Chapter10 - Gravitation

- **Discovery of gravity**
- **Newton's law of gravitation**

GRAVITATION

Why are they racing

Even a simple task like drinking water would become difficult without gravity.

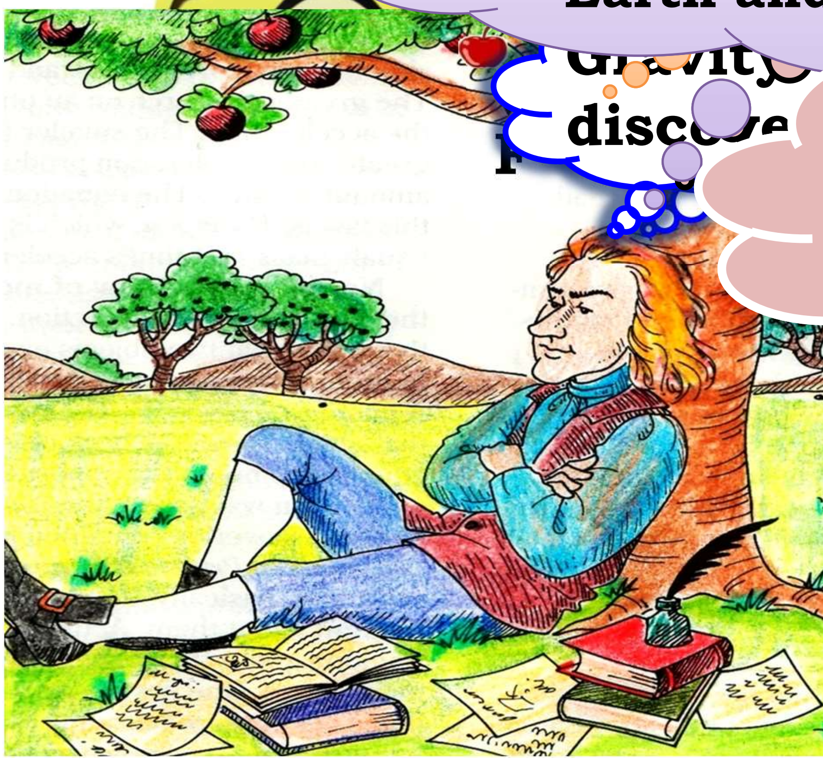
Can you imagine skipping without gravity

GRAVITATI
BETWEEN A

Newton thought further, gravitational force is not just between the Earth and an apple.

ction

B



Gravity
discove

So if th
bodies. A
is a force of

Now this
is absu

But we don't see this happening

Lets see why..???

Man sh
attracted
is passing by

Planets around earth should get attracted to each other

NEWTON'S LAW OF GRAVITATION

Every object in the universe attracts every other object with a force, which is **directly proportional** to the product of their masses and **inversely proportional** to the ... between their centers .

In comparison ...
mass of ...
force ...

Hence we do not attract object around us

If one physical quantity ...

If distance is more force of attraction is less.

