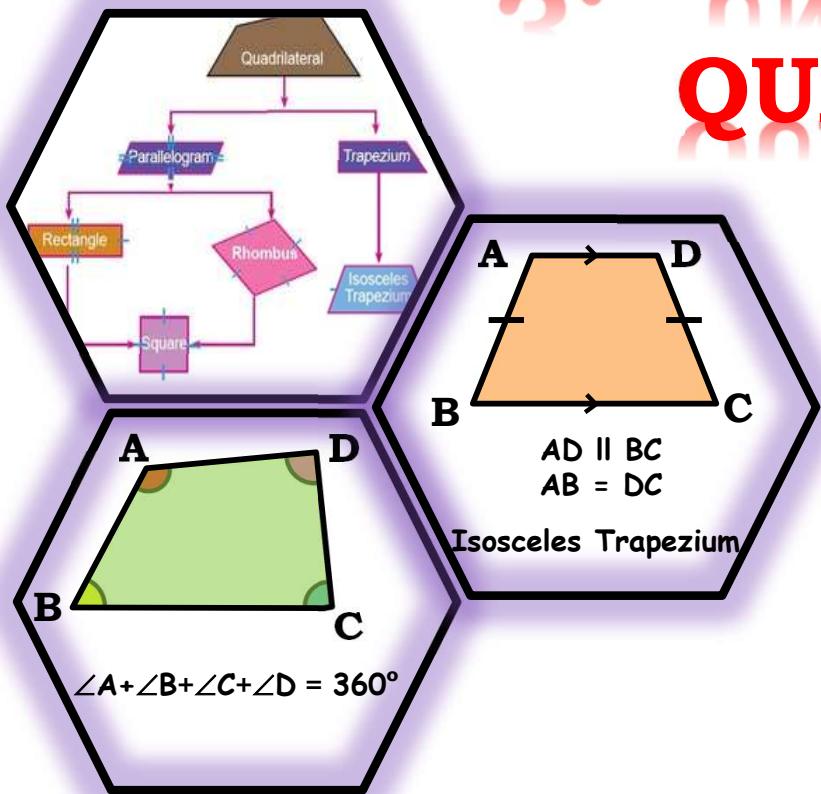


3. UNDERSTANDING QUADRILATERALS

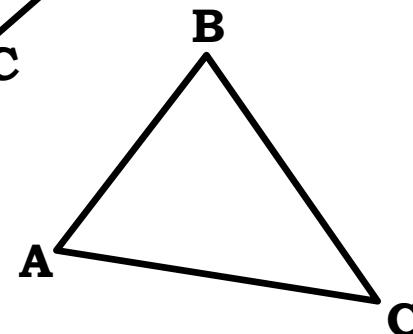
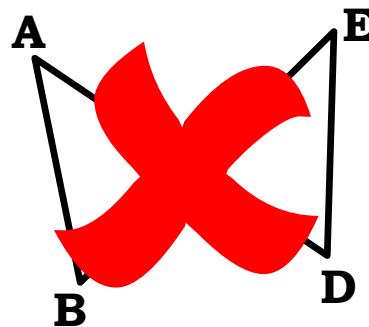
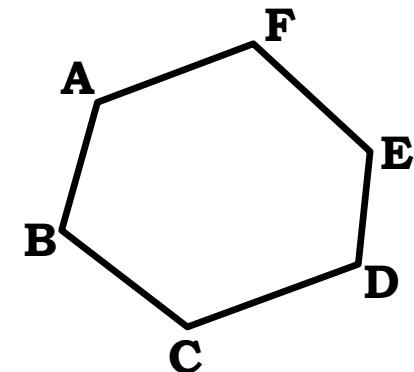
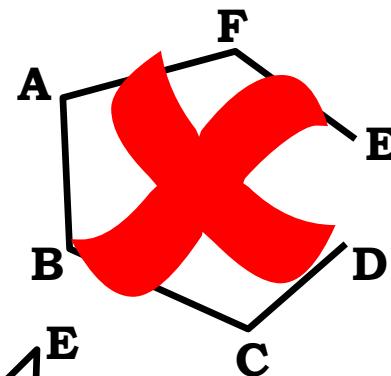
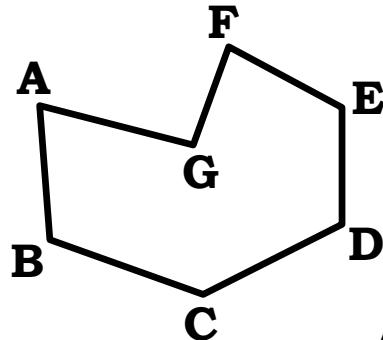


Lecture 1

Module 1

POLYGON

- It is a closed plane figure, bounded by straight-line segments.
- The line segments forming a polygon intersect only at end-points and each end-point is shared by only two line-segments.



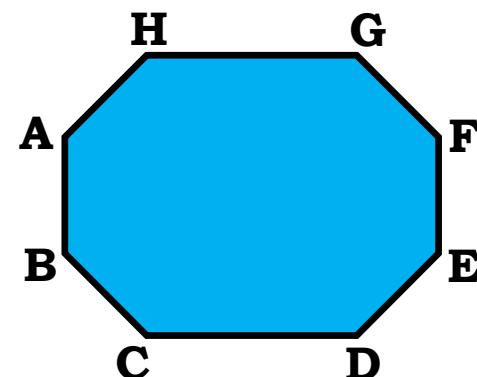
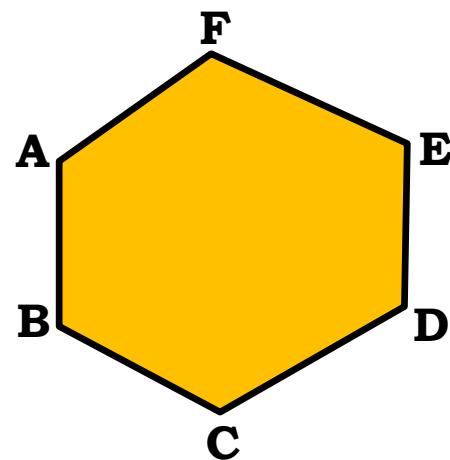
POLYGON

**CONVEX
POLYGON**

**CONCAVE
POLYGON**

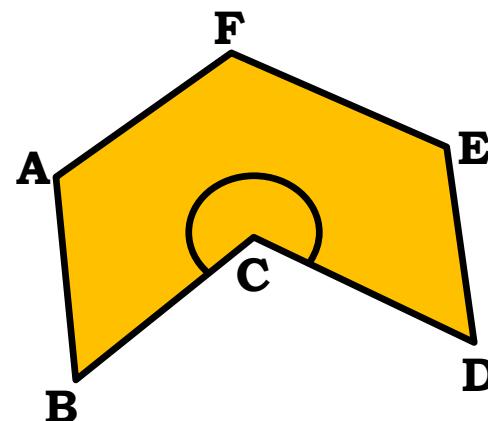
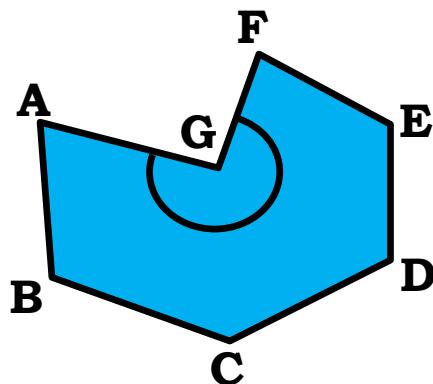
CONVEX POLYGON

- If each angle of a polygon is less than 180° , it is called a convex polygon.



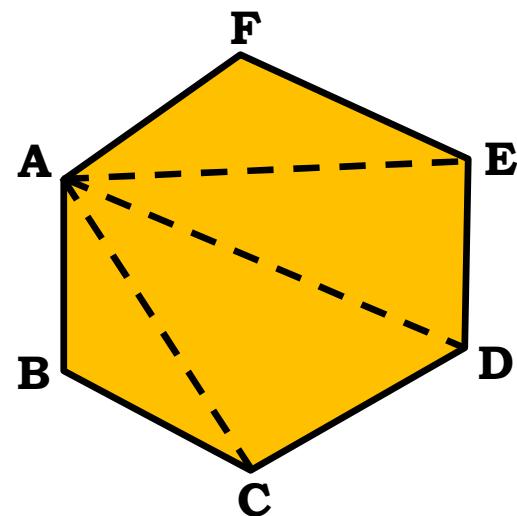
CONCAVE POLYGON

- If at least one angle of a polygon is more than 180° , it is called a concave or re-entrant polygon.

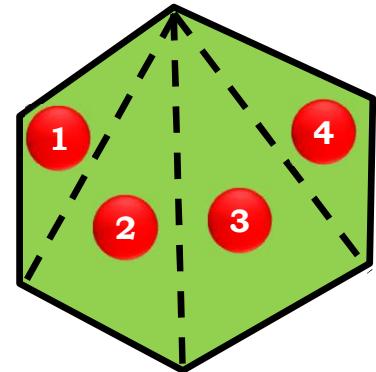
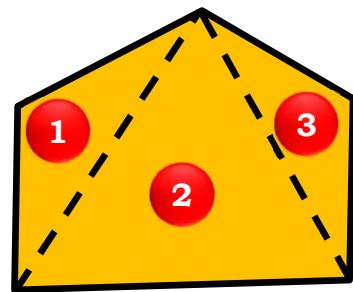
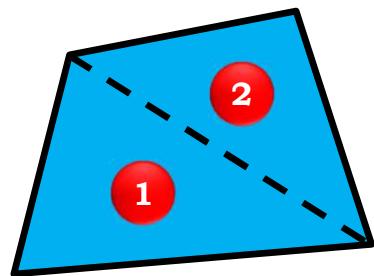


DIAGONAL OF A POLYGON

- ❖ A line segment joining any two non-consecutive vertices of a polygon is called its diagonal.
- ❖ In the adjoining fig.,
AC, AD and AE are diagonals
of hexagon ABCDEF.
- ❖ More diagonals can be
drawn through the
vertices B, C, D, E and F.



SUM OF ANGLES OF A POLYGON



	No. of Sides	No. of Triangles
Quadrilateral	4	2
Pentagon	5	3
Hexagon	6	4

$$\text{No. of Triangles} = \text{No. of Sides} - 2$$

No. of triangles = No. of sides - 2

If polygon has 'n' sides,

Number of triangle formed = $(n - 2)$

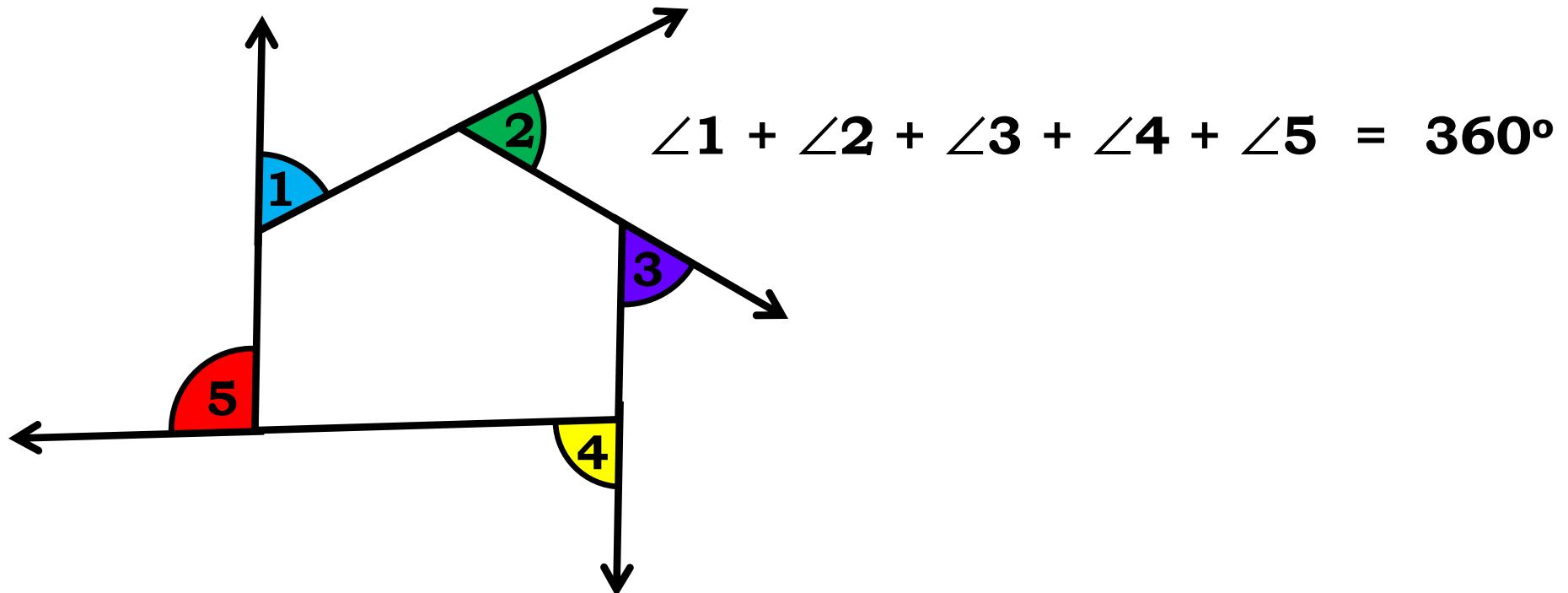
Sum of angles of a triangle = 180°

$$\therefore \text{Sum of angles of } (n - 2) \text{ triangles} = (n - 2) \times 180^\circ \\ = (2n - 4) \times 90^\circ$$

Sum of Interior angles of a polygon with 'n' sides = $(2n - 4) \times 90^\circ$

SUM OF EXTERIOR ANGLES OF A POLYGON

If the sides of a polygon are produced in order,
the sum of exterior angles so formed is always 360° .



Module 2

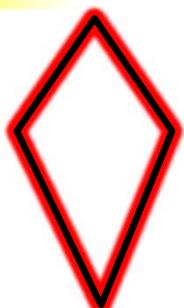
Q. Given here are some figures.

Classify each of them on the basis of the following.

