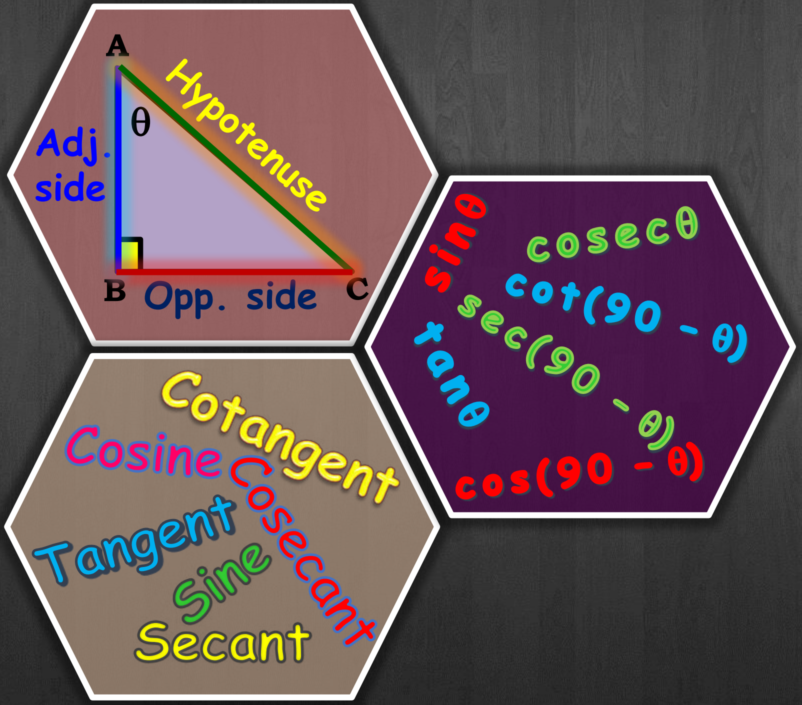
8. INTRODUCTION TO TRIGONOMETRY



Module 1

TRIGONOMETRY

TRI	→	Three
GONA	→	Sides
METRON	→	Measure

Trigonometry deals with the measurements of sides and angles of a triangle.

UNDERSTAND!

- **For ∠X**

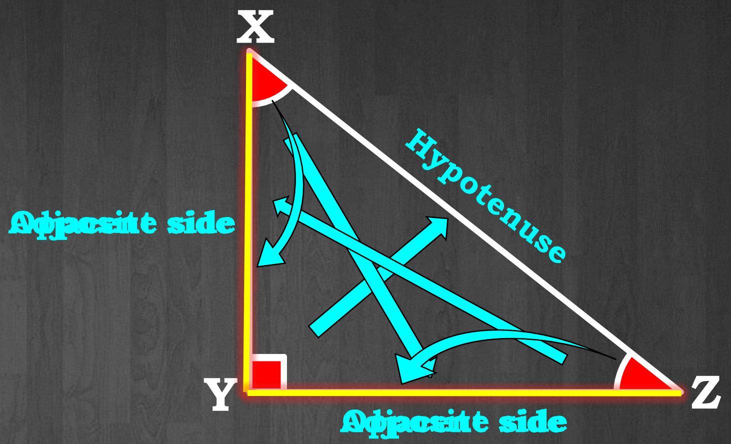
Opposite side → YZ

Adjacent side → XY

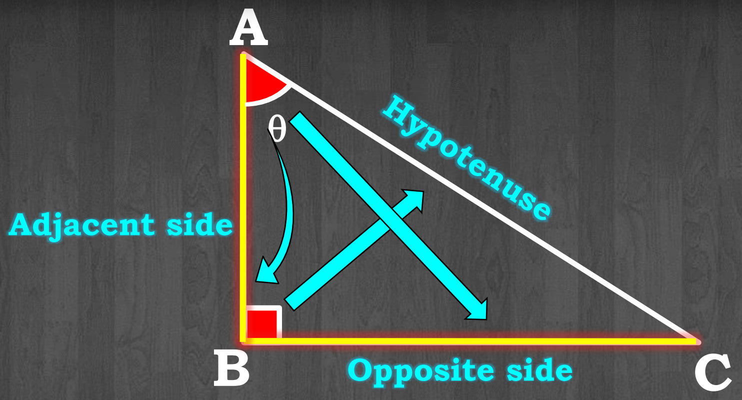
- **For ∠Z**

Opposite side → XY

Adjacent side → YZ



Trigonometric ratios of angle θ in a right-angled triangle



$$\text{Sine } \theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\text{Cosine } \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

$$\text{Tangent } \theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\text{Cosecant } \theta = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

$$\text{Secant } \theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}}$$

$$\text{Cotangent } \theta = \frac{\text{Adjacent side}}{\text{Opposite side}}$$

Module 2

EXERCISE 8.1

Q.1) In ΔABC, right-angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine (i) sin A, cos A (ii) sin C, cos C.

Solution:

In ΔABC, $\angle B = 90°$

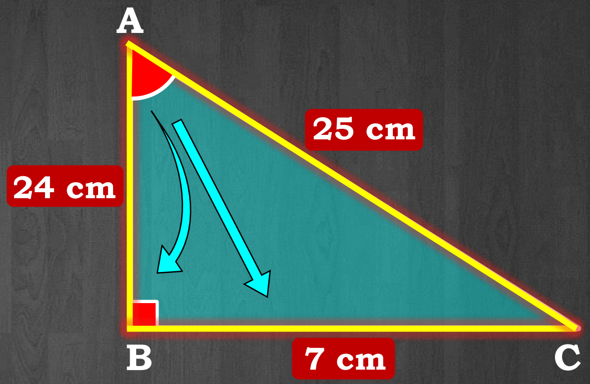
$AC^2 = AB^2 + BC^2$ *[Pythagoras theorem]*

$\therefore AC^2 = (24)^2 + (7)^2$

$\therefore AC^2 = 576 + 49$

$\therefore AC^2 = 625$

$\therefore AC = 25$ cm



(i) $\sin A = \dfrac{BC}{AC}$

$\therefore \boxed{\sin A = \dfrac{7}{25}}$

$\cos A = \dfrac{AB}{AC}$

$\therefore \boxed{\cos A = \dfrac{24}{25}}$

EXERCISE 8.1

Q.1) In ΔABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine (i) sin A, cos A (ii) sin C, cos C.

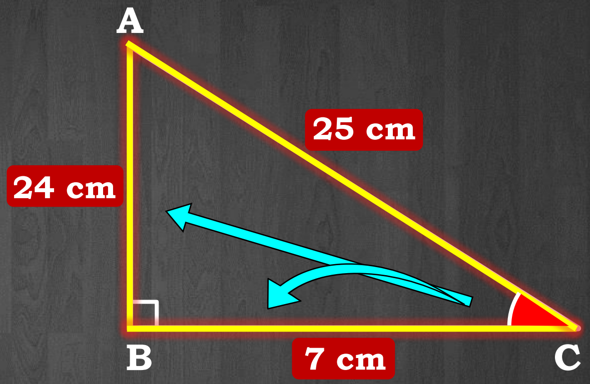
Solution:

(ii) $\sin C = \dfrac{AB}{AC}$

$\therefore$ $\boxed{\sin C = \dfrac{24}{25}}$

$\cos C = \dfrac{BC}{AC}$

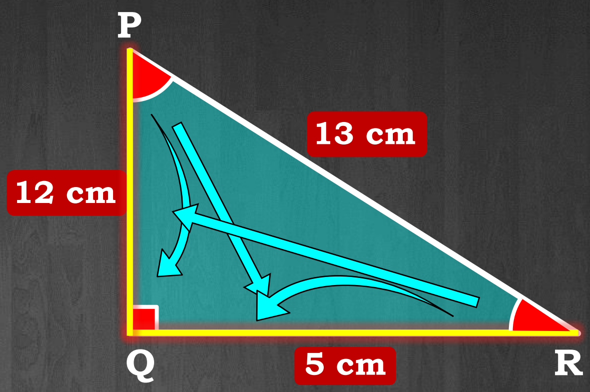
$\therefore$ $\boxed{\cos C = \dfrac{7}{25}}$



Module 3

EXERCISE 8.1

Q.2) In the given figure, find tan P – cot R.



Solution:

In ΔPQR, $\angle Q = 90^\circ$

$$PR^2 = PQ^2 + QR^2 \quad \textit{[Pythagoras theorem]}$$

$$\therefore \quad (13)^2 = (12)^2 + QR^2$$

$$\therefore \quad 169 - 144 = QR^2$$

$$\therefore \quad QR^2 = 25$$

$$\therefore \quad QR = 5 \text{ cm}$$

$$\tan P = \frac{QR}{PQ}$$

$$\therefore \quad \tan P = \frac{5}{12}$$

$$\cot R = \frac{QR}{PQ}$$

$$\therefore \quad \cot R = \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12}$$

$$\therefore \quad \tan P - \cot R = 0$$

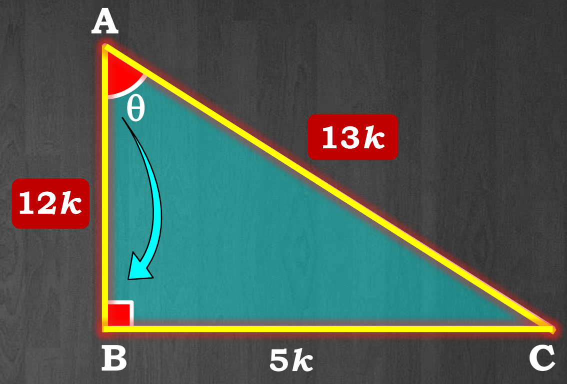
Module 4

EXERCISE 8.1

Q.5) $\sec\theta = \dfrac{13}{12}$, calculate all other trigonometric ratios.

Solution:

Consider ΔABC in which $\angle B = 90°$ and $\angle A = \theta$.



$$\sec\theta = \frac{13}{12} \quad \text{...(i)}$$

$$\sec\theta = \frac{AC}{AB} \quad \text{...(ii)}$$

$$\therefore \quad \frac{AC}{AB} = \frac{13}{12}$$ *[From (i) and (ii)]*

Let the non zero common multiple be k.

$\therefore \quad AC = 13k, \quad AB = 12k$

$$AC^2 = AB^2 + BC^2$$ *[Pythagoras theorem]*

$$\therefore \quad (13k)^2 = (12k)^2 + BC^2$$

$$\therefore \quad 169k^2 - 144k^2 = BC^2$$

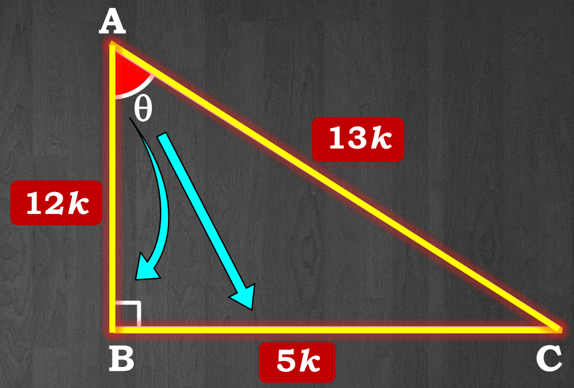
$$\therefore \quad BC^2 = 25k^2$$

$$\therefore \quad BC = 5k$$

EXERCISE 8.1

Q.5) $\sec\theta = \dfrac{13}{12}$, calculate all other trigonometric ratios.

Solution:



$$\sin\theta = \frac{BC}{AC}$$

$$\therefore \sin\theta = \frac{5\cancel{k}}{13\cancel{k}}$$

$$\therefore \boxed{\sin\theta = \frac{5}{13}}$$

$$\cos\theta = \frac{AB}{AC}$$

$$\therefore \cos\theta = \frac{12\cancel{k}}{13\cancel{k}}$$

$$\therefore \boxed{\cos\theta = \frac{12}{13}}$$

$$\tan\theta = \frac{BC}{AB}$$

$$\therefore \tan\theta = \frac{5\cancel{k}}{12\cancel{k}}$$

$$\therefore \boxed{\tan\theta = \frac{5}{12}}$$

$$\operatorname{cosec}\theta = \frac{AC}{BC}$$

$$\therefore \operatorname{cosec}\theta = \frac{13\cancel{k}}{5\cancel{k}}$$

$$\therefore \boxed{\operatorname{cosec}\theta = \frac{13}{5}}$$

$$\cot\theta = \frac{AB}{BC}$$

$$\therefore \cot\theta = \frac{12\cancel{k}}{5\cancel{k}}$$

$$\therefore \boxed{\cot\theta = \frac{12}{5}}$$

Module 5

EXERCISE 8.1

Q.4) Given $15 \cot A = 8$, find sin A and sec A.

Solution:

Consider ΔABC in which $\angle B = 90°$.

$15 \cot A = 8$

$$\boxed{\cot A} = \frac{8}{15} \quad \text{...(i)}$$

$$\boxed{\cot A} = \frac{AB}{BC} \quad \text{...(ii)}$$

$$\therefore \quad \frac{AB}{BC} = \frac{8}{15} \quad \text{[From (i) and (ii)]}$$

Let the non zero common multiple be k.

$\therefore$ $AB = 8k$, $BC = 15k$

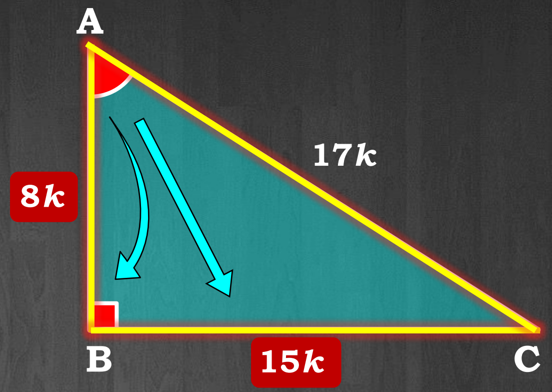
$$AC^2 = AB^2 + BC^2 \quad \text{[Pythagoras theorem]}$$

$$\therefore \quad AC^2 = (8k)^2 + (15k)^2$$

$$\therefore \quad AC^2 = 64k^2 + 225k^2$$

$$\therefore \quad AC^2 = 289k^2$$

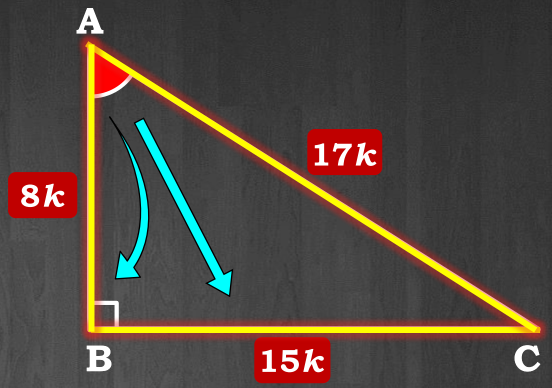
$$\therefore \quad AC = 17k$$



EXERCISE 8.1

Q.4) Given 15 cot A = 8, find sin A and sec A.

Solution:



$$\sin A = \frac{BC}{AC}$$

$$\therefore \quad \sin A = \frac{15\cancel{k}}{17\cancel{k}}$$

$$\therefore \quad \boxed{\sin A = \frac{15}{17}}$$

$$\sec A = \frac{AC}{AB}$$

$$\therefore \quad \sec A = \frac{17\cancel{k}}{8\cancel{k}}$$

$$\therefore \quad \boxed{\sec A = \frac{17}{8}}$$

Thank You

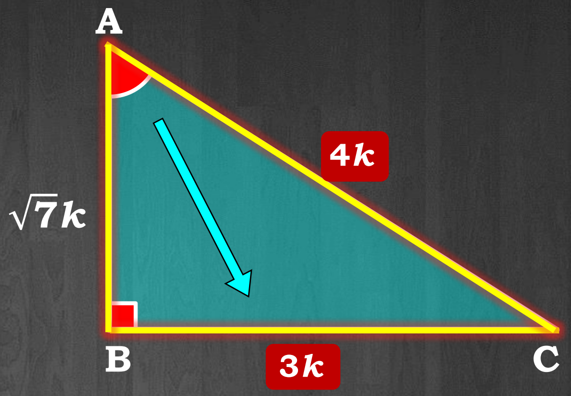
Module 6

EXERCISE 8.1

Q.3) If $\sin A = \dfrac{3}{4}$, calculate cos A and tan A.

