No. 1

QUADRATIC EQUATIONS

- **Introduction To Quadratic Equations**

Quadratic Equation

What is a quadratic equation ?

Quadratic Equation is an equation in one variable with degree 2

$9x^2 + 5x + 7 = 9$ → **Degree 2** → **Quadratic**

Th
Wha

Highest index is 5

4

Qu

General form of Quadratic Equation:-

Degree 2

$7x^2 + xy - 5y = 0$

$ax^2 + bx + c = 0$ ⇒ **Degree 2**

Then, = 0

th

Then, equation will be ax² + bx = 0

where a, b and c are real numbers; a ≠

If a

two variables

It is a Quadratic Equation

Zero is a real no

$0x^2 + bx + c = 0$

$\therefore\ bx + c = 0 \longrightarrow$ Degree 1

Which is not a quadratic equation

1) Which of the following are quadratic equations?

i) $11 = -4x^2 - x^3$

Sol: $11 = -4x^2 - x^3$

$\therefore\ x^3 + 4x^2 + 11 = 0$

The given equation is not in the form of $ax^2 + bx + c = 0$

So it is not a quadratic equation.

Highest index of variable is 3

ii) $(y - 2)(y + 2) = 0$

Sol: $(y - 2)(y + 2) = 0$

$\therefore\ (y)^2 - (2)^2 = 0$

$\therefore\ y^2 - 4 = 0$

$\therefore\ y^2 + 0y - 4 = 0$

The given equation is in the form of $ay^2 + by + c = 0$

So it is a quadratic equation in variable y

Represent Middle term as 0y

No. 2

QUADRATIC EQUATIONS

. **Stating whether the given equation is a Quadratic Equation or not ?**

Q) Which of the following are quadratic equations?

iii) $z - \frac{7}{z} = 4z + 5$

Sol: $z - \frac{7}{z} = 4z + 5$

Multiplying throughout by z, we get,

$$z(z) - \cancel{z} \times \frac{7}{\cancel{z}} = z(4z) + z(5)$$

$$z^2 - 7 = 4z^2 + 5z$$

$$\therefore z^2 - 4z^2 - 5z - 7 = 0$$

$$\therefore -3z^2 - 5z - 7 = 0$$

The given equation is in the form of $az^2 + bz + c = 0$

So it is a quadratic equation in variable z

iv) $\frac{q^2 - 4}{q^2} = -3$

Sol: $\frac{q^2 - 4}{q^2} = -3$

$$\therefore q^2 - 4 = -3q^2$$

$$\therefore q^2 + 3q^2 - 4 = 0$$

$$\therefore 4q^2 - 4 = 0$$

$$\therefore 4q^2 + 0q - 4 = 0$$

The given equation in the form of $aq^2 + bq + c = 0$

So it is a quadratic equation in variable q

Arrange equation such that we get RHS = 0

Represent Middle term as 0q

Q) Which of the following are quadratic equations ?

v) $(x + 2)^3 = 2x\,(x^2 - 1)$

Sol : $(x + 2)^3 = 2x\,(x^2 - 1)$

$\therefore \quad x^3 + 3x^2(2) + 3x(2)^2 + 2^3 = 2x^3 - 2x$

$\therefore \quad x^3 + 6x^2 + 3x(4) + 8 = 2x^3 - 2x$

$\therefore \quad x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$

Arrange equation in such a way that we get R.H.S as 0

$\therefore \quad x^3 - 2x^3 + 6x^2 + 12x + 2x + 8 = 0$

$\therefore \quad -x^3 + 6x^2 + 14x + 8 = 0$

The given equation is not in the form of $ax^2 + bx + c = 0$.

So it is not a quadratic equation.