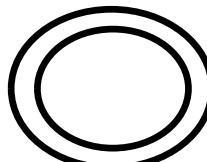
(a) Simple curve



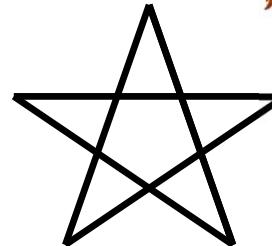
(1)



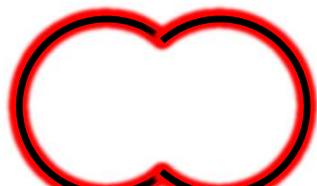
(2)



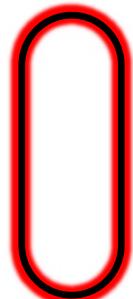
(3)



(4)



(5)



(6)



(7)



A **Simple curve** is a curve that does not cross itself



(8)

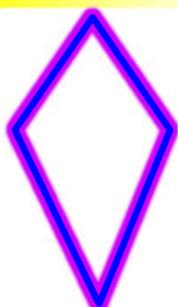
Q. Given here are some figures.

Classify each of them on the basis of the following.

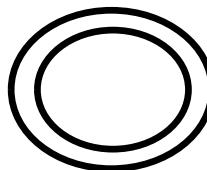
(b) Simple closed curve



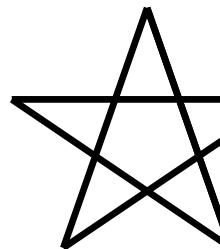
(1)



(2)

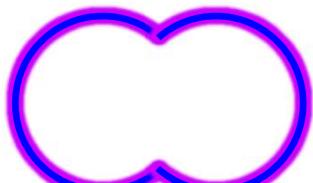


(3)

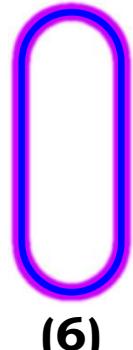


(4)

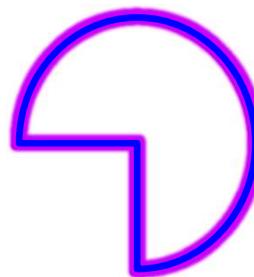
A closed curve which does not cross itself is called a **Simple closed curve**.



(5)



(6)



(7)



(8)

Q. Given here are some figures. Classify each of them.

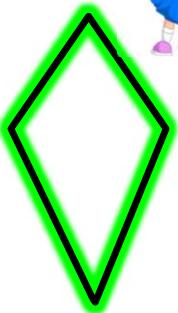


A plane figure with at least three straight sides is called a **Polygon**.

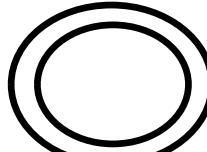
(c) Polygon



(1)



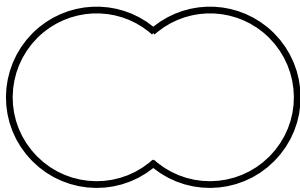
(2)



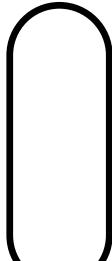
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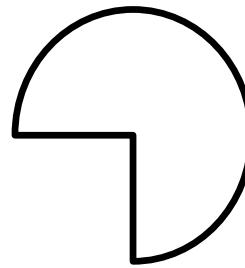
(4)



(5)



(6)



(7)



(8)

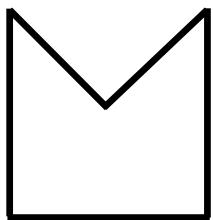
Q. Given here are some figures.

Classify each of them as Convex or Concave.

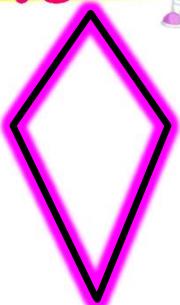
(d) Convex polygon



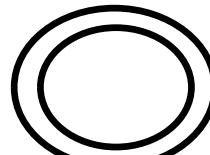
If each angle of a polygon
is less than 180° then it is called a **Convex polygon**.



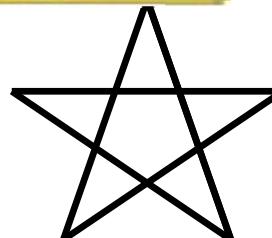
(1)



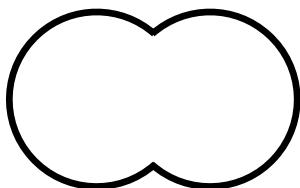
(2)



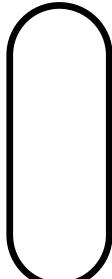
(3)



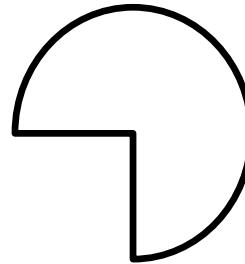
(4)



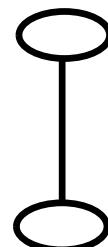
(5)



(6)



(7)



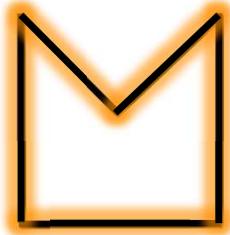
(8)

**Q. Given here are some figures.
Classify each of them.**

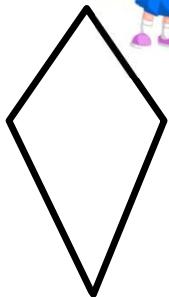
(e) **Concave polygon**

If at least one angle of
a polygon is more than
 180° then it is called a
Concave polygon

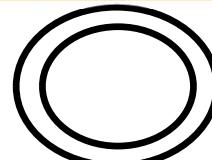
following.



(1)



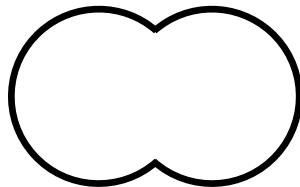
(2)



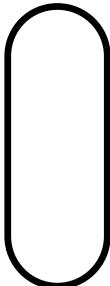
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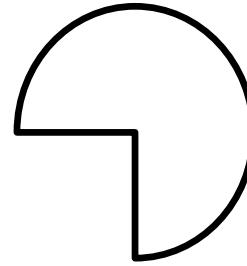
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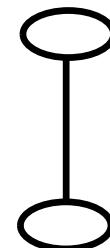
(5)



(6)



(7)

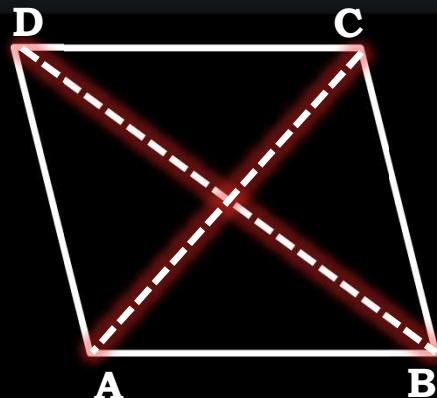


(8)

Q. How many diagonals does each of the following have?

(a) A convex quadrilateral

Sol.

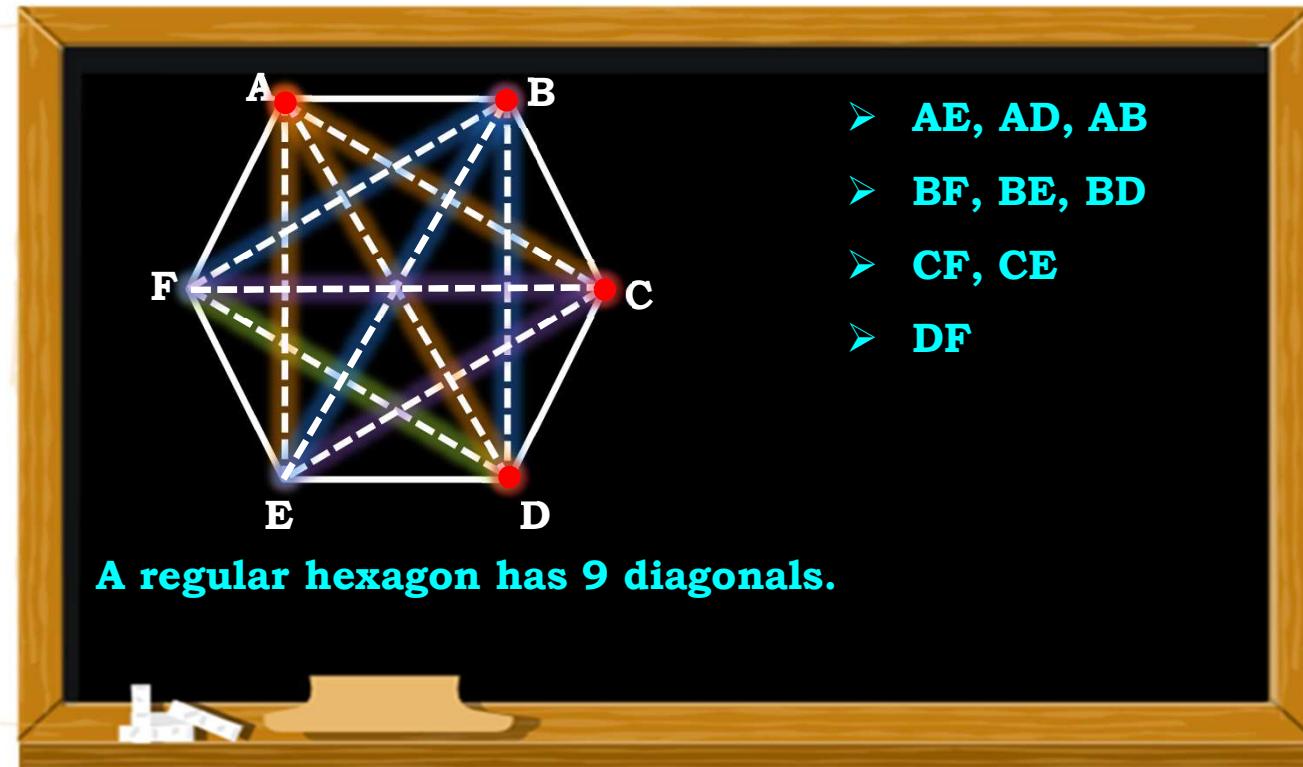


A convex quadrilateral has two diagonals.

- 1. AC**
- 2. BD**

Q. How many diagonals does each of the following have?
(b) A regular hexagon

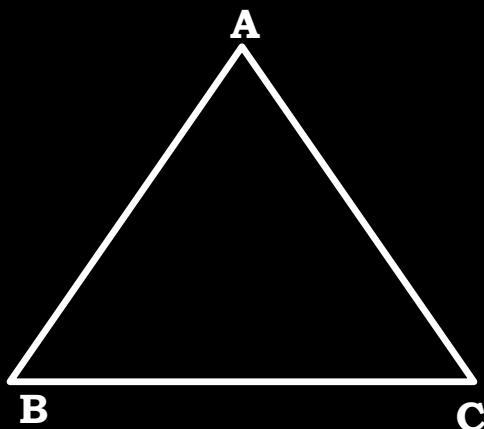
Sol.



Q. How many diagonals does each of the following have?

(c) A triangle

Sol.



A triangle has no diagonal.

Module

3

Q. What is the sum of the measures of the angles of a convex quadrilateral? Will the property hold if the quadrilateral is not convex?

Sol. Let, ABCD be a convex quadrilateral

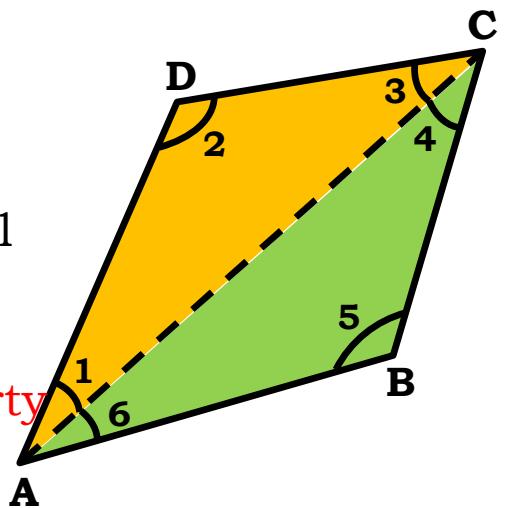
Draw a diagonal AC which divides the quadrilateral in two triangles.

$$\left. \begin{array}{l} \angle 1 + \angle 2 + \angle 3 = 180^\circ \dots(i) \\ \angle 4 + \angle 5 + \angle 6 = 180^\circ \dots(ii) \end{array} \right\} \text{[By Angle sum property of triangle]}$$

Adding (i) and (ii)

consider $\triangle ABC$ $\angle 4 + \angle 5 + \angle 6 = 360^\circ$

\therefore Sum of the angles of convex quadrilateral is 360° .



Q. What is the sum of the measures of the angles of a convex quadrilateral? Will the property hold if the quadrilateral is not convex?

Sol. Let ABCD be a non-convex quadrilateral

Draw BD which divides the quadrilateral in two triangles.

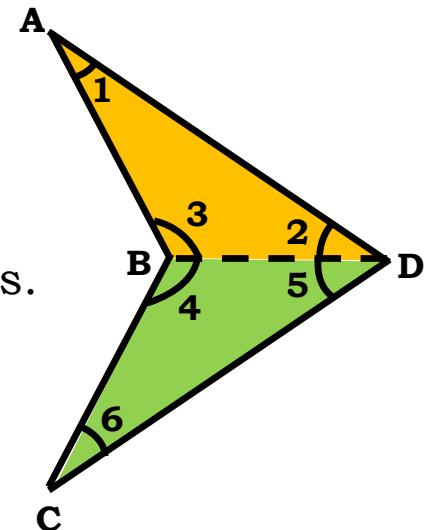
$$\left. \begin{array}{l} \angle 1 + \angle 2 + \angle 3 = 180^\circ \dots \text{(iii)} \\ \angle 4 + \angle 5 + \angle 6 = 180^\circ \dots \text{(iv)} \end{array} \right\} \text{[By Angle sum property of triangle]}$$

Adding (iii) and (iv)

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360^\circ$$

consider $\triangle BDC$

Sum of the angles of non-convex quadrilateral is 360° .



