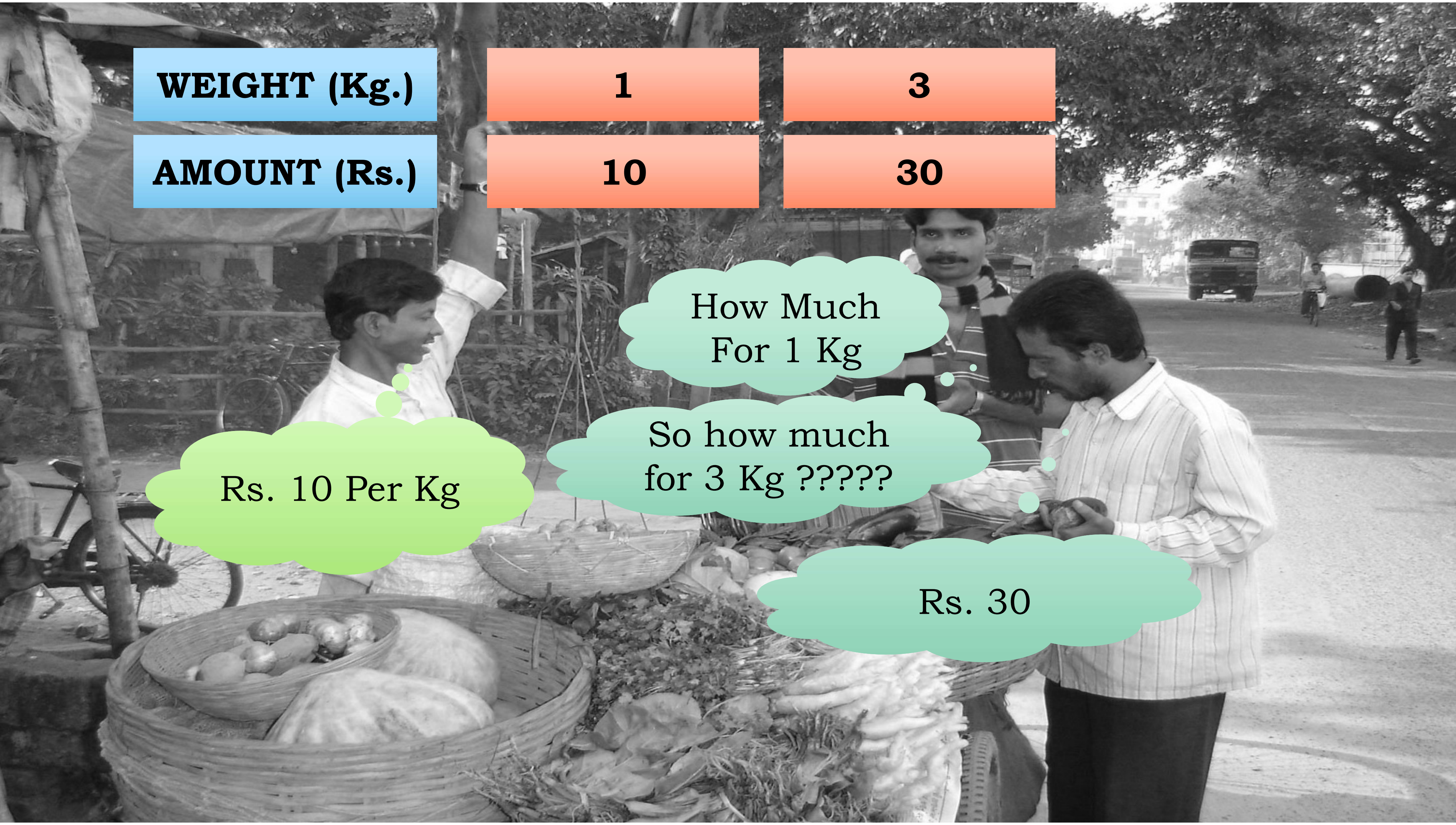
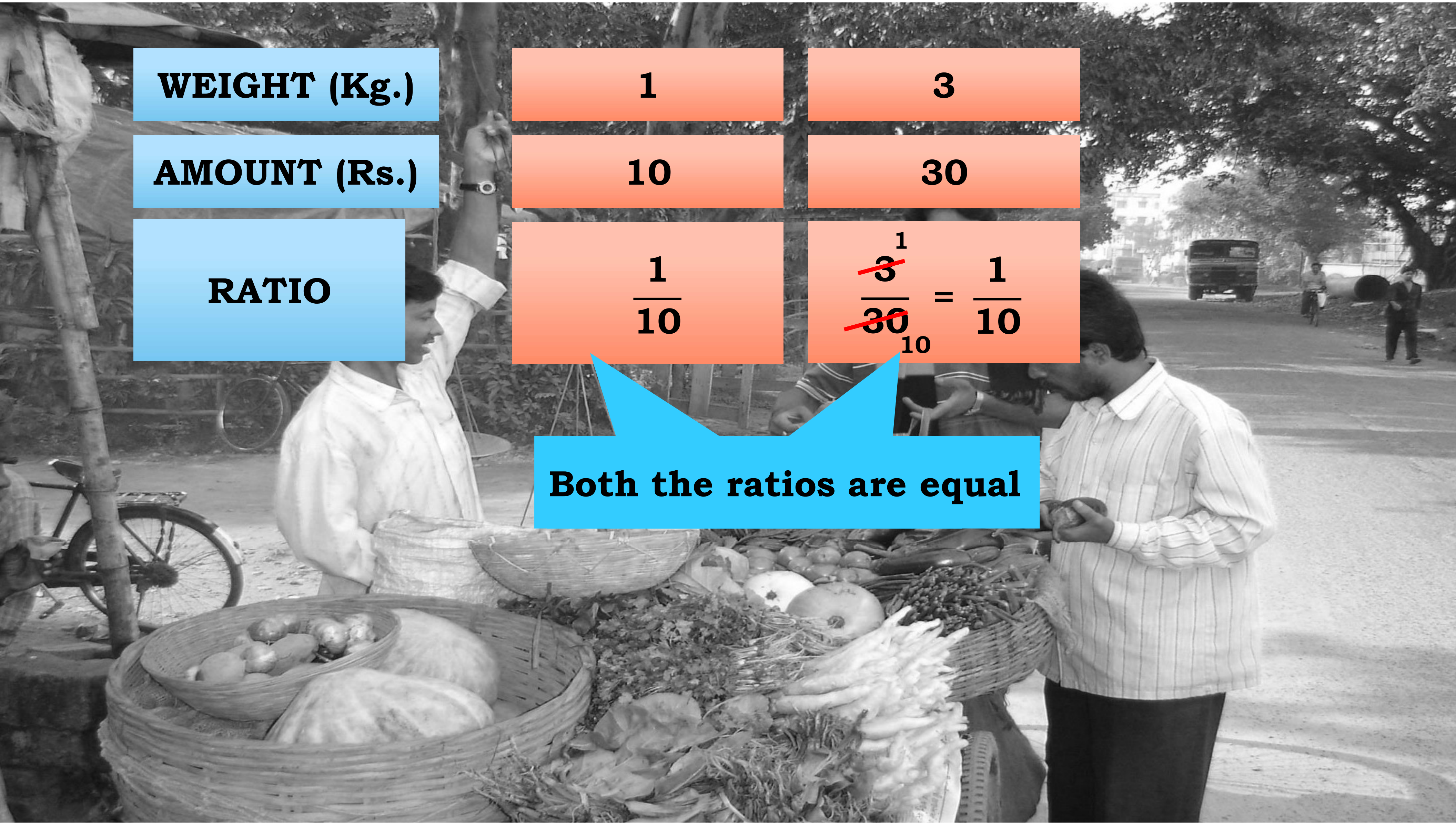
13. Direct and Inverse Proportions

