Module 1

SURFACE AREAS AND VOLUMES

- **CUBOID - Introduction**

CUBOID

Examples :

Let us see few examples of cuboid

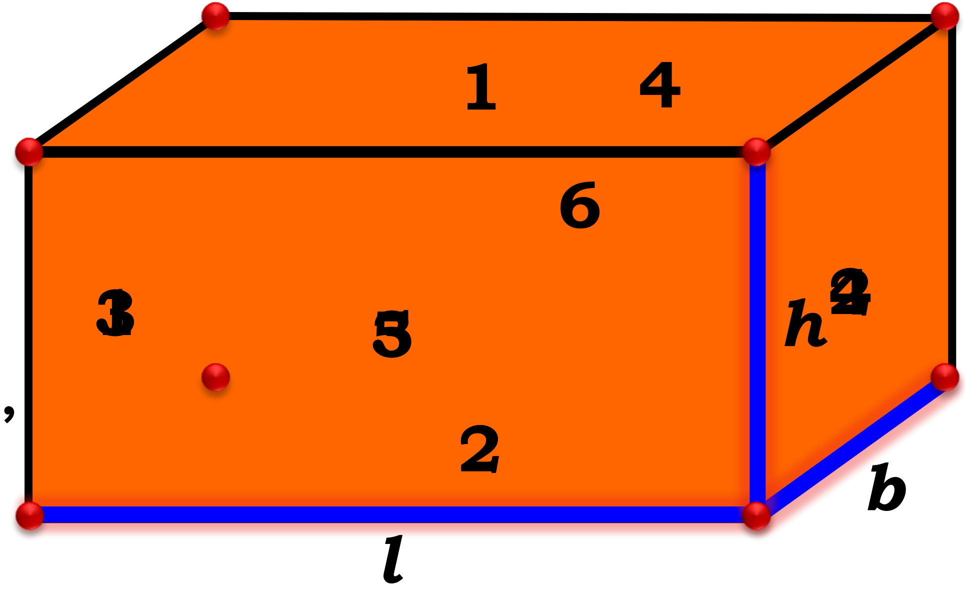
Brick

Boxes

Fish Aquarium

CUBOID

- A cuboid has six faces.
    - Two horizontal faces.
    - Four vertical faces.
- A cuboid has eight corners.
- A cuboid has three dimensions, which are length (*l*), breadth (*b*) and height (*h*)

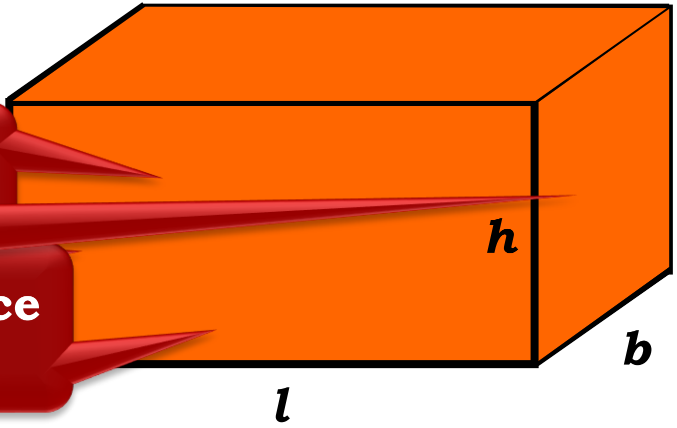


Let us see geometrical figure of cuboid

Formulae

Vertical Surface Area = $2\,(l + b)\,h$

Total Surface Area



Area of the shaded face
$= b \times h$

Area of rectangle = Product of adjacent sides

Total surface area = Vertical surface + Area of 2 base faces

$$= \boxed{2\,(l + b)\,h} + 2\,(lb)$$

$$= 2lh + 2bh + 2lb$$

$$= 2(lb + bh + lh)$$

$$= 2\,(l + b)h$$

Formulae

Continuing the process

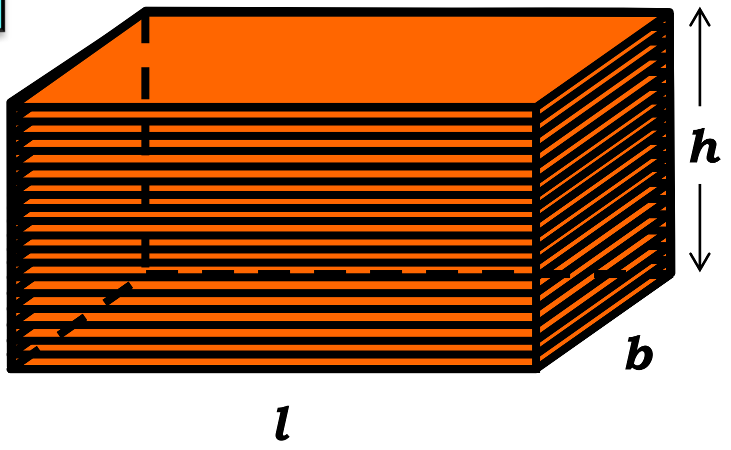
if w … gles

What is the figure so formed ?

Cuboid

What is the space occupied ?

∴ Volume of cuboid = $l \times b \times h$



Volume of cuboid $= l \times b \times h$

$l \times b$

$2 \times l \times b$

$3 \times l \times b$

⋮

$h \times l \times b$

Module 2

SURFACE AREAS AND VOLUMES

- Sums based on Cuboid

Q. The dimensions of a cuboid in cm are 16 × 14 × 20. Find its total surface area.

Sol.

For given cuboid,

length (l) = 16 cm

breadth (b) = 14 cm

height (h) = 20 cm

What is formula for finding total surface area of cuboid ?

$2(lb+bh+lh)$

Total surface area of cuboid = $2(\boxed{lb} + \boxed{bh} + \boxed{lh})$

$= 2\,(16 \times 14 + 14 \times 20 + 16 \times 20)$

$= 2\,(224 + 280 + 320)$

$= 2\,(824)$

$= 1648 \text{ cm}^2$

∴ Total surface area of the cuboid is 1648 cm^2.

Volume

Q. The cuboid water tank has length 2 m, breadth 1.6 m and height 1.8 m. Find the capacity of the tank in litres.

Sol.

length (l) = 2 m

breadth (b) = 1.6 m

height (h) = 1.8 m

We know that,
1 m^3 = 1000 litres

Volume of cuboid water tank = $\boxed{l} \times \boxed{b} \times \boxed{h}$

What is formula for finding volume of cuboid ?

$l \times b \times h$

$= 2 \times 1.6 \times 1.8$

$= 5.76\ m^3$

$= 5.76 \times 1000$ litres

[$1m^3$ = 1000 litres]

= 5760 litres

∴ Volume of cuboid water tank is 5760 litres.

Module 3

SURFACE AREAS AND VOLUMES

- Sum based on Cuboid

Q. A fish tank is in the form of a cuboid whose external measures are 80 cm × 40 cm × 30 cm. The base, side faces and back face are to be covered with a coloured paper. Find the area of the paper needed.

Area of the paper needed = Area of base + Area of two side faces + Area of back face

Sol.

For cuboid fish tank,

length (l) = 80 cm

breadth (b) = 40 cm

height (h) = 30 cm

Area of base $= \boxed{l} \times \boxed{b}$

$= 80 \times 40$

What is the formula to find area of base ?

Ar ... $2 \times b \times \boxed{h}$

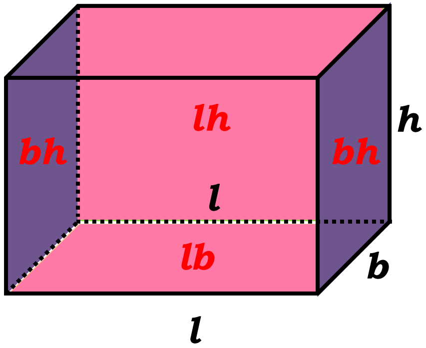
$= 2 \times 40 \times 30$

$= 2400 \text{ cm}^2$

What is the formula to find area of back face ?

nd

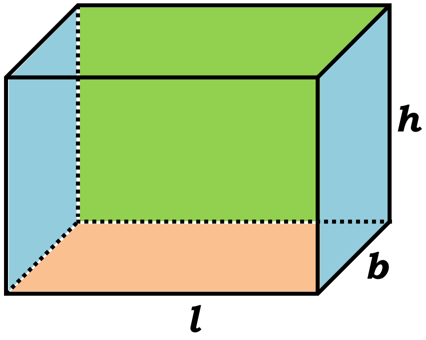
m²



Q. A fish tank is in the form of a cuboid whose external measures are 80 cm × 40 cm × 30 cm. The base, side faces and back face are to be covered with a coloured paper. Find the area of the paper needed.

Area of the paper needed = Area of base + Area of two side faces + Area of back face

Sol.



Area of the paper needed

= Area of base

+ Area of two side faces

+ Area of back face

= 3200 + 2400 + 2400

= 8000 cm^2

Area of base = 3200 cm^2

∴ Area of the pa

Area of two side faces = 2400 cm^2

Area of back face = 2400 cm^2

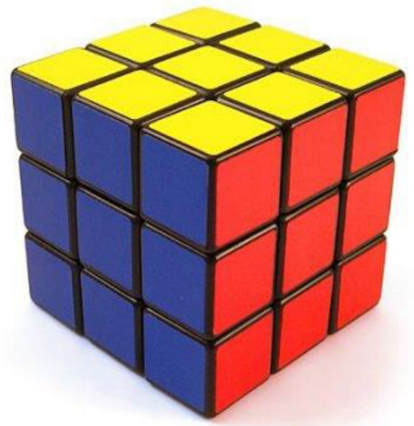
Module 4

SURFACE AREAS AND VOLUMES

- CUBE – Introduction

CUBE

- Cube is a special type of cuboid in which all surfaces are square in shape



Let us see few examples of cube

Rubix cube

Dice

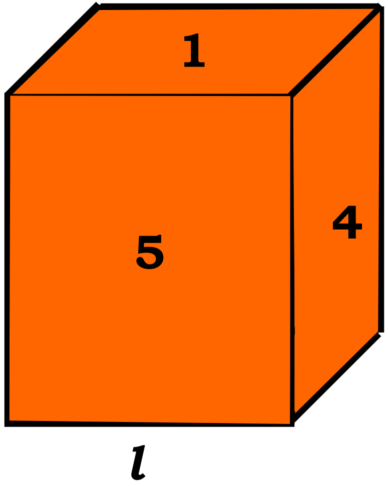
CUBE

- A cube has six faces.

Let us see geometrical figure of cube

All the sides of a cube are equal

Let 'l' be the length of each side



Formulae

1. Vertical surface area $= 4l^2$
2. Total surface area $= 6l^2$
3. Volume of

Let

Are

Area of shaded face
$= l^2$

Area of squar

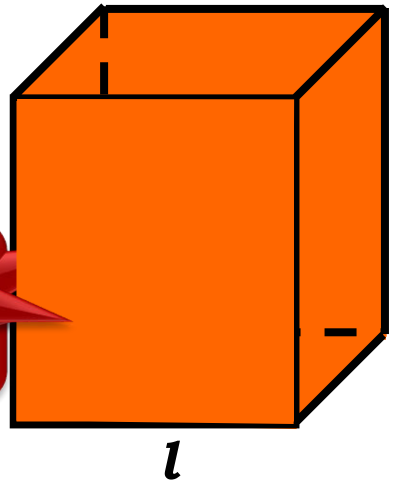
Volun
capacity of the cube

Volume of a cube $= l \times b \times h$

$= l \times l \times l$

$= l^3$

square face



Module 5

SURFACE AREAS AND VOLUMES

- **Sums based on Cube**

Q. The side of a cube is 60 cm. Find the total surface area of the cube.

Sol.

length (l) = 60 cm

Total surface area of the cube = $6\boxed{l^2}$

$= 6 \times (60)^2$

$= 6 \times 3600$

$= 21600\ \text{cm}^2$

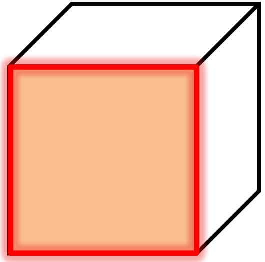
What is formula for finding Total surface area of cube?

$6l^2$

$\therefore$ Total surface area of the cube is $21600\ \text{cm}^2$.

Q. Perimeter of one face of cube is 24 cm.
Find (i) the total area of the 6 faces (ii) the volume of the cube.

Sol.



l

Perimeter of one face of cube = Perimeter of square

What is formula for finding Perimeter of square ?

$\therefore\ 4l$

$\therefore\quad 24 = 4l$

$$l = \frac{24}{4}$$

$\therefore\ \boxed{l = 6 \text{ cm}}$

Total surface area of cube $= 6\boxed{l^2}$

$= 6 \times (6)^2$

$= 6 \times 36$

$= 216 \text{ cm}^2$

Volume of cube $= \boxed{l^3}$

$= (6)^3$

$= 216 \text{ cm}^3$

$\therefore$ Total surface area of the cube is 216 cm^2 and Volume of the cube is 216 cm^3

Q. The volume of a cube is 1000 cm³. Find its total surface area.

Sol.

Volume of a cube = l^3

$\therefore$ 1000 = l^3

$\therefore$ $\boxed{l = 10 \text{ cm}}$ [Taking cube roots]

What is formula for finding Total surface area of cube?

$6l^2$

Total surface area of a cube = $6\boxed{l^2}$

$= 6 \times (10)^2$

$= 6 \times 100$

$= 600 \text{ cm}^2$

$\therefore$ Total surface area of the cube is 600 cm².

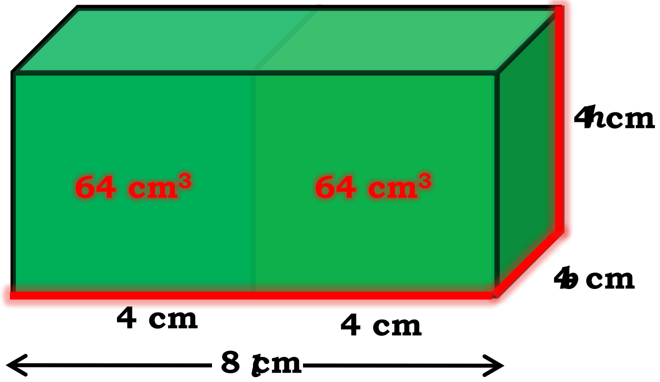
Module 6

SURFACE AREAS AND VOLUMES

- Sum based on Cube and Cuboid

Q. 2 cubes each of volume 64 cm³ are joined end to end. Find the surface area of the resulting cuboid.

Hint: find : l , b and h of cuboid



Sol. Volume of one cube = 64 cm³

$\therefore \quad a^3 = 64$

$\therefore \quad a = 4$ cm

After joining the cubes,

l = 8 cm, b = 4cm, h = 4cm

Surface area = 2 (lb + bh + lh)

What is the formula to

What is the formula to

find volume of cube ?

2 (lb + bh + lh)

a^3

$\underline{4 \times 4}$ + $\underline{8 \times 4}$)

6 + 32)

= 2 × 80

= 160cm²

$\therefore$ Surface area of the cuboid is 160 cm²

Thank You

Module 7

SURFACE AREAS AND VOLUMES

- CYLINDER - Introduction

RIGHT CIRCULAR CYLINDER

Examples :

Roller wheels

Let us see few examples of right circular cylinder

Pipes

Cylindrical candles

RIGHT CIRCULAR CYLINDER

- The right circular cylinder has three faces.
  - Two circular faces
  - One curved face
- Cylinder has radius ‘r’ and height ‘h’.

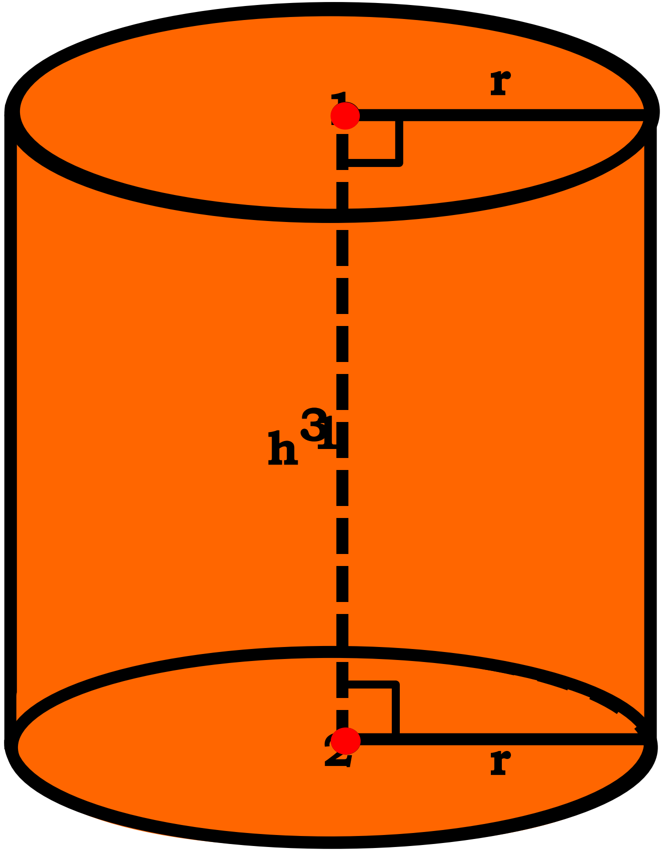
Distance between the centers

of t

h

Height is the

perpendicular distance



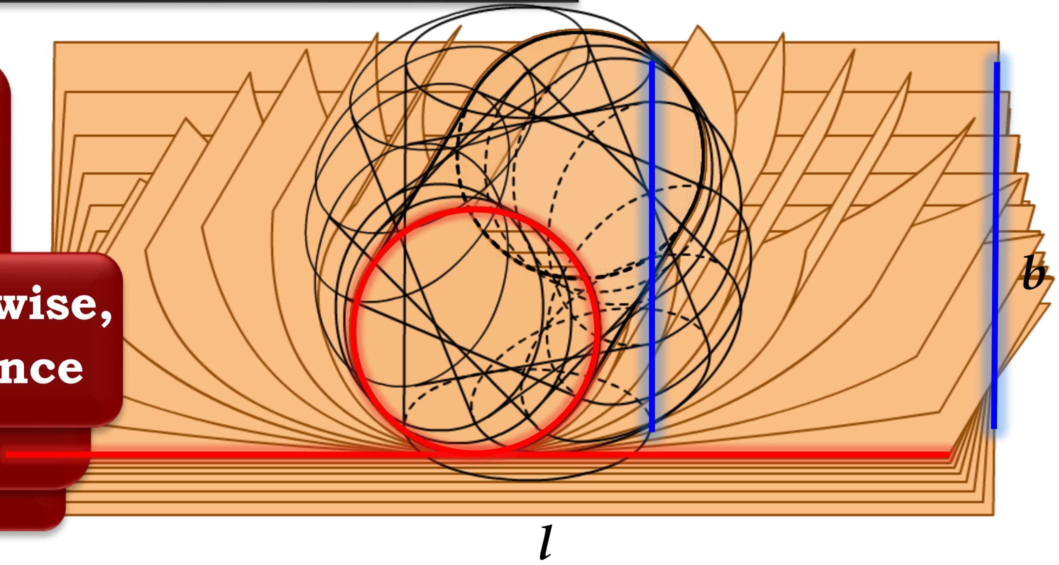
CURVED SURFACE AREA OF CYLINDER

Let us first understand formula for curved surface area of cylinder

CSA of cylinder = $2\pi r h$

When we fold the sheet lengthwise, length of sheet = Circumference

Breadth of sheet = h



Area of the sheet = CSA of cylinder

$\boxed{l} \times \boxed{b}$ = CSA of cylinder

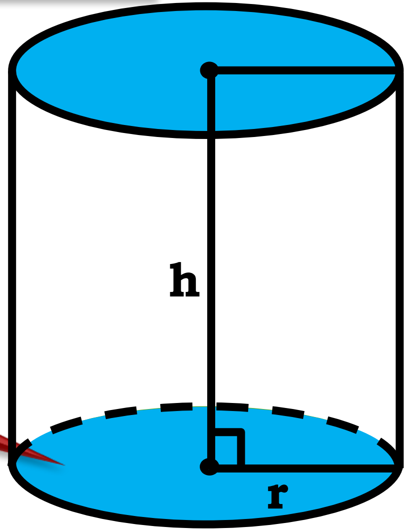
$2\pi r \times h$ = CSA of cylinder

TOTAL SURFACE AREA OF CYLINDER

Total Surface Area = $2 \pi r (r + h)$

The circular faces have equal radii and are parallel

Area of circular face = πr^2



$$TSA = \boxed{\text{CSA of a cylinder}} + \boxed{\text{Area of 2 circular faces}}$$

$$= 2\pi rh + 2 \times \boxed{\text{Area of a circular face}}$$

$$= 2\pi rh + 2\pi r^2$$

$$= 2 \pi r(h + r)$$

VOLUME OF CYLINDER

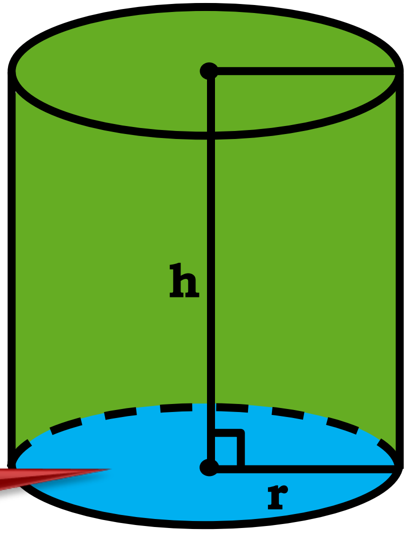
Volume $= \pi r^2 h$

Volume of the cylinder is the capacity of the cylinder

The circular faces of the cylinde

and are parallel to each other

Area of circular face $= \pi r^2$



Volume of cylinder = Area of base × Height

$= \pi r^2 \times h$

Module 8

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder

Q. The radius of a right circular cylinder is 3 cm, height is 7 cm. Find (i) curved surface area (ii) total surface area (iii) volume of the right circular cylinder. $\left(\pi = \frac{22}{7}\right)$

Sol.

Radius (r) = 3 cm

Height (h) = 7 cm

Curved surface area of cylinder $= 2\pi rh$

What is formula for finding curved surface area of cylinder ? $2\pi rh$

$= 2 \times \frac{22}{\cancel{7}} \times 3 \times \cancel{7}$

$= 2 \times 22 \times 3$

$\therefore$ Curved surface area of cylinder $= 132 \text{ cm}^2$

Total surface area of cylinder $= 2\pi r(r + h)$

What is formula for finding total surface area of cylinder ? $2\pi r(r+h)$

$= 2 \times \frac{22}{7} \times 3 \times (3 + 7)$

$= \frac{2 \times 22 \times 3 \times 10}{7}$

$= \frac{1320}{7}$

$\therefore$ Total surface area of cylinder $= 188.57 \text{ cm}^2$

Q. The radius of a right circular cylinder is 3 cm, height is 7 cm. Find (i) curved surface area (ii) total surface area (iii) volume of the right circular cylinder. $\left(\boxed{\pi = \frac{22}{7}}\right)$

Sol.

Radius (r) = 3 cm

Height (h) = 7 cm

What is formula for finding volume of cylinder

$\pi r^2 h$

Volume of cylinder $= \boxed{\pi r^2}\boxed{h}$

$$= \frac{22}{\cancel{7}} \times 3 \times 3 \times \cancel{7}$$

$$= 22 \times 9$$

$\therefore$ **Volume of cylinder = 198 cm^3**

$\therefore$ **Curved surface area is 132 cm^2, Total surface area is 188.57 cm^2 and Volume is 198 cm^3**

Module 9

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder

Q. The radius and height of a cylinder are in the ratio 3 : 7 and its volume is 1584 cm³. Find its radius.

Sol.

$r : h = 3 : 7$

Let the common multiple be 'x'.

$\therefore$ Radius (r) = $3x$, height (h) = $7x$

Volume of cylinder = 1584 cm³

Volume of cylinder = $\pi r^2 h$

$$\therefore \quad 1584 = \pi r^2 h$$

$$\therefore \quad 1584 = \frac{22}{\cancel{7}} \times (3x)^2 \times \cancel{7}x$$

$$\therefore \quad 1584 = 22 \times 9 \times x^2 \times x$$

$$\therefore \quad \frac{\overset{\overset{8}{\cancel{176}}}{\cancel{1584}}}{\cancel{22} \times \cancel{9}} = x^3$$

$$\therefore \quad x^3 = 8$$

$$\therefore \quad \boxed{x = 2}$$

What is formula for finding volume of cylinder ?

$\pi r^2 h$

Radius (r) = $3x$

$= 3 \times 2$

$\therefore \quad r = 6$ cm

$\therefore$ Radius of given cylinder is 6 cm

Module 10

SURFACE AREAS AND VOLUMES

- Cone-Introduction
- Sum based on Cone

RIGHT CIRCULAR CONE

Examples :

Ice Cream cone

Let us see some examples of Right Circular Cone

Conical candles

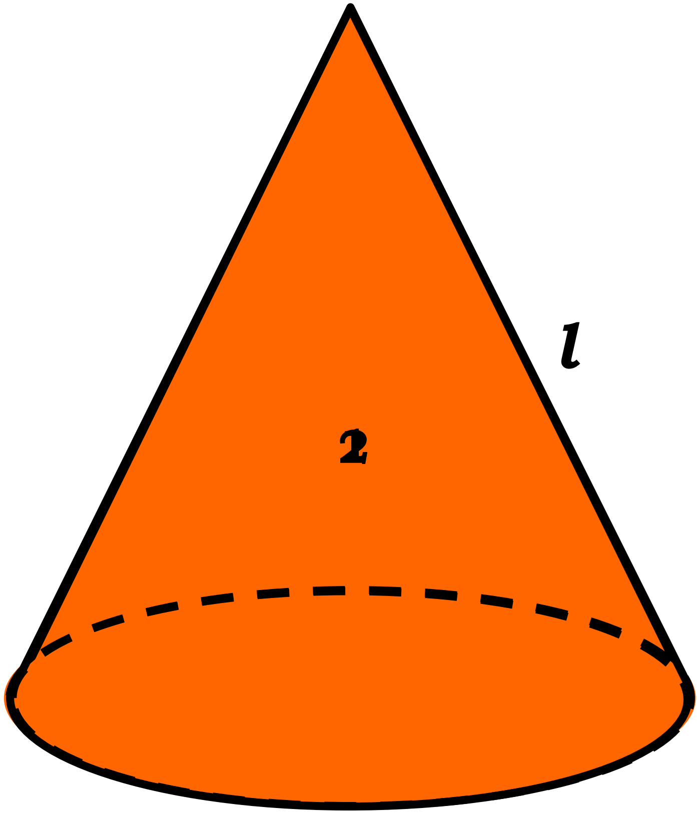
Sand cone

RIGHT CIRCULAR CONE

- A Right Circular Cone has two faces.
  - O
  - O

Let us see geometrical figure of right circular cone

- A cone has radius (r), height (h) and slant height (*l*)

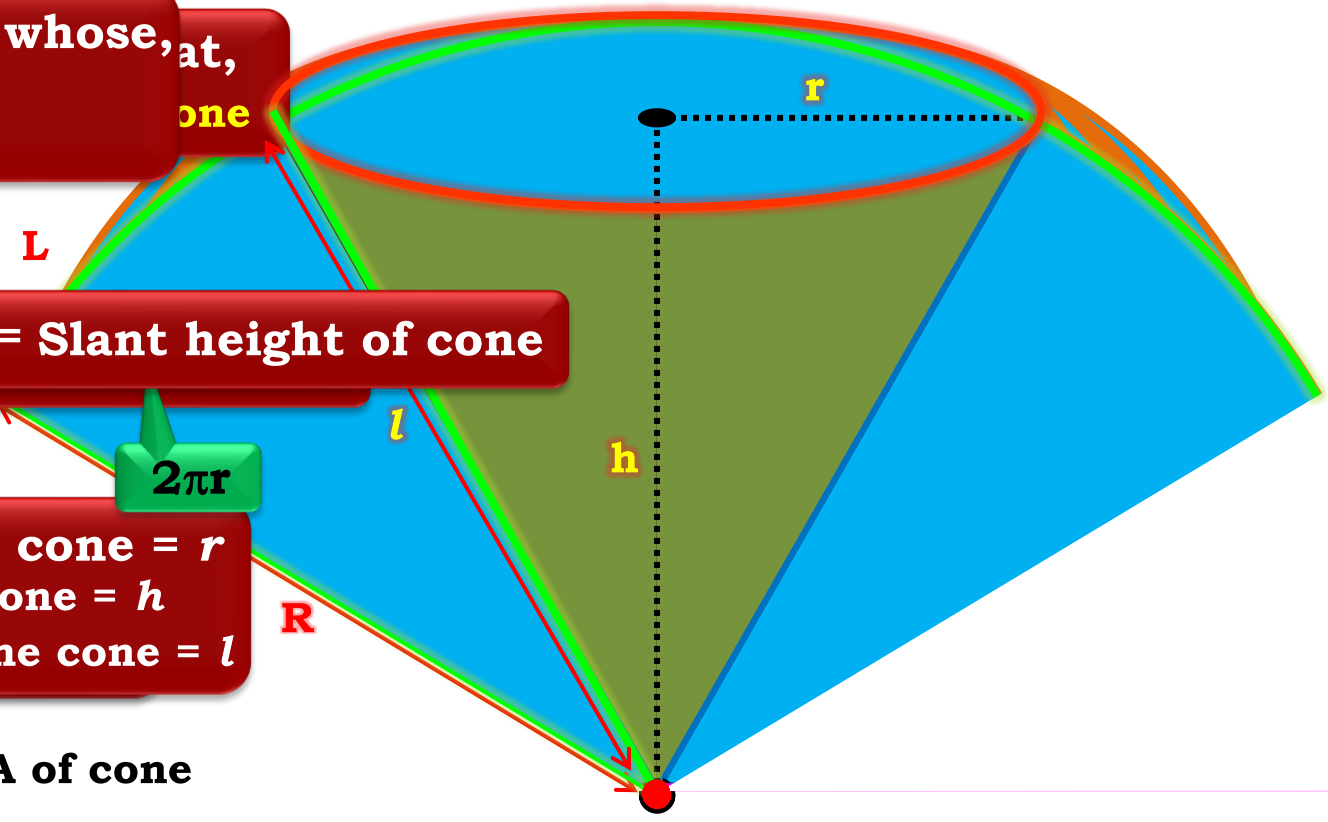


CURVED SURFACE AREA CONE

Observe a sector whose,
Radius = R
length of arc = L

Radius of sector = Slant height of cone

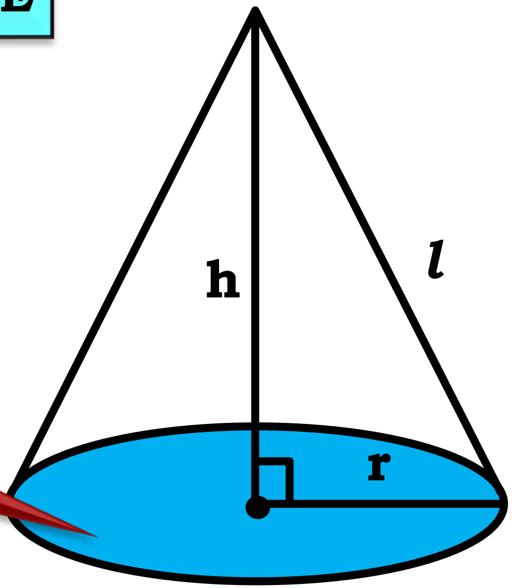
Radius of the cone = r
Height of the cone = h
Slant height of the cone = l



$$\frac{\cancel{2}\pi r \times l}{\cancel{2}} = \text{CSA of cone}$$

TOTAL SURFACE AREA OF CONE

Total surface area $= \pi r (r + l)$



Area of circular face $= \pi r^2$

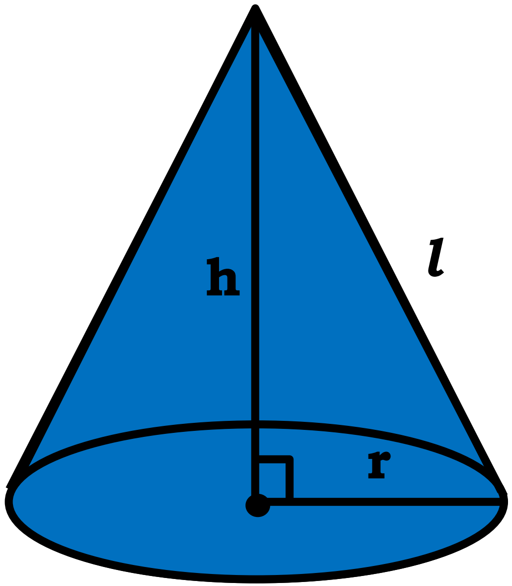
$$\text{TSA} = \boxed{\text{CSA}} + \boxed{\text{Area of circular face}}$$

$$= \pi r l + \pi r^2$$

$$= \pi r (l + r)$$

VOLUME OF CONE

$$\textbf{Volume} = \frac{1}{3} \times \pi r^2 h$$



Q. The curved surface area of a cone is 4070 cm^2 and its diameter is 70 cm. What is its slant height ?

Sol.

Curved surface area of a cone = 4070 cm^2

To find : l

Diameter (d) = 70 cm

What is formula for curved surface area of cone?