Q. What can you say about the angle sum of a convex polygon with number of sides?

Sol. (a) 7

$$\begin{aligned}\text{Angle sum of a convex polygon} &= (n - 2) \times 180^\circ \\ &= (7 - 2) \times 180 \\ &= 5 \times 180\end{aligned}$$

$\therefore \text{Angle sum of a convex polygon} = 900^\circ$

Angle sum of a convex polygon = $(n - 2) \times 180^\circ$

Where 'n' is number of sides



Q. What can you say about the angle sum of a convex polygon with number of sides?

Sol. (b) 8

$$\begin{aligned}\text{Angle sum of a convex polygon} &= (n - 2) \times 180^\circ \\ &= (8 - 2) \times 180^\circ \\ &= 6 \times 180^\circ\end{aligned}$$

$\therefore \text{Angle sum of a convex polygon} = 1080^\circ$

Angle sum of a convex polygon $= (n - 2) \times 180^\circ$

Where 'n' is number of sides



Module 4

Q. What can you say about the angle sum of a convex polygon with number of sides?

Sol. (c) 10

$$\begin{aligned}\text{Angle sum of a convex polygon} &= (n - 2) \times 180^\circ \\ &= (10 - 2) \times 180^\circ \\ &= 8 \times 180^\circ\end{aligned}$$

$\therefore \text{Angle sum of a convex polygon} = 1440^\circ$

Angle sum of a convex polygon $= (n - 2) \times 180^\circ$

Where 'n' is number of sides



Q. What can you say about the angle sum of a convex polygon with number of sides?

Sol. (d) n

$$\text{Angle sum of a convex polygon} = (n - 2) \times 180^\circ$$

Angle sum of a convex polygon = $(n - 2) \times 180^\circ$

Where 'n' is number of sides



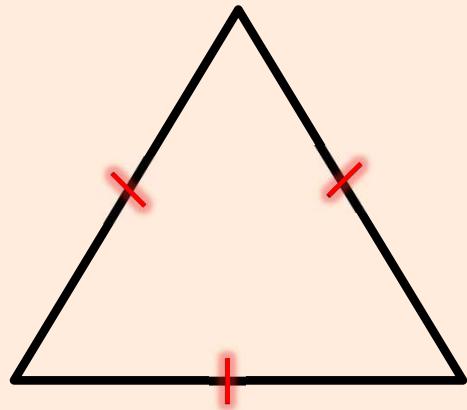
Q. What is a regular polygon? State the name of a regular polygon of:

(i) 3 sides

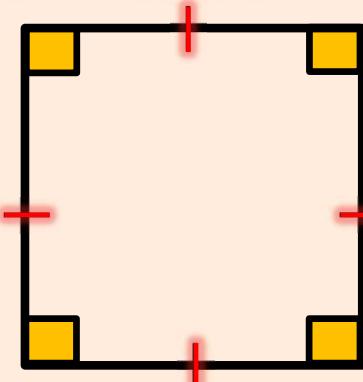
(ii) 4 sides

(iii) 6 sides

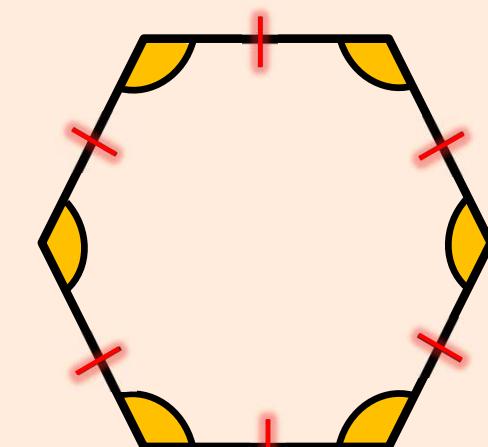
Sol. A polygon having all sides of equal length and the interior angles of equal size is known as **regular polygon**.



Regular Polygon having three sides is called a **Equilateral triangle**.



Regular Polygon having four sides is called a **Square**



Regular Polygon having six sides is called a **hexagon**.

Module 5

Exercise

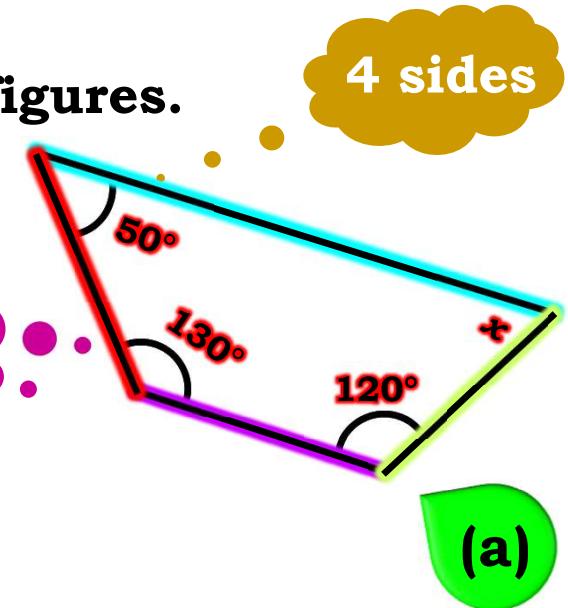
Q. Find the angle measure x in the following figures.

Sol. The sum of the interior angles of a quadrilateral is 360°

Quadrilateral

$$\begin{aligned}50^\circ + 130^\circ + 120^\circ + x &= 360^\circ \\300^\circ + x &= 360^\circ \\x &= 360^\circ - 300^\circ\end{aligned}$$

$x = 60^\circ$



The sum of the interior angles of a quadrilateral is 360°



Q. Find the angle measure x in the following figure.

Sol. The sum of the interior angles of a quadrilateral is 360° .

$$x + 70^\circ + 60^\circ + 90^\circ = 360^\circ$$

$$x + 220^\circ = 360^\circ$$

$$x = 360^\circ - 220^\circ$$

$$\boxed{x = 140^\circ}$$

The sum of the interior angles of a quadrilateral is 360°



Exercise

Fourth angle is forming linear pair with 90°

4 sides



(b)

$$180^\circ - 90^\circ = 90^\circ$$

Module 6

Exercise

Q. Find the angle measure x in the following figures.

Sol. The sum of the interior angles of a pentagon is 540°

Pent

$$\therefore 180^\circ - 70^\circ = 110^\circ$$

$$180^\circ - 60^\circ = 120^\circ$$

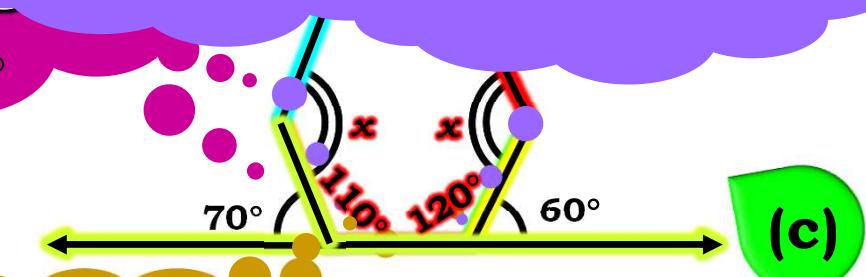
$$30^\circ + x + x + 120^\circ + 110^\circ = 540^\circ$$

$$2x + 260^\circ = 540^\circ$$

$$2x = 540^\circ - 260^\circ$$

$$x = \frac{280^\circ}{2} = 140^\circ$$

$x = 140^\circ$ Angle from linear pair with 70°



(c)

The sum of the interior angles of a pentagon is 540°



Exercise

Q. Find the angle measure x in the following figures.

Sol. The sum of the interior angles of a pentagon is 540°

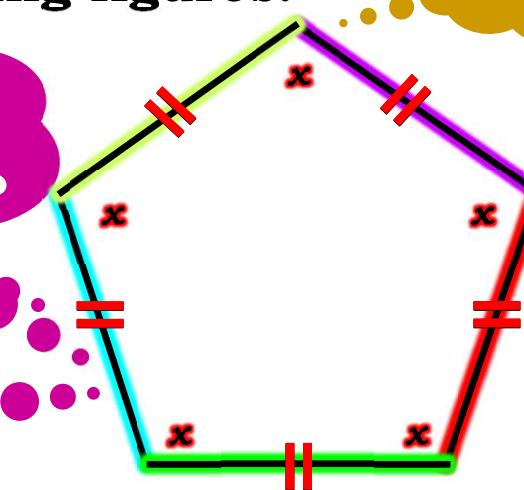
It means all sides are equal ?

$$\underline{x + x + x + x + x} = 540^\circ$$

$$5x = 540^\circ$$

$$x = \frac{540^\circ}{5} = 108^\circ$$

$$x = 108^\circ$$



(d)

∴ It is a regular Pentagon

In regular polygon all sides and **angles** are **equal**

The sum of the interior angles of a pentagon is 540°



Module 7

The sum of the interior opposite angles is equal to the exterior angle



Sol. $\therefore 90^\circ + x = 180^\circ$ [Linear pair]

$$x = 180^\circ - 90^\circ$$

$x = 90^\circ$

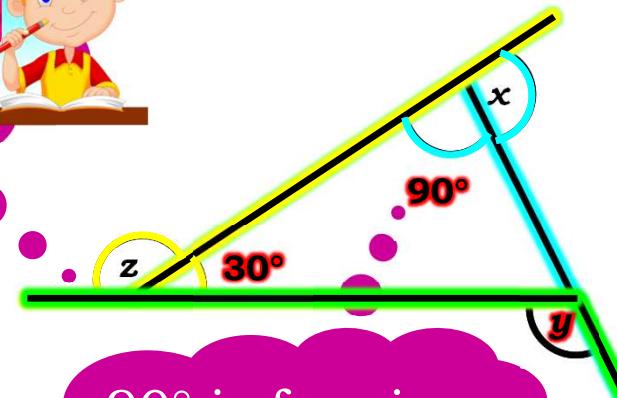
$$\therefore z + 30^\circ = 180^\circ$$
 [Linear pair]

$$z = 180^\circ - 30^\circ$$

$z = 150^\circ$

$$30^\circ + 90^\circ = y$$

$y = 120^\circ$



90° is forming linear pair with x

$$\therefore x + y + z$$

$$90^\circ + 120^\circ + 150^\circ$$

360°

(a)



Q. Find $x + y + z + w$

Sol. The sum of the interior angles of a quadrilateral is 360°

$$60^\circ + 80^\circ + 120^\circ + \angle 1 = 360^\circ$$

$$\angle 1 + 260^\circ = 360^\circ$$

$$\angle 1 = 360^\circ - 260^\circ = 100^\circ$$

$$\therefore 120^\circ + x = 180^\circ \quad [\text{Linear pair}]$$

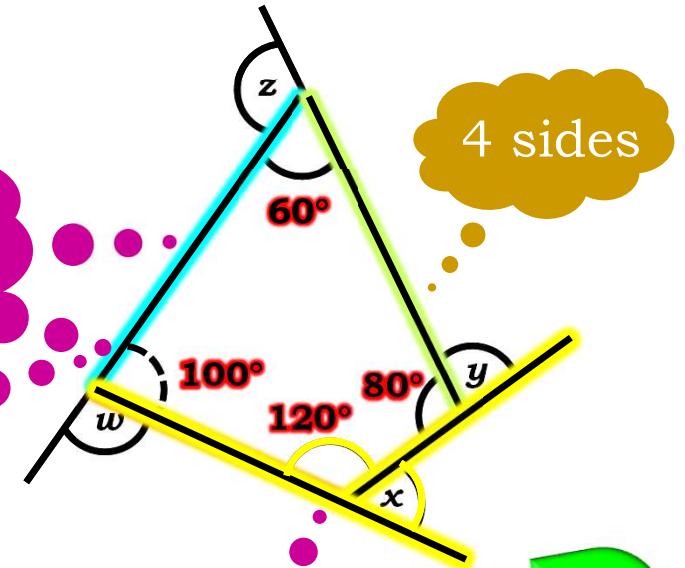
$$x = 180^\circ - 120^\circ$$

The sum of the interior angles of a quadrilateral is 360°



Exercise

\therefore It is an **Quadrilateral**



(b)

120° is forming linear pair with x

Q. Find $x + y + z + w$

Sol. $\therefore 80^\circ + y = 180^\circ$

$$y = 180^\circ - 80^\circ$$

$$y = 100^\circ$$

$$\therefore 60^\circ + z = 180^\circ \quad [\text{Linear pair}]$$

$$z = 180^\circ - 60^\circ$$

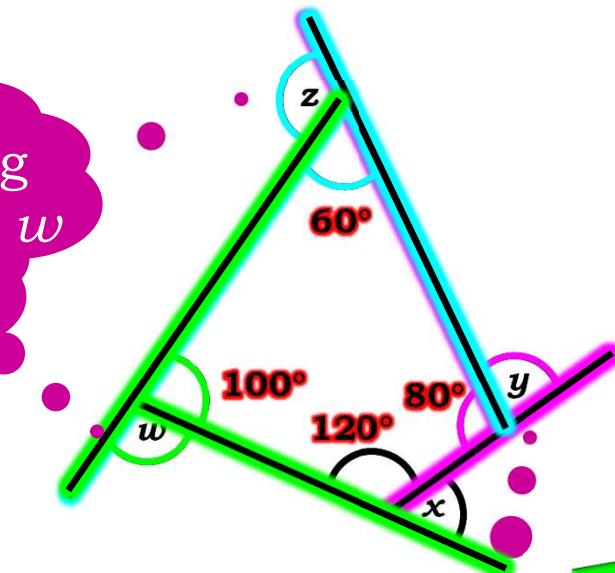
$$z = 120^\circ$$

$$\therefore 100^\circ + w = 180^\circ \quad [\text{Linear pair}]$$

$$w = 180^\circ - 100^\circ$$

Exercise

100° is forming
linear pair with w
linear pair with z



$$\therefore x + y + z + w$$

$$60^\circ + 100^\circ + 120^\circ + 20^\circ$$

80° is forming
linear pair with y

$$360^\circ$$



Lecture 2

Module 8

Exercise

Q. The three angles of a quadrilateral are 76° , 54° and 108° . Find the measure of the fourth angle.

Sol:

Let the fourth angle be x

The sum of measure of angles of a quadrilateral is 360°

$$76^\circ + 54^\circ + 108^\circ + x = 360^\circ$$

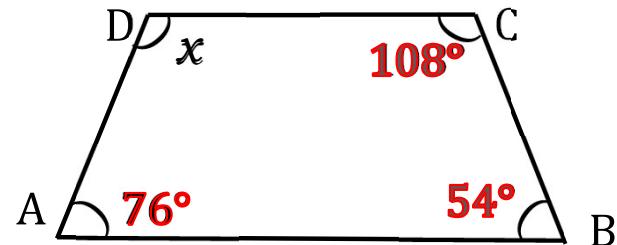
$$238^\circ + x = 360^\circ$$

$$x = 360^\circ - 238^\circ$$

$$\boxed{x = 122^\circ}$$

\therefore The fourth angle is 122°

The sum of measure of angles of a quadrilateral is 360°



Exercise

Q. The angles of a quadrilateral are in the ratio 3 : 5 : 7 : 9. Find the measure of each of these angles.

Sol: Let the measures of angles of the given quadrilateral be $(3x)^0$, $(5x)^0$, $(7x)^0$, $(9x)^0$.

$$(3x) + (5x) + (7x) + (9x) = 360^\circ$$

$$(24x) = 360^\circ$$

$$\therefore x = \frac{360}{24} = 15$$

$$x = 15^\circ$$



The sum of measure of angles of a quadrilateral is 360°



Exercise

Q. The angles of a quadrilateral are in the ratio $3 : 5 : 7 : 9$. Find the measure of each of these angles.

Sol: So, the measure of angles of the given quadrilateral are

$$\angle A = 3x$$

$$\angle A = 3 \times 15$$

$$\angle A = 45^\circ$$

$$\angle C = 7x$$

$$\angle C = 7 \times 15$$

$$\angle C = 105^\circ$$

$$\angle B = 5x$$

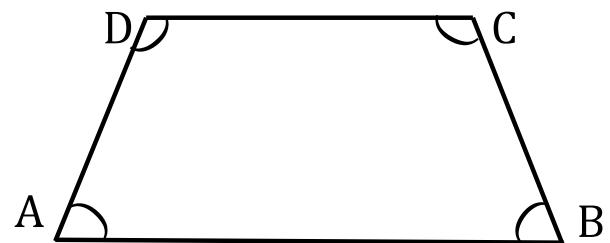
$$\angle B = 5 \times 15$$

$$\angle B = 75^\circ$$

$$\angle D = 9x$$

$$\angle D = 9 \times 15$$

$$\angle D = 135^\circ$$



i.e., $45^\circ, 75^\circ, 105^\circ$ and 135° .