Check whether the following are quadratic equations:

vi) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Sol : $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\therefore$ $x^3 - 4x^2 - x + 1 = x^3 - 3x^2(2) + 3(x)(2)^2 - 2^3$

$\therefore$ $x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 3(x)(4) - 8$

$\therefore$ $\cancel{x^3} - 4x^2 - x + 1 = \cancel{x^3} - 6x^2 + 12x - 8$

$\therefore$ $-4x^2 + 6x^2 - x - 12x + 1 + 8 = 0$

$\therefore$ $2x^2 - 13x + 9 = 0$

Arrange equation in such a way that we get R.H.S as 0

The given equation is in the form of $ax^2 + bx + c = 0$.

So it is a quadratic equation in variable x.

No. 3

QUADRATIC EQUATIONS

. Stating whether the given equation is a Quadratic Equation or not ?

Q.) Check whether the following are quadratic equations :

i) $(x + 1)^2 = 2\,(x - 3)$

Sol : $(x + 1)^2 = 2\,(x - 3)$

$\therefore\quad x^2 + \cancel{2x} + 1 = \cancel{2x} - 6$

$\therefore\quad x^2 + 1 + 6 = 0$

$\therefore\quad x^2 + 7 = 0$

$\therefore\quad x^2 + 0x + 7 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$.

So it is a quadratic equation.

Highest index of variable is 2

b + b^2

term as 0x

Q.) Check whether the following are quadratic equations :

(ii) $x^2 - 2x = (-2)(3 - x)$

Sol : $x^2 - 2x = (-2)(3 - x)$

$\therefore \quad x^2 - 2x = -6 + 2x$

$\therefore \quad x^2 - 2x - 2x + 6 = 0$

$\therefore \quad x^{2} - 4x + 6 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$.

So it is a quadratic equation.

Arrange equation such that we get RHS = 0

Q.) Check whether the following are quadratic equations :

dividing throughout by -1

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

Sol : $(x - 2)(x + 1) = (x - 1)(x + 3)$

$\therefore \quad x(x + 1) - 2(x + 1) = x(x + 3) - 1(x + 3)$

$\therefore \quad \cancel{x^2} + x - 2x - 2 = \cancel{x^2} + 3x - x - 3$

$\therefore \quad -x - 2 = 2x - 3$

$\therefore \quad -x - 2x - 2 + 3 = 0$

$\therefore \quad -3x + 1 = 0$

$\therefore \quad 3x - 1 = 0$

The given equation is not in the form of $ax^2 + bx + c = 0$.

So it is not a quadratic equation.

No. 4

QUADRATIC EQUATIONS

. Stating whether the given equation is a Quadratic Equation or not ?

Q.) Check whether the following are quadratic equations :

(iv) $(x - 3)(2x + 1) = x(x + 5)$

Highest index of variable is 2

Sol : $(x - 3)(2x + 1) = x(x + 5)$

$\therefore x(2x + 1) - 3(2x + 1) = x^2 + 5x$

$\therefore 2x^2 + x - 6x - 3 = x^2 + 5x$

$\therefore 2x^2 - 5x - 3 = x^2 + 5x$

$\therefore 2x^2 - x^2 - 5x - 5x - 3 = 0$

i.e. $x^2 - 10x - 3 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$.

So it is a quadratic equation.

Q.) Check whether the following are quadratic equations :

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

Arrange Equation such that we get RHS as 0

Sol : $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\therefore\ 2x(x - 3) - 1(x - 3) = x(x - 1) + 5(x - 1)$

$\therefore\ 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\therefore\ 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\therefore\ 2x^2 - x^2 - 7x - 4x + 3 + 5 = 0$

i.e. $x^2 - 11x + 8 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$.

So it is a quadratic equation.

Q.) Check whether the following are quadratic equations :

(vi) $x^2 + 3x + 1 = (x - 2)^2$

Arrange equation such that we get RHS = 0

b^2

Sol : $x^2 + 3x + 1 = (x - 2)^2$

$\therefore \cancel{x^2} + 3x + 1 = \cancel{x^2} - 4x + 4$

$\therefore 3x + 4x + 1 - 4 = 0$

i.e. $7x - 3 = 0$

The given equation is not in the form of $ax^2 + bx + c = 0$.

So it is not a quadratic equation.

