MATHS

$(a+b)^2$

ab+

Polynomials

1. What is a polynomial?

A **polynomial** $p(x)$ in one variable x is an algebraic expression in x of the form

$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots\dots + a_2 x^2 + a_1 x + a_0$, where

i. x is a variable

ii. $a_0, a_1, a_2, \dots\dots a_n$ are respectively the coefficients of x^0, x^1, x^2, x^3, …… x^n.

iii. Each of $a_n x^n, a_{n-1} x^{n-1}, a_{n-2} x^{n-2}, \dots\dots a_2 x^2, a_1 x, a_0$, with $a_n \neq 0$, is called the term of a polynomial.

2. The highest exponent of the variable in a polynomial determines the **degree** of the polynomial.

3. Types of polynomials

i. A polynomial of degree zero is called a **constant polynomial**. Examples: $-9x^0$, $\frac{5}{14}$.

ii. A polynomial of degree one is called a **linear polynomial**. It is of the form $ax + b$.
Examples: $x - 2$, $4y + 89$, $3x - z$.

iii. A polynomial of degree two is called a **quadratic polynomial**. It is of the form $ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$.
Examples: $x^2 - 2x + 5$, $x^2 - 3x$ etc.

iv. A polynomial of degree 3 is called a **cubic polynomial** and has the general form $ax^3 + bx^2 + cx + d$.
For example: $x^3 + 2x^2 - 2x + 5$ etc.

4. Value of the polynomial

If p(x) is a polynomial in x, and k is a real number then the value obtained after replacing x by k in p(x) is called the value of p(x) at x = k which is denoted by p(k).

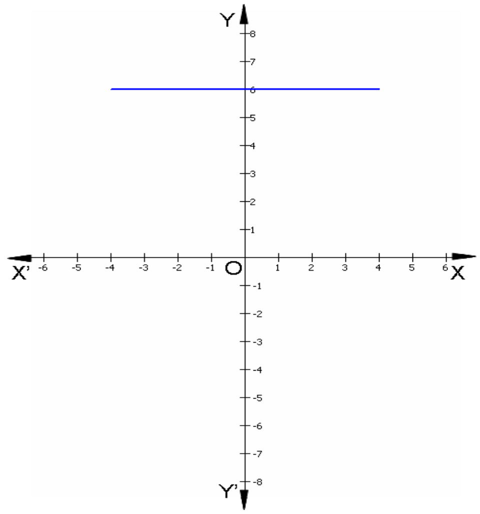
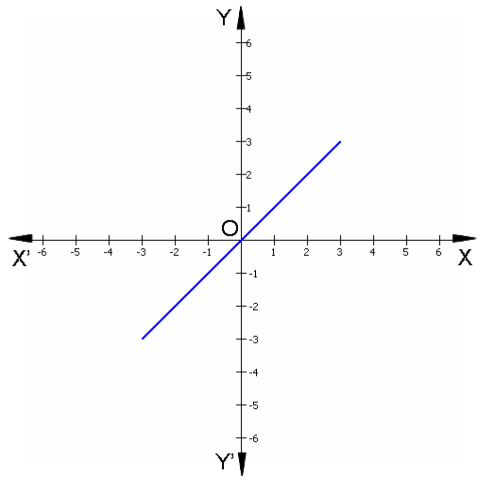
5. Zero of a polynomial

- A real number k is said to be the **zero** of the polynomial $p(x)$, if $p(k) = 0$.
- Zeroes of the polynomial can be obtained by solving the equation $p(x) = 0$.
- It is possible that a polynomial may not have a real zero at all.
- For any linear polynomial $ax + b$, the zero is given by the expression $(-b/a)$ = –(constant term)/(Coefficient of x).

