anodiam
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Circles

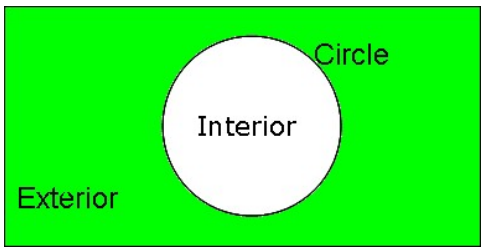
1. What is a Circle?

A **circle** is a collection (set) of all those points in a plane, each one of which is at a constant distance from a fixed point in the plane.

The fixed point is called the **center** and the constant distance is called the **radius** of the circle.

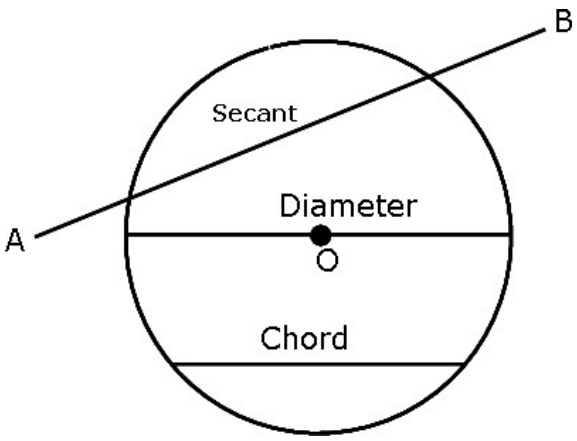
2. Division of a plane using circle

- A circle divides the plane on which it lies into three parts: inside the circle, the circle and outside the circle.
- All the points lying inside a circle are called its **interior points** and all those points which lie outside the circle are called its **exterior points**.
- The collection (set) of all interior points of a circle is called the **interior of the circle** while the collection (set) of all exterior points of a circle is called the **exterior of the circle**.



3. Chord, diameter and secant of a circle

- A line can meet a circle at the most in two points and the line segment joining two points on a circle is called a **chord** of the circle.
- A chord passing through the center of the circle is called a **diameter** of the circle. A diameter of the circle is its longest chord. It is equal to two times the radius.
- A line which meets a circle in two points is called a **secant** of the circle.



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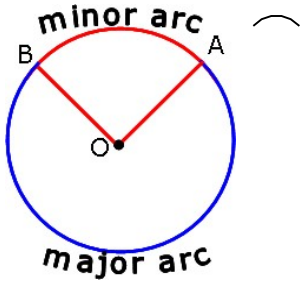
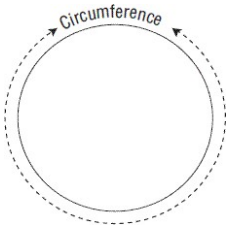
4. Arc of the circle

- A (continuous) part of a circle is called an **arc** of the circle. The arc of a circle is denoted by the symbol ‘ ’.
- When an arc is formed, it divides the circle into two pieces (between the points A and B), the smaller one and the longer one. The smaller one is called the **minor arc** of the circle, and the greater one is called the **major arc** of the circle.

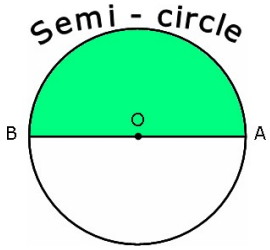


5. Circumference and Semi-circle

- The length of the complete circle is called the **circumference** of the circle.



- One-half of the whole arc (circumference) of a circle is called a **semi-circle**.



6. Central angle and Degree measure

- Any angle whose vertex is centre of the circle is called a **central angle**.
- The **degree measure of a minor arc** is the measure of the central angle subtended by an arc.
- The degree measure of a circle is 360°. The degree measure of a semi-circle is 180° (half of the circle).
- The degree measure of a major arc is (360° - θ°), where θ° is the degree measure of the corresponding minor arc.