Exercise

Q. A quadrilateral has three acute angles, each measuring 75° . Find the measure of the fourth angle.

Sol:

Let the measures of angles of the given quadrilateral be x°

$$\begin{array}{rcl} 75^\circ + 75^\circ + 75^\circ + x & = & 360^\circ \\ \hline 225^\circ + x & = & 360^\circ \end{array}$$

$$x = 360^\circ - 225^\circ$$

$$x = 135^\circ$$

\therefore The fourth angle is 135°



The sum of measure of angles of a quadrilateral is 360°



Module 9

**Q. The measures of two adjacent angles of a quadrilateral are 125° and 35° and the other two angles are equal.
Find the measure of each of the equal angles.**

Sol : Let ABCD be the quadrilateral such that

$$\angle B = 125^\circ, \angle C = 35^\circ \text{ and } \angle A = \angle D$$

By angle sum property of a quadrilateral, we have

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

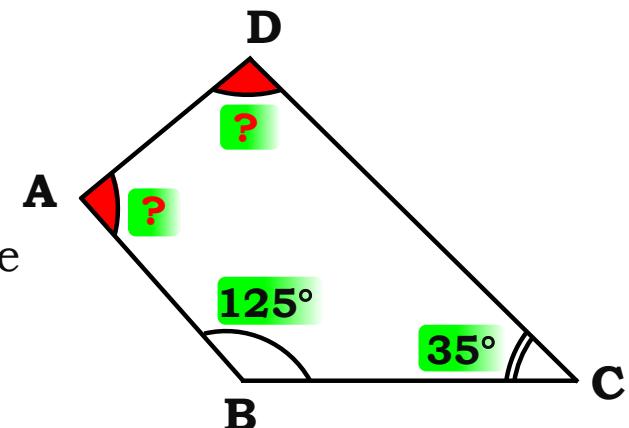
$$\therefore \angle A + 125^\circ + 35^\circ + \angle A = 360^\circ \quad [\because \angle A = \angle D]$$

$$\therefore 160^\circ + 2 \angle A = 360^\circ$$

$$\therefore 2 \angle A = 360^\circ - 160^\circ$$

$$\therefore 2 \angle A = 200^\circ$$

$$\therefore \angle A = \left(\frac{200}{2}\right)^\circ = 100^\circ$$



The sum of the interior angles of a quadrilateral is 360°



Q. ABCDE is a regular pentagon. The bisector of $\angle A$ of the pentagon meets the side CD in M. Show that $\angle AMC = 90^\circ$.

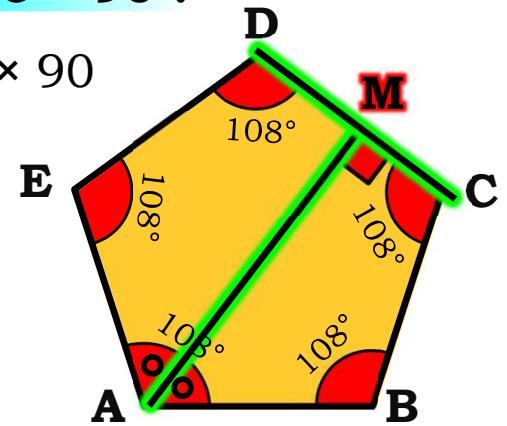
Sol : Sum of interior angles of regular polygon = $(2n - 4) \times 90$

$$\begin{aligned}\text{Measure of each interior angle} &= \frac{2n - 4}{n} \times 90 \\ &= \frac{(2 \times 5) - 4}{5} \times 90 \\ &= \frac{10 - 4}{5} \times 90\end{aligned}$$

Pentagon has 5 sides

$$\begin{aligned}&= \frac{6}{5} \times 18 \\ &= 6 \times 18\end{aligned}$$

$$\therefore \text{Measure of each interior angle} = 108^\circ$$



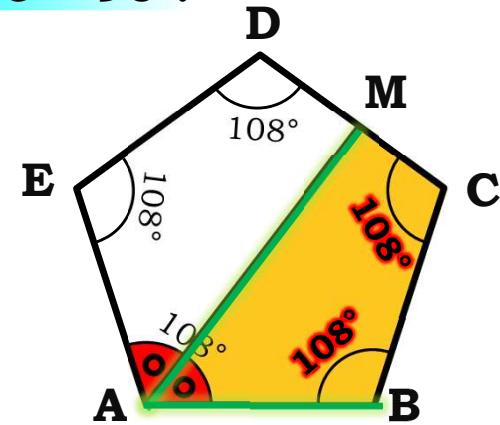
Q. ABCDE is a regular pentagon. The bisector of $\angle A$ of the pentagon meets the side CD in M. Show that $\angle AMC = 90^\circ$.

Sol : $\angle BAE = 108^\circ$

$$\begin{aligned}\angle MAB &= \frac{1}{2} \times \angle BAE \\ &= \frac{1}{2} \times 108\end{aligned}$$

$$\therefore \angle MAB = 54^\circ$$

Consider $\square ABCM$



In $\square ABCM$,

$$\angle MAB + \angle ABC + \angle BCM + \angle AMC = 360^\circ$$

$$\therefore \underline{54} + 108 + 108 + \angle AMC = 360$$

$$\therefore 270 + \angle AMC = 360$$

The sum of the interior angles of a quadrilateral is 360°



Module

10



FORMULAE

Sum of interior angles = $(n - 2) \times 180$ or $(2n - 4) \times 90$

Measure of each interior angle of regular polygon = $\frac{(n - 2) \times 180}{n}$ or $\frac{(2n - 4) \times 90}{n}$

Sum of exterior angles = 360°

For 'n' sided regular polygon :

Measure of each exterior angle = $\frac{360}{n}$

When interior angle is given :

Measure of each exterior angle = $180^\circ -$ Measure of each interior angle

Q. How many sides does a regular polygon have if each of its interior angles is 165° ?

Sol. Measure of an interior angle = 165°

Let, the number of sides be n

$$\text{Exterior angle} = 180 - \text{interior angle}$$

$$= 180 - 165$$

$$\therefore \text{Exterior angle} = 15^\circ$$

$$\text{Number of side} = \frac{360^\circ}{\text{Each exterior angle}}$$

Q. How many sides does a regular polygon have if each of its interior angles is 165° ?

Sol. Let, the number of sides be n

$$\text{Exterior angle} = 15$$

$$\text{Number of side} = \frac{360^\circ}{\text{Each exterior angle}}$$

$$\text{Number of side} = \frac{360}{\text{Each exterior angle}}$$

$$= \frac{\cancel{360}^{24}}{\cancel{15}_1}$$

$$\therefore \text{Number of side} = 24$$

Q. How many sides has a regular polygon, each angle of which is of measure 108° ?

Sol. Measure of an interior angle = 108°

Let, the number of sides be n

$$\text{Exterior angle} = 180 - \text{interior angle}$$

$$= 180 - 108$$

$$\therefore \text{Exterior angle} = 72^\circ$$

$$\text{Number of side} = \frac{360^\circ}{\text{Each exterior angle}}$$

Q. How many sides has a regular polygon, each angle of which is of measure 108° ?

Sol. Let, the number of sides be n

$$\text{Exterior angle} = 72$$

$$\text{Number of side} = \frac{360^\circ}{\text{Each exterior angle}}$$

$$\text{Number of side} = \frac{360}{\text{Each exterior angle}}$$

$$= \frac{\cancel{360}^5}{\cancel{72}^1}$$

$$\therefore \text{Number of side} = 5$$

Module

11

Q. Is it possible to have a regular polygon with measure of each exterior angle as 22° ?

Sol.

$$\text{Exterior angle of regular polygon} = \frac{360}{\text{Number of sides}}$$

$$22 = \frac{360}{\text{Number of sides}}$$

$$\therefore \text{Number of sides} = \frac{360}{22}$$

$$\therefore \text{Number of sides} = 16.3636$$

Side cannot be decimal or fraction so it is not possible to have a regular polygon with measure of each interior angle as 22°

Q. Is it possible to have a regular polygon with measure of each interior angle as 22° ?

Sol.

$$\text{Interior angle of regular polygon} = \frac{2n - 4}{n} \times 90$$

$$22 = \frac{2n - 4}{n} \times 90$$

$$\therefore 22n = 90 \times (2n - 4)$$

$$\therefore 22n = 180n - 360$$

$$\therefore 180n - 22n = 360$$

$$\therefore 158n = 360$$

$$\therefore \text{Number of sides} = \frac{360}{158}$$

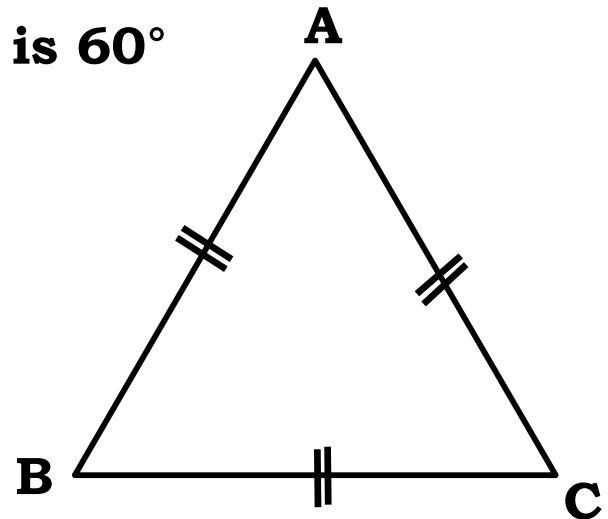
$$\therefore \text{Number of sides} = 2.27$$

Side cannot be decimal or fraction so it is not possible to have a regular polygon with measure of each interior angle as 22°

Q. What is the minimum interior angles possible for a regular polygon? Why?

Sol. Interior angle of an Equilateral triangle is 60°

\therefore Minimum interior angle possible for a regular polygon is 60°



A regular polygon with least number of sides is an Equilateral triangle



Q. What is the maximum exterior angle possible for a regular polygon?

Sol. **Minimum interior angle for a regular polygon = 60°**

Maximum exterior angle for a regular polygon

$$= 180^\circ - \text{Minimum interior angle}$$

$$= 180 - 60$$

$$= 120^\circ$$

∴ Maximum exterior angle possible for a regular polygon is 120°

Module

12

Exercise

Q. Find x in the following figures.

Sol. The sum of the measures of the exterior angles of any polygon is 360° .

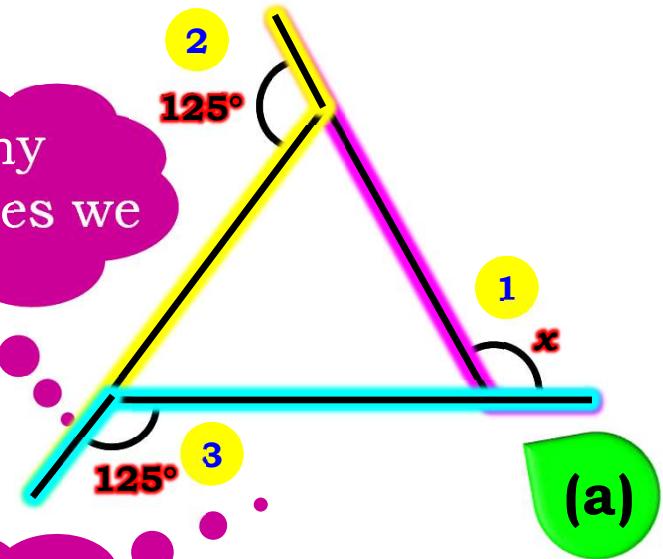
$$x + 125^\circ + 125^\circ = 360^\circ$$

$$x + 250^\circ = 360^\circ$$

$$x = 360^\circ - 250^\circ$$

$$x = 110^\circ$$

Three exterior angles



(a)

The sum of the measures of the exterior angles of any polygon is 360°



Q. Find x in the following figures

Sol. The sum of the measures of the exterior angles of any polygon is 360°

$$70^\circ + 90^\circ + 60^\circ + 90^\circ + x = 360^\circ$$

$$310^\circ + x = 360^\circ$$

$$x = 360^\circ - 310^\circ$$

$$x = 50^\circ$$

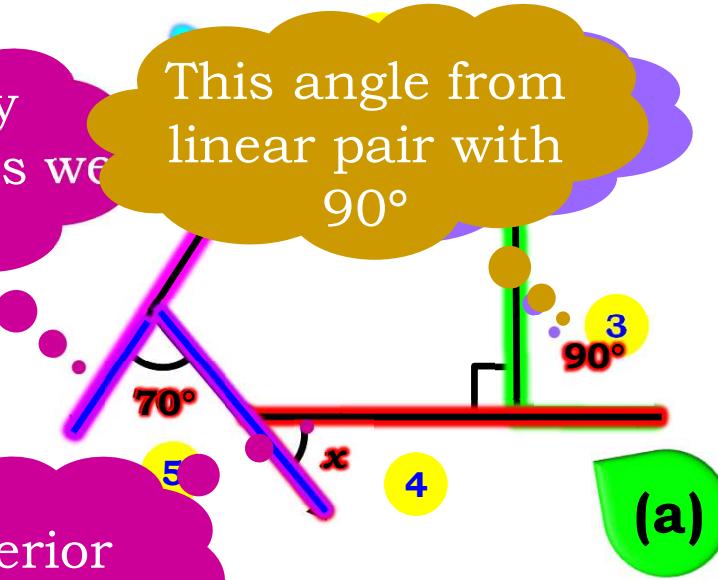
Five exterior angles



Exercise

How many

This angle from linear pair with 90°



(a)

The sum of the measures of the exterior angles of any polygon is 360°



Exercise

Find measure of each exterior angle of regular polygon of

i. **9 Sides**

Sol. Number of sides = 9

$$\therefore \text{Measure of an exterior angles} = \frac{40}{9 - 1}$$

Measure of
each exterior=

$\frac{360^\circ}{\text{Number}}$

of sides

Measure of each exterior angle = 40°

Exercise

**Q. Find measure of each exterior angle of regular polygon of
ii. 15 Sides**

Sol. Number of sides = 15

$$\therefore \text{Measure of an exterior angles} = \frac{24}{15 - 1}$$

Measure of
each exterior =
angle

$$\frac{360^\circ}{\text{Number of sides}}$$

Measure of each exterior angle = 24°

Exercise

Q. How many sides does a regular polygon have if the measure of an exterior angle is 24° ?

Sol.

