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in given figure, Find the perimeter and area of the shaded region.

Sol. Radius (r_1) = 6 cm

$$\therefore \text{Diameter (PS)} = 12 \text{ cm}$$

$$\therefore PQ = QR = RS = \frac{PS}{3} \\ = \frac{12}{3}$$

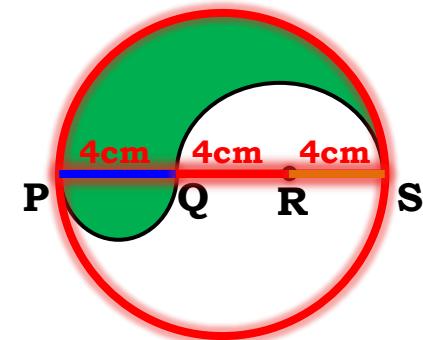
$$\therefore PQ = QR = RS = 4 \text{ cm}$$

$$QS = QR + RS$$

$$= (4 + 4) \text{ cm}$$

$$QS = 8 \text{ cm}$$

Diameter PS is divided into three equal parts

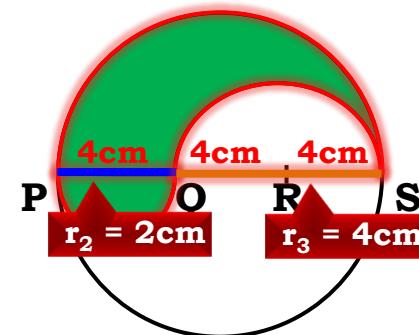


Q. F. What is the formula to find circumference of a semicircle ?

radius 6 cm. The lengths PQ, QR and RS are drawn on PQ and QS as diameter, Find the perimeter and area of the shaded region.

Sol.

$$\begin{aligned}
 \text{Perimeter of shaded region} &= \text{circumference of semi-circle with diameter PS} \\
 &\quad + \text{circumference of semi-circle with diameter PQ} \\
 &\quad + \text{circumference of semi-circle with diameter QS} \\
 &= (\pi r_1 + \pi r_2 + \pi r_3) \\
 &= \pi(r_1 + r_2 + r_3) \\
 &= \pi(6 + 2 + 4) \\
 &= \frac{22}{7} \times 12 \\
 &= \frac{264}{7} = 37.71 \text{ cm}
 \end{aligned}$$

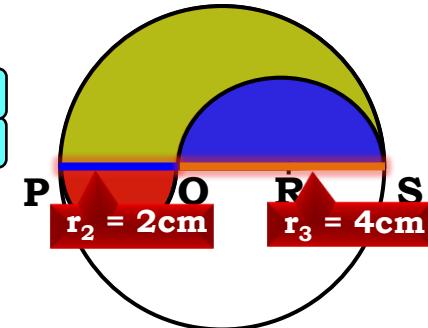


∴ Perimeter of shaded region is 37.71cm

Q. **Two concentric circles have a common center O and radius 6 cm. The lengths PQ, QR and RS are drawn on PQ and QS as diameters of semicircles. If the angles are drawn on PQ and QS as diameters of semicircles, find the perimeter and area of the shaded region.**

Sol. Area of shaded region =

$$\begin{aligned}
 & \text{ar(semi-circle with diameter PS)} \\
 & + \text{ar(semi-circle with diameter PQ)} \\
 & - \text{ar(semi-circle with diameter QS)} \\
 = & \frac{1}{2} \times \pi r_1^2 + \frac{1}{2} \times \pi r_2^2 - \frac{1}{2} \times \pi r_3^2 \\
 = & \frac{1}{2} \times \pi (r_1^2 + r_2^2 - r_3^2) \\
 = & \frac{1}{2} \times \pi (6^2 + 2^2 - 4^2) \\
 = & \frac{1}{2} \times \frac{22}{7} (36 + 4 - 16) \\
 = & \frac{1}{2} \times \frac{22}{7} \times 24^{12} \\
 = & \frac{264}{7} = 37.71 \text{ cm}^2
 \end{aligned}$$



\therefore Area of shaded region is 37.71 cm^2

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Module

AREAS RELATED TO CIRCLE

- **Sum based on Area of circle and semicircle**

Q. Seg AB and seg CD are perpendicular diameters of a circle with radius 7 cm. Find the area of the shaded region.

Sol.

$$\text{ar(shaded region)} = \text{ar (smaller circle)} + \text{ar (semi-circle CBD)} - \text{ar}(\Delta CBD)$$

Diameter of smaller circle = 7 cm

∴

$$\text{its radius (r)} = \frac{7}{2} \text{ cm}$$

$$\text{ar (smaller circle)} = \pi r^2$$

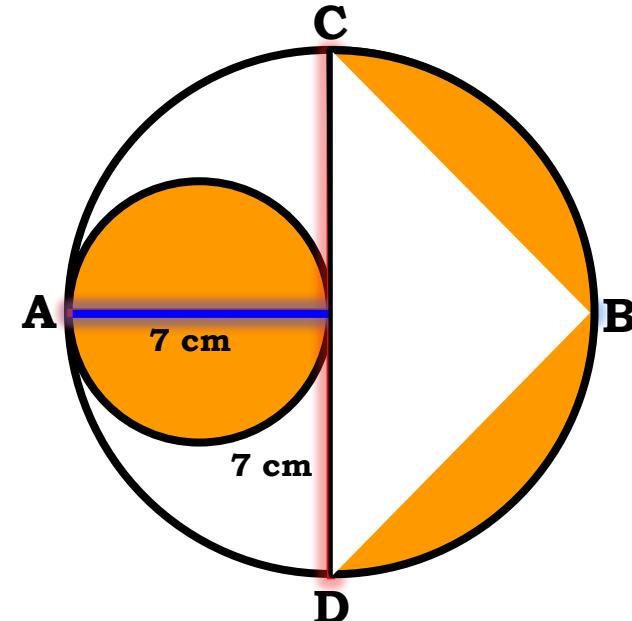
$$OA = OB = OC = OD = 7 \text{ cm}$$

What is the formula to find area of circle?

$\frac{1}{2}$

$$= 38.5 \text{ cm}^2$$

$$\therefore \text{ar (smaller circle)} = 38.5 \text{ cm}^2$$



Q. Seg AB and seg CD are perpendicular diameters of a circle with radius 7 cm. Find the area of the shaded region.

Sol.

$$\text{ar (shaded region)} = \text{ar (smaller circle)} + \text{ar (semi-circle CBD)} - \text{ar} (\Delta CBD)$$

$$\text{ar (smaller circle)} = 38.5 \text{ cm}^2$$

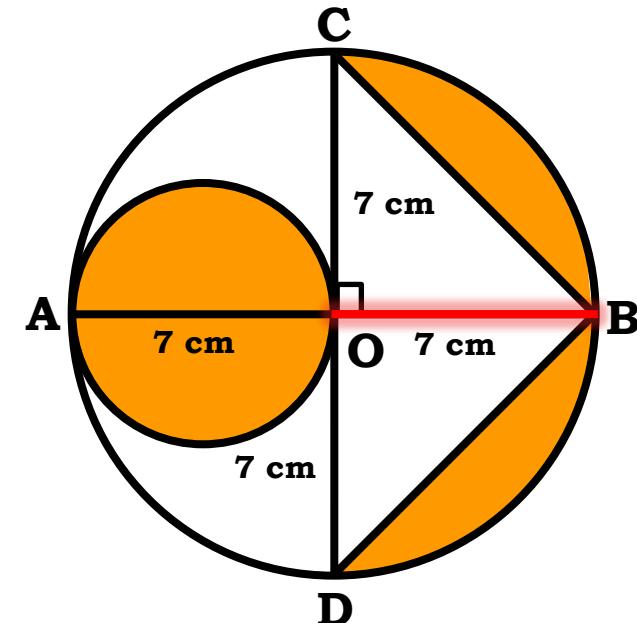
$$\text{Radius of semicircle CBD (R)} = 7 \text{ cm}$$

$$\text{Area of the semicircle CBD} = \frac{1}{2} \times \pi R^2$$

What is the formula to find
area of semi-circle ?

$$= 11 \times 7$$

$$\therefore \text{Area of the semicircle CBD} = 77 \text{ cm}^2$$



Q. Seg AB and seg CD are perpendicular diameters of a circle with radius 7 cm. Find the area of the shaded region.

Sol.

$$\text{ar (shaded region)} = \text{ar (smaller circle)} + \text{ar (semi-circle CBD)} - \text{ar } (\Delta CBD)$$

$$\text{ar (semicircle CBD)} = 77 \text{ cm}^2$$

Which is the formula to
find area of triangles?

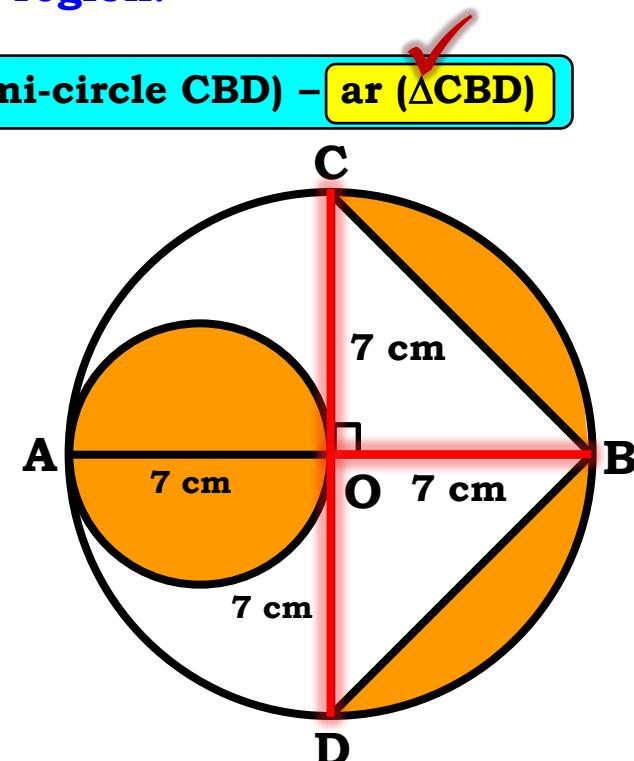
$$\text{ar } (\Delta CBD) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times CD \times OB$$

$$= \frac{1}{2} \times \cancel{14} \times 7$$

$$= 7 \times 7$$

$$\therefore \text{ar } (\Delta CBD) = 49 \text{ cm}^2$$



Q. Seg AB and seg CD are perpendicular diameters of a circle with radius 7 cm. Find the area of the shaded region.

Sol.

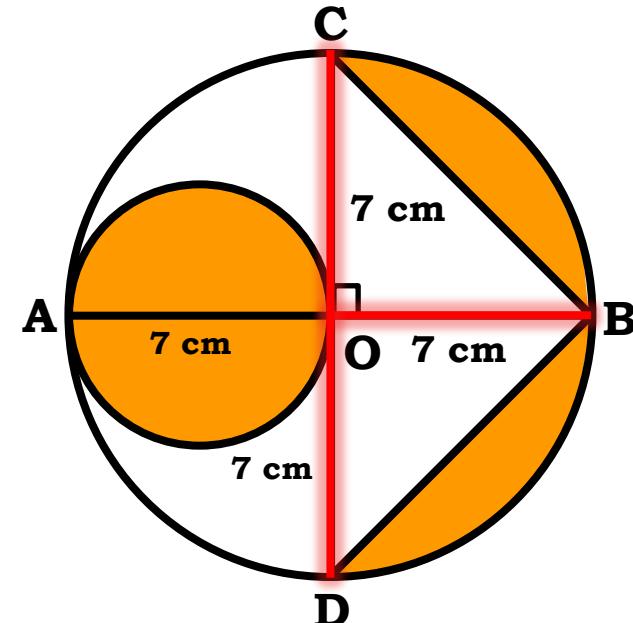
$$\text{ar (shaded region)} = \text{ar (smaller circle)} + \text{ar (semi-circle CBD)} - \text{ar} (\Delta CBD)$$

$$\text{ar (smaller circle)} = 38.5 \text{ cm}^2$$

$$\text{ar (semicircle CBD)} = 77 \text{ cm}^2$$

$$\text{ar} (\Delta CBD) = 49 \text{ cm}^2$$

$$\begin{aligned}\text{ar (shaded region)} &= \text{ar (smaller circle)} + \\ &\quad \text{ar (semi-circle CBD)} - \\ &\quad \text{ar} (\Delta CBD) \\ &= 38.5 + 77 - 49 \\ &= 38.5 + 28 \\ &= 66.5 \text{ cm}^2\end{aligned}$$



∴ Area of shaded region is 66.5 cm^2

Thank You

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Module

AREAS RELATED TO CIRCLE

- **Sum based on finding
Area of shaded region**

Q. In the adjoining figure,

$$PR = 6 \text{ units}, PQ = 8 \text{ units}$$

Semicircles

sides PR, RQ and PQ as diameters.

Find the area of the shaded portion.

What is formula for finding
 $\frac{1}{2} \times \text{base} \times \text{height}$
Area of triangle?

Sol.

Diameter $PR = 6 \text{ units}$

\therefore Its radius (r_1) = 3 units

Diameter $PQ = 8 \text{ units}$

\therefore Its radius (r_2) = 4 units

In $\triangle PRQ$,

$\angle RPQ = 90^\circ$ [Angle inscribed
in a semicircle]

$$\therefore RQ^2 = PR^2 + PQ^2$$

[Pythagoras theorem]

$$\therefore RQ^2 = (6)^2 + (8)^2$$

$$\therefore RQ^2 = 36 + 64$$

$$\therefore RQ^2 = 100$$

$$\therefore RQ = 10 \text{ units}$$

[Taking square roots]

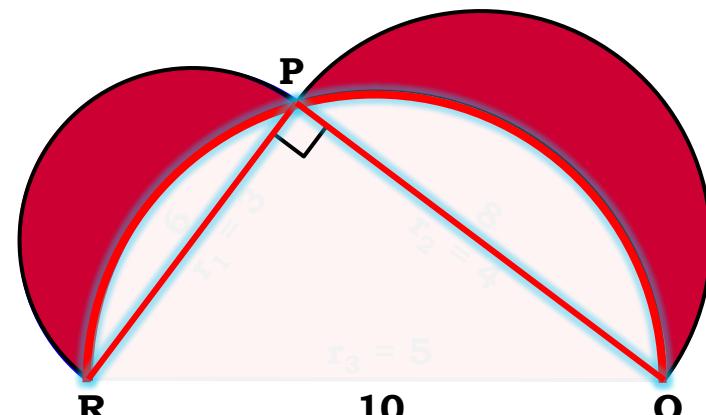
Diameter $RQ = 10 \text{ units}$

\therefore Its radius (r_3) = 5 units

$$A(\triangle PRQ) = \frac{1}{2} \times PR \times PQ$$

$$\therefore A(\triangle PRQ) = \frac{1}{2} \times 6 \times 8$$

$$\therefore A(\triangle PRQ) = 24 \text{ sq. units}$$



ar(shaded region) =
ar(semicircle with
diameter PR)
+ ar(semicircle with
diameter PQ)
+ ar($\triangle PRQ$)
- ar(semicircle with
diameter RQ)

Q. Find the area of the shaded portion.

Sol.

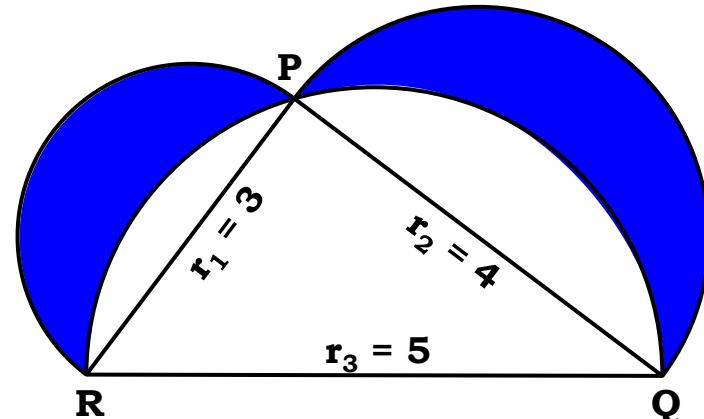
$$r_1 = 3, r_2 = 4, \\ \text{ar}(\Delta PRQ) = 24$$

What is formula for
finding area of semicircle ?

Area of the shaded region

$$\begin{aligned} &= \text{ar(semicircle with diameter PR)} \\ &+ \text{ar(semicircle with diameter PQ)} \\ &- \text{ar(semicircle with diameter RQ)} + \text{ar}(\Delta PRQ) \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \pi r_1^2 + \frac{1}{2} \pi r_2^2 - \frac{1}{2} \pi r_3^2 + 24 \\ &= \frac{1}{2} \pi (r_1^2 + r_2^2 - r_3^2) + 24 \\ &= \frac{1}{2} \pi (3^2 + 4^2 - 5^2) + 24 \\ &= \frac{1}{2} \pi (9 + 16 - 25) + 24 \end{aligned} \quad \begin{aligned} &= 24 + \frac{1}{2} \pi (25 - 25) \\ &= 24 + \frac{1}{2} \pi (0) \\ &= 24 \text{ sq. units} \end{aligned}$$



$$\begin{aligned} \text{ar(shaded region)} &= \text{ar(semicircle with diameter PR)} \\ &+ \text{ar(semicircle with diameter PQ)} \\ &+ \text{ar}(\Delta PRQ) \\ &- \text{ar(semicircle with diameter RQ)} \end{aligned}$$

∴ Area of the shaded region is 24 sq. units

Module

7

AREAS RELATED TO CIRCLE

- Sum based on finding
Area of shaded region

Q. Find the area of the shaded region, if $PQ = 24 \text{ cm}$, $PR = 7 \text{ cm}$ and O is the centre of the circle.

$$\text{ar (shaded region)} = \text{ar (semicircle RPQ)} - \text{ar } (\Delta PQR)$$

Sol. $\angle QPR = 90^\circ$ [Angle in a semi-circle]

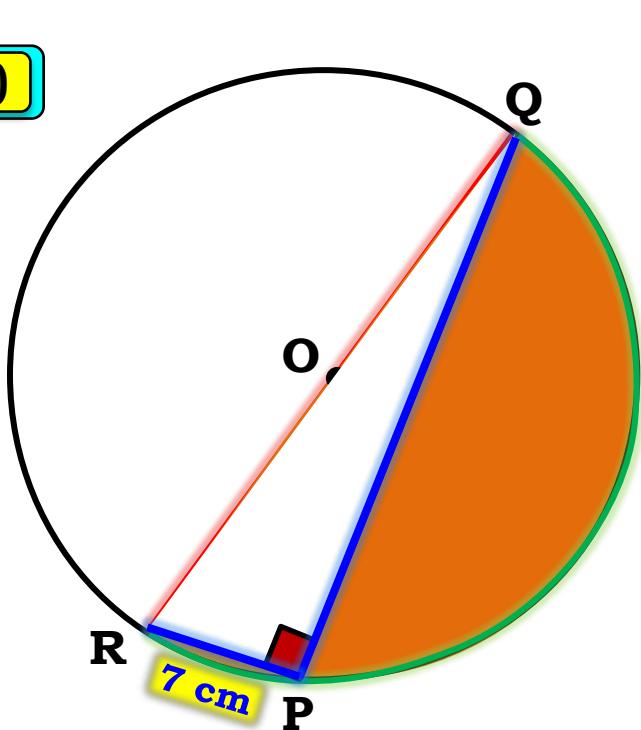
$$\text{ar } (\Delta PQR) = \frac{1}{2} \times \text{Product of perpendicular sides}$$

$$= \frac{1}{2} \times RP \times PO$$

What is the formula to find
 $\frac{1}{2} \times \text{Product of perpendicular sides}$
area of right angle triangle?

$$= 12 \times 7$$

$$\therefore \text{ar } (\Delta PQR) = 84 \text{ cm}^2$$



Q. Find the area of the shaded region, if $PQ = 24 \text{ cm}$, $PR = 7 \text{ cm}$ and O is the centre of the circle.

$$\text{ar (shaded region)} = \text{ar (semicircle RPQ)} - \text{ar } (\triangle PQR)$$

Sol: In $\triangle PQR$, $\angle QPR = 90^\circ$

To find: r

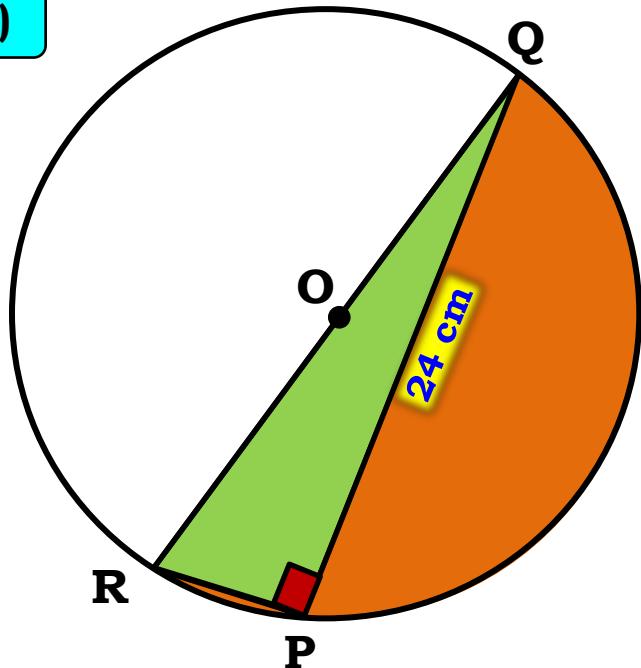
$$\therefore QR^2 = QP^2 + PR^2 \quad \text{Consider } \triangle PQR \quad [\text{Pythagoras theorem}]$$

$$\therefore QR^2 = (24)^2 + (7)^2 \quad \text{Let us apply Pythagoras theorem}$$

$$\therefore QR^2 = 576 + 49 \quad \text{What is the formula to find area of semi-circle?}$$

$$\therefore QR^2 = 625$$

$$\therefore QR = 25 \text{ cm} \quad [\text{Taking square roots}]$$



$$\text{ar } (\triangle PQR) = 84 \text{ cm}^2$$

Q. Find the area of the shaded region, if $PQ = 24 \text{ cm}$, $PR = 7 \text{ cm}$ and O is the centre of the circle.

$$\text{ar (shaded region)} = \text{ar (\text{semicircle } RPQ)} - \text{ar (\triangle PQR)}$$