$\pi r l$

$\therefore$ $\boxed{\text{Radius (r)}} = \dfrac{\cancel{70}^{35}}{\cancel{2}}$ $\boxed{= 35 \text{ cm}}$

Curved surface area of cone = $\boxed{\pi r l}$

$$\therefore 4070 = \frac{22}{7} \times 35 \times l$$

$$\therefore \frac{\cancel{4070}^{\cancel{814}\ 37} \times \cancel{7}}{\cancel{22} \times \cancel{35}_{\cancel{5}}} = l$$

$$l = 37 \text{ cm}$$

$\therefore$ Slant height of a cone is 37 cm.

Module 11

SURFACE AREAS AND VOLUMES

- SPHERE

Sphere

Let us see few examples of sphere

Football

Globe

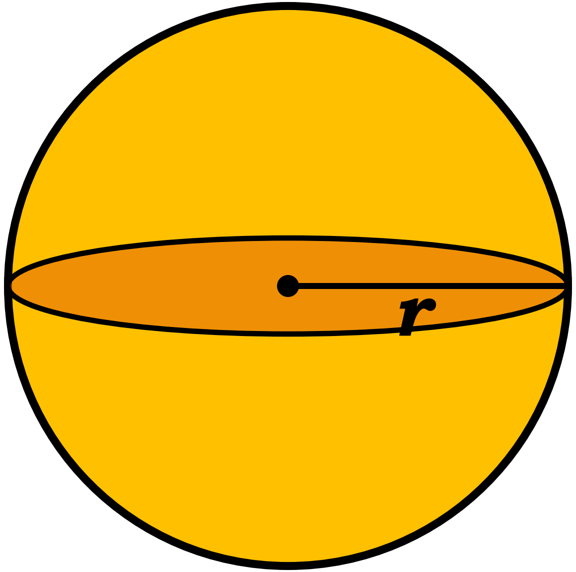
Metallic Ball

Sphere

Formulae :

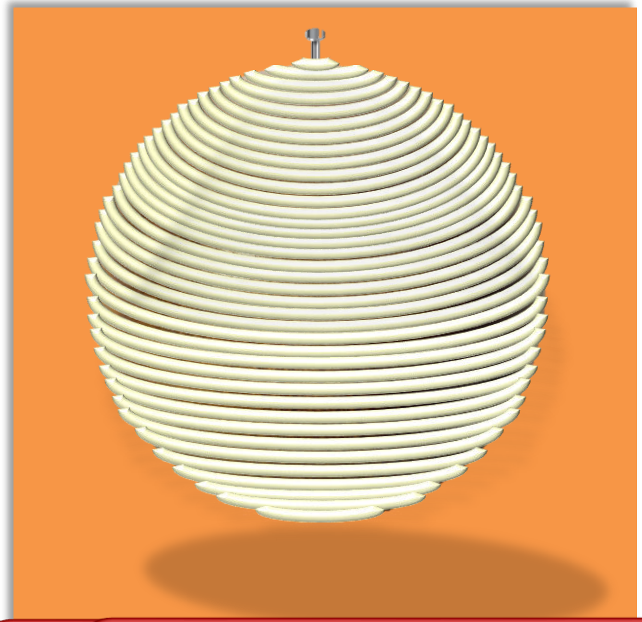
1. $\boxed{\text{Surface Area} = 4\,\pi\,r^2}$

2. Volume $= \dfrac{4}{3}\,\pi\,r^3$



Let us see an activity to understand the formula for total surface area of sphere

Surface area of sphere



Wh part
of t ng in
pl ing
a all,
till e ball

Mark the starting and
fin string,
ar string
from the surface of the ball.

Let radius = 10 cm

Activity :

Total surface area of sphere

$= \text{Length of string}$

$= 4\pi r^2$

$= 4 \times 3.14 \times 10^2$

$= 1256$

Total surface area of sphere	=	1256 cm^2

Length of string used in remaining string = 628

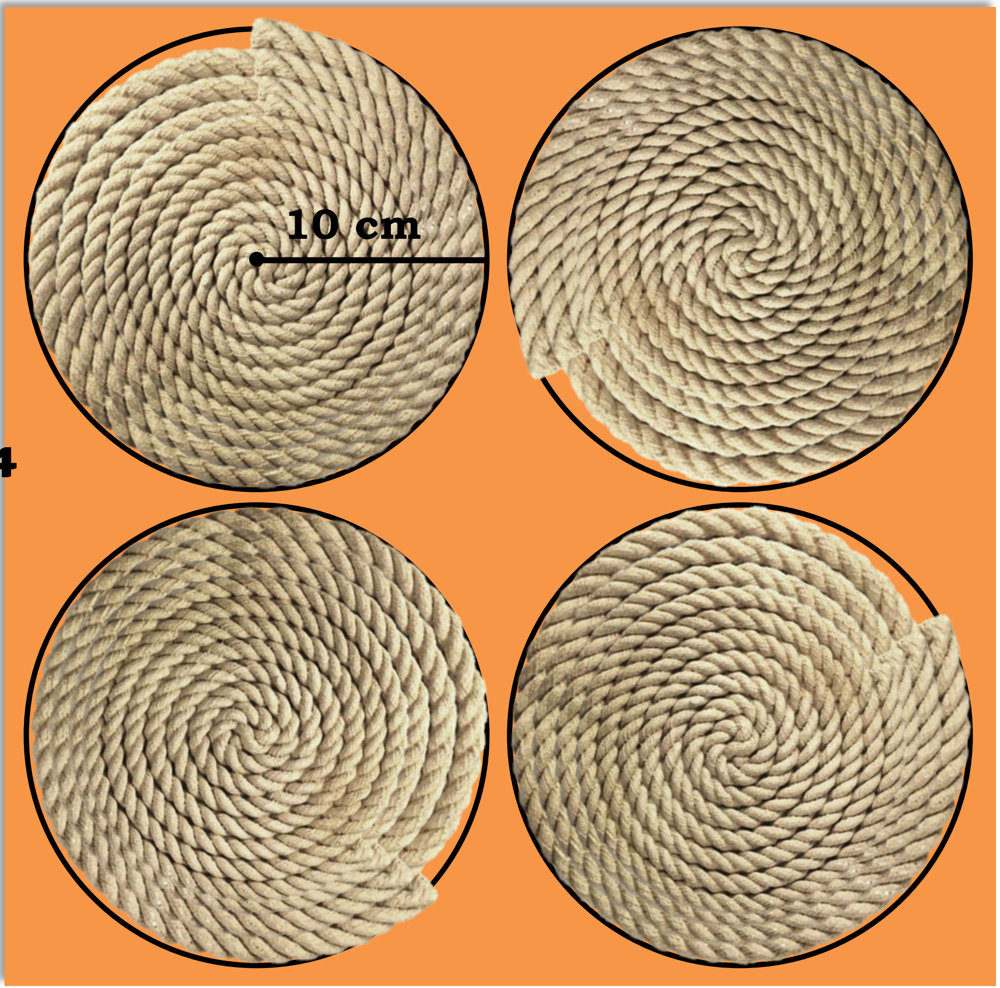
$= 3.14 \times 10^2$

$= 314$

Length of remaining string $= 1256 - 314$

$= 942$

Start filling the next circle with the string



Module 12

SURFACE AREAS AND VOLUMES

- Sums based on Sphere

Q. Find the volume and surface area of sphere of radius 4.2 cm. ($\pi = 22/7$)

Sol. $\boxed{\text{Radius of sphere} = 4.2\text{cm}}$

What is formula for finding volume of sphere ?

$\frac{4}{3}\pi r^3$

$$\text{Volume of sphere} = \frac{4}{3}\boxed{\pi}\boxed{r}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2$$

$$= \frac{4}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \frac{\overset{2}{\cancel{\overset{14}{\cancel{42}}}}}{10} \times \frac{42}{10} \times \frac{42}{10}$$

$$= \frac{4 \times 22 \times 2 \times 42 \times 42}{10 \times 10 \times 10}$$

$$= \frac{310464}{1000}$$

$$= 310.464 \text{ cm}^3$$

$\therefore$ **Volume of sphere is 310.464 cm^3**

Q. Find the volume and surface area of sphere of radius 4.2 cm. ($\pi = 22/7$)

Sol. $\boxed{\text{Radius of sphere} = 4.2\text{cm}}$

What is formula for finding surface area of sphere ?

$4\pi r^2$

Surface area of sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times 4.2 \times 4.2$$

$$= 4 \times \frac{22}{\cancel{7}} \times \frac{\overset{6}{\cancel{42}}}{10} \times \frac{42}{10}$$

$$= \frac{4 \times 22 \times 6 \times 42}{10 \times 10}$$

$$= \frac{22176}{100}$$

$$= 221.76 \text{ cm}^2$$

$\therefore$ Surface area of sphere is 221.76 cm^2

Q. The surface area of sphere is 616 cm^2. What is its volume ? ($\pi = 22/7$)

Hint :To find r.

Sol.

Surface area of sphere = 616cm^2

Surface area of sphere = $4\boxed{\pi}r^2$

$$\therefore \quad 616 = 4\pi r^2$$

$$\therefore \quad 616 = 4 \times \frac{22}{7} \times r^2$$

$$\therefore \quad \frac{\overset{7\ \cancel{154}}{\cancel{616}} \times 7}{\cancel{4} \times \cancel{22}} = r^2$$

$$\therefore \quad r^2 = 49$$

$$= 7 \text{ cm}$$

What is formula for finding surface area of sphere ?

$4\pi r^2$

Q. The surface area of sphere is 616 cm^2. What is its volume ?
(π = 22/7)

Sol.

$\boxed{r = 7\text{ cm}}$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times 7$$

$$= \frac{4 \times 22 \times 7 \times 7}{3}$$

$$= \frac{4312}{3}$$

$$= 1437.33 \text{ cm}^3$$

$\therefore$ **Volume of the sphere is 1437.33 cm^3**

Thank You

Module 13

SURFACE AREAS AND VOLUMES

- **HEMISPHERE- Introduction**

Half Hemisphere

- Hemisphere is half of a sphere

Let us consider examples of hemisphere

Examples :

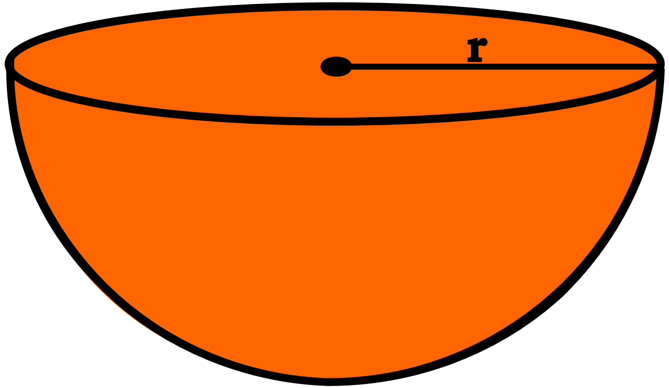
Half-cut sweet lime

Hemispherical bowls

Hemisphere

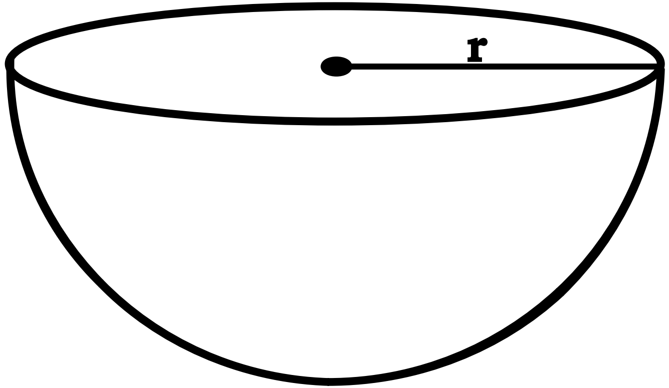
- A Hemisphere has two faces.
    - One circular face
    - One curved face

Let the radius be r



CURVED SURFACE AREA OF HEMISPHERE

$$\textbf{CSA} = 2\pi r^2$$



We know that,
Hemisphere is half
of sphere

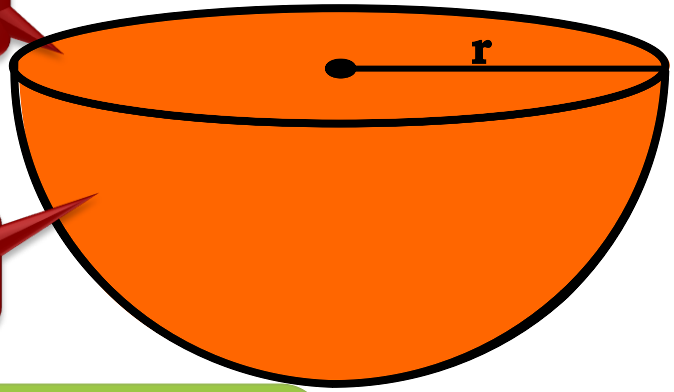
$$\text{CSA} = \frac{\boxed{\text{Curved surface area of sphere}}}{2}$$

$$= \frac{\cancel{4}^{\,2}\pi r^2}{\cancel{2}}$$

TOTAL SURFACE AREA OF HEMISPHERE

TSA = 3

Area of circular face = πr^2



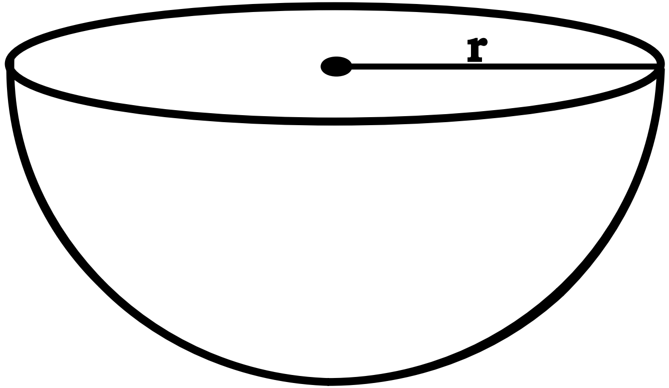
Curved surface area = $2\pi r^2$

TSA = $\boxed{\text{Curved surface area}}$ + $\boxed{\text{Area of circular face}}$

$$= 2\pi r^2 + \pi r^2$$

VOLUME OF HEMISPHERE

$$\text{Volume} = \frac{2}{3}\pi r^3$$



We know that,
Hemisphere is half of sphere

$$\text{Volume} = \frac{1}{2} \times \boxed{\text{Volume of sphere}}$$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{4}^{\,2}}{3}\pi r^3$$

Module 14

SURFACE AREAS AND VOLUMES

- **Sum based on Hemisphere**

Q. The curved surface area of a hemisphere is $905\frac{1}{7}\ cm^2$, What is its volume?

Sol.

Hint : To find: r

CSA of hemisphere $= 905\frac{1}{7}\ cm^2$

$$= \frac{(905 \times 7) + 1}{7}$$

$$= \frac{6335 + 1}{7}$$

$$\boxed{\text{CSA of hemisphere} = \frac{6336}{7}\ cm^2}$$

$$\boxed{\text{CSA of hemisphere} = 2\pi r^2}$$

What is formula for finding surface area of Hemisphere ?

$2\pi r^2$

$$\therefore \frac{6336}{7} = 2 \times \frac{22}{7} \times r^2$$

$$\therefore \frac{\overset{\overset{144}{\cancel{3168}}}{\cancel{6336}} \times \cancel{7}}{\cancel{7} \times \cancel{2} \times \cancel{22}} = r^2$$

$$\therefore r^2 = 144$$

$$\therefore r = 12\ cm$$

[Taking square roots]

Q. The curved surface area of a hemisphere is $905\frac{1}{7}$ cm^2,

What is its volume?

$\boxed{r = 12\text{ cm}}$

Sol.

Volume of hemisphere $= \dfrac{2}{3}\boxed{\pi r}^3$

$$= \frac{2}{\cancel{3}} \times \frac{22}{7} \times \overset{4}{\cancel{12}} \times 12 \times 12$$

$$= \frac{2 \times 22 \times 4 \times 12 \times 12}{7}$$

$$= \frac{25344}{7}$$

$$= 3620.57\text{ cm}^3$$

$$\begin{array}{r|l} & 3620.57 \\ \hline 7 & 25344 \\ & -21 \\ \hline & \quad 43 \\ & -\ 42 \\ \hline & \quad\ 14 \\ & -\ \ 14 \\ \hline & \quad\ \ 040 \\ & -\quad\ 35 \\ \hline & \qquad\ 50 \\ & -\qquad 49 \\ \hline & \qquad\quad 1 \end{array}$$

$\therefore$ **Volume of the hemisphere is 3620.57 cm^3**

Module 15

SURFACE AREAS AND VOLUMES

- **Sum based on Cylinder and Hemisphere**

Q. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter [2 π r h] sphere is 1[2 π r²]e total height of the vessel is 13cm. F[...] inner surface are[...] the vessel.

Inner surface area of vessel = CSA of cylinder (S_1) + CSA of hemisphere (S_2)

We know that,
In Hemisphere,
Radius = Height

Sol. Diameter = 14 cm

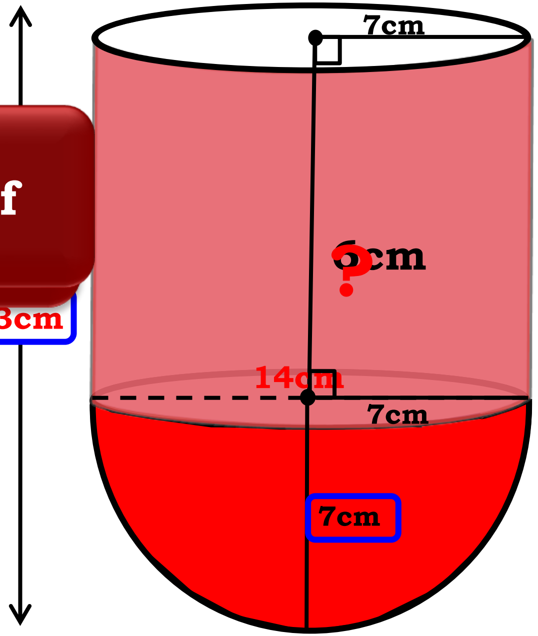
$\therefore$ Radius $= \frac{d}{2} = \frac{\cancel{14}^{7}}{\ldots} = 7$ cm

What is the formula to find curved surface area of cylinder?

– Height of Hemisphere

= 13 – 7

= 6 cm



Q. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter … sphere is 1… e total height of the vessel is 13cm. F… e inner surface ar… e vessel.

$2\pi r h$ $\quad$ $2\pi r^2$

Inner surface area of vessel = CSA of cylinder (S_1) + CSA of hemisphere (S_2)

Sol.

$$\text{CSA of Cylinder } (S_1) = 2\pi \boxed{r}\boxed{h}$$

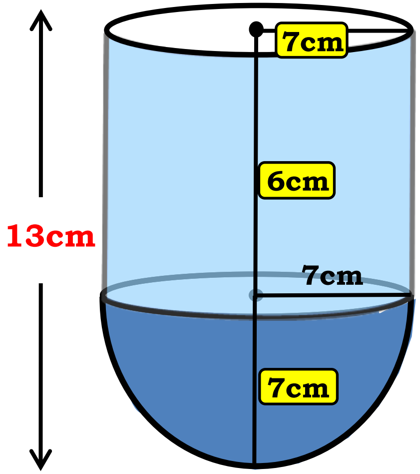
$$= 2 \times \pi \times 7 \times 6$$

$$\therefore \text{CSA of Cylinder } (S_1) = 84\pi \text{ cm}^2$$

$$\text{CSA of Hemisphere } (S_2) = 2\pi \boxed{r}^2$$

$$= 2 \times \pi \times 7 \times 7$$

$$\therefore \text{CSA of Hemisphere } (S_2) = 98\pi \text{ cm}^2$$



Q. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter … sphere is 1… e total height of the vessel is 13cm. F… surface ar… vessel.

$2\pi r h$ **$2\pi r^2$**

Inner surface area of vessel = CSA of cylinder (S_1) + CSA of hemisphere (S_2)

$S_1 = 84\pi$

$S_2 = 98\pi$

Sol.

Inner surface area of the vessel

$= \boxed{S_1} + \boxed{S_2}$

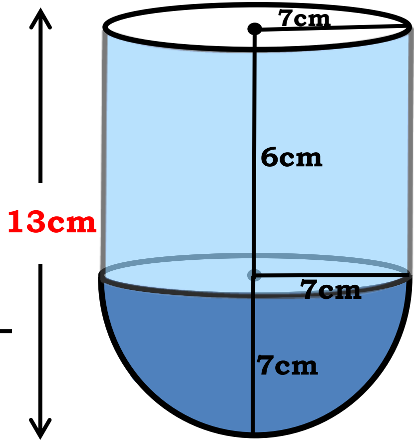
$= 84\pi + 98\pi$

$= 182 \times \pi$

$= \cancel{182}^{\,26} \times \dfrac{22}{\cancel{7}}$

$= 26 \times 22$

$= 572\ cm^2$



$\therefore$ Inner surface area of the vessel is $572\ cm^2$

Module 16

SURFACE AREAS AND VOLUMES

- Sum based on Cone and Hemisphere

Q. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The $\pi r l$ ight of the to $2 \pi r^2$ cm.
Find the total surface area of th

Total surface area of toy = CSA of cone (S_1) + CSA of hemisphere (S_2)

Sol.

Height of cone

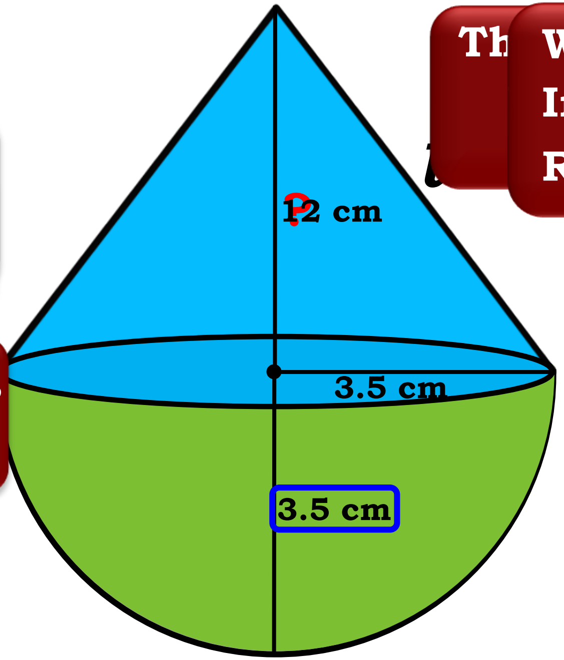
What is the formula to find

What is the ~~formula~~ to find slant height (l)?

$l = \sqrt{r^2 + h^2}$

For getting the slant height (l),
Let us first find height (h)

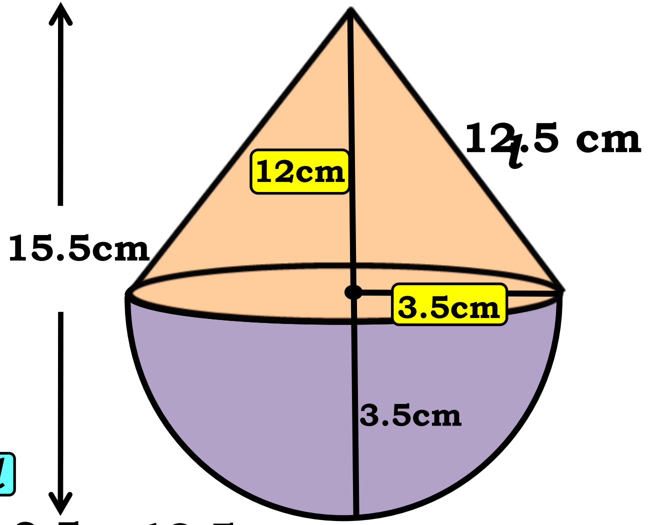
We know that,
In Hemisphere,
Radius = Height



Q. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Total surface area of toy = CSA of cone (S_1) + CSA of hemisphere (S_2)

$\pi r l$ $2 \pi r^2$



Sol.

Slant height (l) $= \sqrt{r^2 + h^2}$

$= \sqrt{(3.5)^2 + 12^2}$

$= \sqrt{12.25 + 144}$

$= \sqrt{156.25}$

$\therefore$ $\boxed{l = 12.5 \text{ cm}}$

Surface area of cone (S_1) $= \pi r l$

$= \pi \times 3.5 \times 12.5$

$\therefore$ Surface area of cone (S_1) $= 43.75\pi \text{cm}^2$

Q. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The $\pi r l$ ight of the to $2\pi r^2$ cm. Find the total surface area of th y.

Total surface area of toy = CSA of cone (S_1) + CSA of hemisphere (S_2)

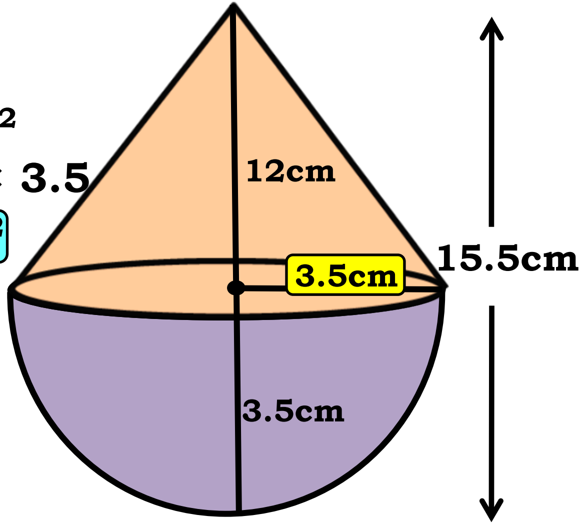
Sol.

Surface area of hemisphere(S_2) $= 2\pi \boxed{r^2}$

$= 2\pi \times (3.5)^2$

$= 2\pi \times 3.5 \times 3.5$

$\therefore$ Surface area of hemisphere(S_2) = $24.5\pi cm^2$



$S_1 = 43.75\pi$

Total surface area of the toy $= \boxed{S_1} + \boxed{S_2}$

$= 43.75\pi + 24.5\pi$

$= 68.25\pi$

$= \overset{9.75}{\cancel{68.25}} \times \frac{22}{\cancel{7}}$

$= 214.25\ cm^2$

$\therefore$ Total surface area of the toy is 214.25 cm^2.

Module 17

SURFACE AREAS AND VOLUMES

- Sum based on Cube and Hemisphere

Q. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest dia[…] the hem[…] can have ? Find the surface area o[…]id.

$6l^2$ πr^2 $2\pi r^2$

Surface area of the solid = TSA (cube) − Base Area (hemisphere) + CSA (hemisphere)

Sol. Greatest diameter = Side of a square = 7 cm

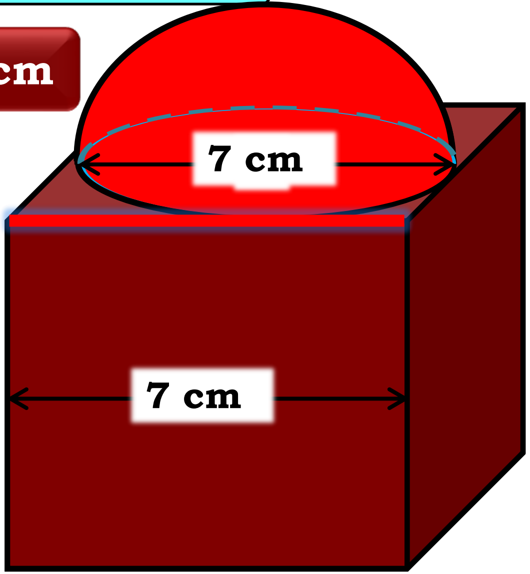
Diameter = 7 cm

$\therefore r = \frac{d}{2} = \frac{7}{2}$ cm

[…]re = 7 cm

W[…] to […]

What is the formula to find area of a circle?



Q. A cubical block of side 7 cm is surmounted by a hemisphere.
What is the greatest dia $6l^2$ he hem πr^2 e can have ? $2 \pi r^2$
Find the surface area o id.

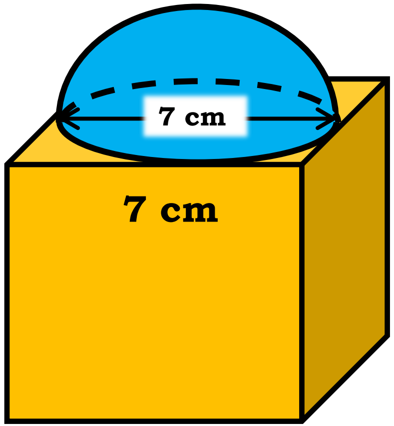
Surface area of the solid = TSA (cube) − Base Area (hemisphere) + CSA (hemisphere)

Sol. TSA of cube $= 6\boxed{l^2}$

$= 6 \times 7^2$

$= 6 \times 49$

$= 294 \text{ cm}^2$



Q. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest di... he hem... e can have ? Find the surface area o... id.

$6l^2$ $\quad \pi r^2 \quad 2\pi r^2$

Surface area of the solid = TSA (cube) − Base Area (hemisphere) + CSA (hemisphere)

TSA (cube) = 294

Radius = $\frac{7}{2}$ cm

Sol.

Surface area of the solid

$$= \text{TSA (cube)} - \text{Base Area (hemisphere)} + \text{CSA (hemisphere)}$$

$$= 294 - \pi r^2 + 2\pi r^2$$

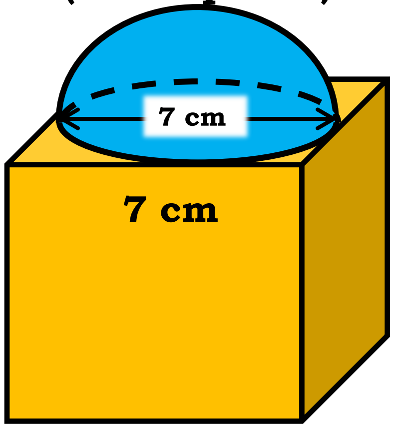
$$= 294 + \pi r^2$$

$$= 294 + \frac{\cancel{22}^{11}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{2}$$

$$= 294 + \frac{77}{2}$$

$$= 294 + 38.5$$

$$= 332.5 \text{ cm}^2$$



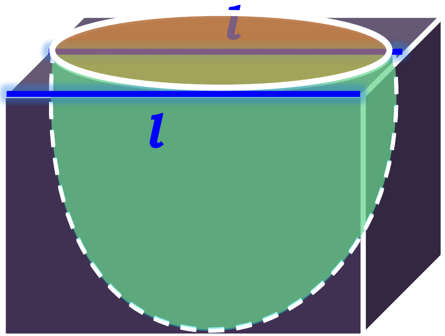
$\therefore$ **Surface area of the solid is 332.5 cm²**

Module 18

SURFACE AREAS AND VOLUMES

- **Sum based on Cube and Hemisphere**

Q. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter 'l' of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.



Sol. Edge of the cube = Diameter of hemisphere = l

Radius of hemisphere (r) = $\frac{l}{2}$

Surface area of the remaining solid

= SA (cube) − A (hemispherical top) + CSA (hemisphere)

$= 6l^2 - \pi r^2 + 2\pi r^2$ $\quad = 6l^2 + \frac{\pi l^2}{4}$

= 6 $\quad \frac{24l^2 + \pi l^2}{}$

Radius (r) = $\frac{\text{Diameter}}{2}$

What is the formula to find curved surface area of hemisphere?

$2\pi r^2$

+ π) sq.units

∴ Surface area of the remaining solid = $\frac{1}{4} l^2 (24 + \pi)$ sq.units

Module 19

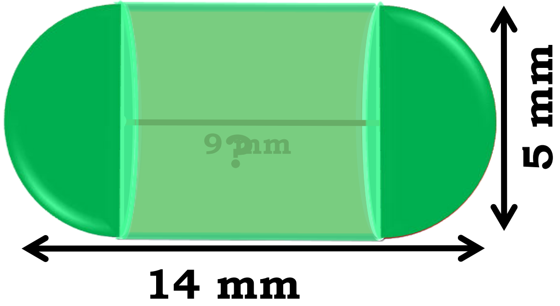
SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Hemisphere

Q. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14mm and the diam... capsule is ... Find its surface area.

$2\pi r h$ $\quad$ $2\pi r^2$

Total surface area of capsule = CSA of cylinder (S_1) + 2 CSA of hemisphere(S_2)



Sol. Diameter = 5 mm

$\therefore$ $\boxed{\text{Radius } (r) = \frac{d}{2} = \frac{5}{2} = 2.5 \text{ mm}}$

Length of the cylinder (h) $= 14 - 2.5 - 2.5$

$\therefore$ $\boxed{h = 9\text{mm}}$

CSA of the cylinder (S_1) $= 2\pi \boxed{r}\boxed{h}$

$= \ldots \pi \times 2.5 \times 9$

$\therefore$ $\ldots$ mm^2

We know that,
In Hemisphere,
Radius = Height

... to find
... ght of

$\ldots = 2\pi \boxed{r}^2$

$= 2 \times 2 \times \pi \times (2.5)^2$

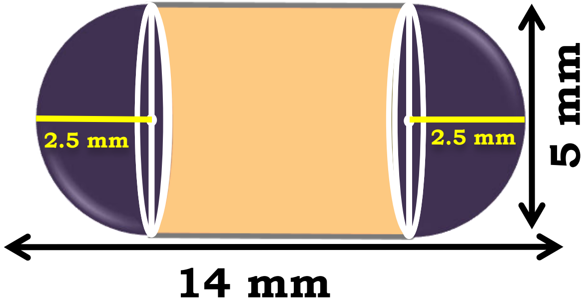
$\therefore$ CSA of 2 hemispheres (S_2) $= 25\pi$ mm^2

Q. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14mm and the diam capsule is Find its surface area.