Solution:

Consider ΔABC in which $\angle B = 90^\circ$.



$$\sin A = \frac{3}{4} \quad \ldots(i)$$

$$\sin A = \frac{BC}{AC} \quad \ldots(ii)$$

$$\therefore \quad \frac{BC}{AC} = \frac{3}{4} \qquad \textit{[From (i) and (ii)]}$$

Let the non zero common multiple be k.

$\therefore \quad BC = 3k, \quad AC = 4k$

$$AC^2 = AB^2 + BC^2 \qquad \textit{[Pythagoras theorem]}$$

$$\therefore \quad (4k)^2 = AB^2 + (3k)^2$$

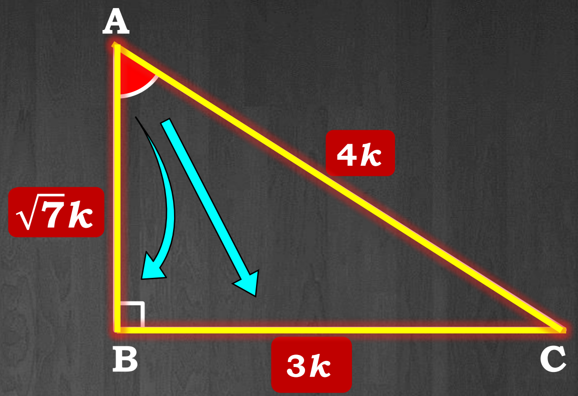
$$\therefore \quad 16k^2 - 9k^2 = AB^2$$

$$\therefore \quad AB^2 = 7k^2$$

$$\therefore \quad AB = \sqrt{7}k \quad \ldots(iii)$$

EXERCISE 8.1

Q.3) If $\sin A = \frac{3}{4}$, calculate **cos A** and **tan A**.



Solution:

$$\cos A = \frac{AB}{AC}$$

$$\therefore \quad \cos A = \frac{\sqrt{7}\cancel{k}}{4\cancel{k}}$$

$$\therefore \quad \boxed{\cos A = \frac{\sqrt{7}}{4}}$$

$$\tan A = \frac{BC}{AB}$$

$$\therefore \quad \tan A = \frac{3\cancel{k}}{\sqrt{7}\cancel{k}}$$

$$\therefore \quad \boxed{\tan A = \frac{3}{\sqrt{7}}}$$

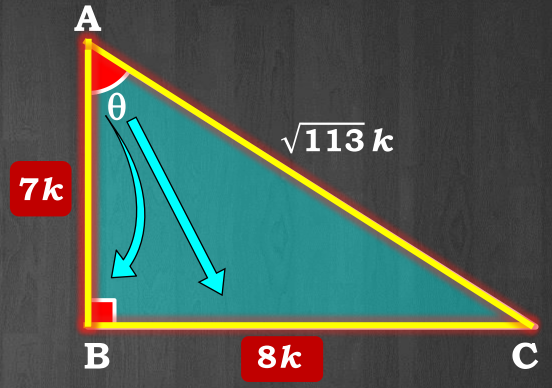
Module 7

EXERCISE 8.1

Q.7) If $\cot\theta = \frac{7}{8}$, Evaluate: (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

Solution:

Consider ΔABC in which $\angle B = 90°$ and $\angle A = \theta$.



$\cot\theta = \frac{7}{8}$ ...(i)

$\cot\theta = \frac{AB}{BC}$ ...(ii)

$\therefore \quad \frac{AB}{BC} = \frac{7}{8}$ *[From (i) and (ii)]*

Let the non zero common multiple be k.

$AB = 7k$, $BC = 8k$

$AC^2 = AB^2 + BC^2$ *[Pythagoras theorem]*

$\therefore \quad AC^2 = (7k)^2 + (8k)^2$

$\therefore \quad AC^2 = 49k^2 + 64k^2$

$\therefore \quad AC^2 = 113k^2$

$\therefore \quad AC = \sqrt{113}\,k$

EXERCISE 8.1

Q.7) If $\cot\theta = \dfrac{7}{8}$, Evaluate: (i) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

Solution:

$$\sin\theta = \frac{BC}{AC}$$

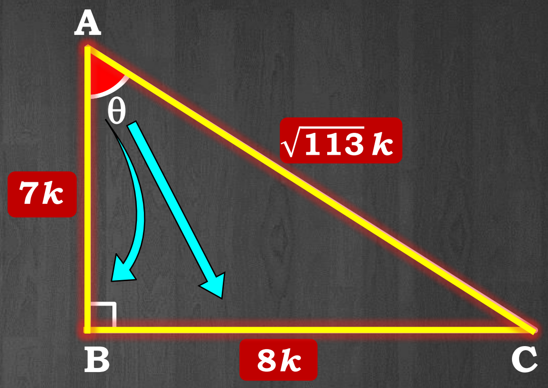
$$\therefore \quad \sin\theta = \frac{8\cancel{k}}{\sqrt{113}\,\cancel{k}}$$

$$\therefore \quad \sin\theta = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{AB}{AC}$$

$$\therefore \quad \cos\theta = \frac{7\cancel{k}}{\sqrt{113}\,\cancel{k}}$$

$$\therefore \quad \cos\theta = \frac{7}{\sqrt{113}}$$



EXERCISE 8.1

Q.7) If $\cot\theta = \frac{7}{8}$, Evaluate: (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

Solution:

$$\sin\theta = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{7}{\sqrt{113}}$$

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \left[1-\left(\frac{8}{\sqrt{113}}\right)^2\right] \div \left[1-\left(\frac{7}{\sqrt{113}}\right)^2\right]$$

$$= \left(1-\frac{64}{113}\right) \div \left(1-\frac{49}{113}\right)$$

$$= \left(\frac{113-64}{113}\right) \div \left(\frac{113-49}{113}\right) = \frac{49}{\cancel{113}} \times \frac{\cancel{113}}{64}$$

$$\therefore \boxed{\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{49}{64}}$$

EXERCISE 8.1

Q.7) If $\cot\theta = \dfrac{7}{8}$, Evaluate: (ii) $\cot^2\theta$

Solution:

$$\cot\theta = \frac{7}{8} \quad \text{...(i)}$$

$$\cot^2\theta = \left(\frac{7}{8}\right)^2$$

$$\therefore \quad \boxed{\cot^2\theta = \frac{49}{64}}$$

Module 8

EXERCISE 8.1

Q.8) If $3 \cot A = 4$, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

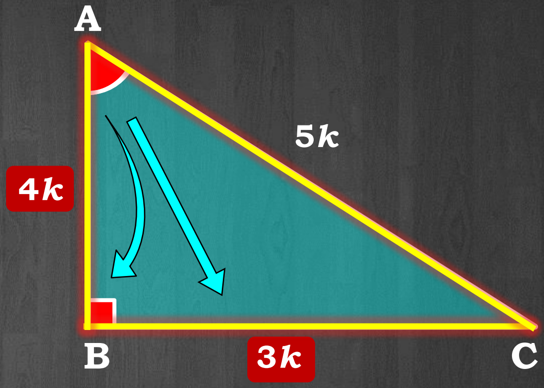
Solution:

Consider ΔABC in which $\angle B = 90^\circ$.

$3 \cot A = 4$

$\boxed{\cot A} = \frac{4}{3}$ ...(i)

$\boxed{\cot A} = \frac{AB}{BC}$ ...(ii)



$\therefore \quad \frac{AB}{BC} = \frac{4}{3}$ *[From (i) and (ii)]*

Let the non zero common multiple be k.

$AB = 4k$, $BC = 3k$

$AC^2 = AB^2 + BC^2$ *[Pythagoras theorem]*

$\therefore \quad AC^2 = (4k)^2 + (3k)^2$

$\therefore \quad AC^2 = 16k^2 + 9k^2$

$\therefore \quad AC^2 = 25k^2$

$\therefore \quad AC = 5k$

EXERCISE 8.1

Q.8) If $3 \cot A = 4$, check whether $\dfrac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Solution:

$$\sin A = \frac{BC}{AC}$$

$$\therefore \quad \sin A = \frac{3\cancel{k}}{5\cancel{k}}$$

$$\therefore \quad \sin A = \frac{3}{5}$$

$$\cos A = \frac{AB}{AC}$$

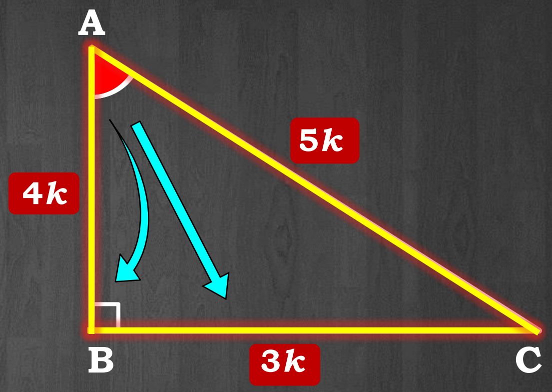
$$\therefore \quad \cos A = \frac{4\cancel{k}}{5\cancel{k}}$$

$$\therefore \quad \cos A = \frac{4}{5}$$

$$\tan A = \frac{BC}{AB}$$

$$\therefore \quad \tan A = \frac{3\cancel{k}}{4\cancel{k}}$$

$$\therefore \quad \tan A = \frac{3}{4}$$



EXERCISE 8.1

Q.8) If 3 cot A = 4, check whether $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Solution:

$$\text{L.H.S} = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$= \left[1-\left(\frac{3}{4}\right)^2\right] \div \left[1+\left(\frac{3}{4}\right)^2\right]$$

$$= \left(1-\frac{9}{16}\right) \div \left(1+\frac{9}{16}\right)$$

$$= \left(\frac{16-9}{16}\right) \div \left(\frac{16+9}{16}\right) = \frac{7}{16} \div \frac{25}{16}$$

$$\therefore \quad \text{L.H.S} = \frac{7}{\cancel{16}} \times \frac{\cancel{16}}{25} = \frac{7}{25} \quad \ldots\text{(iii)}$$

$\sin A = \dfrac{3}{5}$

$\cos A = \dfrac{4}{5}$

$\tan A = \dfrac{3}{4}$

EXERCISE 8.1

Q.8) If $3 \cot A = 4$, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Solution:

$$\text{R.H.S} = \cos^2 A - \sin^2 A$$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$\therefore \quad \text{R.H.S} = \frac{7}{25}$$

$$\therefore \quad \text{L.H.S} = \text{R.H.S} \qquad \ldots\text{(iv)}$$

$$\therefore \quad \boxed{\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A}$$

$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$

$$\text{L.H.S} = \frac{7}{25}$$

Module 9

EXERCISE 8.1

Q.9) In triangle ABC, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$

i) $\sin A \cos C + \cos A \sin C$

Solution:

In ΔABC, $\angle B = 90°$

$\tan A = \frac{1}{\sqrt{3}}$ ...(i)

$\tan A = \frac{BC}{AB}$ ...(ii)

$\therefore \frac{BC}{AB} = \frac{1}{\sqrt{3}}$ *[From (i) and (ii)]*

Let the non zero common multiple be k.

$AB = \sqrt{3}k$, $BC = k$

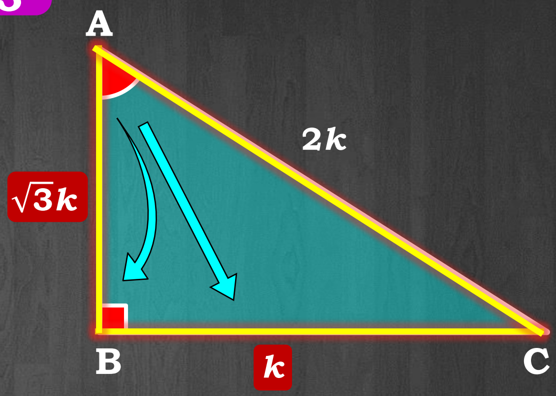
$AC^2 = AB^2 + BC^2$ *[Pythagoras theorem]*

$\therefore AC^2 = (\sqrt{3}k)^2 + (1k)^2$

$\therefore AC^2 = 3k^2 + k^2$

$\therefore AC^2 = 4k^2$

$\therefore AC = 2k$



EXERCISE 8.1

Q.9) In triangle ABC, right angled at B, if $\tan A = \dfrac{1}{\sqrt{3}}$

i) $\sin A \cos C + \cos A \sin C$

Solution:

$$\sin A = \frac{BC}{AC}$$

$$\therefore \quad \sin A = \frac{1\cancel{k}}{2\cancel{k}}$$

$$\therefore \quad \sin A = \frac{1}{2}$$

$$\cos C = \frac{BC}{AC}$$

$$\therefore \quad \cos C = \frac{1\cancel{k}}{2\cancel{k}}$$

$$\therefore \quad \cos C = \frac{1}{2}$$

$$\cos A = \frac{AB}{AC}$$

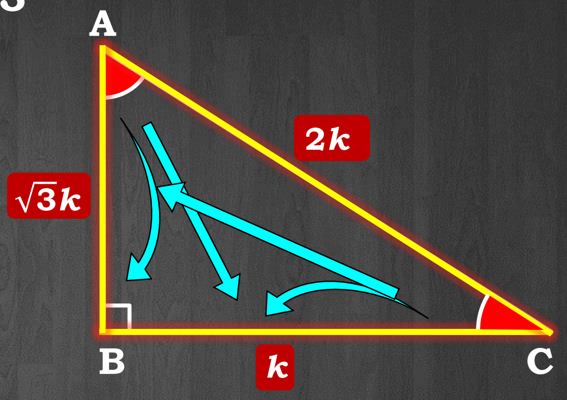
$$\therefore \quad \cos A = \frac{\sqrt{3}\cancel{k}}{2\cancel{k}}$$

$$\therefore \quad \cos A = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{AB}{AC}$$

$$\therefore \quad \sin C = \frac{\sqrt{3}\cancel{k}}{2\cancel{k}}$$

$$\therefore \quad \sin C = \frac{\sqrt{3}}{2}$$

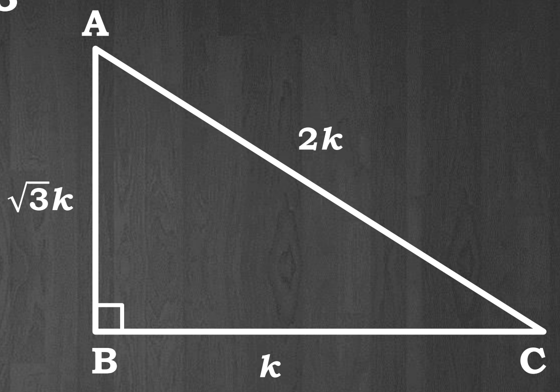


EXERCISE 8.1

Q.9) In triangle ABC, right angled at B, if $\tan A = \dfrac{1}{\sqrt{3}}$

i) sin A cos C + cos A sin C

Solution:



$\sin A \cos C + \cos A \sin C$

$\sin A = \dfrac{1}{2}$

$\cos C = \dfrac{1}{2}$

$\cos A = \dfrac{\sqrt{3}}{2}$

$\sin C = \dfrac{\sqrt{3}}{2}$

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{1+3}{4}$$

$$= \frac{4}{4}$$

$\therefore$ $\boxed{\sin A \cos C + \cos A \sin C = 1}$

EXERCISE 8.1

Q.9) In triangle ABC, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$

ii) $\cos A \cos C - \sin A \sin C$

Solution:

$\cos A \cos C - \sin A \sin C$

$= \left(\frac{\sqrt{3}}{2} \times \frac{1}{2}\right) - \left(\frac{1}{2} \times \frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

$= 0$

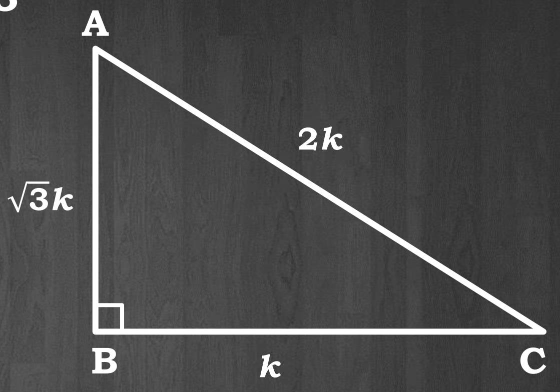
$\sin A = \frac{1}{2}$

$\cos C = \frac{1}{2}$

$\cos A = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\sqrt{3}}{2}$

$\therefore \sin A \cos C + \cos A \sin C = 0$



Module 10

EXERCISE 8.1

Q.10) In Δ PQR, right angled at Q, PR + QR = 25, PQ = 5cm. Determine the value of sin P, cos P and tan P.