No. 5

QUADRATIC EQUATIONS

. Converting statements in Equation Form

Q) Express the following statements mathematically

The area of a rectangular plot is 528 m².The length of the plot is one more than twice its breadth. We need to find the length and breadth of the plot.

Sol: Let the breadth of the rectangular plot be x metres.

$\therefore$ Length $= (2x + 1)$ metres

$\therefore$ Area of the rectangle $= l \times b$

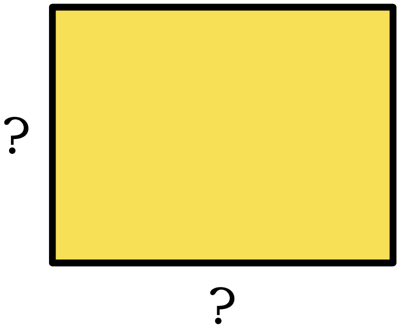
$\therefore$ $528 =$

$\therefore$ $(2x + 1)x = 528$

$\therefore$ $\boxed{2x^2 + x - 528 = 0}$

Is the required representation of problem mathematically

By opening the bracket



Q. Represent the following situations in the form of quadratic equations :

iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Sol : Let Rohan's present age be x years.

	Rohan	**Mother**
Present age	**20 years** **x years**	**(20 + 26) years** **46 years** **(x + 26) years**
Age 3 years from now	**(x + 3) years**	**x + 26 + 3** **= (x + 29) years**

According to given condition

$(x + 3)(x + 29) = 360$

$\therefore\ x(x + 29) + 3(x + 29) = 360$

$\therefore\ x^2 + 29x + 3x + 87 = 360$

$\therefore\ x^2 + 32x + 87 = 360$

0

∴ Rohan's mother's age = Rohan's age + 26

$\therefore\ \boxed{x^2 + 32x - 273 = 0}$

No. 6

QUADRATIC EQUATIONS

. Converting statements in Equation Form

Express the following statements mathematically

Q (ii) **The product of two consecutive integers is 306. We need to find the integers.**

Sol: Let first integer = x

$\therefore$ Second integer = $(x + 1)$

According to given condition,

$x\,(x + 1) = 306$

$\therefore$ $x^2 + x = 306$

$\therefore$ $\boxed{x^2 + x - 306 = 0}$

Is the required representation of problem mathematically.

Two consecutive integers
10, 10 + 1

Q. Represent the following situations in the form of quadratic equations :

iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Sol : Let the speed of the train be x km/hr

∴ The new speed of the train is (x – 8)km/hr

	Old	**New**
Speed	**x km/hr**	**(x – 8) km/hr**
Distance	**480 km**	**480 km**
Time $= \dfrac{\text{Distance}}{\text{Speed}}$	$\left(\dfrac{480}{x}\right)$ **hrs**	$\left(\dfrac{480}{x-8}\right)$ **hrs**

According to given condition

What we have to find in this sum ?

$$\left(\frac{480}{x-8}\right) - \left(\frac{480}{x}\right) = 3$$

If old time = 10hrs

New time = 10 + 3 = 13hrs

13 – 10 = 3

∴ New time – Old time = 3

$$\therefore \quad 480\left[\frac{1}{x-8} - \frac{1}{x}\right] = 3$$

$$\therefore \quad \left[\frac{x-(x-8)}{x(x-8)}\right] = \frac{3}{480}$$

$$\therefore \quad \frac{\cancel{x} - \cancel{x} + 8}{x^2 - 8x} = \frac{1}{160}$$

$$\therefore \quad \frac{8}{x^2 - 8x} = \frac{1}{160}$$

$$\therefore \quad x^2 - 8x = 8 \times 160$$

$$\therefore \quad x^2 - 8x = 1280$$

$$\therefore \quad x^2 - 8x - 1280 = 0$$

$$\therefore \quad \boxed{x^2 - 8x - 1280 = 0}$$

No. 7

QUADRATIC EQUATIONS

- Methods for solving Quadratic Equations
- Factorization Method

Methods to solve a Quadratic Equation

1] Factorisation Method

2] Completing the Square Method

3] Formula Method

1] Factorisation Method

Q) Solve the following quadratic equations by factorization method

34x

From last two '11' is
mmon along with
3rd term sign

Take common
from first two

Sta
form

11 × 3 = 33

1 × 33

+ 33 = 34

1st

Middle no.

no.