$2 \pi r h$ $2 \pi r^2$

Total surface area of capsule = CSA of cylinder (S_1) + 2 CSA of hemisphere(S_2)

Sol.



TSA of the capsule $= \boxed{S_1} + \boxed{S_2}$

$= 45\pi + 25\pi$

$= 70\pi$

$= \cancel{70}^{10} \times \dfrac{22}{\cancel{7}}$

$= 220 \text{ mm}^2$

$\boxed{S_1 = 45\pi}$

$\boxed{S_2 = 25\pi}$

$\therefore$ Surface area of the capsule is 220 mm^2

Module 20

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Cone

Q. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the t e rate of er m^2.
(Note that the base of the te t will not be cov ed with canvas.)

$\pi r l$ $\quad$ $2\pi r h$

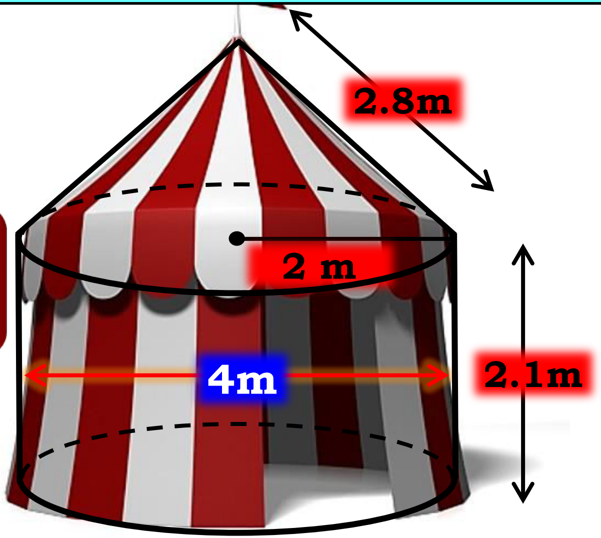
TSA of tent = CSA of cone (S_1) + CSA of cylinder (S_2)

Sol. Diameter = 4 m

$\therefore$ Radius $= \dfrac{d}{2} = \dfrac{\overset{2}{\cancel{4}}}{\cancel{2}} = 2$ m

W d

i.e. we need to total surface area of the tent



Q. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the $\pi\, r\, l$ **the rate of** $2\, \pi\, r\, h$ **er m².**
(Note that the base of the **nt will not be cov** **ed with canvas.)**

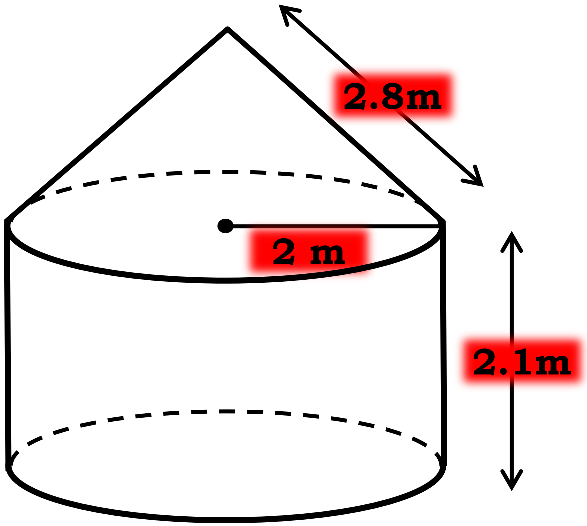
TSA of tent = CSA of cone (S_1) + CSA of cylinder (S_2)

Sol.

$$\text{CSA of cone } (S_1) = \pi \boxed{r} \boxed{l}$$

$$= \pi \times 2 \times 2.8$$

$$\therefore \text{ CSA of cone } (S_1) = 5.6\pi \text{ m}^2$$



$$\text{CSA of cylinder } (S_2) = 2\, \pi \boxed{r} \boxed{h}$$

$$= 2 \times \pi \times 2 \times 2.1$$

$$\therefore \text{ CSA of cylinder } (S_2) = 8.4\pi \text{m}^2$$

Q. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs. 500 per m^2. (Note that the base of the tent will not be covered with canvas.)

TSA of tent = CSA of cone (S_1) + CSA of cylinder (S_2)

Sol. TSA of tent $= \boxed{S_1} + \boxed{S_2}$

$= 5.6\pi + 8.4\pi$

$= 14 \times \pi$

Total cost of the canvas of the tent = TSA of tent × Rate

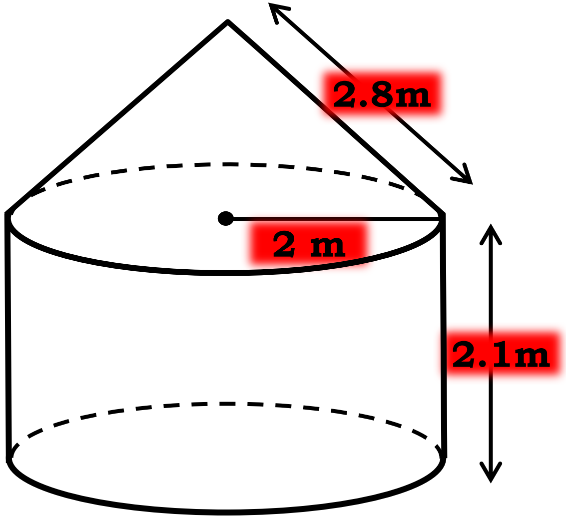
$\therefore$ TSA of tent = $44 m^2$

Total cost of the canvas

$= \boxed{\text{TSA of tent}} \times \boxed{\text{rate}}$

$= 44 \times 500$

$\therefore$ Total cost of the canvas = Rs. 22000



Thank You

Module 21

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Cone

Q. From a **solid cylinder** whose **height is 2.4 cm** and **diameter 1.4 cm**, a conical cavity of the same height and same diameter is hollowed out. Find the total surface are … e rema … l to the ne … m².

πr^2 $\quad$ $2\pi r h$ $\quad$ $\pi r l$

TSA (remaining solid) = Area of base + CSA of cylinder (S_1) + CSA of cone (S_2)

Sol. Diameter = 1.4 cm

$\therefore \boxed{r} = \dfrac{d}{2} = \dfrac{\cancel{1.4}^{\,0.7}}{\cancel{2}} = \boxed{0.7 \text{ cm}}$

Slant height (l) = $\sqrt{\boxed{r}^2 + \boxed{h}^2}$

$= \sqrt{(0.7)^2 + (2.4)^2}$

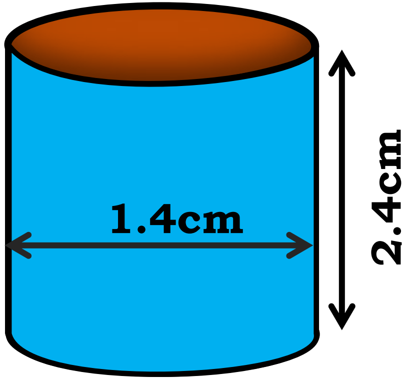
$= \sqrt{0.49 + 5.76}$

What is the formula to find slant height (l)?

$l = \sqrt{r^2 + h^2}$

$= \sqrt{\dfrac{625}{100}} = \dfrac{25}{10}$

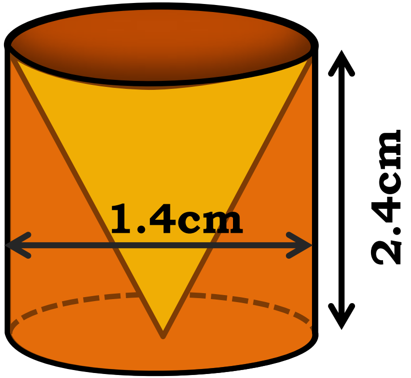
$\therefore \quad l = 2.5$ cm



Q. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface are πr^2 e rema $2 \pi r h$ l to the n $\pi r l$ n^2.

TSA (remaining solid) = Area of base + CSA of cylinder (S_1) + CSA of cone (S_2)

Sol. $\boxed{r = 0.7 \text{ cm}}$, $\boxed{h = 2.4 \text{ cm}}$, $\boxed{l = 2.5 \text{ cm}}$



Area of the base of cylinder

$= \pi \boxed{r^2}$

$= \pi \times (0.7)^2$

$= 0.49\pi \text{ cm}^2$

CSA of cyl. (S_1) $= 2\pi \boxed{rh}$

$= 2 \times \pi \times 0.7 \times 2.4$

$= 0.14 \times \pi \times 2.4$

$\therefore$ CSA of cyl. (S_1) $= 3.36\pi \text{ cm}^2$

CSA of cone (S_2) $= \pi \boxed{r} \boxed{l}$

$= \pi \times 0.7 \times 2.5$

$\therefore$ CSA of cone (S_2) $= 1.75\pi \text{ cm}^2$

Q. **From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface are** $\pi\, r^2$ **e rema** $2\,\pi\, r\, h$ **l to the n** $\pi\, r\, l$ **n².**

TSA (remaining solid) = Area of base + CSA of cylinder (S_1) + CSA of cone (S_2)

Sol.

TSA of the remaining solid

$$= \boxed{\text{Area of the base}} + \boxed{S_1} + \boxed{S_2}$$

$$= 0.49\pi + 3.36\pi + 1.75\pi$$

$$= 5.6\pi$$

$$= \cancel{5.6}^{\,0.8} \times \frac{22}{\cancel{7}}$$

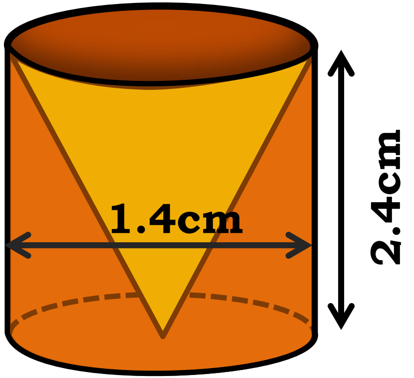
$$= 0.8 \times 22$$

$$= 17.6 \text{ cm}^2$$

$$\boxed{S_1 = 3.36\pi}$$

$$\boxed{S_2 = 1.75\pi}$$

$$\boxed{\text{Area of base} = 0.49\pi}$$



$\therefore$ **TSA of the remaining solid is 18 cm²**

Module 22

SURFACE AREAS AND VOLUMES

- **Sum based on Cylinder and hemisphere**

Q. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10cm, and … of radius 3.5 cm, find the total surface area o… article.

$2\,\pi\, r\, h$ ✓✓

$2\,\pi\, r^2$ ✓

TSA of article = CSA of cylinder (S_1) + 2 CSA of hemisphere (S_2)

Sol.

CSA of the cyl. (S_1) $= 2\,\pi\,\boxed{r}\,\boxed{h}$

$= \underline{2} \times \pi \times \underline{3.5} \times 10$

$= 7 \times \pi \times 10$

$\therefore$ CSA of the cyl. (S_1) $= 70\pi\ cm^2$

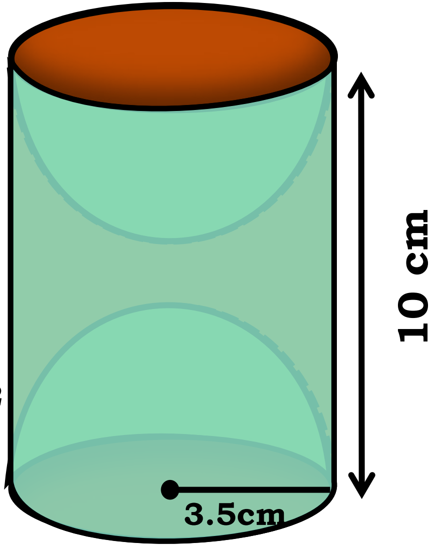
What is the formula to find curved surface area of hemisphere ?

CS… $\boxed{r}^2$

$\times\ \pi \times \underline{(3.5)^2}$

$= 4 \times \pi \times 12.25$

$\therefore$ CSA of 2 hemispheres (S_2) $= 49\pi\ cm^2$



Q. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10cm, and … s of radius 3.5 cm, find the total surface area o… …ticle.

$2\pi r h$

$2\pi r^2$

TSA of article = CSA of cylinder (S_1) + 2 CSA of hemisphere (S_2)

Sol.

$$\text{TSA of the article} = \boxed{S_1} + \boxed{S_2}$$

$$= 70\pi + 49\pi$$

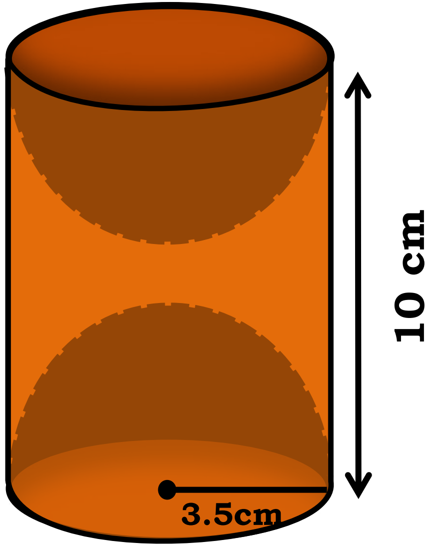
$$= 119\pi$$

$$= \cancel{119}^{17} \times \frac{22}{\cancel{7}}$$

$$= 374 \text{ cm}^2$$

$\boxed{S_1 = 70\pi}$

$\boxed{S_2 = 49\pi}$



∴ **TSA of the article is 374 cm²**

Module 23

SURFACE AREAS AND VOLUMES

- Sum based on Cone and Hemisphere

Q. A solid is in the shape of a cone standing on a hemisphere with both their radii bei … cm and the … the cone is equal to its radiu … volume of th … erms of π.

$\frac{1}{3}\pi r^2 h$

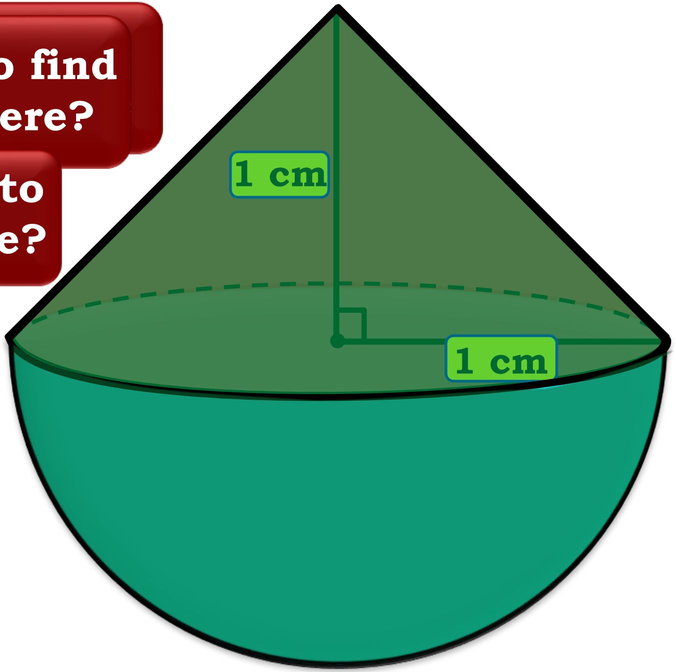
$\frac{2}{3}\pi r^3$

Volume of solid = Volume of cone (V_1) + Volume of hemisphere (V_2)

Sol.

What is the formula to find volume of a hemisphere?

What is the formula to find volume of a cone?



Q. A solid is in the shape of a cone standing on a hemisphere with both their radii bei[...] cm and th[...] the cone is equal to its radiu[...] volume of th[...] erms of π.

$\frac{1}{3}\pi r^2 h$

$\frac{2}{3}\pi r^3$

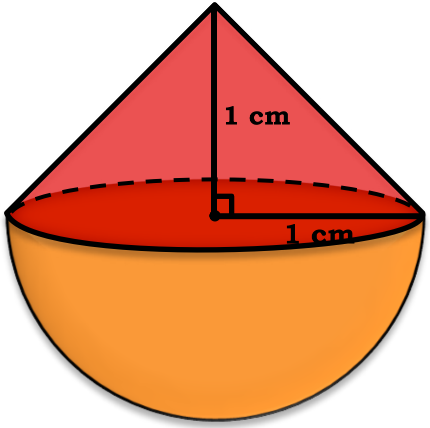
Volume of solid = Volume of cone (V_1) + Volume of hemisphere (V_2)

Sol. Now $\boxed{h = r = 1\text{cm.}}$

$$\text{Volume of cone } (V_1) = \frac{1}{3} \times \pi \boxed{r^2} \boxed{h}$$

$$= \frac{1}{3} \times \pi \times 1 \times 1 \times 1$$

$$\therefore \text{ Volume of cone } (V_1) = \frac{1}{3}\pi \text{ cm}^3$$



$$\text{Volume of hemisphere } (V_2) = \frac{2}{3} \times \pi \boxed{r^3}$$

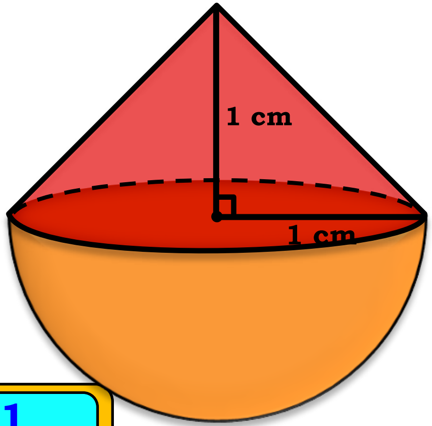
$$= \frac{2}{3} \times \pi \times 1 \times 1 \times 1$$

$$\therefore \text{ Volume of hemisphere } (V_2) = \frac{2}{3}\pi \text{ cm}^3$$

Q. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Volume of solid = Volume of cone (V_1) + Volume of hemisphere (V_2)

Sol.



Volume of the solid $= \boxed{V_1} + \boxed{V_2}$

$= \frac{1}{3}\pi + \frac{2}{3}\pi$

$= \frac{\pi + 2\pi}{3}$

$= \frac{\cancel{3}\pi}{\cancel{3}}$

$= \pi\ cm^3$

$V_1 = \frac{1}{3}\pi$

$V_2 = \frac{2}{3}\pi$

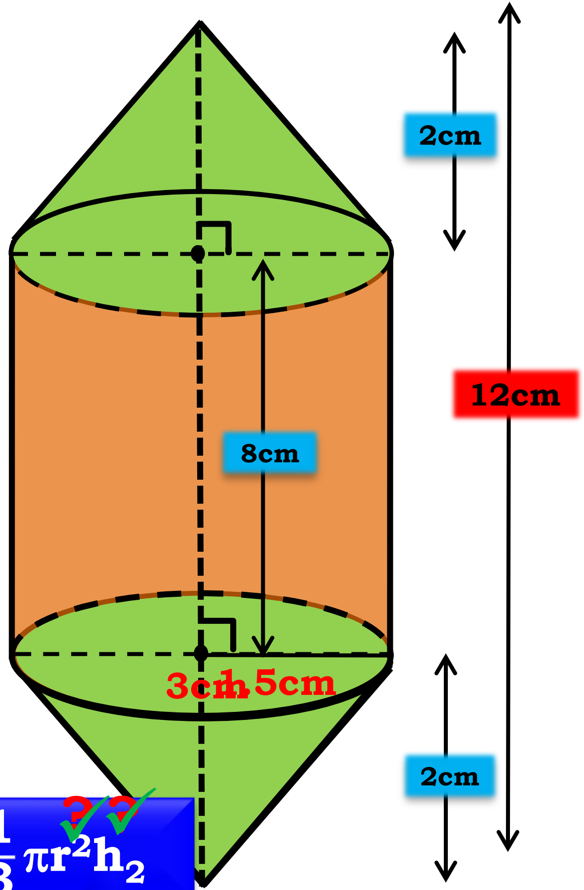
$\therefore$ Volume of the solid $= \pi\ cm^3$

Module 24

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Cone

Q. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same)



Let us find the height of the cylinder

Length of the cyl. $(h_1) = 12 - 2 - 2$

$= 8$ cm

$\pi r^2 h_1$ $\qquad$ $\frac{1}{3}\pi r^2 h_2$

Vol. of air in the model = Vol. of cyl. (V_1) + 2 × Vol. of cone (V_2)

Q. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and in... model to be nearly the same)

$\pi r^2 h_1$ $\frac{1}{3}\pi r^2 h_2$

Vol. of air in the model = Vol. of cyl. + 2 × Vol. of cone (V_2)

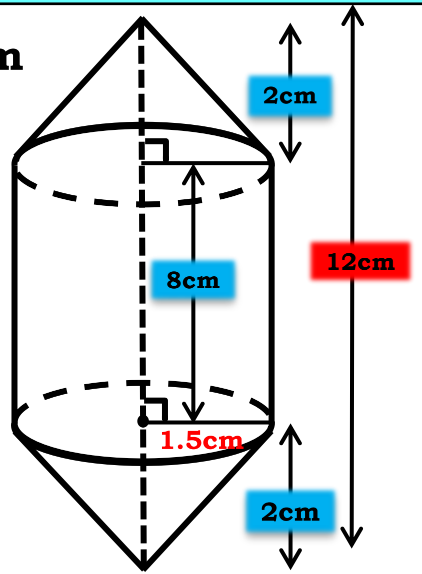
Sol. $\boxed{r = 1.5 \text{ cm}}$, $\boxed{h_1 = 8 \text{ cm}}$, $h_2 = 2$ cm

Vol. of cyl. (V1) $= \pi \boxed{r^2}\boxed{h_1}$

$= \pi \times \underline{1.5 \times 1.5} \times 8$

$= \pi \times 2.25 \times 8$

$\therefore$ Vol. of cyl. (V1) $= 18\pi \text{ cm}^3$

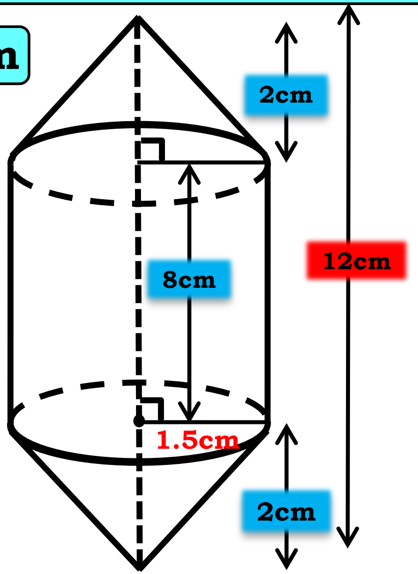


Q. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and in ... model to be nearly the same)

$\pi r^2 h_1$ $\quad$ $\frac{1}{3}\pi r^2 h_2$

Vol. of air in the model = Vol. of cyl. + 2 × Vol. of cone (V_2)

Sol. $\boxed{r = 1.5\text{ cm}}$, $h_1 = 8$ cm, $\boxed{h_2 = 2\text{ cm}}$



$$\text{Vol. of 2 cones } (V_2) = 2 \times \frac{1}{3}\pi \boxed{r^2}\boxed{h_2}$$

$$= 2 \times \frac{1}{3} \times \pi \times 1.5 \times 1.5 \times 2$$

$$= \cancel{2} \times \frac{1}{\cancel{3}} \times \pi \times \frac{\cancel{3}}{\cancel{2}} \times \frac{3}{\cancel{2}} \times \cancel{2}$$

$$= 3\pi \text{cm}^3$$

Q. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and in... model to be nearly the same)

$\pi r^2 h_1$

$\frac{1}{3}\pi r^2 h_2$

Vol. of air in the model = Vol. of cyl. + 2 × Vol. of cone (V_2)

Sol.

Vol. of air in model $= \boxed{V_1} + \boxed{V_2}$

$= 18\pi + 3\pi$

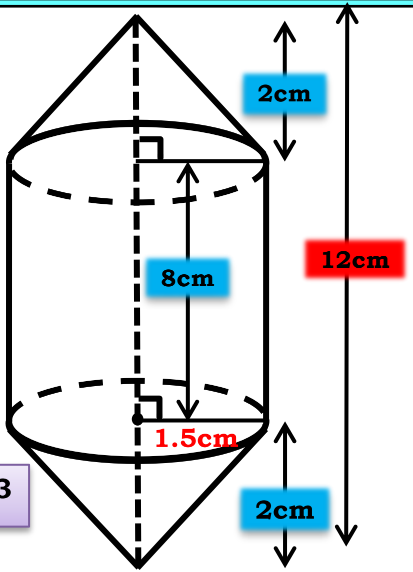
$= 21\pi$

$= \cancel{21}^{3} \times \frac{22}{\cancel{7}}$

$= 66 \text{ cm}^3$

$V_1 = 18\pi \text{ cm}^3$

$V_2 = 3\pi \text{ cm}^3$



∴ Volume of air in the model is 66 cm^3

Module 25

SURFACE AREAS AND VOLUMES

- **Sum based on Cylinder and Hemisphere**

Q. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be ... in 45 gulab jamuns, each sh... e a cylinder with t... herical ends with length 5 cm and ... r 2.8 cm.

$\pi r^2 h$

$\frac{2}{3} \times \pi r^3$

Volume of 1 'Gulab jamun' = Volume of cylinder (V_1) + 2 × Volume of hemisphere (V_2)

Sol.

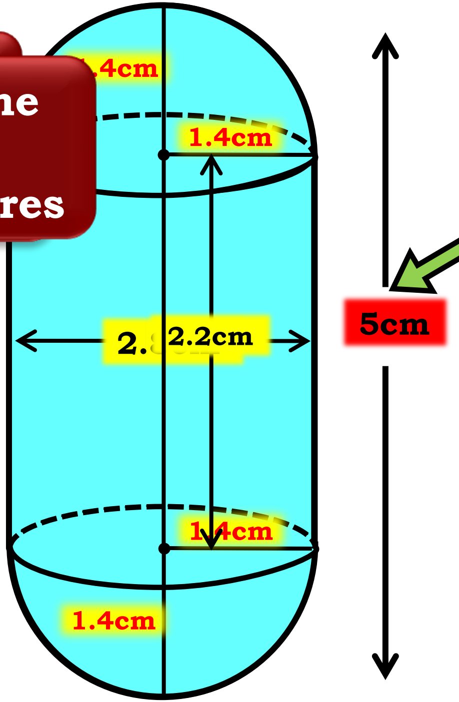
Each 'gulab jamun' is in the shape of a cylinder surmounted by 2 hemispheres

Height of cylinder

$= 5 - 1.4 - 1.4$

$= 5 - 2.8$

$= 2.2$ cm



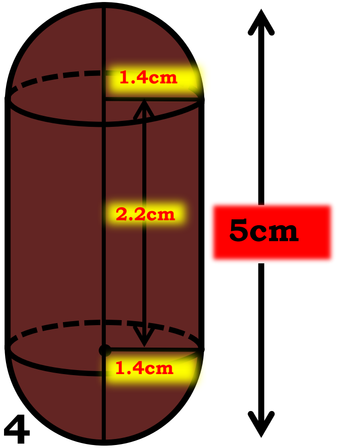
Q. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would b in 45 gulab jamuns, each sh e a cylinder with t herical ends with length 5 cm and d r 2.8 cm.

$\pi r^2 h$

$\frac{2}{3} \times \pi r^3$

Volume of 1 'Gulab jamun' = Volume of cylinder (V_1) + 2 × Volume of hemisphere (V_2)

Sol. $\boxed{r = 1.4 \text{ cm}}$, $\boxed{h = 2.2 \text{ cm}}$



$$\text{Volume of cylinder}(V_1) = \pi \boxed{r}^2 \boxed{h}$$

$$= \pi \times \underline{1.4 \times 1.4} \times 2.2$$

$$= \pi \times 1.96 \times 2.2$$

$$\therefore \text{Volume of cylinder}(V_1) = 4.312\pi \text{ cm}^3$$

$$\text{Volume of hemisphere} = \frac{2}{3} \times \pi \boxed{r}^3$$

$$= \frac{2}{3} \times \pi \times \underline{1.4 \times 1.4 \times 1.4}$$

$$= \frac{\underline{2} \times \pi \times \underline{2.744}}{3}$$

$$\therefore \text{Volume of hemisphere} = \frac{5.488\pi}{3}$$

Q. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

Volume of 1 'Gulab jamun' = Volume of cylinder (V_1) + 2 × Volume of hemisphere (V_2)

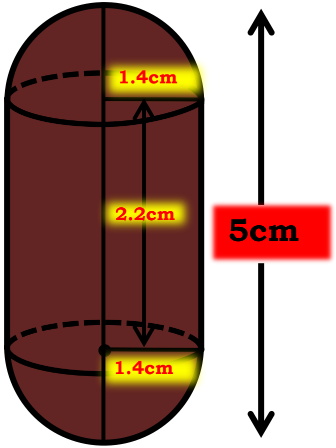
Sol.

$\therefore$ Volume of hemisphere $= \dfrac{5.488\pi}{3}$

Volume of 2 hemispheres (V_2) $= 2 \times \dfrac{5.488\pi}{3}$

$= \dfrac{10.976\pi}{3}$

$\therefore$ Volume of 2 hemispheres (V_2) $= 3.66\pi\ cm^3$



V (1 Gulab Jamun) $= \boxed{V_1} + \boxed{V_2}$

$= 4.312\pi + 3.66\pi$

$\boxed{V_1 = 4.312\pi}$

ulab Jamun) $= 7.972\pi$

Module 26

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Hemisphere (Part II)

Q. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

Sol. $\boxed{V\ (1\ \text{Gulab Jamun}) = 7.972\pi}$

$$V(45\ \text{Gulab Jamuns}) = 45 \times 7.972\pi$$

$$= 45 \times 7.97 \times \frac{22}{7}$$

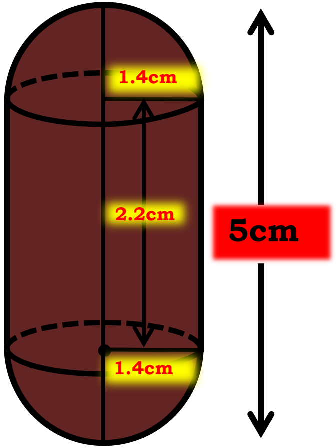
$$= 358.65 \times \frac{22}{7}$$

7890.3

$\therefore$ V(45 Gulab

Volume of 45 'gulab jamuns' =

45 × Volume of 1 gulab jamun



Q. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

Sol.

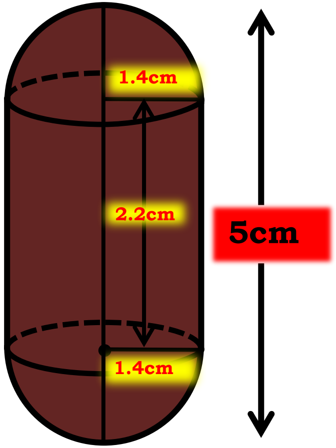
$$V(\text{45 Gulab Jamuns}) = 1127.14 \text{ cm}^3$$

$$30\% \text{ of } V(\text{45 Gulab Jamuns}) = \frac{3\cancel{0}}{10\cancel{0}} \times 1127.14$$

$$= \frac{3 \times 1127.14}{10}$$

$$= \frac{3381.42}{10}$$

$$= 338.14 \text{ cm}^3$$



$\therefore$ 30% of V(45 Gulab Jamuns) is 338.14 cm³

Module 27

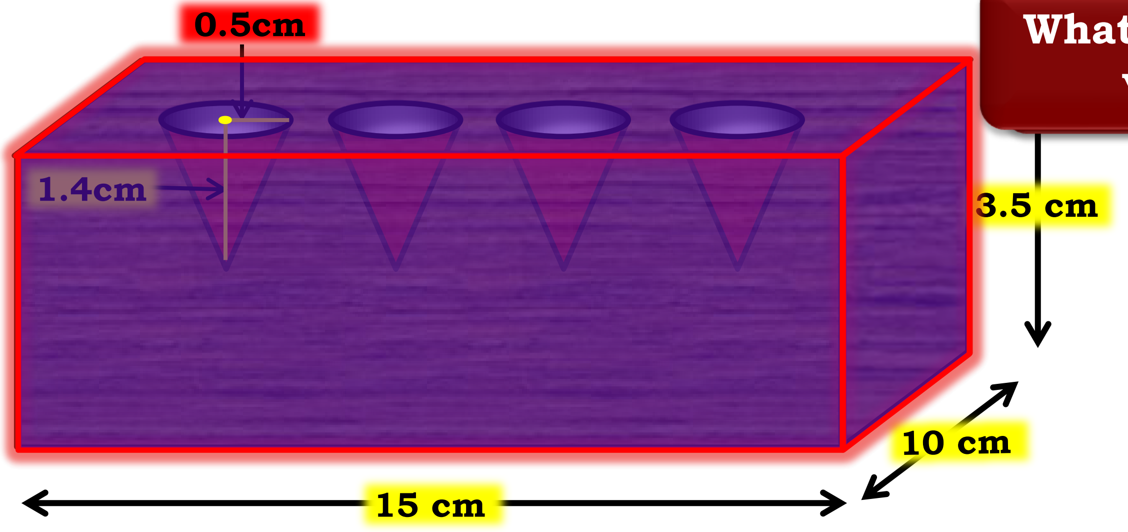
SURFACE AREAS AND VOLUMES

- Sum based on Cuboid and Cone

Q. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and depth is 1.4 cm. Fin $l\ b\ h_1$ lume of wo $\frac{1}{3}\pi r^2 h_2$ tire stand.