Solution:

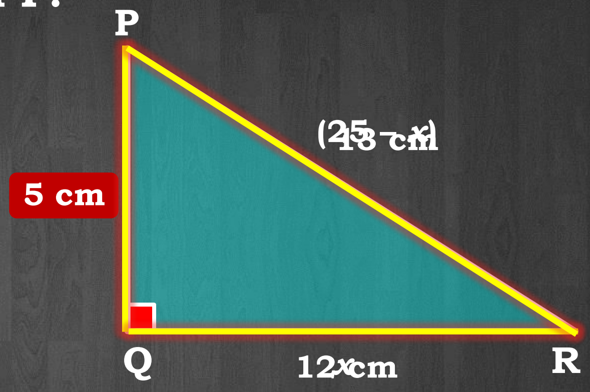
In ΔPQR, $\angle Q = 90^\circ$

$PR + QR = 25$

Let $\boxed{QR = x}$

$\therefore \quad x + PR = 25$

$\therefore \quad \boxed{PR = (25 - x)\text{ cm}}$



$PR^2 = PQ^2 + QR^2$ [*Pythagoras theorem*]

$\therefore \quad (25 - x)^2 = (5)^2 + (x)^2$

$\therefore \quad 625 - 50x + \cancel{x^2} = 25 + \cancel{x^2}$

$\therefore \quad -50x = 25 - 625$

$\therefore \quad x = \dfrac{-\cancel{600}^{\,12}}{-\cancel{50}}$

$\therefore \quad \boxed{x = 12}$

$QR = \boxed{x}$

$\therefore \quad QR = 12\text{ cm}$

$PR = 25 - \boxed{x}$

$= 25 - 12$

$\therefore \quad PR = 13\text{ cm}$

EXERCISE 8.1

Q.10) In Δ PQR, right angled at Q, PR + QR = 25, PQ = 5cm. Determine the value of sin P, cos P and tan P.

Solution:

$$\sin P = \frac{QR}{PR}$$

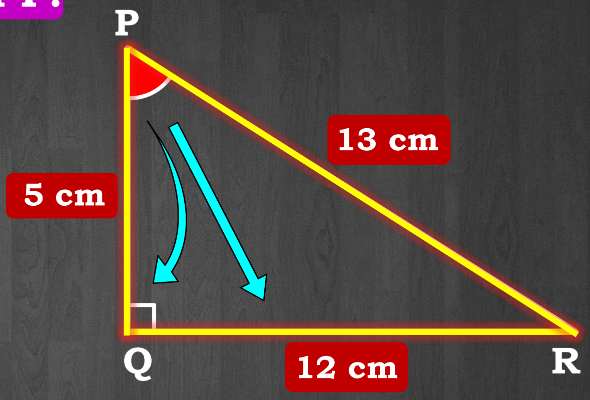
$$\therefore \quad \boxed{\sin P = \frac{12}{13}}$$

$$\cos P = \frac{PQ}{PR}$$

$$\therefore \quad \boxed{\cos P = \frac{5}{13}}$$

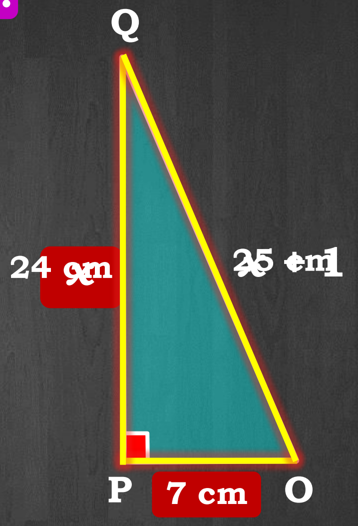
$$\tan P = \frac{QR}{PQ}$$

$$\therefore \quad \boxed{\tan P = \frac{12}{5}}$$



Solved Example:

In Δ OPQ, right angled at P, OP = 7 cm and OQ – PQ = 1 cm.
Determine the values of sin Q and cos Q.



Solution:

In ΔOPQ, $\angle OPQ = 90^\circ$

$OQ - PQ = 1$

Let $PQ = x$

$\therefore \quad OQ - x = 1$

$\therefore \quad \boxed{OQ = x + 1}$

$\boxed{OQ^2} = \boxed{OP^2} + PQ^2$ *(Pythagoras theorem)*

$\therefore \quad (x + 1)^2 = 7^2 + x^2$

$\therefore \quad 1 + \cancel{x^2} + 2x = 49 + \cancel{x^2}$

$\therefore \quad 1 + 2x = 49$

$\therefore \quad 2x = 48$

$\therefore \quad \boxed{x = 24}$

$PQ = x$

$\therefore \quad PQ = 24$ cm

$OQ = \boxed{x} + 1$

$\therefore \quad OQ = 24 + 1$

$\therefore \quad OQ = 25$ cm

Solved Example:

In Δ OPQ, right angled at P, OP = 7 cm and OQ – PQ = 1 cm. Determine the values of sin Q and cos Q.

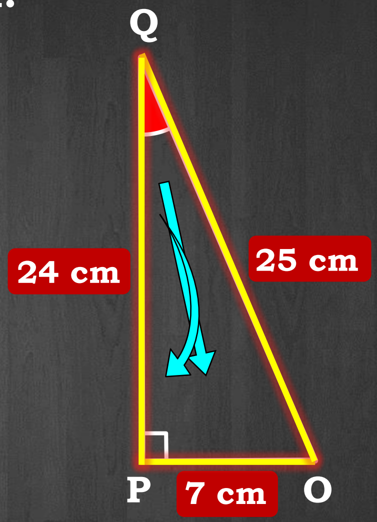
Solution:

$$\sin Q = \frac{OP}{OQ}$$

$$\therefore \quad \boxed{\sin Q = \frac{7}{25}}$$

$$\cos Q = \frac{PQ}{OQ}$$

$$\therefore \quad \boxed{\cos Q = \frac{24}{25}}$$



Thank You

Module 11

EXERCISE 8.1

Q.11) State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

Justification:

Consider ΔABC in which $\angle ABC = 90°$

$$\tan A = \frac{BC}{AB}$$

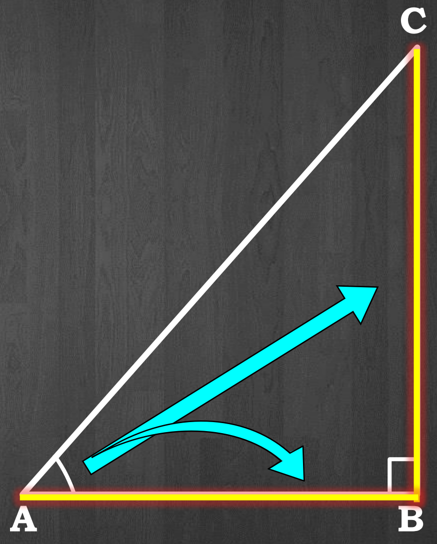
If $BC < AB$, then $\tan A < 1$

If $BC = AB$, then $\tan A = 1$

If $BC > AB$, then $\tan A > 1$

$\therefore$ tan A is not always less than 1.

Hence, the given statement is false.



EXERCISE 8.1

Q.11) State whether the following are true or false. Justify your answer.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

Justification:

In a right angled triangle,

$$\sec A = \frac{\text{Hypotenuse}}{\text{Adjacent side of } \angle A}$$

We know that hypotenuse is the longest side in a right angled triangle.

$\therefore$ sec A can never be less than 1.

$$\sec A = \frac{12}{5} > 1$$

$\therefore$ $\sec A = \frac{12}{5}$ for some angle A.

Hence, the given statement is true.

EXERCISE 8.1

Q.11) State whether the following are true or false. Justify your answer.

(iii) cos A is the abbreviation used for the cosecant of angle A.

Justification:

cos A is the abbreviation used for cosine of $\angle A$.

Abbreviation used for cosecant of $\angle A$ is cosec A.

Hence, the given statement is false.

EXERCISE 8.1

Q.11) State whether the following are true or false. Justify your answer.

(iv) cot A is the product of cot and A.

Justification:

cot A is not the product of 'cot' and $\angle A$.

'cot' separated from $\angle A$ has no meaning.

Hence, the given statement is false.

EXERCISE 8.1

Q.11) State whether the following are true or false. Justify your answer.

(v) $\sin\theta = \frac{4}{3}$ for some angle θ.

Justification:

In a right angled triangle,

$$\sin\theta = \frac{\text{Opposite side of } \theta}{\text{Hypotenuse}}$$

But, hypotenuse is the longest side in a right angled triangle.

$\therefore$ $\sin\theta$ can never be greater than 1.

$$\sin\theta = \frac{4}{3} > 1$$

$\therefore$ $\sin\theta = \frac{4}{3}$ is not possible for any θ.

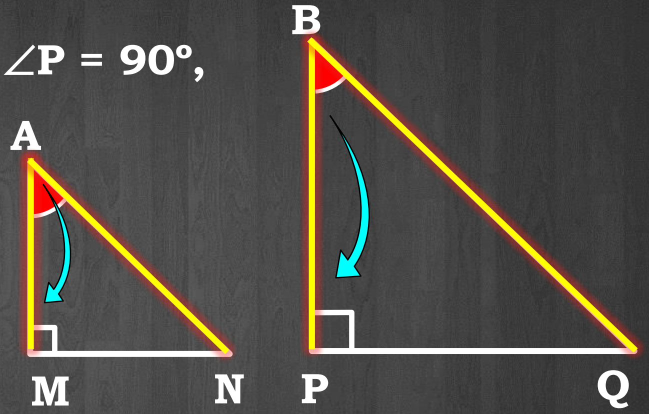
Hence, the given statement is false.

Module 12

EXERCISE 8.1

Q.6) If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then prove that $\angle A = \angle B$.

Proof: Consider ΔAMN and ΔBPQ in which $\angle M = \angle P = 90°$, $\angle A$ and $\angle B$ are acute.



$\cos A = \cos B$

$\cos A = \dfrac{AM}{AN}$

$\cos B = \dfrac{BP}{BQ}$

$\therefore \quad \dfrac{AM}{AN} = \dfrac{BP}{BQ} \Rightarrow \dfrac{AM}{BP} = \dfrac{AN}{BQ}$

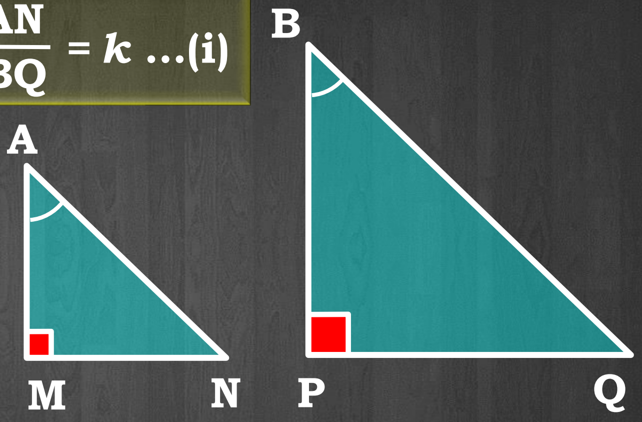
Let $\dfrac{AM}{BP} = \dfrac{AN}{BQ} = k \qquad \ldots(i)$

$\therefore \quad AM = k\,BP, \; AN = k\,BQ \; \ldots(ii)$

EXERCISE 8.1

Q.6) If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then prove that $\angle A = \angle B$.

$$\frac{AM}{BP} = \frac{AN}{BQ} = k \quad \ldots(i)$$



Proof: $AM = k\,BP$, $AN = k\,BQ$ ...(ii)

$AN^2 = AM^2 + MN^2$ *[Pythagoras theorem]*

$\therefore (kBQ)^2 = (kBP)^2 + MN^2$

$\therefore MN^2 = k^2BQ^2 - k^2BP^2$

$\therefore MN^2 = k^2(BQ^2 - BP^2)$ ...(iii)

$BQ^2 = BP^2 + PQ^2$ *[Pythagoras theorem]*

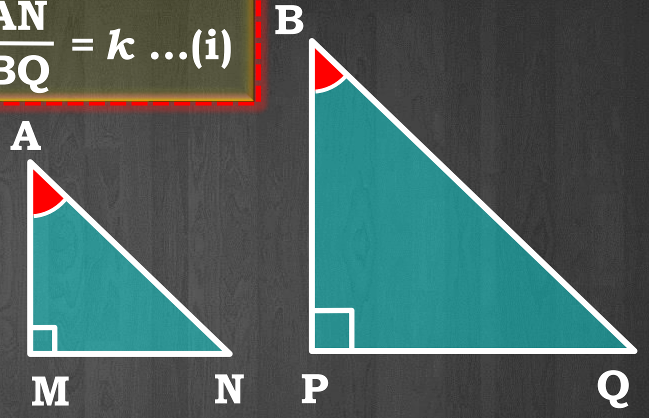
$\therefore PQ^2 = BQ^2 - BP^2$ ...(iv)

Now, $\dfrac{MN^2}{PQ^2} = \dfrac{k^2(BQ^2 - BP^2)}{(BQ^2 - BP^2)}$ *[Dividing (iii) and (iv)]*

EXERCISE 8.1

Q.6) If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then prove that $\angle A = \angle B$.

$$\frac{AM}{BP} = \frac{AN}{BQ} = k \quad \ldots(i)$$



Proof: Now, $\dfrac{MN^2}{PQ^2} = \dfrac{k^2\cancel{(BQ^2 - BP^2)}}{\cancel{(BQ^2 - BP^2)}}$

$\therefore \quad \dfrac{MN^2}{PQ^2} = k^2$

$\therefore \quad \boxed{\dfrac{MN}{PQ} = k} \quad \ldots(v)$ *[Taking square roots]*

In ΔAMN and ΔBPQ,

$\dfrac{AM}{BP} = \dfrac{AN}{BQ} = \dfrac{MN}{PQ}$ [From (i) and (v)]

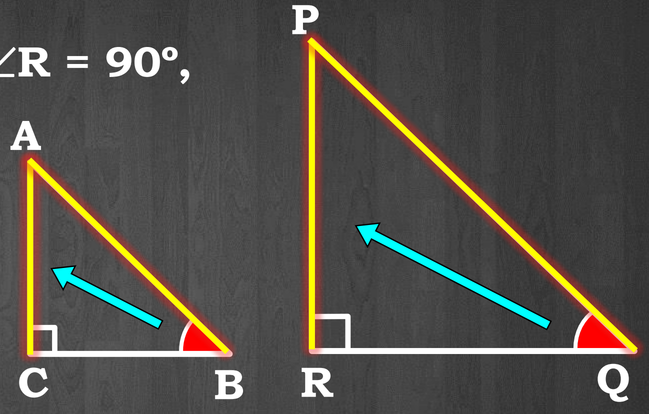
$\therefore \quad \Delta AMN \sim \Delta BPQ$ *[By SSS similarity criterion]*

$\therefore \quad \boxed{\angle A = \angle B}$ *[corresponding angles of similar triangles]*

Module 13

Solved Example:

If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then prove that $\angle B = \angle Q$.