Measure of an exterior angle = 24°

Let, the number of sides be x



$$24 = \frac{360}{x}$$
$$\therefore x = \frac{360}{24} \quad 15$$

Measure of each exterior angle = $\frac{360^\circ}{\text{Number of sides}}$

Number of sides = 15

Module

13

Q. Prove that the interior angle of a regular pentagon is three times the exterior angle of a regular decagon

To prove : Interior angle of a regular pentagon = $3 \times$ Exterior angle of a regular decagon

Proof : Interior angle of a regular pentagon = $\frac{2n - 4}{n} \times 90$

$$\begin{aligned}
 &= \frac{(2 \times 5) - 4}{5} \times 90 \\
 &= \frac{10 - 4}{5} \times 90 \\
 &= \frac{6}{5} \times 90 \\
 &= 6 \times 18
 \end{aligned}$$

Pentagon has 5 sides

\therefore Interior angle of a regular pentagon = $108^\circ \dots (i)$

Measure of each interior angle = $\frac{2n - 4}{n} \times 90$

Where 'n' sides of number

Measure of each exterior angle = $\frac{360^\circ}{n}$

Where 'n' sides of number

Q. Prove that the interior angle of a regular pentagon is three times the exterior angle of a regular decagon

To prove : Interior angle of a regular pentagon = $3 \times$ Exterior angle of a regular decagon

Measure of each exterior angle = $\frac{360^\circ}{n}$

Proof : Interior angle of a regular pentagon = 108° ... (i)

Where 'n' sides of number

$$\begin{aligned}\text{Exterior angle of a regular decagon} &= \frac{360^\circ}{n} \\ &= \frac{360^\circ}{10} \\ &= 36^\circ\end{aligned}$$

Decagon has 10 sides

$$\therefore 3 \times \text{Exterior angle of a regular decagon} = 3 \times 36$$

$$\therefore 3 \times \text{Exterior angle of a regular decagon} = 108^\circ \quad \dots \text{(ii)}$$

Hence, interior angle of a regular pentagon is three times the exterior angle of a regular decagon

Q. The exterior angle of a regular polygon is one-third of its interior angle. How many sides has the polygon ?

Sol : $\text{Exterior angle} = \frac{1}{3} (\text{Interior angle})$

$\therefore \frac{360}{n} = \frac{1}{3} \times \left(\frac{2n - 4}{n} \times 90 \right)$

$\therefore \frac{360}{n} = 30 \left(\frac{2n - 4}{n} \right)$

$\therefore \frac{360}{n} = 60 \left(\frac{n - 2}{n} \right)$

$\therefore 360 = 60(n - 2)$

Let there be n sides
of the polygon.

$\therefore n = 8$

The polygon has 8 sides.

Measure of each exterior angle $= \frac{360^\circ}{n}$

Where 'n' sides of number

Measure of each interior angle $= \frac{2n - 4}{n} \times 90$

Where 'n' sides of number

Lecture 3

Module

14

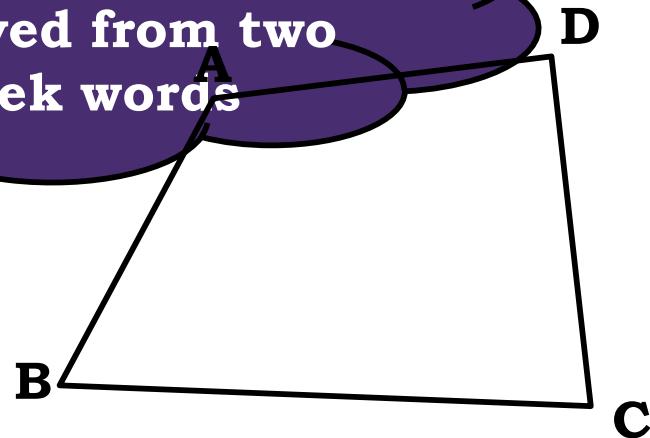
QUADRILATERALS

⇒ FOUR

⇒ SIDES

Definition : Any four sided closed figure is called a Quadrilateral

The word QUADRILATERAL
is derived from two
Greek words



ANGLE SUM PROPERTY OF A QUADRILATERAL

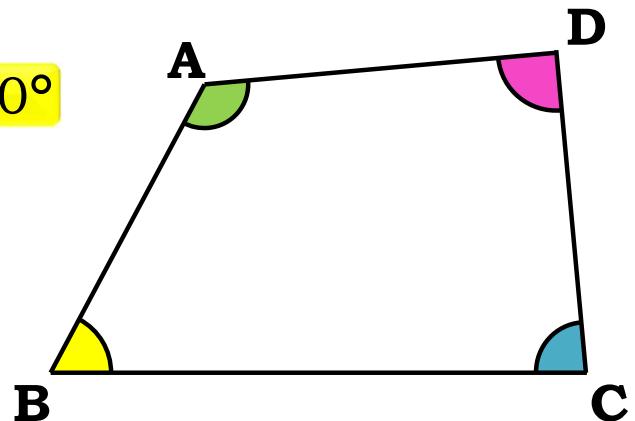
❖ Statement:

Sum of all the angles of a quadrilateral is 360°

In $\square ABCD$,

$\angle A + \angle B +$

Name all the angles
 $\angle A, \angle B, \angle C$ & $\angle D$
of $\square ABCD$



Types of Quadrilaterals

➤ **PARALLELOGRAM**

➤ **RECTANGLE**

➤ **RHOMBUS**

➤ **SQUARE**

➤ **TRAPEZIUM**

➤ **KITE**

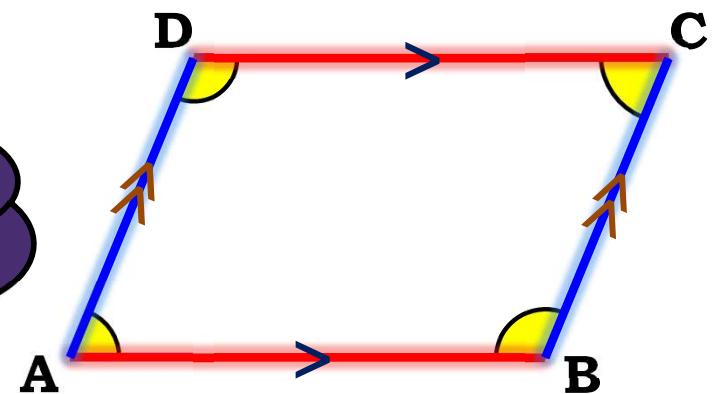
➤ PARALLELOGRAM

Definition : A quadrilateral is called a **Parallelogram** if its opposite sides are parallel.

□ABCD is a Parallelogram

∴ AB || CD and AD || BC

Similarly, all
Sides are parallel.
 $\angle A + \angle B = 180^\circ$
and $\angle C + \angle D = 180^\circ$



Interior angles
Adjacent angles of a parallelogram are supplementary

PROPERTIES OF A PARALLELOGRAM

- Opposite sides are equal

$$AB = CD$$

$$AD = BC$$

Name the pairs
of opposite sides

- Opposite angles are equal

$$\angle A = \angle C$$

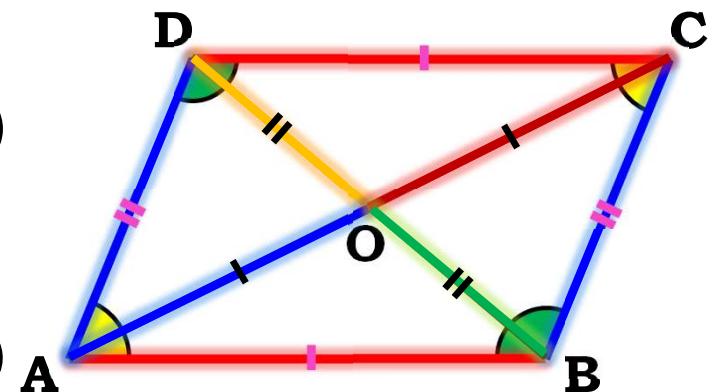
$$\angle B = \angle D$$

Name the pairs
of opposite angles

- Diagonals bisect each other

$$OA = OC$$

$$OB = OD$$



$OA = OC$ &
 $OB = OD$

Module

15

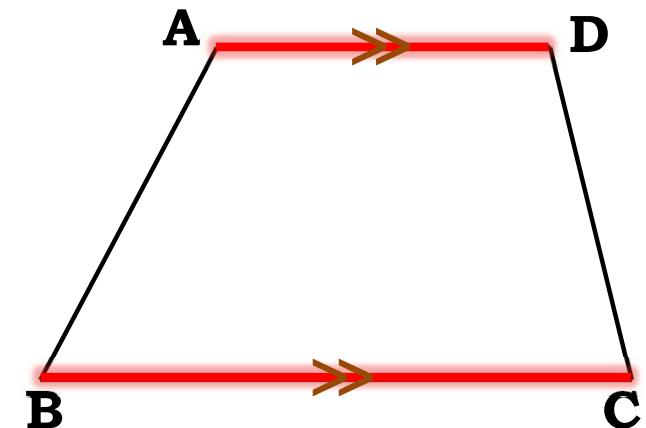
➤ TRAPEZIUM

Definition : A quadrilateral is said to be a trapezium, if only one pair of opposite sides is parallel.

$$AD \parallel BC$$

□ABCD is a Trapezium

$$AD \parallel BC$$



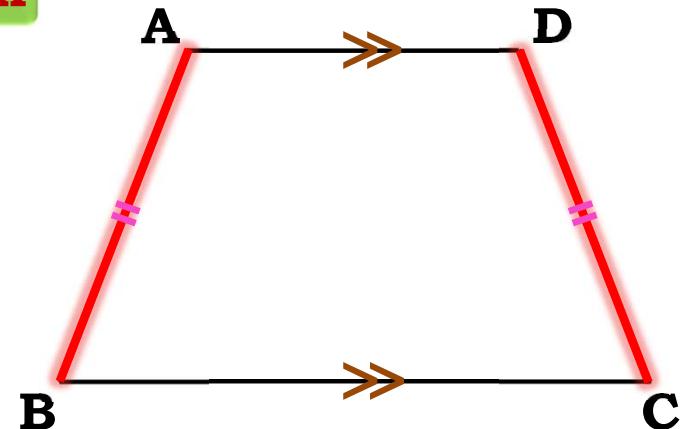
➤ ISOSCELES TRAPEZIUM

Definition : A Trapezium in which the non-parallel sides are equal, is called an **isosceles trapezium**

=

Which are
AB & **DC**
non-parallel sides ?

□ABCD is an Isosceles Trapeziun



➤ KITE

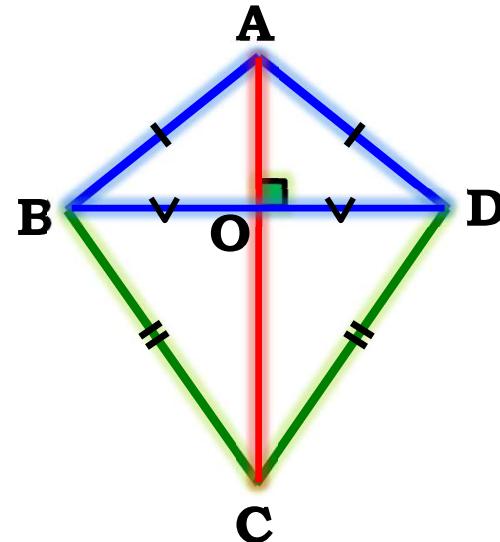
In $\square ABCD$,

$$AB = AD$$

$$BC = DC$$

and $AB \neq BC$

$\therefore \square ABCD$ is a Kite



PROPERTY OF A KITE

➤ Longer diagonal is the Perpendicular Bisector of the shorter diagonal.

$$AC \perp BD$$

$$OB = OD$$

Module

16

Q. Given a parallelogram ABCD. Complete each statement along with the property used.

Sol.