Volume of wood in the stand = Volume of cuboid (V_1) – Volume of 4 cones (V_2)

What is the formula to find volume of a cone?



Q. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and depth is 1.4 cm. Fin ume of wo tire stand.

$l\ b\ h_1$

$\frac{1}{3}\pi r^2 h_2$

Vol. of wood in the entire stand = Volume of cuboid (V_1) – Volume of 4 cones (V_2)

Sol. Vol. of the cuboid(V_1) $= \boxed{l}\,\boxed{b}\,\boxed{h_1}$

$= 15 \times \underline{10 \times 3.5}$

$= 15 \times 35$

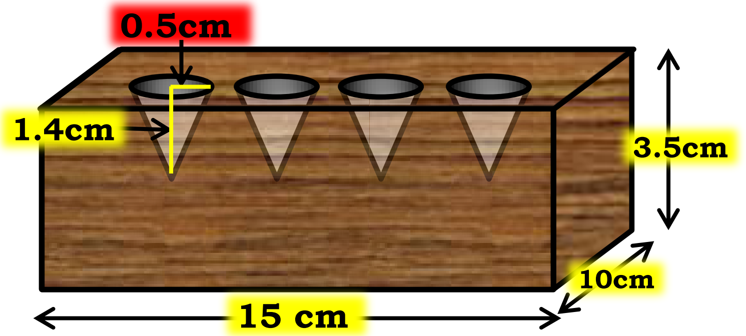
$\therefore$ Vol. of the cuboid(V_1) $= 525\ cm^3$

Vol. of the 4 cones(V_2) $= 4 \times \frac{1}{3}\pi \boxed{r^2}\boxed{h_2}$

$= 4 \times \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

$= 4 \times \frac{1}{3} \times \frac{22}{\cancel{7}} \times \frac{5}{10} \times \frac{5}{10} \times \frac{\cancel{14}^{\,2}}{10}$

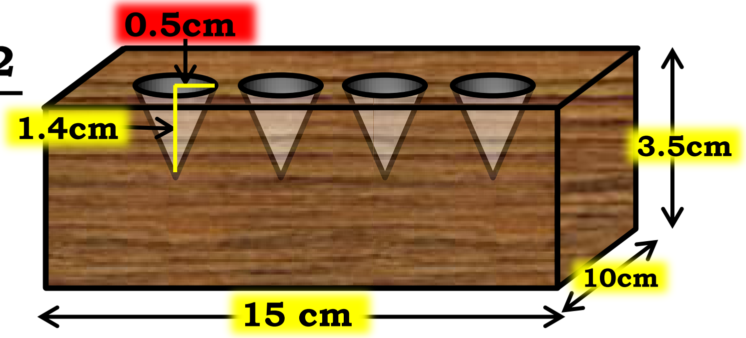
$\therefore$ Vol. of the 4 cones(V_2) $= \frac{4 \times 22 \times 5 \times 5 \times 2}{3 \times 1000}$



Q. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and depth is 1.4 cm. Find the volume of wood in the entire stand.

Vol. of wood in the entire stand = Volume of cuboid (V_1) – Volume of 4 cones (V_2)

Sol.



Vol. of the 4 cones(V_2) $= \dfrac{4 \times 22 \times 5 \times 5 \times 2}{3 \times 1000}$

$= \dfrac{44\cancel{00}}{3 \times 10\cancel{00}}$

$= \dfrac{44}{3 \times 10}$

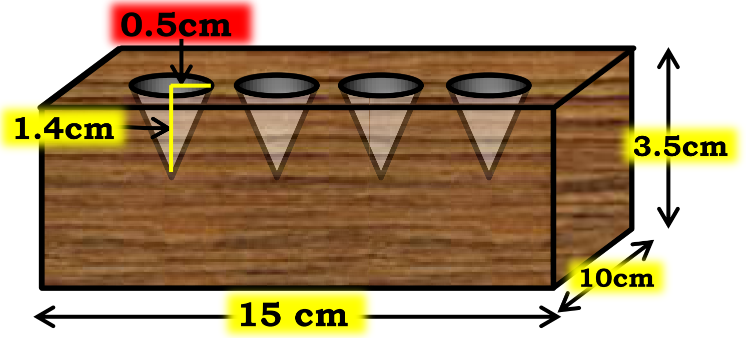
$= \dfrac{14.66}{10}$

$\therefore$ Vol. of the 4 cones(V_2) $=$ 1.47 cm^3

Q. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and depth is 1.4 cm. Find the volume of wood in the entire stand.

Vol. of wood in the entire stand = Volume of cuboid (V_1) – Volume of 4 cones (V_2)

Sol.



Volume of the wood in the entire stand $= \boxed{V_1} - \boxed{V_2}$

$\boxed{V_1 = 525 \text{ cm}^3}$ $= 525 - 1.47$

$\boxed{V_2 = 1.47 \text{ cm}^3}$ $= 523.53 \text{ cm}^3$

$\therefore$ Volume of the wood in the entire stand is 523.53 cm^3

Module 28

SURFACE AREAS AND VOLUMES

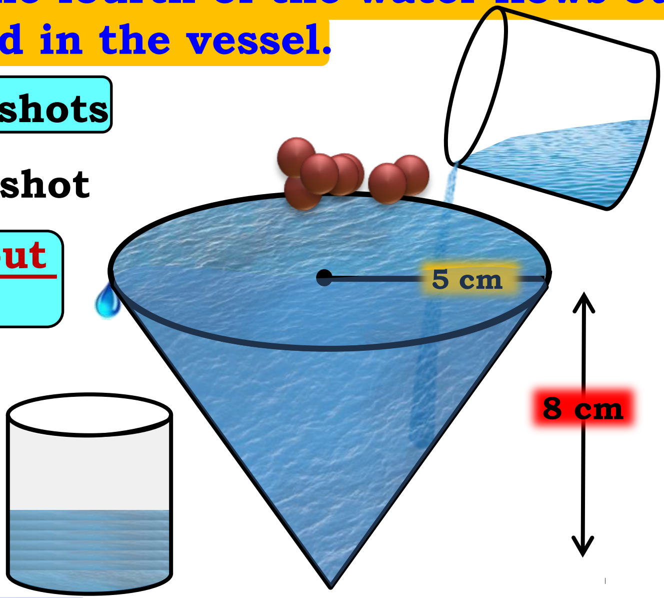
- Sum based on Cone and Sphere

Q. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Sol. Vol. of water flows out = $\boxed{\text{Vol. of } \mathbf{N} \text{ lead shots}}$

Vol. water flows out = $\mathbf{N} \times$ Vol. 1 lead shot

$$\boxed{\mathbf{N} = \frac{\text{Vol. of water flows out}}{\text{Vol. 1 lead shot}}}$$



Water flows out

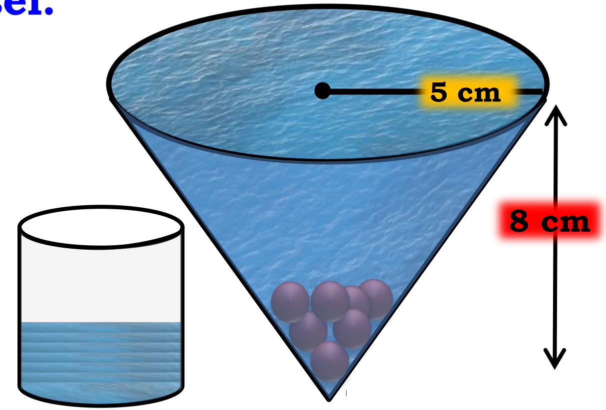
…ing to Arch…

lead shots

…,

Volume of water displaced = Volume of submerged body

Q. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.



Sol.

$$N = \frac{\text{Vol. of water flows out}}{\text{Vol. 1 lead shot}}$$

Vol. water flows out $= \frac{1}{4} \times \boxed{\text{Vol. of water in the cone}}$

$= \frac{1}{4} \times \frac{1}{3} \pi \boxed{r^2}\boxed{h}$

$= \frac{1}{4} \times \frac{1}{3} \times \pi \times 5 \times 5 \times \overset{2}{\cancel{8}}$

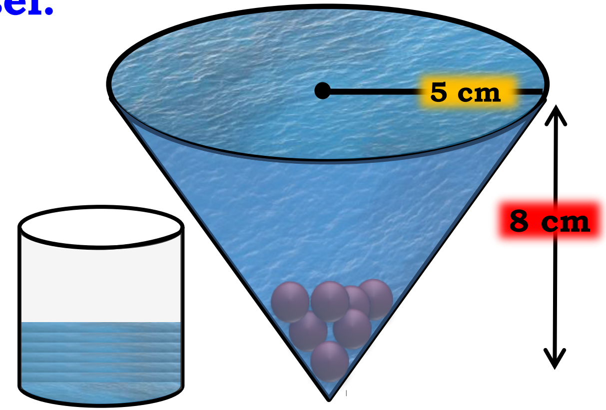
What is the formula to find volume of a cone?

$\frac{1}{3} \times \pi r^2 h$

$5 \times \pi$

$\therefore$ Vol. water flows out $= \frac{50\pi}{3}$ cm^3 ...(i)

Q. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.



Sol.

$$N = \frac{\text{Vol. of water flows out}}{\text{Vol. 1 lead shot}}$$

$$\text{Vol. water flows out} = \frac{50\pi}{3}\ \text{cm}^3 \qquad \ldots\text{(i)}$$

Lead shot is in the form of sphere,

$$\text{Vol. of one lead shot} = \frac{4}{3}\pi \boxed{r}^3$$

$$= \frac{4}{3} \times \pi \times (0.5)^3$$

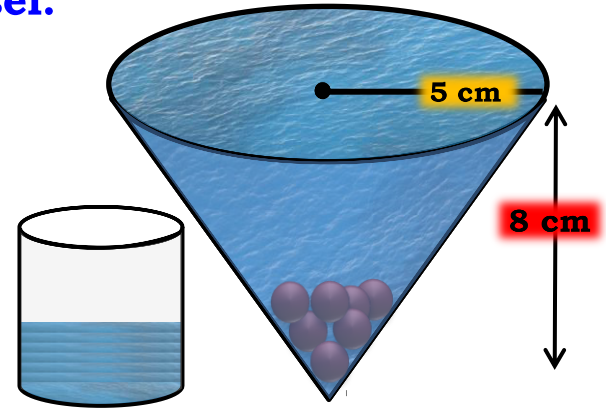
$$\times \pi \times 0.125$$

What is the formula to find volume of sphere?

$\frac{4}{3} \times \pi r^3$

$$\therefore \quad \text{Vol. of one lead shot} = \frac{0.5\pi}{3}\ \text{cm}^3 \qquad \ldots\text{(ii)}$$

Q. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.



Sol.

$$\boxed{N = \frac{\text{Vol. of water flows out}}{\text{Vol. 1 lead shot}}}$$

$$N = \frac{\text{Vol. of water flows out}}{\text{Vol. 1 lead shot}}$$

$$= \frac{50\pi}{3} \div \frac{0.5\pi}{3}$$

$$= \frac{50\cancel{\pi}}{\cancel{3}} \times \frac{\cancel{3}}{0.5\cancel{\pi}}$$

$$\boxed{\text{Vol. water flow out} = \frac{50\pi}{3} \quad ...(i)}$$

$$\boxed{\text{Vol. of 1 lead} = \frac{0.5\pi}{3} \quad ...(ii)}$$

$$= \frac{\cancel{50} \times 10}{\cancel{5}}$$

$$= 100$$

$\therefore$ **Number of lead shots is 100**

Module 29

SURFACE AREAS AND VOLUMES

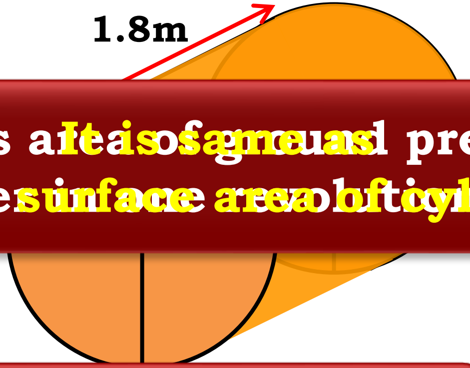
- **Sum based on Cylinder**

Q. A roller of diameter 0.9m and length 1.8m is used to press the ground. Find the area of ground pressed by it in 500 revolutions. ($\pi = 3.14$)

Sol. Radius $= \dfrac{0.9}{2} = 0.45$ m

What is shape of roller?

Cylinder



1.8m

0.45m

What is area of ground pressed by roller in one revolution?

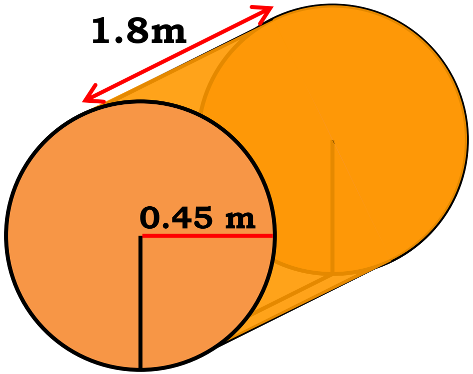
It is same as curved surface area of cylinder

Let us find area pressed by the roller in one revolution

CSA of roller

er

Q. A roller of diameter 0.9m and length 1.8m is used to press the ground. Find the area of ground pressed by it in 500 revolutions. (π = 3.14)



Sol. Radius = $\dfrac{0.9}{2}$ = 0.45 m

Area of ground pressed in 1 rev = CSA of roller

= $2\pi rh$

.14 × 0.45 × 1.8

× 0.81

What is the formula to find curved surface area of cylinder?

$2\pi rh$

$\therefore$ Area of ground pressed in 1 rev = 5.0868 m^2

Area of ground pressed in 500 revolutions

= 500 × Area of ground pressed in 1 rev

= 500 × 5.0868

= 2543.4 m^2

$\therefore$ Area of ground pressed in 500 revolutions is 2543.4 m^2

Thank You

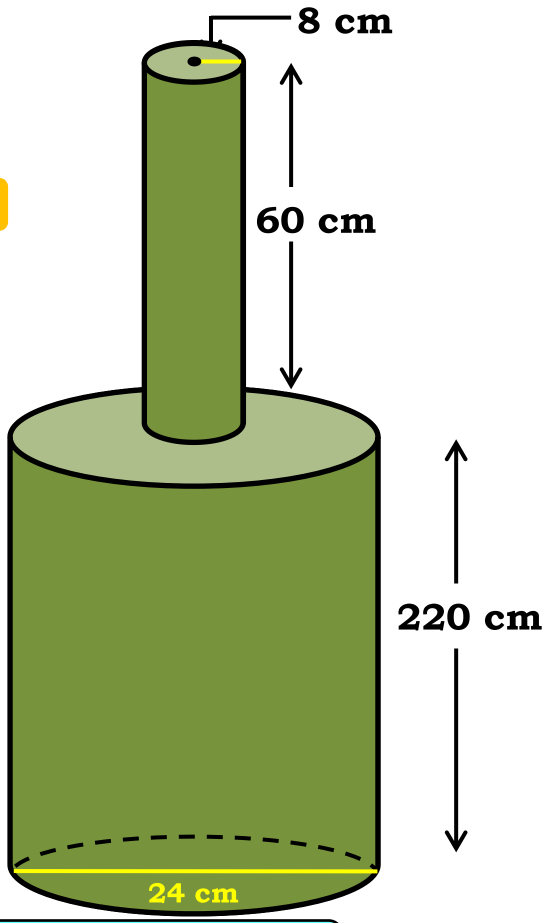
Module 30

SURFACE AREAS AND VOLUMES

- **Sum based on Cylinder**

Q. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8g mass.

(Use π = 3.14)



Sol.

Volume	Weight
1 cm^3 $\Rightarrow$	8 gm

For bigger cylinder,

$r_1 = \frac{24}{2} = 12$ cm

What is the formula to find volume of cylinder?

What does weight represent?

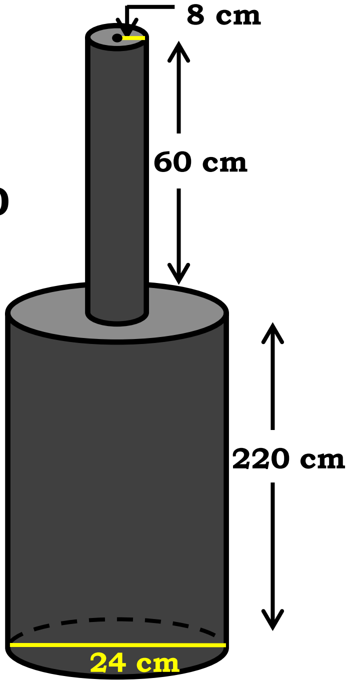
Let us calculate volume of the pole

$\pi r_1^2 h_1$ $\quad$ $\pi r_2^2 h_2$

V (pole) = Vol. of bigger cyl. (V_1) + Vol. of smaller cyl. (V_2)

Q. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 $\pi\, r_1^{\,2}\, h_1$ n has approximately $\pi\, r_2^{\,2}\, h_2$ (Use π = 3.14)

V (pole) = Vol. of bigger cyl. (V_1) + Vol. of smaller cyl. (V_2)



Sol. $\quad r_1 = \dfrac{24}{2} = \boxed{12\text{ cm}}$

Vol. of the bigger cyl. (V_1) $= \pi r_1^{\,2} h_1$

$= \pi \times \underline{12 \times 12} \times 220$

$= \pi \times 144 \times 220$

$\therefore$ Vol. of the bigger cyl. (V_1) $= 31680\pi\text{ cm}^3$

Vol. of the smaller cyl. (V_2) $= \pi r_2^{\,2} h_2$

$= \pi \times \underline{8 \times 8} \times 60$

$= \pi \times 64 \times 60$

$\therefore$ Vol. of the smaller cyl. (V_2) $= 3840\pi\text{ cm}^3$

Q. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8g mass. (Use $\pi = 3.14$)

V (pole) = Vol. of bigger cyl. (V_1) + Vol. of smaller cyl. (V_2)

Sol. Volume of the pole $= \boxed{V_1} + \boxed{V_2}$

$= 31680\pi + 3840\pi$

∴ Volume of the pole $= 35520\pi\ cm^3$

Mass of $1cm^3$ iron = 8gm

∴ Mass of the pole $= 8 \times 35520\pi$

Vol. of bigger cyl. $= 31680\pi$

$= 8 \times 35520$ ...

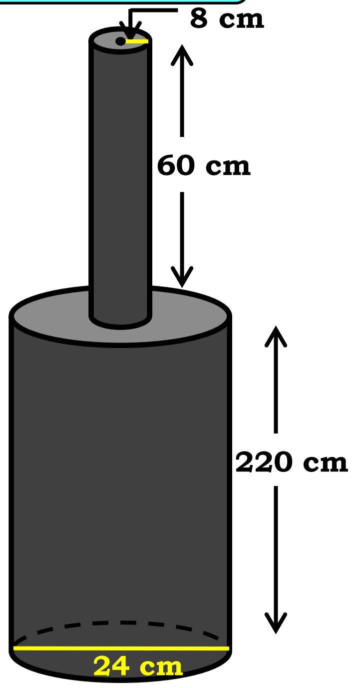
Vol. of smaller cyl. $= 3840\pi$

÷ 1000kg

= 892.2624 kg

Volume	Weight
1 cm^3	8 gm
$35520\pi\ cm^3$	?

∴ Mass of the pole is 892.26 kg.



Module 31

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder

Q. An ink container of cylindrical shape is filled with ink upto 91%. Ball pen refills of length 12 cm and inner diameter 2 mm are filled with ink upto 84 %. If the height and radius of the ink container are 14 cm and 6 cm resp., find the number of refills that can be filled with this ink.

$$\text{No. of refills} = \frac{\text{Volume of ink inside container}}{\text{Volume of ink inside refill}}$$

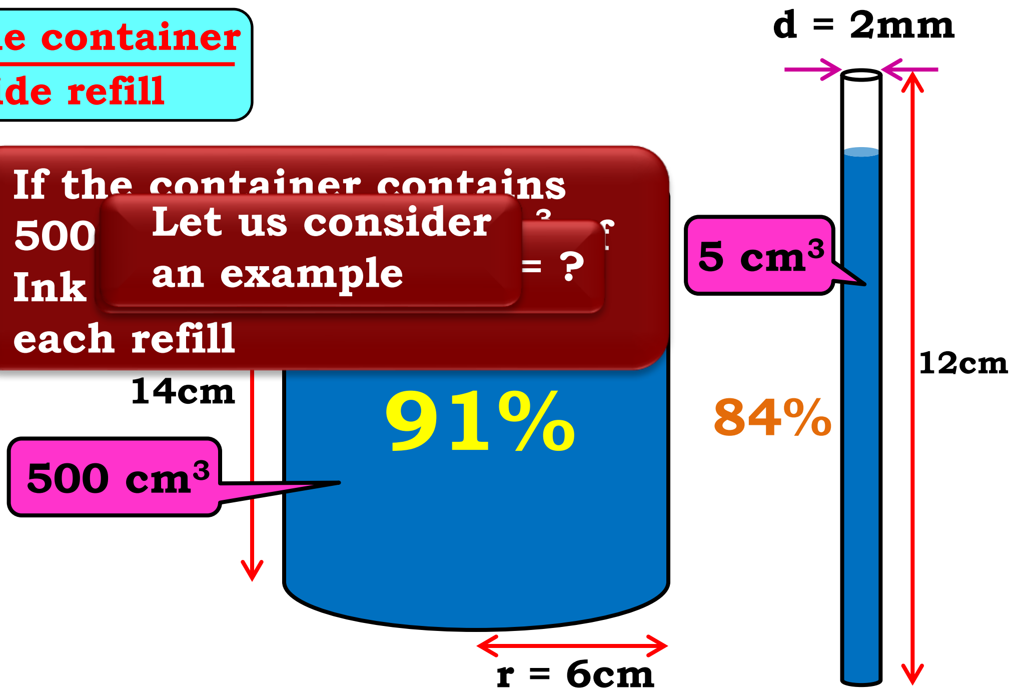
Sol.

$$\text{No. of refills} = \frac{500}{5} = 100$$

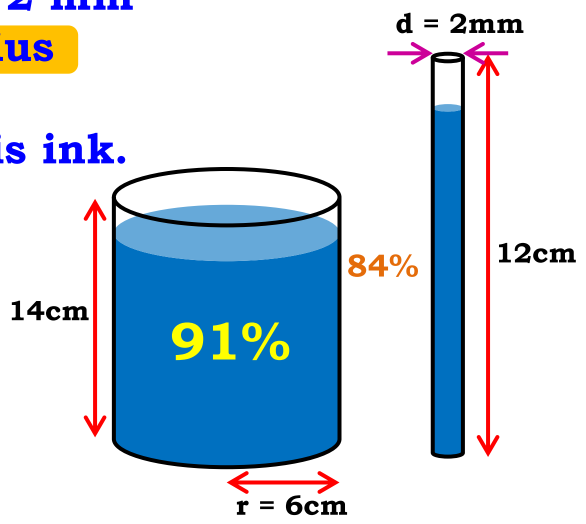
If the container contains
500
Ink
each refill

Let us consider
an example

= ?



Q. An ink container of cylindrical shape is filled with ink upto 91%. Ball pen refills of length 12 cm and inner diameter 2 mm are filled with ink upto 84 %. If the height and radius of the ink container are 14 cm and 6 cm resp., find the number of refills that can be filled with this ink.



$$\text{No. of refills} = \frac{\text{Volume of ink inside container}}{\text{Volume of ink inside refill}}$$

Sol. Volume of container $= \pi \boxed{r}^2 \boxed{h}$

$= \pi \times (6^2) \times 14$

$= \pi \times 36 \times 14$

$= \boxed{504\pi \text{ cm}^3}$

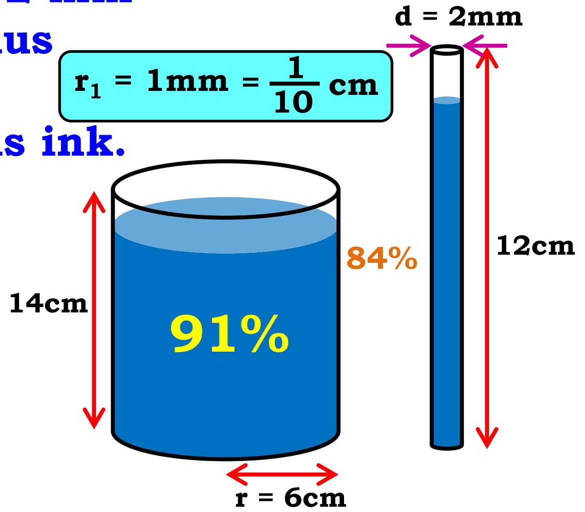
What is the formula to find volume of cylinder?

le container

$= 91\% \times \boxed{\text{Volume of container}}$

$$= \frac{91}{100} \times 504\pi \text{ cm}^3$$

Q. An ink container of cylindrical shape is filled with ink upto 91%. Ball pen refills of length 12 cm and inner diameter 2 mm are filled with ink upto 84 %. If the height and radius of the ink container are 14 cm and 6 cm resp., find the number of refills that can be filled with this ink.



$$\text{No. of refills} = \frac{\text{Volume of ink inside container}}{\text{Volume of ink inside refill}}$$

Sol. Volume of refill $= \pi \boxed{r_1}^2 h_1$

$$= \pi \times \left(\frac{1}{10}\right)^2 \times 12$$

$$= \pi \times \frac{1}{100} \times 12$$

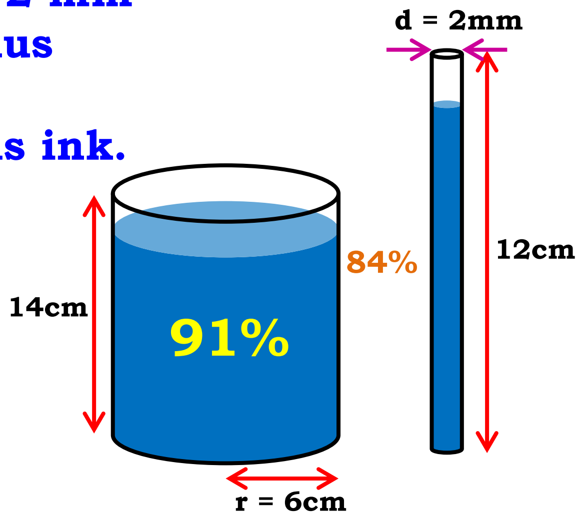
$$\therefore \quad \text{Volume of refill} = \frac{12\pi}{100} \text{ cm}^3$$

$$\text{Vol. of ink inside container} = \frac{91}{100} \times 504\,\pi$$

Vol. of ink inside refill $= 84\% \times \boxed{\text{Volume of refill}}$

$$= \frac{84}{100} \times \frac{12\pi}{100} \text{ cm}^3$$

Q. An ink container of cylindrical shape is filled with ink upto 91%. Ball pen refills of length 12 cm and inner diameter 2 mm are filled with ink upto 84 %. If the height and radius of the ink container are 14 cm and 6 cm resp., find the number of refills that can be filled with this ink.



$$\text{No. of refills} = \frac{\text{Volume of ink inside container}}{\text{Volume of ink inside refill}}$$

Sol.

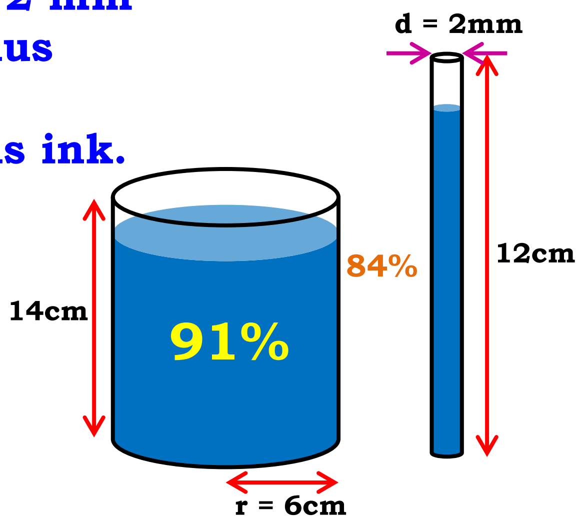
$$\text{No. of refills} = \frac{\text{Volume of ink inside container}}{\text{Volume of ink inside refill}}$$

$$= \left(\frac{91}{100} \times 504\,\pi\right) \div \left(\frac{84}{100} \times \frac{12\,\pi}{100}\right)$$

$$\text{Vol. of ink inside container} = \frac{91}{100} \times 504\,\pi$$

$$\text{Vol. of ink inside refill} = \frac{84}{100} \times \frac{12\,\pi}{100}$$

Q. An ink container of cylindrical shape is filled with ink upto 91%. Ball pen refills of length 12 cm and inner diameter 2 mm are filled with ink upto 84 %. If the height and radius of the ink container are 14 cm and 6 cm resp., find the number of refills that can be filled with this ink.



$$\text{No. of refills} = \frac{\text{Volume of ink inside container}}{\text{Volume of ink inside refill}}$$

Sol.

$$= \left(\frac{91}{100} \times 504\,\pi\right) \div \left(\frac{84}{100} \times \frac{12\,\pi}{100}\right)$$

$$= \left(\frac{91}{\cancel{100}} \times \cancelto{42}{504}\,\cancel{\pi}\right) \times \left(\frac{\cancel{100}}{\cancelto{2}{84}} \times \frac{\cancelto{50}{100}}{\cancel{12}\,\cancel{\pi}}\right)$$

$$= 91 \times 50$$

$$= 4550$$

$\therefore$ **Number of refills that can be filled are 4550**

Module 32

SURFACE AREAS AND VOLUMES

- **Sum based on Cone, Cylinder and Hemisphere**

Q. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

$\pi r^2 h_1$

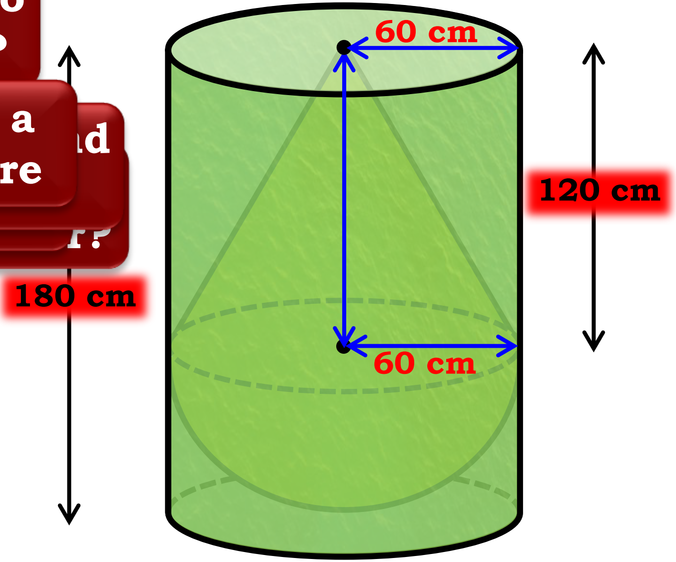
Vol. of cone + Vol. of hemisphere

$\frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3$

Volume of water left = Volume of cylinder (V_1) – Volume of solid (V_2)

What is the formula to find volume of cone?

The solid consists of a cone and a hemisphere



Q. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

$\pi r^2 h_1$ $\qquad$ $\frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3$

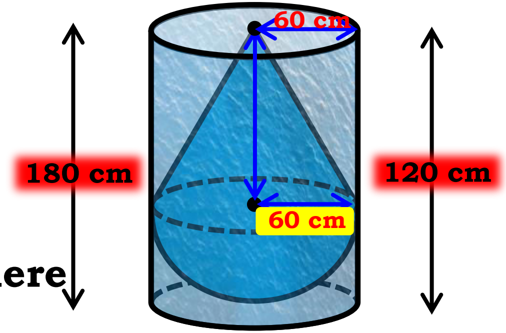
Volume of water left = Volume of cylinder (V_1) – Volume of solid (V_2)

Sol. Vol. of cylinder (V_1) = $\pi \boxed{r^2}\boxed{h_1}$

$= \pi \times (60)^2 \times 180$

$= \pi \times 3600 \times 180$

$\therefore$ Vol. of cylinder (V_1) = $6,48,000\pi$ cm^3



Vol. of solid (V_2) = Vol. of cone + Vol. of hemisphere

$$= \frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi \boxed{r^2}(\boxed{h_2} + 2\boxed{r})$$

$$= \frac{1}{3}\pi \times (60)^2 \times [120 + (2 \times 60)]$$

Q. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

$\pi r^2 h_1$

$\frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3$

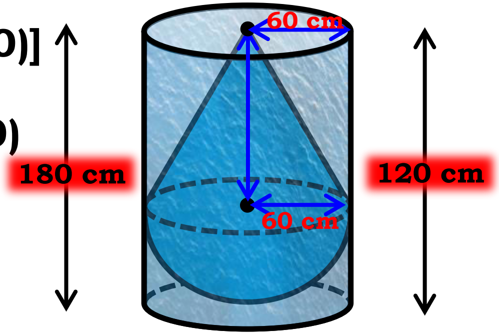
Volume of water left = Volume of cylinder (V_1) – Volume of solid (V_2)

Sol. Vol. of solid $(V_2) = \frac{1}{3}\pi \times (60)^2 \times [120 + (2 \times 60)]$

$= \frac{1}{3}\pi \times 3600 \times (120 + 120)$

$= \frac{1}{\cancel{3}}\pi \times \overset{1200}{\cancel{3600}} \times 240$

$= \pi \times 1200 \times 240$



$\therefore$ $\boxed{\text{Vol. of solid } (V_2) = 288000\,\pi \text{ cm}^3}$

$\boxed{V_1 = 648000\pi}$

Vol. of water left $= \boxed{V_1} - \boxed{V_2}$

$= 6{,}48{,}000\pi - 2{,}88{,}000\pi$

$\therefore$ Vol. of water left $= 3{,}60{,}000\pi \text{ cm}^3$

Q. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Volume of water left = Volume of cylinder (V_1) – Volume of solid (V_2)

Sol. Vol. of water left = $3,60,000\pi$ cm^3

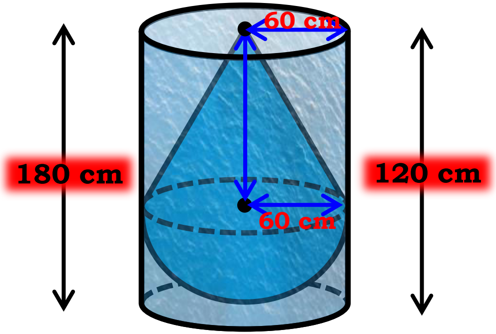
$$= \frac{3,6\cancel{0,000}}{100\cancel{0000}}\pi \text{ m}^3$$

$$= \frac{36}{100}\pi$$

$$= \frac{36}{100} \times \frac{22}{7}$$

$$= \frac{\boxed{36 \times 22}}{100 \times 7} = \frac{792}{100 \times 7}$$

$$= \frac{113.14}{100} = 1.131 \text{ m}^3$$



$\therefore$ **Volume of water left in the cylinder is 1.131 m^3**

Module 33

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Sphere

Q. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child fi... its ...ume to be 345 cm^3. Check whethe... ...orrect, taking ...s the inside measurements. (π = 3.1...