Proof: Consider ΔACB and ΔPRQ in which $\angle C = \angle R = 90°$, $\angle B$ and $\angle Q$ are acute.

$$\sin B = \sin Q$$

$$\sin B = \frac{AC}{AB}$$

$$\sin Q = \frac{PR}{PQ}$$

$$\therefore \quad \frac{AC}{AB} = \frac{PR}{PQ} \Rightarrow \frac{AC}{PR} = \frac{AB}{PQ}$$

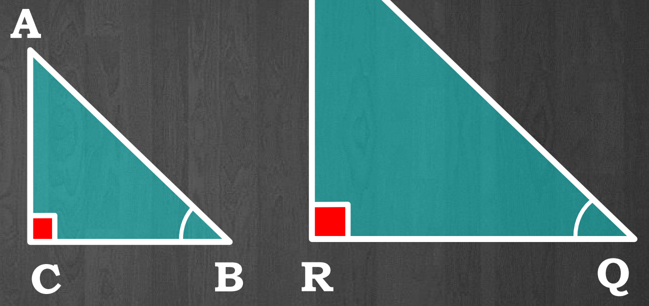
Let $\frac{AC}{PR} = \frac{AB}{PQ} = k$ ...(i)

$\therefore \quad AC = k\,PR, \; AB = k\,PQ$ ...(ii)

Solved Example:

If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then **prove that $\angle B = \angle Q$.**

$$\frac{AC}{PR} = \frac{AB}{PQ} = k \quad \ldots(i)$$



Proof: $AC = k\,PR,\ AB = k\,PQ \ldots(ii)$

$AB^2 = AC^2 + BC^2$ *[Pythagoras theorem]*

$\therefore\ (kPQ)^2 = (kPR)^2 + BC^2$

$\therefore\ BC^2 = k^2\,PQ^2 - k^2\,PR^2$

$\therefore\ \boxed{BC^2 = k^2\,(PQ^2 - PR^2)} \quad \ldots(iii)$

$PQ^2 = PR^2 + QR^2$ *[Pythagoras theorem]*

$\therefore\ \boxed{QR^2 = PQ^2 - PR^2} \quad \ldots(iv)$

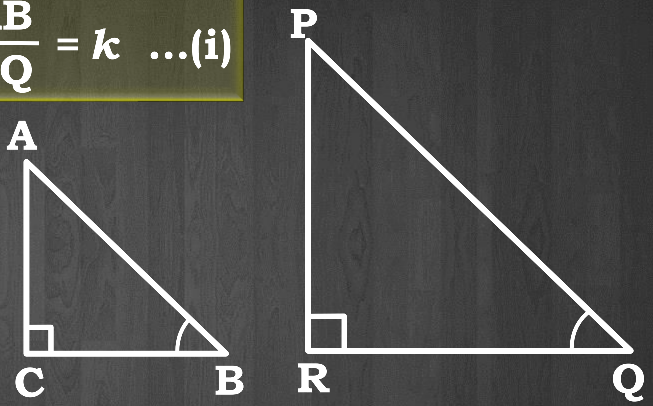
Now, $\dfrac{BC^2}{QR^2} = \dfrac{k^2\,\cancel{(PQ^2 - PR)^2}}{\cancel{(PQ^2 - PR)^2}}$ *[Dividing (iii) and (iv)]*

$\therefore\ \dfrac{BC^2}{QR^2} = k^2$

Solved Example:

If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then prove that $\angle B = \angle Q$.

$$\frac{AC}{PR} = \frac{AB}{PQ} = k \quad \text{...(i)}$$



Proof:

$$\therefore \quad \frac{BC^2}{QR^2} = k^2$$

$$\therefore \quad \frac{BC}{QR} = k \quad \text{...(v)} \quad \text{(Taking square roots)}$$

In ΔACB and ΔPRQ,

$$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR} \quad \text{...[From (i) and (v)]}$$

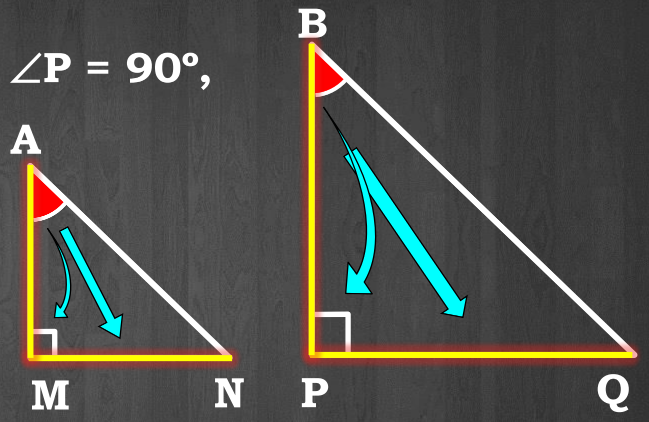
$\therefore \Delta ACB \sim \Delta PRQ$ (By SSS similarity criterion)

$\therefore \angle B = \angle Q$ ...(Corresponding angles of similar triangles)

Module 14

Example:

If $\angle A$ and $\angle B$ are acute angles such that **tan A = tan B** then **prove $\angle A = \angle B$.**



Proof: Consider ΔAMN and ΔBPQ in which $\angle M = \angle P = 90°$, $\angle A$ and $\angle B$ are acute.

$\tan A = \tan B$ *[Given]*

$$\tan A = \frac{MN}{AM}$$

$$\tan B = \frac{PQ}{BP}$$

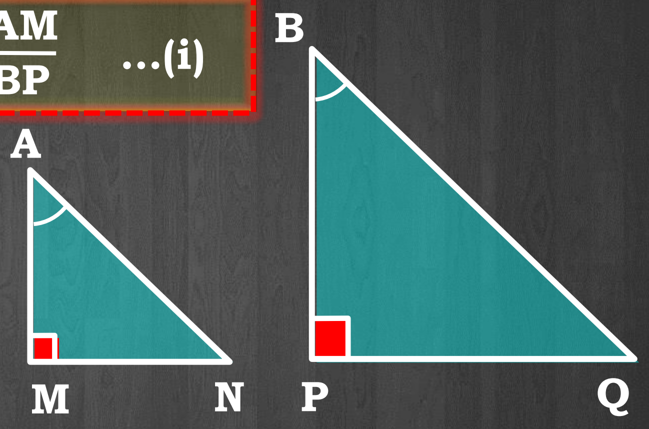
$$\therefore \quad \frac{MN}{AM} = \frac{PQ}{BP}$$

$$\frac{MN}{PQ} = \frac{AM}{BP} \quad \ldots(i)$$

Example:

If ∠A and ∠B are acute angles such that tan A = tan B then prove ∠A = ∠B.

$$\frac{MN}{PQ} = \frac{AM}{BP} \quad \ldots(i)$$



Proof:

In ΔAMN and ΔBPQ,

$$\frac{MN}{PQ} = \frac{AM}{BP} \qquad \text{[From (i)]}$$

$$\angle M = \angle P \qquad \text{[Each } 90^\circ\text{]}$$

$\therefore$ $\Delta AMN \sim \Delta BPQ$ *[By SAS similarity criterion]*

$\therefore$ $\angle A = \angle B$ *[Corresponding angles of similar triangles]*

Thank You

Module 15

Relation Between Trigonometric Ratios

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta} \quad \Big| \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta} \quad \Big| \quad \sin\theta \times \operatorname{cosec}\theta = 1$$

$$\cos\theta = \frac{1}{\sec\theta} \quad \Big| \quad \sec\theta = \frac{1}{\cos\theta} \quad \Big| \quad \cos\theta \times \sec\theta = 1$$

$$\tan\theta = \frac{1}{\cot\theta} \quad \Big| \quad \cot\theta = \frac{1}{\tan\theta} \quad \Big| \quad \tan\theta \times \cot\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \Big| \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Module 16

Working	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

$$\sec\theta = \frac{1}{\cos\theta}$$

Module 17

EXERCISE 8.2

Q.1) Evaluate the following:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

Solution:

$$\sin 60° \cos 30° + \sin 30° \cos 60° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{3+1}{4}$$

$$= \frac{4}{4}$$

$$\therefore \boxed{\sin 60° \cos 30° + \sin 30° \cos 60° = 1}$$

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.1) Evaluate the following:

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

Solution:

$$2\tan^2 45° + \cos^2 30° - \sin^2 60° = 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \cancel{\frac{3}{4}} - \cancel{\frac{3}{4}}$$

$$= 2$$

$$\therefore \quad \boxed{2\tan^2 45° + \cos^2 30° - \sin^2 60° = 2}$$

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Module 18

EXERCISE 8.2

Q.2) Choose the correct option and justify your choice :

(ii) $\dfrac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

(A) $\tan 90^\circ$ (B) 1 (C) $\sin 45^\circ$ (D) 0 ✓

Justification:

$$\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{1 - (1)^2}{1 + (1)^2}$$

$$= \frac{1 - 1}{1 + 1}$$

$$= \frac{0}{2}$$

$$\therefore \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = 0$$

Hence, option (D) is correct.

Ratio \ θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
$\cot\theta$	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
$\operatorname{cosec}\theta$	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.2) Choose the correct option and justify your choice :

(i) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A) sin 60° (B) cos 60° (C) tan 60° (D) sin 30°

Justification:

$$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2}{\sqrt{3}} \div \left[1 + \left(\frac{1}{\sqrt{3}}\right)^2\right]$$

$$= \left(\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) \div \left(1 + \frac{1}{3}\right)$$

$$= \frac{2\sqrt{3}}{3} \div \left(\frac{3+1}{3}\right)$$

$$= \frac{2\sqrt{3}}{3} \div \frac{4}{3}$$

$$= \frac{\cancel{2}\sqrt{3}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}\,2}$$

$$= \frac{\sqrt{3}}{2}$$

$$\therefore \frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \sin 60^\circ$$

Hence, option (A) is correct.

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.2) Choose the correct option and justify your choice :

(iv) $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ} =$

(A) $\cos 60^\circ$ (B) $\sin 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

Justification:

$$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \frac{2}{\sqrt{3}} \div \left[1-\left(\frac{1}{\sqrt{3}}\right)^2\right]$$

$$= \left(\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) \div \left(1-\frac{1}{3}\right)$$

$$= \frac{2\sqrt{3}}{3} \div \left(\frac{3-1}{3}\right)$$

$$= \frac{2\sqrt{3}}{3} \div \frac{2}{3}$$

$$= \frac{\cancel{2}\sqrt{3}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{2}}$$

$$= \sqrt{3}$$

$$\therefore \frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \tan 60^\circ$$

Hence, option (C) is correct.

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.2) Choose the correct option and justify your choice :

(iii) sin 2A = 2 sin A is true when A =

(A) 0° (B) 30° (C) 45° (D) 60°

Justification:

If A = 0°

$\sin 2A = \sin(2 \times 0^\circ) = \sin 0^\circ = 0$

$2 \sin A = 2 \sin 0^\circ = 2 \sin 0^\circ = 0$

$\therefore \sin 2A = 2 \sin A$, when $A = 0^\circ$

Hence, option (A) is correct.

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Thank You

Module 19

EXERCISE 8.2

Q.1) Evaluate the following:

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

Solution:

$$\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \left(\frac{1}{\sqrt{2}}\right) \div \left(\frac{2}{\sqrt{3}} + 2\right)$$

$$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}\right) \div \left[\left(\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) + 2\right]$$

$$= \left(\frac{\sqrt{2}}{2}\right) \div \left(\frac{2\sqrt{3}}{3} + 2\right)$$

$$= \left(\frac{\sqrt{2}}{2}\right) \div \left(\frac{2\sqrt{3} + 6}{3}\right)$$

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.1) Evaluate the following:

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

Solution:

$$\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \frac{\sqrt{2}}{2} \times \frac{3}{2\sqrt{3} + 6}$$

$$\left(\frac{\sqrt{2}}{2}\right) \div \left(\frac{2\sqrt{3} + 6}{3}\right)$$

$$= \frac{3\sqrt{2}}{12 + 4\sqrt{3}} \times \frac{12 - 4\sqrt{3}}{12 - 4\sqrt{3}}$$

[Rationalising the denominator]

$$= \frac{3\sqrt{2}\,(12 - 4\sqrt{3})}{(12)^2 - (4\sqrt{3})^2}$$

$$= \frac{36\sqrt{2} - 12\sqrt{6}}{144 - 48}$$

EXERCISE 8.2

Q.1) Evaluate the following:

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

Solution:

$$\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \frac{36\sqrt{2} - 12\sqrt{6}}{144 - 48}$$

$$= \frac{\cancel{12}(3\sqrt{2} - \sqrt{6})}{\cancel{96}\,8}$$

$$= \frac{(3\sqrt{2} - \sqrt{6})}{8}$$

$$\therefore \boxed{\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \frac{(3\sqrt{2} - \sqrt{6})}{8}}$$

Module 20

EXERCISE 8.2

Q.1) Evaluate the following:

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

Solution:

$$\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°} = \left(\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}\right) \div \left(\frac{2}{\sqrt{3}} + \frac{1}{2} + 1\right)$$

$$= \left[\frac{1}{2} + 1 - \left(\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right)\right] \div \left[\left(\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) + \frac{1}{2} + 1\right]$$

$$= \left(\frac{1}{2} + 1 - \frac{2\sqrt{3}}{3}\right) \div \left(\frac{2\sqrt{3}}{3} + \frac{1}{2} + 1\right)$$

$$= \left(\frac{3 + 6 - 4\sqrt{3}}{6}\right) \div \left(\frac{4\sqrt{3} + 3 + 6}{6}\right)$$

$$= \frac{9 - 4\sqrt{3}}{6} \times \frac{6}{4\sqrt{3} + 9}$$

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.1) Evaluate the following:

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

Solution:

$$\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ} = \frac{9 - 4\sqrt{3}}{\cancel{6}} \times \frac{\cancel{6}}{4\sqrt{3} + 9}$$

$$= \frac{9 - 4\sqrt{3}}{9 + 4\sqrt{3}}$$

$$= \frac{9 - 4\sqrt{3}}{9 + 4\sqrt{3}} \times \frac{9 - 4\sqrt{3}}{9 - 4\sqrt{3}}$$ *[Rationalising the denominator]*

$(a - b)^2 = a^2 - 2\,ab + b^2$

$$= \frac{(9 - 4\sqrt{3})^2}{(9)^2 - (4\sqrt{3})^2}$$

$$= \frac{81 - 72\sqrt{3} + 48}{81 - 48}$$

EXERCISE 8.2

Q.1) Evaluate the following:

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

Solution:

$$\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°} = \frac{81 - 72\sqrt{3} + 48}{81 - 48}$$

$$= \frac{129 - 72\sqrt{3}}{33}$$

$$= \frac{\cancel{3}\,(43 - 24\sqrt{3})}{\cancel{33}\ 11}$$

$$\therefore \boxed{\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°} = \frac{43 - 24\sqrt{3}}{11}}$$

Module 21

EXERCISE 8.2

Q.1) Evaluate the following:

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Solution:

$$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ} = \left[5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2\right] \div \left[\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right]$$

$$= \left(5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1\right) \div \left(\frac{1}{4} + \frac{3}{4}\right)$$

$$= \left(\frac{5}{4} + \frac{16}{3} - 1\right) \div \left(\frac{1+3}{4}\right)$$

$$= \frac{15 + 64 - 12}{12} \div \frac{4}{4}$$

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.1) Evaluate the following:

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Solution:

$$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ} = \frac{67}{12} \div 1$$

$$= \frac{67}{12}$$

$$\therefore \boxed{\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ} = \frac{67}{12}}$$

Module 22

EXERCISE 8.2

Q.4) State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$

Justification:

Let $A = 30^\circ$, $B = 60^\circ$

$\sin(A + B) = \sin(30^\circ + 90^\circ) = \sin 90^\circ = 1$

$\sin A + \sin B = \sin 30^\circ + \sin 60^\circ$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{1 + \sqrt{3}}{2}$$

$\therefore \sin(A + B) \neq \sin A + \sin B$ when $A = 30^\circ$, $B = 60^\circ$

Hence, the given statement is false.

