Lecture_08

No. 51

Arithmetic Progressions

- Sums based on a_n and S_n formula

14) Find the sum of the odd numbers between 0 to 50.

Sol: The odd numbers between 0 to 50 are as follows: 1, 3, 5, 7, ... 49.

These numbers form an A.P. wi

Let $a_n = 49$

We know that for an A.P. ,

$a_n = a + (n-1)d$

$\therefore 49 = 1 + (n-1)2$

$\therefore 49 = 1 + 2n - 2$

$\therefore 49 = 2n - 1$

$\therefore 50 = 2n$

$\therefore n = 25$

$S_n = \frac{n}{2}[a + a_n]$

$\therefore S_{25} = \frac{25}{2}[1 + 49]$

To find number of terms check which term is 49. Because its the last term

Subs value of a & d

25×25

$\therefore S_{25} = 625$

$\therefore$ **Sum of odd numbers between 0 to 50 is 625**

find S_{25}

No. 52

Arithmetic Progressions

- **Relationship between a_n and S_n**

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

$1, \quad 3, \quad 5, \quad 7, \quad 9, \quad \ldots$

$S_1 = a_1$ $\qquad$ $a_1 = S_1$

$S_2 = a_1 + a_2$ $\qquad$ $a_2 = S_2 - S_1$

$S_3 = a_1 + a_2 + a_3$ $\qquad$ $a_3 = S_3 - S_2$

$S_4 = a_1 + a_2 + a_3 + a_4$ $\qquad$ $a_4 = S_4 - S_3$

S_n = Sum of first n terms

No. 53

Arithmetic Progressions

- **Sum Based on relationship between a_n and S_n**

11) If the sum of the first n terms of an AP is $4n - n^2$, What is the first term ? Find a... sum of the first two terms? What is the second term? Find a₂ ... d, 10^{th} and n^{th} ...

Find $a_1 = S_1$

Find S_2

Sol: $S_n = 4n - n^2$

$S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

$a_1 = S_1 = 3$

To find S_1 put n = 1

$S_2 = \ldots = 8 - 4 = 4$

$a_2 = \ldots = 4 - 3 = 1$

To find S_2 put n = 2

Lets find value of d

$d = \ldots = 1 - 3 = -2$

$a_3 = a + 2d = 3 + 2(-2) = 3 - 4 = -1$

$a_{10} = a + 9d = 3 + 9(-2) = 3 - 18 = -15$

$a_n = a + (n - 1)\, d$

$= 3 + (n - 1)(-2)$

$= 3 - 2n + 2$

$\therefore a_n = 5 - 2n$

No. 54

Arithmetic Progressions

- **Word problems based on S_n formula**

15) A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs.200 for the first day, Rs.250 for the seco[illegible]day, Rs.300 for the third day, etc. the penalty for each succeeding day b[illegible] for the preceding day. How much money the contractor ha[illegible] he has delayed the work by 30 days?

Lets make a list of amount of penalty for each succeeding day

Sol: Amount of pena[illegible]ng day are as follows:

200, 250, 300, . . .

These numbers form an A.P. with a = 200 ar[illegible]50

We need to find penalty for 30 days i.e. S_{30}

Penalty for delayed work by 30 days = S_{30}

We know that,

For S_{30} substitute, n = 30, a = 200 & d = 50

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{30} = \frac{30}{2}[2(200) + (30-1)(50)]$$

$$= 15\,[400 + (29)(50)]$$

$$= 15\,[400 + 1450]$$

$$= 15\,[1850]$$

$$\therefore S_{30} = 27750$$

∴ Penalty for delayed work by 30 days is Rs.27750.

No. 55

Arithmetic Progressions

- **Word problems based on S_n formula**

16) A sum of Rs.700 is to be used to give seven cash prizes to students of a sch[illegible] in overall performance. If each prize is Rs.20 less than [illegible] the value of each of the prizes.

That means total of seven cash prizes given is Rs.700 i.e. $S_7 = 700$

These amounts forms an AP

Sol: Since, value of [illegible] its preceding prize

∴ Value of each succes[illegible] $d = -20$

Total amount to be u[illegible]zes $S_7 = 700$

For example:
If,
1st prize: Rs.200
then,
2nd prize: Rs.180
3rd prize: Rs.160

16) A sum of Rs.700 is to be used to give seven cash prizes to students of a school for their overall performance. If each prize is Rs.20 less than its preceding prize, find the value of each of the prizes.

Sol: Since, value of each prize is Rs.20 less than its preceding prize.

$\therefore$ Value of each successive prizes form an A.P. where $d = -20$

Total amount of cash prize given = $S_7 = 700$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)$$

For given value of S_7 700

$$\therefore S_7 = \frac{7}{2}[2a + (7-1)(-20)]$$

$$\therefore 700 = \frac{7}{2}[2a + (6)(-20)]$$

$$\therefore 700 \times \frac{2}{7} = [2a - 120]$$

$$\therefore 200 = 2a - 120$$

$$\therefore 200 + 120 = 2a$$

$$\therefore 2a = 320$$

each prize is Rs.20 less than its preceding prize.

$\therefore$ Value of each prize are Rs.160, 140, 120, 100, 80, 60 & 40 respectively.