Sol: $QR = 25 \text{ cm}$

To find: r

$$\therefore \text{radius} = \frac{25}{2} \text{ cm}$$

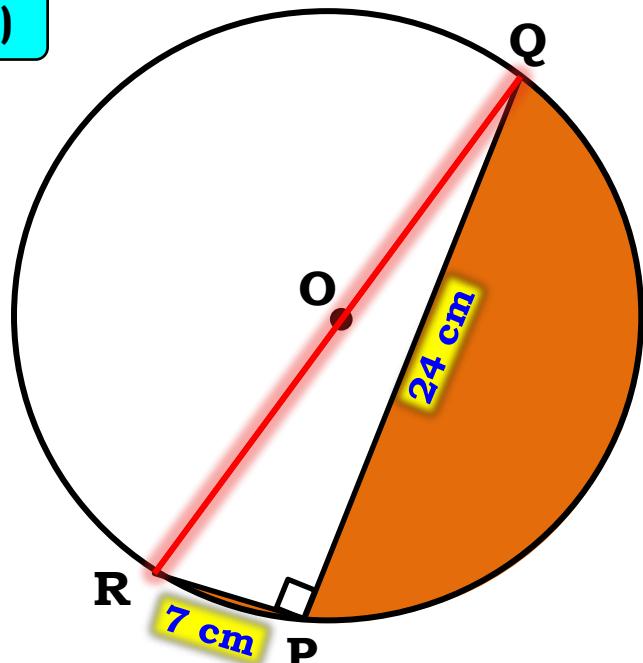
ar(shaded region)
 QR is the diameter of circle

$$= \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$$

$$= \frac{11 \times 25 \times 25}{4 \times 7}$$

$$= \frac{6875}{28}$$

$$= 245.53 \text{ cm}^2$$



$$\text{ar (\triangle PQR)} = 84 \text{ cm}^2$$

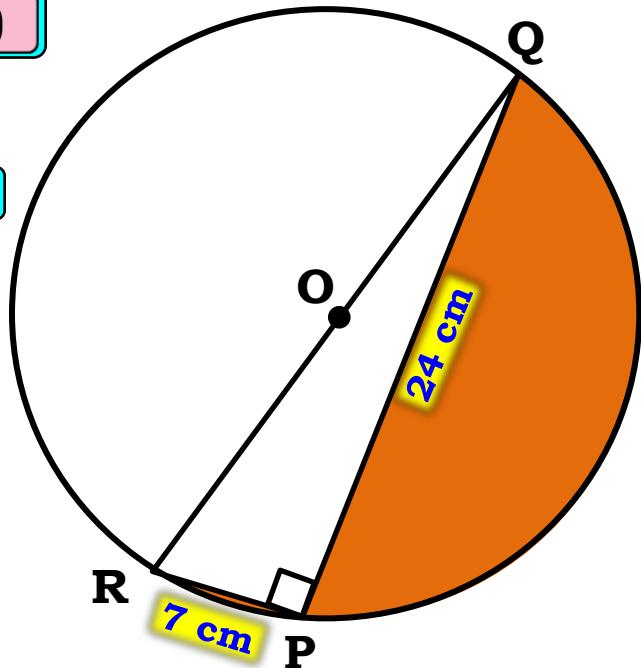
Q. Find the area of the shaded region, if $PQ = 24 \text{ cm}$, $PR = 7 \text{ cm}$ and O is the centre of the circle.

$$\text{ar(shaded region)} = \text{ar(semicircle RPQ)} - \text{ar}(\Delta PQR)$$

Sol:

$$\begin{aligned}\text{ar (shaded region)} &= \text{ar (semicircle RPQ)} - \text{ar} (\Delta PQR) \\ &= 245.53 - 84 \\ &= 161.53 \text{ cm}^2\end{aligned}$$

\therefore ar (shaded portion) is 161.53 cm^2



$$\text{ar} (\Delta PQR) = 84 \text{ cm}^2$$

$$\text{ar (semicircle RPQ)} = 245.53 \text{ cm}^2$$

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Module

AREAS RELATED TO CIRCLE

- Sum based on finding
Area of shaded region

Q. Find the area of the shaded region in adjacent figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

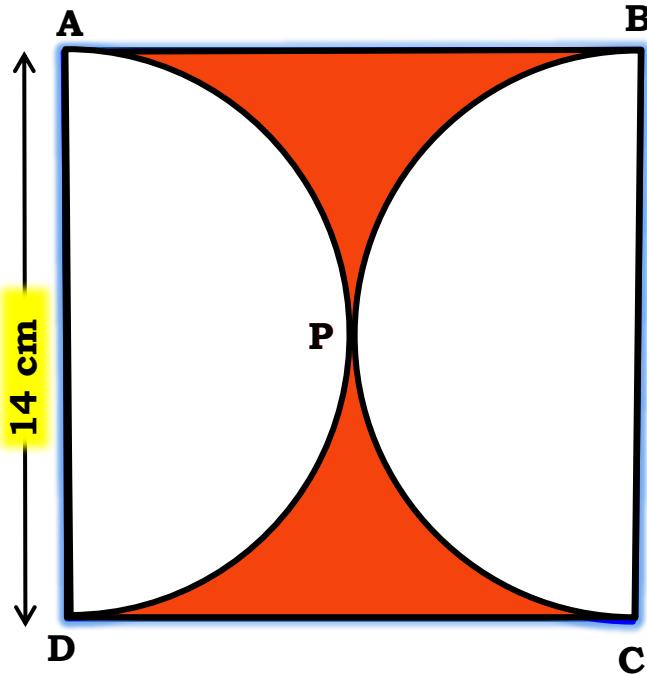
Sol. $\text{ar}(\text{shaded region}) = \text{ar}(\square \text{ABCD}) - 2 \text{ar}(\text{semi-circles})$

Side of the square = 14 cm

$$\begin{aligned}\text{ar}(\square \text{ABCD}) &= (\text{side})^2 \\ &= (14)^2\end{aligned}$$

What is the formula to find
area of a square?

$$\therefore \text{ar}(\square \text{ABCD}) = 196 \text{ cm}^2$$



**Q. Find the area of the shaded region in adjacent figure,
if ABCD is a square of side 14 cm and
APD and BPC are semicircles.**

Sol. $\text{ar (shaded region)} = \text{ar } (\square \text{ABCD}) - 2 \text{ ar (semi-circles)}$

Diameter = 14 cm

∴ Radius (r) = 7 cm

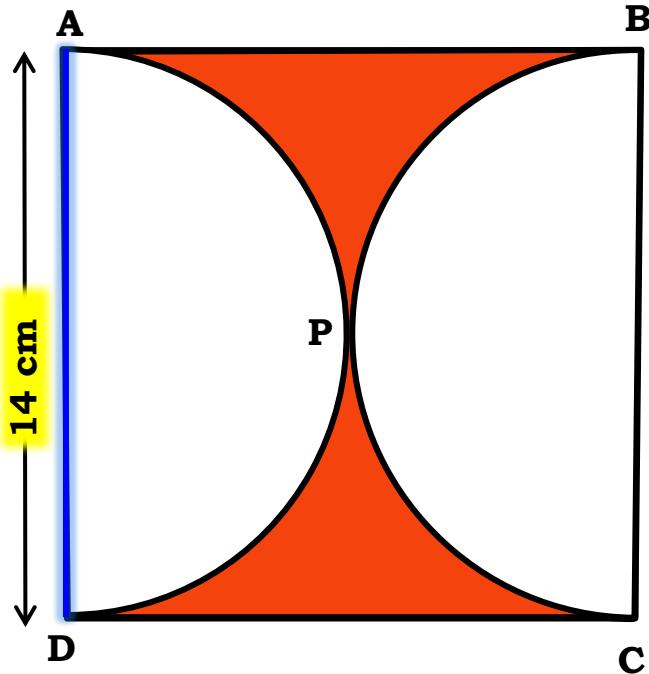
$$2 \text{ ar (semi-circles)} = 2 \times \frac{1}{2} \times \pi r^2$$

$$= \frac{22}{7} \times 7 \times 7$$

What is the formula to find
area of semi-circle?

∴ $2 \text{ ar (semi-circles)} = 154 \text{ cm}^2$

$\text{ar } (\square \text{ABCD}) = 196 \text{ cm}^2$



**Q. Find the area of the shaded region in adjacent figure,
if ABCD is a square of side 14 cm and
APD and BPC are semicircles.**

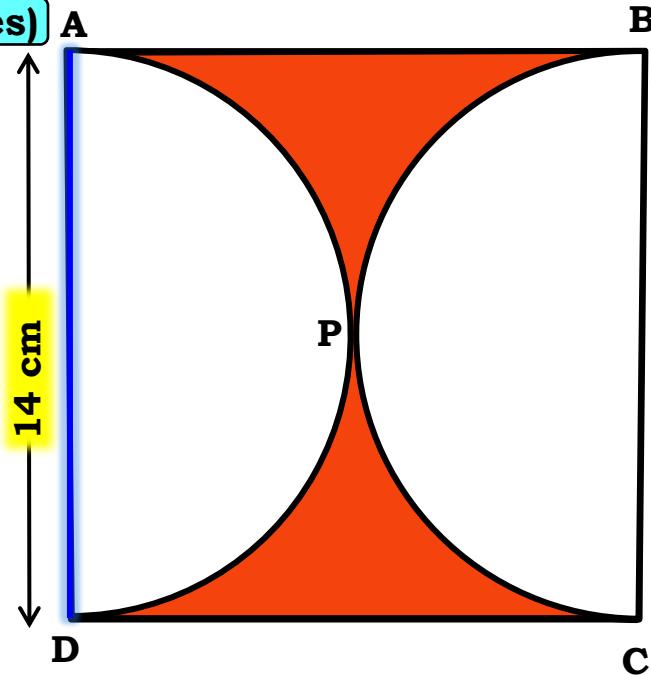
Sol. $\text{ar}(\text{shaded region}) = \text{ar}(\square \text{ABCD}) - 2 \text{ ar}(\text{semi-circles})$

$$\begin{aligned}\text{ar}(\text{shaded region}) &= \text{ar}(\square \text{ABCD}) - 2 \text{ ar}(\text{semi-circles}) \\ &= 196 - 154 \\ &= 42 \text{ cm}^2\end{aligned}$$

\therefore Area of the shaded portion is 42 cm^2

$2 \text{ Ar}(\text{semi-circles}) = 154 \text{ cm}^2$

$\text{Ar}(\square \text{ABCD}) = 196 \text{ cm}^2$



Q. On a square handkerchief, nine circular designs each of radius 7cm are made. Find the area of the remaining portion of the handkerchief.

Sol.

$$\text{ar(remaining portion)} = \text{ar}(\square ABCD) - \text{ar(nine circles)}$$

$$\text{radius of one circle} = 7 \text{ cm}$$

$$\begin{aligned}\text{Side of square} &= 14 + 14 + 14 \\ &= 42 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Area of square} &= (\text{side})^2 \\ &= (42)^2\end{aligned}$$

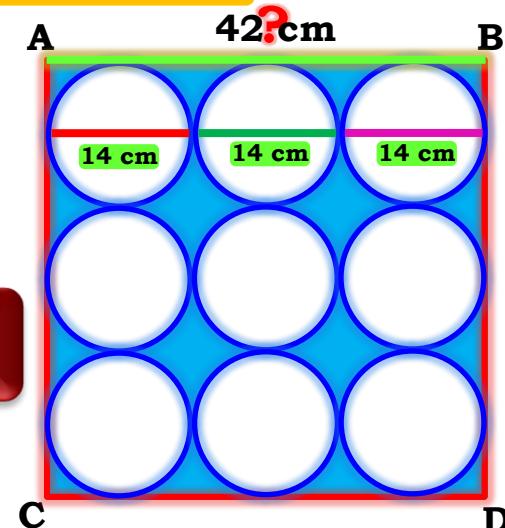
What is formula to
 πr^2 find area of circle?

$$\therefore \frac{\text{Area of square}}{\text{Side of square}} = \frac{\text{Sum of diameter}}{\text{of 3 circles}}$$

$$\begin{aligned}\therefore \text{Diameter} &= 14 \text{ cm} \times 7 \times 7 \\ &= 22 \times 7\end{aligned}$$

$$\therefore \text{Area of one circle} = 154 \text{ cm}^2$$

$$\begin{aligned}\therefore \text{Area of 9 circles} &= 9 \times 154 \\ &= 1386 \text{ cm}^2\end{aligned}$$



Q. On a square handkerchief, nine circular designs each of radius 7cm are made. Find the area of the remaining portion of the handkerchief.

Sol.

$$\text{ar}(\text{remaining portion}) = \text{ar}(\square ABCD) - \text{ar}(9 \text{ circles})$$

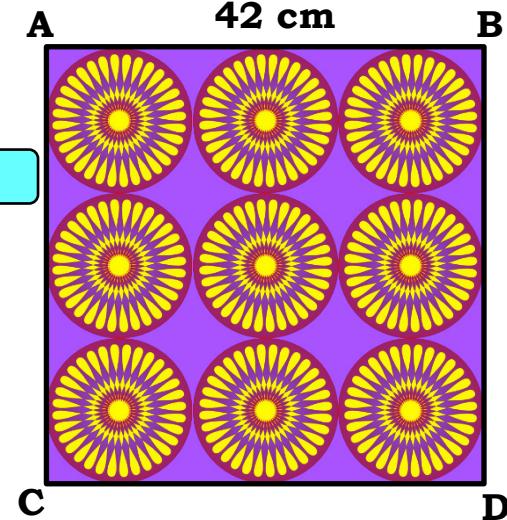
$$\therefore \begin{aligned}\text{Area of the remaining portion} &= \text{ar} (\square ABCD) - \text{ar } (9 \text{ Circles}) \\ &= 1764 - 1386\end{aligned}$$

$$\therefore \begin{aligned}\text{Area of the remaining portion} &= 378 \text{ cm}^2\end{aligned}$$

∴ Area of the remaining portion is 378 cm^2

$$\text{ar } (\square ABCD) = 1764 \text{ cm}^2$$

$$\text{ar } (9 \text{ circles}) = 1386 \text{ cm}^2$$



Module 9

AREAS RELATED TO CIRCLE

- **Sum based on Area of circle
and equilateral triangle**

Q. In a circular table cover of radius 32cm, a design is formed leaving an equilateral triangle ABC in the middle. Find the area of the design (shaded region).

Sol.

$$\text{ar(shaded region)} = \text{ar(circle)} - \text{ar}(\Delta ABC)$$

Let I be the centre of circle.

To find: side

$$\therefore \text{Radius of circle (AI)} = 32 \text{ cm}$$

$$\text{Area of circle} = \pi r^2$$

$$= \frac{22}{7} \times 32 \times 32$$

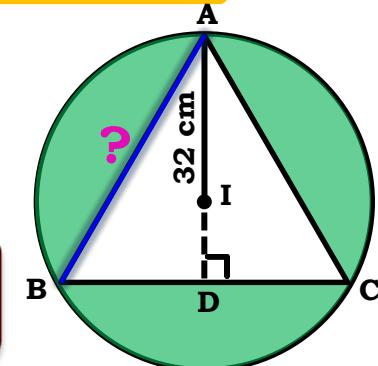
$$\therefore \text{Area of circle} = \frac{22528}{7} \text{ cm}^2$$

∴ A, I, D are collinear

ΔABC is equilateral triangle

∴ I is the centroid of ΔABC.

$$\therefore AI = \frac{2}{3} \times AD \dots (\text{I divides AD in the ratio } 2 : 1)$$



Q. In a circular table cover of radius 32cm, a design is formed leaving an equilateral triangle ABC in the middle.

Find the area

Sol.

ar(shaded region)

$$\therefore AI = \frac{2}{3} \times 32$$

$$\therefore 32 = \frac{2}{3} \times AD$$

$$\therefore \frac{32 \times 3}{2} = AD$$

$$\therefore AD = 48 \text{ cm}$$

$$\therefore \angle B = 60^\circ$$

$$\therefore \sin 60 = \frac{AD}{AB}$$

$$\therefore \frac{\sqrt{3}}{2} \cancel{\times} \frac{48}{AB}$$

Radius

Hypotenuse

Consider $\triangle ABD$

**Opposite side and
Hypotenuse of**

$$\text{Area of circle} = \frac{528}{7} \text{ cm}^2$$

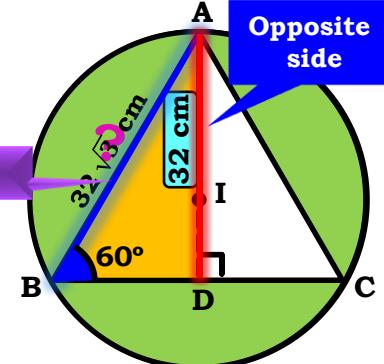
$$\therefore AB = \frac{48 \times 2}{\sqrt{3}}$$

$$\therefore AB = \frac{96}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\therefore AB = \frac{96\sqrt{3}}{3}$$

$$\therefore AB = 32\sqrt{3} \text{ cm}$$

To find: side



Q. In a circular table cover of radius 32cm, a design is formed leaving an equilateral triangle ABC in the middle. Find the area of the design (shaded region).

Sol.

$$\text{ar(shaded region)} = \text{ar(circle)} - \text{ar}(\Delta ABC)$$

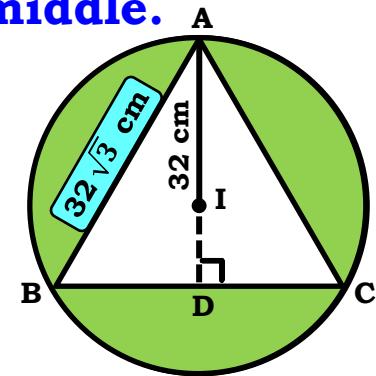
$$\begin{aligned}\therefore \text{ar } (\Delta ABC) &= \frac{\sqrt{3}}{4} \times (\text{side})^2 \\ &= \frac{\sqrt{3}}{4} \times (32\sqrt{3})^2 \\ &= \frac{\sqrt{3}}{4} \times \cancel{32}^8 \times 32 \times 3\end{aligned}$$

$$\therefore \text{ar } (\Delta ABC) = 768\sqrt{3} \text{ cm}^2$$

$$\therefore \text{Area of the design} = \text{Area of circle} - \text{Ar } (\Delta ABC)$$

$$\therefore \text{Area of the design} = \left[\frac{22528}{7} - 768\sqrt{3} \right] \text{ cm}^2$$

$$\therefore \text{Area of the design is } \left[\frac{22528}{7} - 768\sqrt{3} \right] \text{ cm}^2$$



$$\text{Area of circle} = \frac{22528}{7} \text{ cm}^2$$

Module

10

AREAS RELATED TO CIRCLE

- Sum based on Area of circle
and Right-angled triangle

Q. In the adjoining figure, find the area of triangle ABC if I is the center of inscribed circle.

Area of triangle = $\frac{1}{2} \times$ Product of perpendicular sides

\therefore Area of $\triangle ABC = \frac{1}{2} \times AB \times AC$?

Sol.

$$\text{Area of shaded region} = \text{ar}(\triangle ABC) - \text{Area of circle}$$

Applying Pythagoras theorem in $\triangle ABC$, we have

$$BC^2 = AB^2 + AC^2$$

To find: AC

$$\therefore AC^2 = BC^2 - AB^2$$

$$\therefore AC^2 = 10^2 - 6^2$$

$$\therefore AC^2 = 100 - 36$$

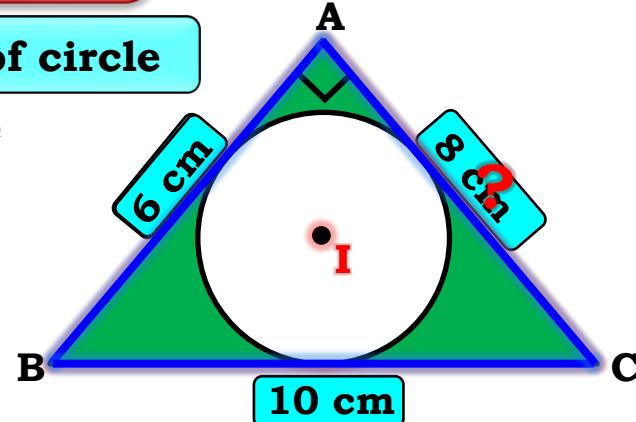
$$\therefore AC^2 = 64$$

$$\therefore AC = 8 \text{ cm}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times AB \times AC$$

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2} \times 6 \times 8$$

$$\therefore \text{Area of } \triangle ABC = 24 \text{ cm}^2$$



What is formula to find area of triangle?

Q. In the adjoining given figure, $\triangle ABC$ is a right angled triangle at A. find the : Area of triangle = $\frac{1}{2} \times b \times h$ AB = 6cm, BC = 10cm and I is the center of incircle of $\triangle ABC$.

Sol. Area of shaded region = ar($\triangle ABC$) - Area of circle

Let radius of the incircle be 'r' cm .

$$\text{Area of } \triangle ABC = \text{Area of } \triangle BIC + \text{Area of } \triangle AIC + \text{Area of } \triangle AIB$$

$$\therefore 24 = \frac{1}{2} \times (BC \times r) + \frac{1}{2} (AC \times r) + \frac{1}{2} (AB \times r)$$

$$\therefore 24 = \frac{1}{2} \times r (BC + AC + AB)$$

$$\therefore 24 = \frac{1}{2} \times r \times (10 + 8 + 6)$$

$$\therefore 24 = \frac{1}{2} \times r^{12}$$

$$\therefore 24 = 12r$$

$$\therefore r = 2$$

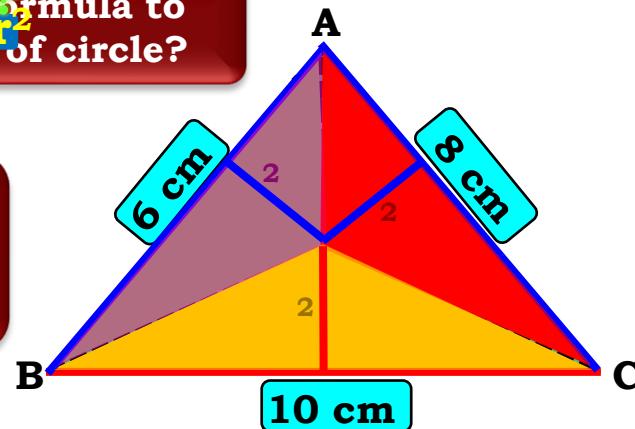
$$\text{Area of circle} = \pi r^2$$

$$= \frac{22}{7} \times 2 \times 2$$

$$\text{Area of circle} = \frac{88}{7} \text{ cm}^2$$

What is formula to
 πr^2
find area of circle?

$\triangle ABC$ is divided into
three triangles
 $\triangle AIB$, $\triangle BIC$, $\triangle AIC$



Q. In the adjoining given figure, ABC is a right angled triangle at A.
 find the area of the shaded region if AB = 6cm, BC = 10cm and I is
 the center of incircle of $\triangle ABC$.

Sol.

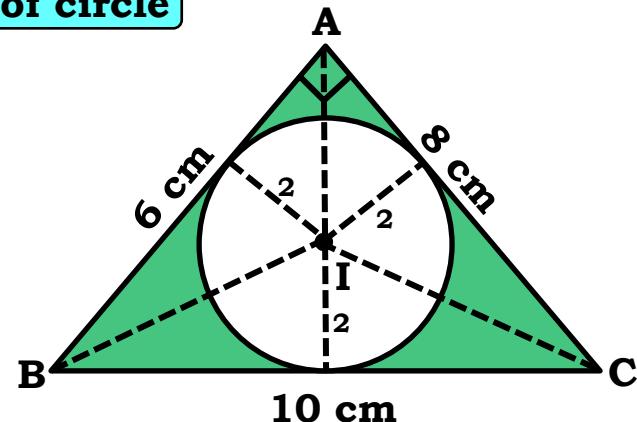
$$\text{Area of shaded region} = \text{ar}(\triangle ABC) - \text{Area of circle}$$

$$\text{Area of the shaded region} = \text{Area of } \triangle ABC - \text{Area of circle}$$

$$\begin{aligned}
 &= 24 - \frac{88}{7} \\
 &= \frac{168 - 88}{7} \\
 &= \frac{80}{7} \text{ cm}^2
 \end{aligned}$$

$$\text{ar}(\triangle ABC) = 24\text{cm}^2$$

$$\therefore \text{Area of the shaded region is } \frac{80}{7} \text{ cm}^2$$



Module

11

AREAS RELATED TO CIRCLE

- Sum based on finding
Area of shaded region

Q. In given figure, a crescent is formed by two circles which touch at A. C is the centre of the large circle.

The width of the crescent at BD is 9 cm and at EF it is 5 cm.

Find : (i) the radii of two circles

(ii) the area of the shaded region. (Use $\pi = 3.14$)

Construction: Draw AE and DE.

