No. 41

Problems based on Numbers

Q. Form the pair of linear equations in the following problems, and find their solutions by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from denominator, fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction ?

What is the condition given to us

Fraction = Numerator / Denominator

According to the first condition,

$$\frac{x + 1}{y - 1} = 1$$

$$x + 1 = y - 1$$

$$x - y = -2 \quad \text{... (i)}$$

According to the second condition,

$$\frac{x}{y + 1} = \frac{1}{2}$$

$$2x = y + 1$$

$$2x - y = 1 \quad \text{... (ii)}$$

Solve (i) and (ii) by Elimination Method

Substituting $x = 3$ in (i), we get

$$y = 5$$

$$\frac{x}{y} = \frac{3}{5}$$

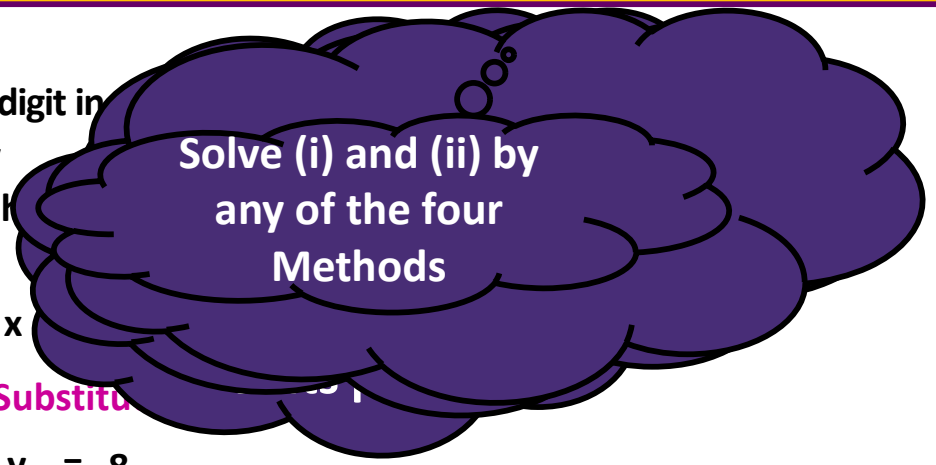
$\therefore$ Required fraction is $\frac{3}{5}$

(iii) The sum of the digits of a two–digit number is 9.
Also, nine times number is twice the number obtained by reversing the order of the digits.
Find the number.

Soln. Let the digit in ten's place be x and the digit in

$\therefore$ Original number = $10x + y$

Number obtained by interchanging th



According to the first condition,

$x + y = 9$ ... (i)

According to the second condition,

$9(10x + y) = 2(10y + x)$

$90x + 9y = 20y + 2x$

$88x - 11y = 0$

Dividing throughout by 11

$8x - y = 0$ ... (ii)

x

Substitu

$y = 8$

$\therefore$ Original Number $= 10x + y$

$= 10(1) + 8$

$= 10 + 8$

$= 18$

$\therefore$ Required two digit number is 18.

No. 42

Q.] The difference between two numbers is 26 and one number is three times the other. Find them.

Sol : Let the two numbers be x and y. (x > y)

According to the first condition,

$$x - y = 26 \quad \ldots (i)$$

According to the second condition,

$$x = 3y \quad \ldots (ii)$$

Substituting (ii) in (i)

$$3y - y = 26$$

$$2y = 26$$

$$y = 13$$

Substituting y = 13 in (ii)

$$x = 3(13)$$

$$x = 39$$

∴ The given numbers are 39 and 13.

Solve the equations by either Substitution or Elimination Method

No. 43

Q. Form the pair of linear equations for the following problem and find their solution by substitution method.

A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

Sol : Let the numerator and the denominator of the fraction be x and y ($y \neq 0$).

$\therefore$ Original fraction is $\frac{x}{y}$ $(y \neq 0)$

According to the 1st condition,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$11x + 22 = 9y + 18$$

$$\therefore 11x = 9y + 18 - 22$$

$$\therefore x = \frac{9y-4}{11} \quad \ldots (i)$$

According to the 2nd condition,

$$\frac{x+3}{y+3} = \frac{5}{6}$$

$$6x + 18 = 5y + 15$$

$$6x - 5y = -3 \quad \ldots (ii)$$

Substituting eqn (i) in eqn (ii)

