Lecture 8

Module 28

Division Algorithm for Polynomials

Statement :

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that, $p(x) = g(x) \times q(x) + r(x)$

where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

Important :

5 is a factor of the 10

For example,

$$g(5)\overline{)\,p(10)}\quad q(2)$$
$$-10$$
$$r(0)$$

- The Division Algorithm for polynomial may be written as :

Dividend = Divisor × Quotient + Remainder

- If Remainder $r(x) = 0$ then the polynomial $g(x)$ is a **factor** of the polynomial $p(x)$.

Exercise 2.3

1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following :

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

Sol. Dividend $= x^3 - 3x^2 + 5x - 3$

Divisor $= x^2 - 2$

$$
\begin{array}{l}
\quad\quad x - 3 \\
\hline
x^2 - 2 \;\big)\; x^3 - 3x^2 + 5x - 3 \\
\quad\quad x^3 \qquad - 2x \\
\quad\quad (-) \qquad (+) \\
\hline
\quad\quad - 3x^2 + 7x - 3 \\
\quad\quad - 3x^2 \qquad + 6 \\
\quad\quad (+) \qquad (-) \\
\hline
\quad\quad 7x - 9
\end{array}
$$

$\frac{x^3}{x^2} = x$ | $x(x^2 - 2) = x^3 - 2x$

$\frac{-3x^2}{x^2} = -3$ | $-3(x^2 - 2) = -3x^2 + 6$

$\therefore$ Quotient $= x - 3$

Remainder $= 7x - 9$

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Exercise 2.3

1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following :

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

Sol. Dividend $= x^4 - 3x^2 + 4x + 5x^0$

$= x^4 + 0x^3 - 3x^2 + 4x + 5$

Divisor $= x^2 + 1 - x = x^2 - x + 1$

$$
\begin{array}{r}
x^2 + x - 3 \\
\hline
x^2 - x + 1 \;\big)\; x^4 + 0x^3 - 3x^2 + 4x + 5 \\
x^4 - x^3 + x^2 \quad (-)\ (+)\ (-) \\
\hline
+ x^3 - 4x^2 + 4x + 5 \\
x^3 - x^2 + x \quad (-)\ (+)\ (-) \\
\hline
-3x^2 + 3x + 5 \\
-3x^2 + 3x - 3 \quad (+)\ (-)\ (+) \\
\hline
8
\end{array}
$$

$\frac{x^4}{x^2} = x^2$ | $x^2(x^2 - x + 1) = x^4 - x^3 + x^2$

$\frac{x^3}{x^2} = x$ | $x(x^2 - x + 1) = x^3 - x^2 + x$

$\frac{-3x^2}{x^2} = -3$ | $-3(x^2 - x + 1) = -3x^2 + 3x - 3$

$\therefore$ Quotient $= x^2 + x - 3$

Remainder $= 8$

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Exercise 2.3

1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following :

(iii) $p(x) = x^4 - 5x + 6,\ g(x) = 2 - x^2$

Sol. Dividend $= x^4 - 5x + 6x^0$

$= x^4 + 0x^3 + 0x^2 - 5x + 6$

Divisor $= 2 - x^2 = -x^2 + 2$

$$
\begin{array}{r|l}
 & -x^2 - 2 \\
\hline
-x^2 + 2 & x^4 + 0x^3 + 0x^2 - 5x + 6 \\
 & \underset{(-)}{x^4} \quad \underset{(+)}{-2x^2} \\
\hline
 & 2x^2 - 5x + 6 \\
 & \underset{(-)}{2x^2} \quad \underset{(+)}{-4} \\
\hline
 & -5x + 10
\end{array}
$$

$\frac{x^4}{-x^2} = -x^2$ | $-x^2(-x^2 + 2) = x^4 - 2x^2$

$\frac{2x^2}{-x^2} = -2$ | $-2(-x^2 + 2) = 2x^2 - 4$

$\therefore$ Quotient $= x^2 + 2$

Remainder $= -5x + 10$