Ratio \ θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
$\cot\theta$	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
$\mathrm{cosec}\,\theta$	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.4) State whether the following are true or false. Justify your answer.

(ii) The value of sin θ increases as θ increases.

Justification:

$\sin 0^\circ = \boxed{0}$

$\sin 30^\circ = \dfrac{1}{2} = \boxed{0.5}$

$\sin 45^\circ = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} = \dfrac{1.41}{2} = \boxed{0.7}$ (Approx.)

$\sin 60^\circ = \dfrac{\sqrt{3}}{2} = \dfrac{1.73}{2} = \boxed{0.87}$ (Approx.)

$\sin 90^\circ = \boxed{1}$

$\therefore$ The value of sin θ increases as θ increases from 0° to 90°.

Hence, the given statement is true.

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.4) State whether the following are true or false. Justify your answer.
(iii) The value of cos θ increases as θ increases.

Justification:

$\cos 0^\circ = \boxed{1}$

$\cos 30^\circ = \frac{\sqrt{3}}{2} = \frac{1.73}{2} = \boxed{0.87}$ **(Approx.)**

$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1.41}{2} = \boxed{0.7}$ **(Approx.)**

$\cos 60^\circ = \frac{1}{2} = \boxed{0.5}$

$\cos 90^\circ = \boxed{0}$

$\therefore$ **Hence, the given statement is false.**

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.4) State whether the following are true or false. Justify your answer.

(iv) sin θ = cos θ for all values of θ.

Justification:

If $\theta = 30^\circ$,

$\sin 30^\circ = \boxed{\frac{1}{2}}$

$\cos 30^\circ = \boxed{\frac{\sqrt{3}}{2}}$

$\therefore \quad \sin 30^\circ \neq \cos 30^\circ$

$\therefore \quad \sin\theta \neq \cos\theta$, when $\theta = 30^\circ$

$\therefore$ **Hence, the given statement is false.**

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.4) State whether the following are true or false. Justify your answer.

(v) cot A is not defined for A = 0°.

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Justification:

$$\cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0}$$

$\frac{1}{0}$ is not defined.

$\therefore$ cot A is not defined.

Hence, the given statement is true.

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Thank You

Module 23

EXERCISE 8.2

Q.3) If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0^\circ < A + B \leq 90^\circ$, $A \geq B$, find A and B.

Solution:

$\tan(A + B) = \sqrt{3}$

But, $\tan 60^\circ = \sqrt{3}$

$\therefore \quad \tan(A + B) = \tan 60^\circ$

$\therefore \quad A + B = 60 \quad \ldots(i)$

$\tan(A - B) = \frac{1}{\sqrt{3}}$

But, $\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\therefore \quad \tan(A - B) = \tan 30^\circ$

$\therefore \quad A - B = 30 \quad \ldots(ii)$

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

EXERCISE 8.2

Q.3) If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0^\circ < A + B \le 90^\circ$, $A \ge B$, find A and B.

Solution:

$$A + B = 60 \quad \ldots(i)$$
$$A - B = 30 \quad \ldots(ii)$$

Adding (i) and (ii), we get

$$A + \cancel{B} + A - \cancel{B} = 60 + 30$$

$$\therefore \quad 2A = 90$$

$$\therefore \quad \boxed{A = 45}$$

Substituting the value of A in (i), we get

$$\boxed{A} + B = 60$$

$$\therefore \quad 45 + B = 60$$

$$\therefore \quad B = 60 - 45$$

$$\therefore \quad \boxed{B = 15}$$

Module 24

Example:

In an acute angled ΔABC, if $\tan(A + B - C) = 1$, $\sec(B + C - A) = 2$, then find the value of C.

Solution:

$$\tan(A + B - C) = 1$$

But, $\tan 45° = 1$

$$\tan(A + B - C) = \tan 45°$$

$$\therefore \quad A + B - C = 45 \quad \ldots(i)$$

$$\sec(B + C - A) = 2$$

But, $\sec 60° = 2$

$$\therefore \quad B + C - A = 60 \quad \ldots(ii)$$

Adding (i) and (ii), we get

$$\cancel{A} + B - \cancel{C} + B + \cancel{C} - \cancel{A} = 45 + 60$$

$$2B = 105$$

$$\therefore \quad B = 52.5 \quad \ldots(iii)$$

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cot θ	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cosec θ	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Example:

In an acute angled ΔABC, if $\tan(A + B - C) = 1$, $\sec(B + C - A) = 2$, then find the value of C.

Solution:

$\boxed{A + B + C = 180}$ ...(iv) (Angle sum property of a triangle)

$B + C - A = 60$ ...(ii)

$B = 52.5$ ...(iii)

Adding (ii) and (iv), we get

$$B + C - \cancel{A} + \cancel{A} + B + C = 60 + 180$$

$$\therefore\ 2B + 2C = 240$$

$$\therefore\ \boxed{B} + C = 120 \quad \text{(Dividing throughout by 2)}$$

$$\therefore\ 52.5 + C = 120 \quad \text{...[From (iii)}$$

$$\therefore\ C = 120 - 52.5$$

$$\therefore\ \boxed{C = 67.5}$$

Module 25

Trigonometric ratios of complementary angles

In ΔABC, $\angle B = 90°$, $\angle A = \theta$, $\angle C = (90 - \theta)$

$$\sin\theta = \frac{BC}{AC} \quad \ldots(i)$$

$$\cos(90 - \theta) = \frac{BC}{AC} \quad \ldots$$

$\therefore$ $\angle A + \angle C = 90°$

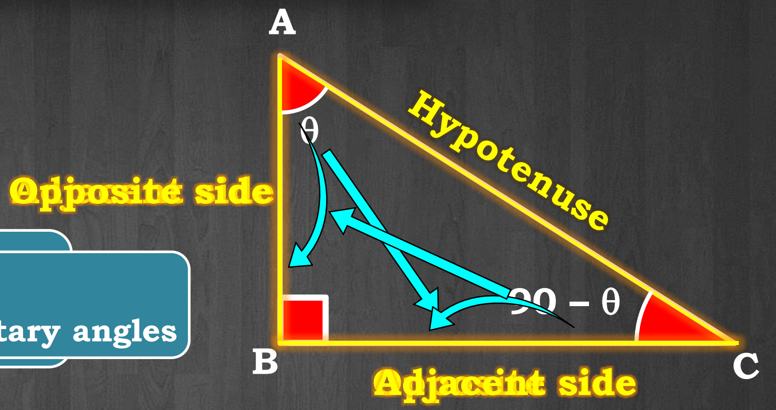
$\therefore$ $\angle A$ and $\angle C$ are complementary angles

$$\therefore \sin\theta = \cos(90 - \theta) \quad \textit{[From (i) and (ii)]}$$

$$\cos\theta = \frac{AB}{AC} \quad \ldots(iii)$$

$$\sin(90 - \theta) = \frac{AB}{AC} \quad \ldots(iv)$$

$$\therefore \cos\theta = \sin(90 - \theta) \quad \textit{[From (iii) and (iv)]}$$



Trigonometric ratios of complementary angles

In ΔABC, $\angle B = 90°$, $\angle A = \theta$, $\angle C = (90 - \theta)$

$\sin\theta = \cos(90 - \theta)$

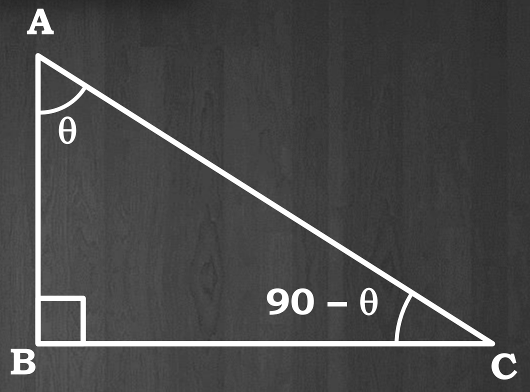
$\cos\theta = \sin(90 - \theta)$

$\tan\theta = \cot(90 - \theta)$

$\cot\theta = \tan(90 - \theta)$

$\operatorname{cosec}\theta = \sec(90 - \theta)$

$\sec\theta = \operatorname{cosec}(90 - \theta)$



Module 26

EXERCISE 8.3

Q.1) Evaluate : (i) $\dfrac{\sin 18^\circ}{\cos 72^\circ}$

$\sin \theta = \cos (90 - \theta)$

Solution:

$$\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\cos (90^\circ - 18^\circ)}{\cos 72^\circ} \quad [\because \sin \theta = \cos (90 - \theta)]$$

$$= \frac{\cancel{\cos 72^\circ}}{\cancel{\cos 72^\circ}}$$

$$= 1$$

$$\therefore \boxed{\frac{\sin 18^\circ}{\cos 72^\circ} = 1}$$

EXERCISE 8.3

Q.1) Evaluate : (ii) $\dfrac{\tan 26^\circ}{\cot 64^\circ}$

$\tan\theta = \cot(90 - \theta)$

Solution:

$$\frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\cot(90^\circ - 26^\circ)}{\cot 64^\circ} \quad [\because \tan\theta = \cot(90 - \theta)]$$

$$= \frac{\cancel{\cot 64^\circ}}{\cancel{\cot 64^\circ}}$$

$$= 1$$

$$\therefore \boxed{\frac{\tan 26^\circ}{\cot 64^\circ} = 1}$$

EXERCISE 8.3

Q.1) Evaluate : (iii) $\cos 48^\circ - \sin 42^\circ$

$\cos \theta = \sin (90 - \theta)$

Solution:

$\cos 48^\circ - \sin 42^\circ$

$= \sin (90^\circ - 48^\circ) - \sin 42^\circ$ [$\because \cos \theta = \sin (90 - \theta)$]

$= \sin 42^\circ - \sin 42^\circ$

$= 0$

$\therefore \boxed{\cos 48^\circ - \sin 42^\circ = 0}$

EXERCISE 8.3

Q.1) Evaluate : (iv) $\text{cosec } 31^\circ - \sec 59^\circ$

Solution:

$\text{cosec } 31^\circ - \sec 59^\circ$

$\text{cosec } \theta = \sec (90 - \theta)$

$= \sec (90^\circ - 31^\circ) - \sec 59^\circ$ $[\because \text{cosec } \theta = \sec (90 - \theta)]$

$= \sec 59^\circ - \sec 59^\circ$

$= 0$

$\therefore$ $\boxed{\text{cosec } 31^\circ - \sec 59^\circ = 0}$

Module 27

EXERCISE 8.3

Q.2) Show that : (i) $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

$\tan\theta = \cot(90 - \theta)$

Proof: L.H.S. $= \tan 48° \tan 23° \tan 42° \tan 67°$

$= \tan 48° \tan 42° \tan 23° \tan 67°$

$= \cot(90° - 48°) \tan 42° \cot(90° - 23°) \tan 67°$

[$\because \tan\theta = \cot(90 - \theta)$]

$= \cot 42° \tan 42° \cot 67° \tan 67°$

$= 1 \times 1$ [$\because \tan\theta \times \cot\theta = 1$]

$= 1$

$\therefore$ L.H.S = R.H.S

$\therefore \tan 48° \tan 23° \tan 42° \tan 67° = 1$

EXERCISE 8.3

Q.2) Show that : (ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

Proof: L.H.S. $= \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$

$\cos\theta = \sin(90 - \theta)$

$= \sin(90^\circ - 38^\circ)\cos 52^\circ - \cos(90^\circ - 38^\circ)\sin 52^\circ$

$\left[\because \sin\theta = \cos(90 - \theta)\ ,\ \cos\theta = \sin(90 - \theta)\right]$

$= \cancel{\sin 52^\circ \cos 52^\circ} - \cancel{\cos 52^\circ \sin 52^\circ}$

$= 0$

$\therefore$ L.H.S = R.H.S

$\therefore \boxed{\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0}$

Thank You

Module 28

EXERCISE 8.3

Q.3) If $\tan 2A = \cot (A - 18^\circ)$, where 2A is an acute angle, find the value of A.

$\tan \theta = \cot (90 - \theta)$

Solution: $\tan 2A = \cot (A - 18^\circ)$

$\therefore \cot (90^\circ - 2A) = \cot (A - 18^\circ)$ $[\because \tan \theta = \cot (90 - \theta)]$

$\therefore 90 - 2A = A - 18$

$\therefore -2A - A = -18 - 90$

$\therefore -3A = -108$

$\therefore A = \dfrac{\cancel{-108}^{\,36}}{\cancel{-3}}$

$\therefore \boxed{A = 36}$

EXERCISE 8.3

Q.5) If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$, where 4A is an acute angle, find the value of A.

$\sec\theta = \operatorname{cosec}(90 - \theta)$

Solution: $\sec 4A = \operatorname{cosec}(A - 20^\circ)$

$\therefore \operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ)$ [$\because \sec\theta = \operatorname{cosec}(90 - \theta)$]

$\therefore 90 - 4A = A - 20$

$\therefore 90 + 20 = A + 4A$

$\therefore 110 = 5A$

$\therefore A = \dfrac{\cancel{110}^{\,22}}{\cancel{5}}$

$\therefore \boxed{A = 22}$

Module 29

Solved Example 10 :

If $\sin 3A = \cos (A - 26^\circ)$, where 3A is an acute angle, find the value of A

Solution:

$\boxed{\sin \theta = \cos (90 - \theta)}$

$$\sin 3A = \cos (A - 26^\circ)$$

$\therefore \quad \cos (90^\circ - 3A) = \cos (A - 26^\circ) \quad [\because \sin \theta = \cos (90 - \theta)]$

$\therefore \quad 90 - 3A = A - 26$

$\therefore \quad 90 + 26 = A + 3A$

$\therefore \quad 116 = 4A$

$\therefore \quad A = \dfrac{\overset{29}{\cancel{116}}}{\cancel{4}}$

$\therefore \quad \boxed{A = 29}$

EXERCISE 8.3

Q.4) If $\tan A = \cot B$, prove that, $A + B = 90°$.

$\tan\theta = \cot(90 - \theta)$

Proof: $\tan A = \cot B$

$\therefore \quad \cot(90° - A) = \cot B \qquad [\because \tan\theta = \cot(90 - \theta)]$

$\therefore \quad 90 - A = B$

$\therefore \quad 90 = B + A$

$\therefore \quad \boxed{A + B = 90°}$

Module 30

EXERCISE 8.3

Q.7 Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45°.

$$\cos\theta = \sin(90 - \theta)$$

Solution:

$$\sin 67^\circ + \cos 75^\circ = \cos(90^\circ - 67^\circ) + \sin(90^\circ - 75^\circ)$$

$$\left[\because \sin\theta = \cos(90-\theta),\ \cos\theta = \sin(90-\theta)\right]$$

$$\therefore \boxed{\sin 67^\circ + \cos 75^\circ = \cos 23^\circ + \sin 15^\circ}$$

Solved Example:

Express $\cot 85^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45°.

$$\cot\theta = \tan(90 - \theta)$$

Solution:

$$\cot 85^\circ + \cos 75^\circ = \tan(90^\circ - 85^\circ) + \sin(90^\circ - 75^\circ)$$

$$\left[\because \cot\theta = \tan(90 - \theta),\ \cos\theta = \sin(90 - \theta)\right]$$

$$\therefore \boxed{\cot 85^\circ + \cos 75^\circ = \tan 5^\circ + \sin 15^\circ}$$

Module 31

EXERCISE 8.3

Q.6) If A, B, C are the interior angles of a triangle ABC,

show that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$.