$$6\left[\frac{9y-4}{11}\right] - 5y = -3$$

Multiplying throughout by 11

$$\therefore \frac{54y-24}{11} - 5y = -3$$

$$\therefore 54y - 24 - 55y = -33$$

$$\therefore -y = -33 + 24$$

$$\therefore -y = -9$$

$$\therefore y = 9$$

Substituting $y = 9$ in eqn (ii)

$$6x - 5(9) = -3$$

$$6x - 45 = -3$$

$$\therefore 6x = 42$$

$$\therefore x = 7$$

$$\therefore \frac{x}{y} = \frac{7}{9}$$

$\therefore$ **Required fraction is $\frac{7}{9}$**

No. 44

Q. Form the pair of linear equations in the following problems and find their solutions by any algebraic methods :

A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Sol : Let the numerator of the fraction be x and the denominator of the fraction be y.

$\therefore$ Original fraction is $\frac{x}{y}$

According to the 1st condition,

$$\frac{x-1}{y} = \frac{1}{3}$$

$$3x - 3 = y$$

$$y = 3x - 3 \quad \ldots (i)$$

According to the 2nd condition,

$$\frac{x}{y+8} = \frac{1}{4}$$

$$4x = y + 8 \quad \ldots (ii)$$

Substituting (i) in (ii), we get,

$$4x = 3x - 3 + 8$$

$$4x - 3x = 5$$

$$x = 5$$

We don't know numerator and denominator of fraction

Substituting x = 5 in equation (i),

$$y = 3(5) - 3$$

$$y = 15 - 3$$

$$y = 12$$

$\therefore$ The required fraction is $\frac{5}{12}$

No. 45

Problems based on numbers

Q. **Two numbers are in the ratio 3 : 4. If 4 is added to each number, their ratio becomes 4 : 5. Find the numbers.**

Sol. Let the two numbers be 'x' and 'y'

As per the first condition

Multiplying (i) by 4

(iii)

.....(iv)

First in this sum will have to either make the coefficient of x same or y same

Here, in the equation (iii) and (iv) the signs are same so, we will subtract these two equations

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umber the equation as (iv)

make

In this sum we will substitute x = 12 in equation (i)

To make the coefficient of y same we will have to multiply equation (i) by 4 and equation (ii) by 3

$\therefore$ 5x – 4y

$\therefore$ $5x - 4y = -4$ .....(ii)

The t nd 16

No. 46

Q. **Two numbers are in the ratio 5 : 6. If 8 is Subtracted to each number, their ratio becomes 4 : 5. Find the numbers.**

Sol. Let the two numbers be 'x' and 'y'

As per the first condition

∴ $5x - 4y = 8$ .....(ii)

Multiplying (i) by 4

.....(iii)

.....(iv)

Here, in the equation (iii) and (iv) the signs are same so, we will subtract these two equations

First in this sum will have to either make the coefficient of x same or y same

In this sum we will substitute x = 40 in equation (i)

To make the coefficient of y same we will have to multiply equation (i) by 4 and equation (ii) by 5

No. 47

Q. The sum of the numerator and denominator of a fraction is 15. The denominator is 3 more than twice the numerator. Determine the fraction.

Sol. Let the numerator of fraction be 'x' and denominator be 'y'

Numerator	Denominator
x	y

As per the first condition

$x + y = 15$ .....(i)

As per the second condition

$y = 2x + 3$ .....(ii)

Here, we can either solve it by equating the coefficients method

It is better to use elimination method because we have (ii) in the form of y = something

Substituting (ii) in (i)

$x + (2x + 3) = 15$

$\therefore \quad 3x + 3 = 15$

$\therefore \quad 3x = 12$

$\therefore \quad x = 4$

Substituting x = 4 in (ii)

$y = 2(4) + 3$

$\therefore \quad y = 11$

$\therefore \quad \frac{x}{y} = \frac{4}{11}$

The Required Fraction is $\frac{4}{11}$

No. 48

Q. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

Sol. Let the numerator of fraction be 'x' and denominator be 'y'

Numerator	Denominator
x	y

What do we need to find ?