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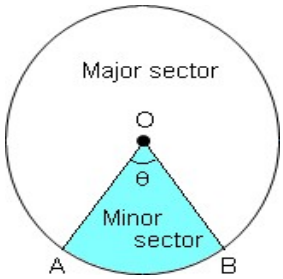
7. Congruent circles and arcs

- Two **circles** are said to be **congruent** if and only if either of them can be superposed on the other so as to cover it exactly.
- Accordingly, two **arcs** of a circle (or of congruent circles) are **congruent** if either of them can be superposed on the other so as to cover it exactly.

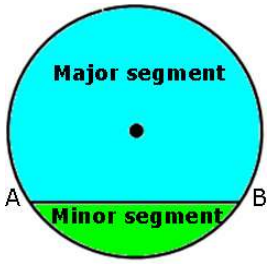
8. Sector of a circle

- The part of the plane region enclosed by an arc of a circle and its two bounding radii is called **sector** of a circle.
- If the central angle of a sector is more than $180°$, then the sector is called a **major sector** and if the central angle is less than $180°$, then the sector is called a **minor sector**.



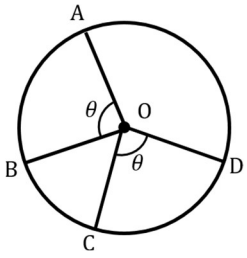
9. Segment of a circle

- A chord of a circle divides it into two parts. Each part is called a **segment** of the circle.
- The part containing the minor arc is called the **minor segment**, and the part containing the major arc is called the **major segment**.

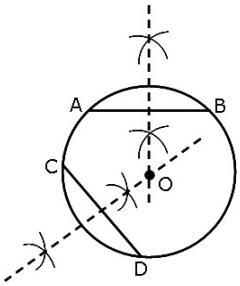


10. Angle subtended by a chord and perpendicular drawn to a chord

- Equal chords of a circle subtend equal angles at the centre.

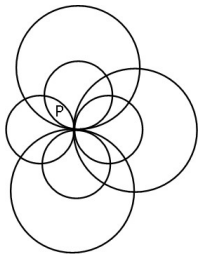


- If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.
- In a circle, perpendicular from the center to a chord bisects the chord.
- A line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
- Perpendicular bisectors of two chords of a circle, intersect each other at the centre of the circle.

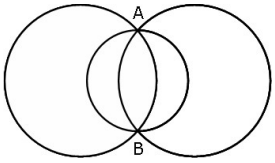


11. Number of circle through one or more point(s)

- An infinite number of circles can be drawn through a given point, say P.



- An infinite number of circles can be drawn through two given points, say A and B.



- One and only one circle can be drawn through three non-collinear points.

12. Distance of chord from the centre

- The length of the perpendicular from a point to a line is the distance of the line from the point.
- Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).
- Chords equidistant from the centre of a circle are equal in length.

13. Angle subtended by an Arc of a circle

- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- If two chords of a circle are equal, then their corresponding arcs are congruent.

- Conversely, if two arcs are congruent, then their corresponding chords are equal.
- Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre.

14. Con-cyclic points

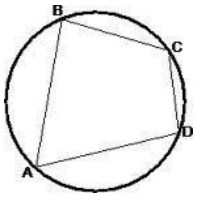
- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points are con-cyclic, i.e., they lie on the same circle.
- Angles in the same segment of a circle are equal.

15. Angle in a semi-circle

- An angle in a semi-circle is a right angle.
- The arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

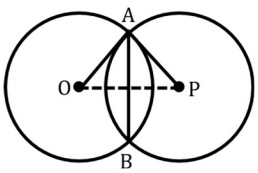
16. Cyclic quadrilaterals

A quadrilateral, all the four vertices of which lie on a circle is called a **cyclic quadrilateral**. The four vertices *A, B, C* and *D* are said to be concyclic points.



17. Properties of cyclic quadrilateral

- The opposite angles of a cyclic quadrilateral are supplementary i.e. their sum is 180°.
- If the sum of any pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.
- Any exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- The quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.
- The line of centres of two intersecting circles subtends equal angles at the two points of intersection.



In the figure, angle OAM = angle PAM.

- If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, then it is a rectangle.
- If the non-parallel sides of a trapezium are equal, then it is cyclic.