Direct Proportions

WEIGHT (Kg.)	1	3
AMOUNT (Rs.)	10	30





WEIGHT (Kg.)	1	3
AMOUNT (Rs.)	10	30
RATIO	$\frac{1}{10}$	$\frac{\overset{1}{\cancel{3}}}{\underset{10}{\cancel{30}}} = \frac{1}{10}$

Both the ratios are equal

DIRECT VARIATIONS

Time (hours)	1	2	3
Distance (km)	60	120	180

$$\frac{180}{3} = 60$$

$$\therefore \frac{\text{Distance}}{\text{Time}} = 60 = 60 = 60$$

This is the case of

In direct Variation ratio of Quantities remain constant

DIRECT VARIATIONS AND DIRECT PROPERTION

WEIGHT Kg.	1	3
AMOUNT Rs.	`10	`30

We MAY say this is a case of direct variation

But how to confirm ?

If the ratio of the two quantities remain constant, this is a case of direct variation

Weight

Amount

If one variable increases and other variable also increases

DIRECT VARIATIONS AND DIRECT PROPERTION

‘y’ varies directly as ‘x’

$$y \propto x$$

$$\therefore \quad \frac{y}{x} = k$$

Let us remove ‘α’(alpha) sign

Whenever we have ‘k’, our first job is to remove ‘k’

In direct variation, the ratio is constant.

Example :

- The price of the scooter was 34000 last year. If it has increased by 20% this year, what is the price now ?

Let us take an example

The original price of the scooter

Sol : Original price of the scoote

Increase in % = 20%

The amount is calculated as

20% of original price = 20% of 34000

$$= \frac{20}{1\cancel{0}\cancel{0}} \times 340\cancel{0}\cancel{0}$$

$$= 20 \times$$

$$= 6800$$

Here, increase % is given. Hence we will calculate the amount of increase

Let us see how to calculate increase/decrease in%

New price = Original price + Increase

= 34000 + 6800

= Rs 40,800

EXERCISE 13.1

❍ **Following are the car parking charges near a railway station upto**

4 hours	Rs. 60
8 hours	Rs. 100
12 hours	Rs. 140
24 hours	Rs. 180

Check if the parking are in direct proportion to the parking

Therefore we will take ratio of the quantities

Let us reduce the ratio in the lowest form

Which are the two quantities that are varying ?

If the parking time increases then the parking charges will also increase

Sol :

Parking time	Parking charges	
4 hours	Rs 60	$\frac{\overset{15}{\cancel{60}}}{\underset{1}{\cancel{4}}} = \boxed{\frac{15}{1}}$
8 hours	Rs 100	$\frac{\overset{25}{\cancel{100}}}{\underset{2}{\cancel{8}}} = \boxed{\frac{25}{2}}$
12 hours	Rs 140	$\frac{\overset{35}{\cancel{140}}}{\underset{2}{\cancel{12}}} = \boxed{\frac{35}{3}}$
24 hours	Rs 180	$\frac{\overset{25}{\cancel{180}}}{\underset{2}{\cancel{24}}} = \boxed{\frac{25}{2}}$

Since $\frac{15}{1} \neq \frac{25}{2} \neq \frac{35}{3} \neq \frac{25}{2}$

$\therefore$ **The parking charges are not in direct proportion with the parking time.**

EXERCISE 13.1

❍ A Mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table. Find the parts of base that need to be added

Parts of red pigment	x_1 1	x_2	x_3	x_4	x_5
Parts of base	y_1 8	y_2	y_3	y_4	y_5

Sol :

Let the red pigment be represented by $x_1, x_2, x_3,..$
And parts of base be denoted by $y_1, y_2, y_3,,..$
As the part of red pigment increase, the required number of bases will also increase
It is a case of direct proportion.

Therefore we will take the ratio of the quantities

i.e. $$\frac{\boxed{x_1}}{\boxed{y_1}} = \frac{x_2}{y_2} = \frac{x_3}{y_3} \;....$$

For $x_1 = 1, \quad y_1 = 8$

$$\therefore \quad \frac{x_1}{y_1} = \frac{1}{8}$$

EXERCISE 13.1

❍ **A Mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table. Find the parts of base that need to be added**

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	….	….	….

Sol :

$x_2 = 4, \ y_2 = ?$

Now, $\dfrac{\boxed{x_2}}{\boxed{y_2}} = \dfrac{1}{8}$

$\therefore \quad \dfrac{4}{y_2} = \dfrac{1}{8}$

$\therefore \quad y_2 = 4 \times 8$

$\therefore \quad \boxed{y_2 = 32}$

EXERCISE 13.1

○ **A Mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table. Find the parts of base that need to be added**

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	? 56	....	....

Sol :

$x_2 = 7,\ y_2 = ?$

Now, $\frac{\boxed{x_3}}{\boxed{y_3}} = \frac{1}{8}$

$\therefore \quad \frac{7}{y_3} = \frac{1}{8}$

$\therefore \quad y_3 = 7 \times 8$

$\therefore \quad \boxed{y_3 = 56}$

EXERCISE 13.1

❍ **A Mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table. Find the parts of base that need to be added**

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	56	? 96	....