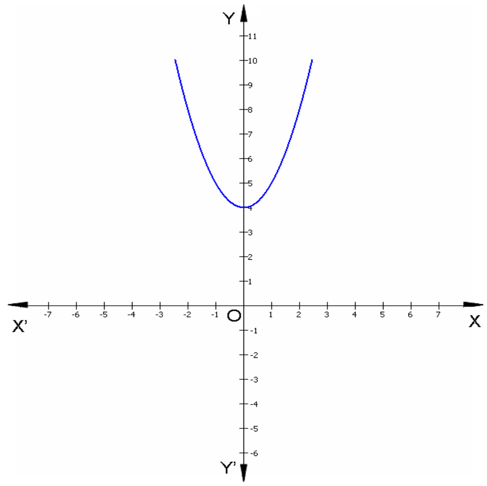
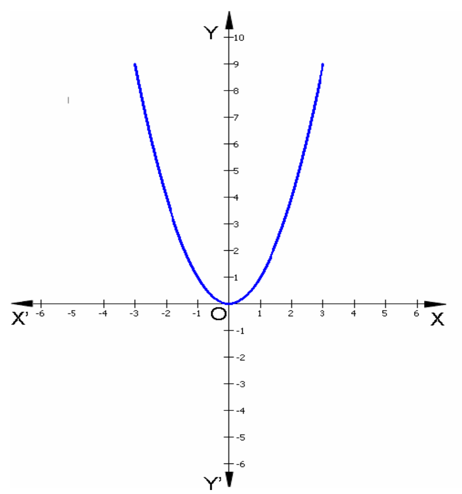
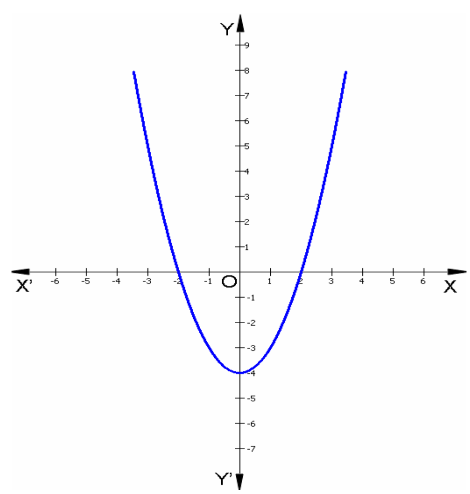
6. Number of zeroes of a polynomial

- The number of real zeros of the polynomial is the number of times its graph touches or intersects x-axis.
- The graph of a polynomial $p(x)$ of degree n intersects or touches the x-axis at at most n points.
- A polynomial of degree n has at most ***n* distinct real zeroes.**

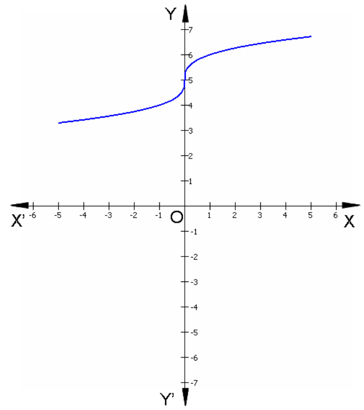
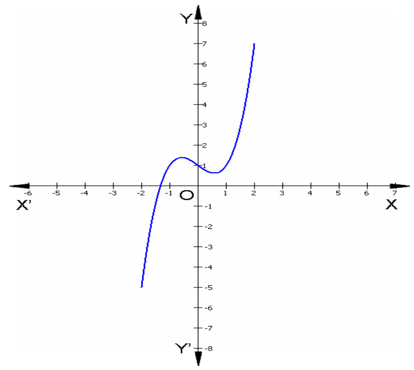
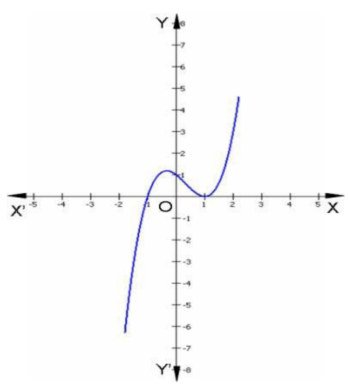
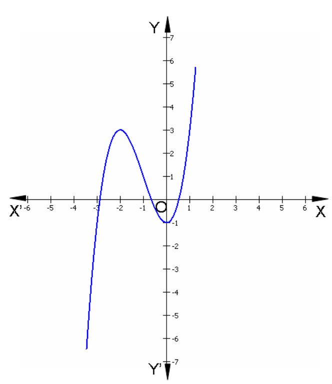
7. A linear polynomial has at most one real zero.