$\pi r_1^2 h$

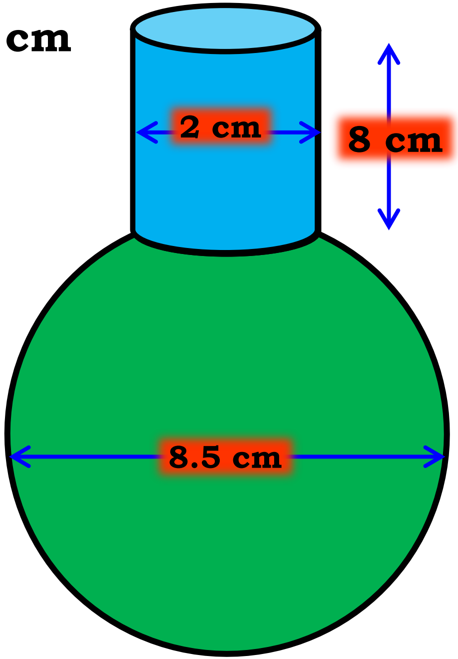
$\frac{4}{3} \times \pi r_2^3$

Volume of glass vessel = Vol. of cylinder (V_1) + Vol. of sphere (V_2)

Sol. Radius of cylinder $(r_1) = \frac{2}{2} = 1$ cm

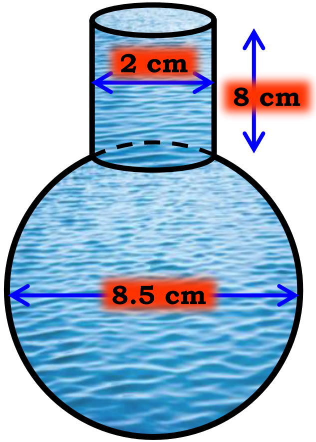
Radius of sphere $(r_2) = \frac{8.5}{2}$

To check whether the child is correct, let us calculate volume of the glass vessel.



Q. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whethe $\pi r_1^2 h$ orrect, taking $\frac{4}{3} \times \pi r_2^3$ s the inside measurements. (π = 3.1

Volume of glass vessel = Vol. of cylinder (V_1) + Vol. of sphere (V_2)



Sol. Radius of cylinder (r_1) $= \frac{2}{2} = \boxed{1 \text{ cm}}$

Radius of sphere (r_2) $= \frac{8.5}{2}$

Vol. of cylinder (V_1) $= \pi \boxed{r_1}^2 \boxed{h}$

$= \pi \times (1)^2 \times 8$

$\therefore$ Vol. of cylinder (V_1) $= 8\pi$ cm^3

Vol. of sphere (V_2) $= \frac{4}{3} \times \pi \boxed{r_2}^3$

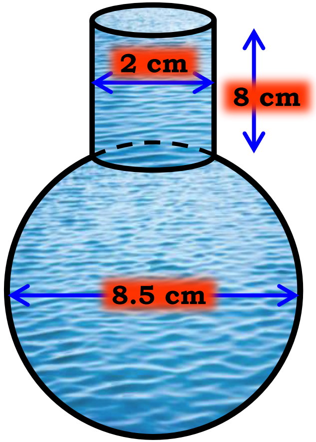
Vol. of sphere (V_2) $= \frac{4}{3} \times \pi \times \frac{8.5}{2} \times \frac{8.5}{2} \times \frac{8.5}{2}$

Q. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whethe orrect, taking s the inside measurements. (π = 3.1

$\pi r^2 h$

$\frac{4}{3} \times \pi r^3$

Volume of glass vessel = Vol. of cylinder (V_1) + Vol. of sphere (V_2)



Sol. Vol. of sphere (V_2) $= \frac{4}{3} \times \pi \times \frac{8.5}{2} \times \frac{8.5}{2} \times \frac{8.5}{2}$

$$V_1 = 8\pi$$

$$= \frac{\cancel{4}^{2}}{3} \times \pi \times \frac{85}{\cancel{20}_{10}} \times \frac{85}{\cancel{20}_{10}} \times \frac{85}{20}$$

$$= \frac{\pi \times 85 \times 85 \times 85}{6 \times 1000}$$

$$= \frac{614125 \times \pi}{6 \times 1000}$$

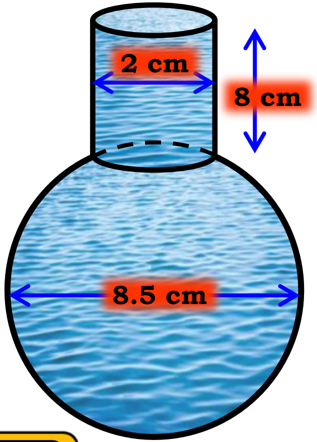
$$= \frac{102354.16\,\pi}{1000}$$

$\therefore$ Vol. of sphere (V_2) $= 102.354\pi$ cm^3

Q. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements. ($\pi = 3.14$)

Volume of glass vessel = Vol. of cylinder (V_1) + Vol. of sphere (V_2)

Sol.



Vol. of glass vessel $= \boxed{V_1} + \boxed{V_2}$

$= 8\pi + 102.354\pi$

$= 110.354\,\boxed{\pi}\,\text{cm}^3$

$= 110.354 \times 3.14$

$V_1 = 8\pi$

$V_2 = 102.354\pi$

$\therefore$ Vol. of glass vessel $= 346.51$ cm^3

A child finds the volume to be 345 cm^3

$\therefore$ The volume found by the child, i.e., 345 cm^3 is incorrect.

Module 34

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder, Hemisphere and Cone

Q. A toy is combination of a cylinder, hemisphere and a cone, each with radius 10 cm. Height of the conical part is 10 cm and total height is 60 cm. Fi tota ea and the toy.

$(\pi = 3.14, \sqrt{2} = 1.41)$

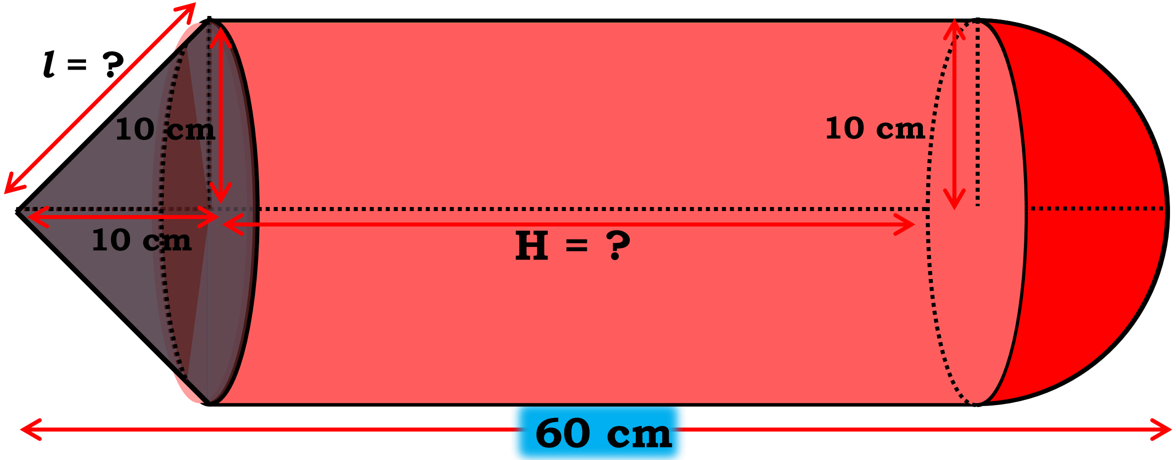
$\pi r l$ ✓ ?

$2 \pi r H$ ✓ ?

$2 \pi r^2$ ✓

TSA of toy = CSA of cone + CSA of cylinder + CSA of hemisphere

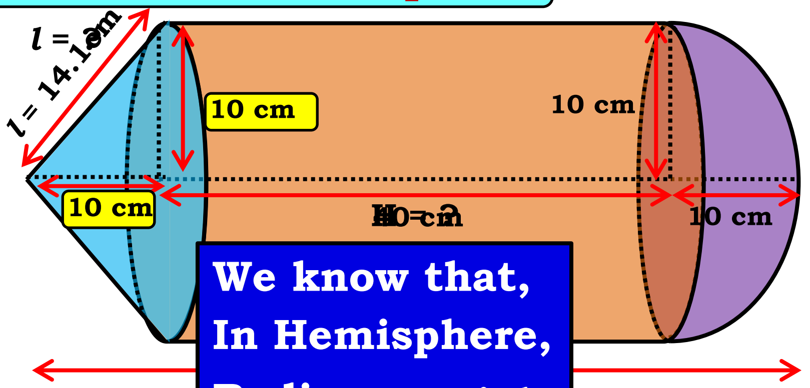
What is formula for CSA of hemisphere ?



Q. A toy is combination of a cylinder, hemisphere and a cone, each with radius 10 cm. Height of the conical part is 10 cm and total height is 60 cm. Find total surface area and ... the toy.
(π = 3.14, $\sqrt{2}$ = 1.41)

$\pi r l$ $\quad$ $2\pi r H$ $\quad$ $2\pi r^2$

TSA of toy = CSA of cone + CSA of cylinder + CSA of hemisphere



Sol. Slant height (l) $= \sqrt{r^2 + h^2}$

$= \sqrt{(10)^2 + (10)^2}$

$= \sqrt{100 + 100}$

$= \sqrt{200}$

What is the formula to find

$l = \sqrt{r^2 + h^2}$

Let us find height of cylinder

... ht of cone

... linder

$\therefore \quad l = 14.1$ cm

We know that,
In Hemisphere,
Radius = Height

Height of cylinder = Total height – height of cone – height of hemisphere

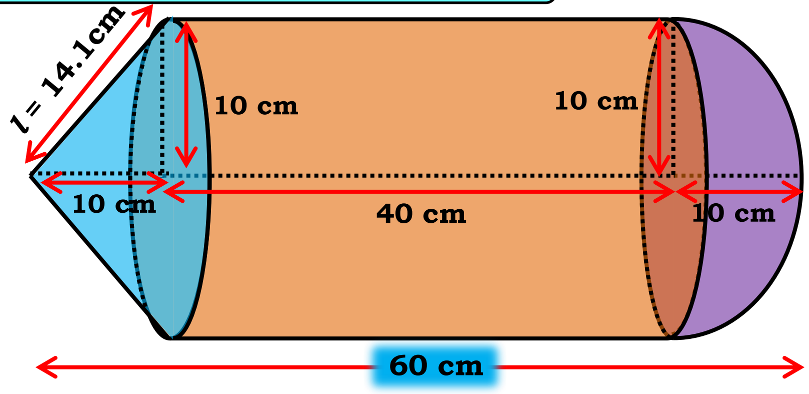
= 60 – 10 – 10

$\therefore$ Height of cylinder = 40 cm

Q. A toy is combination of a cylinder, hemisphere and a cone, each with radius 10 cm. Height of the conical part is 10 cm and total height is 60 cm. Fi[...] tota[...] ea and [...] the toy.
($\pi = 3.14$, $\sqrt{2} = 1.41$)

$\pi r l$ $\quad 2 \pi r H$ $\quad 2 \pi r^2$

TSA of toy = CSA of cone + CSA of cylinder + CSA of hemisphere



Sol. TSA of the toy

= CSA of cone

+ CSA of cylinder

+ CSA of hemisphere

$= \pi r l + 2\pi r H + 2\pi r^2$

$= \pi r(l + 2H + 2r)$

$= 3.14 \times 10 \times (14.1 + 2 \times 40 + 2 \times 10)$

$= 31.4 \times (14.1 + 80 + 20)$

$= 31.4 \times 114.1$

$= 3582.74 \text{ cm}^2$

$r = 10\text{cm}$

$l = 14.1\text{cm}$

$H = 40\text{cm}$

$\therefore$ Total surface area of toy is 358[...]

Module 35

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder, Hemisphere and Cone (Part II)

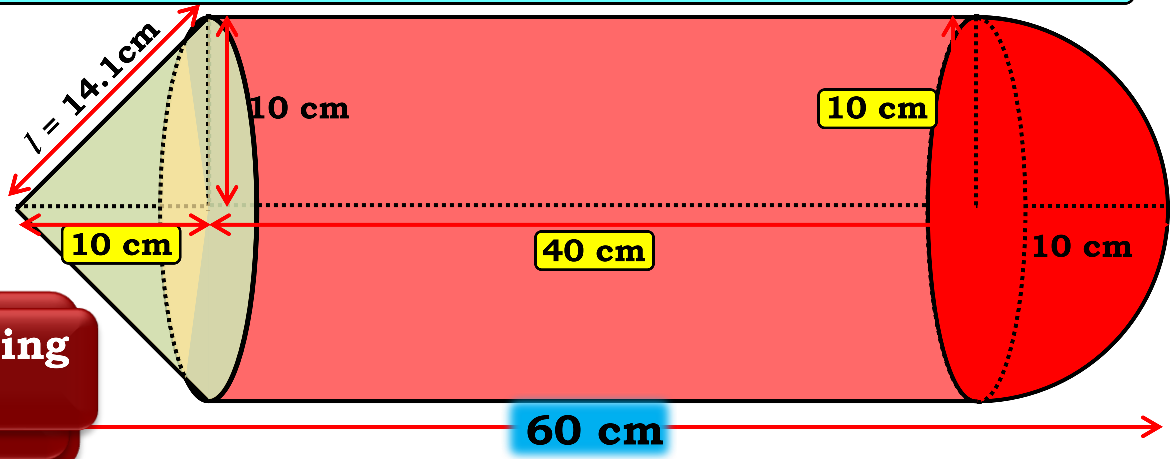
Q. A toy is combination of a cylinder, hemisphere and a cone, each with radius 10 cm. Height of the conical part is 10 cm and total height is 60 c the total s rea and volume y.

$(\pi = 3.14$, $\sqrt{2}$

$\frac{1}{3}\pi r^2 h$ ✓✓ $\pi r^2 H$ ✓✓ $\frac{2}{3}\pi r^3$ ✓

Volume of toy = Volume of cone + Volume of cylinder + Volume of hemisphere

Sol.



Volume of toy

= Volume of cone

+ Volume of cylinder

+ Volume of hemisphere

What is formula for finding volume of cone ?

$$= 3.14 \times 10 \times 10 \times \left[\frac{1}{3} \times 10 + 40 + \frac{2}{3} \times 10\right]$$

Q. A toy is combination of a cylinder, hemisphere and a cone, each with radius 10 cm. Height of the conical part is 10 cm and total height is 60 cm. Find the total surface area and volume of the toy.

Volume of toy = Volume of cone + Volume of cylinder + Volume of hemisphere

Sol.

$$= 3.14 \times 10 \times 10 \times \left[\frac{1}{3} \times 10 + 40 + \frac{2}{3} \times 10\right]$$

$$= \underline{3.14 \times 10 \times 10} \times \left[\frac{10}{\underline{3}} + 40 + \frac{20}{\underline{3}}\right]$$

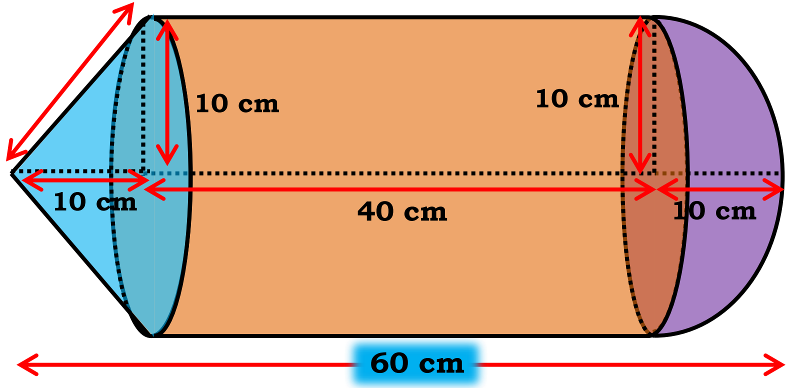
$$= 314 \times \left[\cancel{\frac{30}{3}}^{10} + 40\right]$$

$$= 314 \times (\underline{10 + 40})$$

$$= 314 \times 50$$

$$= 15700 \text{ cm}^3$$

$\therefore$ **Volume of toy is 15700 cm³**



Module 36

SURFACE AREAS AND VOLUMES

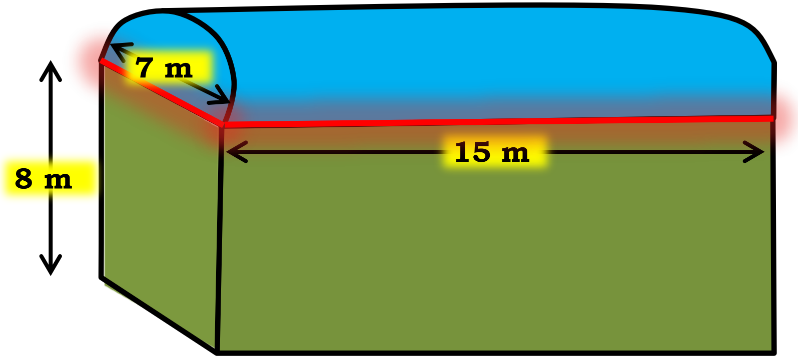
- Sum based on Cylinder and Cuboid

Q. Shanta runs an industry in a shed which is in the shape of a cuboid surmounted by a half cylinder. If the base of the shed is of dimensions 15m × 7m, and the height of the cuboidal portion is 8m, find the volume of air that the shed can hold. Further, suppose the machinery in the shed occupies a total space of 300m^3, and there are 20 workers, each of whom occupy ab 8 m n an average. Then, how much air is in the sl ke

$l\,b\,h_1$ $\quad \frac{1}{2}\pi r^2 h_2$

Volume of air that the shed can hold = Vol. of cuboid + Vol. of half cylinder

Sol. $l = 15$ m, b = 7 m, $h_1 = 8$ m



Height of cylinder(h_2) = l = 15 m

Diameter of cylinder = b = 7 m

$\therefore$ Radius (r) = $\frac{7}{2}$ m

W Diameter of cylinder = breadth of cuboid

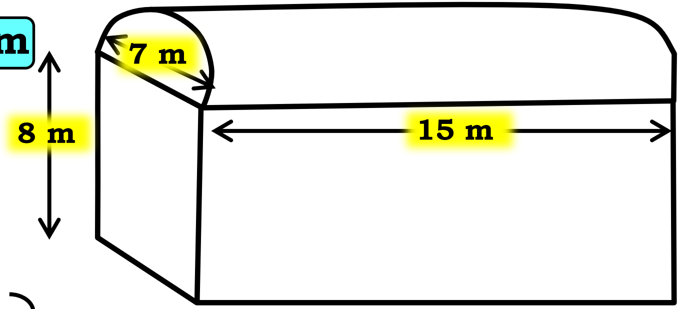
Q. Shanta runs an industry in a shed which is in the shape of a cuboid surmounted by a half cylinder. If the base of the shed is of dimensions 15m × 7m, and the height of the cuboidal portion is 8m, find the volume of air that the shed can hold. Further, suppose the machinery in the shed occupies a total space of 300m^3, and there are 20 workers, each of whom occupy ab [illegible] 8 m [illegible] n an average. Then, how much air is in the sl [illegible] ke [illegible]

$l\,b\,h_1$

$\frac{1}{2}\pi r^2 h_2$

Volume of air that the shed can hold = Vol. of cuboid + Vol. of half cylinder

Sol. $l = 15$ m, $b = 7$ m, $h_1 = 8$ m, $r = \frac{7}{2}$ m, $h_2 = 15$ m



Vol. of air = Vol. of cuboid + Vol. of half cylinder

$$= (\boxed{l} \times \boxed{b} \times \boxed{h_1}) + \left(\frac{1}{2} \times \boxed{\pi} \times \boxed{r^2} \times \boxed{h_2}\right)$$

$$= (\underline{15 \times 7 \times 8}) + \left(\frac{1}{\cancel{2}} \times \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{2} \times \frac{7}{2} \times 15\right)$$

$$= 840 + \left(\frac{11 \times 7 \times 15}{2 \times 2}\right)$$

Q. Shanta runs an industry in a shed which is in the shape of a cuboid surmounted by a half cylinder. If the base of the shed is of dimensions 15m × 7m, and the height of the cuboidal portion is 8m, find the volume of air that the shed can hold. Further, suppose the machinery in the shed occupies a total space of 300m³, and there are 20 workers, each of whom occupy about 0.08 m³ space on an average. Then, how much air is in the shed? (Take π = 22/7)

Volume of air that the shed can hold = Vol. of cuboid + Vol. of half cylinder

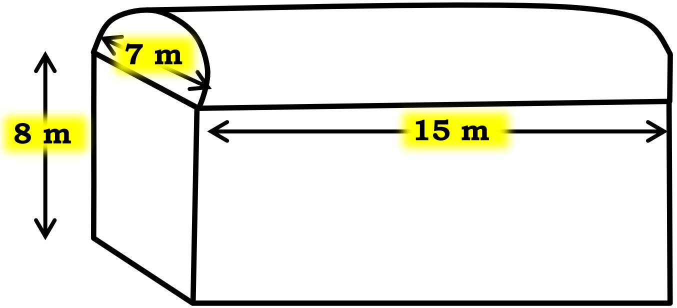
Sol.

$$= 840 + \left(\frac{11 \times 7 \times 15}{2 \times 2}\right)$$

$$= 840 + \left(\frac{1155}{4}\right)$$

$$= 840 + 288.75$$

$$= 1128.75 \text{ m}^3$$

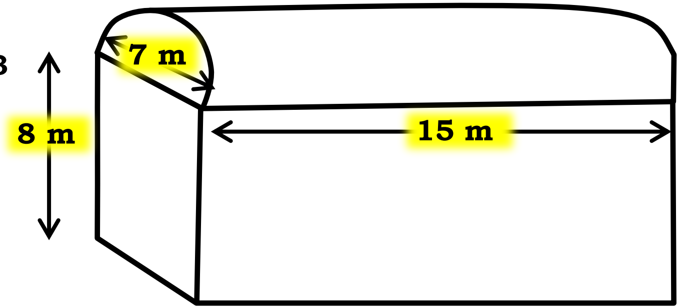


∴ **Volume of air that the shed can hold is 1128.75 m³**

Q. Shanta runs an industry in a shed which is in the shape of a cuboid surmounted by a half cylinder. If the base of the shed is of dimensions 15m × 7m, and the height of the cuboidal portion is 8m, find the volume of air that the shed can hold. Further, suppose the machinery in the shed occupies a total space of 300m³, and there are 20 workers, each of whom occupy about 0.08 m³ space on an average. Then, how much air is in the shed? (Take π = 22/7)

Sol.

Total space occupied by the machinery = $300\ m^3$



The space occupied by 1 worker = $0.08\ m^3$

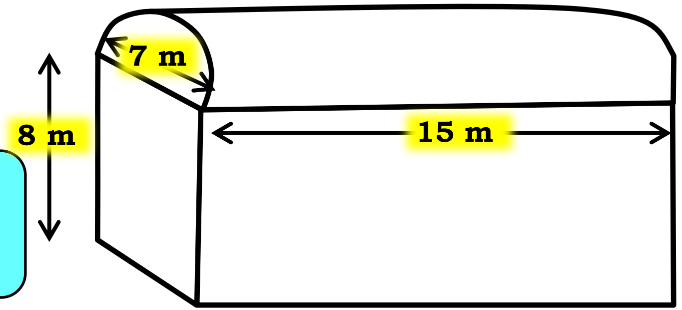
∴ Total space occupied by 20 workers = 20×0.08

$= 1.6\ m^3$

Volume of air that the shed can hold is 1128.75 m³

Q. Shanta runs an industry in a shed which is in the shape of a cuboid surmounted by a half cylinder. If the base of the shed is of dimensions 15m × 7m, and the height of the cuboidal portion is 8m, find the volume of air that the shed can hold. Further, suppose the machinery in the shed occupies a total space of 300m^3, and there are 20 workers, each of whom occupy about 0.08 m^3 space on an average. Then, how much air is in the shed? (Take π = 22/7)

Sol.



Volume of the air, when there are machinery and workers

= Vol. of air – Total space occupied by the machinery – Total space occupied by the workers

= 1128.75 – 300 – 1.6

Volume of air that the shed can hold is 1128.75 m^3

= 827.15 m^3

Total space occupied by the machinery = 300 m^3

∴ Air in the shed is 827.

Total space occupied by the workers = 1.60

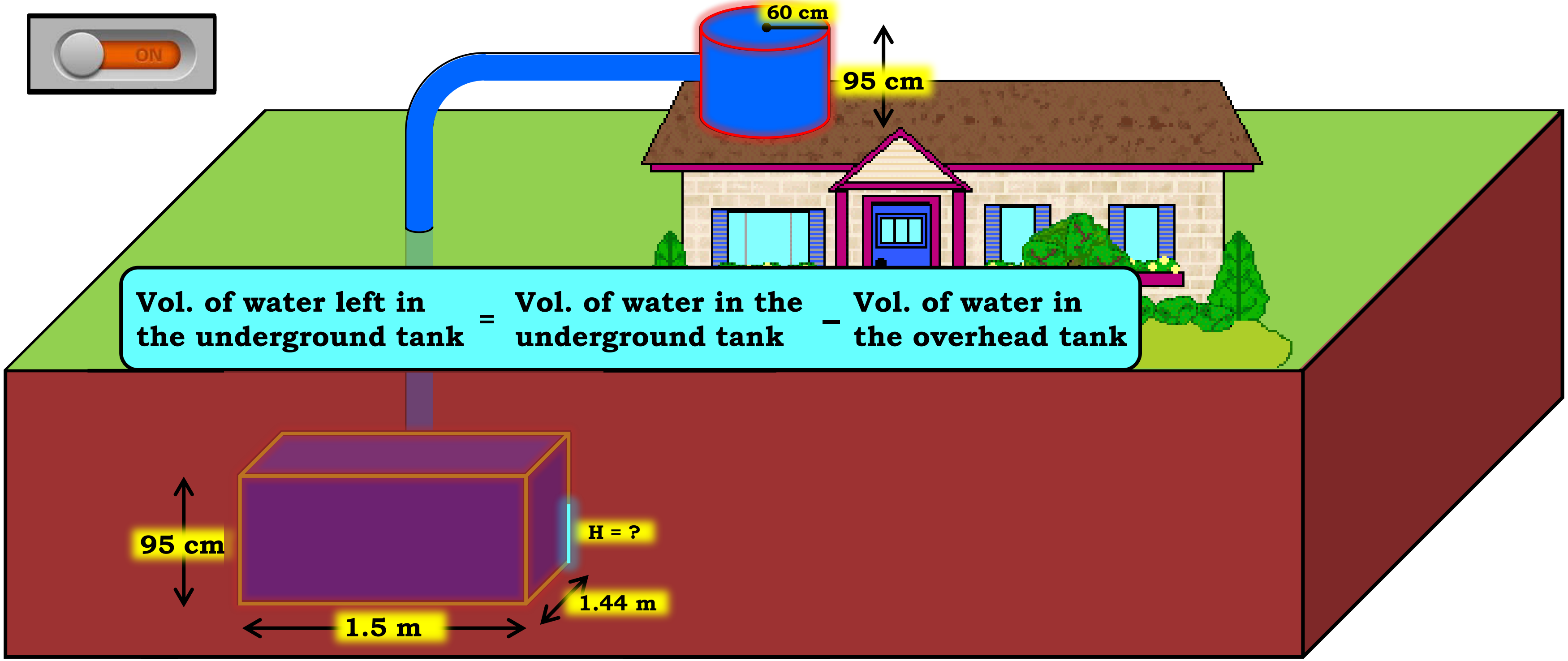
Thank You

Module 37

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Cuboid

Q. Rohan's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from an underground tank which is in the shape of a cuboid. The underground tank has dimensions 1.5m × 1.44m × 95cm. The overhead tank has its radius 60 cm and height 95 cm. Find the height of the water left in the underground tank after the overhead tank has been completely filled with water from underground tank which had been full. Compare the capacity of both the tanks. (Take π = 3.14)



Vol. of water left in the underground tank = Vol. of water in the underground tank – Vol. of water in the overhead tank

Q. Rohan's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from an underground tank which is in the shape of a cuboid. The underground tank has dimensions 1.5m × 1.44m × 95cm. The overhead tank has its radius 60 cm and height 95 cm. Find ... in the underground tank after the overhead tank has been com... underground tank which had been full. Compare the capacity of both the tanks. (Take $\pi = 3.14$)

150 cm 144 cm 95 cm

Sol.

Vol. of water left in the underground tank = Vol. of water in the underground tank − Vol. of water in the overhead tank

$$\therefore \quad \boxed{l} \times \boxed{b} \times H = \boxed{l} \times \boxed{b} \times \boxed{h} - \boxed{\pi}\boxed{r^2}\boxed{h_1}$$

$$\therefore \quad 150 \times 144 \times H = 150 \times \underline{144} \times 95 - 3.14 \times 60 \times 60 \times 95$$

What is the formula to find Volume of cylinder?

$\pi r^2 h$

$$\therefore \quad 1 \ldots 95 - \frac{314}{1\cancel{00}} \times 6\cancel{0} \times 6\cancel{0} \times 95$$

Vol. of water left in the underground tank = Vol. of water in the underground tank − Vol. of water in the overhead tank

$\times \underline{95}$

$$\therefore \quad 150 \times 144 \times H = 36 \times 95 \times (150 \times 4 - 314)$$

Q. Rohan's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from an underground tank which is in the shape of a cuboid. The underground tank has dimensions 1.5m × 1.44m × 95cm. The overhead tank has its radius 60 cm and height 95 cm. Find the height of the water left in the underground tank after the overhead tank has been completely filled with water from underground tank which had been full. Compare the capacity of both the tanks. (Take π = 3.14)

Sol.

$$150 \times 144 \times H = 36 \times 95 \times (\underline{150 \times 4} - 314)$$

$$\therefore \quad 150 \times 144 \times H = 36 \times 95 \times (\underline{600 - 314})$$

$$\therefore \quad 150 \times 144 \times H = 36 \times 95 \times 286$$

$$\therefore \quad H = \frac{\overset{19}{\cancel{95}} \times \cancel{36} \times \overset{143}{\cancel{286}}}{\underset{30}{\cancel{150}} \times \underset{\underset{2}{\cancel{4}}}{\cancel{144}}}$$

$$\therefore \quad H = \frac{19 \times 143}{30 \times 2}$$

$$\therefore \quad H = \frac{2717}{6 \times 10}$$

$$\therefore \quad H = \frac{452.83}{10} = 45.28 \text{ cm}$$

$\therefore$ **Height of the water left in the underground tank is 45.28 cm**

Q. Rohan's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from an underground tank which is in the shape of a cuboid. The underground tank has dimensions 1.5m × 1.44m × 95cm. The overhead tank has its radius 60 cm and height 95 cm. Find t... left in the underground tank after the overhead tank has been comp... underground tank which had been full. Compare the capacity of both the tanks. (Take π = 3.14)

150 c

144 c

95 cm

Sol.

Volume of undergrou...

... of

... of

...nk

Let us compare their volumes

Volume of overhead tank = $\pi r^2 h$

What is the formula of a cylinder ?