Sol :

$x_2 = 12,\ y_2 = ?$

Now, $\frac{\boxed{x_4}}{\boxed{y_4}} = \frac{1}{8}$

$\therefore \quad \frac{12}{y_4} = \frac{1}{8}$

$\therefore \quad y_4 = 12 \times 8$

$\therefore \quad \boxed{y_4 = 96}$

EXERCISE 13.1

❍ **A Mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table. Find the parts of base that need to be added**

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	56	96	1?0

Sol :

$x_2 = 20,\ y_2 = ?$

Now, $\frac{\boxed{x_4}}{\boxed{y_4}} = \frac{1}{8}$

$\therefore \quad \frac{20}{y_4} = \frac{1}{8}$

$\therefore \quad y_4 = 20 \times 8$

$\therefore \quad \boxed{y_4 = 160}$

$\therefore$ **Thus the required part of base are 8, 32, 56, 96, 160.**

EXERCISE 13.1

❍ **In question 2 above, if a part of a red pigment requires 75mL of base, how much red pigment we mix with 1800 ml of base?**

Sol :

We have, $\frac{\boxed{x_1}}{\boxed{y_1}} = \frac{\boxed{1}}{\boxed{75}} = \frac{x_2}{y_2}$

Here, $x_1 = 1$ and $y_1 = 75$

$$\frac{1}{75} = \frac{x_2}{1800}$$

$$\therefore \quad x_2 = \frac{\cancel{1800}^{\cancel{360}\ \cancel{72}\ 24}}{\cancel{75}_{\cancel{15}\ \cancel{3}\ 1}}$$

$\therefore$ **Thus, the required red pigment = 24 parts.**

Which are the two quantities that are varying ?

Parts of red pigment **And** **Parts of base**

case
on

EXERCISE 13.1

❍ A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?

Sol :

Number of bottles filled	Number of hours
840	6
x	5

the tw

are va

Th

Therefore we will

Lets assume it as x

the quantities

of bottles filled

And

Number of hours

For more number of hours, more number of bottles would be filled.

$$\frac{840}{x} = \frac{6}{5}$$

$$\therefore \quad 6x = 5 \times 840$$

$$\therefore \quad x = \frac{5 \times \cancel{840}^{140}}{\cancel{6}_{1}}$$

$$\therefore \quad x = 700$$

∴ **Thus, the required number of bottles = 700**

EXERCISE 13.1

❍ A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm as shown in the diagram. What is the actual length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?

Which are the two quantities that are varying ?

Length And

Lets assume it as x

Sol :

Length	Enlarged length
5	50000
x	20000

Actual length of bacteria $= \dfrac{5}{50000} = \dfrac{5}{10000}$ cm

$= 10^{-4}$ cm

Here length and enlarged length of bacteria are indirect proportion

$$\frac{5}{50000} = \frac{x}{20000}$$

$\therefore \quad 50000x = 5 \times 20000$

$\therefore \quad x = \dfrac{\overset{1}{\cancel{5}} \times \cancel{20000}^{\,2}}{\cancel{50000}\ \cancel{10000}\ 1}$

$\therefore \quad x = 2$

$\therefore$ Hence enlarged length of bacteria is 2 cm.

EXERCISE 13.1

❍ **In model of ship, the mast is $\boxed{9\text{ cm}}$ high, while the mast of actual ship is $\boxed{12\text{ cm}}$ high, if length of ship is $\boxed{28\text{ cm}}$, how long is the model ship ?**

Sol :

Length of the ship	Height of the mast
28	12
x	9

the two
are vary

the ship **And** **Height of the mast**

Th

Therefore we will

Lets assume it as x

the quantities

Since, more the length of the ship,
More would be the length of its mast.

$$\frac{28}{x} = \frac{12}{9}$$

$$\therefore \quad 12x = 28 \times 9$$

$$\therefore \quad x = \frac{\overset{7}{\cancel{28}} \times \cancel{9}\ 3}{\cancel{12}\ \cancel{4}\ 1}$$

$$\therefore \quad x = 21$$

∴ **Thus, the required length of the model = 21 cm**

EXERCISE 13.1

❍ Suppose 2 kg of sugar contains 9 × 106 crystal. How many sugar crystals are there in (i) 5 kg of sugar : (ii) 1.2 kg of sugar ?

Sol :

Number of bottles filled	Numb
2	9
5	

Therefore we will

me it as x

antities

Which are the two quantities that are varying ?

x **Weight of sugar** And **Number of crystal**

For more number of hours, more number

$$\frac{2}{5} = \frac{9 \times 10^6}{x}$$

$$\therefore \quad 2x = 5 \times 9 \times 10^6$$

$$\therefore \quad x = \frac{5 \times 9 \times 10^6}{2}$$

$$\therefore \quad x = \frac{\overset{22.5}{\cancel{45}} \times 10^6}{\underset{1}{\cancel{2}}}$$

$$\therefore \quad x = 22.5 \times 10^6$$

∴ **Thus, the required length of sugar crystal = 22.5×10^6**

EXERCISE 13.1

❍ Suppose 2 kg of sugar contains 9 × 106 crystal. How many sugar crystals are there in (i) 5 kg of sugar : (ii) 1.2 kg of sugar ?

Sol :

Number of bottles filled	Number of hours
2	9×10^6
1.2	y

Th

Therefore we will

Lets assume it as y

the quantities

For more number of hours, more number of bottles would be filled.

$$\frac{2}{1.2} = \frac{9 \times 10^6}{y}$$

∴ $2y = 1.2 \times 9 \times 10^6$

∴ $y = \frac{1.2 \times 9 \times 10^6}{2}$

∴ $y = \frac{\overset{5.4}{\cancel{10.8}} \times 10^6}{\underset{1}{\cancel{2}}}$

∴ $y = 5.4 \times 10^6$

∴ **Thus, the required length of sugar crystal = 5.4×10^6**

EXERCISE 13.1

❍ **Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?**

Sol :

Actual distance in km	Distance covere in map (in cm)
18	1
72	x

Which are the two quantities that are varying ?

Actual distance in km And **Distance covered in map (in cm)**

Here actual distance and distance covered in the map are in direct proportion.

$$\frac{18}{1} = \frac{72}{x}$$

$$\therefore \quad 18 \times x = 72 \times 1$$

$$\therefore \quad x = \frac{\overset{\cancel{24}\ 4}{\cancel{72}} \times 1}{\cancel{18}\ \cancel{2}\ 1}$$

$$\therefore \quad x = 4$$

∴ **Here distance covered in the map is 4**

EXERCISE 13.1

❍ A 5m 60cm high vertical pole casts a shadow 3m 20cm long. Find at the same time
(i) the length of the shadow cast by another pole 10m 50cm high and
(ii) the height of a pole which casts a shadow 5m long.