No. 56

Arithmetic Progressions

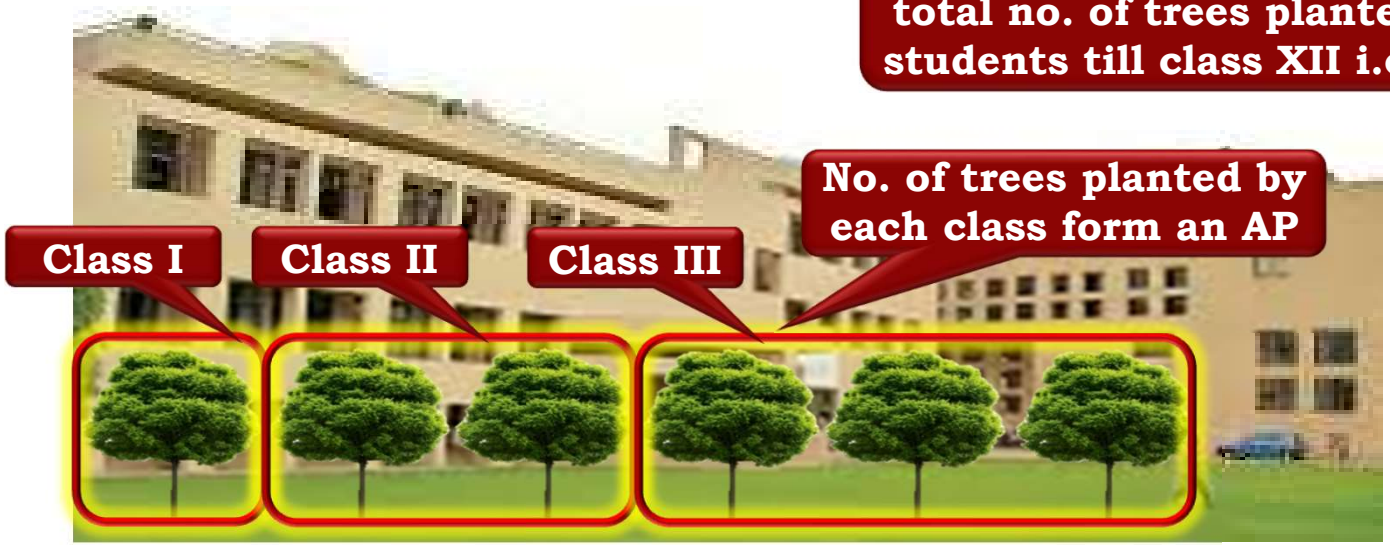
- **Word problems based on S_n formula**

17] In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of tree, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students ?

Sol: The number of trees planted by each class are as follows: 1, 2, 3, … 12

These numbers form an A.P. with … and $a_n = 12$

That means we need to find total no. of trees planted by students till class XII i.e. S_{12}



17] In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of tree, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students ?

Sol: The number of trees planted by each class are as follows: 1, 2, 3, ..., 12

These numbers form an A.P. with $a = 1$, $d = 2 - 1 = 1$, $a_n = 12$

We know that,

$$S_n = \frac{n}{2}[a + a_n]$$

$$\therefore S_{12} = \frac{12}{2}[1 + 12]$$

$$= 6 \quad [13]$$

$$\therefore S_{12} = 78$$

For S_{12} substitute, $n = 12$, $a = 1$ & $d = 1$

we need to find total no. of trees planted by students till class XII i.e. S_{12}

$\therefore$ Total number of trees planted by 1 section of each the class = 78

$\therefore$ Total number of trees planted by 3 sections of each the class = $3 \times 78 = 234$

Total trees planted by the students will be 234.

No. 57

Arithmetic Progressions

- **Word problems based on S_n formula**

18) A spiral is made up of successive semicircles, with centers alternately at A and B, starting with center at A of radii 0.5cm, 1.0cm, 1.5cm,..
What is total length of such a spiral made up of 13 consecutive semicircles?

Sol: Length of

∴ Length of

These num

Means there are two different centers A & B which are used alternately

That means we need to find total length of 13 semicircles i.e. S_{13}

We need to find S_{13}

$0.5\pi, 1\pi, 1.5\pi, \ldots$

$= 0.5\pi$ and $d = 1\pi - 0.5\pi = 0.5\pi$

semicircle 1.5cm

n

cumference
circle = $2\pi r$

semicircle 1cm

∴ Length = 1π cm

ength of
icircle = πr

18) A spiral is made up of successive semicircles, with centers alternately at A and B, starting with center at A of radii 0.5cm,1.0cm,1.5cm,.. What is total length of such a spiral made up of 13 consecutive semicircles?

Sol: Length of a semicircle = πr

$\therefore$ Length of successive semicircles are, 0.5π, 1π, 1.5π, ...

These numbers form an A.P. with $a = 0.5\pi$ & $d = 1\pi - 0.5\pi = 0.5\pi$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

We need to find S_{13}

For S_{13} substitute, n = 13, a = 0.5π & d = 0.5π

$$\therefore S_{13} = \frac{13}{2}[\,2(0.5\pi) + (13-1)(0.5\pi)\,]$$

$$= \frac{13}{2}[\,1\pi + (12)(0.5\pi)\,]$$

$$= \frac{13}{2}[\,1\pi + 6\pi\,]$$

$$= \frac{13}{2}[\,7\pi\,]$$

$$= \frac{13}{\cancel{2}} \times \cancel{7} \times \frac{\cancel{22}^{11}}{\cancel{7}}$$

$$\therefore S_{13} = 143$$

$\therefore$ Total length of spiral made up of 13 consecutive semicircles is 143 cm.

Thank You