Sol. In smaller circle

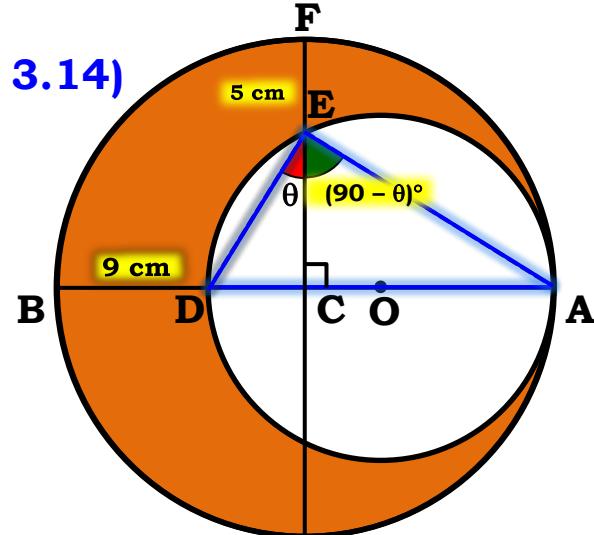
$$\angle AED = 90^\circ \quad [\text{Angle in a semi-circle}]$$

$\angle AED$ is made up of $\angle DEC$ and $\angle AEC$

$$\angle AED = \angle DEC + \angle AEC$$

$$\therefore 90^\circ = \theta + \angle AEC$$

$$\therefore \angle AEC = (90 - \theta)^\circ \dots \text{(ii)}$$



Q. In given figure, a crescent is formed by two circles which touch at A. C is the centre of the large circle.

The width of the crescent at BD is 9 cm and at EF it is 5 cm.

Find : (i) the radii of two circles

(ii) the area of the shaded region. (Use $\pi = 3.14$)

Construction: Draw AE and DE.

$$\angle AEC = (90 - \theta)^\circ \dots \text{(ii)}$$

Sol. In $\triangle DCE$,

Consider $\triangle ECA$

$$\angle DEC = \theta, \angle ECD = 90^\circ,$$

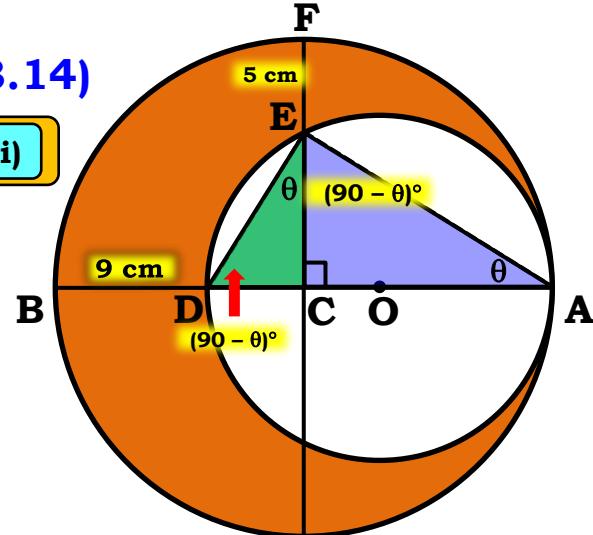
$$\therefore \angle EDC = (90 - \theta)^\circ \dots \text{(iii) [Remaining angle]}$$

In $\triangle ECA$,

$$\angle AEC = (90 - \theta)^\circ \quad \text{[From (ii)]}$$

$$\angle ECA = 90^\circ$$

$$\therefore \angle EAC = \theta \dots \text{(iv) [Remaining angle]}$$



Q. In given figure, a crescent is formed by two concentric circles with common center C. The outer circle touches at A. C is the centre of the inner circle. The width of the crescent at BD is 9 cm. Find :
 (i) the radii of two circles
 (ii) the area of the shaded region. (use $\pi = 3.14$)

$$\begin{aligned}\angle DEC &= \theta \quad \dots \text{(i)} \\ \angle AEC &= (90 - \theta)^\circ \quad \dots \text{(ii)} \\ \angle EDC &= (90 - \theta)^\circ \quad \dots \text{(iii)} \\ \angle EAC &= \theta \quad \dots \text{(iv)}\end{aligned}$$

Construction: Draw AE and DE.

Sol. In $\triangle DCE$ and $\triangle ECA$ Consider $\triangle DCE$ and $\triangle ECA$

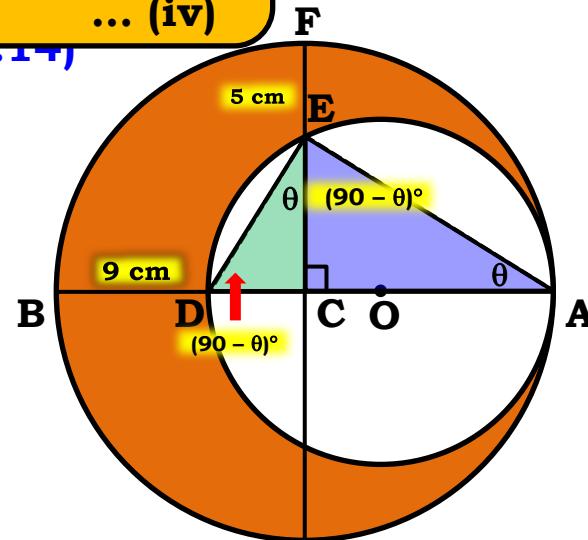
$$\angle DEC = \angle EAC \quad [\text{From (i) and (iv)}]$$

$$\angle EDC = \angle AEC \quad [\text{From (ii) and (iii)}]$$

$\therefore \triangle DCE \sim \triangle ECA$ [AA criterion]

$$\therefore \frac{DC}{EC} \cancel{\times} \frac{EC}{CA} \quad [\text{c.s.s.t}]$$

$$\therefore EC^2 = DC \times CA$$



Q. In given figure, a crescent is formed by two circles which touch at A. C is the centre of the large circle.

The width of the crescent at BD is 9 cm and at EF it is 5 cm.

Find : (i) the radii of two circles

(ii) the area of the shaded region. (Use $\pi = 3.14$)

Construction: Draw AE and DE.

Sol. $EC^2 = DC \times CA$

$$CA = 'R'$$

Now let us find EC and DC in terms of 'R'

Let radius of larger circle be 'R'

$$BD + DC = BC$$

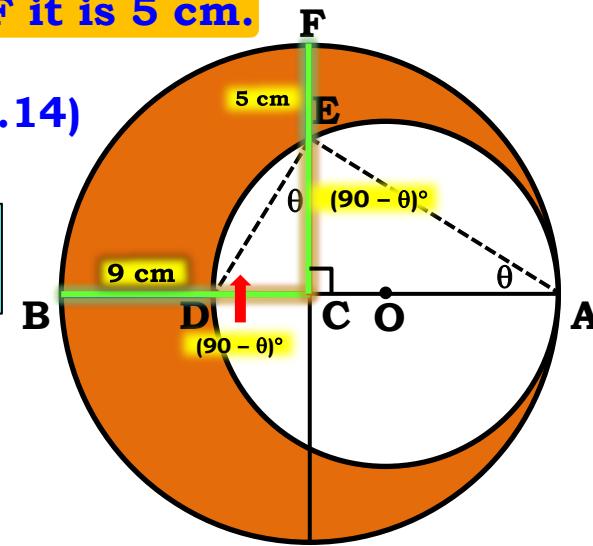
$$\therefore 9 + DC = R$$

$$\therefore DC = (R - 9) \text{ cm}$$

$$CE + EF = CF$$

$$\therefore CE + 5 = R$$

$$\therefore CE = (R - 5) \text{ cm}$$



Module

12

AREAS RELATED TO CIRCLE

- Sum based on finding
Area of shaded region (Part-II)

Q. In given figure, a crescent is formed by two circles which touch at A. C is the centre of the large circle.

The width of the crescent at BD is 9 cm and at EF it is 5 cm.

Find : (i) the radii of two circles

(ii) the area of the shaded region. (Use $\pi = 3.14$)

Construction: Draw AE and DE.

$$\text{Sol. } EC^2 = DC \times CA$$

$$CA = 'R'$$

$$DC = (R - 9) \text{ cm}$$

$$CE = (R - 5) \text{ cm}$$

$$\therefore (R - 5)^2 = (R - 9) \times R$$

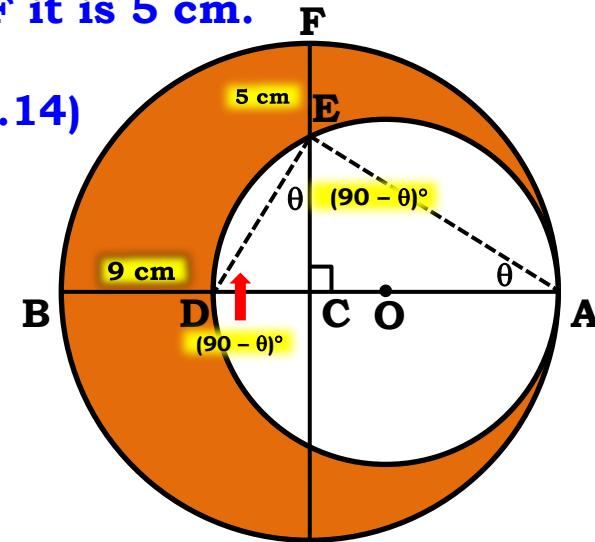
$$(a-b)^2 = a^2 - 2ab + b^2$$

$$\therefore \cancel{R^2 - 10R + 25} = \cancel{R^2 - 9R}$$

$$\therefore 25 = -9R + 10R$$

$$\therefore R = 25$$

$$\text{Diameter of larger circle} = 50 \text{ cm}$$



Q. In given figure, a crescent is formed by two circles which touch at A. C is the centre of the large circle.

The width of the crescent at BD is 9 cm and at EF it is 5 cm.

Find : (i) the radii of two circles

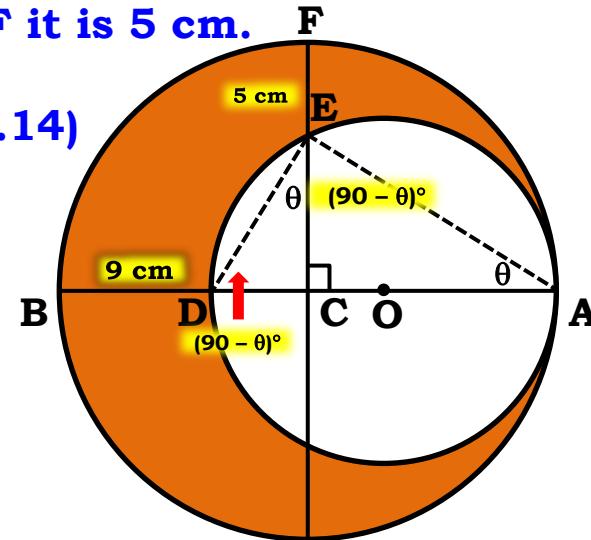
(ii) the area of the shaded region. (Use $\pi = 3.14$)

Construction: Draw AE and DE.

$$\begin{aligned}\text{Sol. Diameter of smaller circle} &= AB - BD \\ &= 50 - 9 \\ &= 41 \text{ cm}\end{aligned}$$

Let radius of smaller circle be 'r'

$$\therefore \text{radius of smaller circle (r)} = \frac{41}{2} \quad \boxed{\text{Diameter of larger circle} = 50 \text{ cm}}$$
$$= 20.5 \text{ cm}$$



Q. In given figure, a crescent is formed by two circles which touch at A. C is the centre of the large circle.

The width of the crescent at BD is 9 cm and at EF it is 5 cm.

Find : (i) the radii of two circles

(ii) the area of the shaded region. (Use $\pi = 3.14$)

$$\text{ar shaded region} = \text{ar (larger circle)} - \text{ar (smaller circle)}$$

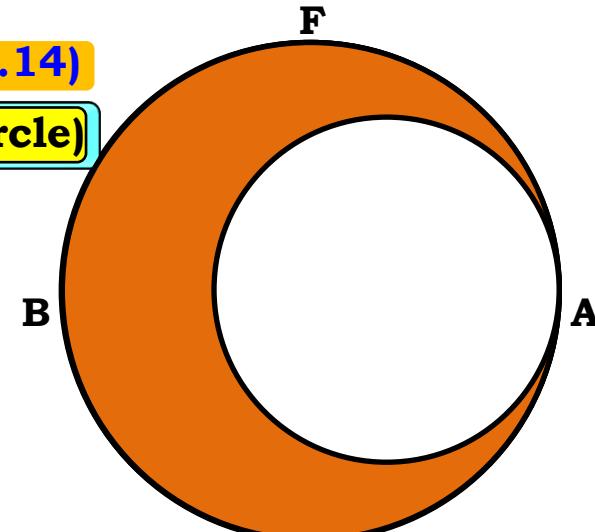
Sol.

$$a^2 - b^2 = (a + b)(a - b)$$

$$\begin{aligned}\text{ar (shaded region)} &= \pi R^2 - \pi r^2 \\ &= \pi (R^2 - r^2) \\ &= \pi (25^2 - 20.5^2)\end{aligned}$$

What is the formula to find $(25 - 20.5)$
area of circle?

$$\begin{aligned}&3.14 \times 15.5 \times 4.5 \\ &= 642.915 \text{ cm}^2\end{aligned}$$



Radius of larger circle (R) = 25 cm

Radius of smaller circle (r) = 20.5 cm

\therefore ar (shaded region) is 642.915 cm^2

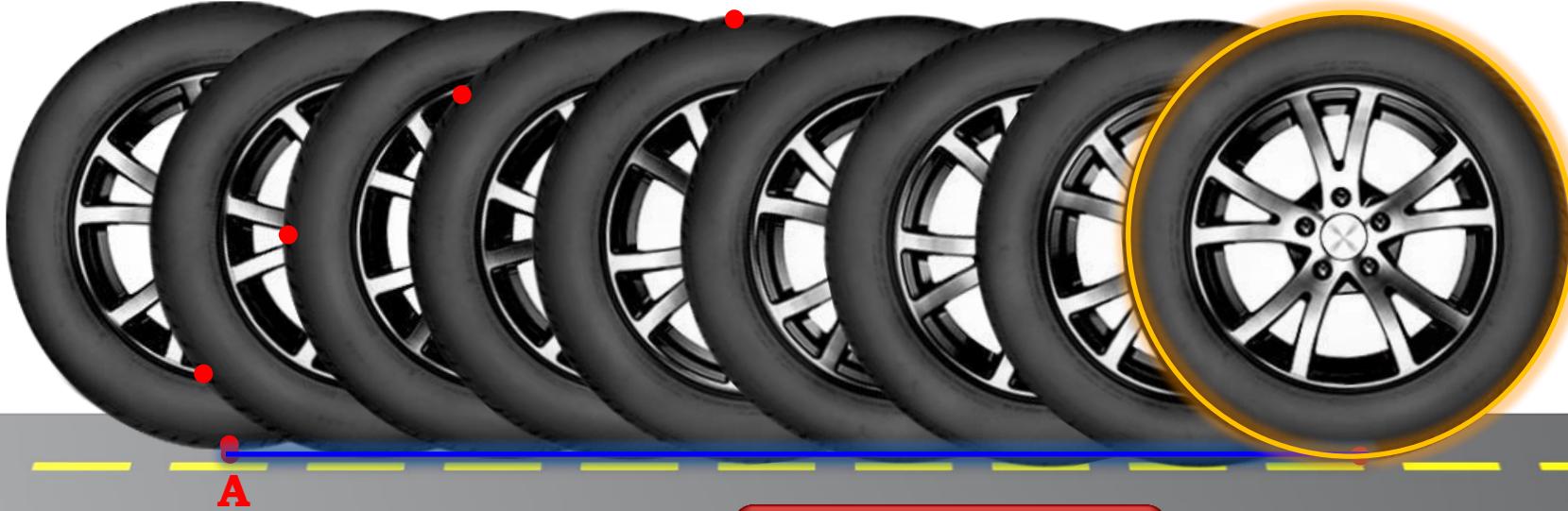
Module

13

AREAS RELATED TO CIRCLE

- **Sum based on circumference of circle**

UNDERSTAND!



Distance covered by the wheel = Circumference of the wheel × No. of rotations

Let us consider one rotation of the wheel

Total Distance covered = $N \times \text{Circumference in 1 revolution}$

∴ No. of rotations (N) = $\frac{\text{Total Distance covered}}{\text{Distance covered in 1 revolution}}$

Q. The wheels of a car are of diameter 80 cm each.

How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

$$\text{No. of revolutions} = \frac{\text{Total distance covered}}{\text{Distance covered in one revolution}}$$

Sol. Diameter of wheel of car = 80 cm

$$\therefore \text{Radius of wheel of car} = \frac{80}{2} = 40\text{cm}$$

Distance travelled in one revolution = Circumference

$$= 2\pi r$$

No. of revolutions = ?

$$\text{Distance covered} \times \frac{\text{Circumference}}{\text{Circumference}} = 2\pi r \times 40$$

$$= 80\pi \text{ cm}$$



Q. The wheels of a car are of diameter 80 cm each.

How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

$$\text{No. of revolutions} = \frac{\text{Total distance covered}}{\text{Distance covered in one revolution}}$$

Sol.

$$1 \text{ km} = 100000 \text{ cm}$$

$$\text{Distance travelled in one revolution} = 80\pi \text{ cm}$$

$$\text{Distance travelled in 1 hour} = 66 \text{ km}$$

$$\therefore \text{Distance travelled in 1 minute} = \frac{66}{60} \text{ km}$$

$$\therefore \text{Distance travelled in 10 minutes} = \frac{\cancel{66}}{\cancel{60}} \times \cancel{10}^{\color{red}11} = 11 \text{ km}$$

$$\therefore \text{Distance travelled in 10 minutes} = 11 \times 100000 \text{ cm}$$

$$(1 \text{ km} = 100000 \text{ cm})$$



Q. The wheels of a car are of diameter 80 cm each.

How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

$$\text{No. of revolutions} = \frac{\text{Total distance covered}}{\text{Distance covered in one revolution}}$$

$$\begin{aligned}\text{Sol. No. of revolutions} &= \frac{\text{Total distance covered}}{\text{Distance covered in one revolution}} \\ &= \frac{11 \times 100000}{80\pi} = \frac{11 \times 100000}{80 \times \frac{22}{7}} \\ &= \frac{11 \times 1000 \times 50}{80 \times 22} \\ \text{Distance travelled in one revolution} &= 80\pi \text{ cm} \\ &= 125 \times 35\end{aligned}$$

$$\text{Total distance covered} = 14375 \text{ m}$$



∴ Wheel makes 4375 revolutions in 10 minutes

Thank You

Module

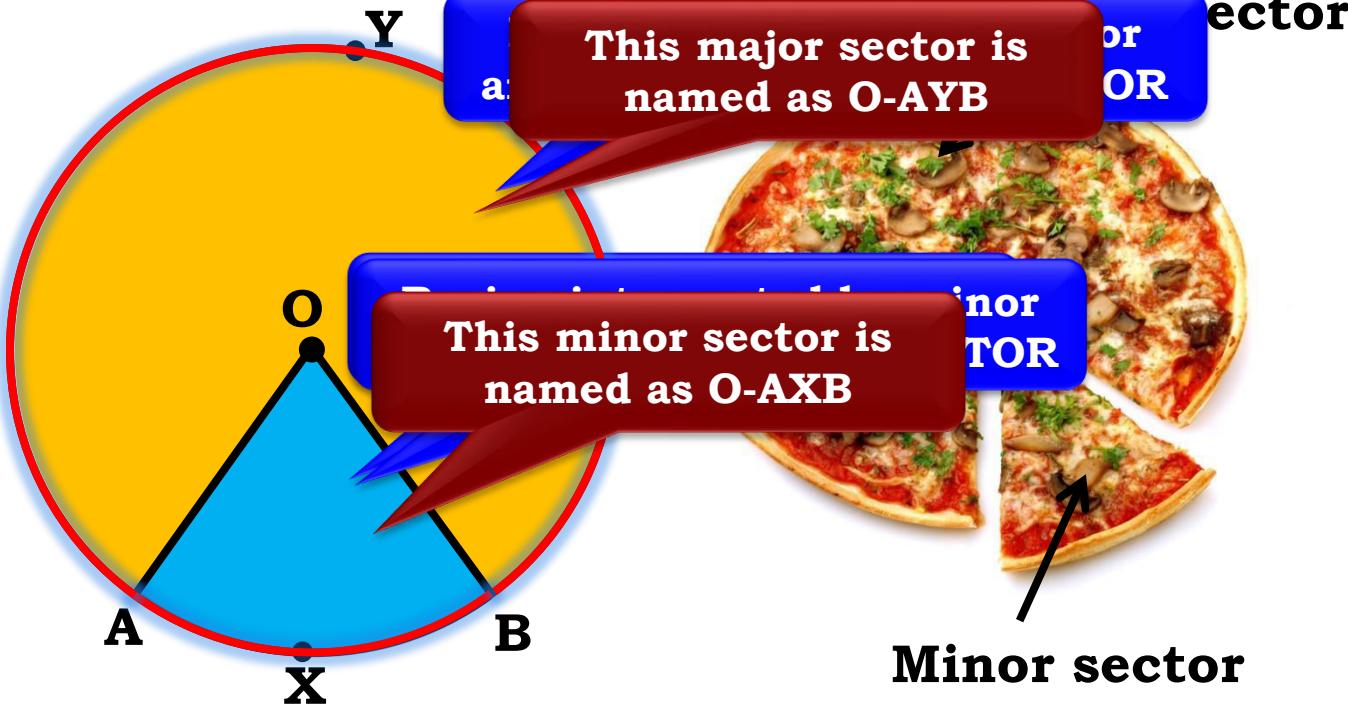
14

AREAS RELATED TO CIRCLE

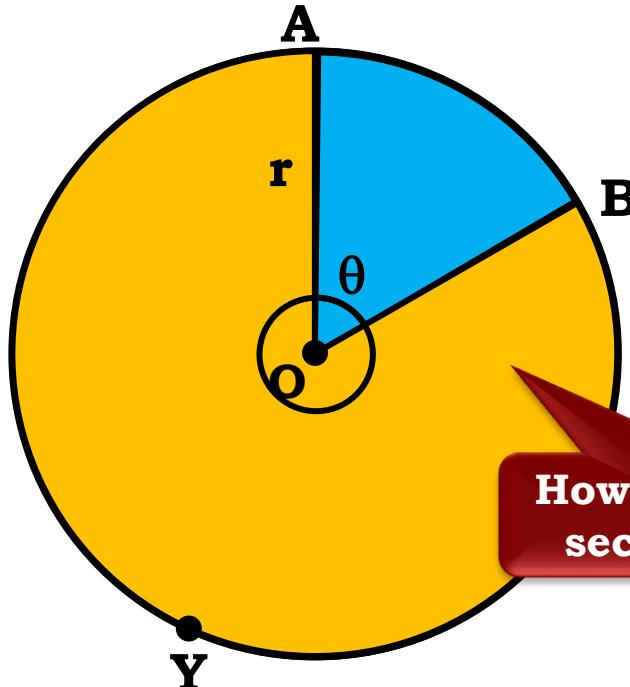
- **Introduction : SECTOR**
- **Formula : Area of sector**

SECTOR

Sector of a circle is the part of the circular region enclosed by two radii and their intercepted arc.



SECTOR



Rotation	Area
360°	πr^2
θ	$A(\text{sector})$

$$Ar(\text{sector}) \times 360^\circ = \theta \times \pi r^2$$

$$Ar(\text{sector}) = \frac{\theta}{360} \times \pi r^2$$

How to find Area of
sector O - AYB ?

$$Ar(O - AYB) = Ar(\text{circle}) - Ar(O - AB)$$

Q. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60° .