Sol :

Height of a pole	Length of a shadow
5m 60cm = 560	3m 20cm = 320
10m 60cm = 1050	x cm

As the highest of the pole increases the length of its shadow also increases in the same ratio.

$$\frac{560}{1050} = \frac{320}{x}$$

∴ $560 \times x = 1050 \times 320$

∴ $x = \dfrac{\overset{4\ \cancel{8}}{\cancel{320}} \times \overset{150}{\cancel{1050}}}{\underset{1\ \cancel{7}\ \cancel{14}}{\cancel{560}}}$

∴ $x = 4 \times 150$

∴ $x = 600$

∴ **Thus, the required length of the shadow is 600 cm of 6m**

Whic

The
c

Therefore we will take the ratio of the quantities

Actu
distance in km

And

d
in map (in cm)

∴ 1m = 100cm
∴ 5m = 500cm
∴ 5m 60cm = 500 + 60
∴ 5m 60cm = 560

∴ 1m = 100cm
∴ 10m = 1000cm
∴ 10m 50cm = 1000 + 50
∴ 10m 50cm = 1050

∴ 1m
∴ 3m
∴ 3m 20cm
∴ 3m 20cm

EXERCISE 13.1

❍ A 5m 60cm high vertical pole casts a shadow 3m 20cm long. Find at the same time
(i) the length of the shadow cast by another pole 10m 50cm high and
(ii) the height of a pole which casts a shadow 5m long.

Sol :

Height of a pole	Length of a shadow
5m 60cm = 560	3m 20cm = 320
y cm	5m = 500

Lets assume it as *y*

As the highest of the pole increases the length of its shadow also increases in the same ratio.

$$\frac{560}{y} = \frac{320}{500}$$

$$\therefore \quad 320 \times y = 560 \times 500$$

$$\therefore \quad x = \frac{\overset{125}{\cancel{560}} \times \overset{7}{\cancel{500}}}{1\,\cancel{4}\,\cancel{320}}$$

$$\therefore \quad x = 125 \times 7$$

$$\therefore \quad x = 875$$

∴ Thus, the required height of a pole is 875cm or 8m 75 cm

EXERCISE 13.1

❍ **A loaded truck travels $\boxed{14 \text{ km}}$ in $\boxed{25 \text{ minutes}}$. If the speed remains the same, how far can it travel in 5 hours?**

Sol : Let distance covered in $\boxed{5 \text{ hours be } x \text{ km.}}$

$\therefore$ 1 hour = 60 minutes

$\therefore$ 5 hours = 5 × 60 = $\boxed{300 \text{ minutes}}$

Distance (in km)	Time (in minutes)
14	25
x	300

Here distance covered and time in direct proportion

$$\therefore \quad \frac{14}{25} = \frac{x}{300}$$

$$\therefore \quad x \times 25 = 14 \times 300$$

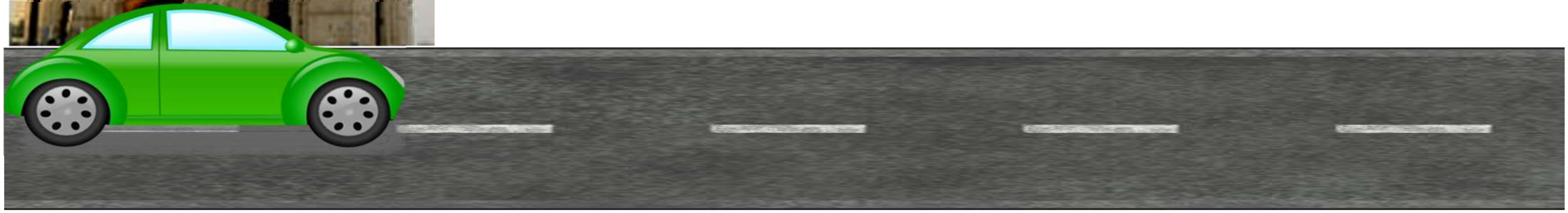
$$\therefore \quad x = \frac{14 \times \cancel{300}\ \cancel{60}\ 12}{1\ \cancel{5}\ \cancel{25}}$$

$$\therefore \quad x = 168 \text{ cm}$$

$\therefore$ **Hence, the distance covered in 5 hours is 168 km.**

SPEED	TIME TAKEN
40 km/hr	4 hours

SPEED	TIME TAKEN
40 km/hr	4 hours
80 km/hr	2 hours



SPEED	TIME TAKEN	PRODUCT
40 km/hr	4 hours	$40 \times 4 = 160$
80 km/hr	2 hours	$80 \times 2 = 160$

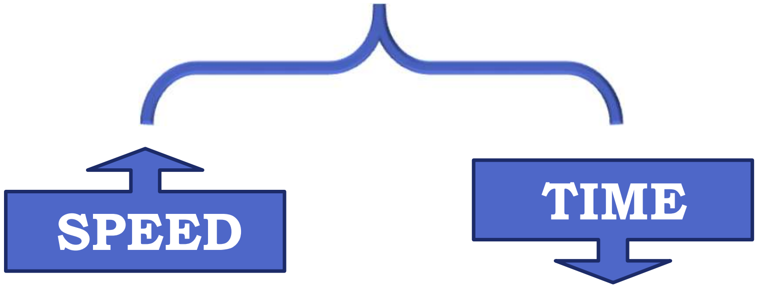
INVERSE VARIATION
OR
INVERSE PROPORTION

We **MAY** say this is a case of inverse variation

If the product of the two quantities remain constant, this is a case of inverse variation

If one variable increases and other variable decreases.

But how to confirm ?



SPEED

TIME

INVERSE VARIATION
OR
INVERSE PROPORTION

$$x \propto \frac{1}{y}$$

'*x*' varies inversely as 'y'

Let us remove '∝'(alpha) sign

$= k$ … **[where k is a non-zero constant of variation]**

$$\therefore \quad x \times y = k$$

Whenever we have 'k', our first job is to remove 'k'

In inverse variation, the product is constant.

Inverse Proportions

EXERCISE 13.2

❍ **Which of the following are in inverse proportion ?**

1. The number of workers on a job and the time to complete the job

Sol : **If number of workers are increased then time to complete the job would decrease.**

Hence, it is not a case if inverse variation

2. The time taken for a journey and the distance travelled in a uniform speed.

Sol : **For longer distance, more time would be required.**

Hence, it is not a case if inverse variation

3. Area of cultivated land and the crop harvested.

Sol : **It is a direct proportion because more are of cultivated land will yield more crops.**

EXERCISE 13.2

❍ **Which of the following are in inverse proportion ?**

4. The time taken for a fixed journey and the speed of the vehicle

Sol : **Time and speed are inverse proportion because if time is less, speed is more.**

5. The population of a country and the area of land per person.

Sol : **For more population, less area per person would be required.**

Hence, it is not a case if inverse variation

EXERCISE 13.2

- In a television game show, the prize money of … ually amongst the winners. Compete the following t… money given to an individual winner is directl… e number of winners ?