$$F = \frac{G\, m_1 m_2}{r^2}$$

Where G is universal constant of gravitation

... the ... the center of the mass

... gravitation is the ... between two unit ... distance apart

Module 02 - Gravitation

- **Unit of gravitation**
- **Importance of gravitation**

Let us Derive the unit for gravitational constant

We know that

$$F = \frac{Gm_1m_2}{r^2}$$

$$G = \frac{Fr^2}{m_1m_2}$$

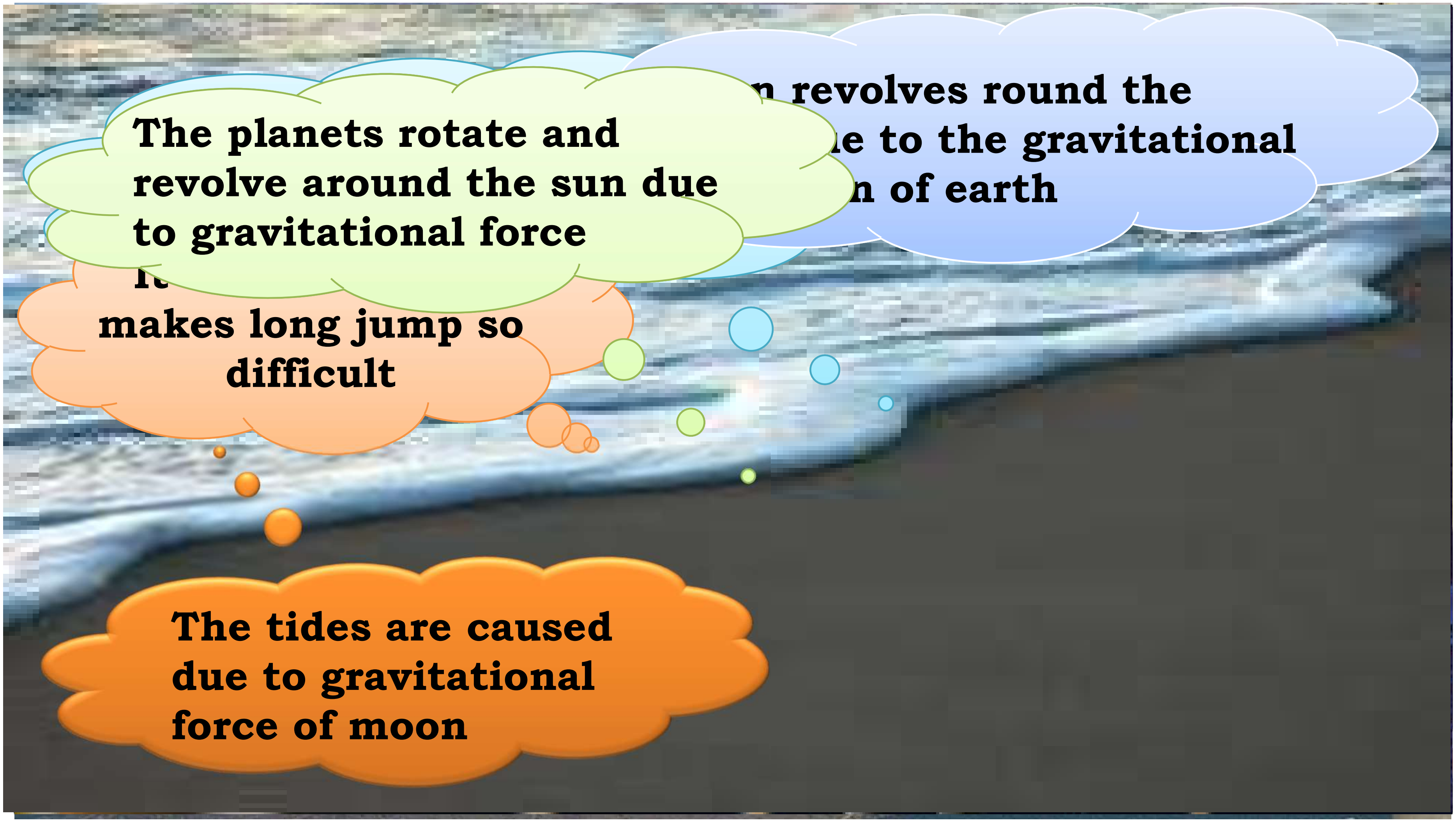
CGS unit of

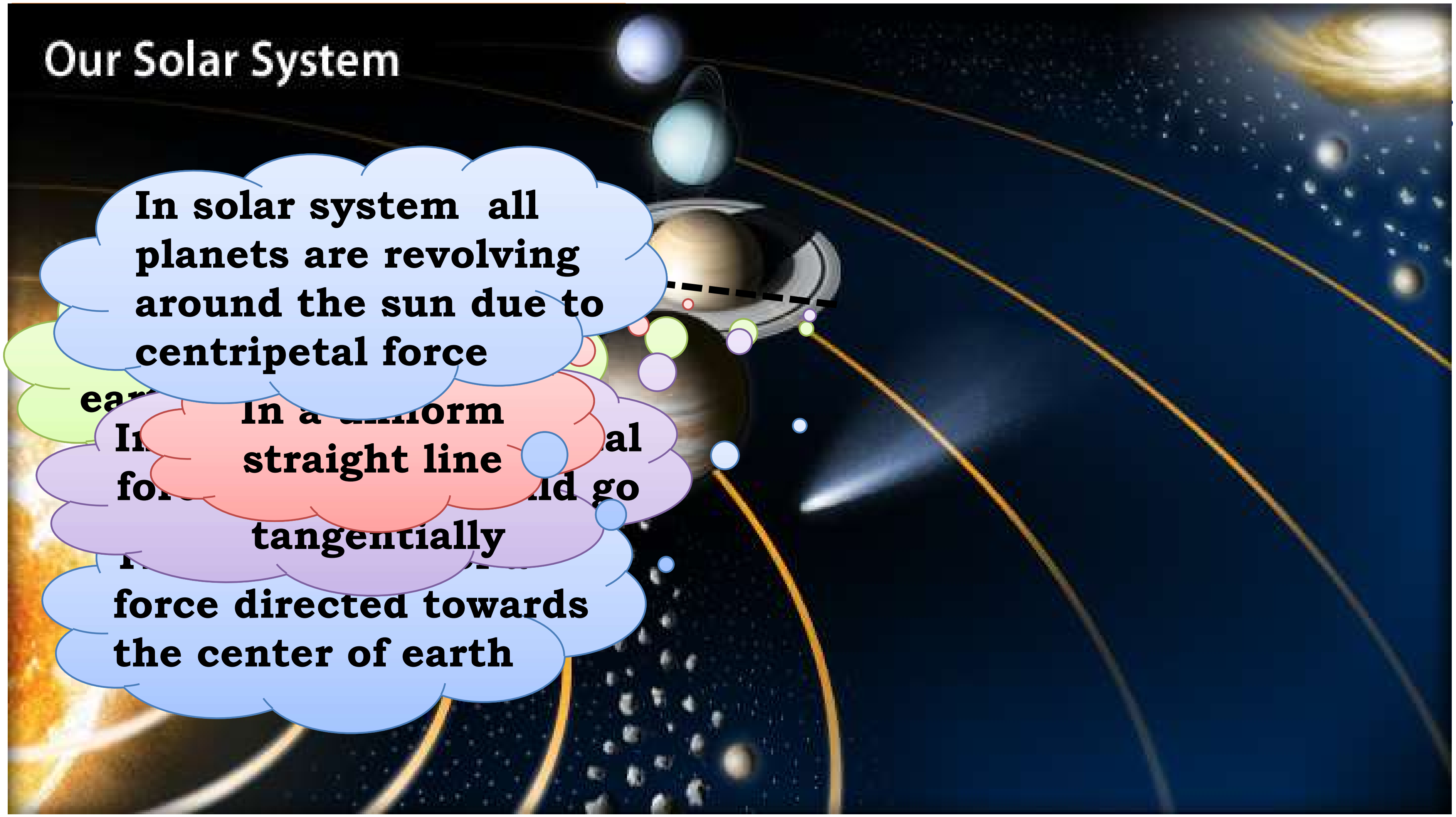
CGS unit of displacement is

CGS unit of mass is

SI unit of G

C.G.S unit of G = ______


revolves round the
to the gravitational
of earth
The planets rotate and revolve around the sun due to gravitational force
makes long jump so difficult
The tides are caused due to gravitational force of moon


Our Solar System
In solar system all planets are revolving around the sun due to centripetal force
In a
straight line
tangentially
force directed towards the center of earth

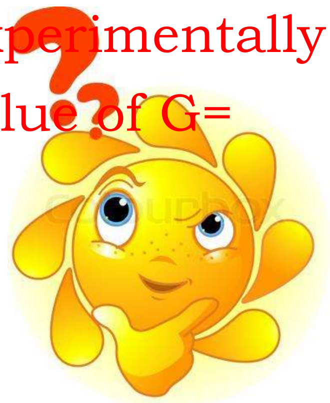
Module 03 - Gravitation

- **Relationship between S.I & C.G.S unit of gravitational constant**
- **Acceleration due to gravity**
- **Value of acceleration due to gravity on the surface of Earth**