Proof:

$A + B + C = 180°$

[Angle sum property of a triangle]

$\therefore \frac{A+B+C}{2} = \frac{180}{2}$

[Dividing throughout by 2]

$\therefore \frac{A}{2} + \frac{B+C}{2} = 90$

$\therefore \frac{B+C}{2} = 90 - \frac{A}{2}$

$\sin(90 - \theta) = \cos\theta$

$\therefore \sin\left(\frac{B+C}{2}\right) = \sin\left(90 - \frac{A}{2}\right)$

$\therefore \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

Module 32

Example

If A, B, C are the interior angles of a triangle ABC,

show that $\cot \dfrac{B+C}{2} = \tan \dfrac{A}{2}$.

Proof:

$$A + B + C = 180^\circ$$

[Angle sum property of a triangle]

$$\therefore \frac{A+B+C}{2} = \frac{180}{2}$$

[Dividing both sides by 2]

$$\therefore \frac{A}{2} + \frac{B+C}{2} = 90$$

$$\therefore \frac{B+C}{2} = 90 - \frac{A}{2}$$

$$\therefore \cot\left(\frac{B+C}{2}\right) = \cot\left(90 - \frac{A}{2}\right)$$

$$\therefore \cot\left(\frac{B+C}{2}\right) = \tan\frac{A}{2}$$

$\cot(90 - \theta) = \tan\theta$

Example

If A, B, C are the interior angles of a triangle ABC,

show that $\operatorname{cosec}\boxed{\dfrac{B+C}{2}} = \sec\boxed{\dfrac{A}{2}}$

Proof:

$$A + B + C = 180^\circ$$

[Angle sum property of a triangle]

$$\therefore \frac{A+B+C}{2} = \frac{180}{2}$$

[Dividing both sides by 2]

$$\therefore \frac{A}{2} + \frac{B+C}{2} = 90$$

$$\therefore \frac{B+C}{2} = 90 - \frac{A}{2}$$

$$\operatorname{cosec}(90 - \theta) = \sec\theta$$

$$\therefore \operatorname{cosec}\left(\frac{B+C}{2}\right) = \operatorname{cosec}\left(90 - \frac{A}{2}\right)$$

$$\therefore \operatorname{cosec}\left(\frac{B+C}{2}\right) = \sec\frac{A}{2}$$

Thank You

Module 33

TRIGONOMETRIC IDENTITIES

In ΔABC, $\angle ABC = 90°$

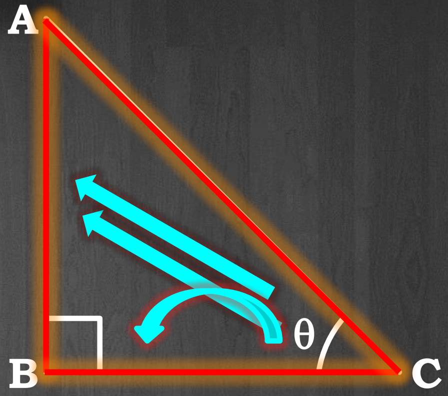
$\therefore \quad AC^2 = AB^2 + BC^2 \quad \ldots$ (i) (Pythagoras theorem)

Dividing equation (i) by AC^2

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$\therefore \quad 1 = \left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2$$

$$\therefore \quad 1 = \sin^2\theta + \cos^2\theta$$



TRIGONOMETRIC IDENTITIES

In ΔABC, $\angle ABC = 90^\circ$

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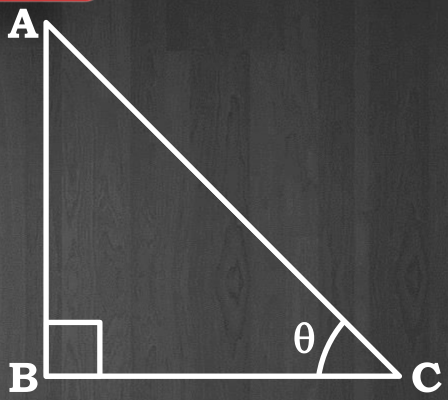
Dividing equation (i) by AB^2

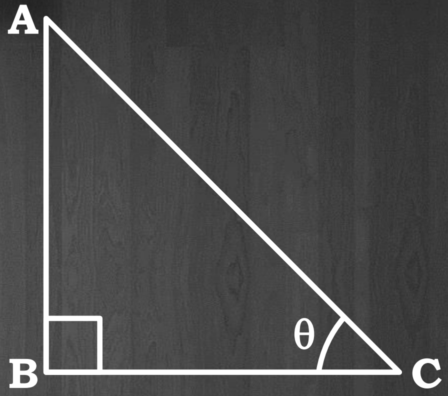
$$\frac{AC^2}{AB^2} = \frac{AB^2}{AB^2} + \frac{BC^2}{AB^2}$$

$$\therefore \left(\frac{AC}{AB}\right)^2 = 1 + \left(\frac{BC}{AB}\right)^2$$

$$\therefore \; \operatorname{cosec}^2\theta = 1 + \cot^2\theta$$



TRIGONOMETRIC IDENTITIES



In ΔABC, $\angle ABC = 90°$

$\therefore \; AC^2 = AB^2 + BC^2 \quad \ldots \text{(i)}$ (Pythagoras theorem)

Dividing equation (i) by BC^2

$$\frac{AC^2}{BC^2} = \frac{AB^2}{BC^2} + \frac{BC^2}{BC^2}$$

$$\therefore \left(\frac{AC}{BC}\right)^2 = \left(\frac{AB}{BC}\right)^2 + 1$$

$$\therefore \; \sec^2\theta = \tan^2\theta + 1$$

TRIGONOMETRIC IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$

OR

$$\sin^2\theta = 1 - \cos^2\theta$$

OR

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\sec^2\theta = 1 + \tan^2\theta$$

OR

$$\sec^2\theta - \tan^2\theta = 1$$

OR

$$\tan^2\theta = \sec^2\theta - 1$$

$$\mathrm{cosec}^2\theta = 1 + \cot^2\theta$$

OR

$$\mathrm{cosec}^2\theta - \cot^2\theta = 1$$

OR

$$\cot^2\theta = \mathrm{cosec}^2\theta - 1$$

Module 34

EXERCISE 8.4

Q.1) Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Solution:

$$\boxed{\tan A = \frac{1}{\cot A}} \quad \ldots \text{(i)}$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\therefore \quad \sec^2 A = 1 + \left(\frac{1}{\cot A}\right)^2 \quad \text{[From (i)]}$$

$$\therefore \quad \sec^2 A = 1 + \frac{1}{\cot^2 A}$$

$$\therefore \quad \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A}$$

$$\therefore \quad \boxed{\sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}} \quad \ldots \text{(ii)} \quad \textit{[Taking square roots]}$$

EXERCISE 8.4

Q.1) Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Solution:

$$\tan A = \frac{1}{\cot A} \quad \ldots \text{(i)}$$

$$\sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A} \quad \ldots \text{(ii)}$$

$$\cos A = \frac{1}{\sec A}$$

$$= 1 \div \frac{\sqrt{1 + \cot^2 A}}{\cot A} \quad \text{[From (ii)]}$$

$$\therefore \quad \cos A = \frac{\cot A}{\sqrt{1 + \cot^2 A}} \quad \ldots \text{(iii)}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\therefore \sin A = \tan A \times \cos A$$

$$= \frac{1}{\cancel{\cot A}} \times \frac{\cancel{\cot A}}{\sqrt{1 + \cot^2 A}} \quad \text{[From (i) and (iii)]}$$

$$\therefore \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

Module 35

EXERCISE 8.4

Q.2) Write all the other trigonometric ratios of $\angle A$ in terms of sec A.

Solution:

$$\cos A = \frac{1}{\sec A} \quad \text{... (i)}$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\therefore \quad \tan^2 A = \sec^2 A - 1$$

$$\therefore \quad \tan A = \sqrt{\sec^2 A - 1} \quad \text{... (ii)}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\therefore \quad \sin A = \tan A \times \cos A$$

$$\therefore \quad \sin A = \sqrt{\sec^2 A - 1} \times \frac{1}{\sec A} \quad \text{[From (i) and (ii)]}$$

$$\therefore \quad \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A} \quad \text{... (iii)}$$

EXERCISE 8.4

Q.2) Write all the other trigonometric ratios of $\angle A$ in terms of sec A.

Solution:

$$\boxed{\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}} \quad \dots \text{(iii)}$$

$$\boxed{\tan A = \sqrt{\sec^2 A - 1}} \quad \dots \text{(ii)}$$

$$\text{cosec } A = \frac{1}{\sin A}$$

$$\therefore \quad \text{cosec } A = 1 \div \left(\frac{\sqrt{\sec^2 A - 1}}{\sec A}\right) \quad \text{[From (iii)]}$$

$$\therefore \quad \boxed{\text{cosec } A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}}$$

$$\cot A = \frac{1}{\tan A}$$

$$\therefore \quad \boxed{\cot A = \frac{1}{\sqrt{\sec^2 A - 1}}} \quad \dots \text{[From (ii)]}$$

Module 36

Solved Example:

Q.12) Express the ratios cos A, tan A and sec A in terms of sin A.

Solution:

$$\sin^2 A + \cos^2 A = 1$$

$$\therefore \quad \cos^2 A = 1 - \sin^2 A$$

$$\therefore \quad \boxed{\cos A = \sqrt{1 - \sin^2 A}} \quad \text{...(i)} \; [\textit{Taking square root}]$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\therefore \quad \boxed{\tan A = \frac{\sin A}{\sqrt{1 - \sin^2 A}}} \quad \text{...(ii)} \; [\textit{From (i)}]$$

$$\sec A = \frac{1}{\cos A}$$

$$\therefore \quad \boxed{\sec A = \frac{1}{\sqrt{1 - \sin^2 A}}} \quad [\textit{From (ii)}]$$

Module 37

EXERCISE 8.4

Q.3) Evaluate : (i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

Solution:

$$\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$= \frac{\cos^2(90^\circ - 63^\circ) + \sin^2 27^\circ}{\sin^2(90^\circ - 17^\circ) + \cos^2 73^\circ}$$

$\left[\because \sin\theta = \cos(90 - \theta), \ \cos\theta = \sin(90 - \theta)\right]$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$$

$$= \frac{1}{1}$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$$\therefore \boxed{\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ} = 1}$$

EXERCISE 8.4

Q.3) Evaluate : (ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

Solution:

$\sin^2\theta + \cos^2\theta = 1$

$$\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

$$= \cos(90^\circ - 25^\circ)\cos 65^\circ + \sin(90^\circ - 25^\circ)\sin 65^\circ$$

$\left[\because \sin\theta = \cos(90 - \theta),\ \cos\theta = \sin(90 - \theta)\right]$

$$= \cos 65^\circ \cos 65^\circ + \sin 65^\circ \sin 65^\circ$$

$$= \cos^2 65^\circ + \sin^2 65^\circ$$

$$= 1 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$\therefore \boxed{\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ = 1}$$

Thank You

Module 38

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

Proof:

$1 - \cos^2\theta = \sin^2\theta$

$$\text{L.H.S} = (\operatorname{cosec}\theta - \cot\theta)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{\cancel{(1-\cos\theta)}\,(1-\cos\theta)}{\cancel{(1-\cos\theta)}\,(1+\cos\theta)}$$

$$= \frac{(1-\cos\theta)}{(1+\cos\theta)}$$

$\therefore$ L.H.S = R.H.S

$$\therefore (\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

Module 39

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

(ii) $\boxed{\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A}} = 2 \sec A$

$(a + b)^2 = a^2 + 2\,ab + b^2$

Proof:

$$\text{L.H.S} = \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) \cdot \cos A}$$

$$= \frac{\underline{\cos^2 A} + 1 + 2 \sin A + \underline{\sin^2 A}}{(1 + \sin A) \cdot \cos A}$$

$$= \frac{(\sin^2 A + \cos^2 A) + 1 + 2\sin A}{(1 + \sin A) \cdot \cos A}$$

$$= \frac{2 + 2\sin A}{(1 + \sin A) \cdot \cos A}$$

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

(ii) $$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=\boxed{2\sec A}$$

Proof:

$$\text{L.H.S} = \frac{2+2\sin A}{(1+\sin A)\,.\,\cos A}$$

$$= \frac{2\,\cancel{(1+\sin A)}}{\cancel{(1+\sin A)}\,.\,\cos A}$$

$$= \frac{2}{\cos A}$$

$$= 2\sec A$$

$\therefore$ L.H.S = R.H.S

$$\therefore \boxed{\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A}$$

Module 40

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

(iv) $\boxed{\dfrac{1 + \sec A}{\sec A}} = \boxed{\dfrac{\sin^2 A}{1 - \cos A}}$

$\boxed{\sec\theta = \dfrac{1}{\cos\theta}}$

$\boxed{1 - \cos^2\theta = \sin^2\theta}$

Proof:

$$\text{L.H.S} = \frac{1 + \sec A}{\sec A}$$

$$= \left(1 + \frac{1}{\cos A}\right) \div \left(\frac{1}{\cos A}\right)$$

$$= \frac{\cos A + 1}{\cancel{\cos A}} \times \frac{\cancel{\cos A}}{1}$$

$$\therefore \boxed{\text{L.H.S} = 1 + \cos A}$$

$$\text{R.H.S} = \frac{\sin^2 A}{1 - \cos A}$$

$$= \frac{1 - \cos^2 A}{1 - \cos A}$$

$$= \frac{\cancel{(1 - \cos A)}\,(1 + \cos A)}{\cancel{1 - \cos A}}$$

$$\therefore \boxed{\text{R.H.S} = (1 + \cos A)}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

i.e $$\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

Module 41

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Proof:

$$\text{L.H.S} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,[2\,(1 - \sin^2\theta) - 1]}$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2 - 2\sin^2\theta - 1)}$$

$$\therefore \text{L.H.S} = \frac{\sin\theta\,\cancel{(1 - 2\sin^2\theta)}}{\cos\theta\,\cancel{(1 - 2\sin^2\theta)}}$$

$$= \boxed{\frac{\sin\theta}{\cos\theta}}$$

$$\boxed{\frac{\sin\theta}{\cos\theta} = \tan\theta}$$

$$= \tan\theta$$

$\therefore$ L.H.S = R.H.S

$$\therefore \boxed{\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta}$$

Module 42

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

(ix) $(\text{cosec } A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

Proof:

$$\text{L.H.S} = (\text{cosec } A - \sin A)(\sec A - \cos A)$$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A}$$

$\therefore$ L.H.S $= \cos A \, . \, \sin A \quad \ldots(i)$

$\text{cosec } \theta = \dfrac{1}{\sin \theta}$

$\sec \theta = \dfrac{1}{\cos \theta}$

$1 - \cos^2\theta = \sin^2\theta$

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

(ix) $(\text{cosec } A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

Proof:

$$\text{R.H.S} = \frac{1}{\tan A + \cot A}$$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$$1 \div \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= 1 \div \left(\frac{\sin^2 A + \cos^2 A}{\cos A \times \sin A}\right)$$

$\sin^2\theta + \cos^2\theta = 1$

$$= 1 \div \left(\frac{1}{\cos A \times \sin A}\right)$$

$\therefore$ R.H.S $= \cos A \, . \, \sin A$ ...(ii)

L.H.S $= \cos A \, . \, \sin A$ ...(i)