We don't know numerator and denominator

This means equal

As per the first condition

$x + y$

$\therefore -2x + x + y =$

$\therefore -x + y =$

$\therefore y = x + 4$

Number it as equation (i)

Number it as equation (ii)

The new Fraction

Numerator	Denominator
x + 3	y + 3

Under the second condition

$\frac{x+3}{y+3} \times \frac{2}{3}$

$+ 3)$

$2y + 6$

$\therefore 3x - 2y = 6 - 9$

$\therefore 3x - 2y = -3$ .....(ii)

Sol. **Substituting (i) in (ii)**

$$3x - 2(x + 4) = -3$$

$$\therefore \quad 3x - 2x - 8 = -3$$

$$\therefore \quad 3x - 2x = -3 + 8$$

$$\therefore \quad x = 5$$

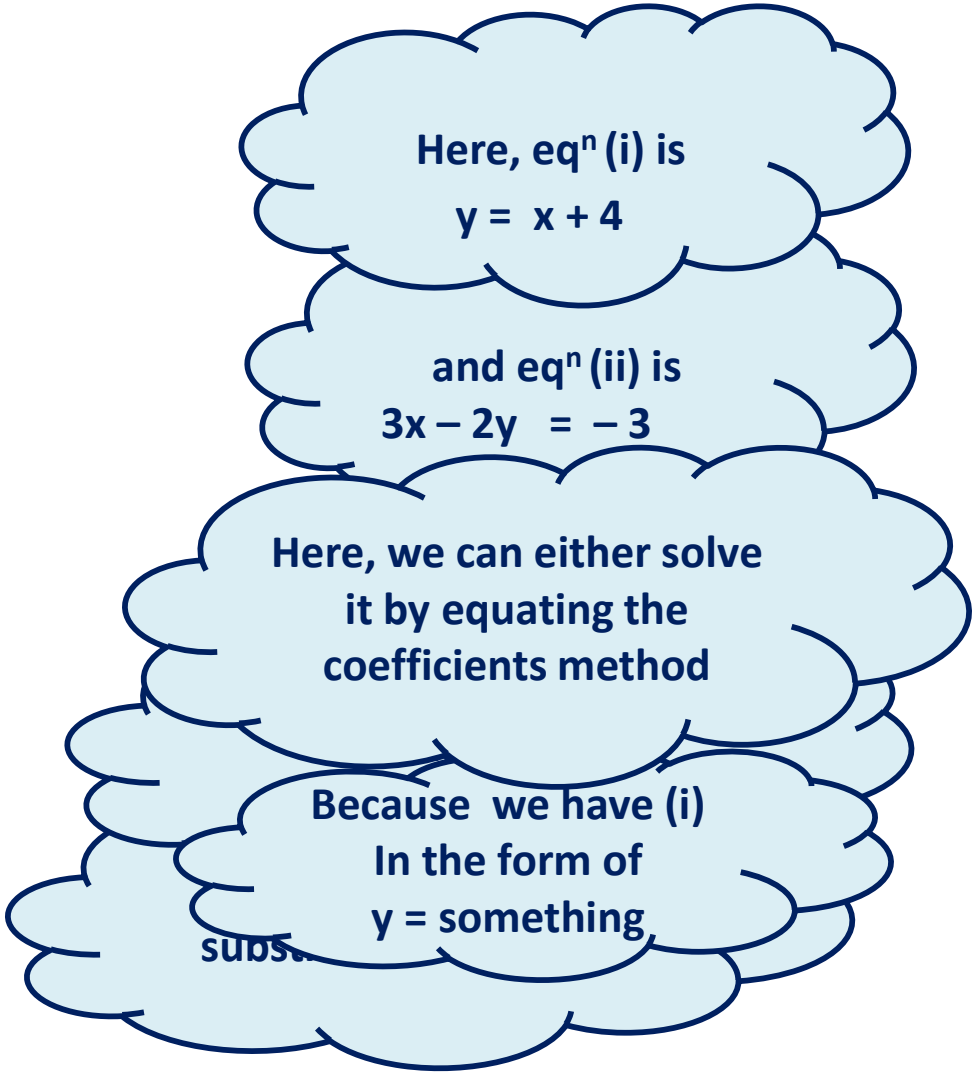
Substituting x = 5 in (i)

$$\therefore \quad y = 5 + 4$$

$$\therefore \quad y = 9$$

$$\therefore \quad \frac{x}{y} = \frac{5}{9}$$

The Required Fraction is $\frac{5}{9}$



No. 49

Q. A two digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Sol. Let the digit in tens place be 'x' and the digit in the unit place be 'y'

Tens	Units
$10 \times x$	$1 \times y$

$\therefore$ Two digit number is =

As per the first condition

$= 4(x + y) + 3$

$10x + y = 4x + 4y + 3$

$10x - 4x + y - 4y = 3$

$6x - 3y = 3$

Dividing throughout by 3

$2x - y = 1$ .....(i)