Sol.

$$r = 6 \text{ cm}$$

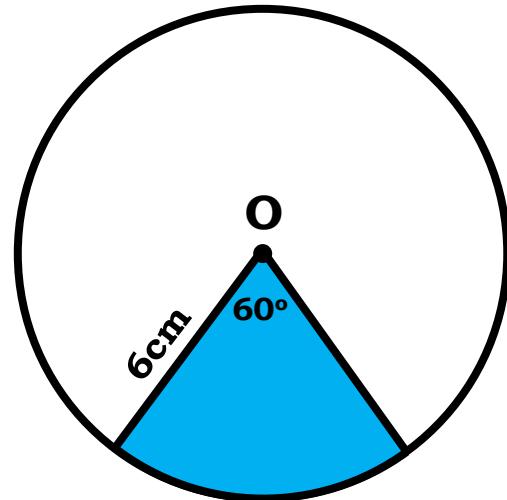
$$\theta = 60^\circ$$

What is formula to find area of sector ?

$$\begin{aligned}\text{Area of sector} &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6 \\ &= \frac{132}{7}\end{aligned}$$

$$\therefore \text{Area of sector} = 18.857 \text{ cm}^2$$

$$\therefore \text{Area of sector is } 18.86 \text{ cm}^2$$



Q. Tick the correct answer in the following :

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\frac{p}{180} \times 2\pi R$

(B) $\frac{p}{180} \times \pi R^2$

(C) $\frac{p}{360} \times 2\pi R$

(D) $\frac{p}{720} \times 2\pi R^2$

Sol. Radius = R

$\theta = p$

Wh Multiply and divide by 2

$$\text{Area of sector} = \frac{\theta}{360} \times \pi R^2$$

$$= \frac{p}{360} \times \pi R^2 \times \frac{2}{2}$$

$$= \frac{p}{720} \times 2\pi R^2$$

∴

Option (D) is correct.

Module

15

AREAS RELATED TO CIRCLE

- Sum based on Area of Sector

Q. If the area of the minor sector is 392.5 sq. cm and the corresponding central angle is 72° , find the radius. ($\pi = 3.14$)

Radius = ?

Sol.

$$\text{Area of the sector} = \frac{\theta}{360} \times \pi r^2$$

Since area of sector is given,

What is formula for finding area of sector ?

$$\therefore 392.5 = \frac{72}{360} \times 3.14 \times r^2$$

$$\therefore \frac{3925}{10} = \frac{72}{360} \times \frac{314}{100} \times r^2$$

$$\therefore \frac{25}{10} \times \frac{5}{360} \times \frac{100}{72} \times \frac{314}{157} = r^2$$

Radius of the circle is 25 cm.

Q. Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Circumference = 22 cm

To find : r

$$\text{Circumference} = 2\pi r$$

$$\therefore 22 = 2 \times \frac{22}{\pi}$$

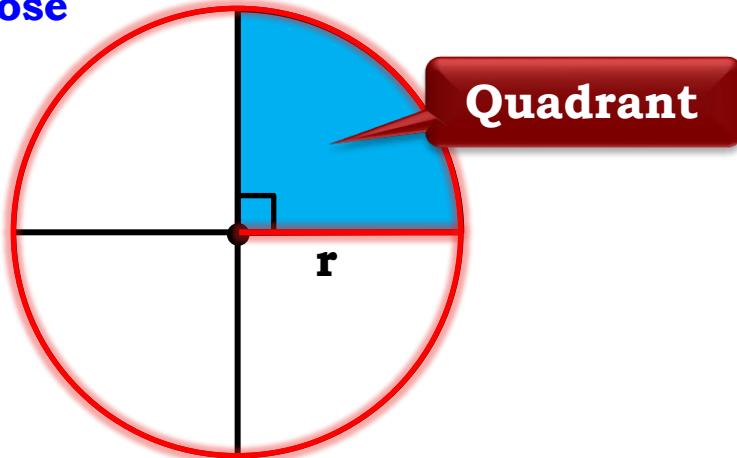
~~$\frac{22}{2} \times \pi$~~
 $\therefore \frac{22}{2} \times \frac{1}{\pi}$

Let us draw two perpendicular diameters

$$\therefore r = \frac{7}{2} \text{ cm}$$

$$\begin{aligned}\text{Area of quadrant of a circle} &= \frac{1}{4} \times \pi r^2 \\ &= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \\ &= \frac{77}{8} = 9.625 \text{ cm}^2\end{aligned}$$

\therefore Area of quadrant of a circle is 9.63 cm^2



Quadrant

Module

16

AREAS RELATED TO CIRCLE

- **Sums based on Area of sector**

Q. An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Sol.

$$\text{Radius} = 45 \text{ cm}$$



$$\text{Angle between two ribs} = \frac{360}{8}$$

$$\theta = 45^\circ$$

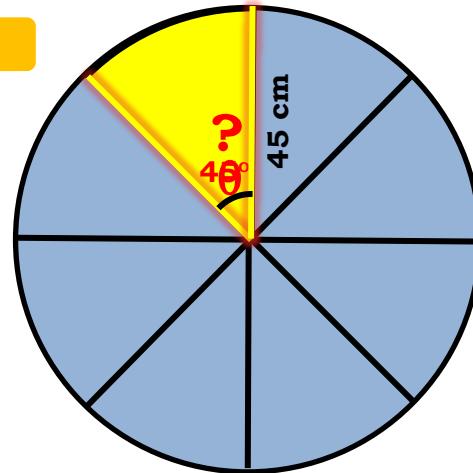
$$\text{Area between two consecutive ribs} = \frac{\theta}{360} \times \pi r^2$$



$$\begin{aligned}
 &= \frac{45}{360} \times \frac{22}{7} \times 45 \times 45 \\
 &\quad \cancel{8} \cancel{4} \quad \cancel{11}
 \end{aligned}$$

$$= \frac{22275}{28}$$

$$= 795.53 \text{ cm}^2$$



The area between two consecutive ribs of the umbrella is 795.53 cm^2

Q. The length of the minute hand of a clock is 14 cm.

Find the area swept by minute hand in 5 minutes.

Sol.

$$\begin{aligned} \text{Area swept by minute hand in 5 minutes} &= \text{Area of sector} \\ &= \frac{\theta}{360} \times \pi r^2 \end{aligned}$$

$$\text{Angle swept by minute hand in 1 min} = \frac{360^\circ}{60} = 6^\circ$$

$$\text{Angle swept by minute hand in 5 min} = 5 \times 6^\circ = 30^\circ$$

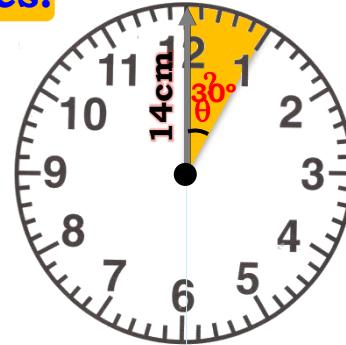
$$\theta = 30^\circ$$

$$\begin{aligned} \text{Area swept by minute hand in 5 minutes} &= \text{Area of sector} \\ &= \frac{\theta}{360} \times \pi r^2 \end{aligned}$$

$$\begin{aligned} &= \frac{30}{360} \times \frac{22}{7} \times 14 \times 14 \\ &\quad \cancel{12} \cancel{8} \cancel{3} \end{aligned}$$

$$= \frac{154}{3} = 51.33 \text{ cm}^2$$

14 cm.
5 minutes.



∴ Area swept by the minute hand in 5 minutes is 51.33 cm^2

Module

17

AREAS RELATED TO CIRCLE

- Sum based on Area of sector

Q. A Horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze. Also find the increase in the grazing area if the rope were 10m long instead of 5m. (Use $\pi = 3.14$)

Sector

Sol. Length of rope = 5 m

∴ Radius of sector (r) = length of rope = 5 m

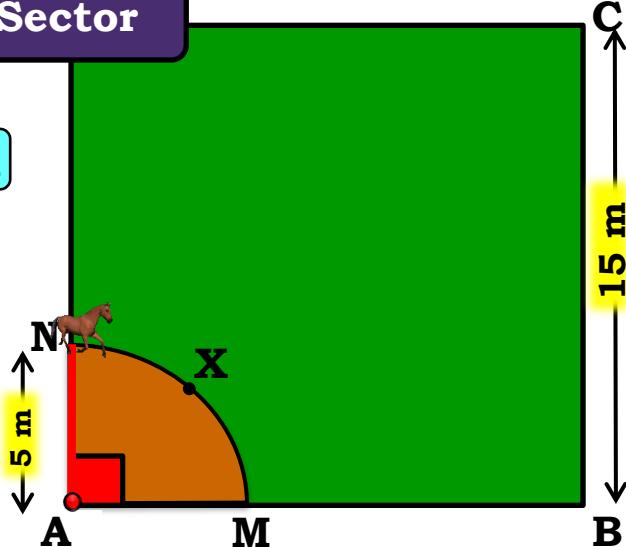
$$\theta = 90^\circ$$

$$\text{ar } (A - MXN) = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{90}{360} \times 3.14 \times 5 \times 5$$

What is the formula to find
area of sector?

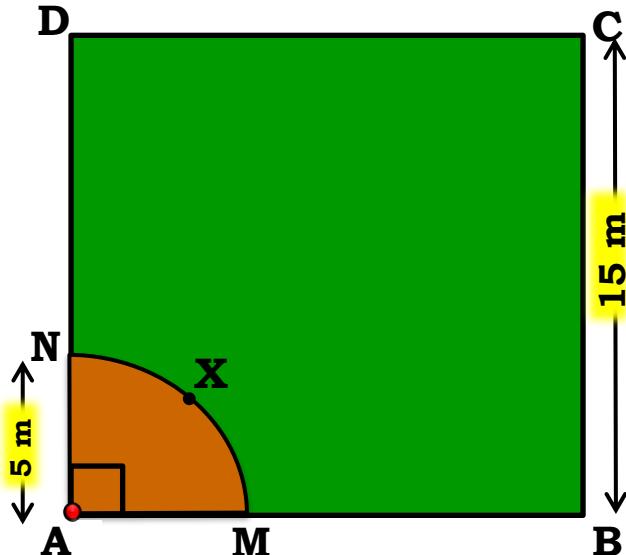
$$\therefore \text{ar } (A - MXN) = \frac{1}{4} \times \frac{314}{100} \times 5 \times 5$$



Q. A Horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope.
Find the area of that part of the field in which the horse can graze.
the increase in the grazing area if the rope were 10m long instead of 5m.
(Use $\pi = 3.14$)

Sol.

$$\begin{aligned}
 \text{ar (A - MXN)} &= \frac{1}{4} \times \frac{\cancel{3.14}}{100} \times 5 \times 5 \\
 &= \frac{\cancel{3925}}{2 \times 100} \\
 &= \frac{1962.5}{100} \\
 \therefore \text{ar (A - MXN)} &= 19.625 \text{ m}^2
 \end{aligned}$$



\therefore Area of the part of the field in which horse can graze is 19.625 m^2

Q. A Horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope.

**Find the area of that part of the field in which the horse can graze.
the increase in the grazing area if the rope were 10m long instead of 5m.
(Use $\pi = 3.14$)**

Sol. Length of rope = 10 m

Radius of sector (r) = length of rope = 10m

$$\theta = 90^\circ$$

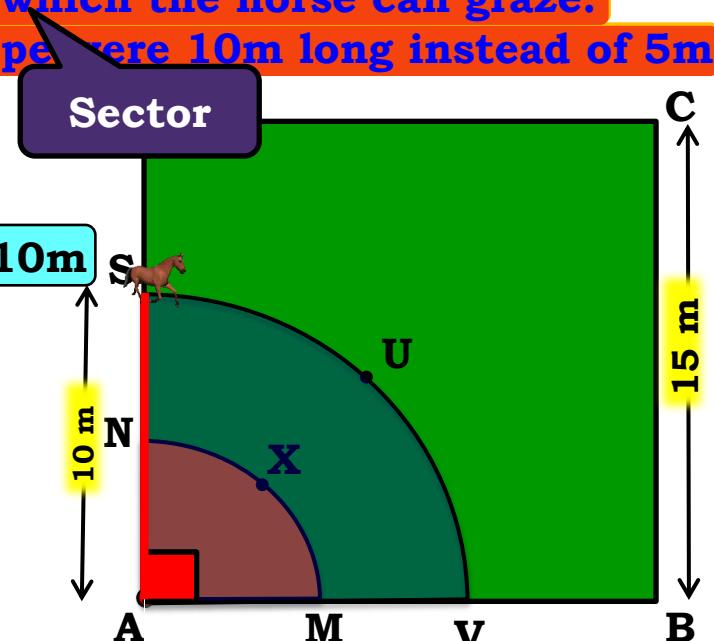
$$ar(A - SUV) = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{1}{360} \times 3.14 \times 10 \times 10$$

What is the formula to find
area of sector?

$$\therefore ar(A - SUV) = 78.5 \text{ m}^2$$

∴ Area of the part of the field in which horse can graze is 78.5 m^2



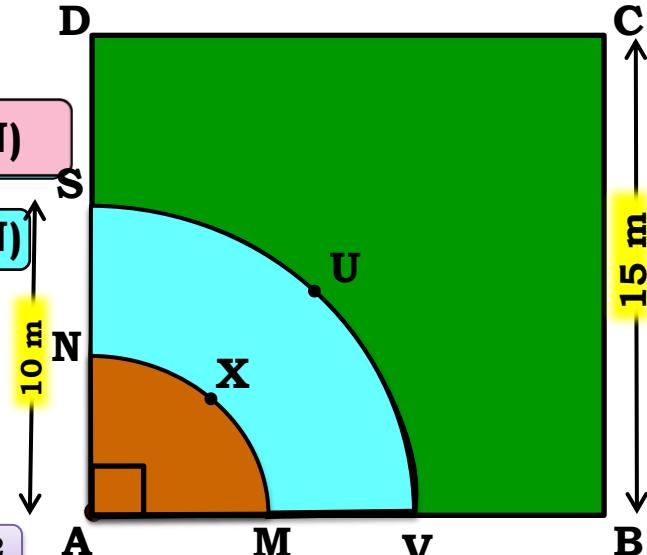
Q. A Horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope.
 Find the area of that part of the field in which the horse can graze.
 the increase in the grazing area if the rope were 10m long instead of 5m.
 (Use $\pi = 3.14$)

Sol.

$$\text{Increase in grazing area} = \text{ar}(A-SUV) - \text{ar}(A-MXN)$$

$$\begin{aligned}\text{Increase in grazing area} &= \text{ar}(A-SUV) - \text{ar}(A-MXN) \\ &= 78.5 - 19.625 \\ &= 58.875 \text{ m}^2\end{aligned}$$

∴ Increase in the grazing area is 58.875 m^2



$$\text{ar}(A-SUV) = 78.5 \text{ m}^2$$

$$\text{ar}(A-MXN) = 19.625 \text{ m}^2$$

Module

18

AREAS RELATED TO CIRCLE

- Sum based on Area of sector
and Right-angled triangle

Q. In the adjoining figure, OACB is a quadrant of a circle (centre O) and radius 3.5 cm. If OD = 2 cm, find the area of the

- (i) Quadrant OACB,
(ii) shaded region.**

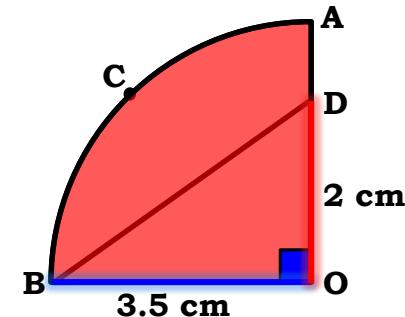
Sol.

$$\theta = 90^\circ$$

$$\text{Radius} = 3.5 \text{ cm}$$

What is formula to find
area quadrant ?

$$\begin{aligned}\therefore \text{ar (O-BCA)} &= \frac{\theta}{360} \times \pi r^2 \\&= \frac{90}{360} \times \frac{22}{7} \times 3.5 \times 3.5 \\&= 5.5 \times 0.5 \times 3.5 \\&= 9.625 \text{ cm}^2\end{aligned}$$



∴ Area of the quadrant OACB is 9.625 cm^2

Q. In the adjoining figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

- (i) quadrant OACB,
(ii) shaded region.

Sol.

$$\text{ar(shaded region)} = \text{ar}(O - BCA) - \text{ar}(\Delta BOD)$$

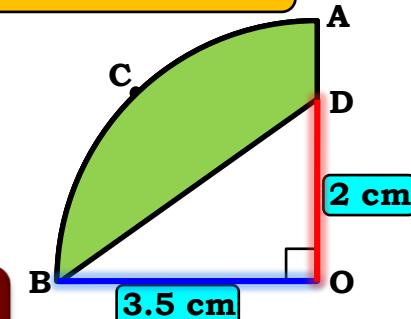
$$\text{ar } (\Delta BOD) = \frac{1}{2} \times BO \times OD$$

$$= \frac{1}{2} \times 3.5 \times 2$$

$$\therefore \text{ar } (\Delta BOD) = 3.5 \text{ cm}^2$$

What is formula to
find area of triangle?

$$\text{ar}(O - BCA) = 9.625 \text{ cm}^2$$



$$\text{Area of the shaded portion} = \text{ar } (O - BCA) - \text{ar } (\Delta BOD)$$

$$= 9.625 - 3.5$$

$$\therefore \text{Area of the shaded portion} = 6.125 \text{ cm}^2$$

$$\therefore \text{Area of the shaded portion is } 6.125 \text{ cm}^2$$

Module

19

AREAS RELATED TO CIRCLE

- Sum based on Area of sector

Q. Find the area of the shaded region, if the radii of the two concentric circles with center O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$.

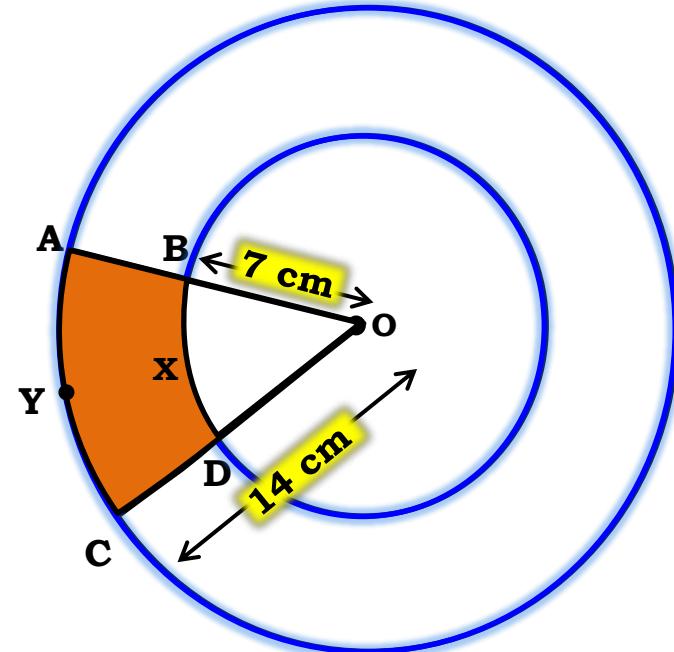
Sol: Radius of smaller circle (r) = 7 cm

Radius of Bigger circle (R) = 14 cm

Central angle (θ) = 40°

$$\begin{aligned} \text{ar (shaded portion)} &= \text{ar}(O - AYC) - \text{ar}(O - BXD) \\ &= \frac{\theta}{360} \times \pi R^2 - \frac{\theta}{360} \times \pi r^2 \\ &= \frac{\theta}{360} \pi (R^2 - r^2) \end{aligned}$$

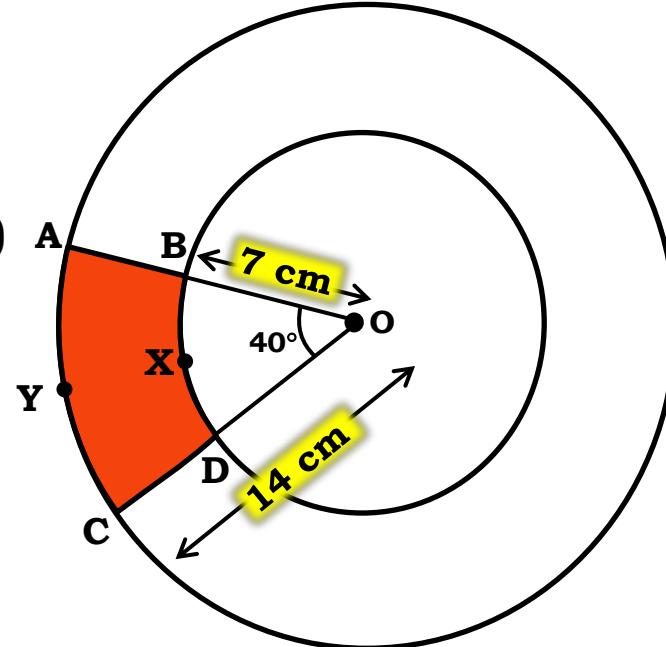
What is the formula to find
ar (shaded portion) $\frac{\theta}{360} \times \pi r^2$?



Q. Find the area of the shaded region, if the radii of the two concentric circles with center O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$.

Sol:

$$\begin{aligned}
 \text{ar (shaded portion)} &= \frac{\frac{1}{9} \times \frac{22}{7} \times (14^2 - 7^2)}{360} \\
 &= \frac{1}{9} \times \frac{22}{7} \times (14 + 7)(14 - 7) \\
 &= \frac{1}{9} \times \frac{22}{7} \times 21 \times 7 \\
 &\quad \boxed{(a^2 - b^2)b(a = ?b)} \\
 &= \frac{22 \times 7}{3} \\
 &= \frac{154}{3} \\
 &= 51.33\text{cm}^2
 \end{aligned}$$



\therefore Area of the shaded portion is 51.33 cm^2

Q. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm with centre O. If $\angle AOB = 30^\circ$, find the area of the shaded region.

Sol. Radius of smaller circle (r) = 7 cm

Radius of Bigger circle (R) = 21 cm

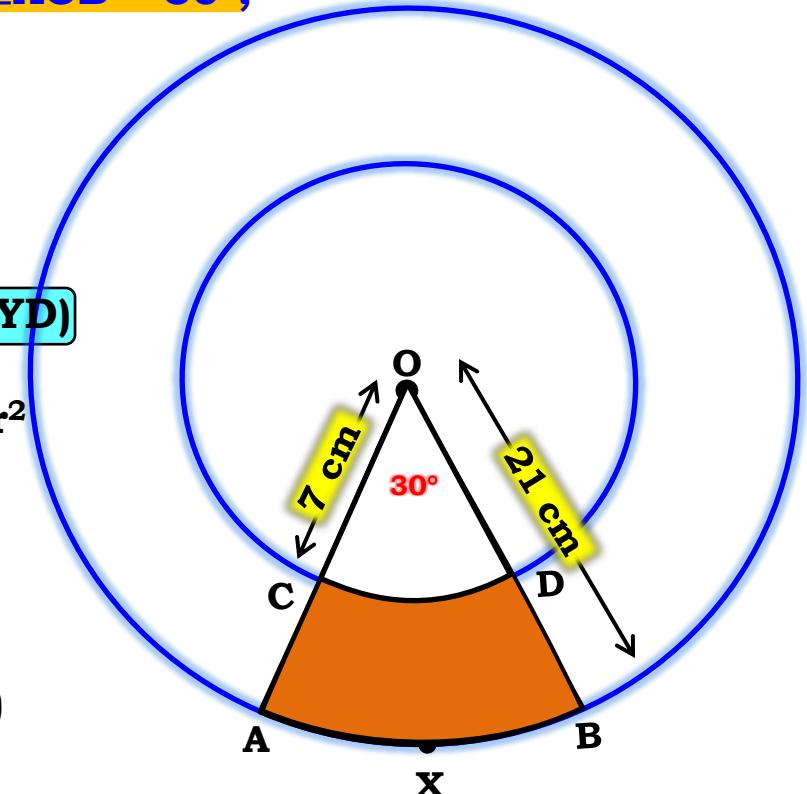
Central angle (θ) = 30°

$$\text{ar (shaded portion)} = \text{ar}(O - AXB) - \text{ar}(O - CYD)$$

$$= \frac{\theta}{360} \times \pi R^2 - \frac{\theta}{360} \times \pi r^2$$

What is the formula to find
area of sector?