Linear Polynomial having **no zero**.	
Linear Polynomial having **one zero**.	

8. A quadratic polynomial has at most two real zeroes.

Quadratic Polynomial having **no zeroes**.	
Quadratic Polynomial having **one zero**.	
Quadratic Polynomial having **two zeroes**.	

9. A cubic polynomial has at most three real zeroes.

Cubic Polynomial having **no zeroes**.	
Cubic Polynomial having **one zero**.	
Cubic Polynomial having **one zeroes**.	
Cubic Polynomial having **three zeroes**.	

10. **Relationship between zeroes and coefficients** of a polynomial:

   i. For a linear polynomial $ax + b$, $a \neq 0$, the zero is $x = \frac{-b}{a}$. It can be observed that:

   $$\frac{-b}{a} = -\frac{\text{constant term}}{\text{Coefficient of } x}$$

   ii. For a **quadratic polynomial** $ax^2 + bx + c$, $a \neq 0$,

   Sum of the zeroes $= -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

   Product of the zeroes $= \frac{c}{a} = \frac{\text{constant term}}{\text{Coefficient of } x^2}$

   iii. For a **cubic polynomial** $ax^3 + bx^2 + cx + d = 0$, $a \neq 0$,

   Sum of zeroes $= \frac{-b}{a} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$,

   Sum of the product of zeroes taken two at a time $= \frac{c}{a} = \frac{\text{Coefficient of x}}{\text{Coefficient of } x^3}$

   Product of zeroes $= -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

11. The quadratic polynomial whose sum of the zeroes = $(\alpha + \beta)$ and product of zeroes = $(\alpha\beta)$ is given by: $\boldsymbol{k\,[x^2 - (\alpha + \beta)\,x + (\alpha\beta)]}$, where k is real.

   If a , b and g are the zeroes of a cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$, a ¹ 0 , then

   $f(x) = k\,(x - a)(x - b)(x - g)$

   $f(x) = k\left\{x^3 - (a + b + g)x^2 + (ab + bg + ga)x - abg\right\}$,

   where k is any non-zero real number.

12. Process of dividing a polynomial $f(x)$ by another polynomial $g(x)$ is as follows:

   Step 1: To obtain the first term of the quotient, divide the highest degree term of the dividend by the highest degree term of the divisor. Then carry out the division process.
   Step 2: To obtain the second term of the quotient, divide the highest degree term of the new dividend by the highest degree term of the divisor. Then again carry out the division process
   Step 3: Continue the process till the degree of the new dividend is less that the degree of the divisor. This will be called the remainder.

13. **Division Algorithm for polynomials:** If $f(x)$ and $g(x)$ are any two polynomials, where $g(x) \neq 0$, then there exists the polynomials $q(x)$ and $r(x)$ such that $f(x) = g(x)\,q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

   So, $q(x)$ is the quotient and $r(x)$ is the remainder obtained when the polynomial $f(x)$ is divided by the polynomial $g(x)$.

14. Factor of the polynomial

If $f(x) = g(x)\, q(x) + r(x)$ and $r(x) = 0$, then polynomial $g(x)$ is a **factor of the polynomial** $f(x)$.

15. Finding zeroes of a polynomial using division algorithm

Division algorithm can also be used to find the **zeroes of a polynomial**. For example, if 'a' and 'b' are two zeroes of a fourth degree polynomial $f(x)$, then other two zeroes can be found out by dividing $f(x)$ by $(x-a)(x-b)$.