Which are the two quantities that are varying ?

Number of winners And **Prize for each winner (in Rs.)**

Number of winners	1	2	~~x_1~~ 4	x_2	x_3	x_4	x_5
Prize for each winner (in Rs.)	1,00,000	50,000	25000	y_2	y_3	y_4	y_5

Sol : If more the numbers of winners, less …

will

to

x_1 =

Let the number of winners be represented by x_1, x_2, x_3

And the prize of each winner represented by y_1, y_2, y_3

$$\therefore \quad x_1 \times y_1 = 1 \times 100000$$

$$\therefore \quad 4 \times y_1 = 1 \times 100000$$

$$\therefore \quad y_1 = \frac{1 \times \cancel{100000}^{25000}}{\cancel{4}_{1}}$$

$$\therefore \quad y_1 = 25000$$

$$\therefore \quad \boxed{y_1 = 25,000}$$

EXERCISE 13.2

- In a television game show, the prize money of Rs. 1,00,000 is to be divided equally amongst the winners. Compete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners ?

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs.)	1,00,000	50,000	25000	20?00			

Sol :

If more the numbers of winners, less is the prize money

Let us reduce it to the lowest form

$x_1 = 5, y_1 = ?$

$\therefore \quad x_1 \times y_1 = 1 \times 100000$

$\therefore \quad 5 \times y_1 = 1 \times 100000$

$\therefore \quad y_1 = \dfrac{1 \times \overset{20000}{\cancel{100000}}}{\underset{1}{\cancel{5}}}$

$\therefore \quad y_1 = 20000$

$\therefore \quad \boxed{y_1 = 20,000}$

EXERCISE 13.2

❍ **In a television game show, the prize money of Rs. 1,00,000 is to be divided equally amongst the winners. Compete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners ?**

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs.)	1,00,000	50,000	25000	20000	12?00		

Sol :

If more the numbers of winners, less is the prize money

The
c

Let us reduce it to the lowest form

$x_1 = 8, y_1 = ?$

$\therefore \quad x_1 \times y_1 = 1 \times 100000$

$\therefore \quad 8 \times y_1 = 1 \times 100000$

$\therefore \quad y_1 = \dfrac{1 \times \overset{12500}{\cancel{100000}}}{\underset{1}{\cancel{8}}}$

$\therefore \quad y_1 = 12,500$

$\therefore \quad \boxed{y_1 = 12,500}$

EXERCISE 13.2

❍ In a television game show, the prize money of Rs. 1,00,000 is to be divided equally amongst the winners. Compete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners ?

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs.)	1,00,000	50,000	25000	20000	12500	10?00	

Sol :

If more the numbers of winners, less is the prize money

The
c

Let us reduce it to
the lowest form

$x_1 = 8, y_1 = ?$

$$\therefore \quad x_1 \times y_1 = 1 \times 100000$$

$$\therefore \quad 10 \times y_1 = 1 \times 100000$$

$$\therefore \quad y_1 = \frac{1 \times \cancel{100000}^{10000}}{\cancel{10}_{1}}$$

$$\therefore \quad y_1 = 10,000$$

$$\therefore \quad \boxed{y_1 = 10000}$$

EXERCISE 13.2

- In a television game show, the prize money of Rs. 1,00,000 is to be divided equally amongst the winners. Compete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners ?

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs.)	1,00,000	50,000	25000	20000	12500	10000	5000

Sol :

If more the numbers of winners, less is the prize money

Let us reduce it to the lowest form

$\boxed{x_1 = 8, y_1 = ?}$

$\therefore \quad x_1 \times y_1 = 1 \times 100000$

$\therefore \quad 20 \times y_1 = 1 \times 100000$

$\therefore \quad y_1 = \dfrac{1 \times \cancel{100000}^{5000}}{\cancel{20}_{1}}$

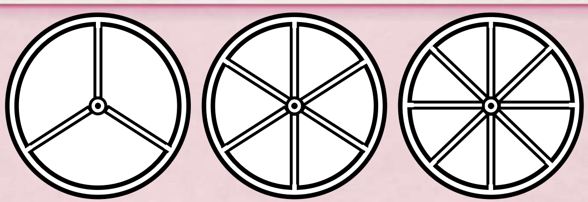
$\therefore \quad y_1 = 5,000$

$\therefore \quad \boxed{y_1 = 5,000}$

EXERCISE 13.2

❍ **Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spoke are equal. Help him by completing the following table.**

Number of spokes	4	6	8	x_2	x_3
Angle between a pair of consecution spokes	90°	60°	4?°	y_2	y_3



(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion ?

$x_1 = 8$, $y_1 = ?$

Sol :

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion

Which are th

that are

Let the number of spokes be represented by x_1, x_2, x_3

ce it to

form

If more the numbers of winners, less is the prize money.

Number of spokes

And

Angle between a pair of consecutive spokes

$$\therefore \quad x_1 \times y_1 = 4 \times 90^\circ$$

$$\therefore \quad 8 \times y_1 = 4 \times 90^\circ$$

$$\therefore \quad y_1 = \frac{\overset{1}{\cancel{4}} \times 90^\circ}{\cancel{8}_{2}}$$

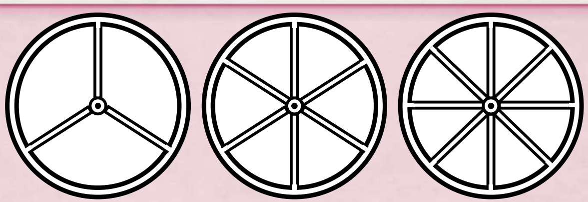
$$\therefore \quad y_1 = 45^\circ$$

$$\therefore \quad \boxed{y_1 = 45^\circ}$$

EXERCISE 13.2

❍ **Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spoke are equal. Help him by completing the following table.**

Number of spokes	4	6	8	10	12
Angle between a pair of consecution spokes	90°	60°	45°	3?°	



(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion ?

will

take the product

$x_1 = 10,\ y_1 = ?$

Sol : **(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion .**

If more the numbers of winners, less is the prize money.

$$\therefore \quad x_1 \times y_1 = 4 \times 90^\circ$$

$$\therefore \quad 10 \times y_1 = 4 \times 90^\circ$$

$$\therefore \quad y_1 = \frac{4 \times 9\cancel{0}^\circ}{1\cancel{0}}$$

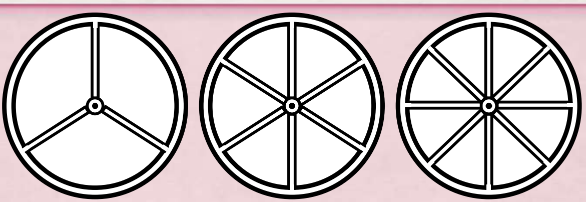
$$\therefore \quad y_1 = 36^\circ$$

$$\therefore \quad \boxed{y_1 = 36^\circ}$$

EXERCISE 13.2

❍ **Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spoke are equal. Help him by completing the following table.**

Number of spokes	4	6	8	10	12
Angle between a pair of consecution spokes	90°	60°	45°	36°	3?°



(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion ?