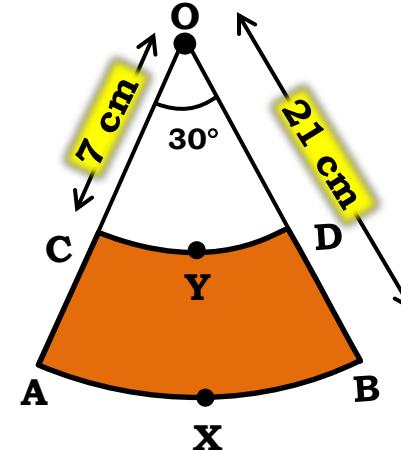
$$\text{ar (shaded portion)} = \frac{30}{360} \times \frac{22}{7} \times (21^2 - 7^2)$$



Q. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm with centre O. If $\angle AOB = 30^\circ$, find the area of the shaded region.

Sol.

$$\begin{aligned}
 \text{ar (shaded portion)} &= \frac{\frac{1}{30}}{\frac{360}{30}} \times \frac{22}{7} \times (21^2 - 7^2) \\
 &= \frac{1}{12} \times \frac{22}{7} \times (21 + 7)(21 - 7) \\
 &= \frac{1}{12} \times \frac{22}{7} \times 28 \times 14 \quad (\text{a}^2 - b^2 = (a+b)(a-b)) \\
 &\cancel{\neq} 3 \\
 &= \frac{11 \times 28}{3} \\
 &= \frac{308}{3} \\
 &= 102.67\text{cm}^2
 \end{aligned}$$



\therefore Area of the shaded portion is 102.67 cm^2

Module 20

AREAS RELATED TO CIRCLE

- Sum based on Area of sector

Q. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find : (i) the total length of the silver wire required. (ii) the area of each sector of the brooch.

Sol.

$$\text{Radius (r)} = \frac{35}{2} \text{ mm}$$

Total length of the wire = Circumference + 5 (Diameters)

$$2\pi r + 5(2r)$$

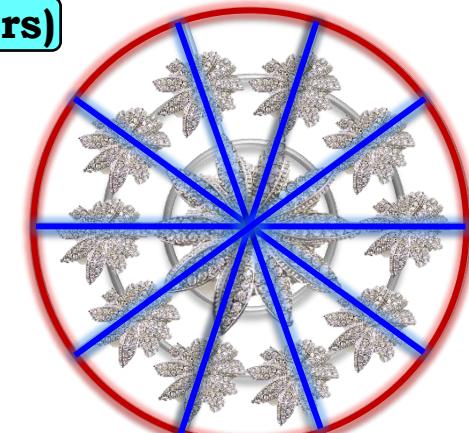
Ques.

What is the formula to find circumference of circle?

$$= 2 \times \frac{35}{2} \times \left[\frac{22 + 35}{7} \right]$$

$$= 2 \times \frac{35}{2} \times \frac{57}{7}$$

$$= 5 \times 57$$



∴ Total length of the wire is 285 mm

Q. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find : (i) the total length of the silver wire required.

Sol.

(ii) the area of each sector of the brooch.

$$\text{Central angle} = \frac{360}{10} = 36^\circ$$

$$\text{Radius (r)} = \frac{35}{2} \text{ mm}$$

$$\text{Area of each sector} = \frac{\theta}{360} \times \pi r^2$$

What is the formula to find area of a sector?

$$\begin{aligned}
 &= \frac{11 \times 35}{4} \\
 &= \frac{385}{4} \\
 &= 96.25 \text{ mm}^2
 \end{aligned}$$



∴ Area of each sector is 96.25 mm^2

Thank You

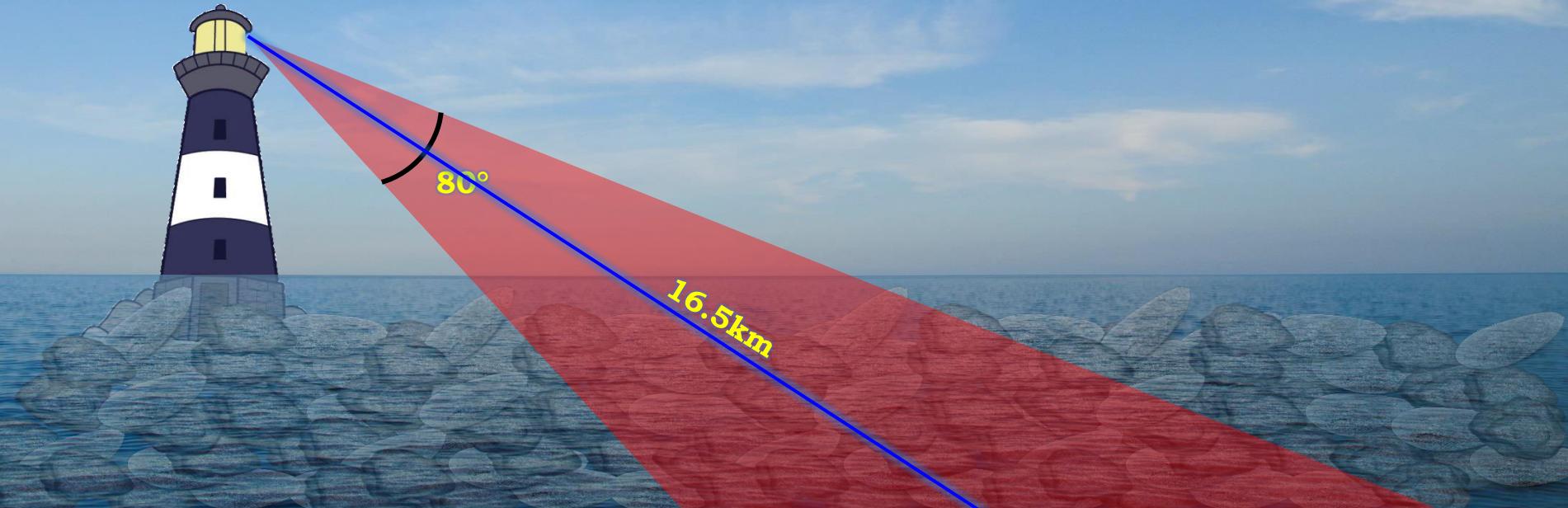
Module 21

AREAS RELATED TO CIRCLE

- **Sums based on Area of sector**

Q. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km.

Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)



Q. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km.

Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)

Sol.

$$\text{Radius } (r) = 16.5 \text{ km}$$

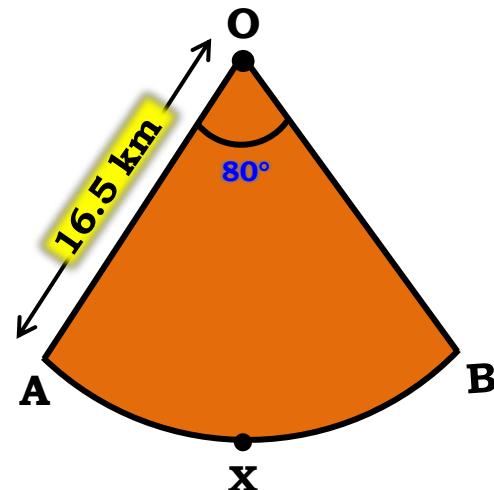
$$\text{Central angle } (\theta) = 80^\circ$$

$$\begin{aligned} \text{ar}(O-AXB) &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{2 \cancel{80}}{9 \cancel{360}} \times 3.14 \times 16.5 \times 16.5 \\ &= \frac{2}{9} \times \frac{314}{100} \times \frac{55}{10} \times \frac{55}{10} \end{aligned}$$

What is the formula to find area of sector?

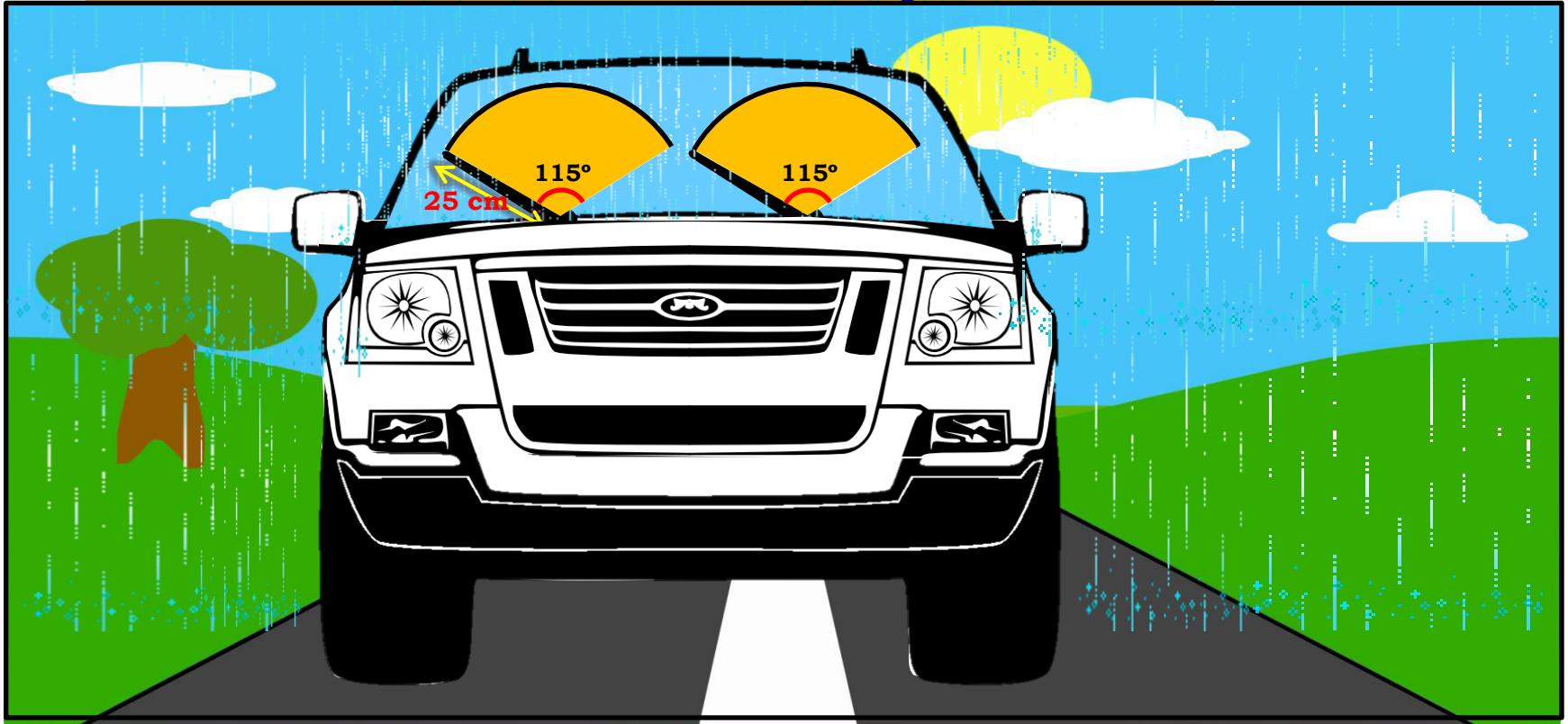
$$\frac{2 \times 314 \times 55 \times 55}{360 \times 10000} = \frac{1899700}{10000}$$

$$= 189.97 \text{ km}^2$$



\therefore Area of sector is 189.97 km^2

Q. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades.



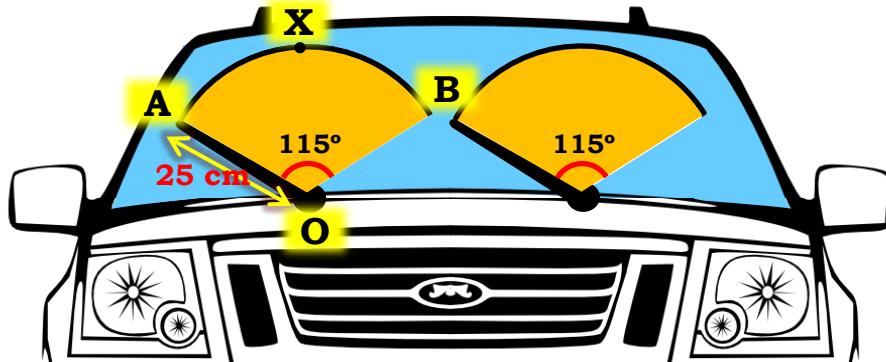
Q. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades.

Sol.

$$r = 25 \text{ cm}$$

$$\theta = 115^\circ$$

$$\begin{aligned} \therefore \text{ar}(O-AXB) &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{115}{360} \times \frac{22}{7} \times 25 \times 25 \\ &= \frac{23}{36} \times \frac{11}{7} \times 25 \times 25 \end{aligned}$$



$$\begin{aligned} \therefore \text{Total area} &= 2 \times \frac{23}{36} \times \frac{11}{7} \times 25 \times 25 \\ &= \frac{23 \times 11 \times 25 \times 25}{18 \times 7} \\ &= \frac{158125}{126} \end{aligned}$$

$$\text{area of sector} = \frac{\theta}{360} \times \pi r^2$$

The total area cleaned at each sweep of the blades is 1254.96 cm^2

$$\therefore \text{Total area} = 1254.96 \text{ cm}^2$$

Module

22

AREAS RELATED TO CIRCLE

- Sum based on finding area of shaded portion

Q. In given figure, a square OABC is inscribed in a quadrant

We know,

$$\text{Diagonal} = \sqrt{2} \times \text{Side}$$

Sol. $\square OABC$ is a square

$$\therefore \angle O = 90^\circ$$

$$r = 20 \text{ cm}, \text{ find the area of the shaded region.}$$

$$\text{ar}(O - PBQ) = \text{ar}(OABC)$$

Draw OB

To find: r

Radius of quadrant = Diagonal of a square

$$\therefore \text{Diagonal} = \sqrt{2} \times \text{Side}$$

So, to find radius we need formula to of quadrant?
Diagonal to find diagonal of square.

Radius of quadrant = Diagonal of a square

Let us draw radius for given quadrant OPBQ

We know,

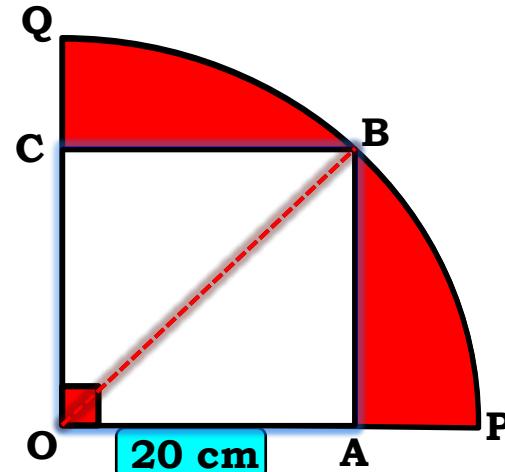
$\square OABC$ is a square

$$\therefore \text{ar}(O - PBQ)$$

$$\begin{aligned} &= \frac{90}{360} \times 3.14 \times 20\sqrt{2} \times 20\sqrt{2} \\ &= 3.14 \times 5 \times 20 \times 2 \end{aligned}$$

$$= 3.14 \times 200$$

$$= 628 \text{ cm}^2$$



Q. In given figure, a square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20 cm, find the area of the shaded region.

$$\text{ar(shaded region)} = \text{ar}(O - PBQ) - \text{ar}(\square OABC)$$

Sol.

$$\begin{aligned}\text{ar}(\square OABC) &= (\text{Side})^2 \\ &= (20)^2\end{aligned}$$

What is formula to
find area of square?

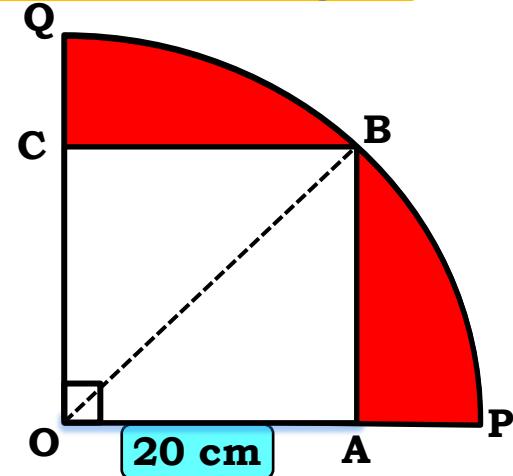
$$\therefore \text{ar}(\square OABC) = 400 \text{ cm}^2$$

$$\begin{aligned}\text{ar(shaded region)} &= \text{ar}(O - PBQ) - \text{ar}(\square OABC) \\ &= 628 - 400\end{aligned}$$

$$\therefore \text{ar(shaded region)} = 228 \text{ cm}^2$$

∴ ar(shaded region) is 228 cm²

$$\text{ar}(O - PBQ) = 228 \text{ cm}^2$$



Module

23

AREAS RELATED TO CIRCLE

- Sum based on finding area of shaded portion

- Q.** In the figure, ABCD is a square of side 14 cm, with centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

$$\text{ar (shaded region)} = \text{ar } (\square ABCD) - 4\pi r^2$$

Sol.

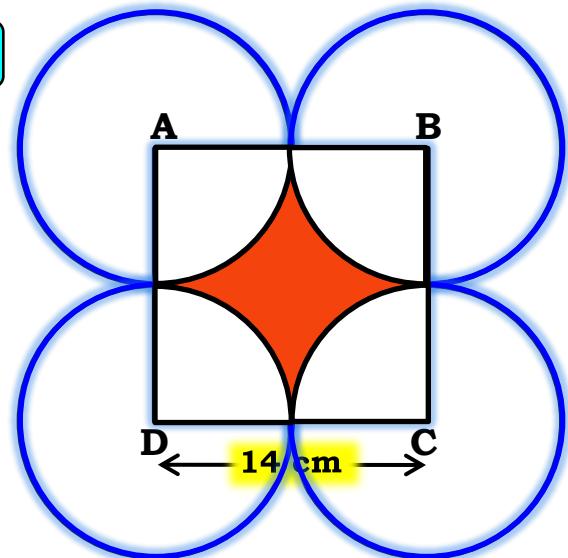
Side of the square = 14 cm

$$\text{Ar } (\square ABCD) = (\text{side})^2$$

What is the formula to find
(Side)²
area of a square?

$$= 196 \text{ cm}^2$$

$$\therefore \text{Ar } (\square ABCD) = 196 \text{ cm}^2$$



Q. In the figure, ABCD is a square of side 14 cm. with centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

$$\text{ar (shaded region)} = \text{ar } (\square ABCD) - 4 \text{ ar (1 sectors)}$$

Sol. Diameter = 14 cm

\therefore Radius (r) = 7 cm

Central angle (θ) = 90°

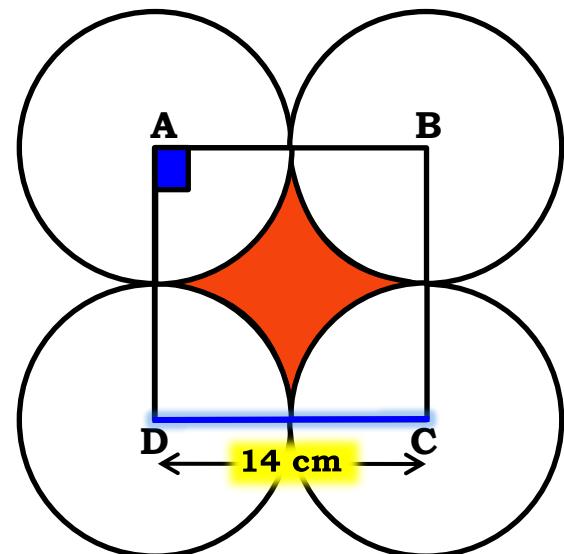
$$\text{Ar (4 sectors)} = 4 \times \frac{\theta}{360} \times \pi r^2$$

$$= 4 \times \frac{90}{360} \times \frac{22}{7} \times 7 \times 7$$

What is the formula to find area of 1 sector?

$$= 154 \text{ cm}^2$$

\therefore Ar (4 sectors) = 154 cm^2



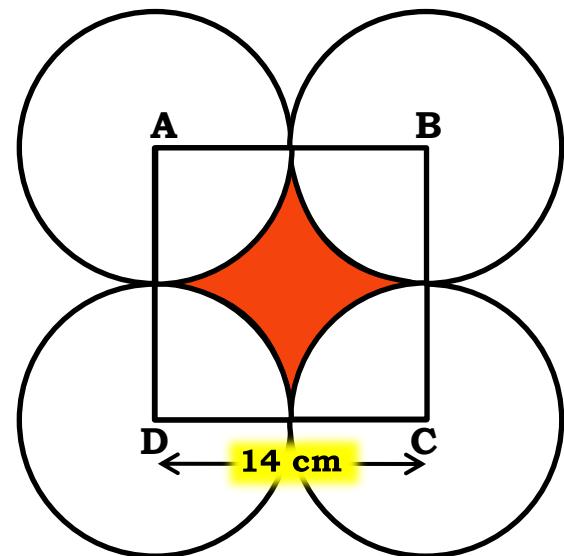
Q. In the figure, ABCD is a square of side 14 cm. with centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

$$\text{ar (shaded region)} = \text{ar } (\square ABCD) - 4 \text{ ar (1 sectors)}$$

Sol.

$$\begin{aligned}\text{ar (shaded region)} &= \text{ar } (\square ABCD) - 4 \text{ ar (1 sectors)} \\ &= 196 - 154 \\ &= 42 \text{ cm}^2\end{aligned}$$

\therefore Ar (shaded region) is 42 cm^2



$$\text{Ar } (\square ABCD) = 196 \text{ cm}^2$$

$$\text{Ar (4 sectors)} = 154 \text{ cm}^2$$

Module 24

AREAS RELATED TO CIRCLE

- Sum based on finding area of shaded portion

Q. The area of an equilateral triangle ABC is 17320.5 cm^2 . With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle. Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)

$$\text{ar (shaded region)} = \text{ar } (\Delta ABC) - 3 \text{ ar (3 sectors)}$$

Sol. $\text{ar } (\Delta ABC) = 17320.5 \text{ cm}^2$

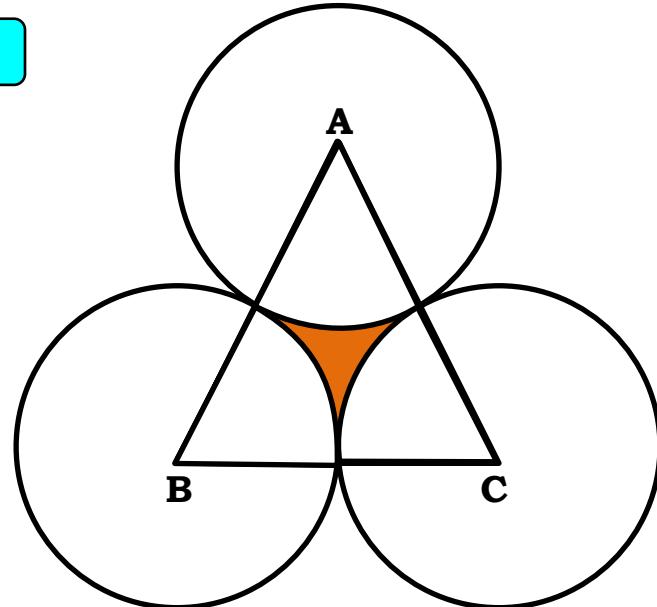
$$\text{ar } (\Delta ABC) = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

From area, let us find side of

\therefore What is the formula to find area of an equilateral triangle?

$$\therefore \frac{1.73205}{4} \times (\text{side})^2 = 17320.5$$

$$\therefore \frac{173205}{4 \times 100000} \times (\text{side})^2 = \frac{173205}{4 \times 10}$$



Q. The area of an equilateral triangle ABC is 17320.5 cm^2 . With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle. Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)

$$\text{ar (shaded region)} = \text{ar } (\Delta ABC) - 3 \text{ ar (1 sector)}$$

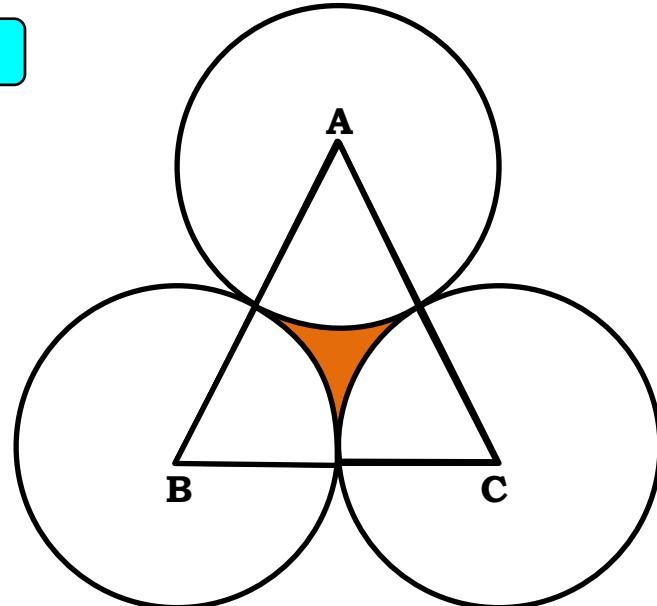
Sol.

$$\therefore \frac{173205}{4 \times 100000} \times (\text{side})^2 = \frac{173205}{4 \times 10}$$

$$\begin{aligned}\therefore (\text{side})^2 &= \frac{173205 \times 4 \times 100000}{10 \times 173205} \\ &= 4 \times 10000\end{aligned}$$

$$\text{side} = 2 \times 100$$

$$\therefore \text{side} = 200 \text{ cm}$$



Q. The area of an equilateral triangle ABC is 17320.5 cm^2 . With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle. Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)

$$\text{ar (shaded region)} = \text{ar}(\Delta ABC) - 3 \text{ ar (1 sector)}$$

$$\begin{aligned}\text{Sol. } \text{Radius (r)} &= \frac{1}{2} \times \text{Side} \\ &= \frac{1}{2} \times 200\end{aligned}$$

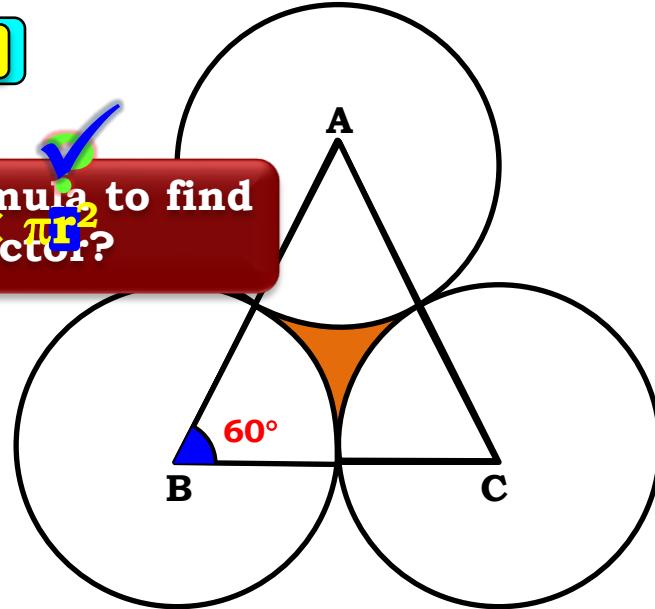
$$\therefore \text{Radius (r)} = 100$$

What is the formula to find
area of $\frac{\theta}{360}$ sector?