Let us find the relationship between SI and CGS unit of gravitation constant

Experimentally it is observed that

Value of G= $6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$



1m = 10^2cm

1kg = 10^3g

$\therefore$ 1kg^2 = 10^6g^2

$$= \frac{6.67 \times 10^{-11}}{\quad}$$

$$= \frac{66.67 \times 10^{-11} \times 10^{9}}{10^{6}} \quad \frac{\text{dyne cm}^2}{\text{g}^2}$$

$$= 6.67 \times 10^{-11} \times 10^{9} \times 10^{-6} \quad \frac{\text{dyne cm}^2}{\text{g}^2}$$

$$= 6.67 \times 10^{-11} \times 10^{3} \quad \frac{\text{dyne cm}^2}{\text{g}^2}$$

Hence

$$= 6.67 \times 10^{-8} \quad \frac{\text{dyne cm}^2}{\text{g}^2}$$

$$6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} = 6.67 \times 10^{-8} \frac{\text{Dyne cm}^2}{\text{g}^2}$$

Acceleration due to gravity (g)

Acceleration = $\dfrac{\text{change in velocity}}{\text{time}}$

NEWTON'S LAW OF GRAVITATION

$$F = \frac{G m_1 m_2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$F = \frac{G M m}{R^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

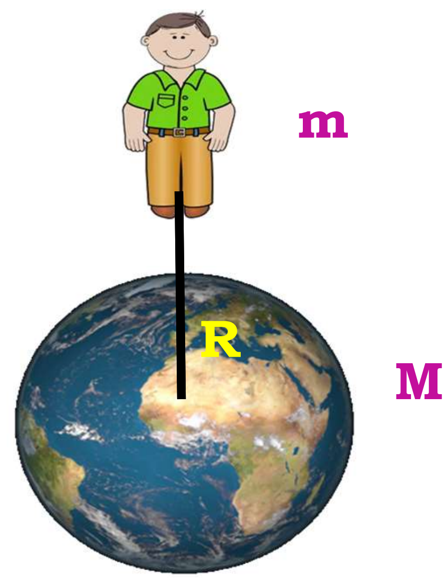
According to Newton's second law

$F = mg$ ………… (2) ($\because$ acceleration due to gravity)

