Lecture_04

No. 26

ARITHMETIC PROGRESSIONS

- **Sums based on ‘a_n’ Formula**

10) The 17^th term of an AP exceeds its 10^th term by 7. Find the co

$a_{17} = a$

$a_{10} = a + 9d$

We need to find d

Sol: For given AP:

$$a_{17} = a_{10} + 7$$

$$\therefore \cancel{a} + 16d = \cancel{a} + 9d + 7$$

$$\therefore 16d - 9d = 7$$

$$\therefore 7d = 7$$

$$\therefore d = 1$$

$\therefore$ **Common difference of AP is 1.**

16) Determine the AP whose third term is 16 and 7th term exceeds the 5th term by 12.

Sol: $a_3 = 16, \quad a_7 = a_5 + 12$

We know that,

$a_3 = a + 2d$

$\therefore \quad 16 = a + 2d \quad .....(i)$

$a_7 = a_5 + 12$

$\therefore \quad \cancel{a} + 6d = \cancel{a} + 4d + 12$

$\therefore \quad 6d - 4d = 12$

$\therefore \quad 2d = 12$

$\therefore \quad d = 6$

Substituting d = 6 in (i)

$a + 2(6) = 16$

$\therefore \quad a + 12 = 16$

$\therefore \quad a = 16 - 12$

$\therefore \quad a = 4$

We need to determine the AP

$a_7 = a + 6d$

$a_5 = a + 4d$

$a_4 = 16 + 6 = 22$

$a_3 = 10 + 6 = 16$

$a_2 = 4 + 6 = 10$

That means, First term of AP is 4 and common difference is 6

$\therefore$ **The required AP is 4, 10, 16, 22, ...**

No. 27

ARITHMETIC PROGRESSIONS

- **Sums based on 'a_n' Formula**

11) Which term of AP: 3, 15, 27, 39, … will be 132 more than its 54^{th} term.

Sol: For given AP

$a_n = a +$ $a_{54} = a + 53d$

We need to find 'n' such that, $a_n = a_{54} + 132$

$a = 3, \quad d = 15 - 3 = 12$

$$a_n = a_{54} + 132$$

$$\therefore \cancel{a} + (n - 1)d = \cancel{a} + 53d + 132$$

$$\therefore (n - 1)(12) = 53\,(12) + 132$$

$$\therefore (n - 1)(12) = 636 + 132$$

$$\therefore (n - 1)(12) = 768$$

$$\therefore (n - 1) = \frac{768}{12}$$

$$\therefore n - 1 = 64$$

$$\therefore n = 65$$

That means, $a_{65} = a_{54} + 132$

$\therefore$ $\boxed{\textbf{65}^{th}\textbf{ term of AP will be 132 more than its 54}^{th}\textbf{ term.}}$

No. 28

ARITHMETIC PROGRESSIONS

- **Sums based on 'a_n' Formula**

13) How many three-digit numbers are divisible by 7?

Sol: Three digit nos. divisible by 7 are: 105, 112, 119, ... 994

... an A.P

$a = 105$, $d = 112 - 105 = 7$, $a_n = 994$

We know that,

$a_n = a + (n - 1)\,d$

$\therefore\ 994 = 105 + (n - 1)(7)$

$\therefore\ 994 - 105 = (n - 1)\,7$

$\therefore\ 889 = (n - 1)\,7$

$\therefore\ 127 = n - 1$

$\therefore\ n = 128$

∴ ... bers divisible by 7.

3 digit numbers

100,
101,
102,
103,
104,
105,
106,
.
.
993,
994,
995,
996,
997,
998,
999.

... divisible by 7

... not divisible by 7

Lets make a list of 3 digit ...

... 3 digit number divisible by 7

Next no. divisible by 7 will be obtained by adding 7 to previous

We need to find 'n' such that, $a_n = 994$

That means, $a_{128} = 994$

14) How many multiples of 4 lie between 10 and 250?

Sol: Multiple of 4 lying between 10 & 250 are: 12, 16, ... 248

which forms an A.P.