$x_1 = 12$, $y_1 = ?$

will

take the product

Sol : (i) **Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion .**

If more the numbers of winners, less is the prize money.

$$\therefore \quad x_1 \times y_1 = 4 \times 90^\circ$$

$$\therefore \quad 12 \times y_1 = 4 \times 90^\circ$$

$$\therefore \quad y_1 = \frac{\overset{1}{\cancel{4}} \times \cancel{90^\circ}\ 30}{\cancel{12}\ \cancel{3}\ 1}$$

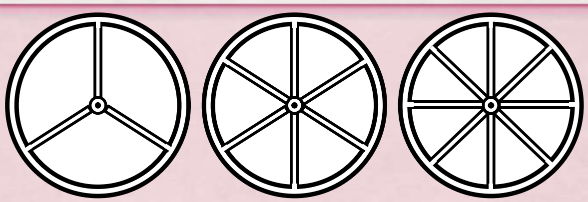
$$\therefore \quad y_1 = 30^\circ$$

$$\therefore \quad \boxed{y_1 = 30^\circ}$$

EXERCISE 13.2

❍ **Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spoke are equal. Help him by completing the following table.**

Number of spokes	4	6	8	10	12
Angle between a pair of consecution spokes	90˚	60˚	45º	36º	30º



(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

[illegible] **will** [illegible]**duct**

$x_1 = 15,\ y_1 = ?$

Sol : (i) **Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.**

If more the numbers of winners, less is the prize money.

$\therefore \quad x_1 \times y_1 = 4 \times 90^{\circ}$

$\therefore \quad 15 \times y_1 = 4 \times 90^{\circ}$

$\therefore \quad y_1 = \frac{4 \times \cancel{90}^{\circ}\ 6}{\cancel{15}\ 1}$

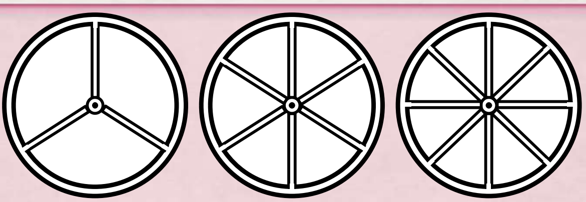
$\therefore \quad y_1 = 24^{\circ}$

$\therefore \quad \boxed{y_1 = 24^{\circ}}$

EXERCISE 13.2

❍ **Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spoke are equal. Help him by completing the following table.**

Number of spokes	4	6	8	10	12
Angle between a pair of consecution spokes	90°	60°	45°	36°	30°



(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40° ?

e will
duct

$x_1 = 40$, $y_1 = ?$

Sol :

(ii) Let the required number of spoke be *n*.

$$\therefore \quad 40 \times n = 4 \times 90^{\circ}$$

$$\therefore \quad n = \frac{\overset{1}{\cancel{4}} \times 9\cancel{0}^{\circ}}{\underset{1}{\cancel{40}}}$$

$$\therefore \quad n = 9^{\circ}$$

∴ **The required number of spoke is 9°.**

EXERCISE 13.2

❍ If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4 ?

Sol :

children	Distance
24	5
20	n

Therefore this is a case of inverse variation

Therefore it is a case of inverse variation

Reduced number of children = 24 – 4 = 20

Since, more the number of children, less is the quantity of sweets.

$$\therefore \quad 20 \times n = 24 \times 5$$

$$\therefore \quad n = \frac{\overset{6}{\cancel{24}} \times \overset{1}{\cancel{5}}}{\cancel{20}\ \cancel{4}\ 1}$$

$$\therefore \quad n = 6$$

∴ **Each student will get 6 sweets.**

EXERCISE 13.2

❍ A farmer has enough food to feed 20 animals in cattle for 6 days. How would the food last were 10 more animals in his cattle ?

Sol :

Animals	Days
20	6
30	x

Whic

Therefore this is a case of inverse variation

Animals And Days

Reduced number of children = 20 + 10 = 30

For more number of animals, the food will last less number of days.

Let

anim

Therefore it is a case of inverse variation

$$\therefore \quad 30 \times x = 20 \times 6$$

$$\therefore \quad x = \frac{{}^{4}\cancel{20} \times \cancel{6}^{1}}{\cancel{30}\ \cancel{5}_{1}}$$

$$\therefore \quad x = 4$$

∴ Therefore, the food will now last for 4 days.

EXERCISE 13.2

- A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

Sol :

Person	Days
3	4
4	x

Which are the two quantities that are varying ?

Person And **Days**

Here the number of persons and the number of days are in inverse proportion.

$$\frac{3}{4} = \frac{x}{4}$$

$$\therefore \quad 4 \times x = 3 \times 4$$

$$\therefore \quad x = \frac{3 \times \cancel{4}^{1}}{\cancel{4}_{1}}$$

$$\therefore \quad x = 3$$

∴ **Here, they will complete the in 3 days.**

EXERCISE 13.2

❍ **A batch of bottles was packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?**

Sol :

Number of bottles in each box	boxes
12	25
20	x

Which are the two quantities that are varying ?

Person **And** **Days**

Here the number of persons and the number of days are in inverse proportion.

$$\frac{12}{20} = \frac{x}{25}$$

$$\therefore \quad 20 \times x = 12 \times 25$$

$$\therefore \quad x = \frac{\overset{3}{\cancel{12}} \times \overset{5}{\cancel{25}}}{\cancel{20}\ \cancel{4}\ 1}$$

$$\therefore \quad x = 15$$

∴ **Here, 15 boxes would be filled.**

EXERCISE 13.2

❍ **A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?**

Sol :

Number of bottles in each box	boxes
63	42
54	x

Which are the two quantities that are varying ?