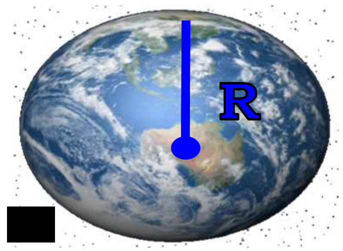
From (1) and (2)

$$\cancel{m}g = \frac{G M \cancel{m}}{R^2}$$

$$\boxed{g = \frac{G M}{R^2}}$$



VALUE OF (g) ON THE SURFACE OF EARTH



$R = 6.4 \times 10^{6}$ m

$M = 6 \times 10^{24}$ kg

$$g = \frac{GM}{R^2}$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^{6})^2}$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{40.96 \times 10^{12}}$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 10^{-12}}{40.96}$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{12}}{40.96}$$

$$= \frac{6.67 \times 6 \times 10}{40.96}$$

$$= \frac{400.2}{10.96}$$

$$= \frac{9.8}{1}$$

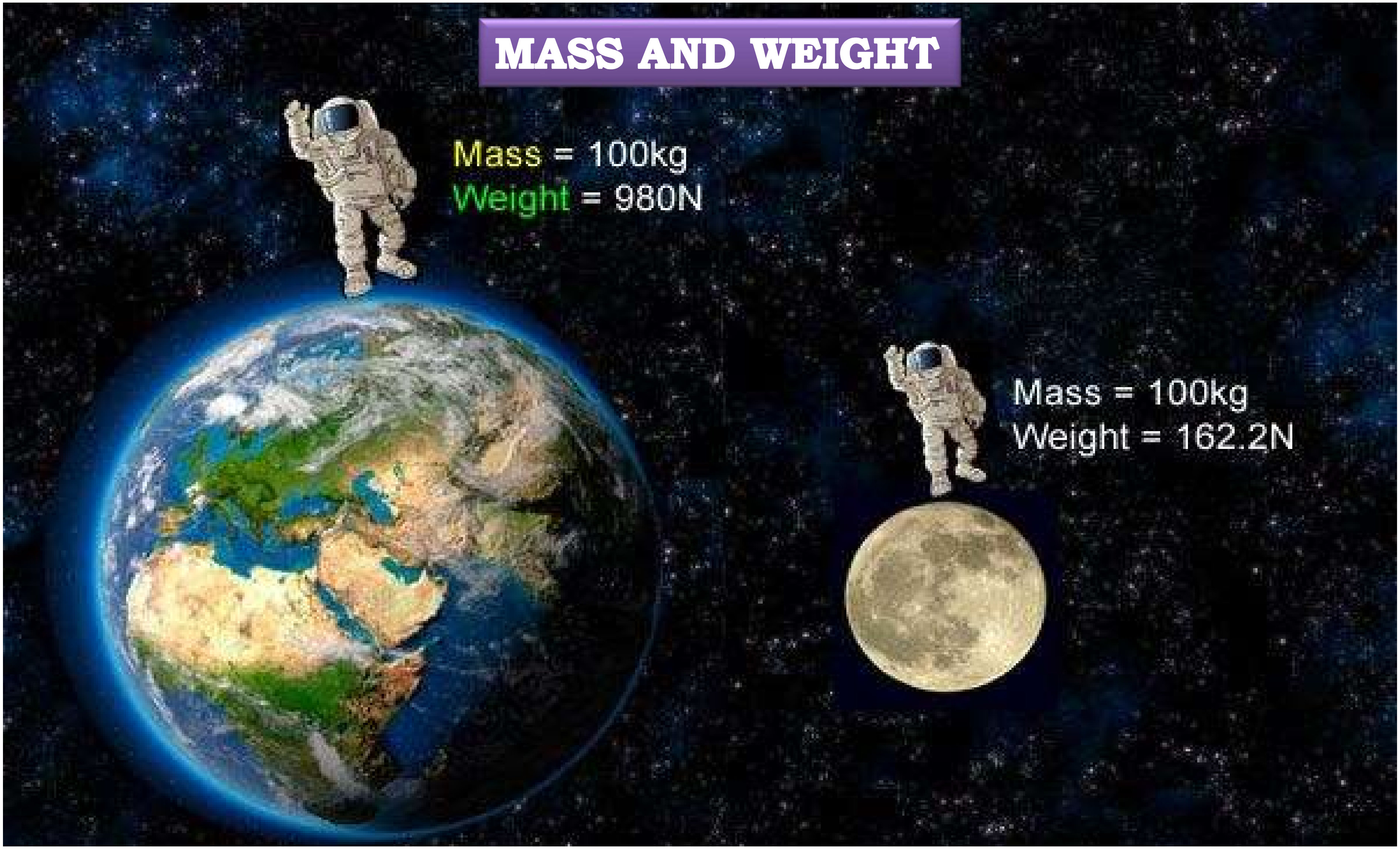
$g = 9.8\ m/s^{2}$

Module 04 - Gravitation

- **Difference between gravitational constant and acceleration due to gravity**
- **Difference between mass and weight**
- **Free fall**

DISTINGUISH BETWEEN G & g

Universal gravitation constant	Acceleration due to gravity of the earth
➢ It the force of attraction between two unit masses placed at unit distance apart from each other.	➢ It is the acceleration caused by earth's gravitational force on an object
➢ Its value is taken as 6.67×10^{-11} Nm^2/kg^2	➢ Its value is taken as 9.8 m/s^2.
➢ It is constant.	➢ It is variable.


MASS AND WEIGHT
Mass = 100kg
Weight = 980N
Mass = 100kg
Weight = 162.2N

MASS AND WEIGHT

Mass	Weight
Mass is the quantity of matter contained in the object	Weight is the force with which the earth attracts it towards its centre
Does not change with place	Changes with place
SI unit kg CGS unit g	SI unit Newton CGS unit dyne
Mass is measured with weighing scale	Weight measured by the formula $W = mg$

FREE FALL

FREE FALL

Force acting on ball

1. Gravitational force
2. Air resistance

Force acting on ball

1. Gravitational force

When the body falls towards the earth, under the influence of earth's gravity alone, its motion is called free fall .