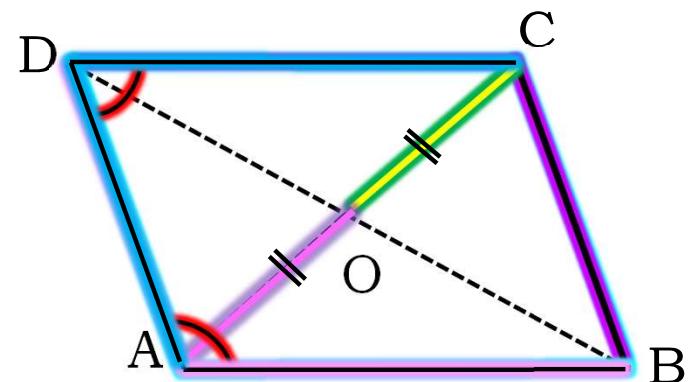
- i. $AD =$ **BC**
- ii. $\angle DCB =$ **$\angle DAB$**
- iii. $OC =$ **OA**
- iv. $m \angle DAB + m \angle CDA =$ **180°**

Opposite angle of $\angle DCB$? $\angle D?$

$\angle DAB$

- Opposite sides are equal
- Opposite angles are equal
- Divide in two equal parts
- Adjacent angles are supplementary
- Diagonal bisects each other

180°



Q. Consider the following parallelogram. Find the values of the unknowns x , y , z .

Sol.

$$\angle D = \angle B$$

[Opposite angles are equal]

$$\therefore y = 100$$

$$\therefore \angle B + \angle C = 180^\circ \quad [\text{Adjacent angles are supplementary}]$$

$$\therefore 100^\circ + x = 180^\circ$$

$$\therefore x = 180 - 100$$

$$\therefore x = 80$$

Name the adjacent angle of $\angle B$

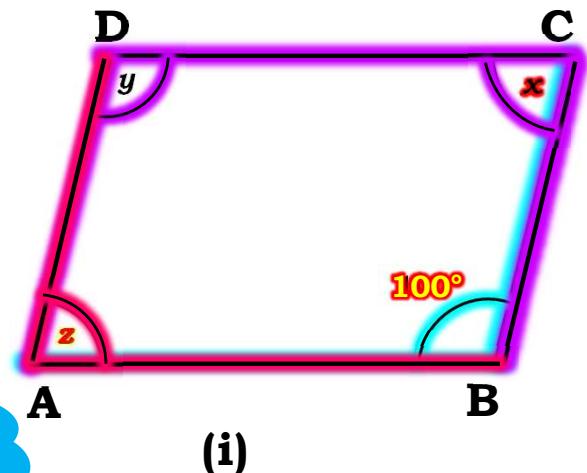
$$\therefore \angle A = \angle C$$

➤ Opposite angles are equal

$$\therefore z = 80$$

➤ Adjacent angles are Supplementary

$$180^\circ$$



(i)

Name the angle opposite to $\angle C$

Q. Consider the following parallelogram. Find the values of the unknowns x , y , z .

Sol. $\therefore \angle A + \angle B = 180^\circ$ [Adjacent angles are supplementary]

$$\therefore 50 + x = 180$$

$$\therefore x = 180 - 50$$

$$\therefore x = 130$$

$$\therefore \angle D = \angle B \quad (\text{Opposite angles are equal})$$

$$\therefore y = 130$$

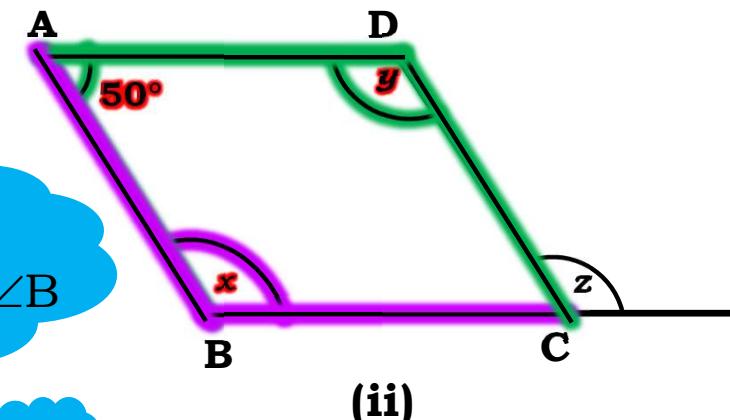
Name the angle opposite to angle $\angle B$

$\angle D$

180°

➤ Opposite angles are equal

➤ Adjacent angles are Supplementary



(ii)

Q. Consider the following parallelogram. Find the values of the unknowns x , y , z .

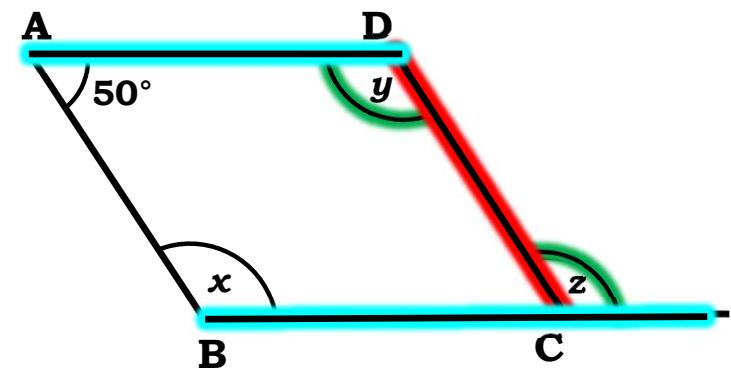
Sol. $\therefore x = 130, y = 130$

$\therefore AD \parallel BC$ and DC is transversal

$\therefore y = z$ [Alternate angles]

$\therefore z = 130$

$\therefore x = 130, y = 130$ and $z = 130$



Module

17

Q. Consider the following parallelogram. Find the values of the unknowns x , y , z .

Sol. $\angle AOD = \angle BOC$ [Vertically opposite angles]

$$\therefore x = 90$$

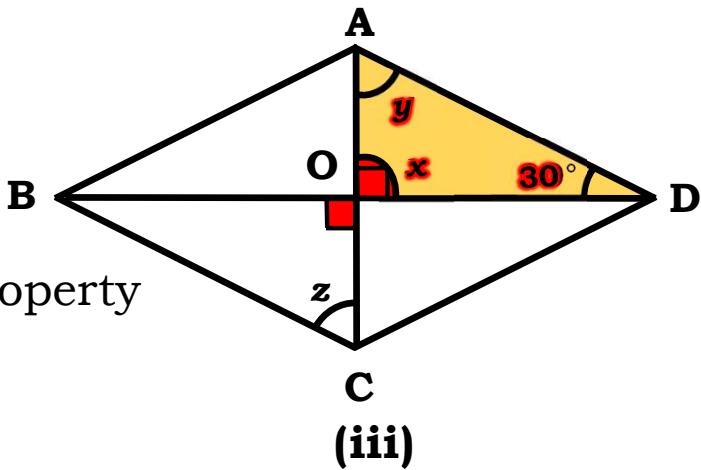
Consider $\triangle AOD$

In $\triangle AOD$,

$\angle OAD + \angle AOD + \angle ADO = 180^\circ$ [Angle sum property of a triangle]

$$\begin{aligned} y + 90 + 30 &= 180^\circ \\ y + 120 &= 180 \\ y &= 180 - 120 \end{aligned}$$

$$\therefore y = 60$$



The sum of the measures of all angles of a triangle is 180°



Q. Consider the following parallelogram. Find the values of the unknowns x , y , z .

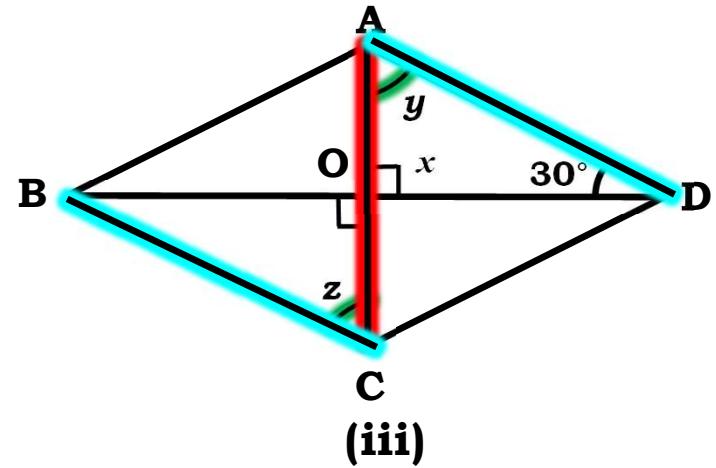
Sol. $\therefore x = 90, y = 60$

$\therefore AD \parallel BC$ and AC is transversal

$\therefore y = z$ [Alternate angles]

$\therefore z = 60$

$\therefore x = 90, y = 60$ and $z = 60$



Q. Consider the following parallelogram. Find the values of the unknowns x , y , z .

Sol. $\therefore \angle P = \angle R$ (Opposite angles are equal)

$\therefore y = 80$

$\therefore \angle Q + \angle R = 180^\circ$ [Adjacent angles are supplementary]

$\therefore x + 80 = 180$

$\therefore x = 180 - 80$

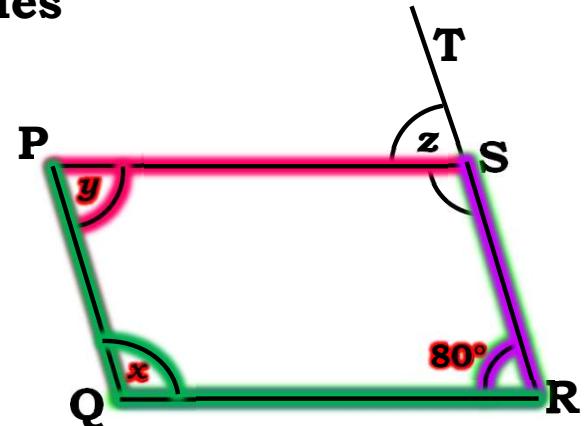
$\therefore x = 100$

Name the adjacent angle of $\angle R$

- Opposite angles are equal

- Adjacent angles are Supplementary

180°



(iv)

Q. Consider the following parallelogram. Find the values of the unknowns x , y , z .

Sol.

$$\therefore x = 100$$

$$\angle S = \angle Q$$

$$\therefore \angle S = 100$$

$\angle PSR$ & $\angle PST$ forms linear pair

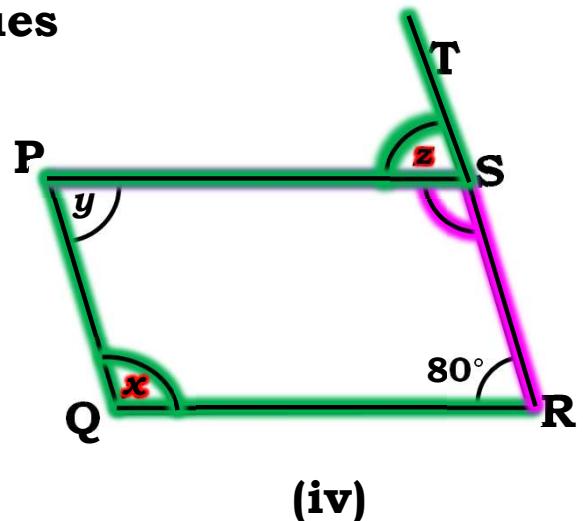
$\angle S$

$$\angle PSR + \angle PST = 180^\circ$$

$$100 + z = 180$$

$$z = 180 - 100$$

$$z = 80$$



Module

18

Q. Consider the following parallelogram. Find the values of the unknowns x , y , z .

Sol. $\angle A = \angle D$ (Opposite angles are equal)

$$\therefore y = 112$$

$\angle A$

In $\triangle ACB$,

$$\angle ABC + \angle CAB + \angle ACB = 180^\circ$$

$$x + 112 + 40 = 180$$

$$x + 152 = 180$$

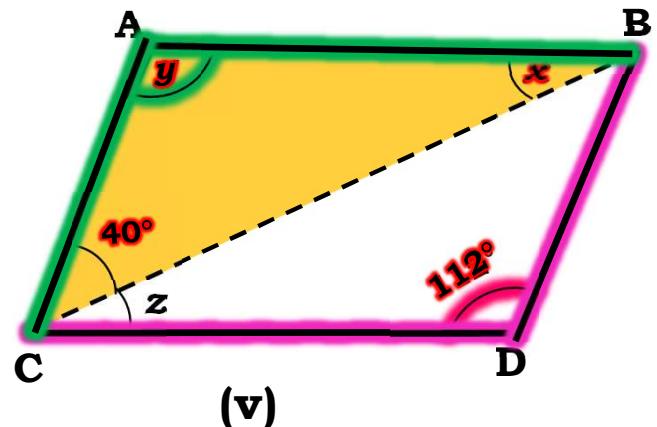
$$x = 180 - 152$$

$$\therefore x = 28$$

Name the angle opposite to $\angle D$

➤ Opposite angles are equal

Measures of all angles is 180°



Q. Consider the following parallelogram. Find the values of the unknowns x , y , z .

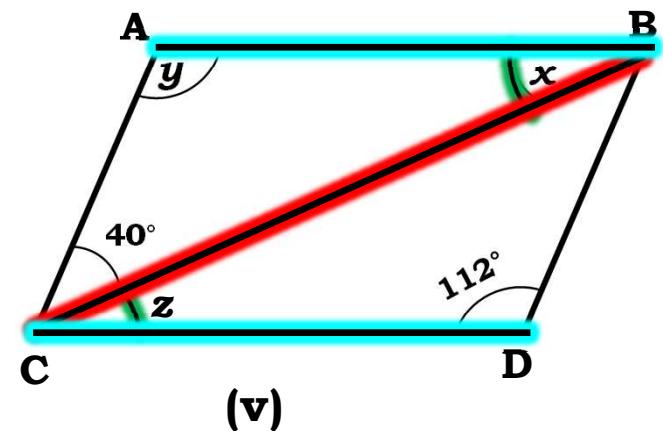
Sol. $\therefore x = 28, y = 112$

$\therefore AB \parallel CD$ and BC is transversal

$\therefore x = z$ [Alternate angles]

$\therefore z = 28$

$\therefore x = 28, y = 112$ and $z = 28$



Q. The measures of two adjacent angles of a parallelogram are in the ratio 3:2. Find the measure of each of the angles of the parallelogram.

Sol. Let ABCD be a parallelogram.