∴ $\ldots 11 = 0$

∴ $\ldots 1) = 0$

$\therefore \quad (3x + 1)\,(x + 11) = 0$

$\therefore\ 3x + 1 = 0 \quad \text{or} \quad x + 11 = 0$

$\therefore\ 3x = -1 \quad \text{or} \quad x = -11$

$\therefore \quad x = -\frac{1}{3} \quad \text{or} \quad x = -11$

Factorise by

Product of two
brackets is zero

ns

factors

To factorise by splitting
middle term

Find product of 3rd no.
with 1st no.

Find two factors of 33 in
such a way that by adding
factors we get middle no.

Since, last sign is +
Give middle sign to both
factors.

∴ The roots of the given quadratic equations are –11 and $-\frac{1}{3}$

Q) Solve the following quadratic equations by factorization method

From first t
is comı

From last two '15'
is common a long
with 3rd term sign

Sta
form

$\therefore\ 2m^2 + 4m + 15m + 30 = 0$

1s middle no. d no.

$\therefore\ 2m(m + 2) + 15(m + 2) = 0$

$\therefore\ (m + 2)\ (2m + 15) = 0$

$\therefore\ m + 2 = 0 \quad \text{or} \quad 2m + 15 = 0$

$\therefore\ m = -2 \quad \text{or} \quad 2m = -15$

$\therefore\ m = -2 \ \text{or}\ m = -\frac{15}{2}$

Factorise by splitting middle term

0
ns
0

To factorise by splitting
iddle term

Find product of 3rd no.
with 1st no.

nd two factors of 60 in
ich a way that by adding
ctors we get middle no.

nce, last sign is +
ive middle sign to both
tors.

30 × 2

60 = 4 × 15

60 ≠ 19

19

3

20 ≠ 19

+4 + +15 = 19

Now signs to be given to both factors

∴ The roots of the given quadratic are –2 and - $\frac{15}{2}$

No. 8

QUADRATIC EQUATIONS

- Factorization Method Continued…

Q) Solve the following quadratic equations by factorization method

iv) x^2

From fi
is c

From last two '11' is common along with 3rd term sign

middle no.

$\therefore \quad x^2 - 12x + 11x - 132 = 0$

$\therefore \quad x(x - 12) + 11(x - 12) = 0$

$\therefore \quad (x - 12)(x + 11) = 0$

$\therefore \quad x - 12 = 0 \quad \text{or} \quad x + 11 = 0$

$\therefore \quad x = 12 \quad \text{or} \quad x = -11$

Standard form

$132 \times 1 = 132$

−12 **+ 11**

Factorise by splitting middle term

s to be
both
factors

Find product of 3rd no. with 1st no.

Find two factors of 132 in such a way that by subtracting factors we get middle no.

Since, last sign is – Give middle sign only to bigger factor and opposite sign to smaller factor

∴ The roots of the given quadratic equations are 12 and - 11

Q) Solve the following quadratic equations by factorization method

v) 8

From firs
is co

From last two ‘3’ is common along with 3rd term sign

n be
rise as

So

Since, last sign is –
middle sign only to
r factor.