$\pi r^2 h$

$\times 60 \times 60 \times 95$

$\times \cancel{60} \times \cancel{60} \times 95$

$= \boxed{314 \times 6 \times 6 \times 95 \text{ cm}^3}$

$$\frac{\text{Volume of underground tank}}{\text{Volume of overhead tank}} = \frac{150 \times 144 \times 95}{314 \times 6 \times 6 \times 95}$$

Q. Rohan's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from an underground tank which is in the shape of a cuboid. The underground tank has dimensions 1.5m × 1.44m × 95cm. The overhead tank has its radius 60 cm and height 95 cm. Find the height of the water left in the underground tank after the overhead tank has been completely filled with water from underground tank which had been full. Compare the capacity of both the tanks. (Take $\pi = 3.14$)

Sol.

$$\frac{\text{Volume of underground tank}}{\text{Volume of overhead tank}} = \frac{150 \times \cancel{144}^{\cancel{24}\ \cancel{4}\ 2} \times \cancel{95}}{\underset{157}{\cancel{314}} \times \cancel{6} \times \cancel{6} \times \cancel{95}}$$

$$= \frac{300}{157}$$

$$\frac{\text{Volume of underground tank}}{\text{Volume of overhead tank}} = 1.91$$

$$\frac{\text{Volume of underground tank}}{\text{Volume of overhead tank}} \cong 2$$

$$\text{Volume of underground tank} \cong 2 \times \text{Volume of overhead tank}$$

∴ $\boxed{\text{Capacity of underground tank is twice of capacity of overhead tank}}$

Module 38

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Sphere

Q. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius ... Find the ... f the ...

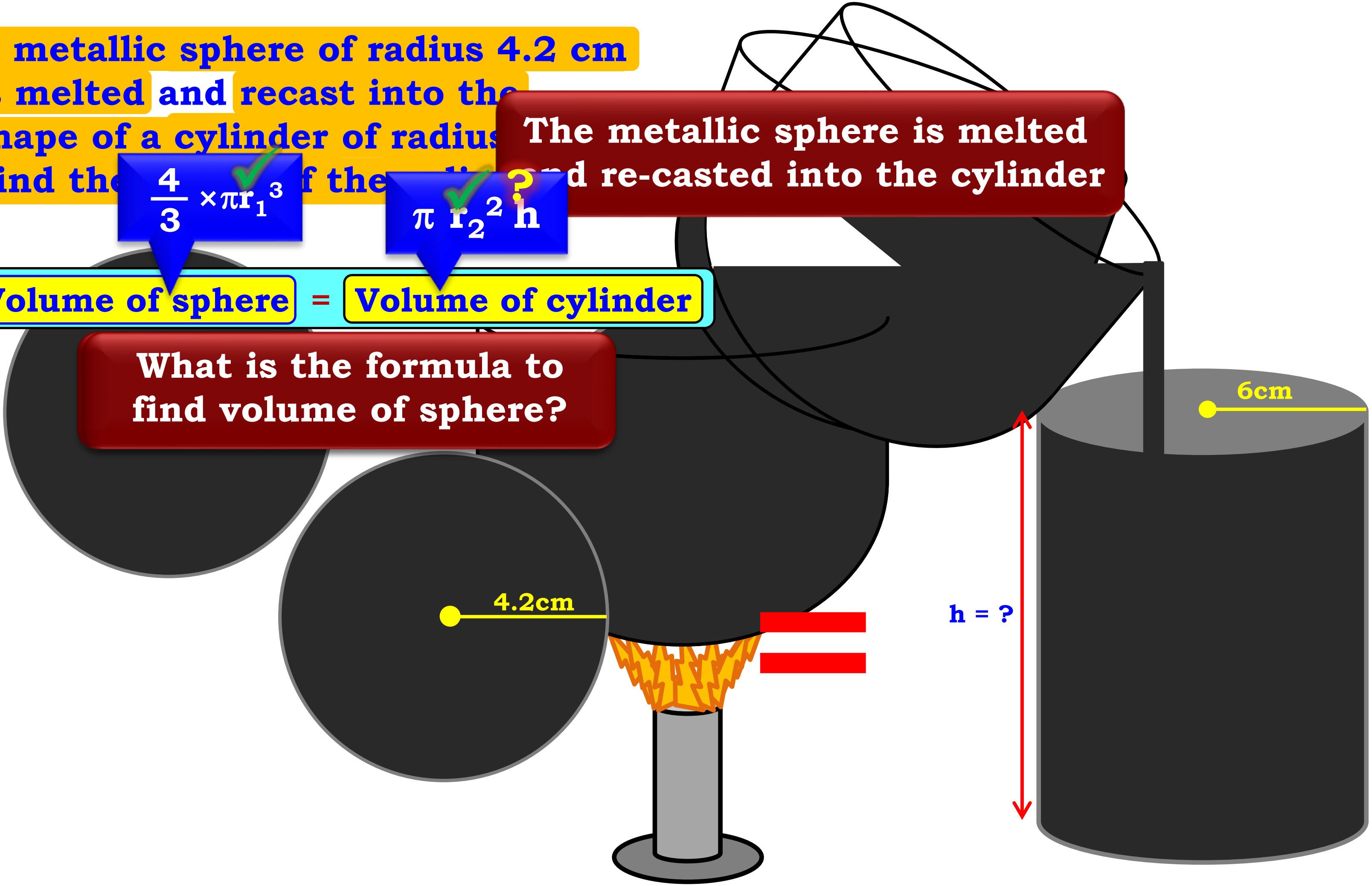
Sol.

$\frac{4}{3} \times \pi r_1^{\,3}$ $\quad$ $\pi\, r_2^{\,2}\, h$?

Volume of sphere = Volume of cylinder

The metallic sphere is melted and re-casted into the cylinder

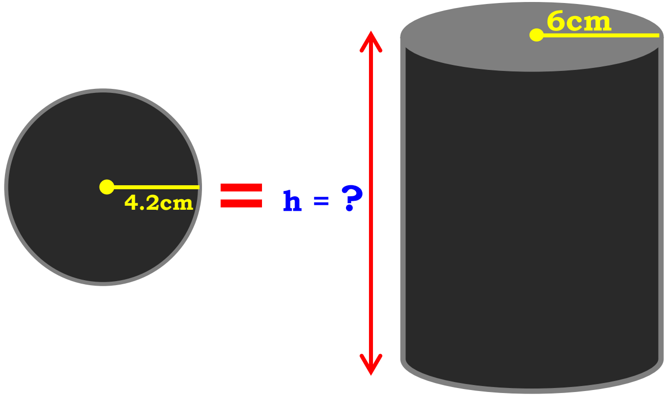
What is the formula to find volume of sphere?



Q. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

$\frac{4}{3} \times \pi r_1^3$ $\quad$ $\pi r_2^2 h$?

Volume of sphere = Volume of cylinder



Sol. $\quad$ **Vol. of the sphere = Vol. of the cylinder**

$$\frac{4}{3} \times \pi \boxed{r_1}^3 = \pi \boxed{r_2}^2 h$$

$$\frac{4}{3} \times \cancel{\pi} \times (4.2)^3 = \cancel{\pi} \times 6 \times 6 \times h$$

$$\therefore \quad \frac{4}{3} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} = 6 \times 6 \times h$$

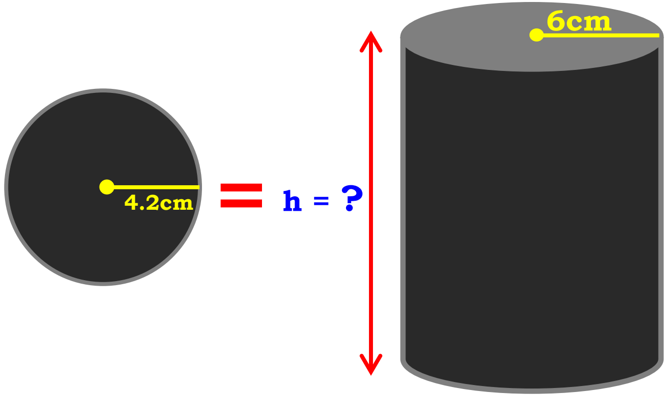
$$\therefore \quad \frac{4 \times \overset{14}{\cancel{42}} \times \overset{7}{\cancel{42}} \times \overset{7}{\cancel{42}}}{\cancel{3} \times \cancel{6} \times \cancel{6} \times 1000} = h$$

$$\therefore \quad h = \frac{4 \times 14 \times 7 \times 7}{1000}$$

Q. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

$\frac{4}{3} \times \pi r_1^{\,3}$ $\qquad$ $\pi\, r_2^{\,2}\, h$

Volume of sphere = Volume of cylinder



Sol.

$$h = \frac{4 \times 14 \times 7 \times 7}{1000}$$

$$= \frac{2744}{1000}$$

$$\therefore \quad h = 2.744 \text{ cm}$$

$\therefore$ **Height of the cylinder is 2.74 cm**

Module 39

SURFACE AREAS AND VOLUMES

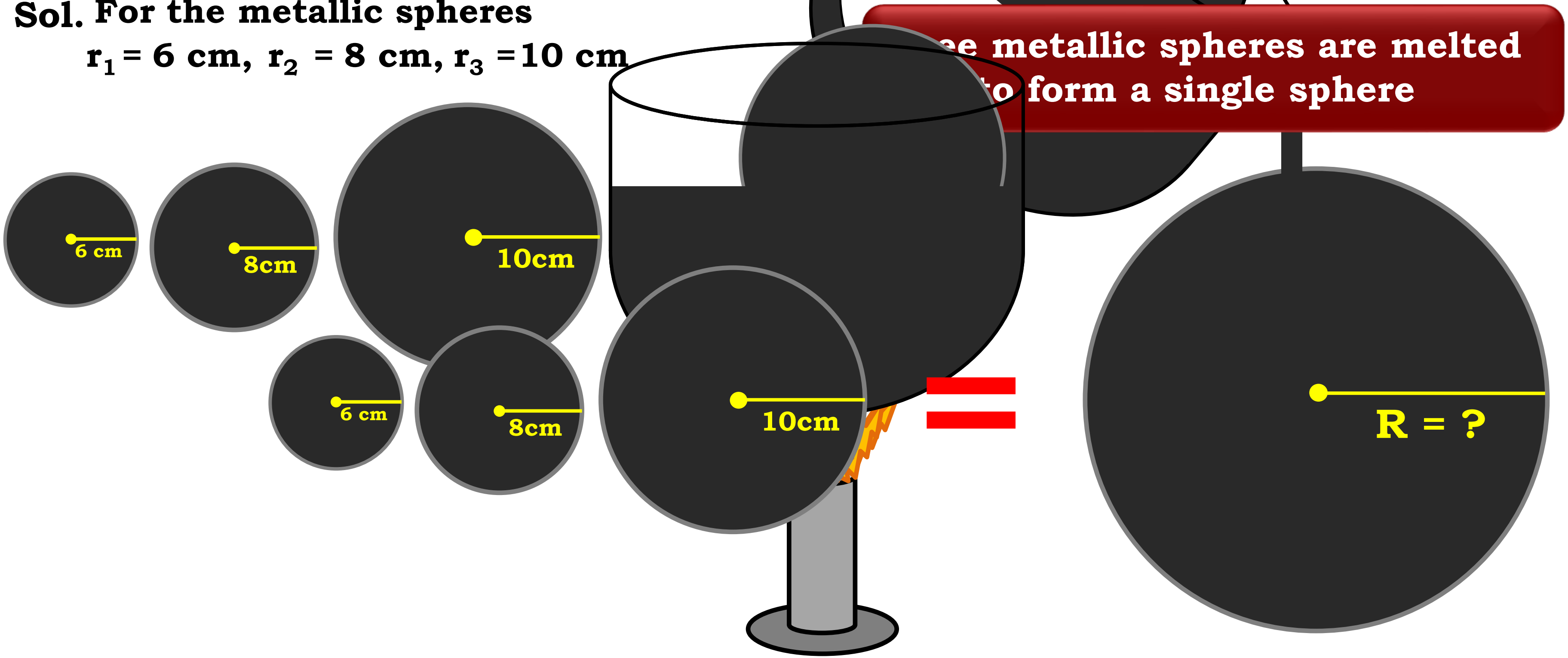
- Sum based on Sphere

Q. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Volume of 3 metallic spheres = Volume of bigger sphere

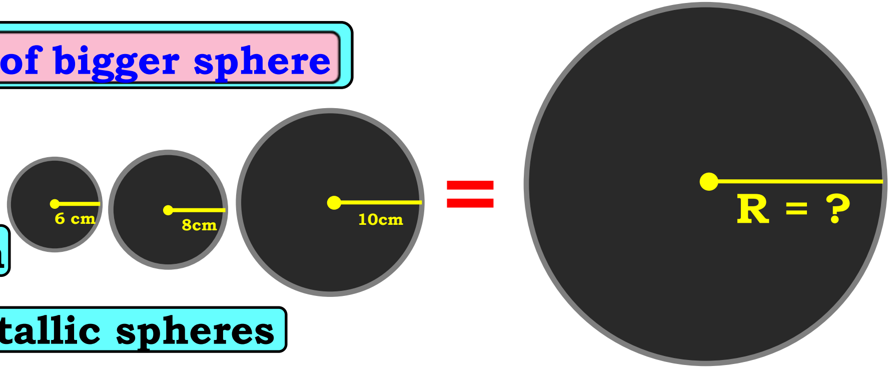
Sol. For the metallic spheres

$r_1 = 6$ cm, $r_2 = 8$ cm, $r_3 = 10$ cm



Q. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Volume of 3 metallic spheres = Volume of bigger sphere



Sol. For the metallic spheres

$r_1 = 6$ cm, $r_2 = 8$ cm, $r_3 = 10$ cm

Let the radius of new sphere be 'R' cm

Volume of bigger sphere = Vol. of 3 metallic spheres

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\therefore \quad \cancel{\frac{4}{3}}\cancel{\pi} R^3 = \cancel{\frac{4}{3}}\cancel{\pi}(r_1^3 + r_2^3 + r_3^3)$$

What is the formula to find volume of sphere?

$\frac{4}{3} \times \pi R^3$

$r_1^3 + r_2^3 + r_3^3$

$6^3 + 8^3 + 10^3$

$$\therefore \quad R^3 = 216 + 512 + 1000$$

Q. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Volume of 3 metallic spheres = Volume of bigger sphere

Sol.

$$R^3 = 216 + 512 + 1000$$

$$\therefore \quad R^3 = 1728$$

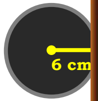
$$\therefore \quad R = \sqrt[3]{1728}$$

$$\therefore \quad R = \sqrt[3]{\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}}$$

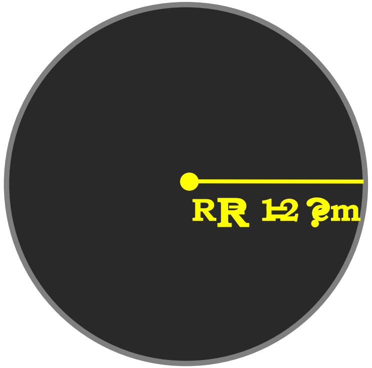
$$\therefore \quad R = 2 \times 2 \times 3$$

$$\therefore \quad R = 12 \text{ cm}$$

$\therefore$ **Radius of the resulting sphere is 12 cm.**



2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1



Module 40

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Cuboid

Q. A 10 m deep well of diameter 1.4 m is dug up in a field and the earth from d... read evenly on the adjoi... The length and breadt... eld are 55 m and 14 m re... e thickness of the earth layer.

$\pi r^2 H$

$l\ b\ h$

Sol.

How much earth did we obtain by digging the cylindrical well ?

...tire earth dug completely ...ills the cuboidal field

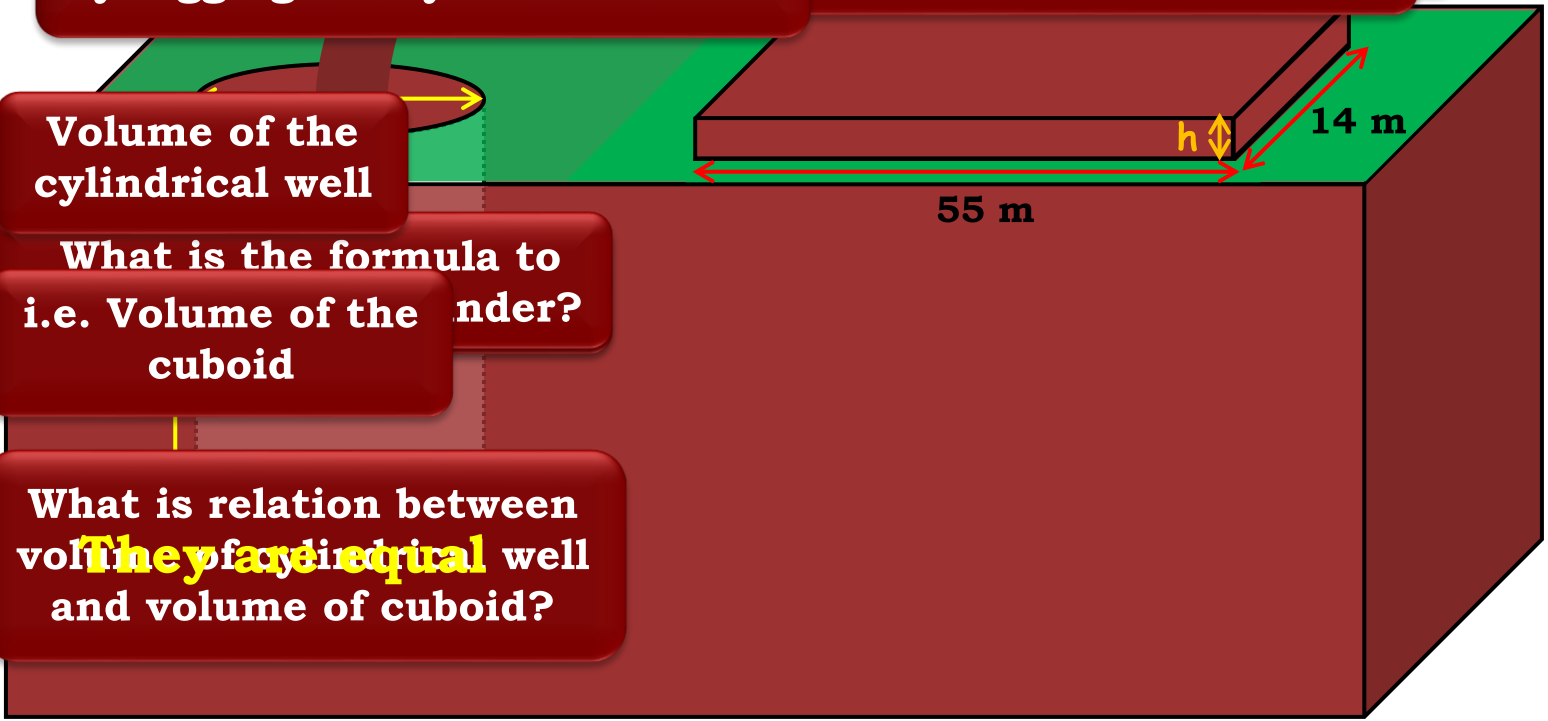
Volume of the cylindrical well

What is the formula to ...nder?

i.e. Volume of the cuboid

What is relation between volume of cylindrical well and volume of cuboid?

They are equal



Q. A 10 m deep well of diameter 1.4 m is dug up in a field and the earth fro… s spread evenly on the adjoi… The length and brea… t field are 55 m and 14 m r… he thickness of the earth layer.

$\pi r^2 H$

? $l\,b\,h$

Volume of cylinder = Volume of cuboid

Sol. Radius $= \dfrac{1.4}{2}$ m

Let the height of the cuboid be h.

Volume of cylinder = Volume of cuboid

$$\pi \times \boxed{r^2} \times \boxed{H} = \boxed{l} \times \boxed{b} \times h$$

$$\therefore \frac{22}{7} \times \frac{1.4}{2} \times \frac{1.4}{2} \times 10 = 55 \times 14 \times h$$

$$\therefore h = \frac{\overset{\cancel{11}}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{14}}{\underset{10}{\cancel{20}}} \times \frac{\overset{\cancel{2}}{\cancel{14}}}{\underset{\cancel{2}}{\cancel{20}}} \times \cancel{10} \times \frac{1}{\underset{5}{\cancel{55}}} \times \frac{1}{\cancel{14}}$$

$$\therefore h = \frac{1}{50}$$

$$\therefore h = 0.02 \text{ m}$$

$\therefore$ **Thickness of the earth layer is 0.02 m.**

Module 41

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Cuboid

Q. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring ... m to ... Find ... of t...

$\pi r^2 h$

Vol. of outer cyl. – Vol. of inner cyl.

$\pi R^2 H$ $\pi r^2 H$

Vol. of cylindrical well = Vol. of embankment

Sol. Diameter = 3 m

$\therefore$ Radius (r) = $\frac{3}{2}$ m

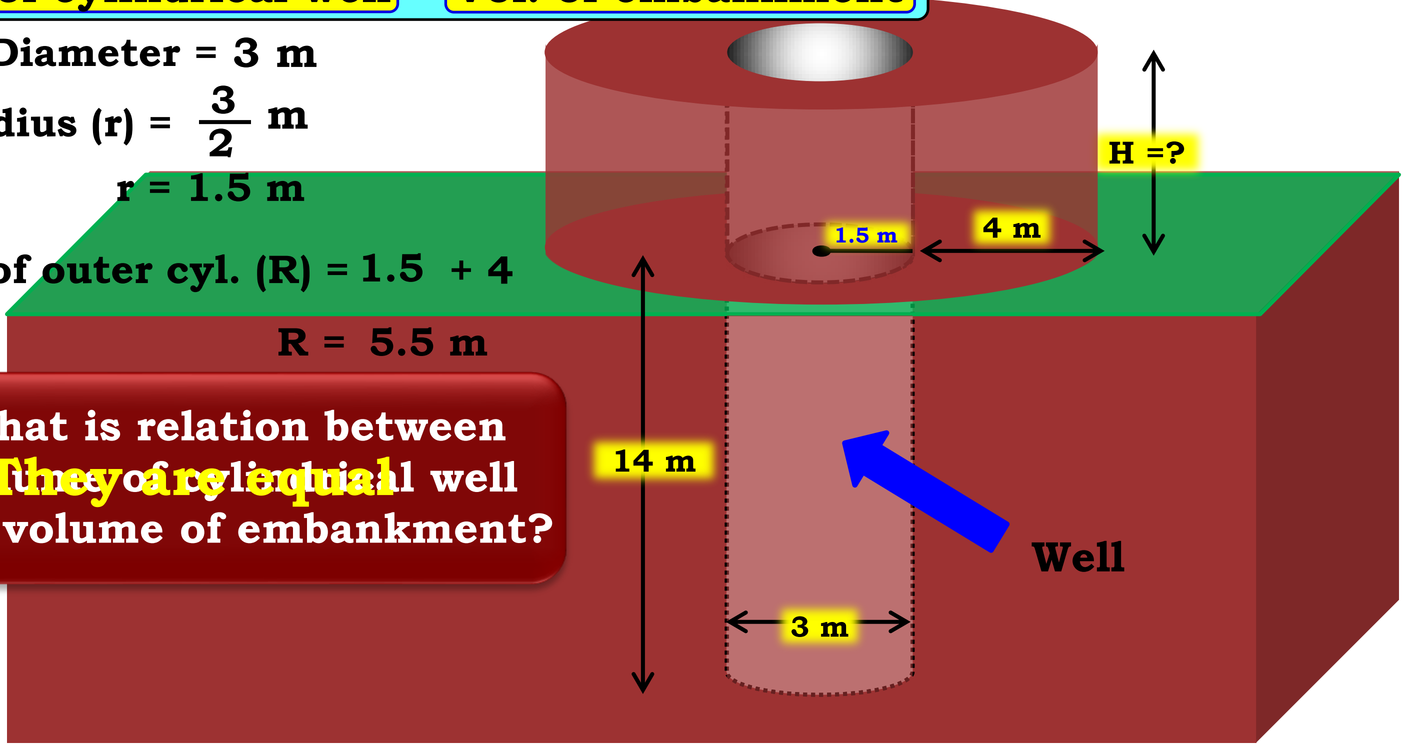
$\therefore$ r = 1.5 m

Radius of outer cyl. (R) = 1.5 + 4

$\therefore$ R = 5.5 m

What is relation between volume of cylindrical well and volume of embankment?

They are equal

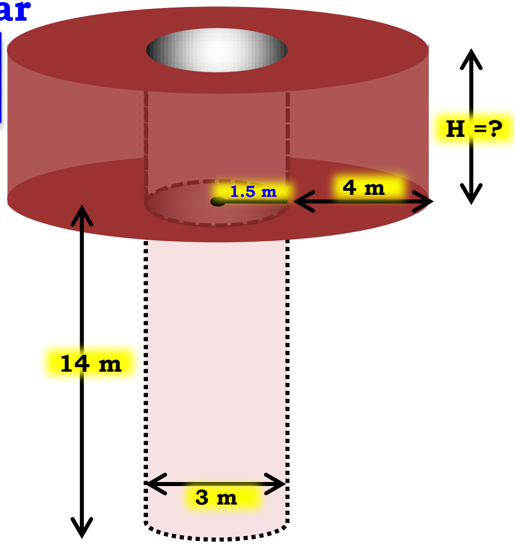


Q. A well of diameter 3 m is dug **14 m deep**. The earth taken out of it has been spread evenly all around it in the shape of a circular ring m to

Find of t

$\pi r^2 h$

$\pi R^2 H \quad - \quad \pi r^2 H$



Vol. of cylindrical well = Vol. of embankment

Sol. $\boxed{r = 1.5\ \text{m}}$

$R = 5.5$ m

$$\text{Vol. of cylindrical well} = \pi \boxed{r^2} \boxed{h}$$

$$= \frac{22}{\cancel{7}} \times (1.5)^2 \times \cancel{14}^{\,2}$$

$$= 22 \times 1.5 \times 1.5 \times 2$$

$$= 22 \times \underline{2.25 \times 2}$$

$$= 22 \times 4.5$$

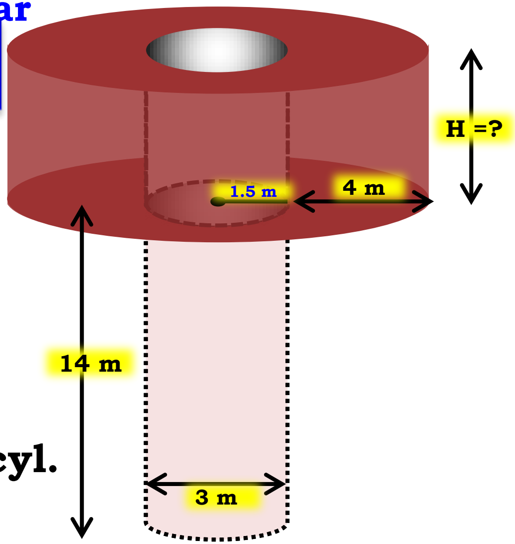
$$\therefore \text{Vol. of cylindrical well} = 99\ \text{cm}^3$$

Q. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring ... m to ... Find ... height of ...

$\pi r^2 h$

$\pi R^2 H \quad - \quad \pi r^2 H$

Vol. of cylindrical well = Vol. of embankment



Sol. $r = 1.5$ m

$R = 5.5$ m

Vol. of cylindrical well = 99 cm³

Vol. of cylindrical well = Vol. of embankment

∴ Vol. of cylindrical well = Vol. of outer cyl. – Vol. of inner cyl.

$$\therefore \quad 99 = \pi R^2 H - \pi r^2 H$$

$$\therefore \quad 99 = \pi (R^2 - r^2) H$$

$$\therefore \quad 99 = \pi [(5.5)^2 - (1.5)^2] H$$

$$\therefore \quad 99 = \pi (5.5 + 1.5)(5.5 - 1.5) H$$

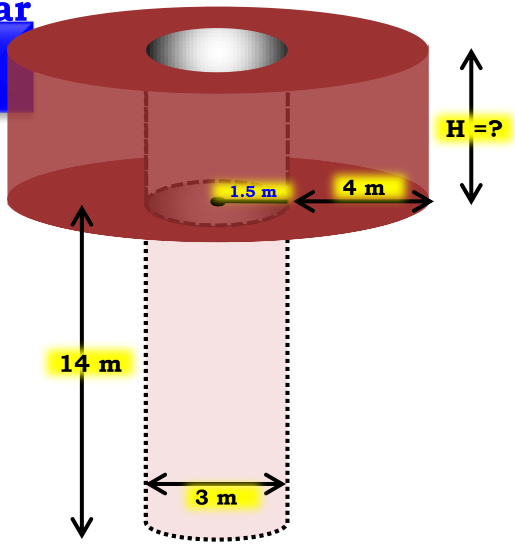
$$\therefore \quad 99 = \pi \times 7 \times 4 \times H$$

$$\therefore \quad 99 = \frac{22}{7} \times 7 \times 4 \times H$$

Q. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring o… to … Find t… of th…

$\pi r^2 h$ $\qquad$ $\pi R^2 H \quad - \quad \pi r^2 H$

Vol. of cylindrical well = Vol. of embankment



Sol. $r = 1.5$ m

$R = 5.5$ m

$$99 = \frac{22}{\cancel{7}} \times \cancel{7} \times 4 \times H$$

$$\therefore \quad 99 = 22 \times 4 \times H$$

$$\therefore \quad H = \frac{\cancel{99}^{\,9}}{{}_{2}\cancel{22} \times 4}$$

$$\therefore \quad H = \frac{9}{8}$$

$$\therefore \quad H = 1.125 \text{ m}$$

$\therefore$ $\boxed{\text{Height of the embankment is } 1.125\text{m}}$

Module 42

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Cone

Q. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the no. of such cones which can be filled with ice cream.

$$\text{Number of cones} = \frac{\text{Vol. of cylinder } (V_1)}{\text{Vol. of ice cream cone } (V_2)}$$

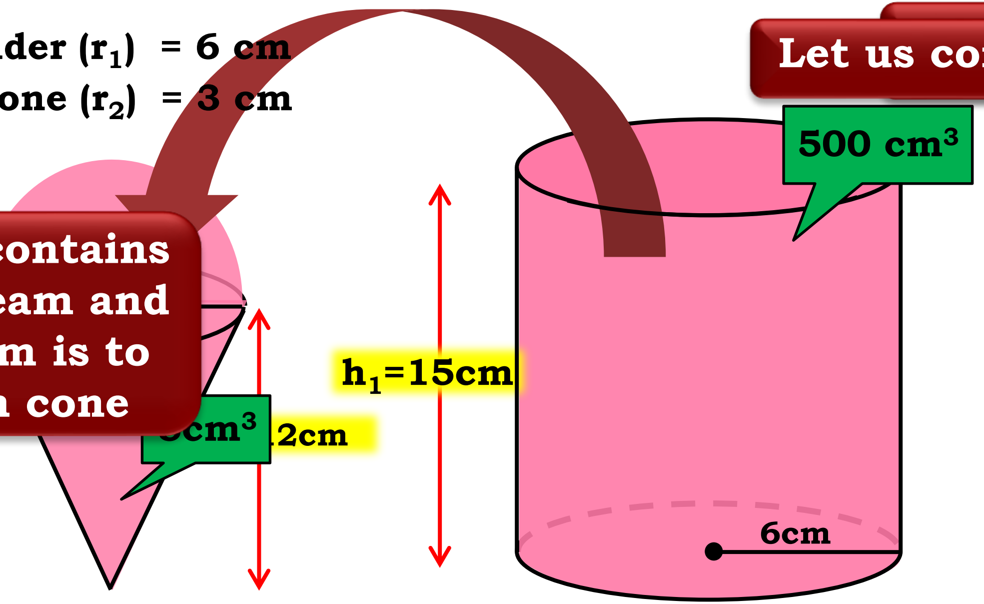
$$\text{No. of cones} = \frac{500}{5} = 100$$

Sol.

Radius of the cylinder (r_1) = 6 cm

Radius of the cone (r_2) = 3 cm

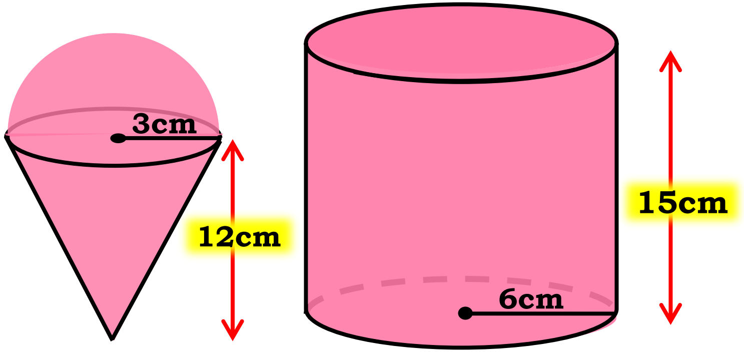
Let us consider an example



If the container contains $500 cm^3$ of ice cream and $5 cm^3$ of ice cream is to be filled in each cone

Q. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the no. of such cones which can be filled with ice cream.

$$\text{Number of cones} = \frac{\text{Vol. of cylinder } (V_1)}{\text{Vol. of ice cream cone } (V_2)}$$



What is the formula to find volume of a hemisphere? $\frac{2}{3}\pi r^3$

Sol.