Let, $\angle A = 3x$, $\angle B = 2x$

$$\therefore \angle A + \angle B = 180^\circ$$

$$\therefore \underline{3x + 2x} = 180$$

$$\therefore 5x = 180$$

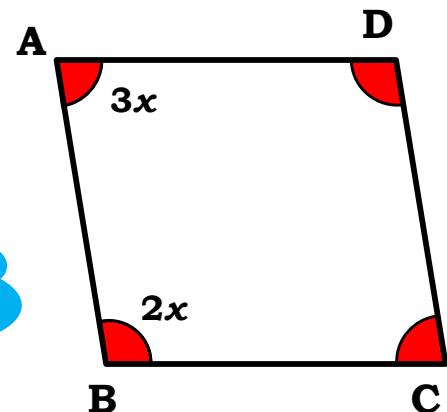
$$\therefore x = \frac{\cancel{180}}{\cancel{5}} \overset{36}{1}$$

$$\therefore x = 36$$

$$\therefore \angle A = 3x = 3 \times 36 = 108^\circ$$

$$\therefore \angle B = 2x = 2 \times 36 = 72^\circ$$

Adjacent angles are supplementary



Q. The measures of two adjacent angles of a parallelogram are in the ratio 3:2. Find the measure of each of the angles of the parallelogram.

Sol. $\therefore \angle A = 108^\circ$

$$\therefore \angle B = 72^\circ$$

$$\angle A = \angle C$$

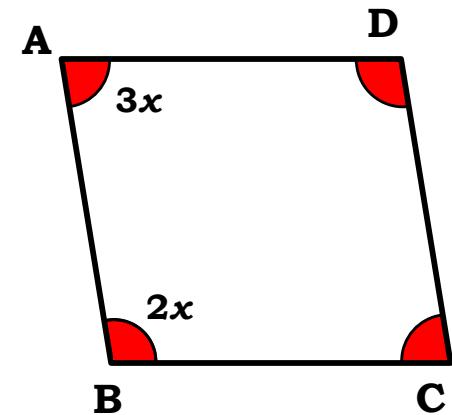
$$\therefore \angle C = 108^\circ$$

$$\angle B = \angle D$$

$$\therefore \angle D = 72^\circ$$

$$\therefore \angle A = 108^\circ, \angle B = 72^\circ, \angle C = 108^\circ \text{ and } \angle D = 72^\circ$$

Opposite angles
are equal



Module

19

Q. Can a quadrilateral ABCD be a parallelogram if

~~(i) $\angle D + \angle B = 180^\circ$?~~

(ii) $AB = DC = 8 \text{ cm}$, $AD = 4 \text{ cm}$ and $BC = 4.4 \text{ cm}$?

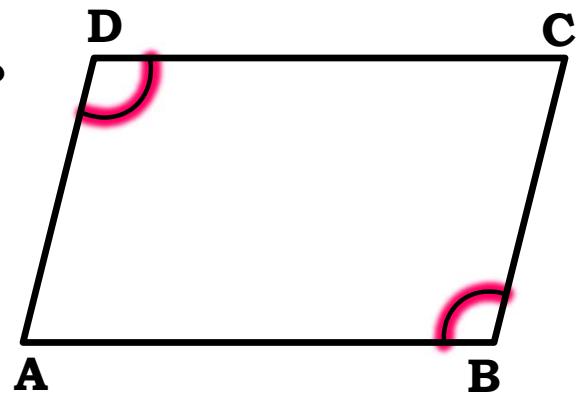
(iii) $\angle A = 70^\circ$ and $\angle C = 65^\circ$?

Sol. $\angle D$ and $\angle B$ are opposite angles

But, in a parallelogram adjacent angles
are supplementary

\therefore If $\angle D + \angle B = 180^\circ$

then $\square ABCD$ is not a parallelogram.



- Opposite sides are equal
- Opposite angles are equal
- Opposite sides are parallel

But, according to given condition $\angle B$ and $\angle D$ are supplementary

Q. Can a quadrilateral ABCD be a parallelogram if

~~(ii) $\angle D + \angle B = 180^\circ$?~~

~~(i) $AB = DC = 8 \text{ cm}$, $AD = 4 \text{ cm}$ and $BC = 4.4 \text{ cm}$?~~

~~(iii) $\angle A = 70^\circ$ and $\angle C = 65^\circ$?~~

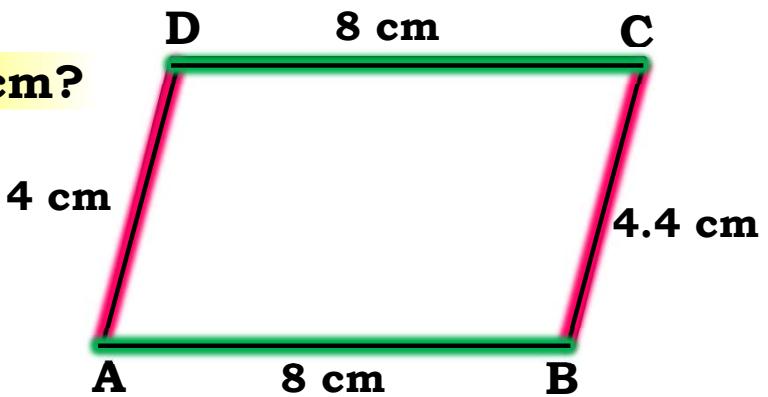
Sol. $AB = DC$

$AD \neq BC$

\therefore According to given condition

$\square ABCD$ can not be a parallelogram.

- Opposite sides are equal
- Opposite angles are equal
- Opposite sides are parallel



Q. Can a quadrilateral ABCD be a parallelogram if

~~(i) $\angle D + \angle B = 180^\circ$?~~

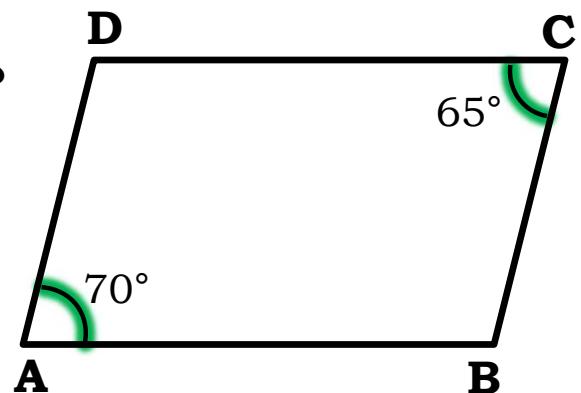
~~(ii) $AB = DC = 8 \text{ cm}$, $AD = 4 \text{ cm}$ and $BC = 4.4 \text{ cm}$?~~

~~(iii) $\angle A = 70^\circ$ and $\angle C = 65^\circ$?~~

Sol. $\angle A \neq \angle C$

\therefore According to given condition

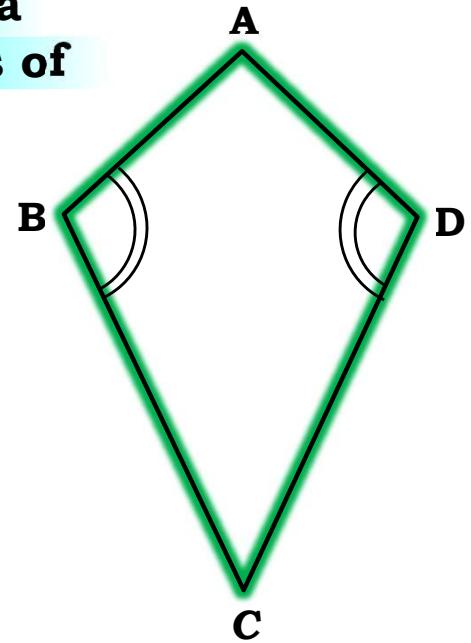
$\square ABCD$ can not be a parallelogram.



- Opposite sides are equal
- Opposite angles are equal
- Opposite sides are parallel

Q. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

Sol. $\square ABCD$ is a quadrilateral
 $\angle B = \angle D$
 $\square ABCD$ is not a Parallelogram



**Q. Two adjacent angles of a parallelogram have equal measure.
Find the measure of the angles of the parallelogram.**

Sol. $\angle A = \angle B = x$

$$\angle A + \angle B = 180^\circ$$

$$\therefore x + x = 180^\circ$$

$$\therefore 2x = 180^\circ$$

$$\therefore x = \frac{180}{2} = 90^\circ$$

$$\therefore \angle A = \angle B = 90^\circ$$

$$\angle A = \angle C \quad [\text{Opposite angles are equal}]$$

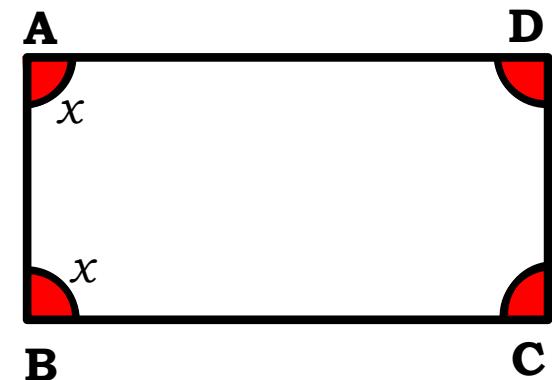
$$\therefore \angle C = 90^\circ$$

$$\angle B = \angle D \quad [\text{Opposite angles are equal}]$$

$$\therefore \angle D = 90^\circ$$

To Find : $\angle A, \angle B, \angle C, \angle D$

- Opposite angles are equal
- Adjacent angles are Supplementary



180°

Module 20

Exercise

Q. Consider the following parallelogram. Find the values of the unknowns x , y , z .

Sol. In $\triangle HOP$,

$$\angle HOP = 180 - (y + z)$$

$$= 180 - 70$$

$$= 110$$

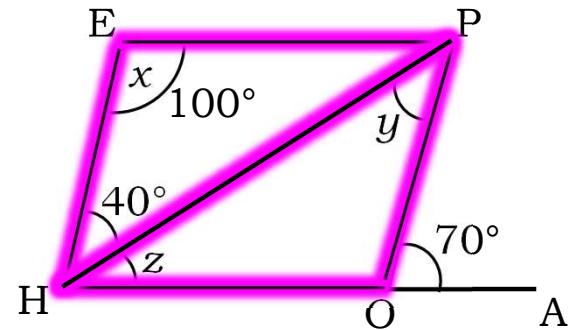
$$\angle HOP = 110$$

$$\angle HOP = x$$

$$\therefore x = 110$$

' $\angle HOP$ is a part of $\triangle HOP$

Opposite angles are equal



The sum of the measure of all angles of a triangle is 180°

Exercise

Q. Consider the following parallelogram. Find the values of the unknowns x , y , z .

Sol. $\therefore EH \parallel OP$ and PH is transversal

$$\angle EHP = \angle HPO$$

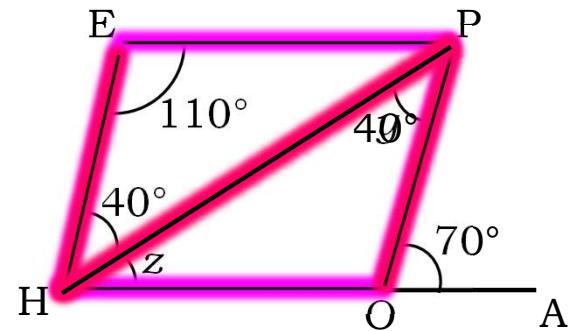
$$\therefore y = 40^\circ$$

$$40^\circ + z = 70^\circ$$

$$z = 70^\circ$$

From the figure, it is observed that
the exterior angle x is equal to the sum of the interior angles
opposite to it.
 $\therefore x = 110^\circ$

$$\therefore x = 110^\circ, y = 40^\circ \text{ and } z = 30^\circ$$



In a triangle, exterior angles is equal to the sum of interior angles opposite.

Exercise

Q. In the following figure, both RISK and CLUE are parallelogram. Find the value of x .

Sol. RISK is a parallelogram.

$$\therefore \angle R + \angle K = 180^\circ$$

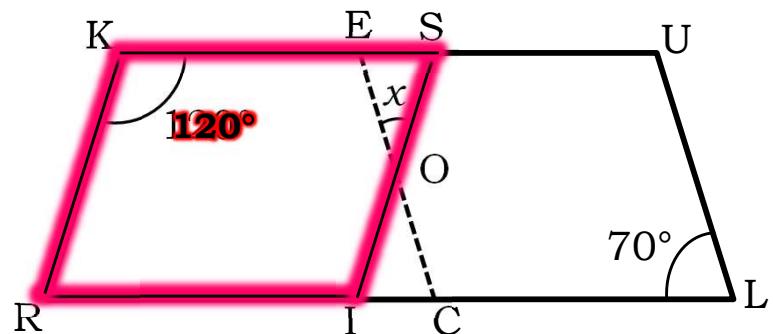
$$\therefore \angle R + 120^\circ = 180^\circ$$

$$\angle R = 180 - 120^\circ$$

$\angle R = 60^\circ$

But $\angle R$ and $\angle S$ are opposite angles.

$\angle S = 60^\circ$



Adjacent angles are supplementary

Opposite angles are equal.