Tens	Units
	$1 \times y$

A two digit number

4 5

↑ ↑

T U

$45 = 10 \times 4 + 5$

$= 40 + 5$

$= 45$

Here the digit are reversed

digit in units place.

ghout by 9

$x - y = -2$ .....(ii)

Subtracting (ii) from (i)

$$\begin{array}{rrr} 2x - \cancel{y} & = & 1 \\ x - \cancel{y} & = & -2 \\ (-)\quad (+) & & (+) \\ \hline x & = & 3 \end{array}$$

Substituting x = 3 in equation (ii)

$$\begin{array}{lrl} & 3 - y & = -2 \\ \therefore & -y & = -2 - 3 \\ \therefore & -y & = -5 \\ \therefore & y & = 5 \end{array}$$

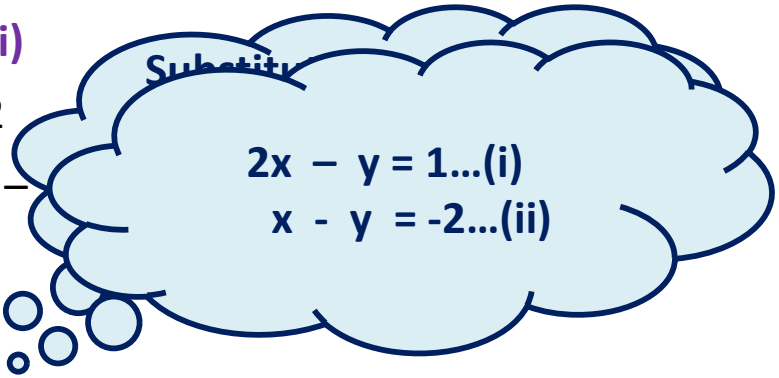
Original number $= 10x + y$

$= 10 \times 3 + 5$

$= 30 + 5$

$= 35$

$\therefore$ **The original number is 35.**



No. 50

Q. **A two digit number is 3 more than six times the sum of its digits. If 18 is added to the number obtained by interchanging the digits, we get the original number. Find the number.**

Sol. Let the digit in tens place be 'x' and the digit in the unit place be 'y'

Tens	Units
$10 \times x$	$1 \times y$

$\therefore$ Two digit number is = **10x + y**

As per the first condition

$= 6(x$

$10x + y = 6$

$10x - 6x + y - 6y = 3$

$4x - 5y = 3$

Tens	Units

A two digit number has two parts.

Here the digit are interchanged.

nging

As per the second condition

Lets make the digit in units place.

We will make the coefficient of variable xsame

nout by 9

$x + y = -2$ .....(ii)

Multiplying (ii) by 4

$-4x + 4y = -8$ ……..(iii)

Adding (i) and (iii)

$$\begin{array}{rrr} \cancel{4x} - 5y & = & 3 \\ \cancel{-4x} + 4y & = & -8 \\ \hline -y & = & -5 \end{array}$$

$\therefore \quad y = 5$

Substituting y = 5 in equation (ii)

$-x + 5 = -2$

$\therefore \quad -x = -2 - 5$

$\therefore \quad -x = -7$

Original nu

Substitute the values of x & y in the

$-x + y = -2$…(ii)

$\therefore$ **The original number is 75.**

No. 51

Q. The sum of a two digit number and the number obtained by interchanging the digits is 99. If the digits differ by 3. Find the number

Sol. Let the digit in ten's place be x and digit in unit's place be y

∴ The number = 10x + y

The number ob[illegible]

the digit = [illegible]

Accordi[illegible]

∴ 10x + y + [illegible]

∴ 11x + [illegible]

Dividing [illegible]

∴ [illegible] ....(i)

Di[illegible]

Number it as equation (ii)

∴ x − y = 3 .....(ii)

So there are

Substitute the values of x & y in the original two digit number

Adding (i) and (ii)

$$\begin{array}{rcl} x + \cancel{y} & = & 9 \\ x - \cancel{y} & = & 3 \\ \hline 2x & = & 12 \end{array}$$

∴ $x = 6$

Substituting x = 6 in (i)

$6 + y = 9$

∴ $y = 9 - 6$

∴ $y = 3$

The two digit number = 10x + y

= 10 (6) + 3

= 60 + 3

= 63

Q. The sum of a two digit number and the number obtained by interchanging the digits is 99. If the digits differ by 3. Find the

Case (ii) :

$x < y$

$\therefore \quad y - x = 3$

$\therefore \quad -x + y = 3 \quad$ .....(iii)