Radius of the cylinder (r_1) = 6 cm

Radius of the cone (r_2) = 3 cm

$$\text{Vol. of the cylinder}(V_1) = \pi \boxed{r_1}^2 \boxed{h_1}$$

$$= \pi \times 6 \times 6 \times 15$$

$$= \pi \times 36 \times 15$$

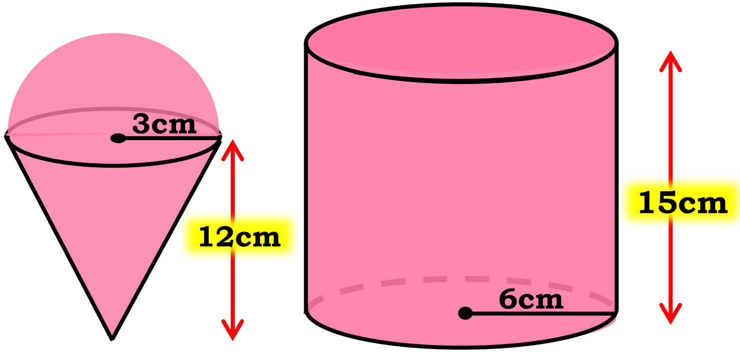
$$\therefore \text{Vol. of the cylinder } (V_1) = 540\,\pi\text{ cm}^3$$

$$\text{Vol. of ice cream cone } (V_2) = \boxed{\text{Vol. of cone}} + \boxed{\text{Vol. of hemisphere}}$$

$$\text{Vol. of ice cream cone } (V_2) = \frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3$$

Q. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the no. of such cones which can be filled with ice cream.

$$\text{Number of cones} = \frac{\text{Vol. of cylinder } (V_1)}{\text{Vol. of ice cream cone } (V_2)}$$



Sol.

Radius of the cylinder (r_1) = 6 cm

Radius of the cone (r_2) = 3 cm

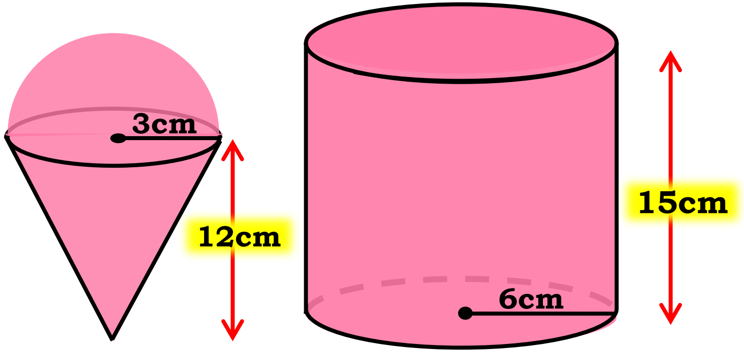
$$\text{Vol. of ice cream cone } (V_2) = \frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3$$

$$= \frac{1}{3}\pi r_2^2 (h_2 + 2r_2)$$

$$= \frac{1}{3}\pi \times (3)^2 \times [(12 + (2 \times 3)]$$

$$\therefore \text{Vol. of ice cream cone } (V_2) = \frac{1}{3}\pi \times 9 \times (12 + 6)$$

Q. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the no. of such cones which can be filled with ice cream.



$$\text{Number of cones} = \frac{\text{Vol. of cylinder } (V_1)}{\text{Vol. of ice cream cone } (V_2)}$$

Sol.

$$\text{Vol. of ice cream cone } (V_2) = \frac{1}{3}\pi \times 9 \times (\underline{12 + 6})$$

$$= \frac{1}{\cancel{3}}\pi \times \overset{3}{\cancel{9}} \times 18$$

$$= \pi \times 3 \times 18$$

$$\therefore \boxed{\text{Vol. of ice cream cone } (V_2) = 54\pi \text{ cm}^3}$$

$$\boxed{V_1 = 540\pi}$$

$$\text{Number of cones} = \frac{\boxed{V_1}}{\boxed{V_2}}$$

$$= \frac{\overset{10}{\cancel{540\pi}}}{\cancel{54\pi}}$$

$$= 10$$

$\therefore$ **10 cones can be filled with ice cream.**

Thank You

Module 43

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Cuboid

Q. How many silver coins, 1.75 cm in diameter and of thickness 2mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Sol. Radius (r) = $\dfrac{1.75}{2}$ cm

Vol. of N silver coins = Vol. of Cuboid

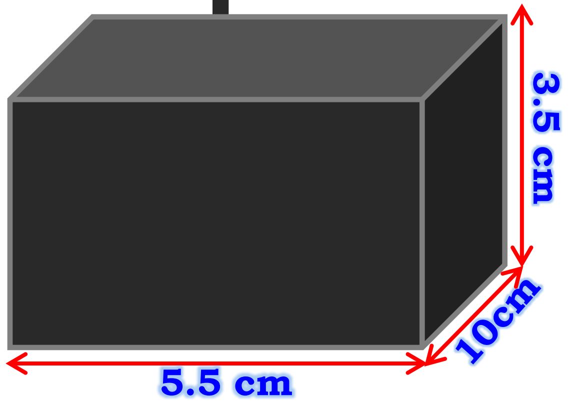
N × Vol. of 1 silver coin = Vol. of Cuboid

$\dfrac{\text{Vol. of cuboid}}{\text{l. of 1 silver coin}}$

Silver coins are melted to form a cuboid

1.75 cm

=



Q. How many silver coins, 1.75 cm in diameter and of thickness 2mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

$$N = \frac{\text{Vol. of cuboid}}{\text{Vol. of 1 silver coin}}$$

What is the shape of silver coins?

Cylinder

Sol. Radius (r) $= \dfrac{1.75}{2}$ cm

Vol. of cuboid $= \boxed{l \times b \times h_1}$

$= 5.5 \times 10 \times 3.5$

$\therefore$ Vol. of cuboid $= \dfrac{55}{10} \times 10 \times \dfrac{35}{10}$ cm³

Vol. of 1 silver coin $= \pi \boxed{r}^2 \boxed{h_2}$

$\therefore$ Vol. of 1 silver coin $= \dfrac{22}{7} \times \dfrac{1.75}{2} \times \dfrac{1.75}{2} \times \dfrac{2}{10}$

Q. How many silver coins, 1.75 cm in diameter and of thickness 2mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

$$N = \frac{\text{Vol. of cuboid}}{\text{Vol. of 1 silver coin}}$$

Sol. Vol. of 1 silver coin $= \frac{22}{7} \times \frac{1.75}{2} \times \frac{1.75}{2} \times \frac{2}{10}$

$\therefore$ Vol. of 1 silver coin $= \frac{22}{7} \times \frac{175}{200} \times \frac{175}{200} \times \frac{2}{10}$

$$N = \frac{\text{Vol. of cuboid } (V_1)}{\text{Vol. of 1 silver coin } (V_2)}$$

$$\therefore \quad N = \frac{\frac{55}{10} \times 10 \times \frac{35}{10}}{\frac{22}{7} \times \frac{175}{200} \times \frac{175}{200} \times \frac{2}{10}}$$

Vol. of cuboid $= \frac{55}{10} \times 10 \times \frac{35}{10}$ cm^3

Q. How many silver coins, 1.75 cm in diameter and of thickness 2mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

$$N = \frac{\text{Vol. of cuboid}}{\text{Vol. of 1 silver coin}}$$

Sol.

$$N = \frac{\frac{55}{10} \times 10 \times \frac{35}{10}}{\frac{22}{7} \times \frac{175}{200} \times \frac{175}{200} \times \frac{2}{10}}$$

$$= \frac{\overset{5}{\cancel{55}}}{\cancel{10}} \times \cancel{10} \times \frac{\cancel{35}}{\cancel{10}} \times \frac{\cancel{7}}{\underset{\cancel{11}}{\cancel{22}}} \times \frac{\overset{\overset{20}{\cancel{40}}}{\cancel{200}}}{\underset{\cancel{35}}{\cancel{175}}} \times \frac{\overset{\overset{\overset{4}{\cancel{8}}}{\cancel{40}}}{\cancel{200}}}{\underset{\underset{\cancel{5}}{\cancel{35}}}{\cancel{175}}} \times \frac{\cancel{10}}{\cancel{2}}$$

$$= 5 \times 20 \times 4$$

$$= 400$$

$\therefore$ **400 coins must be melted.**

Module 44

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Cone

Q. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This [illegible] emptied on the ground and a conical h[illegible]nd is fo[illegible] height of the conical heap is 24 cm, [illegible] radius and slan[illegible] height of the heap.

$\pi r_1^2 h_1$ ✓ ✓

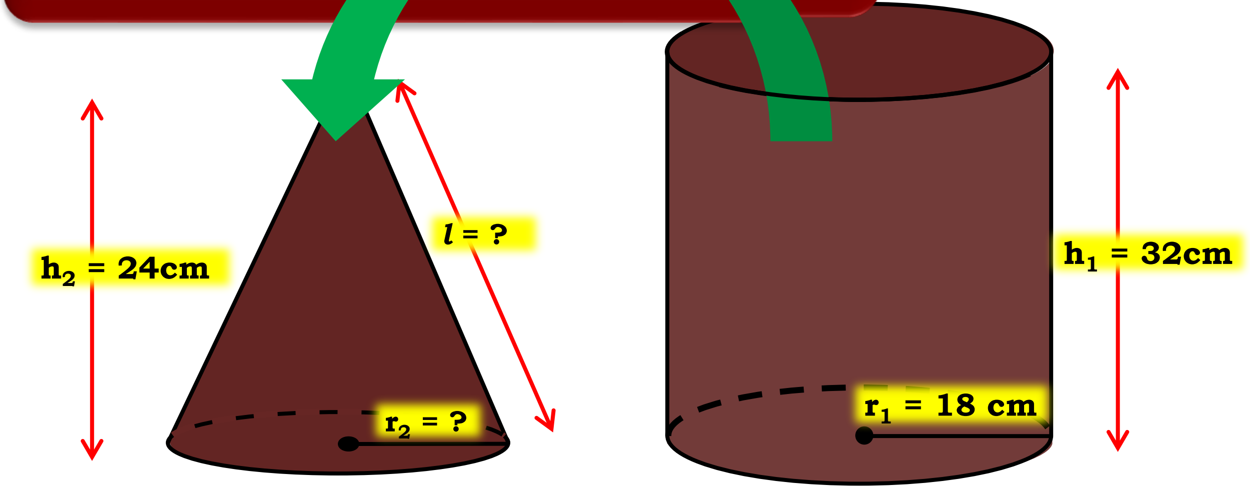
$\frac{1}{3} \times \pi r_2^2 h_2$? ✓

Vol. of cylindrical bucket (V_1) = Vol. of cone (V_2)

What is the relation between the volume of the cylindrical bucket and volume of cone?

They are equal

[illegible] ith [illegible] nd a [illegible] d.

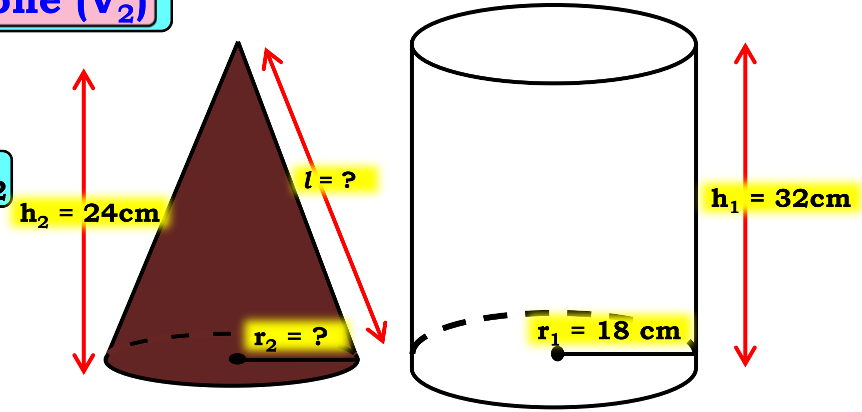


Q. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical h… nd is fo… height of the conical heap is 24 cm, … the radius and slan… eight of the heap.

$\pi r_1^2 h_1$

$\frac{1}{3} \times \pi r_2^2 h_2$?

Vol. of cylindrical bucket (V_1) = Vol. of cone (V_2)



Sol.

Vol. of cylindrical bucket = Vol. of the cone

$$\therefore \quad \cancel{\pi} \times \boxed{r_1}^2 \times \boxed{h_1} = \frac{1}{3} \times \cancel{\pi} \times r_2^2 \times \boxed{h_2}$$

$$\therefore \quad 18 \times 18 \times 32 = \frac{1}{3} \times r_2^2 \times 24$$

$$\therefore \quad \frac{\overset{3}{\cancel{18}} \times 18 \times \overset{8}{\cancel{32}} \times 3}{\underset{\cancel{4}}{\cancel{24}}} = r_2^2$$

$$\therefore \quad r_2^2 = 3 \times \underline{18} \times \underline{8} \times 3$$

$$\therefore \quad r_2^2 = \underline{3 \times 3} \times \underline{3} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3}$$

$$\therefore \quad r_2 = 3 \times 3 \times 2 \times 2$$

Q. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical h... nd is fo... height of the conical heap is 24 cm, ... the radius and slant height of the heap.

$\pi r_1^2 h_1$

$\frac{1}{3} \times \pi r_2^2 h_2$

Vol. of cylindrical bucket (V_1) = Vol. of cone (V_2)

Sol.

$r_2 = 3 \times 3 \times 2 \times 2$

$\therefore \boxed{r_2 = 36 \text{ cm}}$

$\therefore$ Radius of the heap is 36 cm

Now

Now, let us find slant height of the cone

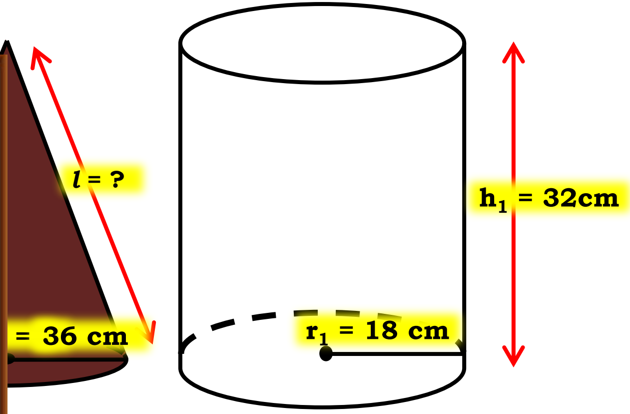
What is the formula to find slant height (l) ?

$l^2 = r^2 + h^2$

$\therefore \quad l = \sqrt{1872}$

$\therefore \quad l = \sqrt{2\times2\times2\times2\times3\times3\times13}$

2	1872
2	936
2	468
2	234
3	117
3	39
13	13
	1



Q. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Vol. of cylindrical bucket (V_1) = Vol. of cone (V_2)

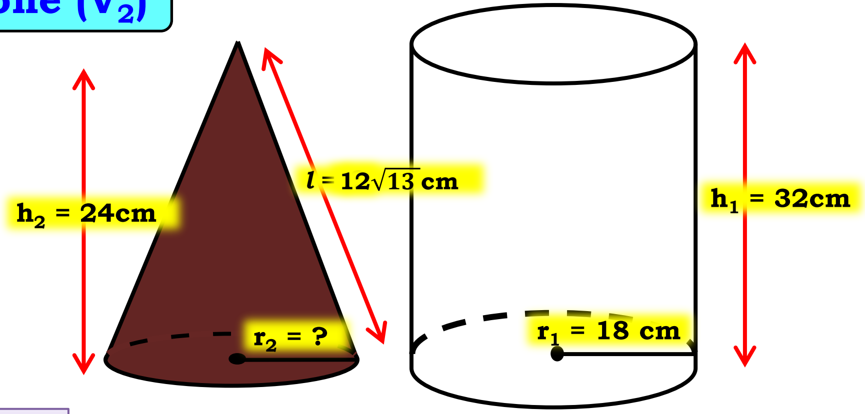
Sol.

$$l = \sqrt{\underline{2\times2}\times\underline{2\times2}\times\underline{3\times3}\times13}$$

$$\therefore \quad l = 2 \times 2 \times 3\sqrt{13}$$

$$\therefore \quad l = 12\sqrt{13}$$

$\therefore$ **Slant height of the heap is $12\sqrt{13}$ cm.**

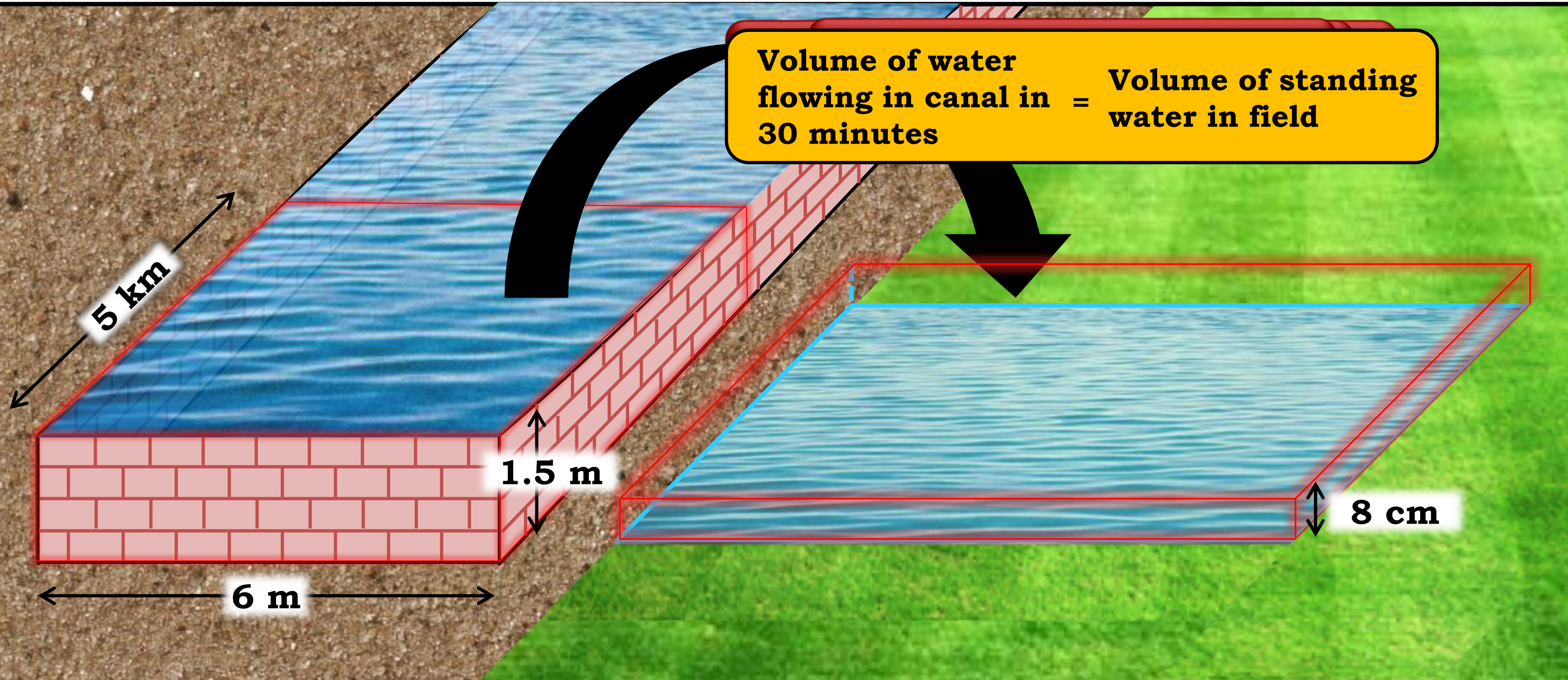


Module 45

SURFACE AREAS AND VOLUMES

- Sum based on Cuboid

Q. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?



Q. W $\frac{8}{100}$ m canal, 6 m wide and 1.5 m deep, is flowing with a sp 0 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Sol. Length of the water flowing in 1 hour = 1

W We know that, a to find
Area = $l \times b$ oid?

∴ Length of the water flowing in 30 minutes (l) = $\frac{}{2}$

= 5 km

= 5×1000 m

∴ Length of the water flowing in 30 minutes (l) = 5000 m

Volume of water flowing in canal in 30 minutes = Volume of standing water in field

∴ $l \times b \times h = l_1 \times b_1 \times h_1$

∴ Volume of water flowing in canal in 30 minutes = Volume of standing water in field

∴ $5000 \times 6 \times 1.5 = \text{Area} \times \frac{8}{100}$

Q. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

1 hectare = 10,000 m²

Sol.

$$5000 \times 6 \times 1.5 = \text{Area} \times \frac{8}{100}$$

$$\therefore \quad \text{Area} = (5000 \times 6 \times 1.5) \div \frac{8}{100}$$

$$= \overset{2500}{\cancel{5000}} \times \overset{3}{\cancel{6}} \times \frac{15}{\cancel{10}} \times \frac{\overset{5}{\cancel{100}}}{\underset{\cancel{4}\ \cancel{2}}{\cancel{8}}}$$

$$= 2500 \times 3 \times 75$$

$$\therefore \quad \text{Area} = 562500 \text{ m}^2$$

$$= \frac{5625\cancel{00}}{100\cancel{00}}$$

$$= \frac{5625}{100}$$

$$= 56.25 \text{ hectares}$$

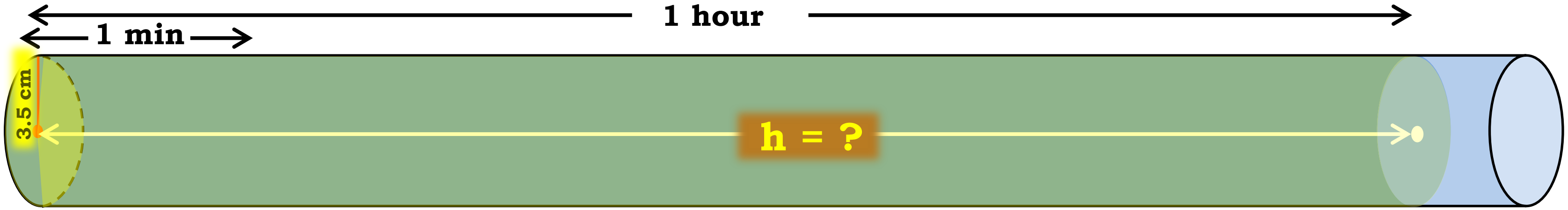
$\therefore$ **Area irrigated will be 562500 m² or 56.25 hectares.**

Module 46

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder

Q. A cylindrical pipe has inner diameter of 7 cm and water flows through it at 192.5 litres per minute. Find the rate of flow in kilometers per hour.



Sol. Radius $= \dfrac{7}{2} = 3.5$ cm

Vo… in = 192.5 litres

What is formula to find volume of cylinder ?

$\pi r^2 h$

V… our $= 192.5 \times 60$ litres

$= 11550 \times 1000 \text{ cm}^3$

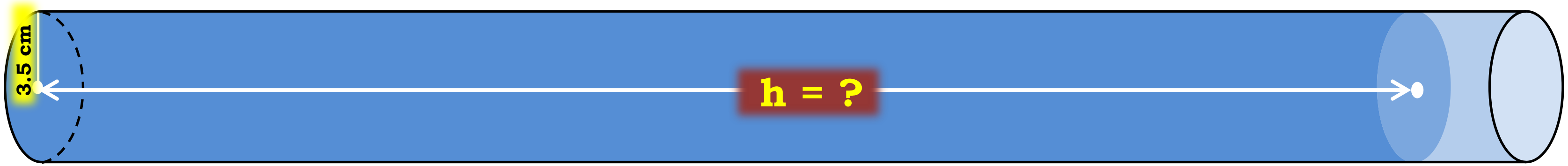
Volume of water flowing in 1 hour $= 11550000 \text{ cm}^3$

Volume of water flowing 1 hour = Vol. of cylinder

$11550000 = \pi r^2 h$

Q. A cylindrical pipe has inner diameter of 7 cm and water flows through it at 192.5 litres per minute. Find the rate of flow in kilometers per hour.

$$\text{Radius} = \frac{7}{2} = 3.5 \text{ cm}$$



Sol.

$$11550000 = \boxed{\pi}\boxed{r}^2 h$$

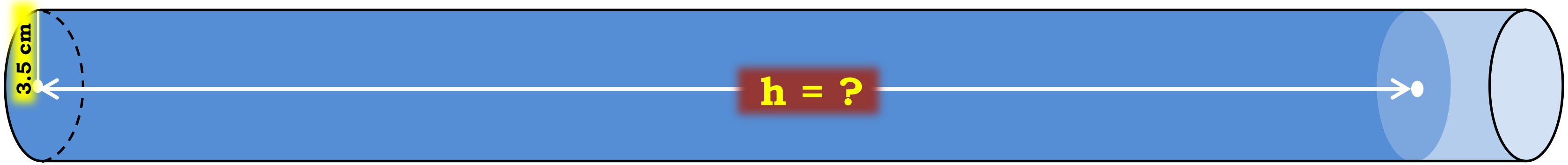
$$\therefore \quad 11550000 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h$$

$$\therefore \quad h = \frac{11550000 \times 7 \times 2 \times 2}{22 \times 7 \times 7}$$

$$\therefore \quad h = \frac{\overset{\overset{15}{\cancel{105}}}{\cancel{1155}} \times 10000 \times \cancel{7} \times \cancel{2} \times 2}{\underset{\cancel{2}}{\cancel{22}} \times \cancel{7} \times \cancel{7}}$$

$$\therefore \quad h = 15 \times 10000 \times 2$$

Q. A cylindrical pipe has inner diameter of 7 cm and water flows through it at 192.5 litres per minute. Find the rate of flow in kilometers per hour.



Sol.

$$h = 15 \times 10000 \times 2$$

$\therefore$ $$h = 300000 \text{ cm}$$

1 km = 100000 cm

$\therefore$ $$h = \frac{3\cancel{00000}}{1\cancel{00000}}$$

$\therefore$ $$h = 3 \text{ km}$$

$\therefore$ The rate of flow of water is 3 km per hour.

Module 47

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Cuboid

Q. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Sol.

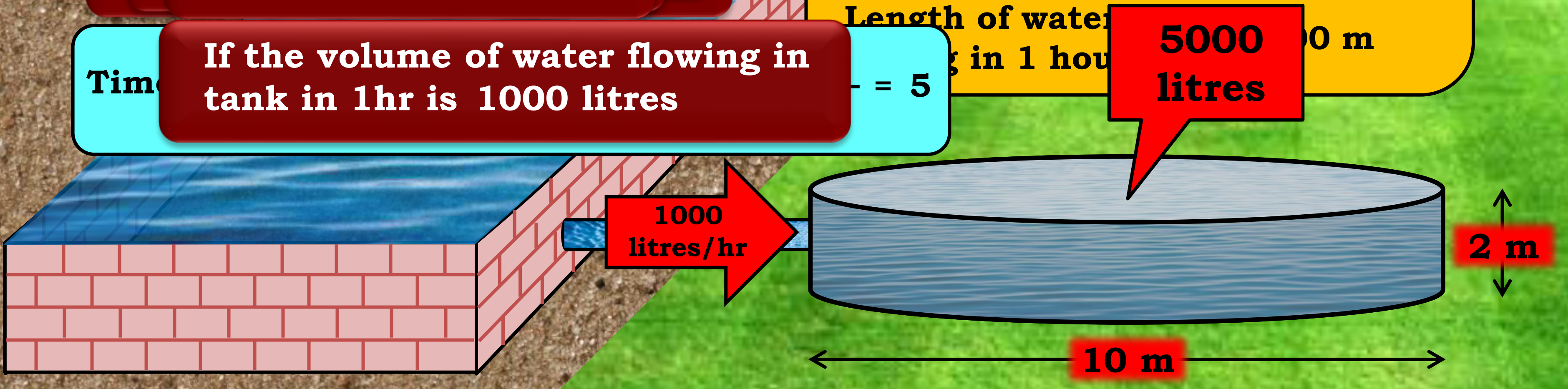
If the volume of cylindrical tank is 5000 litres

Height of tank (H) = 2 m

Then, what will be the time taken ?

If the volume of water flowing in tank in 1hr is 1000 litres

= 5



Q. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

$$\text{Time taken} = \frac{\boxed{\text{Vol. of cylindrical tank}}}{\boxed{\text{Vol. of water flowing in 1hr}}}$$

What is the formula to find Volume of cylinder?

$\pi R^2 h$

Sol. $\boxed{r = \frac{1}{10}\ m}$, $\boxed{R = 5\ m}$, $\boxed{H = 2\ m}$, $\boxed{h = 3000\ m}$

$$\text{Vol. of cylindrical tank} = \pi \boxed{R}^2 \boxed{H}$$

$$= \pi \times 25 \times 2$$

$$\therefore \text{Vol. of cylindrical tank} = 50\pi\ m^3$$

$$\text{Vol. of water flowing in 1hr} = \pi \boxed{r}^2 \boxed{h}$$

$$= \pi \times \frac{1}{1\cancel{0}} \times \frac{1}{1\cancel{0}} \times 30\cancel{0}\cancel{0}$$

$$\therefore \text{Vol. of water flowing in 1hr} = (30\pi)\ m^3$$

Q. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

$$\text{Time taken} = \frac{\text{Vol. of cylindrical tank}}{\text{Vol. of water flowing in 1hr}}$$

Sol. $$\text{Time taken} = \frac{\boxed{\text{Vol. of cylindrical tank}}}{\boxed{\text{Vol. of water flowing in 1hr}}}$$

$$= \frac{5\cancel{0}\cancel{\pi}}{3\cancel{0}\cancel{\pi}}$$

$$= \frac{5}{3}$$

5

Vol. of water in tank = 50 π m^3

Vol. of water flowing in 1hr = 30 π m^3

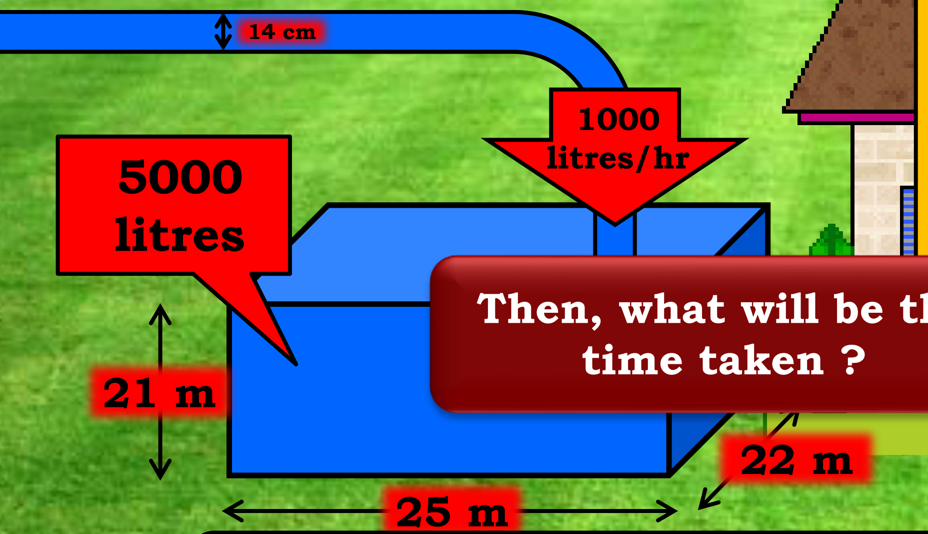
$\therefore$ Time taken to fill the tank is 100 minutes.

Module 48

SURFACE AREAS AND VOLUMES

- Sum based on Cylinder and Cuboid

Q. Water is flowing at the rate of 5 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 25 m long and 22 m wide. Determine the time in which the level of water in the tank will rise by 21 cm. (Take π = 22/7)



For pipe,

Radius of pipe (R) = 7 cm $= \frac{7}{100}$ m

Length of water flowing in 1 hour (H) = 5000 m

For tank,

Length of tank (l) = 25 m

Breadth of tank (b) = 22 m

Height of tank (h) = 21 cm $= \frac{21}{100}$ m

Then, what will be the time taken ?

$$\text{Time taken} = \frac{\text{Vol. of rectangular tank}}{\text{Vol. of water flowing in 1hr}} = \frac{5000}{1000} = 5$$

i.e ... flowing in 1hr (H)

Q. Water is flowing at the rate of 5 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 25 m long and 22 m wide. Determine the time in which the level of water in the tank will rise by 21 cm. (Take π = 22/7)

$$\text{Time taken} = \frac{\text{Vol. of rectangular tank}}{\text{Vol. of water flowing in 1hr}}$$

What is the formula to find Volume of cylinder?