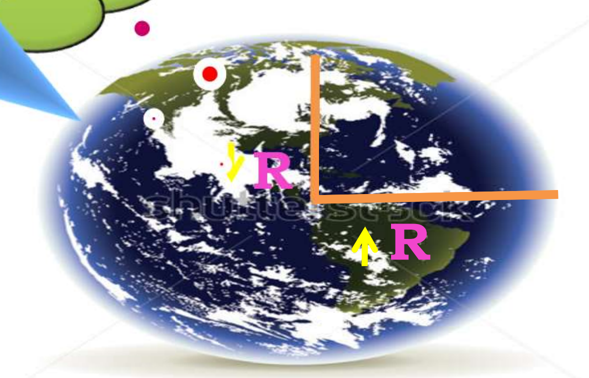
VARIATION OF (g)

With depth

Due to shape of the earth

g’= variation in acceleration due to gravity
g=gravity on the surface of earth
d- depth to which person goes
R- radius of earth.

Gravity is inversely proportional to the distance R
Thus (R)decreases (g) increases and vice versa



FREE FALL IS GOVERNED BY THREE EQUATIONS OF MOTION

1. $v = u + at$ **Changes to** $v = u + gt$

2. $s = ut + \frac{1}{2}at^2$ **Changes to** $s = ut + \frac{1}{2}gt^2$

3. $v^2 = u^2 + 2as$ **Changes to** $v^2 = u^2 + 2gs$

Where 'u' and 'v' are the initial and final velocities and 's' is the distance covered in time 't'.

TYPE - A

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = mg$$

Gravitation

- **Numerical based on Force of gravitation**

1

What happens to the force between two objects, if (i) the mass of one object is doubled? (ii) the distance between the objects is doubled and tripled? (iii) the masses of both objects are doubled?

Formula : $F = \dfrac{Gm_1m_2}{r^2}$

Solution : (i) $F \propto m_1\, m_2$

If mass (i) doubled

$\therefore\ m' = 2m_1$

$\therefore\ F' \propto m'\, m_2$

$F' \propto (2m_1)\,(m_2)$

$F' \propto 2(m_1\, m_2)$

$\therefore$ Force doubled

(ii) distance doubled

$F \propto \dfrac{1}{r^2}$

Now, $r' = 2r$

$\therefore\ F' \propto \dfrac{1}{(r')^2}$

$F' \propto \dfrac{1}{(2r)^2}$

$F' \propto \dfrac{1}{4r^2}$

Force becomes $\dfrac{1}{4}^{th}$

(iii) Mass of both objects is doubled

$F \propto m_1\, m_2$

$m'_1 = 2m_1 \qquad m'_2 = 2m_2$

$F' \propto m'_1\, m'_2$

$F' \propto (2m_1)\,(2m_2)$

$F' \propto 4\, m_1\, m_2$

$F' \propto 4\,F$

Ans : Force becomes four times.

Gravitation

- **Numerical based on Force of gravitation**

2 Gravitational force acts on all objects in proportion to their masses. Why then, a heavy object does not fall faster than a light object?

Ans : If F be the gravitational force on a body of mass m, then

$$F = \frac{GMm}{r^2} = mg$$

But also, $g = \dfrac{GM}{r^2}$

Clearly 'g' is independent of m, Hence, all bodies fall with the same rapidness when there is no air resistance.

3 Calculate the force of gravitation between the earth and the Sun, given that the mass of the earth = 6×10^{24} kg and of the Sun = 2×10^{30} kg. The average distance between the two is 1.5×10^{11} m.

Given :

Mass of Earth (M_E) = 6×10^{24} kg

Mass of Sun (M_S) = 2×10^{30} kg

distance (R) = 1.5×10^{11} m

Gravitational constant (G) = 6.7×10^{-11} Nm^2 kg^{-2}

To find : Force of attraction (F) = ?

Formula : $F = \dfrac{G\,Mm}{R^2}$

Solution : $F = \dfrac{G\,M_S\,M_E}{R^2}$

$$F = \frac{6.7 \times 10^{-11} \times 2 \times 10^{30} \times 6 \times 10^{24}}{(1.5 \times 10^{11})^2}$$

$$F = \frac{6.7 \times 2 \times 6 \times 10^{-11} \times 10^{30} \times 10^{24}}{1.5 \times 1.5 \times 10^{22}}$$

$$= \frac{6.7 \times 2 \times 6 \times 10^{21}}{1.5 \times 1.5}$$

$$= \frac{80.4 \times 10^{21}}{2.25}$$

$$= 35.7 \times 10^{21}$$

$$= 3.57 \times 10^{22}\ \text{N}$$

Ans : The force of gravitation between the earth and the sun is 3.57×10^{22} N

Gravitation

- **Numerical based on Force of gravitation**

4 What is the magnitude of the gravitational force between the earth and a 1 kg object on its surface? (Mass of the earth is 6×10^{24} kg and radius of the earth is 6.4×10^{6} m)

Given : Mass of the body (m) = 1 kg

Mass of the earth (M) = 6×10^{24} kg

Radius of the earth (R) = 6.4×10^{6} m

To find : Force (F) = ?