$a = 12$, $d = 4$, $a_n = 248$

We know that,

$$a_n = a + (n - 1)\, d$$

$$\therefore\ 248 = 12 + (n - 1)(4)$$

$$\therefore\ 248 - 12 = (n - 1)\, 4$$

$$\therefore\ 236 = (n - 1)\, 4$$

$$\therefore\ 59 = n - 1$$

$$\therefore\ n = 60$$

$\therefore$ ... s of 4 lies between 10 and 250.

Multiples of 4

4,
8,
12,
16,
20,
.
.
.
240,
244,
248,
252,
.
.
.

Lying between 10 & 250

Lets make a list of multiples of 4 lie between 10 & 250

If l...

**Similarly,
to find number of terms check which term is 248.
Because, it is the last term.**

We need to find '*n*' such that, $a_n = 248$

That means, $a_{60} = 248$

No. 29

ARITHMETIC PROGRESSIONS

- **Sums based on 'a_n' Formula**

17) Find the 20th term from the last term of the AP: 3, 8, 13, …, 253.

We need to determine a_{20} when AP is written in reverse order

Lets write given AP in reverse order

Sol: Given AP: 3, 8, 13, ..., 253.

a_{20}

Reverse order of given AP:

253, 248, 243, …, 8, 3

a = 253, d = 248 – 253 = – 5

We know that,

$$a_{20} = a + 19d$$

$$= 253 + 19(-5)$$

$$= 253 - 95$$

$$\therefore a_{20} = 158$$

$\therefore$ $\boxed{\textbf{20}^{\textbf{th}} \textbf{ term from the last term of the AP is 158.}}$

No. 30

ARITHMETIC PROGRESSIONS

- **Sums based on 'a_n' Formula**

Q.18) The sum of the 4th and 8th terms of an A.P is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

a_4 = a + 3d

a_8 = a + 7d

Sol:

$a_4 + a_8 = 24$ ..... (given)

$\therefore\ a + 3d + a + 7d = 24$

$\therefore\ 2a + 10d = 24$

Dividing throughout by 2

a_6 = a + 5d

a_{10} = a + 9d

$a + 5d = 12$ ... (i)

$a_6 + a_{10} = 44$ ... (given)

$\therefore\ a + 5d + a + 9d = 44$

$\therefore\ 2a + 14d = 44$

Dividing throughout by 2

$a + 7d = 22$ ... (ii)

Subtracting (i) from (ii)

With the values of a & d lets find a_2 & a_3.

$a + 5d = 12$

(-) (-) (-)

$2d = 10$

$\therefore\ d = 5$

Substituting d = 5 in (i)

$a + 5d = 12$

$\therefore\ a + 5(5) = 12$

$\therefore\ a + 25 = 12$

$\therefore\ a = 12 - 25$

$\therefore\ a = -13$

a_1, a_2, a_3

$a_2 = a + d$

$= -13 + 5$

$= -8$

$a_3 = a + 2d$

$= -13 + 2(5)$

$= -13 + 10$

$= -3$

The first three terms of AP are –13, –8 and –3

No. 31

ARITHMETIC PROGRESSIONS

- **Sums based on 'a_n' Formula**

12) Two APs have the same common difference. The difference between their 100^{th} terms is 100, what is the difference between their 1000^{th} terms?

We need to find $A_{1000} - a_{1000}$

Sol: Let d be the common difference of both APs. and first term be denoted by 'A' and 'a' of two APs respectively.

$A_{100} = A$

$a_{100} = a + 99d$

$$A_{100} - a_{100} = 100$$

$$\therefore\ A + 99d - (a + 99d) = 100$$

$$\therefore\ A + \cancel{99d} - a - \cancel{99d} = 100$$

$$\therefore\ A - a = 100 \quad .....(i)$$

Lets find $A_{1000} - a_{1000}$

$$A_{1000} - a_{1000}$$

$$= A + 999d - (a + 999d)$$

$$= A + \cancel{999d} - a - \cancel{999d}$$

$$= A - a$$

$$= 100 \quad ..... \text{(from i)}$$

$\therefore$ **Difference between their 1000^{th} terms is 100.**

Thank You