$\pi R^2 H$

Sol. For pipe, $\boxed{R} = \frac{7}{100}$ m, $\boxed{H} = 5000$ m,

For tank, $l = 25$m, $b = 22$m, $h = \frac{21}{100}$ m

$$\text{Vol. of rectangular tank} = \boxed{l} \times \boxed{b} \times \boxed{h}$$

$$\text{Vol. of rectangular tank} = 25 \times 22 \times \frac{21}{100} \text{ m}^3$$

$$\text{Vol. of water flowing in 1hr} = \pi \boxed{R}^2 \boxed{H}$$

$$= \pi \times \frac{7}{\cancel{100}_{2}} \times \frac{7}{\cancel{100}} \times \cancel{5000}$$

$$\therefore \text{Vol. of water flowing in 1hr} = \frac{49}{2}\pi \text{ m}^3$$

Q. Water is flowing at the rate of 5 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 25 m long and 22 m wide. Determine the time in which the level of water in the tank will rise by 21 cm. (Take $\pi = 22/7$)

$$\text{Time taken} = \frac{\text{Vol. of rectangular tank}}{\text{Vol. of water flowing in 1hr}}$$

Sol.

$$\text{Time taken} = \frac{\text{Vol. of rectangular tank}}{\text{Vol. of water flowing in 1hr}}$$

$$= 25 \times 22 \times \frac{21}{100} \div \frac{49}{2}\pi$$

$$= 25 \times 22 \times \frac{21}{100} \div \frac{49}{2} \times \frac{22}{7}$$

$$\text{Vol. of water in tank} = 25 \times 22 \times \frac{21}{100}\ m^3$$

$$\text{Vol. of water flowing in 1hr} = \frac{49}{2}\pi\ m^3$$

$\therefore$ The time in which the level of water in the tank will rise by 21 cm is 1.5 hours

Thank You

Module 49

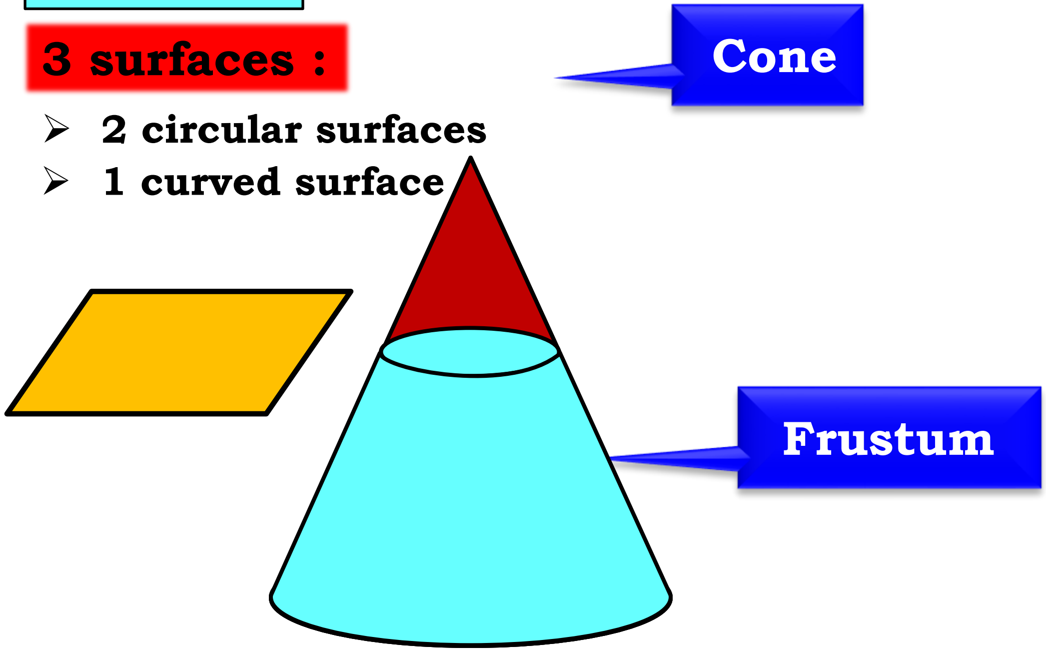
SURFACE AREAS AND VOLUMES

- **Frustum**

Frustum

3 surfaces :

- 2 circular surfaces
- 1 curved surface



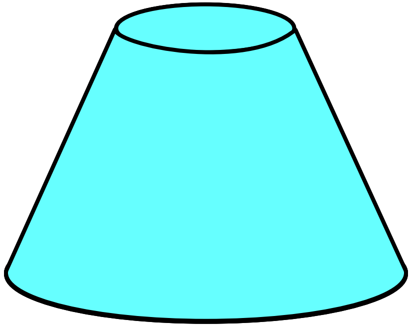
Formulae

Curved surface area $= \pi(r_1 + r_2)l$

Total surface area $=$ A(two circular surfaces) $+$ C.S.A.

$$= \pi r_1^2 + \pi r_2^2 + \pi(r_1 + r_2)l$$

Volume $= \frac{1}{3}\pi(r_1^2 + r_2^2 + r_1.r_2)h$

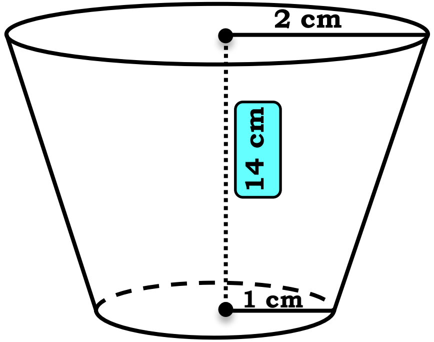


Module 50

SURFACE AREAS AND VOLUMES

- **Sums based on Frustum of Cone**

Q. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.



Sol. $\boxed{r_1 = \frac{4}{2} = 2\text{ cm}}$

$\boxed{r_2 = \frac{2}{2} = 1\text{ cm}}$

Volume of frustum $= \frac{1}{3}\pi h\,(r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3} \times \frac{22}{7} \times 14 \times (2^2 + 1^2 + \underline{2 \times 1})$

$\frac{22}{7} \times 14 \times (\underline{4 + 1 + 2})$

Let the radius of the bottom circular face be r_2

$= \frac{\;}{3} \times \frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}} \times 7$

$= \frac{1}{3} \times 22 \times 2 \times 7$

$= \frac{308}{3} = 102.67\text{ cm}^3$

$\therefore$ **Capacity of the glass is 102.67 cm^3**

Q. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Sol.

$$\boxed{2\pi r_1} = \boxed{18} \qquad \ldots(i)$$

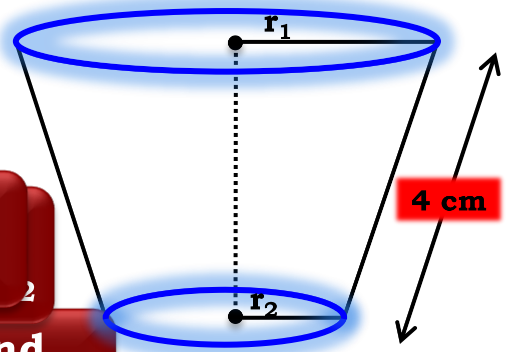
$$2\pi r_2 = 6 \qquad \ldots(ii)$$

To find : $r_1 + r_2$

Let the radius of the top circular face be r_1

What is the formula to find curved surface area of frustum?

$\pi l \boxed{(r_1 + r_2)}$



Adding (i) & (ii)

$$2\pi r_1 + 2\pi r_2 = 18 + 6$$

$$2\pi r_1 + 2\pi r_2 = 24$$

$$2\pi (r_1 + r_2) = 24$$

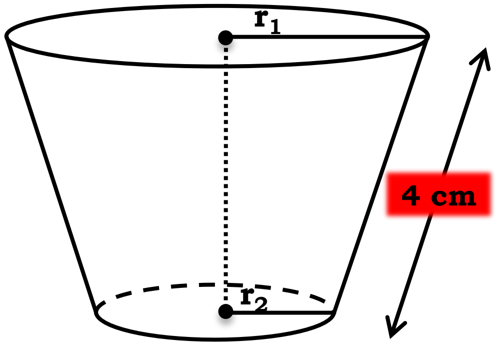
$$\therefore \quad r_1 + r_2 = \frac{\cancel{24}^{\,12}}{\cancel{2}\pi}$$

$$\therefore \quad r_1 + r_2 = \frac{12}{\pi}$$

Q. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Sol.

$$\boxed{r_1 + r_2 = \frac{12}{\pi}}$$



CSA of the frustum $= \pi \boxed{l} \boxed{(r_1 + r_2)}$

$= \cancel{\pi} \times 4 \times \dfrac{12}{\cancel{\pi}}$

$= 4 \times 12$

$= 48 \text{ cm}^2$

$\therefore$ **Curved surface area of the frustum is 48 cm²**

Module 51

SURFACE AREAS AND VOLUMES

- Sum based on Frustum of cone

Q. A *fez*, the cap used by the Turks, is shaped like the frustum of a cone If its radius on the open side is 10 cm, radius at the upper base is 4 ... slan... t is 15 cm, find the area of material use... it.

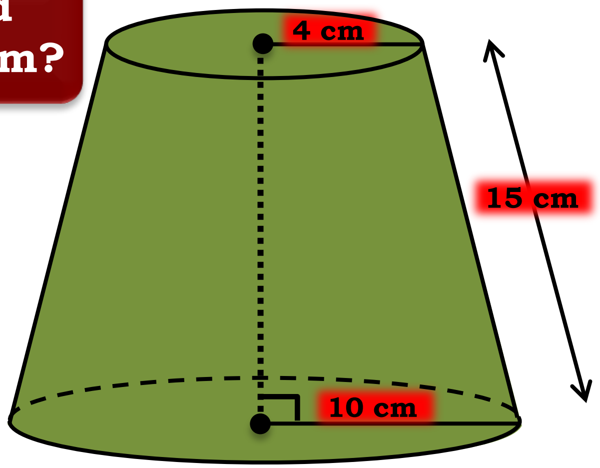
$\pi l\,(r_1 + r_2)$ $\quad$ $\pi\, r_2^{\,2}$

Area of material used = CSA of (Frustum) + Area of top

Sol. $r_1 = 10$ cm, $r_2 = 4$ cm, $l = 15$ cm

What is the formula to find curved surface area of frustum?

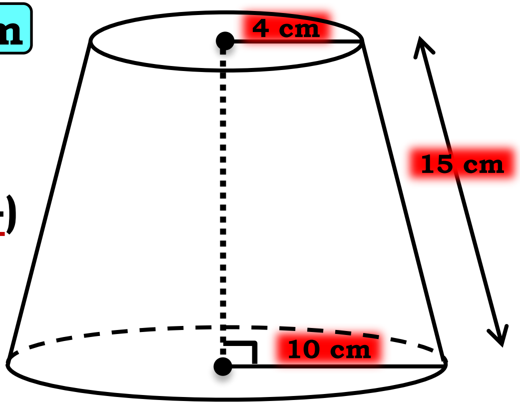
What is the formula to find area of top?



Q. A *fez*, the cap used by the Turks, is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 $\pi l\,(r_1 + r_2)$ slan πr_2^2 t is 15 cm, find the area of material use g it.

Area of material used = CSA of (Frustum) + Area of top

Sol. $r_1 = 10$ cm, $r_2 = 4$ cm, $l = 15$ cm



$$\text{CSA of (Frustum)} = \pi \boxed{l}\,(\boxed{r_1} + \boxed{r_2})$$

$$= \frac{22}{7} \times 15 \times (10 + 4)$$

$$= \frac{22}{\cancel{7}} \times 15 \times \overset{2}{\cancel{14}}$$

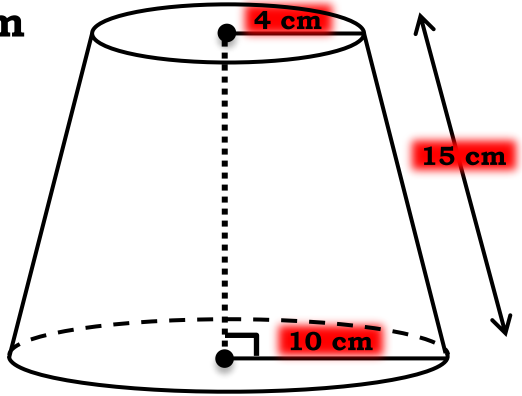
$$= 22 \times 15 \times 2$$

$$\therefore \text{CSA of (Frustum)} = 660 \text{ cm}^2$$

Q. A *fez*, the cap used by the Turks, is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 $\pi l\,(r_1 + r_2)$ slan $\pi\, r_2^{\,2}$ t is 15 cm, find the area of material use g it.

Area of material used = CSA of (Frustum) + Area of top

Sol. $r_1 = 10$ cm, $\boxed{r_2 = 4 \text{ cm}}$, $l = 15$ cm



$$\text{Area of top} = \pi \boxed{r_2}^2$$

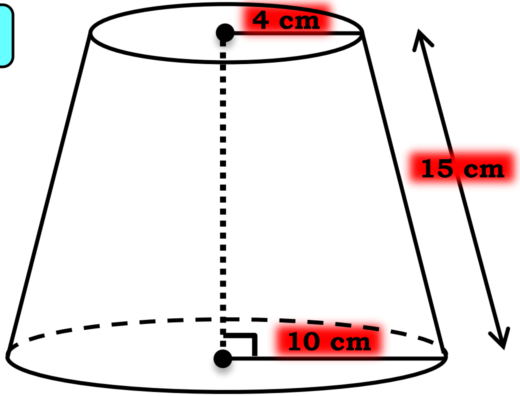
$$= \frac{22}{7} \times (4)^2$$

$$= \frac{22}{7} \times 16$$

$$\therefore \text{Area of top} = \frac{352}{7} \text{ cm}^2$$

Q. A *fez*, the cap used by the Turks, is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 $\pi l\,(r_1 + r_2)$ slan $\pi\, r_2^{\,2}$ t is 15 cm, find the area of material use it.

Area of material used = CSA of (Frustum) + Area of top



Sol. $r_1 = 10$ cm, $r_2 = 4$ cm, $l = 15$ cm

Area of material used = $\boxed{\text{CSA of (Frustum)}} + \boxed{\text{Area of top}}$

$$= 660 + \frac{352}{7}$$

$$= \frac{4972}{7}$$

$$= 710.28\text{cm}^2$$

$\boxed{\text{CSA of (Frustum)} = 660 \text{ cm}^2}$

∴ is 710.28 cm^2

$\boxed{\text{Area of top} = \frac{352}{7} \text{ cm}^2}$

Module 52

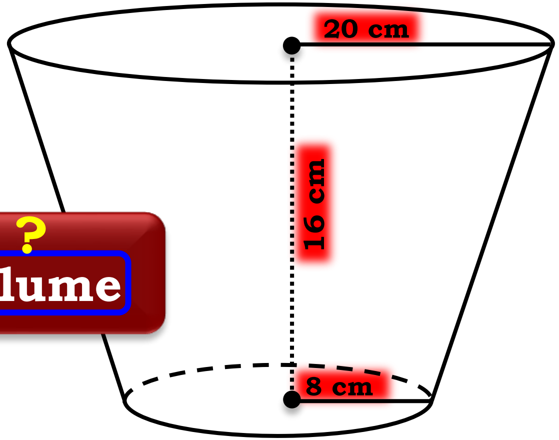
SURFACE AREAS AND VOLUMES

- Sum based on Frustum of Cone

Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm^2. (Take π = 3.14)

Sol. $h = 16$ cm, $r_1 = 8$ cm, $r_2 = 20$ cm

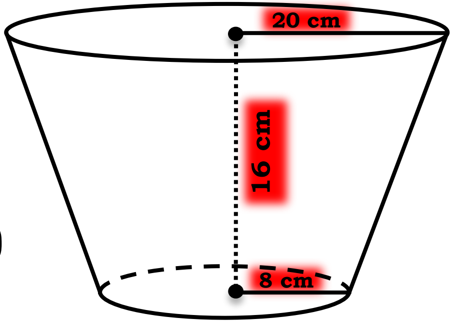
To find : Vol.



Cost = $\boxed{\text{Rate}} \times \boxed{\text{Volume}}$

Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm². (Take $\pi = 3.14$)

Sol. $h = 16$ cm, $r_1 = 8$ cm, $r_2 = 20$ cm **To find : Vol.**



Vol. of frustum of a cone $= \frac{1}{3} \pi h \left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times 3.14 \times 16 \times (8^2 + 20^2 + \underline{8 \times 20})$

What is the formula to find volume of the frustum of cone?

$\frac{1}{3} \pi h \left(r_1^2 + r_2^2 + r_1 r_2\right)$

$\times 16 \times (\underline{64 + 400 + 160})$

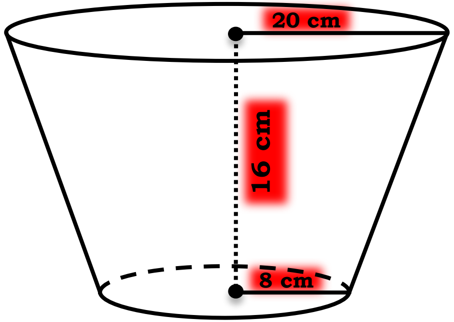
$= \frac{1}{\cancel{3}} \times \frac{314}{100} \times 16 \times \overset{208}{\cancel{624}}$

Vol. of frustum of a cone $= \frac{314 \times 16 \times 208}{100}$

Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm^2. (Take π = 3.14)

Sol. $h = 16$ cm, $r_1 = 8$ cm, $r_2 = 20$ cm

To find : Vol.



Vol. of frustum of a cone $= \dfrac{314 \times 16 \times 208}{100}$

1 litre = 1000 cm^3

$= \dfrac{1044992}{100}$

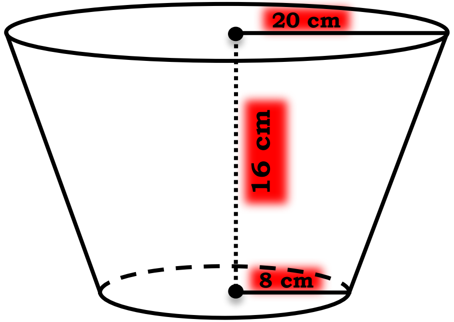
$= 10449.92\ cm^3$

$= \dfrac{10449.92}{1000}$ litres

$= 10.44992$ litres

$\therefore$ Vol. of frustum of a cone = 10.45 litres

Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm^2. (Take π = 3.14)



Sol. $h = 16$ cm, $r_1 = 8$ cm, $r_2 = 20$ cm

Vol. of frustum of a cone = 10.45 litres

Rate = Rs. 20 per litre

Cost = Rate × Volume

Cost = Rate × Volume × 10.45

= Rs. 209

∴ Cost of the milk which can completely fill the container is Rs. 209

Module 53

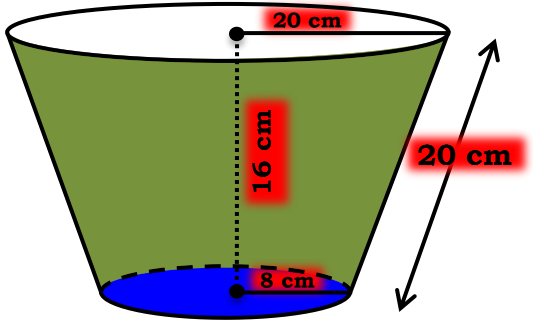
SURFACE AREAS AND VOLUMES

- **Sum based on Frustum of Cone (PART II)**

Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre ... he ... metal sheet used to make the container, if it ... er ... . (Take $\pi = 3.14$)

$\pi l\,(r_1 + r_2)$ $\quad \pi r_1^{\,2}$

Area of material used = CSA of (Frustum) + Area of base



Sol. $h = 16$ cm, $r_1 = 8$ cm, $r_2 = 20$ cm

$\therefore\ l = \sqrt{(r_1 - r_2)^2 + h^2}$

Cost = Rate × Area ?

$= \sqrt{(8-20)^2 + (16)^2}$

What is the formula to find area of base?

What is ... curved surface area of frustum?

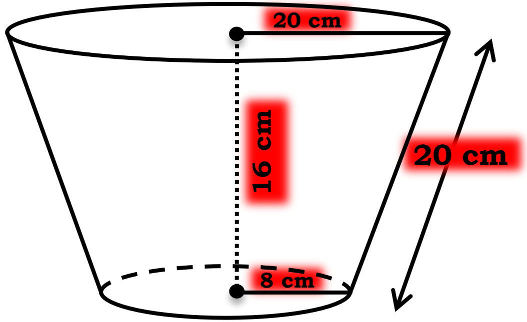
height of the cone

$= \sqrt{400}$

$\therefore\ l = 20$ cm

Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre $\pi l\,(r_1 + r_2)$ he $\pi\, r_1^{\,2}$ metal sheet used to make the container, if it ... er 1 ... . (Take $\boxed{\pi = 3.14}$)

Area of material used = CSA of (Frustum) + Area of base



Sol. $h = 16$ cm, $\boxed{r_1 = 8 \text{ cm}}$, $\boxed{r_2 = 20 \text{ cm}}$

Area of material used = CSA of (Frustum) + Area of base

$$= \pi \boxed{l}\,(\boxed{r_1} + \boxed{r_2}) + \pi \boxed{r_1}^2$$

$$= \pi \times 20 \times (8 + 20) + \pi \times (8)^2$$

$$= \pi \times [20 \times 28 + 64]$$

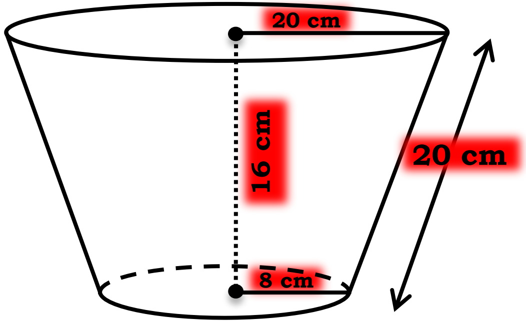
$$= \pi \times [560 + 64]$$

$$= \boxed{\pi} \times 624$$

$$= 3.14 \times 624$$

$\therefore$ Area of material used $= 1959.36 \text{ cm}^2$

Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm^2. (Take π = 3.14)



Sol. Area of material used = 1959.36 cm^2

Rate = Rs. 8 per 100 cm^2

Rate = Rs. $\frac{8}{100}$ per cm^2

Cost = Rate × Area

$= \frac{8}{100} \times 1959.36$

$= \frac{15674.88}{100}$

= Rs.156.75

Cost = Rate × Area

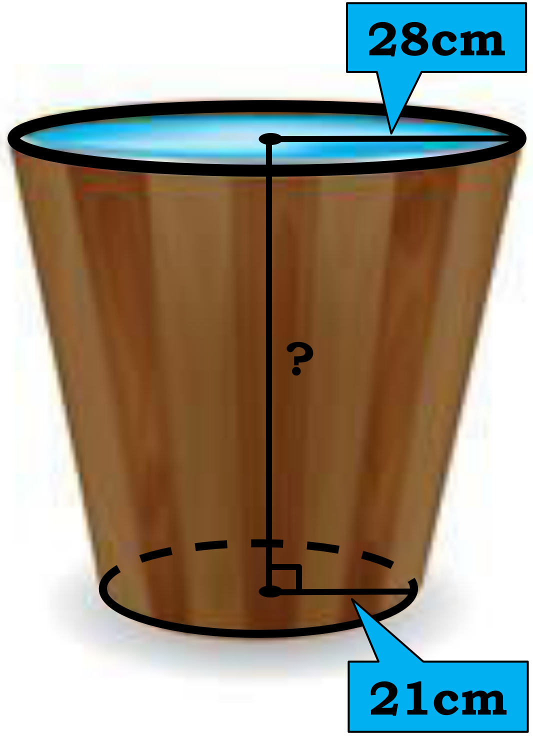
∴ Cost of metal sheet used to make the container is Rs. 156.75

Module 54

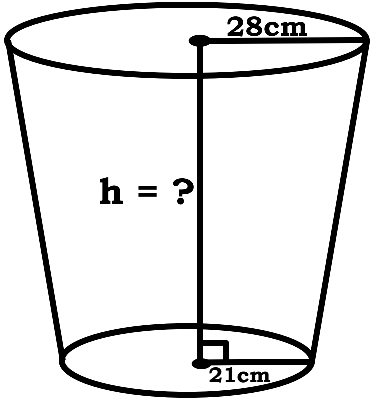
SURFACE AREAS AND VOLUMES

- Sum based on Frustum of cone

Q. A bucket in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.



Q. A bucket in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.



Sol. Let the height of the bucket be h cm.

$r_1 = 28$ cm, $r_2 = 21$ cm,

Volume of the bucket = 28.490 litres

= 28.490 × 1000

Let us convert litres into cm³

Volume of the bucket = 28490 cm³

Volume of the frustum $= \frac{1}{3}\pi h\left(r_1^2 + r_1 r_2 + r_2^2\right)$

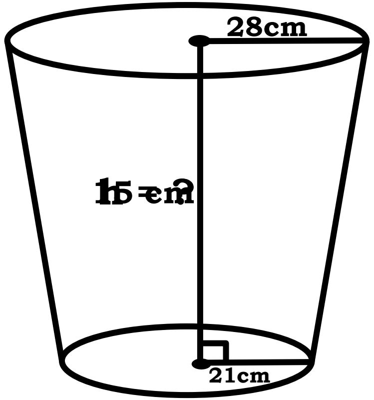
$28490 = \frac{1}{3} \times \frac{22}{7} \times h \times (28^2 + 28 \times 21 + 21^2)$

What is the formula to find volume of the frustum?

$\frac{1}{3}\pi h\left(r_1^2 + r_1 r_2 + r_2^2\right)$

$\frac{22}{21} \times h \times (784 + 588 + 441)$

Q. A bucket in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.



Sol.

$$28490 = \frac{22}{21} \times h \times (\underline{784 + 588 + 441})$$

$$\therefore \quad 28490 = \frac{22}{21} \times h \times 1813$$

$$\therefore \quad h = \frac{\overset{2590}{\cancel{28490}} \times \overset{3}{\cancel{21}}}{{}_{2}\cancel{22} \times \cancel{1813}\ {}_{259}}$$

$$\therefore \quad h = \frac{\overset{5\ \cancel{10}}{\cancel{2590}} \times 3}{\cancel{2} \times \cancel{259}}$$

$$\therefore \quad h = 15 \text{ cm}$$

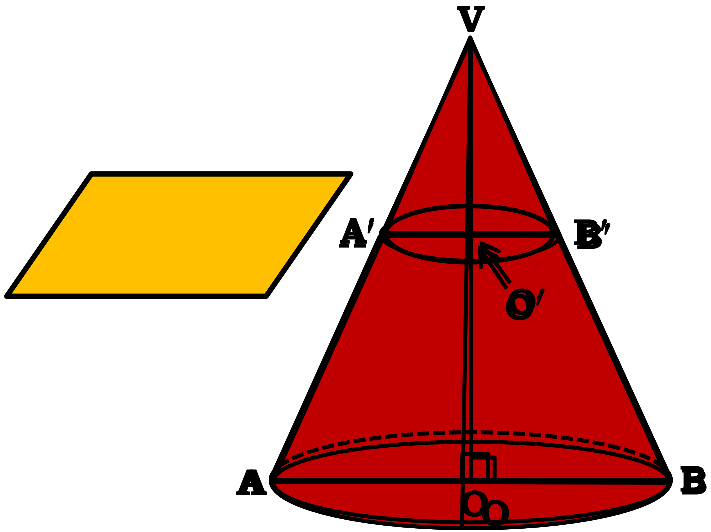
$\therefore$ **Height of the bucket is 15 cm**

Module 55

SURFACE AREAS AND VOLUMES

- Sum based on Frustum of cone

Q. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the line-segment into which the cone's altitude is divided by the plane.



Q. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the cur... ...ne, find the ratio of the line-segment into wh... ...vided by the plane.

Consider ΔVOA and $\Delta VO'A'$

Sol. Let VAB ... slant height L and base radius R.

Let VA′B′ be a small cone of height h, slant height l and base radius r.

To find: $\frac{\boxed{VO'}}{\boxed{O'O}}$ i.e $\frac{h}{H-h}$

... ΔVOA and $\Delta VO'A'$,

$O'O = \boxed{VO} - \boxed{VO'}$

$\therefore \; O'O = H - h$

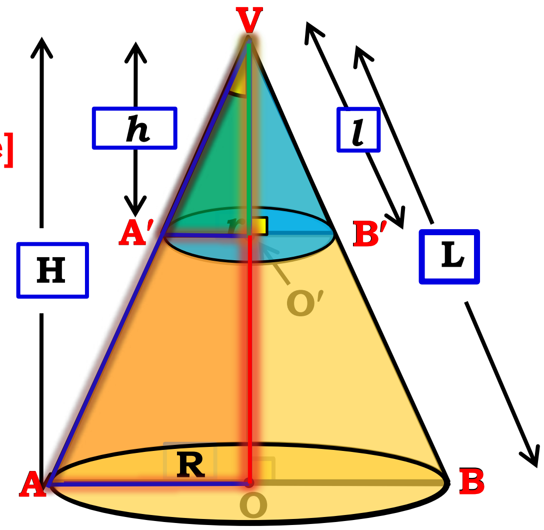
... VO' [common angle]

... $'A'$ [∵ each 90°]

$\therefore \; \Delta VOA \sim \Delta VO'A'$ [By AA test]

$$\frac{VO}{VO'} = \frac{OA}{O'A'} = \frac{VA}{VA'} \quad \text{[c.s.s.t.]}$$

$$\frac{H}{h} = \frac{R}{r} = \frac{L}{l} \quad \text{....(i)}$$



Q. A hollow cone is cut by a plane parallel to the base and the

u… ed surface of the remainder is $\frac{8}{9}$ of

t… find the ratio of the line-segment

i… ed by the plane.

What is the formula to find curved surface area of the cone ?

$\pi r l$

$$\frac{h}{H-h} = ?$$

$$\frac{H}{h} = \frac{R}{r} = \frac{L}{l}$$

Sol. CSA of the ABB′A′ = $\boxed{\text{CSA of VAB}} - \boxed{\text{CSA of VA′B′}}$

$$= \pi RL - \pi r l$$

$\therefore$ $\boxed{\text{CSA of the ABB′A′} = \pi(RL - rl)}$

$\boxed{\text{CSA of the ABB′A′}} = \frac{8}{9} \times \text{CSA of VAB}$

$$\therefore \cancel{\pi}(RL - rl) = \frac{8}{9} \times \cancel{\pi}RL$$

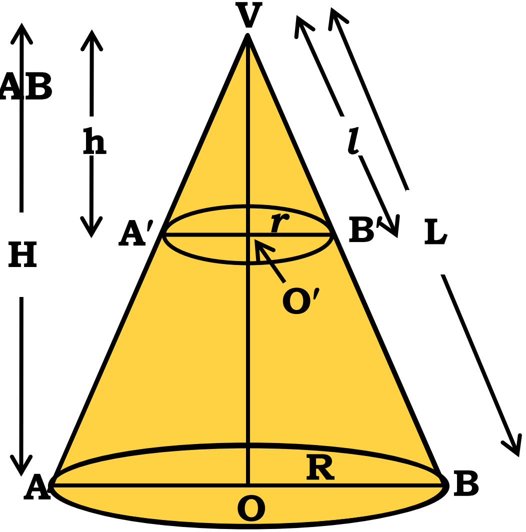
$$\therefore 9(RL - rl) = 8RL$$

$$\therefore 9RL - 9rl = 8RL$$

$$\therefore 9RL - 8RL = 9rl$$

$$\therefore RL = 9rl$$

$$\therefore \frac{RL}{rl} = \frac{9}{1}$$



Q. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the line-segment into which the cone's altitude is divided by the plane.

$$\frac{h}{H-h} = ?$$

$$\frac{H}{h} = \frac{R}{r} = \frac{L}{l}$$

Sol.

$$\frac{RL}{rl} = \frac{9}{1}$$

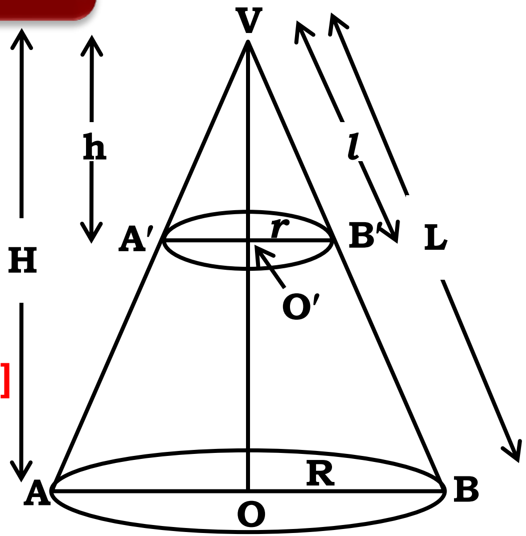
We want ratio in terms of height

$$\therefore \quad \boxed{\frac{R}{r}} \times \boxed{\frac{L}{l}} = \frac{9}{1}$$

$$\therefore \quad \frac{H}{h} \times \frac{H}{h} = \frac{9}{1}$$

$$\therefore \quad \frac{H^2}{h^2} = \frac{9}{1}$$

$$\therefore \quad \frac{H}{h} = \frac{3}{1}$$ [Taking square root]



Q. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the line-segment into which the cone's altitude is divided by the plane.

$$\frac{h}{H-h} = ?$$

Sol.

$$\frac{H}{h} = \frac{3}{1}$$ [Taking

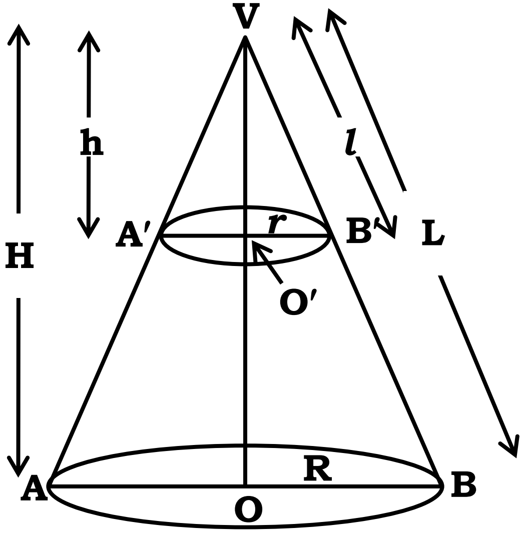
Subtracting 1 on both side

$$\therefore \quad \frac{H}{h} - 1 = \frac{3}{1} - 1$$

$$\therefore \quad \frac{H-h}{h} = \frac{3-1}{1}$$

$$\therefore \quad \frac{H-h}{h} = \frac{2}{1}$$

$$\therefore \quad \boxed{\frac{h}{H-h} = \frac{1}{2}}$$



Thank You