Formula : $F = \dfrac{GMm}{R^2}$

Solution : $F = \dfrac{GMm}{R^2}$

$$\therefore F = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 1}{(6.4 \times 10^{6})^2}$$

$$= \frac{6.67 \times 6 \times 10^{-11+24-12}}{6.4 \times 6.4}$$

$$= \frac{6.67 \times 6 \times 10}{6.4 \times 6.4}$$

$$= 9.77 N \cong 9.8 N.$$

Ans : The magnitude of gravitational force between the earth and 1 kg object on it's surface is 9.8 N

5 Why is the weight of an object on the moon $1/6^{th}$ its weight on the earth?

Given : Mass of the Earth (M_E) $= 5.98 \times 10^{24}$ kg

Mass of the Moon (M_M) $= 7.36 \times 10^{22}$ kg

Radius of the Earth (R_E) $= 6.4 \times 10^{6}$ m

Radius of the Moon (R_M) $= 1.74 \times 10^{6}$ m

To find : Prove that $W_M = \frac{1}{6} W_E$

Formula : $W = F = \frac{G\,Mm}{R^2}$

Solution : $W = \frac{G\,Mm}{R^2}$

For moon, $W_M = \frac{G\,M_M m}{R_M^2}$ ……… (1)

For earth, $W_E = \frac{G\,M_E m}{R_E^2}$ ……… (2)

Dividing 1 and 2

$$\frac{W_M}{W_E} = \frac{\dfrac{\cancel{G}\,M_M \cancel{m}}{R_M^2}}{\dfrac{\cancel{G}\,M_E \cancel{m}}{R_E^2}} = \frac{M_M}{R_M^2} \times \frac{R_E^2}{M_E}$$

$$\therefore \frac{W_M}{W_E} = \frac{7.36 \times 10^{22} \times (6.4 \times 10^{6})^2}{5.98 \times 10^{24} \times (1.74 \times 10^{6})^2}$$

$$= \frac{7.36 \times 10^{22} \times 40.96 \times \cancel{10^{12}}}{5.98 \times 10^{24} \times 3.0276 \times \cancel{10^{12}}}$$

Ans : Weight of an object on the moon is 1/6 of its weight on the Earth.

Gravitation

- **Numerical based on Mass And Weight**

6 Gravitational force on the surface of the moon is only 1/6 as strong as gravitational force on the earth. What is the weight in newton's of a 10 kg object on the moon and on the earth ?

Given : Mass of the object on the moon = 10 kg

Mass of the object on the earth = 10 kg

(ii) $W_M = \frac{1}{6} W_E$

To find : (i) Weight of the object on the earth (W_E) = ?

(ii) Weight of the object on the moon (W_M) = ?

Formula: (i) $W_E = mg$

Solution : Weight of the object on the earth (W_E) = mg

$= 10 \times 9.8$

$= 98$ N

Weight of the object on the moon (W_M) $= \frac{1}{6} W_E = \frac{1}{6} \times 98 = 16.33$ N

Ans : The weight of the object of mass 10 kg on the moon is 16.33N

Gravitation

- Numerical based on kinematical equations

Type - B

NUMERICAL

$$v = u + gt$$

$$s = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gs$$

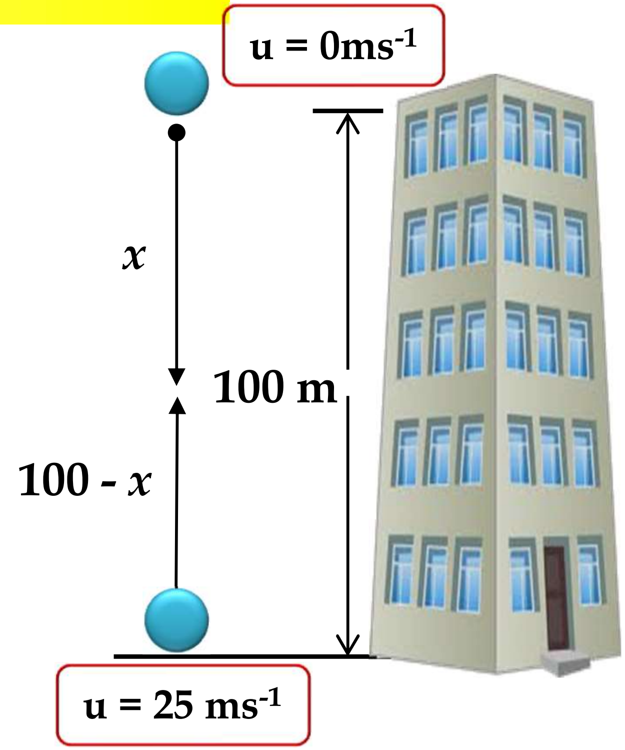
1 A stone is allowed to fall from the top of a tower 100 m high and at the same time another stone is projected vertically up wards from the ground with a velocity of 25 m/s. Calculate when and where the two stones will meet?