Adding (i) and (iii), we get

$$\begin{array}{rcl} x + y & = & 9 \\ -x + y & = & 3 \\ \hline 2y & = & 12 \end{array}$$

$\therefore \quad y = 6$

Substituting y = 6 in

$\therefore \quad x + 6 = 9$

$\therefore \quad x = 9 - 6$

$\therefore \quad x = 3$

Number it as equation (iii)

We know,

$x + y = 9$ .....(i)

Substitute the values of x & y in the original two digit number

$= 10x + y$

$= 10\,(3) + 6$

$= 30 + 6$

$= 36$

$\therefore$ **The two digit number is either 63 or 36.**

No. 52

Q. 10 students of Class X took part in a Mathem
number of girls is 4 more than the number o
find the number of boys and girls who took p

Sol : Let the number of boys be x and the numbe

According to the first condition,

$x + y = 10$ ... (i)

According to the second condition,

$y = x + 4$ ... (ii)

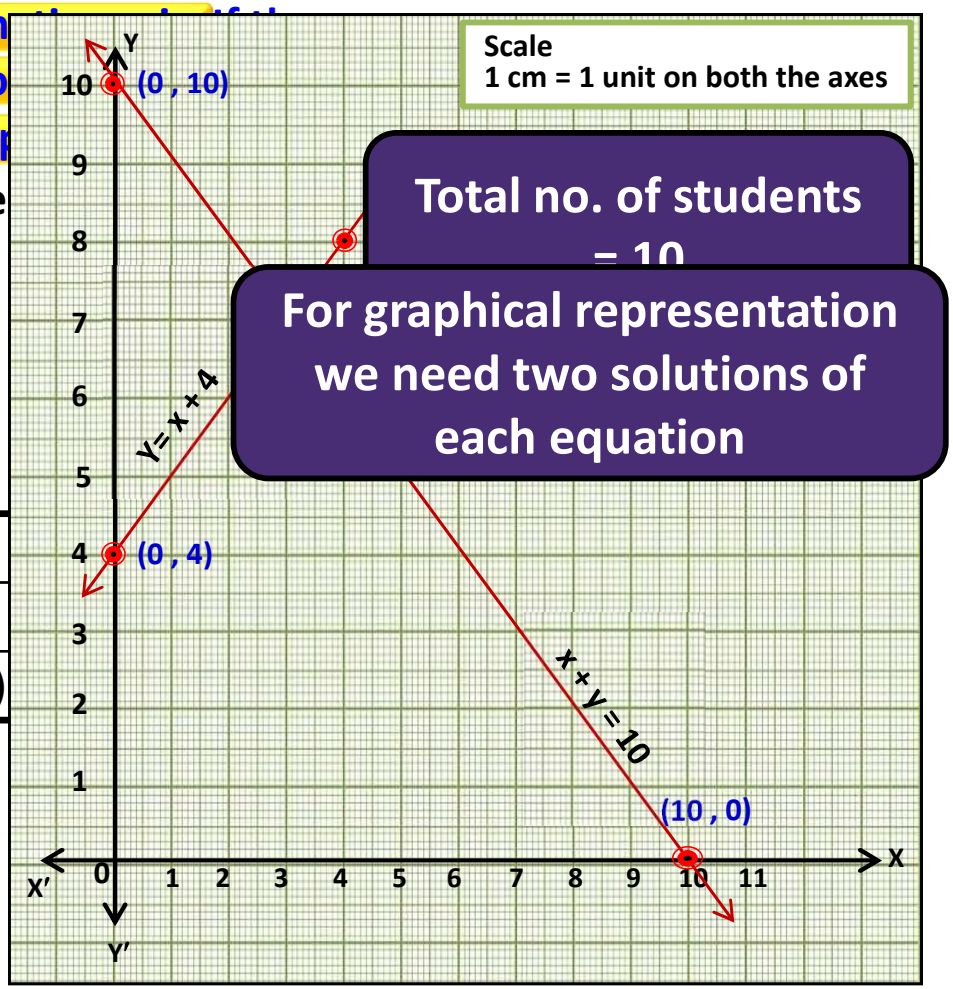
$x + y = 10$

x	0	10
y	10	0
(x, y)	(0, 10)	(10, 0)

$y = x + 4$

x	0	4
y	4	8
(x, y)	(0,4)	(4,8)

∴ 3 Boys and 7 Girls took part in quiz.



Thank You