We know that, angle of an equilateral triangle = 60°

$$\text{ar (3 sectors)} = \frac{60}{360} \times \pi r^2$$

$$\text{ar (3 sectors)} = 3 \times \frac{60}{360} \times 3.14 \times 100 \times 100$$



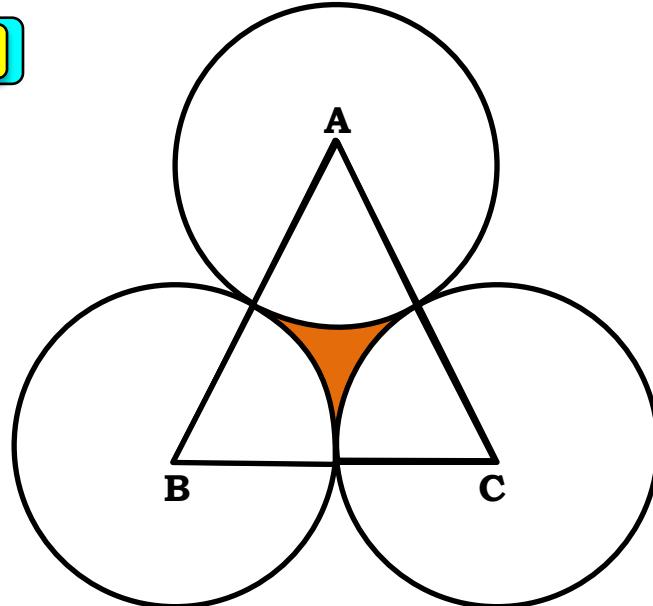
$$\text{Side} = 200 \text{ cm}$$

Q. The area of an equilateral triangle ABC is 17320.5 cm². With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle. Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)

ar (shaded region) = ar (ΔABC) - 3 ar (1 sector)

Sol.

$$\begin{aligned}
 \text{ar (3 sectors)} &= \cancel{3} \times \frac{\cancel{160}}{\cancel{360}} \times 3.14 \times 100 \times 100 \\
 &= \frac{1}{2} \times \frac{157}{100} \times 100 \times 100 \\
 &= 15700 \text{ cm}^2
 \end{aligned}$$



Q. The area of an equilateral triangle ABC is 17320.5 cm^2 . With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle. Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)

$$\text{ar (shaded region)} = \text{ar } (\Delta ABC) - 3 \text{ ar (1 sector)}$$

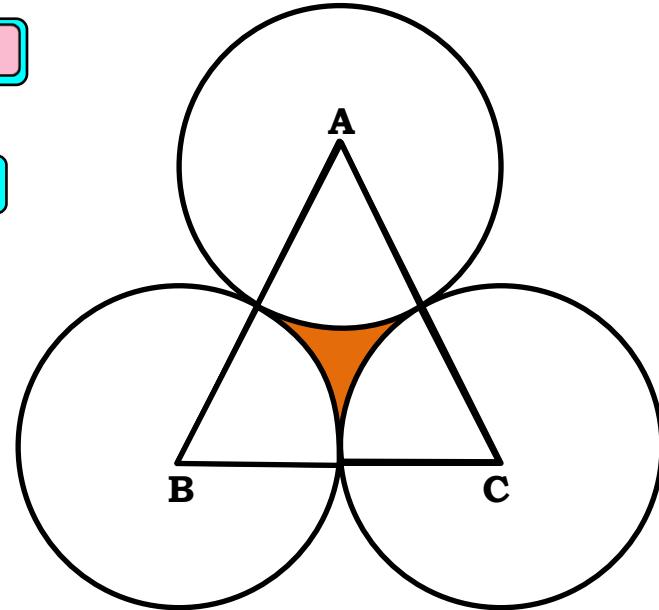
Sol.

$$\text{ar (shaded region)} = \text{ar } (\Delta ABC) - 3 \text{ ar (1 sector)}$$

$$= 17320.5 - 15700$$

$$= 1620.5 \text{ cm}^2$$

∴ Area of the shaded portion is 1620.5 cm^2



$$3 \text{ ar (1 sectors)} = 15700 \text{ cm}^2$$

Module

25

AREAS RELATED TO CIRCLE

- Sum based on finding area of shaded portion

Q. Find the area of the shaded region, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

Sol.

$$\text{ar(shaded region)} = \text{ar(circle)} + \text{ar}(\Delta AOB) - \text{ar}(O - CXD)$$

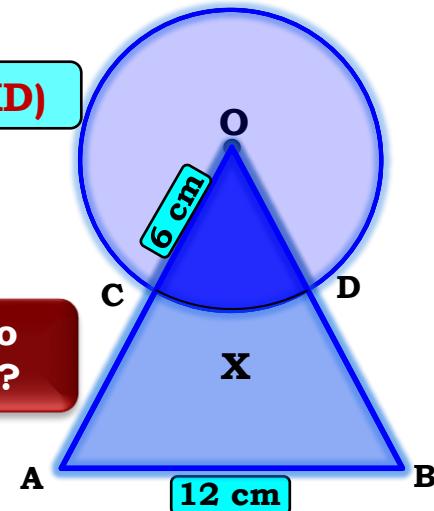
$$\text{Area of circle} = \pi r^2$$

$$= \frac{22}{7} \times 6 \times 6 \\ = \frac{792}{7} \quad 113.14$$

What is formula to
find $\text{ar}(\text{Side})^2$
equilateral triangle ?

$$\therefore \text{Area of circle} = 113.14 \text{ cm}^2$$

What is formula to
find area of circle?



$$\text{Area of } \Delta AOB = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times \cancel{12} \times \cancel{12}$$

$$\therefore \text{Area of } \Delta AOB = 36\sqrt{3} \text{ cm}^2$$

Q. Find the area of the shaded region, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

Sol.

$$\text{ar(shaded region)} = \text{ar(circle)} + \text{ar}(\Delta AOB) - \text{ar}(O - CXD)$$

$$\text{ar}(O - CXD) = \frac{\theta}{360} \times \pi r^2$$

We know that, in equilateral triangle each angle is 60°

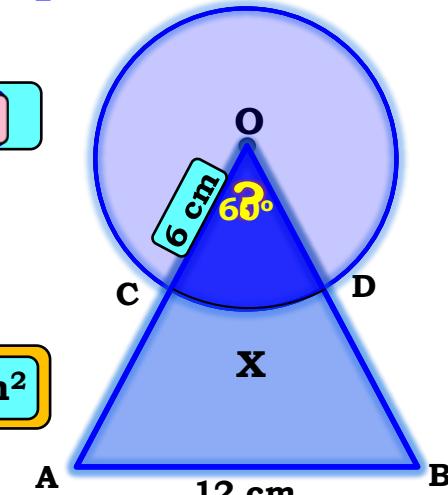
$$= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{132}{7} \text{ cm}^2$$

$$\text{ar(circle)} = 113.14 \text{ cm}^2$$

$$\therefore \text{ar}(O - CXD) = 18.85 \text{ cm}^2$$

$$\text{ar}(\Delta AOB) = 36\sqrt{3}$$



$$\begin{aligned}\therefore \text{Area of the shaded portion} &= \text{Area of circle} + \text{ar}(\Delta AOB) - \text{ar}(O - CXD) \\ &= 113.14 + 36\sqrt{3} - 18.85\end{aligned}$$

$$\therefore \text{Area of the shaded portion} = [94.29 + 36\sqrt{3}] \text{ cm}^2$$

$$\therefore \text{Area of the shaded portion is } [94.29 + 36\sqrt{3}] \text{ cm}^2$$

Thank You

Module 26

AREAS RELATED TO CIRCLE

- Sum based on finding area of shaded portion

Q. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut in the middle. Find the area of the remaining portion of the square.

$$\text{ar (shaded region)} = \text{ar}(\square ABCD) - 4 \text{ ar (quadrant)} - \text{ar (smaller circle)}$$

Sol. Side of the square = 4 cm

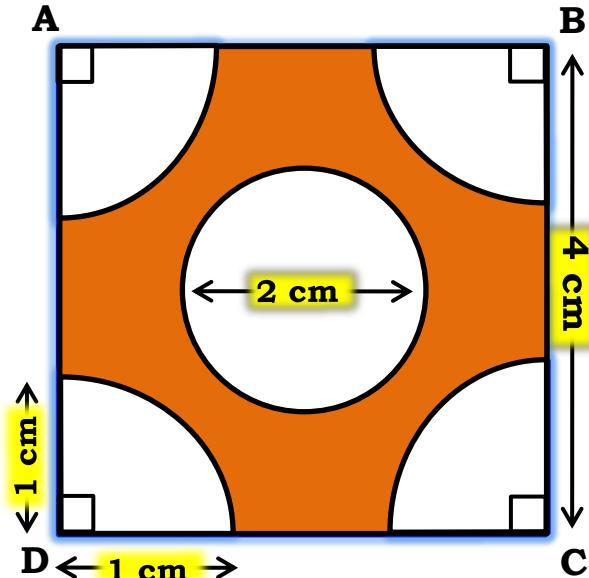
Radius of quadrant (r) = 1 cm

$$\text{Radius of circle (R)} = \frac{2}{2} = 1 \text{ cm}$$

$$\begin{aligned}\text{ar } (\square ABCD) &= (\text{side})^2 \\ &= (4)^2\end{aligned}$$

What is the formula to find
(Side)²
area of a square?

∴



Q. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut in the middle. Find the area of the remaining portion of the square.

$$\text{ar (shaded region)} = \text{ar}(\square ABCD) - 4 \text{ ar (quadrant)} - \text{ar (smaller circle)}$$

Sol.

$$\begin{aligned}\text{ar (4 quadrants)} &= \cancel{4} \times \frac{1}{\cancel{4}} \times \pi r^2 \\ &= \pi r^2\end{aligned}$$

What is the formula to find
area of quadrant?

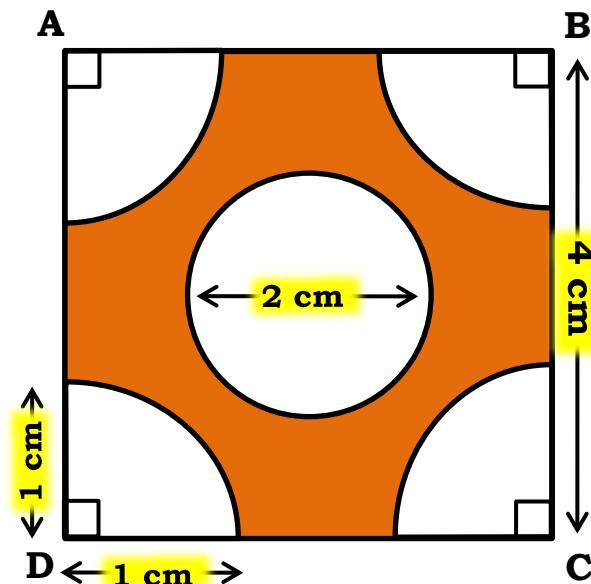
$$\begin{aligned}&\times 1 \\ &= \frac{\pi r^2}{4} \text{ cm}^2\end{aligned}$$

$$\therefore \text{ar (4 quadrants)} = \frac{22}{7} \text{ cm}^2$$

Radius of quadrant (r) = 1 cm

Radius of circle (R) = 1 cm

ar ($\square ABCD$) = 16 cm 2



Q. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut in the middle. Find the area of the remaining portion of the square.

$$\text{ar (shaded region)} = \text{ar}(\square ABCD) - 4 \text{ ar (quadrant)} - \text{ar (smaller circle)}$$

Sol. $\text{ar (smaller circle)} = \pi r^2$

$$= \frac{22}{7} \times 1 \times 1$$

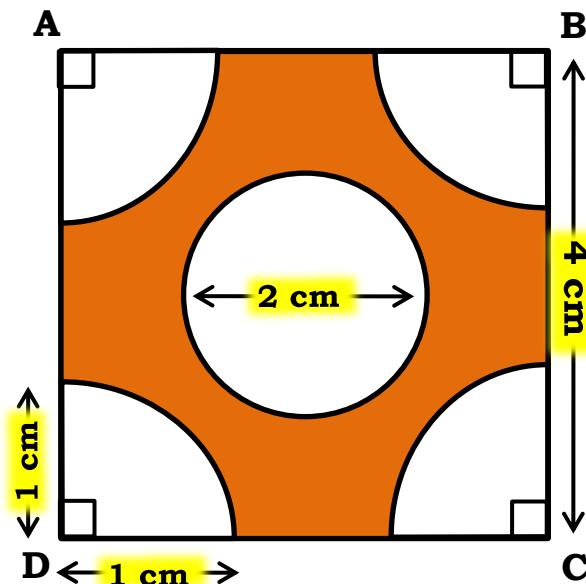
What is the formula to find
 πr^2
area of circle?

$$\therefore \text{ar (smaller circle)} = \frac{22}{7} \text{ cm}^2$$

Radius of circle (R) = 1 cm

$\text{ar (4 quadrants)} = \frac{22}{7} \text{ cm}^2$

$\text{ar}(\square ABCD) = 16 \text{ cm}^2$



Q. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut in the middle. Find the area of the remaining portion of the square.

$$\text{ar (shaded region)} = \text{ar } (\square ABCD) - 4 \text{ ar (quadrant)} - \text{ar (smaller circle)}$$

Sol. $\text{ar (shaded region)} = \text{ar } (\square ABCD) - 4 \text{ ar (quadrant)} - \text{ar (smaller circle)}$

$$\begin{aligned} &= 16 - \frac{22}{7} - \frac{22}{7} \\ &= 16 - \left[\frac{22}{7} + \frac{22}{7} \right] \\ &= 16 - \frac{44}{7} \end{aligned}$$

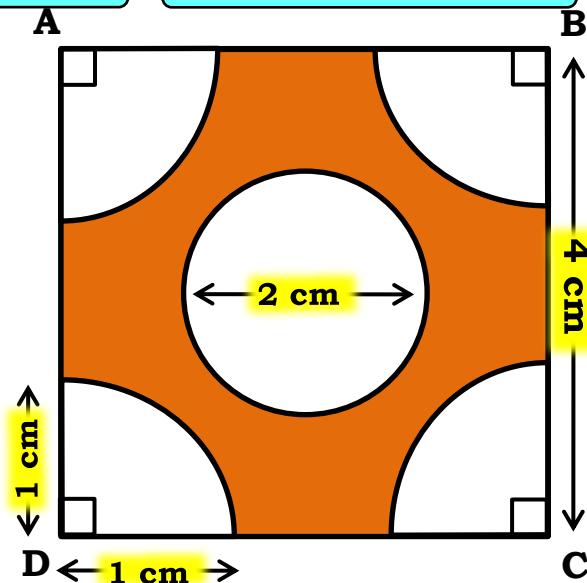
$$\text{ar } (\square ABCD) = 16 \text{ cm}^2$$

$$112 - 44$$

$$\text{ar (4 quadrants)} = \frac{22}{7} \text{ cm}^2$$

$$\text{ar (smaller circle)} = \frac{22}{7} \text{ cm}^2$$

∴ Area of the shaded portion is 9.71 cm^2



Module

27

AREAS RELATED TO CIRCLE

- Sum based on finding area of shaded portion

Q. In given figure, ABCD is a trapezium with $AB \parallel DC$ and $\angle BCD = 60^\circ$. If BFEC is a sector of a circle with center C and $AB = BC = 7 \text{ cm}$ and $DE = 4 \text{ cm}$, then find the area of the shaded region (use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.732$). 

$$\text{Area of shaded region} = \text{ar}(\square ABCD) - \text{ar}(C - BFE)$$

Let us draw  
BL \perp To find:  **DC and BL**

Sol. BC = CE = 7cm

DC =

Why are CF_1 and CF_2 given PC? $\therefore \text{They are equal}$

∴ DC =

Opposite side → BL

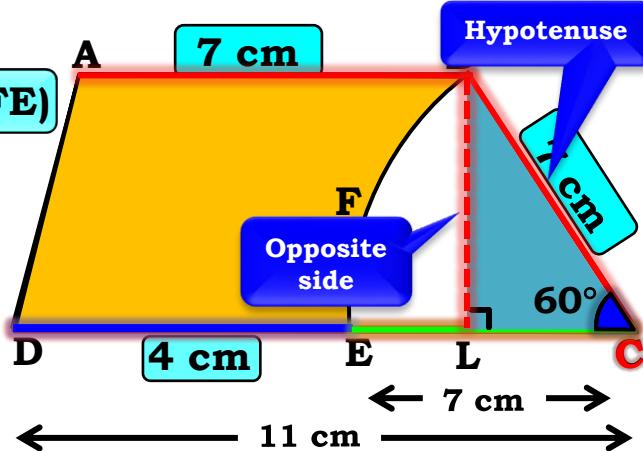
Hypotenuse → BC

$$\frac{1}{2} \times [\text{Sum of parallel sides}] \times \text{height}$$

area of trapezium?

$$\frac{1}{2} \times [AB + DC] \times BL$$

$$\mathbf{BL} = \frac{7\sqrt{3}}{2}$$



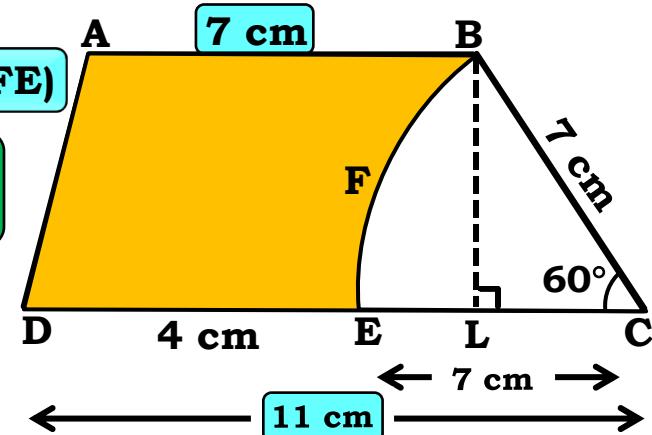
Q. In given figure, ABCD is a trapezium with $AB \parallel DC$ and $\angle BCD = 60^\circ$. If BFEC is a sector of a circle with center C and $AB = BC = 7 \text{ cm}$ and $DE = 4 \text{ cm}$, then find the area of the shaded region (use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.732$).

$$\text{Area of shaded region} = \text{ar}(\square ABCD) - \text{ar}(C - BFE)$$

Sol.

$$\begin{aligned}\therefore \text{ar}(\square ABCD) &= \frac{1}{2} \times (\text{AB} + \text{DC}) \times \text{BL} \\ &= \frac{1}{2} (7 + 11) \times \frac{7\sqrt{3}}{2} \\ &= \frac{1}{2} \times 18^{\cancel{9}} \times \frac{7\sqrt{3}}{2} \\ &= \frac{9 \times 7\sqrt{3}}{2}\end{aligned}$$

$$\therefore \text{ar}(\square ABCD) = \frac{63\sqrt{3}}{2} \text{ cm}^2$$



Q. In given figure, ABCD is a trapezium with $AB \parallel DC$ and $\angle BCD = 60^\circ$. If BFEC is a sector of a circle with center C and $AB = BC = 7 \text{ cm}$ and $DE = 4 \text{ cm}$, then find the area of the shaded region (use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.732$).

$$\text{Area of shaded region} = \text{ar}(\square ABCD) - \text{ar}(C - BFE)$$

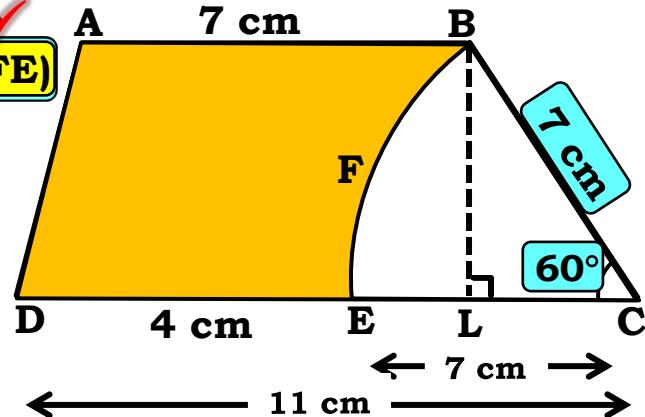
Sol.

$$\begin{aligned}\text{ar}(C - BFE) &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{60}{360} \times \frac{22}{7} \times 7 \times 7\end{aligned}$$

What is formula to
find area of sector?