$\therefore$ L.H.S = R.H.S ...[From (i) and (ii)]

$\therefore (\text{cosec } A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

Thank You

Module 43

Solved Example:

Prove: $$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

Proof:

$$\text{L.H.S} = \frac{\cot A - \cos A}{\cot A + \cos A}$$

$$= \left(\frac{\cos A}{\sin A} - \cos A\right) \div \left(\frac{\cos A}{\sin A} + \cos A\right)$$

$$= \left[\cancel{\cos A}\left(\frac{1}{\sin A} - 1\right)\right] \div \left[\cancel{\cos A}\left(\frac{1}{\sin A} - 1\right)\right]$$

$$= \left(\frac{1}{\sin A} - 1\right) \div \left(\frac{1}{\sin A} - 1\right)$$

$$= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

$$\therefore \quad \text{L.H.S} = \text{R.H.S}$$

$$\therefore \quad \boxed{\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}}$$

Module 44

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

(vi) $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

Proof: R.H.S $= \sec A + \tan A$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$\therefore \quad \text{R.H.S} = \frac{1 + \sin A}{\cos A} \quad \ldots \text{(i)}$$

$$\text{L.H.S.} = \sqrt{\frac{1 + \sin A}{1 - \sin A}}$$

$$= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$$

$$= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}}$$

$\cos^2\theta = 1 - \sin^2\theta$

$$\therefore \text{L.H.S} = \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} \quad \ldots \text{(ii)}$$

$\therefore$ L.H.S = R.H.S [*From (i) & (ii)*]

$$\therefore \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

Module 45

Q. $\sqrt{\dfrac{\operatorname{cosec} x - 1}{\operatorname{cosec} x + 1}} = \dfrac{1}{\sec x + \tan x}$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\therefore \operatorname{cosec}^2\theta - 1 = \cot^2\theta$

$(a - b)(a + b) = a^2 - b^2$

Proof. L.H.S $= \sqrt{\dfrac{(\operatorname{cosec} x - 1)}{(\operatorname{cosec} x + 1)}}$

$= \sqrt{\dfrac{(\operatorname{cosec} x - 1)}{(\operatorname{cosec} x + 1)} \times \dfrac{(\operatorname{cosec} x - 1)}{(\operatorname{cosec} x - 1)}}$

$= \sqrt{\dfrac{(\operatorname{cosec} x - 1)^2}{\operatorname{cosec}^2 x - 1}}$

$= \sqrt{\dfrac{(\operatorname{cosec} x - 1)^2}{\cot^2 x}}$

L.H.S $= \dfrac{\operatorname{cosec} x - 1}{\cot x}$

Q. $\sqrt{\dfrac{\operatorname{cosec} x - 1}{\operatorname{cosec} x + 1}} = \dfrac{1}{\sec x + \tan x}$

Proof. L.H.S $= \dfrac{\operatorname{cosec} x - 1}{\cot x}$

$$= \frac{\operatorname{cosec} x}{\cot x} - \frac{1}{\cot x}$$

$$= (\operatorname{cosec} x \div \cot x) - \frac{1}{\cot x}$$

$$= \left[\frac{1}{\sin x} \div \frac{\cos x}{\sin x}\right] - \frac{1}{\cot x}$$

$$= \left[\frac{1}{\cancel{\sin x}} \times \frac{\cancel{\sin x}}{\cos x}\right] - \boxed{\frac{1}{\cot x}}$$

$$= \boxed{\frac{1}{\cos x}} - \tan x$$

$$= \sec x - \tan x$$

$\dfrac{1}{\cos\theta} = \sec\theta$

$\dfrac{1}{\cot\theta} = \tan\theta$

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Q. $\sqrt{\dfrac{\text{cosec } x - 1}{\text{cosec } x + 1}} = \dfrac{1}{\sec x + \tan x}$

$(a - b)(a + b) = a^2 - b^2$

$\sec^2\theta - \tan^2\theta = 1$

Proof. L.H.S $= \sec x - \tan x$

$$= \frac{(\sec x - \tan x)(\sec x + \tan x)}{(\sec x + \tan x)}$$

$$= \frac{\sec^2 x - \tan^2 x}{(\sec x + \tan x)}$$

$$\text{L.H.S} = \frac{1}{(\sec x + \tan x)}$$

$\therefore$ L.H.S = R.H.S

$$\therefore \sqrt{\frac{(\text{cosec } x - 1)}{(\text{cosec } x + 1)}} = \frac{1}{(\sec x + \tan x)}$$

Module 46

Ex. 8.4 Q.5(viii)

Q. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

Soln. L.H.S. $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$(a + b)^2$ $(a + b)^2$

$= \sin^2 A + 2\sin A \operatorname{cosec} A + \ldots$

$= \sin^2 A + 2.$ ...

Lets sto... i.e. more com...

We want tan and cot in the R.H.S.

Identify the alg... Exp...

$\sec^2 A = 1 + \tan^2 A$

$= 5 + 1 + \cot^2 A + \ldots$

$= 7 + \tan^2 A + \ldots$

$= \text{R.H.S.}$

We will convert cosec and sec into cot and tan

$\therefore$ $(\sin A + \operatorname{cosec} A)^2 \ldots + \cot^2 A$

Module 47

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

(x) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\dfrac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

Proof:

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{1}{\cos^2 A} \div \frac{1}{\sin^2 A}$$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$

$$\therefore \frac{1 + \tan^2 A}{1 + \cot^2 A} = \tan^2 A \quad \ldots(i)$$

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

(x) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\dfrac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$

Proof:

$$\dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \boxed{\tan^2 A} \quad \ldots\text{(i)}$$

$$\left(\dfrac{1 - \tan A}{1 - \cot A}\right)^2$$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$$= \left(1 - \dfrac{\sin A}{\cos A}\right)^2 \div \left(1 - \dfrac{\cos A}{\sin A}\right)^2$$

$$= \left(\dfrac{\cos A - \sin A}{\cos A}\right)^2 \div \left(\dfrac{\sin A - \cos A}{\sin A}\right)^2$$

$a - b = -(b - a)$

$$= \left(\dfrac{-(\sin A - \cos A)}{\cos A}\right)^2 \div \left(\dfrac{\sin A - \cos A}{\sin A}\right)^2$$

$$= \dfrac{\cancel{(\sin A - \cos A)^2}}{\cos^2 A} \times \dfrac{\sin^2 A}{\cancel{(\sin A - \cos A)^2}}$$

$$= \dfrac{\sin^2 A}{\cos^2 A}$$

$\dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$$\therefore \left(\dfrac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A \quad \ldots\text{(ii)}$$

From (i) and (ii),

$$\therefore \dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\dfrac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$$

Thank You

Module 48

Ex. 8.4 Q.5 (iii)

Show that : $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta.\operatorname{cosec}\theta$

$$\text{L.H.S.} = \frac{\boxed{\tan\theta}}{1-\boxed{\cot\theta}} + \frac{\boxed{\cot\theta}}{1-\boxed{\tan\theta}}$$

Let us conv
in terms o

θ

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

?

$$= \frac{\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\cos\theta}{\sin\theta}} + \frac{\dfrac{\cos\theta}{\sin\theta}}{1-\dfrac{\sin\theta}{\cos\theta}}$$

$$= \frac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta - \sin\theta}{\cos\theta}}$$

$$= \left[\frac{\sin\theta}{\cos\theta} \div \frac{(\sin\theta - \cos\theta)}{\sin\theta}\right] + \left[\frac{\cos\theta}{\sin\theta} \div \frac{(\cos\theta - \sin\theta)}{\cos\theta}\right]$$

Ex. 8.4 Q.5 (iii)

Show that : $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta.\operatorname{cosec}\theta$

$$\text{L.H.S.} = \left[\frac{\sin\theta}{\cos\theta} \div \frac{(\sin\theta - \cos\theta)}{\sin\theta}\right] + \left[\frac{\cos\theta}{\sin\theta} \div \frac{(\cos\theta - \sin\theta)}{\cos\theta}\right]$$

$$= \left[\frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{(\sin\theta - \cos\theta)}\right] + \left[\frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{(}\right]$$

Taking (sin θ – cos θ) common

$$= \left[\frac{\sin^2\theta}{\cos\theta\ (\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta\ \boxed{(\cos\theta - \sin\theta)}}\right]$$

$$= \left[\frac{\sin^2\theta}{\cos\theta\ (\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{-\sin\theta\ (\sin\theta - \cos\theta)}\right]$$

$$= \frac{1}{(\sin\theta - \cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

Ex. 8.4 Q.5 (iii)

(4) Show that : $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta.\operatorname{cosec}\theta$

$$\text{L.H.S.}=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}\right]$$

$$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\boxed{\sin^3\theta-\cos^3\theta}}{\sin\theta.\cos\theta}\right]$$

$(a^3$ … $^2)$

We know,

$\sin^2\theta+\cos^2\theta=1$

$$=\frac{1}{\cancel{(\sin\theta-\cos\theta)}}\left[\frac{\cancel{(\sin\theta-\cos\theta)}\,(\sin^2\theta+\sin\theta\cdot\cos\theta+\cos^2\theta)}{\sin\theta.\cos\theta}\right]$$

$$=\frac{1+(\sin\theta.\cos\theta)}{\sin\theta.\cos\theta}$$

$$=\frac{1}{\sin\theta.\cos\theta}+\frac{\cancel{\sin\theta.\cos\theta}}{\cancel{\sin\theta.\cos\theta}}$$

Ex. 8.4 Q.5 (iii)

(4) Show that : $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta.\ \text{cosec}\,\theta$

$$\frac{1}{\cos\theta}=\sec\theta$$

$$\text{L.H.S.}=\frac{1}{\sin\theta\,.\,\cos\theta}+1$$

$$=\boxed{\frac{1}{\sin\theta}}\times\boxed{\frac{1}{\cos\theta}}+1$$

$$=\text{cosec}\,\theta\times\sec\theta+1$$

$$\boxed{\text{L.H.S.}=1+\sec\theta\times\text{cosec}\,\theta}$$

$\therefore$ L.H.S. = R.H.S.

$$\boxed{\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta.\ \text{cosec}\,\theta}$$

Module 49

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$, using $\operatorname{cosec}^2 A = 1 + \cot^2 A$.

$\operatorname{cosec} A = \dfrac{1}{\sin A}$ $\qquad$ $\cot A = \dfrac{\cos A}{\sin A}$

Proof:

$$\text{L.H.S} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

$$= \frac{\dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\sin A} - \dfrac{1}{\sin A}}$$

[Dividing numerator and denominator by sin A]

$$= \frac{\cot A - \boxed{1} + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{(1 + \cot A - \operatorname{cosec} A)}$$

EXERCISE 8.4

Q.5) Prove the following identities where the angles involved are acute angles for which the expressions are defined.

v) $$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \boxed{\operatorname{cosec} A + \cot A}$$ using $\operatorname{cosec}^2 A = 1 + \cot^2 A$.

$a^2 - b^2 = (a + b)(a - b)$

Proof:

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{(1 + \cot A - \operatorname{cosec} A)}$$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{(1 + \cot A - \operatorname{cosec} A)}$$

$$= \frac{(\operatorname{cosec} A + \cot A)\ [1 - (\operatorname{cosec} A - \cot A)]}{1 + \cot A - \operatorname{cosec} A}$$

$$= \frac{(\cot A + \operatorname{cosec} A)\ \cancel{(1 - \operatorname{cosec} A + \cot A)}}{\cancel{(1 + \cot A - \operatorname{cosec} A)}}$$

$$= \cot A + \operatorname{cosec} A$$

$\therefore$ L.H.S. = R.H.S.

$$\therefore \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

Module 50

Solved Example:

Prove that $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$ using $\sec^2\theta = 1 + \tan^2\theta$.

$\sec\theta = \dfrac{1}{\cos\theta}$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Proof:

$$\text{L.H.S.} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

$$= \frac{\dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\cos\theta} + \dfrac{1}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\cos\theta} - \dfrac{1}{\cos\theta}}$$

[Dividing numerator and denominator by sin A]

$$= \frac{\tan\theta - \boxed{1} + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$\sec^2\theta - \tan^2\theta = 1$

$$= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{(1 + \tan\theta - \sec\theta)}$$

Solved Example:

Prove that $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$ using $\sec^2\theta = 1 + \tan^2\theta$.

Proof:

$a^2 - b^2 = (a + b)(a - b)$

$$= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{(1 + \tan\theta - \sec\theta)}$$

$$= \frac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{(1 + \tan\theta - \sec\theta)}$$

$$= \frac{(\tan\theta + \sec\theta)[1 - (\sec\theta - \tan\theta)]}{1 + \tan\theta - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta)\cancel{(1 - \sec\theta + \tan\theta)}}{\cancel{(1 + \tan\theta - \sec\theta)}}$$

$$= \frac{\sec\theta + \tan\theta}{\boxed{1}}$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$\sec^2\theta - \tan^2\theta = 1$

Solved Example:

Prove that $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$ using $\sec^2\theta = 1 + \tan^2\theta$.

Proof:

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$a^2 - b^2 = (a + b)(a - b)$

$$= \frac{\cancel{(\sec\theta + \tan\theta)}}{(\sec\theta - \tan\theta)\cancel{(\sec\theta + \tan\theta)}}$$

$$= \frac{1}{(\sec\theta - \tan\theta)}$$

$\therefore$ L.H.S. = R.H.S.

$$\therefore \boxed{\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}}$$

Thank You

Module 51

Example:

Prove that $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \dfrac{1 + \cos A}{\sin A}$

Proof:

$$\text{R.H.S} = \frac{1 + \cos A}{\sin A}$$

$$= \frac{1}{\sin A} + \frac{\cos A}{\sin A}$$

$$\boxed{\text{R.H.S} = \operatorname{cosec} A + \cot A} \quad \text{... (i)}$$

$\operatorname{cosec}^2 A = 1 + \cot^2 A.$

$\operatorname{cosec} A = \dfrac{1}{\sin A}$ $\quad$ $\cot A = \dfrac{\cos A}{\sin A}$

$$\text{L.H.S} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

$$= \frac{\dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\sin A} - \dfrac{1}{\sin A}}$$

[Dividing numerator and denominator by sin A]

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

Example:

Prove that $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \dfrac{1 + \cos A}{\sin A}$

Proof:

$$\text{R.H.S} = \operatorname{cosec} A + \cot A \quad \dots \text{(i)}$$

$$= \frac{\cot A - \boxed{1} + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{(1 + \cot A - \operatorname{cosec} A)}$$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{(1 + \cot A - \operatorname{cosec} A)}$$

$$= \frac{(\operatorname{cosec} A + \cot A)\ [1 - (\operatorname{cosec} A - \cot A)]}{1 + \cot A - \operatorname{cosec} A}$$

$$= \frac{(\cot A + \operatorname{cosec} A)(1 - \operatorname{cosec} A + \cot A)}{(1 + \cot A - \operatorname{cosec} A)}$$

Example:

Prove that $$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{1 + \cos A}{\sin A}$$

Proof:

$$\boxed{\text{R.H.S} = \operatorname{cosec} A + \cot A \quad \ldots \text{(i)}}$$

$$= \frac{(\cot A + \operatorname{cosec} A)\,\cancel{(1 - \operatorname{cosec} A + \cot A)}}{\cancel{(1 + \cot A - \operatorname{cosec} A)}}$$

$$= \boxed{\operatorname{cosec} A + \cot A \quad \ldots \text{(ii)}}$$

$\therefore$ L.H.S. = R.H.S. [From (i) and (ii)]

$$\therefore \boxed{\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{1 + \cos A}{\sin A}}$$

Module 52

EXERCISE 8.4

Q.4) Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

Justification:

$\sec^2\theta - \tan^2\theta = 1$

$$9\sec^2 A - 9\tan^2 A = 9\,(\sec^2 A - \tan^2 A)$$

$$= 9\,(1) \quad [\because \sec^2\theta - \tan^2\theta = 1]$$

$$\therefore \quad 9\sec^2 A - 9\tan^2 A = 9$$

Hence, option (B) is correct.