21 can be factorise as

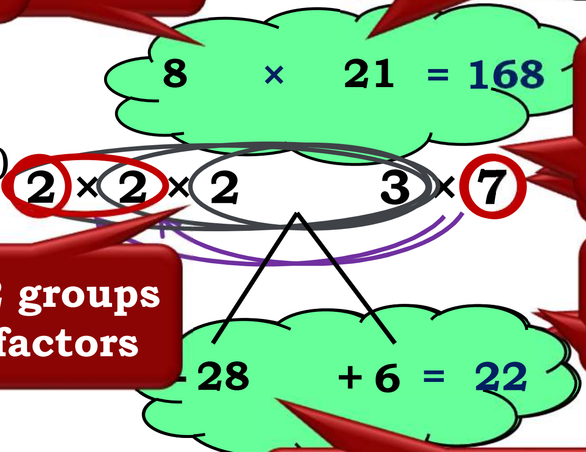
$\therefore \quad 8x^2 – 28x + 6x – 21 = 0$

$\therefore \quad 4x\ (2x – 7) + 3\ (2x – 7) = 0$

$\therefore \quad (2x – 7)\ (4x + 3) = 0$

$\therefore \quad 2x – 7 = 0$ or $4x$

$\therefore \quad x = \frac{7}{2}$ or $x = -\frac{3}{4}$



Take biggest no. & 2 of the smallest as one group

Make 2 groups of all factors

Remaining no. as other group

To add or subtract as per last sign

∴ **The roots of the given quadratic eq**

are $\frac{7}{2}$ and $-\frac{3}{4}$

No. 9

QUADRATIC EQUATIONS

- **Factorization Method Continued…**

Q) Solve the following quadratic equations by factorization method

viii) $10x^2 + 3x - 4 = 0$

Sol: $\boxed{10}x^2 \; + \; \boxed{3}x \; - \; \boxed{4} = 0$

$\therefore \quad 10x^2 + 8x - 5x - 4 = 0$

$\therefore \quad 2x(\underline{5x + 4}) - 1(\underline{5x + 4}) = 0$

$\therefore \quad (5x + 4)\,(2x - 1) = 0$

$\therefore \quad 5x + 4 = 0 \quad \text{or} \quad 2x - 1 = 0$

$\therefore \quad 5x = -4 \quad \text{or} \quad 2x = 1$

$\therefore \quad x = -\dfrac{4}{5} \quad \text{or} \quad x = \dfrac{1}{2}$

$\therefore$ The roots of the given quadratic equations are $-\dfrac{4}{5}$ and $\dfrac{1}{2}$

1st middle no. no.

Stanc form

Factorise by splitting middle te or

'–' sign means subtracting

Now signs to be given to both factors

4 × 10

40 = 5 × 8

40

– 5 + 8

40 – 1 ≠ 3

20 – 2 ≠ 3

10 – 4 ≠ 3

8 – 5 = 3

To factorise by splitting iddle term

Fi t of 3rd no.

rs of 40 in at by tors we get

n is – Give ddle sign only to bigger factor & opposite sign to smaller factor

Q) Solve the following quadratic equations by factorization method

ix) $\frac{1}{9}x^2 - \frac{2}{3}x + 1 = 0$

Sol : $\frac{1}{9}x^2 - \frac{2}{3}x + 1 = 0$

To Remove ‘9 & 3’ from denominator multiply by LCM of 9 & 3

LCM of 9 & 3 is 9

Multipl 9, we get,

$= 9 \times 0$

From fi
is co

From last two ‘3’ is common along with 3rd term sign

0

‘+’ sign means adding

$\therefore \quad x^2 - 3x - 3x + 9 = 0$

$\therefore \quad x(x-3) - 3(x-3) = 0$

$\therefore \quad (x-3)(x-3) = 0$

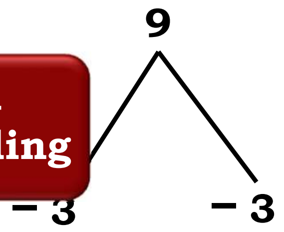
$\therefore \quad (x-3) = 0 \quad \text{or} \quad (x-3) = 0$

$\therefore \; x = 3 \quad \text{or} \quad x = 3$

Find product of 3rd no. with 1st no.

Find two factors of 9 in such a way that by adding factors we get middle no.

ince, last sign is +
Give middle sign to both factors.



∴ The roots of the given quadratic equation is 3

Thank You