3

$$\therefore \text{ar}(C - BFE) = \frac{77}{3} \text{ cm}^2$$



Q. In given figure, ABCD is a trapezium with $AB \parallel DC$ and $\angle BCD = 60^\circ$. If BFEC is a sector of a circle with center C and $AB = BC = 7 \text{ cm}$ and $DE = 4 \text{ cm}$, then find the area of the shaded region (use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.732$).

$$\text{Area of shaded region} = \text{ar}(\square ABCD) - \text{ar}(C - BFE)$$

Sol.

$$\text{Area of shaded region} = \text{ar}(\square ABCD) - \text{ar}(C - BFE)$$

\therefore

$$= \left[\frac{63\sqrt{3}}{2} - \frac{77}{3} \right]$$

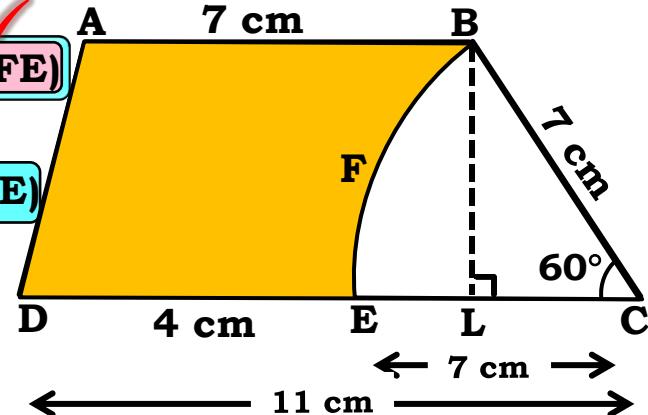
$$= \left[\frac{31.5 \times 1.732}{2} - \frac{77}{3} \right]$$

$$= (31.5 \times 1.732) - 25.666$$

$$= 54.558 - 25.666$$

$$\therefore \text{Area of shaded region} = 28.89 \text{ cm}^2$$

$$\therefore \text{Area of shaded region is } 28.89 \text{ cm}^2$$



$$\text{ar}(C - BFE) = \frac{77}{3} \text{ cm}^2$$

$$\text{ar}(\square ABCD) = \frac{63\sqrt{3}}{2} \text{ cm}^2$$

Module 28

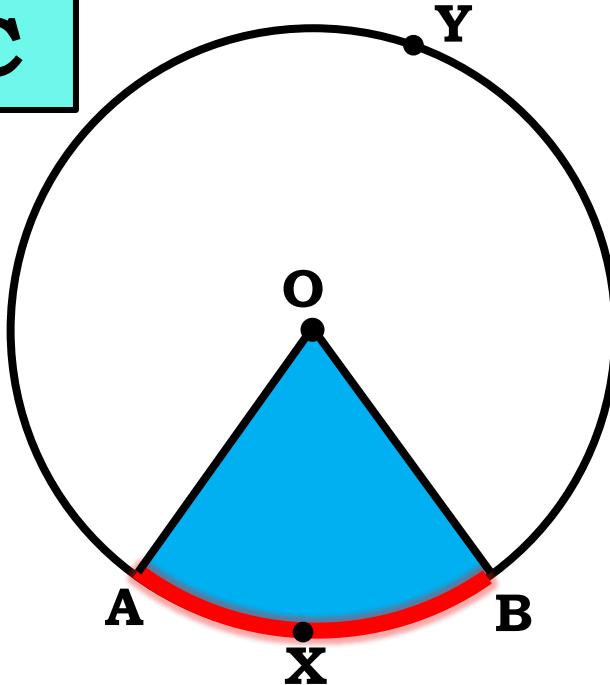
AREAS RELATED TO CIRCLE

- **Length of an Arc**

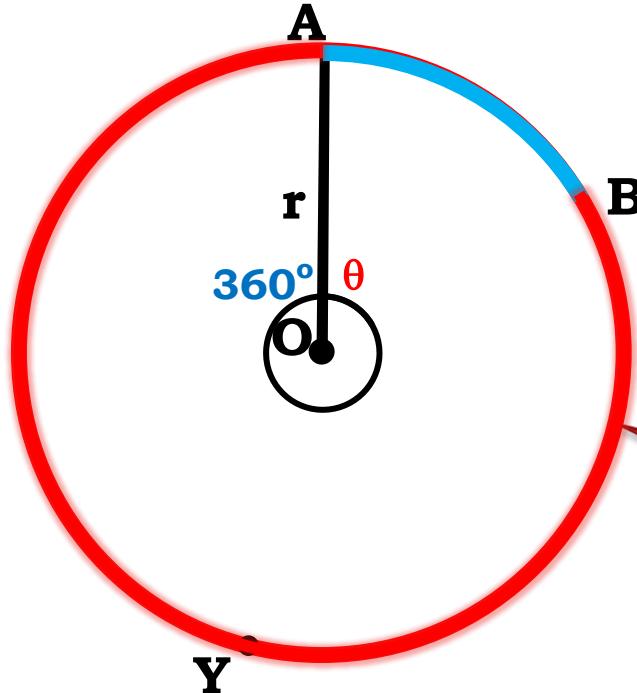
LENGTH OF ARC

Consider sector O-AXB

Let us understand the formula
to find length of arc



LENGTH OF ARC



Rotation	Length
360°	$2\pi r$
θ	P

$$l \times 360^\circ = \theta \times 2\pi r$$

$$l = \frac{\theta}{360} \times 2\pi r$$

How to find length of arc AYB?

$$l(\text{arc AYB}) = \text{circumference} - l(\text{arc AB})$$

$$\theta = ?$$

Q. Find the angle subtended at the centre of the circle by an arc, radius of circle

Let us keep 'θ' on RHS and move all the numbers on LHS

$$\pi = \frac{22}{7}$$

Sol. :

$$\text{Length of arc } (l) = \frac{\theta}{360} \times 2\pi r$$

$$\therefore 6.05 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 5.5$$

$$\therefore \frac{605}{100} = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times \frac{55}{10}$$

$$\therefore \frac{605}{100} \times \frac{\cancel{360} \times 7 \times 10}{\cancel{2} \times \cancel{22} \times \cancel{55}} = \theta$$

$$\therefore \theta = 9 \times 7$$

$$\therefore \theta = 63^\circ$$

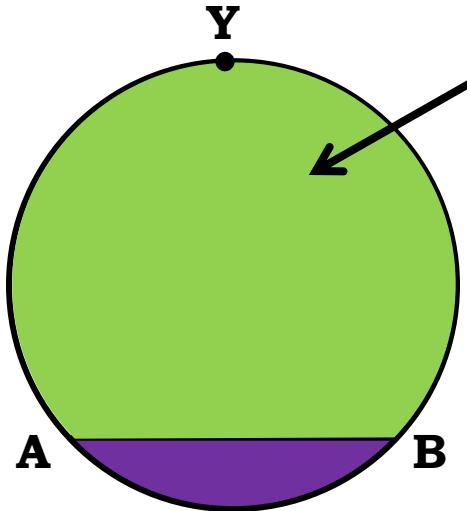
Angle subtended at the centre of the circle by an arc is 63°

Module 29

AREAS RELATED TO CIRCLE

- **Segment of a circle**

Segment of a circle



Major segment

The part of the circular region enclosed by a chord and its corresponding arc is called a Segment of a circle

Let us draw chord AB

$$\text{Area of Segment} = \text{Area of sector} - \text{Area of triangle}$$

$$\text{Area of Major Segment} = \text{Area of circle} - \text{Area of minor segment}$$

Q. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major segment. ? (Use $\pi = 3.14$)

Area of minor segment = $\text{ar}(O - AXB) - \text{ar}(\Delta OAB)$

Sol.

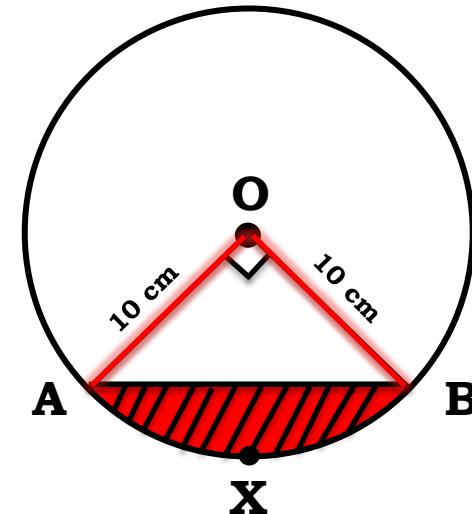
$$r = 10 \text{ cm}$$

$$\theta = 90^\circ$$

What is formula to
find $\frac{\theta}{360} \times \pi r^2$ of sector?

$$\begin{aligned}\text{Area of sector } (O - AXB) &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{90}{360} \times 3.14 \times 10 \times 10\end{aligned}$$

$$\therefore \text{Area of sector } (O - AXB) = 78.5 \text{ cm}^2$$



Q. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major segment. (Use $\pi = 3.14$) ?

Area of minor segment = $ar(O - AXB)$ - $ar(\Delta OAB)$

Sol.

$$\text{Area of } \triangle OAB = \frac{1}{2} \times AO \times OB$$

$$= \frac{1}{2} \times \cancel{10}^5 \times 10$$

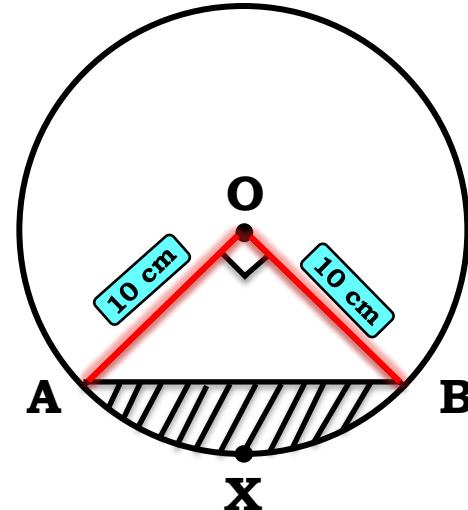
What is formula to
find area of triangle?

$$A(O - AXB) = 78.5 \text{ cm}^2$$

$$\text{Area of } \triangle OAB = 50 \text{ cm}^2$$

$$\begin{aligned}\text{Area of minor segment} &= A(O - AXB) - A(\triangle OAB) \\ &= 78.5 - 50 \\ &= 28.5 \text{ cm}^2\end{aligned}$$

\therefore Area of minor segment is 28.5 cm^2



Q. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major segment. ? (Use $\pi = 3.14$)

Area of major segment = [Area of circle] - Area of minor segment

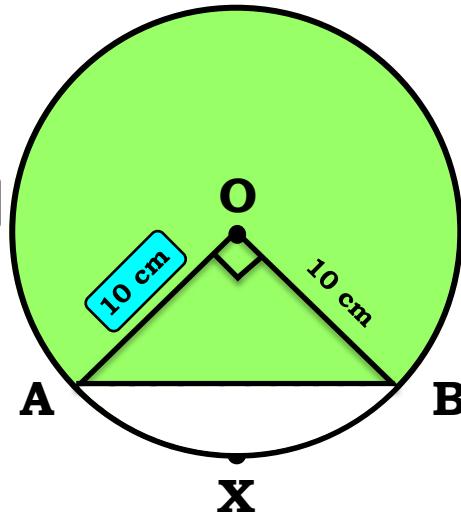
Sol.

$$\text{Area of circle} = \pi r^2$$

$$\text{Area of circle} = 3.14 \times 10 \times 10$$

$$\therefore \text{Area of circle} = 314 \text{ cm}^2$$

What is formula to
 πr^2
find area of circle?



Area of major segment = [Area of circle] - Area of minor segment

$$= 314 - 28.5$$

Area of minor segment = 28.5 cm^2

$\therefore \text{Area of major segment is } 285.5 \text{ cm}^2$

Module

30

AREAS RELATED TO CIRCLE

- **Sum based on length of an arc,
Area of a sector and segment**

Q. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find :

- (i) The length of the arc
- (ii) Area of the sector formed by the arc
- (iii) Area of the segment formed by the corresponding chord.

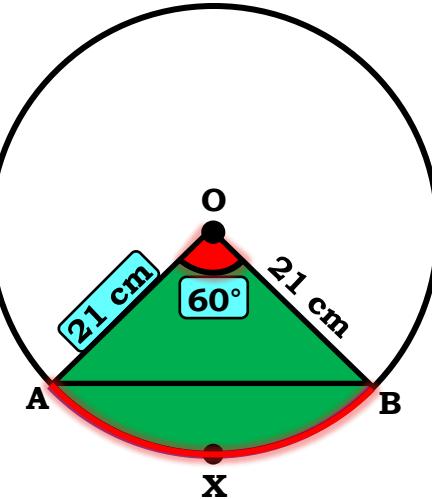
Sol.

$$l(\text{arc AXB}) = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$$

$$= 22 \text{ cm}$$

What is formula to find area of sector?



Length of arc is 22 cm.

$$A(O - AXB) = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21$$

$$= 231 \text{ cm}^2$$

Area of sector (O - AXB) is 231 cm^2 .

Q. In a circle of radius 21 cm, an arc subtends an angle of 60° at the center. Find

(i) $\triangle OAB$ is equilateral triangle

(ii) Area of the segment formed by the arc

(iii) Area of the segment formed by the corresponding chord.

What is formula to find area of equilateral triangle ?

$$\text{Area of minor segment} = \text{ar}(O - AXB) - \text{ar}(\triangle OAB)$$

Sol.

In $\triangle OAB$,

$$OA = OB \quad [\text{radius of same circle}]$$

$$\angle A = \angle B = x \quad [\text{Angles opposite to equal sides are equal}]$$

$$\angle A + \angle B + \angle O = 180^\circ$$

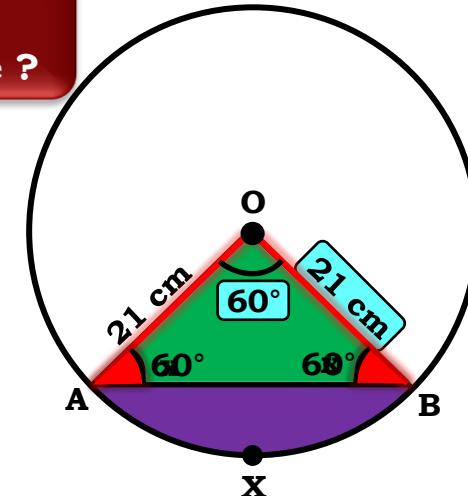
$$\therefore x + x + 60 = 180$$

$$\therefore 2x = 180 - 60$$

$$\text{ar}(O - AXB) = 231 \text{ cm}^2$$

$$x = \frac{120}{2}$$

$$x = 60$$



$\therefore \triangle OAB$ is an equilateral triangle

$$\text{ar} (\triangle OAB) = \frac{\sqrt{3}}{4} \times (\text{Side})^2$$

$$= \frac{\sqrt{3}}{4} \times 21 \times 21$$

$$\therefore \text{ar} (\triangle OAB) = \frac{441\sqrt{3}}{4} \text{ cm}^2$$

Q. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find :

- (i) The length of the arc
- (ii) Area of the sector formed by the arc
- (iii) **Area of the segment formed by the corresponding chord.**

$$\text{Area of minor segment} = \text{ar}(O - AXB) - \text{ar}(\triangle OAB)$$

Sol.

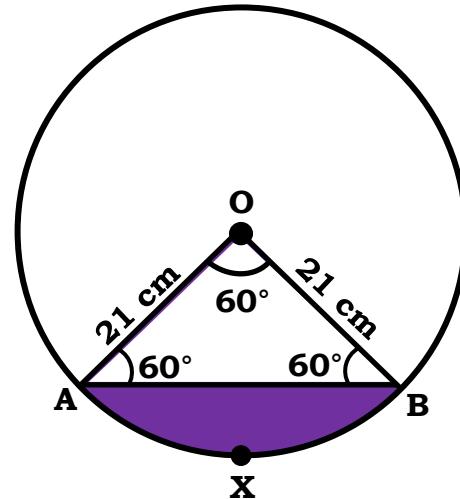
$$\text{Area of minor segment} = \text{ar}(O - AXB) - \text{ar}(\triangle OAB)$$

$$\therefore \text{Area of Minor Segment} = 231 - \frac{441}{4}\sqrt{3} \text{ cm}^2$$

∴ Area of Minor Segment

$$\text{ar}(O - AXB) = 231 \text{ cm}^2$$

$$\text{ar}(\triangle OAB) = \frac{441}{4}\sqrt{3} \text{ cm}^2$$



Thank You

Module

31

AREAS RELATED TO CIRCLE

- Sum based on Area of minor and major segment

Q. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.
 (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Sol.

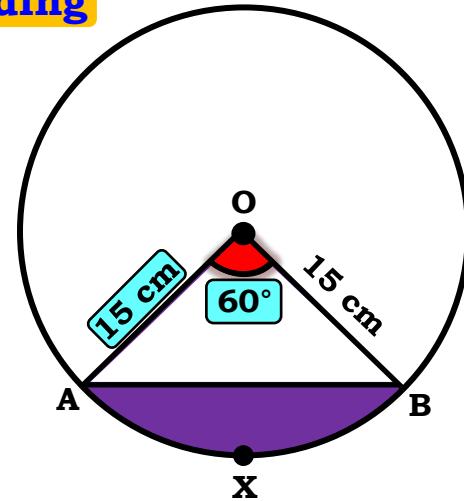
$$\text{Area of minor segment} = \text{ar}(O - AXB) - \text{ar}(\Delta OAB)$$

$$\begin{aligned} A(O - AXB) &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{60}{360} \times 3.14 \times 15 \times 15 \end{aligned}$$

2 What is formula to find area of sector?

$$= 1.57 \times 5 \times 15$$

$$\therefore A(O - AXB) = 117.75 \text{ cm}^2$$



Q. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of minor and major segments of the circle.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

$\therefore \triangle OAB$ is equilateral triangle

Area of minor segment = $ar(O - AXB) - ar(\triangle OAB)$

Sol.

In $\triangle OAB$,

$$OA = OB$$

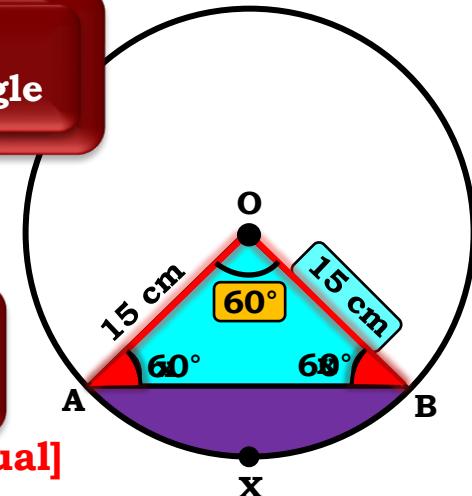
$$\angle A = \angle B = x$$

[radii of same circle]

[Angles opposite to equal sides are equal]

What is formula to find $ar(\triangle)$ of equilateral triangle ?

$$\begin{aligned} \angle A + \angle B + \angle O &= 180^\circ && \therefore \triangle OAB \text{ is an equilateral triangle} \\ x + x + 60 &= 180 && ar (\triangle OAB) = \frac{\sqrt{3}}{4} \times (\text{Side})^2 \\ 2x + 60 &= 180 && = \frac{1.73}{4} \times 15 \times 15 \\ 2x &= 180 - 60 && = \frac{1.73}{4} \times 225 \\ 2x &= 120 && = \frac{1.73}{4} \times 56.25 \\ x &= \frac{120}{2} && \therefore ar (\triangle OAB) = 97.3125 \text{ cm}^2 \\ x &= 60 && \end{aligned}$$



Q. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Sol. Area of major segment = $\text{ar(circle)} - \text{ar(minor segment)}$

$$\text{Area of Minor Segment} = \text{ar}(O - AXB) - \text{ar}(\Delta OAB)$$

= ?
What is formula to
find area of circle?

$$\therefore \text{Area of Minor Segment} = 2$$

Area of Minor Segment is 20.4375 cm^2

$$\text{Area of circle} = \pi r^2$$

$$\text{ar}(O - AXB) = 117.75 \text{ cm}^2 \quad 3.14 \times 15 \times 15$$

$$\therefore \text{Area of circle} = 706.5 \text{ cm}^2$$

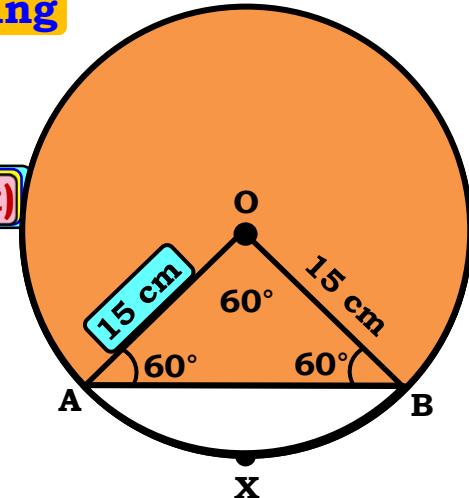
$$\text{ar}(\Delta OAB) = 97.3125 \text{ cm}^2$$

$$\text{Area of major Segment} = \text{Area of circle} - \text{Area of minor segment}$$

$$= 706.5 - 20.4375$$

$$\therefore \text{Area of major Segment} = 686.0625 \text{ cm}^2$$

Area of major Segment is 686.0625 cm^2



Module

32

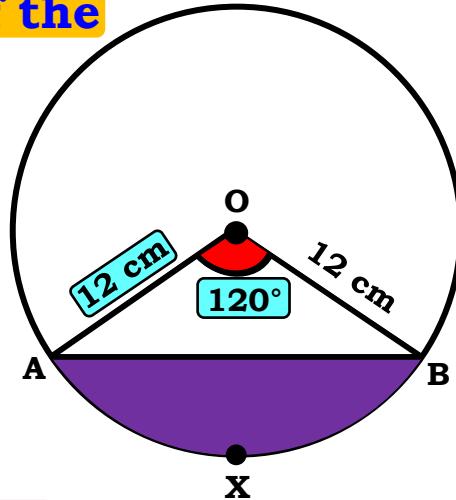
AREAS RELATED TO CIRCLE

- **Sum based on Segment**

Q. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.
 (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

$$\text{Area of minor segment} = \text{ar}(O - AXB) - \text{ar}(\Delta OAB)$$

$$\begin{aligned}\text{Sol. ar } (O - AXB) &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{120}{360} \times 3.14 \times 12 \times 12 \\ &= 3.14 \times 4 \times 12\end{aligned}$$



$$\therefore \text{ar } (O - AXB) = 150.72 \text{ cm}^2$$

What is formula to
 $\frac{\theta}{360} \times \pi r^2$
 find area of sector?

Q. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.
 (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

$$\text{Area of minor segment} = \text{ar}(O - AXB) - \text{ar}(\Delta OAB)$$

Sol. In $\triangle OLA$ and $\triangle OLB$

To find: AB and OL

$$\angle OLA = \angle OLB$$

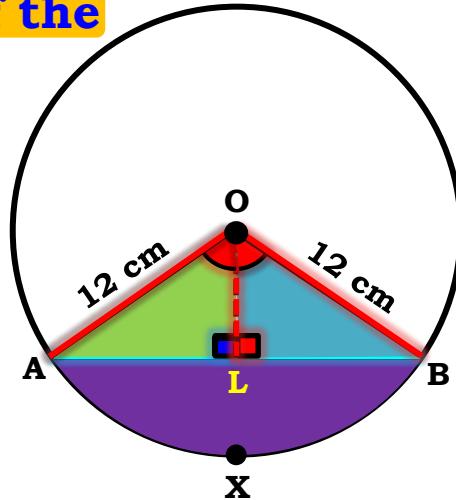
What is $\frac{1}{2} \times \text{base} \times \text{height}$?
 find area of triangle?

$$OA = OB$$

$$OL = OL$$

$$\triangle AOL \cong \triangle BOL$$

$$\therefore \angle AOL = \angle BOL \quad [\text{c.p.c.t}]$$



$$\text{ar}(O - AXB) = 150.72 \text{ cm}^2$$

Q. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

$$\text{Area of minor segment} = \text{ar}(O - AXB) - \text{ar}(\Delta OAB)$$

To find: AB and OL

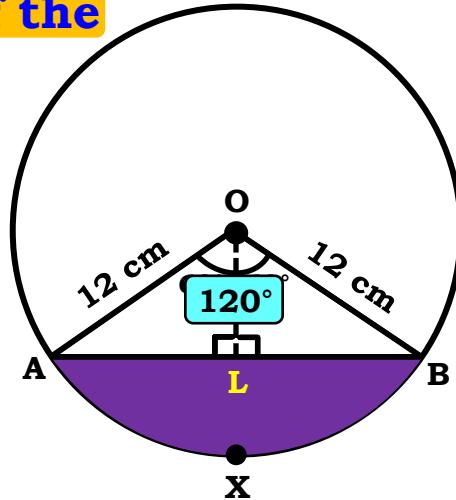
Sol.