EXERCISE 8.4

Q.4) Choose the correct option. Justify your choice.

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

(A) $\sec^2 A$ (B) -1 (C) $\cot^2 A$ (D) $\tan^2 A$ ✓

$\dfrac{\sin\theta}{\cos\theta} = \tan\theta$

Justification:

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{1}{\cos^2 A} \div \frac{1}{\sin^2 A}$$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$\therefore \frac{1 + \tan^2 A}{1 + \cot^2 A} = \tan^2 A$$

Hence, option (D) is correct.

EXERCISE 8.4

Q.4) Choose the correct option. Justify your choice.

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) $\sec A$ (B) $\sin A$ (C) $\operatorname{cosec} A$ (D) $\cos A$

Justification:

$$(\sec A + \tan A)(1 - \sin A)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1 + \sin A)}{\cos A}(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A} \quad [\because \cos^2\theta = 1 - \sin^2\theta]$$

$$= \cos A$$

$\therefore (\sec A + \tan A)(1 - \sin A) = \cos A$

Hence, option (D) is correct.

$1 - \sin^2\theta = \cos^2\theta$

Module 53

EXERCISE 8.4

Q.4) Choose the correct option. Justify your choice.

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$

(A) 0 (B) 1 (C) 2 (D) – 1

Justification:

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(1 + \frac{\sin\theta + 1}{\cos\theta}\right)\left(1 + \frac{\cos\theta - 1}{\sin\theta}\right)$$

$$= \left(\frac{\underbrace{\cos\theta + \sin\theta}_{a} + \underbrace{1}_{b}}{\cos\theta}\right)\left(\frac{\underbrace{\sin\theta + \cos\theta}_{a} - \underbrace{1}_{b}}{\sin\theta}\right)$$

$$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta \cdot \cos\theta}$$

EXERCISE 8.4

Q.4) Choose the correct option. Justify your choice.

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$

(A) 0 (B) 1 (C) 2 (D) – 1

$\sin^2\theta + \cos^2\theta = 1$

Justification:

$$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta \cdot \cos\theta}$$

$$= \frac{\sin^2\theta + 2\sin\theta \cdot \cos\theta + \cos^2\theta - 1}{\sin\theta \cdot \cos\theta}$$

$$= \frac{\cancel{1} + 2\sin\theta \cdot \cos\theta - \cancel{1}}{\sin\theta \cdot \cos\theta}$$

$$= \frac{2\cancel{\sin\theta \cdot \cos\theta}}{\cancel{\sin\theta \cdot \cos\theta}} = 2$$

$\therefore \quad (1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) = 2$

Hence, option (C) is correct.

Module 54

Trigonometric Ratios of 45°

Consider ΔABC in which $\angle ABC = 90°$ and $\angle A = 45°$.

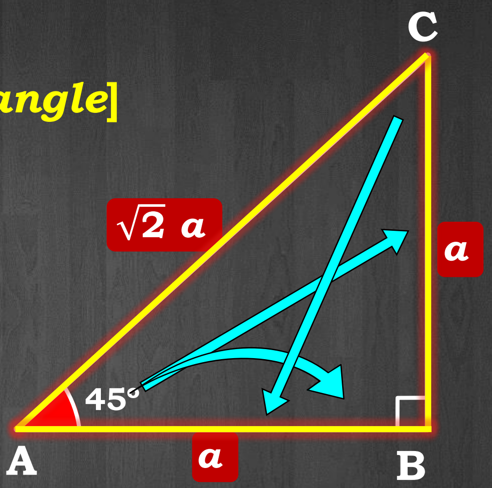
$\angle A + \angle B + \angle C = 180°$ *[Angle sum property of a triangle]*

$\therefore \quad 45 + 90 + \angle C = 180°$

$\therefore \quad \angle C = 180 - 135 = 45°$

$\therefore \quad \angle A = \angle C$

$\therefore \quad AB = BC$ *[sides opposite to equal angles are equal]*



Let, $AB = BC = a$

$AC^2 = AB^2 + BC^2$

[Pythagoras theorem]

$\therefore \quad AC^2 = a^2 + a^2$

$\therefore \quad AC^2 = 2a^2$

$\therefore \quad AC = \sqrt{2}\, a$

$\sin 45° = \dfrac{BC}{AC}$

$\therefore \quad \sin 45° = \dfrac{\cancel{a}}{\sqrt{2}\, \cancel{a}}$

$\therefore \quad \boxed{\sin 45° = \dfrac{1}{\sqrt{2}}}$

$\cos 45° = \dfrac{AB}{AC}$

$\therefore \quad \cos 45° = \dfrac{\cancel{a}}{\sqrt{2}\, \cancel{a}}$

$\therefore \quad \boxed{\cos 45° = \dfrac{1}{\sqrt{2}}}$

Trigonometric Ratios of 45°

$$\tan 45^\circ = \frac{BC}{AB}$$

$$\therefore \quad \tan 45^\circ = \frac{\cancel{a}}{\cancel{a}}$$

$$\therefore \quad \boxed{\tan 45^\circ = 1}$$

$$\operatorname{cosec} 45^\circ = \frac{AC}{BC}$$

$$\therefore \quad \operatorname{cosec} 45^\circ = \frac{\sqrt{2}\,\cancel{a}}{\cancel{a}}$$

$$\therefore \quad \boxed{\operatorname{cosec} 45^\circ = \sqrt{2}}$$

$$\sec 45^\circ = \frac{AC}{AB}$$

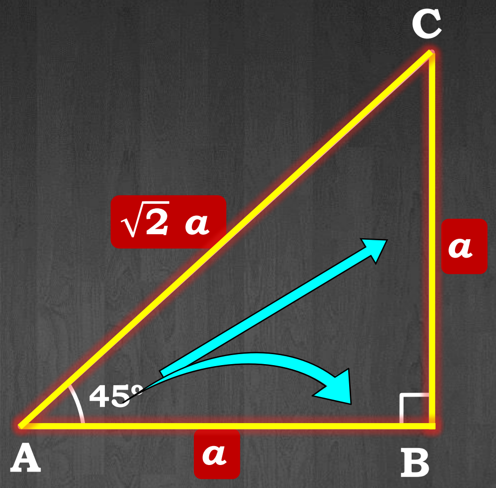
$$\therefore \quad \sec 45^\circ = \frac{\sqrt{2}\,\cancel{a}}{\cancel{a}}$$

$$\therefore \quad \boxed{\sec 45^\circ = \sqrt{2}}$$

$$\cot 45^\circ = \frac{AB}{BC}$$

$$\therefore \quad \cot 45^\circ = \frac{\cancel{a}}{\cancel{a}}$$

$$\therefore \quad \boxed{\cot 45^\circ = 1}$$



Module 55

Trigonometric Ratios of 30° and 60°

Consider an **equilateral ΔABC such that $AD \perp BC$.**

$\therefore\ AB = BC = AC$ and $\angle A = \angle B = \angle C = 60°$

In ΔADB and ΔADC,

$\angle ADB = \angle ADC = 90°$

$AB = AC$ [Sides of an equilateral triangle]

$AD = AD$ [Common Side]

$\therefore\ \Delta ABD \cong \Delta ABC$ [By RHS congruence criterion]

$\therefore\ \angle BAD = \angle CAD$ (c.p.c.t)

$\therefore\ \angle BAD = \angle CAD = \frac{1}{2}\angle A$

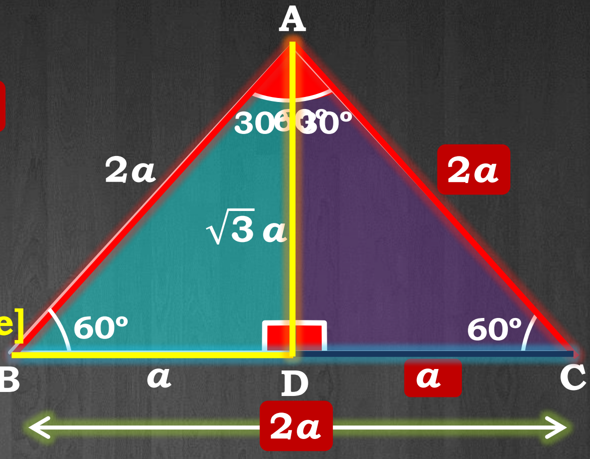
$\therefore\ \angle BAD = \angle CAD = 30°$

$BD = CD$ (c.p.c.t)

Let, $BD = CD = a$

$\therefore\ BC = 2a$

$\therefore\ AB = BC = AC = 2a$



In ΔADC, $\angle ADC = 90°$

$AC^2 = AD^2 + CD^2$ [*Pythagoras theorem*]

$\therefore\ (2a)^2 = AD^2 + a^2$

$\therefore\ AD^2 = 4a^2 - a^2$

$\therefore\ AD^2 = 3a^2$

$\therefore\ AD = \sqrt{3}\,a$

Trigonometric Ratios of 30° and 60°

$$\sin 30^\circ = \frac{CD}{AC}$$

$$\therefore \sin 30^\circ = \frac{\cancel{a}}{2\cancel{a}}$$

$$\therefore \boxed{\sin 30^\circ = \frac{1}{2}}$$

$$\cos 30^\circ = \frac{AD}{AC}$$

$$\therefore \cos 30^\circ = \frac{\sqrt{3}\,\cancel{a}}{2\cancel{a}}$$

$$\therefore \boxed{\cos 30^\circ = \frac{\sqrt{3}}{2}}$$

$$\tan 30^\circ = \frac{CD}{AD}$$

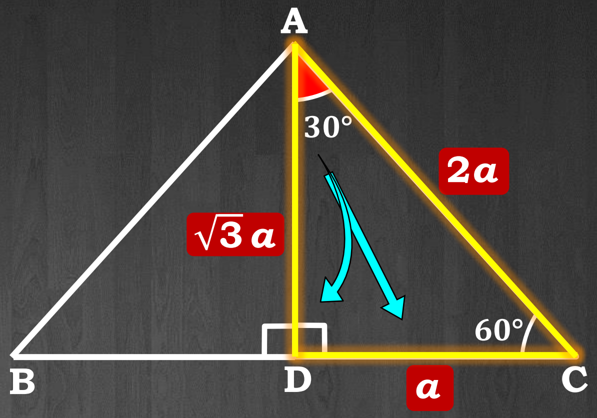
$$\therefore \tan 30^\circ = \frac{\cancel{a}}{\sqrt{3}\,\cancel{a}}$$

$$\therefore \boxed{\tan 30^\circ = \frac{1}{\sqrt{3}}}$$

$$\text{cosec } 30^\circ = \frac{AC}{CD}$$

$$\therefore \text{cosec } 30^\circ = \frac{2\cancel{a}}{\cancel{a}}$$

$$\therefore \boxed{\text{cosec } 30^\circ = 2}$$



Trigonometric Ratios of 30° and 60°

$$\sec 30^\circ = \frac{AC}{AD}$$

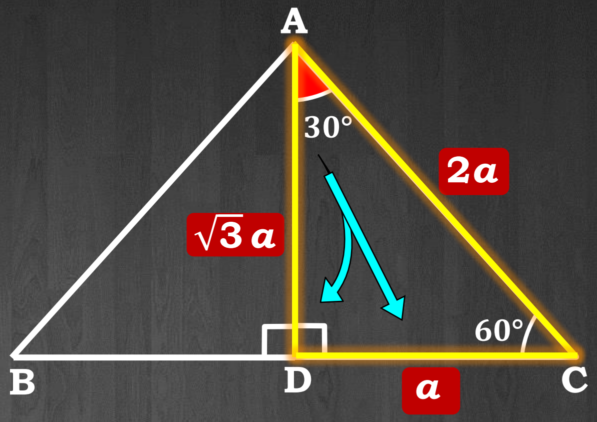
$$\therefore \sec 30^\circ = \frac{2\cancel{a}}{\sqrt{3}\,\cancel{a}}$$

$$\therefore \boxed{\sec 30^\circ = \frac{2}{\sqrt{3}}}$$

$$\cot 30^\circ = \frac{AD}{CD}$$

$$\therefore \cot 30^\circ = \frac{\sqrt{3}\,\cancel{a}}{2\cancel{a}}$$

$$\therefore \boxed{\cot 30^\circ = \frac{\sqrt{3}}{2}}$$



Trigonometric Ratios of 30° and 60°

$$\sin 60° = \frac{AD}{AC}$$

$$\therefore \quad \sin 60° = \frac{\sqrt{3}\,\cancel{a}}{2\cancel{a}}$$

$$\therefore \quad \boxed{\sin 60° = \frac{\sqrt{3}}{2}}$$

$$\cos 60° = \frac{CD}{AC}$$

$$\therefore \quad \cos 60° = \frac{\cancel{a}}{2\cancel{a}}$$

$$\therefore \quad \boxed{\cos 60° = \frac{1}{2}}$$

$$\tan 60° = \frac{AD}{CD}$$

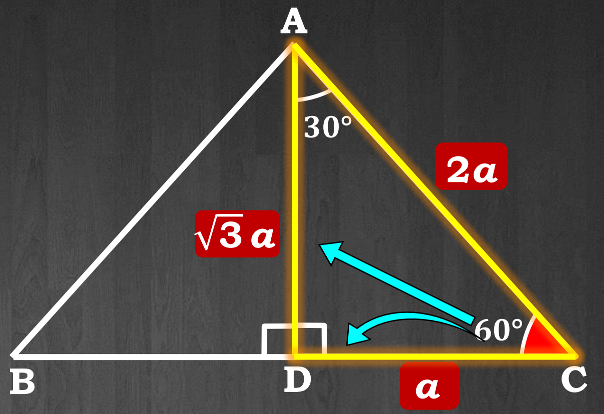
$$\therefore \quad \tan 60° = \frac{\sqrt{3}\,\cancel{a}}{\cancel{a}}$$

$$\therefore \quad \boxed{\tan 60° = \sqrt{3}}$$

$$\operatorname{cosec} 60° = \frac{AC}{AD}$$

$$\therefore \quad \operatorname{cosec} 60° = \frac{2\cancel{a}}{\sqrt{3}\,\cancel{a}}$$

$$\therefore \quad \boxed{\operatorname{cosec} 60° = \frac{2}{\sqrt{3}}}$$



Trigonometric Ratios of 30° and 60°

$$\sec 60^\circ = \frac{AC}{CD}$$

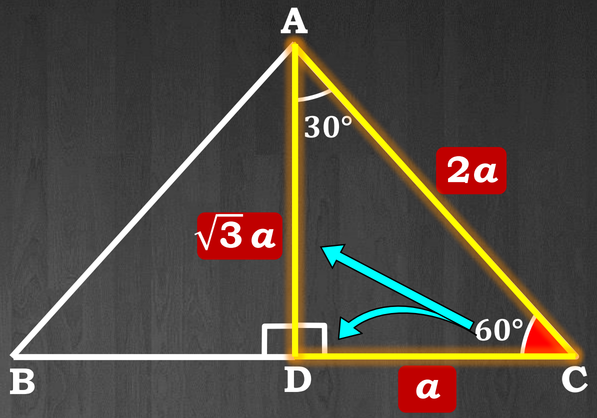
$$\therefore \sec 60^\circ = \frac{2\cancel{a}}{\cancel{a}}$$

$$\therefore \boxed{\sec 60^\circ = 2}$$

$$\cot 60^\circ = \frac{CD}{AD}$$

$$\therefore \cot 60^\circ = \frac{\cancel{a}}{\sqrt{3}\,\cancel{a}}$$

$$\therefore \boxed{\cot 60^\circ = \frac{1}{\sqrt{3}}}$$



Thank You