Number of bottles in each box **And** **Boxes**

Here the number of persons and the number of days are in inverse proportion.

$$\frac{63}{54} = \frac{x}{42}$$

$$\therefore \quad 54 \times x = 63 \times 42$$

$$\therefore \quad x = \frac{\overset{3}{\cancel{63}} \times \overset{21}{\cancel{42}}}{\cancel{54}\ \cancel{27}\ 1}$$

$$\therefore \quad x = 49$$

∴ **Here, 49 machines would be required.**

EXERCISE 13.2

❍ **A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. How long will it take when the car travels at the speed of 80 km/h**

Sol :

Speed km/h	Number of hours
60	2
80	x

Which are the two quantities that are varying ?

Speed km/h **And** **Number of hours**

Here the number of persons and the number of days are in inverse proportion.

$$\frac{60}{80} = \frac{x}{2}$$

$$\therefore \quad 80 \times x = 60 \times 2$$

$$\therefore \quad x = \frac{\overset{3}{\cancel{60}} \times \cancel{2}^{1}}{{}_{2}\cancel{4}\,\cancel{80}}$$

$$\therefore \quad x = \frac{3}{2}$$

$\therefore$ **Thus, the required number of hours $\frac{3}{2}$ hours**

EXERCISE 13.2

○ **Two persons could fit new windows in a house in 3 days**

(i) One of the persons fell ill before the work started. How long would the job take now?

Which are the two quantities that are varying ?

Speed km/h **And** **Number of hours**

Sol :

Persons	Days
2	3
1	x

Here the number of persons and the number of days are in inverse proportion.

$$\frac{2}{1} = \frac{x}{3}$$

$$\therefore \quad 1 \times x = 2 \times 3$$

$$\therefore \quad x = \frac{2 \times 3}{1}$$

$$\therefore \quad x = \frac{6}{1}$$

$$\therefore \quad x = 6$$

Q. Two persons could fit new windows in a house in 3 days (ii) How many persons would be needed to fit the windows in one day?

Sol. Let the number of persons be x.

Persons	Days
2	x
3	1

Here, the number of persons and the number of days are in inverse proportion.

$$\therefore \quad \frac{2}{1} = \frac{x}{3}$$

$$= x \times 1 = 2 \times 3$$

$$= \ x = \frac{2 \times 3}{1} = 6 \text{ persons}$$

Q. A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

Sol. Let the duration of each period be x.

Period	Duration of period (in minutes)
8	45
9	x

Here the number of periods and the duration of periods are in inverse proportion.

$$\therefore \frac{8}{9} = \frac{x}{45}$$

$$x \times 9 = 8 \times 45$$

$$x = \frac{8 \times 45}{9} = 40 \text{ cm}$$

Hence, the duration of each period would be 40 minutes.

Time and Work

INTRODUCTION

Q.No.1 **Suppose A can finish a piece of work in 8 days.**

⇒ **Then work done by A in 1 day**

Eq. **Suppose that the work done by A in 1 day is $\frac{1}{6}$**

⇒ **Then, time taken by A to finish the whole work = 6 days**

GENERAL RULES

Q.No.1 **Suppose A can finish a piece of work in n days.**

⇒ Then, work done by A in 1 day = $\frac{1}{n}$

Lets take an example to understand the rule

Eq. **Suppose that the work done by A in 1 day is $\frac{1}{6}$**

⇒ Then, time taken by A to finish the whole work = 6 days

EXAMPLE

❍ **A alone can finish a piece of work in 12 days and B alone can do it in 15 days. If both of them work at it together, how much time will they take to finish it ?**

Sol :

Time taken by (A + B) to finish the work = 12 days

Time taken by B alone to finish the work = 15 days

Work done by A in 1 day $= \boxed{\dfrac{1}{12}}$

Work done by A in 1 day $= \boxed{\dfrac{1}{15}}$

Work done by $(\boxed{A} + \boxed{B})$ **in 1 day** $= \left(\dfrac{1}{12} + \dfrac{1}{15}\right)$

$$= \frac{\cancel{9}^{3}}{\cancel{60}_{20}} = \frac{3}{20}$$

Time taken by (A + B) to finish the work $= \boxed{\dfrac{20}{3}}$ **days, i.e.** $6\,\dfrac{2}{3}$ **days**

6
3) 20
– 9
11

∴ **Hence, both can finish the work in** $6\,\dfrac{2}{3}$ **days**

EXAMPLE

❍ **A and B together can do a piece of work in 12 days, while B alone can finish it in 30 days. In how many days can A alone finish the work?**

Sol : **Time taken by (A + B) to finish the work = 12 days**

Time taken by B alone to finish the work = 30 days

∴ **(A + B)'s 1 day's work** $= \boxed{\dfrac{1}{12}}$

∴ **B's 1 day's work** $= \boxed{\dfrac{1}{30}}$

∴ **A's 1 day's work** = **(A + B)'s 1 day's work** – **B's 1 day's work**

$$= \left(\frac{1}{12} - \frac{1}{30}\right)$$

$$= \frac{\cancel{3}^{1}}{\cancel{60}_{20}} = \frac{1}{20}$$

∴ **Alone can finish the work in 20 days.**

A cistern or a water rank is connected with two types of pipes

Inlet :

The pipe which
fills the tank
is called as inlet

Outlet :

The pipe which
fills the tank
is called as inlet

Rule : 1

Suppose a pipe fills a tank in n hours

Then, part of the tank filled in 1 hour $= \frac{1}{n}$

i.e., work done by the inlet in 1 hour $= \frac{1}{n}$

A cistern or a water rank is connected with two types of pipes

Inlet :
The pipe which
fills the tank
is called as inlet

Outlet :
The pipe which
fills the tank
is called as inlet

Rule : 2

Suppose an outlet empties a full tank in m hours

Then, part of the tank emptied in 1 hour $= \dfrac{1}{\mathrm{m}}$

i.e., work done by the inlet in
1 hour $= \dfrac{1}{m}$

Additional sums

EXERCISE

❍ **If x and y vary directly, find the missing entries in the following table :**

(i)

x	2.5	….	….	21
y	5	8	24	….

Sol : **(i)** **It is given that x and y are in direct variation. Therefore, the ratio of the corresponding values of x and y remain constant.**

We have, $\frac{2.5}{5} = \frac{1}{2}$

So, x and y are in direct variation with the constant of variation equal to $\frac{1}{2}$

This means that x is half of y or y is twice of x.

Thus, the required entries are

$$\frac{x}{8} = \frac{1}{2}$$
$$x = \frac{1}{2} \times 8$$
$$x = 4$$

$$\frac{x}{24} = \frac{1}{2}$$
$$x = \frac{1}{2} \times 24$$
$$x = 12$$

$$\frac{21}{x} = \frac{1}{2}$$
$$21 = \frac{1}{2} \times x$$
$$x = 21 \times 2$$

Thus, the required entries are 4, 12 and 42.