\therefore

$$\angle AOL = \angle BOL \quad [\text{c.p.c.t}]$$

$$\begin{aligned}\angle AOL &= \angle BOL = \frac{1}{2} \times \angle AOB \\ &= \frac{1}{2} \times \cancel{120}^{60}\end{aligned}$$

$$\angle AOL = \angle BOL = 60^\circ$$



$$\text{ar}(O - AXB) = 150.72 \text{ cm}^2$$

Q. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Sol.

$$\text{ar}(O - AXB) = 150.72 \text{ cm}^2$$

In $\triangle AOL$,

For $\angle AOL$,
Opposite side $\rightarrow AL$
Hypotenuse $\rightarrow AO$

Ratio of opposite side
and hypotenuse reminds
us of 'sin'

$$\therefore \frac{AL}{AO} = \frac{6}{12} = \frac{1}{2}$$

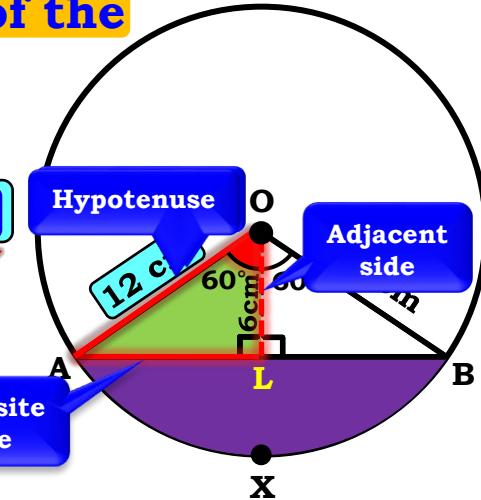
$$\sin 60^\circ = \frac{AL}{AO}$$

To find: AB and OL

$\frac{\sqrt{3}}{2}$ AL
Observe $\angle AOL$
 $\frac{1}{2}\sqrt{3}$

For $\angle AOL$,
Adjacent side $\rightarrow OL$
Hypotenuse $\rightarrow AO$

Ratio of adjacent side
and hypotenuse reminds
us of 'cos'



Q. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

$$\text{Area of minor segment} = \text{ar}(O - AXB) - \text{ar}(\Delta OAB)$$

Sol. $\text{ar}(O - AXB) = 150.72 \text{ cm}^2$

$$\text{ar}(\Delta OAB) = \frac{1}{2} \times \text{AB} \times \text{OL}$$

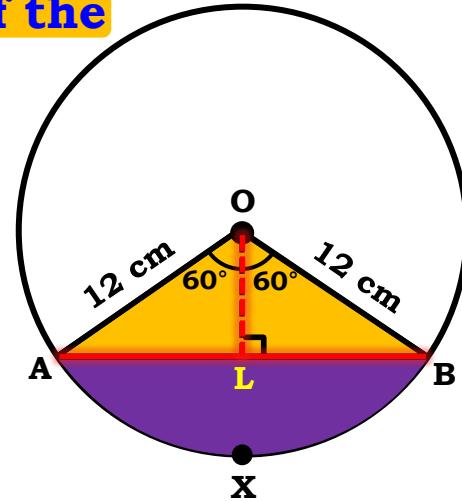
$$= \frac{1}{2} \times 12\sqrt{3} \times \cancel{6}^3$$

$$= 36\sqrt{3}$$

$$= 36 \times 1.73$$

$$\therefore \text{ar}(\Delta OAB) = 62.28 \text{ cm}^2$$

$$\begin{aligned}\text{OL} &= 6 \text{ cm} \\ \text{AB} &= 12\sqrt{3} \text{ cm}\end{aligned}$$



Q. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Area of minor segment = $\text{ar}(O - AXB) - \text{ar}(\Delta OAB)$

Sol.

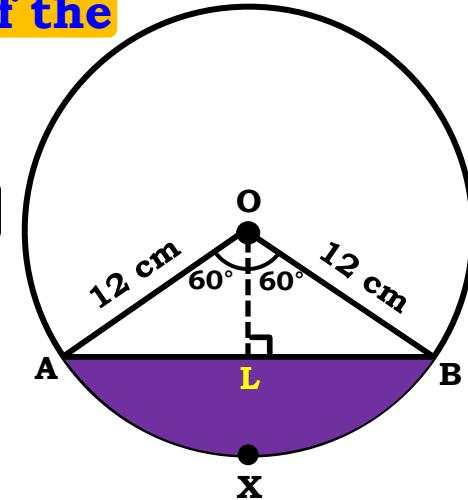
$$\begin{aligned}\text{Area of Minor Segment} &= \text{ar}(O - AXB) - \text{ar}(\Delta OAB) \\ &= 150.72 - 62.28\end{aligned}$$

$$\therefore \text{Area of Minor Segment} = 88.44 \text{ cm}^2$$

∴ Area of Minor Segment is 88.44 cm^2

$\text{ar}(O - AXB) = 150.72 \text{ cm}^2$

$\text{ar}(\Delta OAB) = 62.28 \text{ cm}^2$



Module

33

AREAS RELATED TO CIRCLE

- Sum based on Area of segment

Q. A round table cover has six equal designs as shown

If the radius of the table is 28 cm, what is the formula to find area of one sector? (of making 6 sectors) ($\pi = \frac{22}{7}$, $\sqrt{3} = 1.7$)

$$\text{ar } (\text{AXB}) = \text{ar } (\text{O-AXB}) - \text{ar } (\triangle \text{OAB})$$

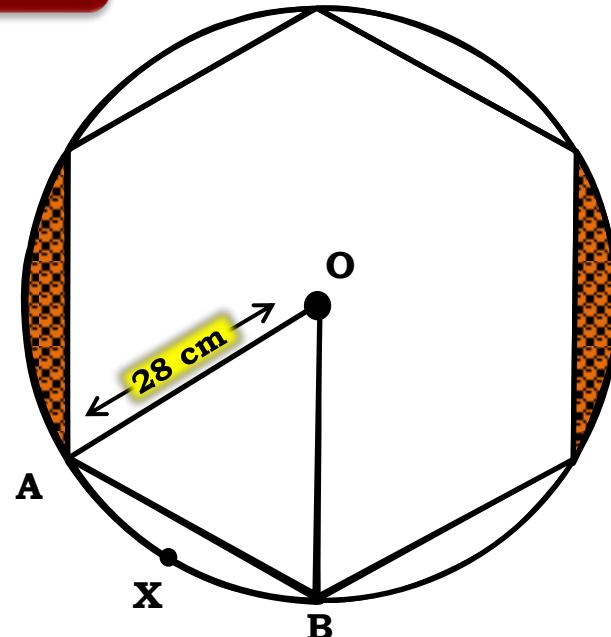
Sol. Radius of quadrant (r) = 28 cm

Let us first find area of 1 segment

$$\text{Cost} = \text{Rate} \times \text{Area}$$

$$\begin{aligned} \text{ar } (\text{O-AXB}) &= \frac{60}{360} \times \pi r^2 \\ &= \frac{1}{6} \times \frac{22}{7} \times 28 \times 28 \\ &= \frac{22 \times 2 \times 28}{3} = \frac{1232}{3} \\ &= 410.67 \text{ cm}^2 \end{aligned}$$

$$\therefore \text{ar } (\text{O-AXB}) = 410.67 \text{ cm}^2$$



Q. A road 14 m wide is to be made in the shape of a sector of a circle of radius 28 cm as shown in. If the cost of making the road is Rs 0.35 per cm². What is the formula to find area of an equilateral triangle? What is the cost of making the designs at the rate of Rs 0.35 per cm². (Use $\sqrt{3} = 1.7$)

$$\text{ar } (\text{AXB}) = \text{ar } (\text{O} - \text{AXB}) - \text{ar } (\Delta \text{OAB})$$

Sol. Radius of quadrant (r) = 28 cm

ΔOAB is an equilateral triangle

$$\text{ar } (\Delta \text{OAB}) = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

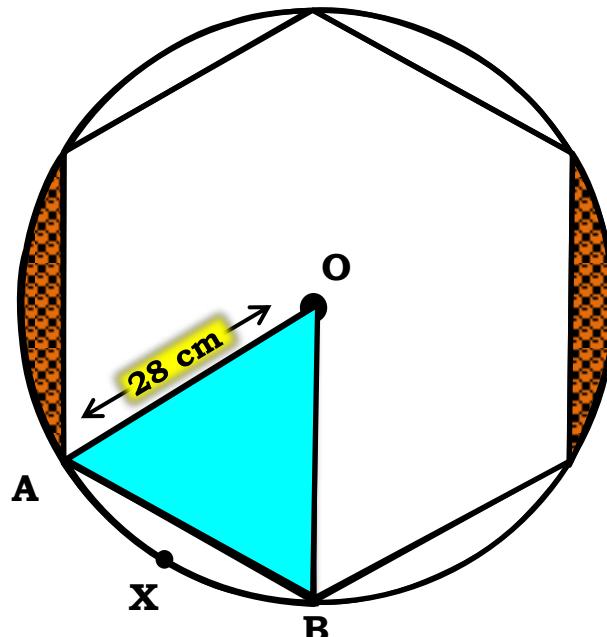
$$= \frac{\sqrt{3}}{4} \times (28)^2$$

$$= \frac{\sqrt{3}}{4} \times \cancel{28}^7 \times 28$$

$$= \sqrt{3} \times 7 \times 28$$

$$= 1.7 \times 7 \times 28$$

$$\therefore \text{ar } (\Delta \text{OAB}) = 333.2 \text{ cm}^2$$



**Q. A round table cover has six equal designs as shown in.
If the radius of the cover is 28 cm, find the cost of making
the designs at the rate of Rs 0.35 per cm². (Use $\sqrt{3} = 1.7$)**

$$\text{ar (AXB)} = \text{ar (O - AXB)} - \text{ar (\Delta OAB)}$$

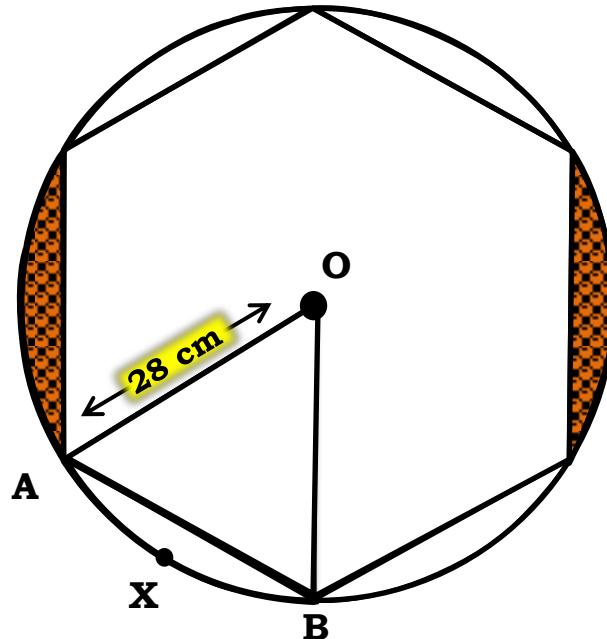
$$\begin{aligned}\text{Sol. ar (AXB)} &= \text{ar (O - AXB)} - \text{ar (\Delta OAB)} \\ &= 410.67 - 333.2\end{aligned}$$

$$\text{ar (AXB)} = 77.47 \text{ cm}^2$$

$$\begin{aligned}\text{Area of one design} &= 77.47 \\ \therefore \text{Area of 6 segment} &= 77.47 \times 6 \\ &= 464.82 \text{ cm}^2\end{aligned}$$

$$\text{ar (\Delta OAB)} = 333.2 \text{ cm}^2$$

$$\begin{aligned}&- 0.35 \quad 464.82 \\ \therefore \text{ar (O-AXB)} &= 410.67 \text{ cm}^2 \\ &- 162.68\end{aligned}$$



\therefore Total cost of making designs is 162.68

Module 34

AREAS RELATED TO CIRCLE

- Sum based on finding area of shaded portion

Q. ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter.
Find the area of the shaded region.

?

Area of shaded region = ar(semi-circle) – ar(minor segment)

Sol. In $\triangle ABC$,
 $\angle A = 90^\circ$

What is formula to find
area of semi-circle?

To find: R

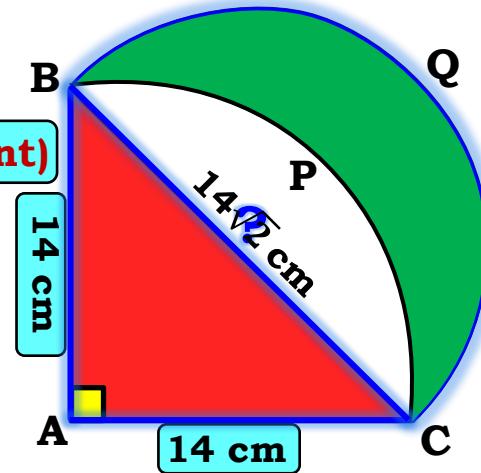
$$AB^2 + AC^2 = BC^2 \quad [\text{By Pythagoras theorem}]$$

$$\therefore (14)^2 + (14)^2 = BC^2$$

$$\therefore BC^2 = 196 + 196$$

$$\therefore BC^2 = 2 \times 196$$

$$\therefore BC = 14\sqrt{2} \text{ cm}$$



Q. ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter.

Find the area of the shaded region.



Area of shaded region = ar(semi-circle) – ar(minor segment)

Sol.

$$\therefore \text{Radius of semi-circle} = \frac{\text{BC}}{2}$$

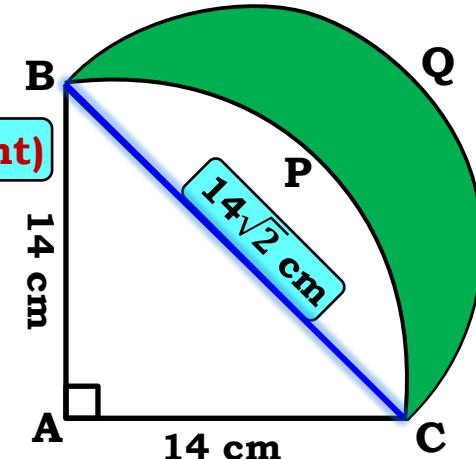
BC is diameter of semi-circle

$$= \frac{7}{2} \times \frac{14\sqrt{2}}{2}$$

To find: R

$$\therefore \text{Radius of semi-circle} = 7\sqrt{2} \text{ cm}$$

$$\begin{aligned}\text{Area of semi-circle} &= \frac{1}{2} \times \pi R^2 \\ &= \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2} \\ &= \frac{11}{2} \times \sqrt{2} \times 7\sqrt{2} \\ &= 77 \times 2 \\ \therefore \text{Area of semi-circle} &= 154 \text{ cm}^2\end{aligned}$$



- Q. ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter.

Find the area of the shaded region.

Area of shaded region = ar(semi-circle) - ar(minor segment)

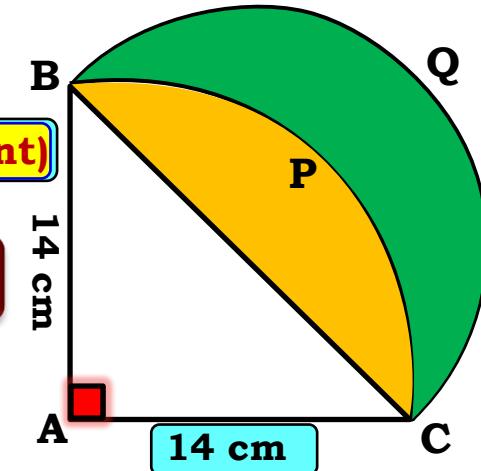
Sol.

What is formula to find area of sector?

ar(A - BPC) - ar($\triangle ABC$)

$$\begin{aligned} \text{ar } (A - BPC) &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{90}{360} \times \frac{22}{7} \times 14^2 \\ &= 22 \times 7 \end{aligned}$$

$$\therefore \text{ar } (A - BPC) = 154 \text{ cm}^2$$



Area of semi-circle = 154 cm^2

Module

35

AREAS RELATED TO CIRCLE

- Sum based on finding area of shaded portion (Contd...)

Q. ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter.

Find the area of the shaded region.

Area of shaded region = ar(semi-circle) - ar(minor segment)

Sol.

What is formula to find area of triangle?

ar(A - BPC) - ar($\triangle ABC$)

$$\text{ar } (\triangle ABC) = \frac{1}{2} \times \text{AC} \times \text{AB}$$

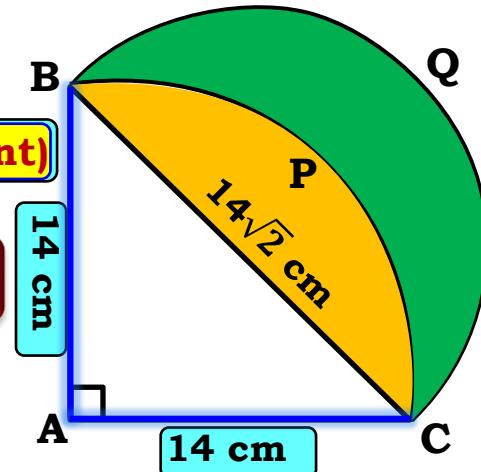
$$= \frac{1}{2} \times \cancel{14}^7 \times 14$$

$$= 7 \times 14$$

$$\therefore \text{ar } (\triangle ABC) = 98 \text{ cm}^2$$

$$\text{ar } (A - BPC) = 154 \text{ cm}^2$$

$$\text{Area of semi-circle} = 154 \text{ cm}^2$$



Q. ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter.

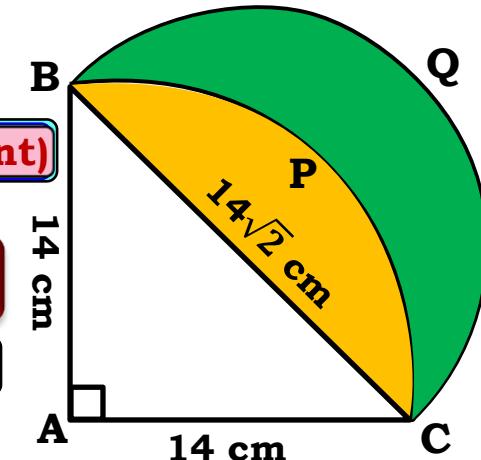
Find the area of the shaded region.

Area of shaded region = ar(semi-circle) – ar(minor segment)

Sol.

ar(A – BPC) – ar(ΔABC)

$$\begin{aligned} \text{ar(minor segment BPC)} &= \text{ar}(A - BPC) - \text{ar}(\Delta ABC) \\ &= 154 - 98 \end{aligned}$$



∴ ar(minor segment BPC) = 56 cm²

$$\begin{aligned} \text{Area of shaded region} &= \text{ar(semi-circle)} - \text{ar(minor segment)} \\ &= 154 - 56 \end{aligned}$$

ar (ΔABC) = 98 cm²

∴ Area of shaded region = 98 cm²

∴ Area of shaded region is 98 cm²

ar (A – BPC) = 154 cm²

Area of semi-circle = 154 cm²

Module

36

AREAS RELATED TO CIRCLE

- **Sum based on Area of segment**

Area of the shaded region
 $= 2 \times \text{area of segment AXC}$

Q. Calculate the area of the shaded region in the adjoining figure where $\square ABCD$ is a square with side 8 cm each. ($\pi = 3.14$)

Sol.

$$\text{ar(segment AXC)} = \text{ar(D-AXC)} - \text{ar}(\triangle ADC)$$

R

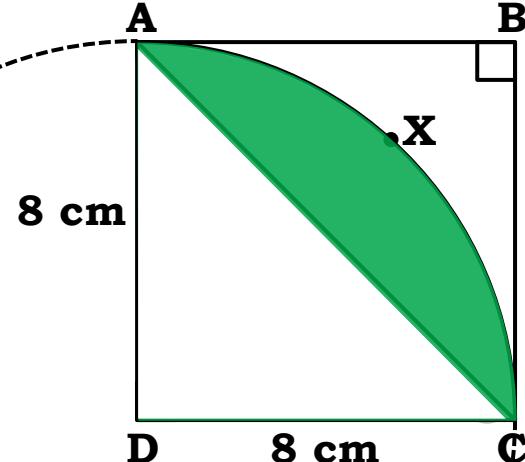
Shaded region is made up of two identical segments

$$\text{ar(D-AXC)} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{90}{360} \times 3.14 \times 8^2$$

\therefore

$$\text{ar(D-AXC)} = 50.24 \text{ cm}^2$$



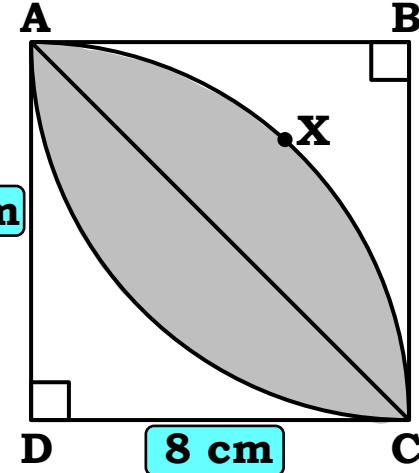
Area of the shaded region
 $= 2 \times$ area of segment AXC

Q. Calculate the area of the shaded region in the adjoining figure where $\square ABCD$ is a square with side 8 cm each. ($\pi = 3.14$)

Sol.

$$\text{ar}(\text{segment AXC}) = \text{ar}(\text{sector D-AXC}) - \text{ar}(\triangle ADC)$$

$$\begin{aligned}\text{ar } (\triangle ADC) &= \frac{1}{2} \times \text{AD} \times \text{DC} \\ &= \frac{1}{2} \times 8 \times 8\end{aligned}$$



\therefore

$\frac{1}{2} \times$ What is the formula to find
Product of perpendicular sides
area of right angled triangle?

$$\therefore \text{ar}(\text{segment AXC}) = 18.24 \text{ cm}^2$$

$\text{ar}(\text{sector D-AXC}) = 50.24 \text{ cm}^2$

**Area of the shaded region
= $2 \times$ area of segment AXC**

**Q. Calculate the area of the shaded region
in the adjoining figure where $\square ABCD$
is a square with side 8 cm each. ($\pi = 3.14$)**

Sol.

Area of the shaded region

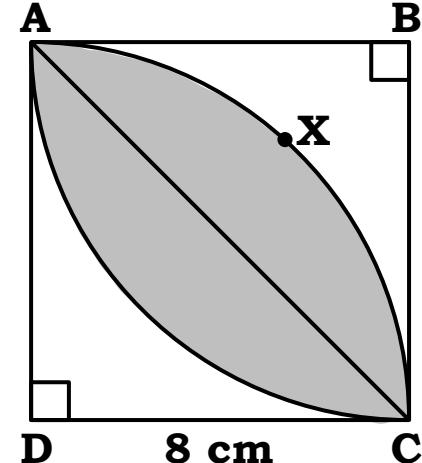
$$= 2 \times \text{area of segment AXC}$$

$$= 2 \times 18.24$$

$$= 36.48 \text{ cm}^2$$

\therefore Area of the shaded region is 36.48 cm^2

ar(segment AXC) = 18.24 cm^2



Thank You