Given : Displacement (s) = 100 m

To find : Time and distance when the stones meet = ?

Formula : $s = ut + \frac{1}{2}gt^2$

Solution : Suppose the two stones meet at a height x from the top of a tower after time t from the start.



For the downward motion of stone A

$u = 0ms^{-1}$, $g = +10\ ms^{-2}$, $s = (x)$

$$s = ut + \frac{1}{2}gt^2$$

$$\therefore (x) = 0 + \frac{1}{2} \times 10 \times t^2$$

$$\therefore (x) = \frac{1}{2}10t^2 \quad \ldots\ldots (i)$$

1 A stone is allowed to fall from the top of a tower 100 m high and at the same time another stone is projected vertically up wards from the ground with a velocity of 25 m/s. Calculate when and where the two stone will meet?

Solution : **For upward motion of stone B**

$u = +\,25\,ms^{-1}$, $s = (100 - x)$, $g = -\,10\ ms^{-2}$

As the object goes upwards

$$s = ut + \frac{1}{2} g t^2$$

$$\therefore \quad (100 - x) = 25\,t - \frac{1}{2} \times 10 \times t$$

$$\therefore \quad 100 - \cancel{\frac{1}{2} 10t^2} = 25t - \cancel{\frac{1}{2} 10t^2}$$

$$\therefore \quad 100 = 25t$$

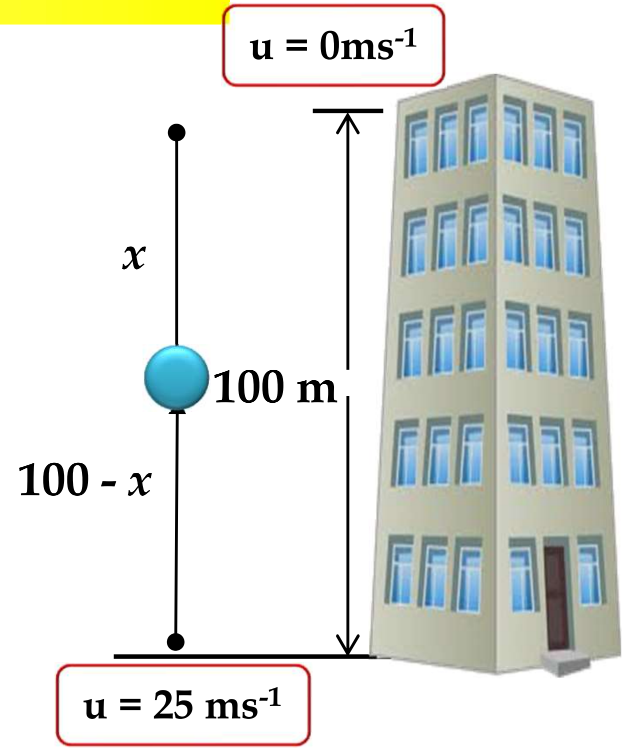
$$\therefore \quad t = 4\ s$$

From equation (i),

$$(x) = \frac{1}{2}\, 10 \times (4)^2$$

$$\therefore \quad x = 5 \times 4 \times 4$$

$$\therefore \quad x = 80\ m$$



Ans : The two stones meet after 4s at a height of 80 m from the top of the tower and are 20 m from surface of the earth.

Gravitation

- **Numerical based on kinematical equations**

2 A ball thrown up vertically returns to the thrower after 6 s. Find the velocity with which it was thrown up.

Given : Time of ascent = Time of descent = $\frac{6}{2}$ = 3 s

To find : Initial velocity (u) = ?

Formula : $v = u + gt$

Solution : (a) For upward motion of the ball,

$v = 0\,ms^{-1}$, $t = 3\,s$, $g = -9.8\,ms^{-2}$

$v = u + gt$

$\therefore \quad 0 = u + (-9.8 \times 3)$

$\therefore \quad 0 = u - 9.8 \times 3$

$\therefore \quad u = 9.8 \times 3$

$\therefore \quad u = 29.4\,ms^{-1}$

Ans : The ball was thrown with the velocity of $29.4\,ms^{-1}$

t = 6 sec

3 **A boy drops a coin from the top of a building which is 49 m high. Find the velocity with which the coin strikes the ground.**

Given : Initial velocity (u) $= 0$

Displacement (s) $= 49$m.