EXERCISE

❍ **If x and y vary directly, find the missing entries in the following table :**

(ii)

x	….	9	15	….
y	3	4.5	7.5	13.25

Sol :

(ii) We have, $\frac{9}{4.5} = \frac{15}{4.5} = 2$

So, x and y are in direct variation such that x is twice of y.

$$\frac{x}{y} = 2$$

$$\frac{x}{3} = 2$$

$$x = 2 \times 3$$

$$\frac{x}{y} = 2$$

$$\frac{x}{13.25} = 2$$

$$x = 2 \times 13.25$$

$$26.50$$

Thus, the missing entries are 6 and 26.50

EXERCISE

- A car travels 432 km on 48 liters of petrol. How far would it travel on 20 liters of petrol ?

Sol : Suppose the car travels x km on 20 liters of petrol. Then, the above information car be put in the following tabular form

Petrol (in liters)	Distance (in km)
48	432
20	x

Which are the two quantities that are varying ?

Speed km/h And Number of hours

We observe that the lesser that the lesser he petrol consumed, the smaller the number of kilometers travelled. So, it is a case of direct variation.

$$\frac{48}{20} = \frac{432}{x}$$

$$\therefore \quad 48 \times x = 20 \times 432$$

$$\therefore \quad x = \frac{\overset{10}{\cancel{20}} \times \overset{18}{\cancel{\overset{216}{\cancel{432}}}}}{\underset{1}{\cancel{\underset{12}{\cancel{\underset{24}{\cancel{48}}}}}}}$$

$$\therefore \quad x = 180$$

∴ Hence, the car would travel 180 km on 20 liters of petrol.

EXERCISE 13.1

❍ **Reema types 540 words during half an hour. How many words would she type in 6 minutes?**

Sol : Suppose she types x word in 6 minutes. Then, the given information can be exhibited in the following tabular form.

Number of words	Time (in minutes)
540	30
x	6

Wh

Th

Therefore we will

Lets assume it as x

the quantities

Length

the ship

And

the mast

Since in more time words can be typed. So, it is a case of direct variation.

$$\frac{540}{x} = \frac{30}{6}$$

∴ $30 \times x = 540 \times 6$

∴ $x = \frac{\cancel{540} \times \cancel{6}^{2}}{\cancel{30}_{1}}$

∴ $x = 108$

∴ **Hence, she types 108 words in 6 minutes.**

EXERCISE

- **The amount of extension in an elastic spring varies directly as the weight hung on it. If a weight of 150 gm produces an extension of 2.9 cm, then what weight would produce an extension of 17.4 cm ?**

Sol : Let the required weight be x gram. Then, the above information can be exhibited in the following tabular form :

Weight (in gram)	150	x
Distance (in km)	2.9	17.4

It is given that the amount of extension in the spring varies directly as the weight hung on it. So, it is a case of direct variation.

$\therefore$ Ratio of petrol consumed = Ratio of distance travelled

$$\therefore \quad 150 : x = 2.9 : 17.4$$

$$\therefore \quad \frac{150}{x} = \frac{2.9}{17.4}$$

$$\therefore \quad \frac{150}{x} = \frac{1}{6}$$

$$\therefore \quad 150 \times 6 = 1 \times x$$

$$\therefore \quad x =$$

$\therefore$ Hence, a weight of 900 gram would produce an extension of 17.4 cm.

EXERCISE

❍ **If a and b vary inversely, fill in table blanks :**

(i)

a	8	2	….	5	1
b	10	….	20	….	80

Sol : (i) **Since a and b vary inversely, so the product *ab* remains constant and is equal to 8 × 10 = 80**

∴ **First blank space is to be filled by** $\frac{80}{2} = 40$

Second blank space is to be filled by $\frac{80}{20} = 4$

Third blank space is to be filled by $\frac{80}{5} = 16$

EXERCISE

❍ **If a and b vary inversely, fill in table blanks :**

(ii)

a	16	32	8	128
b	4	….	….	0.5

Sol : (ii) **Proceeding as in (i), we find that**

$\therefore$ **First blank space is to be filled by** $\frac{64}{32} = 2$

Second blank space is to be filled by $\frac{64}{8} = 8$

EXERCISE

❍ If 52 men can do a piece of work in 35 days, in how many days 28 men will do it ?

Sol : Suppose 28 men will do the piece of work in x days. The given information can be exhibited in the following tabular form.

Petrol (in liters)	Distance (in km)
52	35
28	x

Which are the two quantities that are varying ?

Speed km/h And **Number of hours**

Clearly, less is the number of men, more will be the number of days to finish the work. It is therefore, the case of inverse variation.

$$\frac{52}{28} = \frac{x}{35}$$

$$\therefore \quad 28 \times x = 52 \times 35$$

$$\therefore \quad x = \frac{\overset{13\ \cancel{26}}{\cancel{52}} \times \overset{5}{\cancel{35}}}{\underset{1\ \cancel{7}\ \cancel{14}}{\cancel{28}}}$$

$$\therefore \quad x = 65$$

∴ **Hence, 28 men will do the work in 65 days.**

EXERCISE 13.1

- **Shalu cycles to her school at an average speed of 12km/hr. It takes her 20 minutes to reach the school. If she wants to reach her school in 15 minutes, what should be her average speed?**

Sol : Let the required speed be x km/hr. Then, the given information may be presented in the following tabular form :

Speed (in km/ hr)	Time (in minutes)
12	20
x	15

Wh

Th

Therefore we will

Lets assume it as x

the quantities

Length of the ship

And

Height of the mast

We note that more the speed, less will be the time taken to cover the given distance. So, it is case of inverse variation.

$$\frac{12}{x} = \frac{15}{20}$$

$$\therefore \quad 15 \times x = 12 \times 20$$

$$\therefore \quad x = \frac{\overset{4}{\cancel{12}} \times \overset{4}{\cancel{20}}}{\cancel{15}\ \cancel{3}\ 1}$$

$$\therefore \quad x = 16$$

∴ **Hence Shalu's average speed should be 16km/hr.**

EXERCISE

❍ **If x and y vary inversely as each other, and $x = 10$ when $y = 6$. Find y when $x = 15$**

Sol : **Since x and y vary inversely as each other, therefore the product xy always remains constant.**

$$\therefore \quad 10 \times 6 = 15 \times y$$

$$\therefore \quad 60 = 15y$$

$$\therefore \quad \frac{\overset{4}{\cancel{\overset{12}{\cancel{60}}}}}{\underset{1}{\cancel{\underset{3}{\cancel{15}}}}} = y$$

$$\therefore \quad y = 4$$


Thank you.