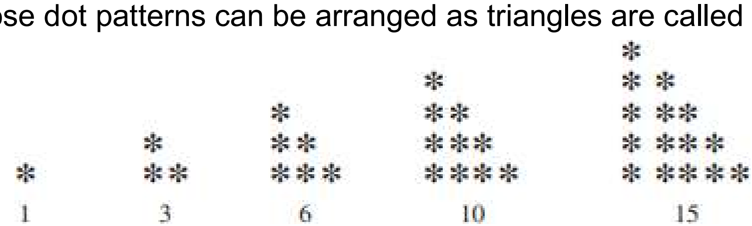
Squares and Square Roots

1. **Square number**
   - Square of a number is obtained when it is multiplied by itself twice. Thus, square of x = (x $\times$ x), denoted by x^2.
   - Some of the square numbers are 1, 4, 9, 16, 25, …
   - A natural number n is a perfect square if n can be expressed as m^2, for some natural number m. The numbers 1, 4, 9, 16, 25, … are perfect squares.

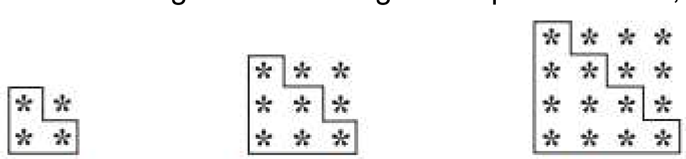
2. **Steps to find whether a given natural number is a perfect square or not:**
   i. Step 1: Get the natural number.
   ii. Step 2: Find the prime factorization of the given natural number.
   iii. Step 3: Group the factors in pairs in such a way that both the factors in each pair are equal.
   iv. Step 4: Check if any factor is left over. If no factor is left over in grouping, then the given number is a perfect square. Otherwise, it is not a perfect square.
   v. Step 5: To find the square root of a given number, take one factor from each group and multiply them.

3. **Properties of square numbers:**
   i. A number ending in 2, 3, 7 or 8 is never a perfect square.
   ii. A number ending with an odd number of zeroes is never a perfect square.
   iii. The number of zeroes at the end of a perfect square is always even.
   iv. Squares of even numbers are even.
   v. Squares of odd numbers are odd.
   vi. If a number has 1 or 9 in the unit's place, then its square ends in 1.
   vii. If a square number ends in 6, the number whose square it is, will have either 4 or 6 in the unit's place.

4. **Triangular numbers**
   - The numbers whose dot patterns can be arranged as triangles are called the triangular numbers.



   - Adding any two consecutive triangular numbers give a square number, for example:



$1 + 3 = 4 = 2^2$ $\quad$ $3 + 6 = 9 = 3^2$ $\quad$ $6 + 10 = 16 = 4^2$

Anodiam
Education that enlightens!

5. **Numbers between square numbers**
There are 2n non perfect square numbers between the squares of the numbers n and (n + 1).
For n = 4, n + 1 = 5
$n^2 = 4^2 = 16$, $(n + 1)^2 = 5^2 = 25$
$n^2 – (n + 1)^2 = 25 – 16 = 9$
There are 8 (2n) non perfect square numbers between 4^2 and 5^2.

6. **Adding consecutive odd numbers**
The square of a natural number 'n' is equal to the sum of the first 'n' odd natural numbers.
1 [one odd number] $= 1 = 1^2$
1 + 3 [sum of first two odd numbers] $= 4 = 2^2$
1 + 3 + 5 [sum of first three odd numbers] $= 9 = 3^2$
1 + 3 + 7 + 9 […] $= 16 = 4^2$
And so on…

7. There are no natural numbers m and n such that $m^2 = 2n^2$

8. **Square of an odd number**
The square of any odd number can be expressed as the sum of two consecutive positive integers.
$3^2 = 9 = 4 + 5$
$5^2 = 25 = 12 + 13$
$7^2 = 49 = 24 + 25$
$9^2 = 81 = 40 + 41$ and so on….
Moreover, if n is the square of an odd number m then the two consecutive numbers whose sum is n are $\frac{n-1}{2}$ and $\frac{n+1}{2}$.
The first odd number is 3 and its square is 9 which can be written as 4 + 5

9. **Some useful square identities:**
If a and b are two natural numbers, then,
   i. $(a + 1)(a - 1) = a^2 - 1$
   ii. $(a + b)^2 = a^2 + b^2 + 2ab$
   iii. $(a - b)^2 = a^2 + b^2 - 2ab$
Note: Square of big numbers can be calculated using these three identities.

10. **Calculating the square of a number with unit digit 5**
Consider a number with unit digit 5, say, (a5).
$(a5)^2 = (10a + 5)^2$
$= 10a(10a + 5) + 5(10a + 5)$
$= 100a^2 + 50a + 50a + 25$
$= 100a(a + 1) + 25$

= a(a + 1) hundred + 25
Hence, $(a5)^2$ = a(a + 1) hundred + 25.
For example: $35^2 = 3(3 + 1)\ 100 + 25 = 3(4)100 + 25 = 1225$.

11. Pythagorean triplet

- A triplet (a, b, c) of three natural numbers a, b and c is called a Pythagorean triplet if $a^2 + b^2 = c^2$.
- For any natural number m greater than 1, $(2m, m^2 – 1, m^2 + 1)$ is a Pythagorean triplet.

12. What is square root?

- Square root is the inverse operation of square.
- Square of 2 is 4, and so, the square root of 4 is 2.
- Finding the number with the known square is known as finding the square root.

13. Square root of a number

The square root of a number 'x' is that number which when multiplied by itself gives 'x' as the product. We denote the square root of x by $\sqrt{x}$.

14. Finding square roots through different methods

- **Repeated subtraction**
  We stated above that the square of a number is the sum of first n odd natural numbers. So, square root of a square number can be obtained by subtracting the successive odd natural numbers starting from 1 till we get 0.
  Example: To find $\sqrt{49}$
  49 – 1 = 48, 48 – 3 = 45, 45 – 5 = 40, 40 – 7 = 33, 33 – 9 = 24, 24 – 11 = 13, 13 – 13 = 0
  We subtracted 7 successive odd natural numbers.
  Thus, 7 is the square root of 49.

- **Prime factorization**
  Express the number as the product of prime numbers, group the common primes in a pair, take one prime from each pair and then multiply to get the square root.
  Calculation of square root of 9604 using prime factorization method:

  $9604 = \underline{2 \times 2} \times \underline{7 \times 7} \times \underline{7 \times 7}$

  $\therefore \sqrt{9604} = 2 \times 7 \times 7 = 98$

  Note: If one or more primes are not in pairs, the number is not a perfect square.

- **Division method**
  Steps to perform division:
  i. Place a bar over every pair of digits starting from the one's digit.
  ii. Find the largest number whose square is less than or equal to the number under the left-most bar (take this as dividend) and take this as a divisor. Divide and get the remainder.
  iii. Bring down the number under the next bar and place it to the right of the remainder and this

will act as the new dividend.

iv. Double the quotient and write it with a blank on its right.

v. Find the largest digit to fill the blank which also becomes the new digit in quotient such that the product of new quotient and new divisor gives a number less than or equal to the dividend.

vi. Continue this process till we get the remainder as 0. The quotient becomes the square root of the number.

Example: Square root of 841

	29
2	$\overline{8}\ \overline{41}$
	-4
49	441
	-441
	0

$\therefore \sqrt{841} = 29$

Note: This method can also be used to find the square root of a non-perfect square or decimal number.

15. For positive numbers a and b, we have:

i. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

ii. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$