Acceleration due to gravity (g) $= 9.8\ m/s^2$

To find : Final velocity (v) $= ?$

Formula : $v^2 = u^2 + 2gs$

Solution : $v^2 = u^2 + 2gs$

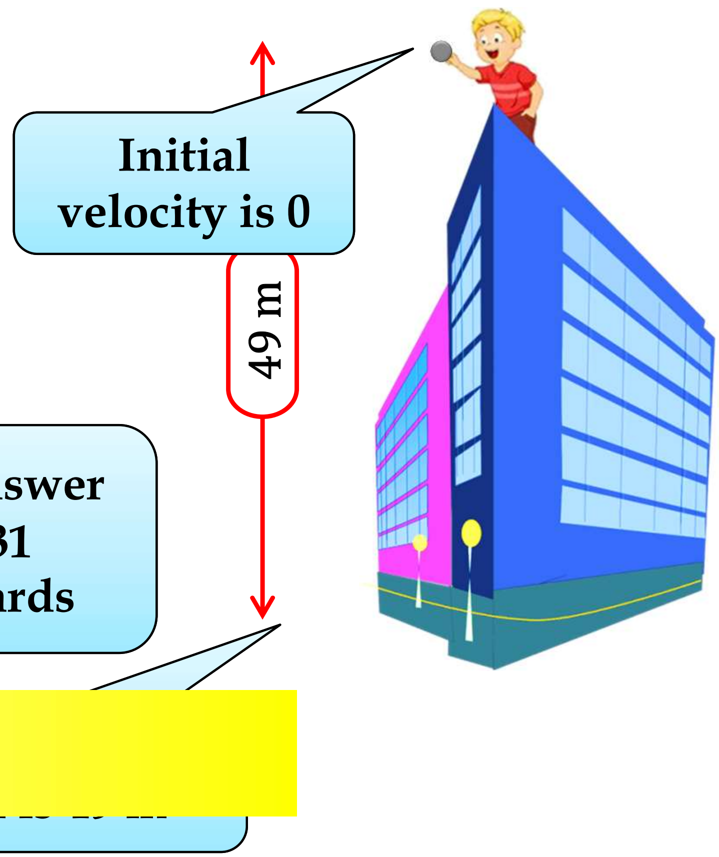
$v^2 = (0)^2 + 2 \times 9.8$

$v^2 = 961$

$v = \sqrt{961}$

But we take the answer as 31 as it is directed downwards

$\sqrt{961}$ Can be ± 31



Ans : **The velocity with which the coin strikes the ground is 31 m/s.**

Gravitation

- **Numerical based on kinematical equations**

4

A ball is thrown vertically upwards with a velocity of 49 m/s. Calculate (i) the maximum height to which it rises, (s) (ii) the total time (t) it takes to return to the surface of the earth. ($g = -9.8 ms^{-2}$)

Given :

$u = 49\ m\ s^{-1}$

$g = -9.8 ms^{-2}$

$v = 0\ m\ s^{-1}$

To find : (i) The maximum height to which it rises (s) = ?

(ii) The total time it takes to return to the surface of the earth (t) = ?

Formula : $v^2 = u^2 + 2\ gs$

Solution : $v^2 = u^2 + 2\ gs$

$(0)^2 = (49)^2 + 2 \times (-9.8)\ s$

$\therefore\ 0 = 49 \times 49 - 2 \times -$

$\therefore\ (49 \times 49) = 2 \times 9.8\ s$

$$\therefore\ s = \frac{\cancel{49}^{\,1} \times 49 \times \cancel{10}^{\,5}}{2 \times \cancel{98}_{\,\cancel{2}\,1}}$$

$$\therefore\ s = \frac{49 \times 5}{2} = 122.5\ m$$

(ii) $v = u + gt$

$0 = 49 - 9.8 \times t$

$$\therefore\ t = \frac{49}{9.8}$$

$\therefore\ t = 5\ s$

$\therefore$ Time of ascent = Time of descent

v = 0 m/s

u = 49 m/s

Ans :

The ball rises to height of 122.5m and it takes 10 seconds to return to the surface

Gravitation

- **Numerical based on kinematical equations**

5 A stone is released from the top of a tower of height 19.6m. Calculate its final velocity just before touching the ground.

Given : Initial velocity (u) = $0 ms^{-1}$

Displacement (s) = 19.6m

To find : Final velocity (v) = ?

Formula : $v^2 - u^2 = 2\,gs$

Solution : $v^2 - u^2 = 2\,gs$

$\therefore\ v^2 - (0)^2 = 2 \times 9.8 \times 19.6$

$\therefore\ v^2 = 19.6 \times 19.6$

$\therefore\ v = 19.6\ m\ s^{-1}$

Ans : The final velocity just before touching the ground is $19.6\ m\ s^{-1}$

6

A stone is thrown vertically upward with an Initial velocity of 40 m/s. Taking g = 10m/ s^2, find the maximum height reached by the stone. What is the net displacement and the total distance covered by the stone?

Given :

$u = 40 \text{ m s}^{-1}$

$g = -10 \text{ ms}^{-2}$

$v = 0 \text{ m s}^{-1}$

To find :

(i) The maximum height reached by the stone (s) = ?

(ii) The total distance covered by the stone = ?

(iii) The net displacement = ?

Formula : $v^2 = u^2 + 2\,gs$

Solution :

$v^2 = u^2 + 2\,gs$

$(0)^2 = (40)^2 = 2 \times (-10)\,s$

$$s = \frac{-(40 \times 40)}{-20} = 80 \text{ m}$$

(ii) Total distance covered = s + s = 80 + 80 = 160 m

(iii) Net displacement = 80 - 80 = 0 m