Exercise

Q. In the following figure, both RISK and CLUE are parallelogram. Find the value of x.

Sol. CLUE is a parallelogram.

$$\angle E = \angle L = 70^\circ$$

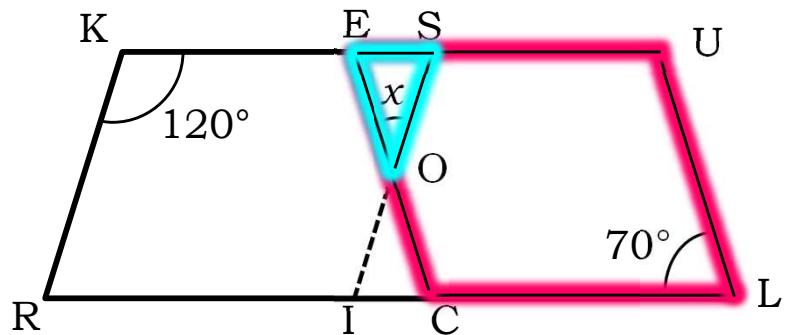
In $\triangle ESO$

$$\angle E + \angle S + x = 180^\circ$$

$$70^\circ + 60^\circ + x = 180^\circ$$

$$x = 180^\circ - 60^\circ \quad x \text{ is part of } \triangle ESO$$

$$\boxed{x = 50^\circ}$$



The sum of the measure of all angles of the triangle 180°



Module 21

**Q. The following figure GUNS is a parallelogram. Find x and y .
(Lengths are in cm)**

Sol. In parallelogram GUNS,

$$GS = UN \quad [\text{Opposite sides of parallelogram are equal}]$$

$$\therefore 3x = 18$$

$$\therefore x = \frac{18}{3}$$

$$\therefore x = 6 \text{ cm}$$

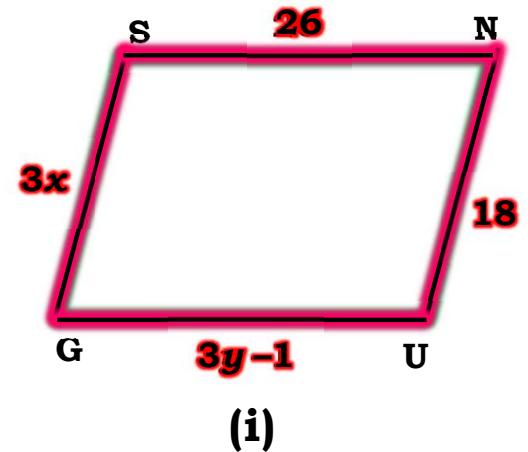
Also, $GU = SN$

$$\therefore 3y - 1 = 26$$

$$\therefore 3y = 26 + 1$$

$$\therefore 3y = 27$$

$$\therefore y = \frac{27}{3}$$



$$\therefore y = 9 \text{ cm}$$

$$\therefore x = 6 \text{ cm and } y = 9 \text{ cm}$$

Opposite sides of parallelogram are equal

**Q. The following figure RUNS is a parallelogram. Find x and y .
(Lengths are in cm)**

Sol. In parallelogram RUNS,

$$y + 7 = 20 \quad [\text{Diagonals of parallelogram bisects each other}]$$

$$\therefore y = 20 - 7$$

$$\therefore y = 13 \text{ cm}$$

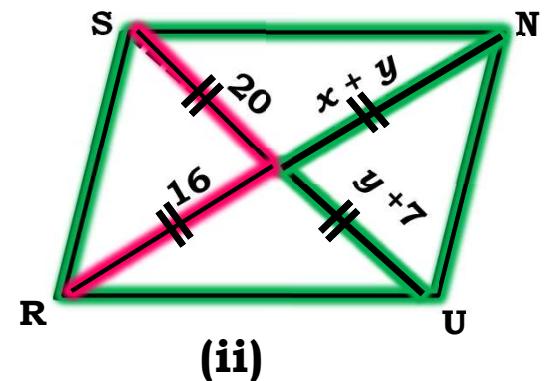
$$x + y = 16$$

$$\therefore x + 13 = 16$$

$$\therefore x = 16 - 13$$

$$\therefore x = 3 \text{ cm}$$

$$\therefore x = 3 \text{ cm and } y = 13 \text{ cm}$$



Q. Explain how this figure is a trapezium.

Which of its two sides are parallel?

Sol.

$$\angle M + \angle L = 100$$

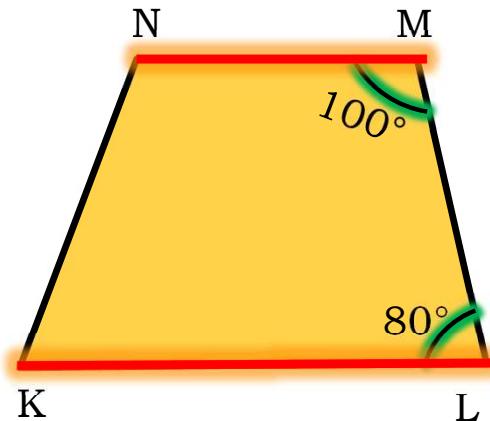
A quadrilateral having one pair of opposite sides parallel is called trapezium

$$\angle M + \angle L = 180$$

$\therefore MN \parallel KL$ [Interior angles]

Since, one pair of opposite side is parallel

$\therefore \square MNKL$ is a trapezium.



4

Lecture

Module 22

Exercise

Q. In the given figure $\square ABCD$ is a square. Find the measure of $\angle CAD$.

Sol: In $\triangle CDA$

$$AD = DC \quad \bullet \bullet \bullet$$

$$\therefore \angle DAC = \angle DCA = \gamma$$

$$\angle DAC + \angle DCA + \angle ADC = 180$$

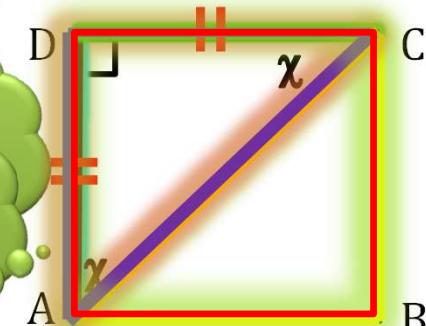
$$\underline{x} + \underline{x} + 90 = 180$$

$$2x + 90 = 180$$

$$2x =$$

Opposite sides are equal
 $\angle CAD$ is
Let see what is
Given is a
AC divides $\square ABCD$ into two triangles

$$\therefore x = \frac{180 - 90}{2} = 45^\circ$$



The sum of measure of angles of a triangles is 180°



Exercise

Q. Find $m \angle C$ in the adjoining figure if $\overline{AB} \parallel \overline{DC}$.

Sol:

$ABED$ is a trapezium

Let see
what is
given.

$$\therefore m \angle A + m \angle C = 180^\circ$$

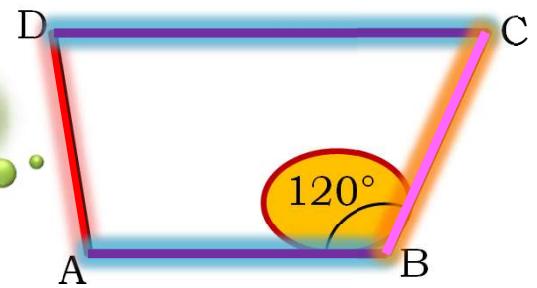
$$m \angle C = 180^\circ - m \angle A$$

$$m \angle C = 180^\circ - 120^\circ$$

$$\boxed{m \angle C = 60^\circ}$$

❖ Interior angles are
supplementary.

Given is a
trapezium



Module 23

Exercise

Q. Find the measure of $\angle P$ and $\angle S$ if $\overline{SP} \parallel \overline{RQ}$ in figure.

Sol: PQRS is a trapezium

$$\overline{SP} \parallel \overline{RQ}$$

PQ is a transversal.

$$m \angle P + m \angle Q = 180^\circ$$

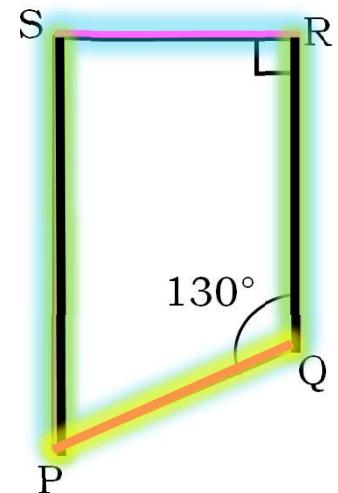
$$m \angle P + 130^\circ = 180^\circ$$

$$m \angle P = 180^\circ - 130^\circ$$

$m \angle P = 50^\circ$

❖ Interior angles are supplementary.

Now let us
find $m\angle S$



Exercise

Q. Find the measure of $\angle P$ and $\angle S$ if $\overline{SP} \parallel \overline{RQ}$ in figure.

Sol:

$$\overline{SP} \parallel \overline{RQ}$$

RS is a transversal.

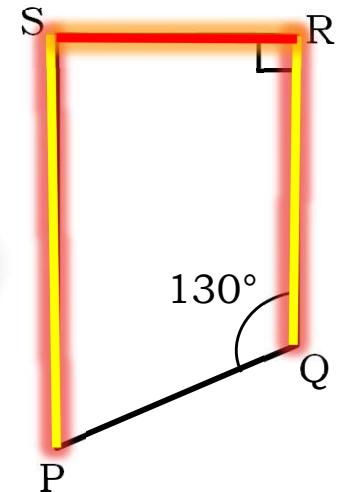
$$m \angle S + m \angle R = 180^\circ$$

$$m \angle S + 90^\circ = 180^\circ$$

$$m \angle S = 180^\circ - 90^\circ$$

$m \angle S = 90^\circ$

❖ Interior angles are supplementary.



Module 24

Q. In a quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively. Prove that $\angle AOB = \frac{1}{2}(\angle C + \angle D)$.

$$\text{Sol : } \angle 1 = \frac{1}{2} \angle A, \quad \angle 2 = \frac{1}{2} \angle B$$

In $\triangle AOB$,

$$\angle AOB + \angle OAB + \angle ABC = 180^\circ$$

$$\angle AOB + \angle 1 + \angle 2 = 180^\circ$$

$$\therefore \angle AOB = 180^\circ - (\angle 1 + \angle 2)$$

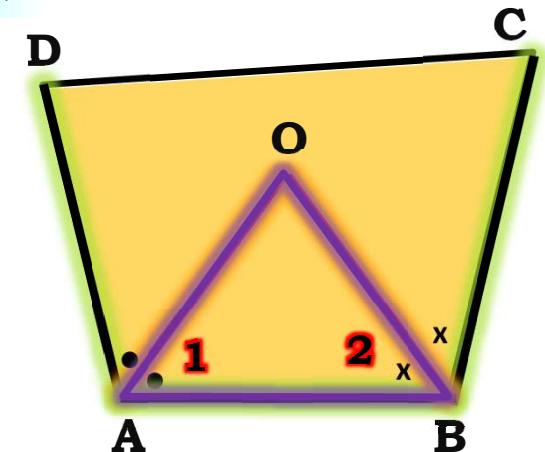
$$\therefore \angle AOB = 180^\circ - \left(\frac{1}{2} \angle A + \frac{1}{2} \angle B \right)$$

$$\therefore \angle AOB = 180^\circ - \frac{1}{2} (\angle A + \angle B) \dots \text{(i)}$$

In $\square ABCD$,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

The sum of the interior angles of a quadrilateral is 360°



Q. In a quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively. Prove that $\angle AOB = \frac{1}{2}(\angle C + \angle D)$.

Sol : $\angle AOB = 180^\circ - \frac{1}{2} (\angle A + \angle B) \dots \text{(i)}$

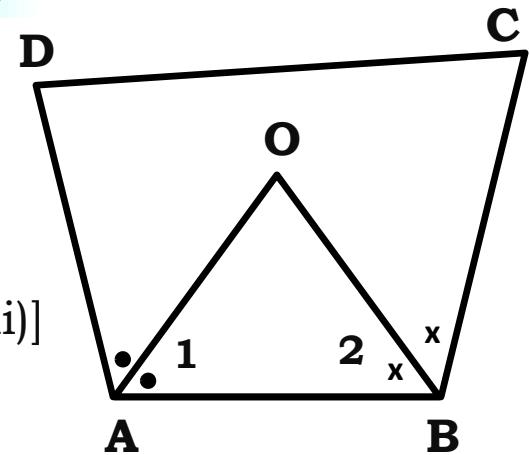
$\angle A + \angle B = 360^\circ - (\angle C + \angle D) \dots \text{(ii)}$

$$\angle AOB = 180^\circ - \frac{1}{2} [360^\circ - (\angle C + \angle D)] \quad [\text{From (i) and (ii)}]$$

$$\therefore \angle AOB = 180^\circ - \frac{1}{2} \times \cancel{360^\circ}^{\color{red}180^\circ} + \frac{1}{2} (\angle C + \angle D)$$

$$\therefore \angle AOB = \cancel{180^\circ} - \cancel{180^\circ} + \frac{1}{2} (\angle C + \angle D)$$

$$\therefore \angle AOB = \frac{1}{2} (\angle C + \angle D)$$



Module 25

Exercise

Q. In the adjacent figure, the bisectors of $\angle A$ and $\angle B$ meet in a point P.

If $\angle C=100^\circ$ and $\angle D=60^\circ$, find the measure of $\angle APB$.

Sol:

$$\angle A + \angle B + \angle C + \angle D = 360$$

$$\angle A + \angle B + 60 + 100 = 360$$

$$\angle A + \angle B + 160 = 360$$

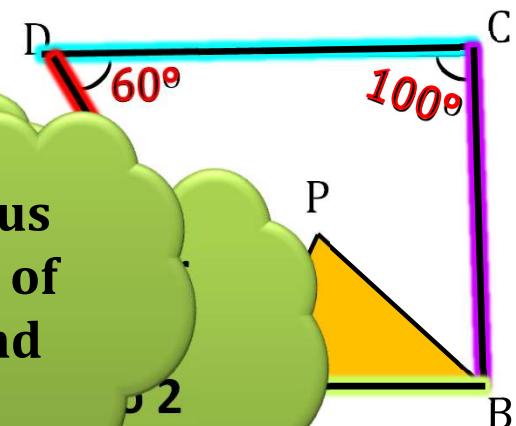
$$\angle A + \angle B = 360$$

$$\angle A + \angle B = 200$$

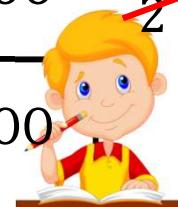
$$\therefore \frac{1}{2} \angle A + \frac{1}{2} \angle B = 200 \times \frac{1}{2}$$

Now, let us
find sum of
 $\angle PAB$ and
 $\angle PBA$

quadrilaterals



The sum of $\frac{1}{2} \angle A + \frac{1}{2} \angle B = 100$
quadrilaterals is 360°



Exercise

Q. In the adjacent figure, the bisectors of $\angle A$ and $\angle B$ meet in a point P.
If $\angle C=100^\circ$ and $\angle D=60^\circ$, find the measure of $\angle APB$.

Sol:

$$\therefore \angle PAB + \angle PBA = 100$$

In $\triangle APB$,

$$\angle BAP + \angle ABP + \angle APB = 180$$

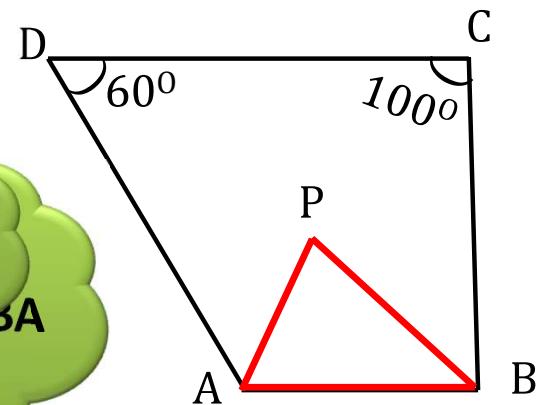
$$\frac{1}{2} \angle A = \angle DAP = \angle PAB$$

$\angle APB$ is part of $\triangle APB$

$$\angle APB = 180 - 100$$

$$\angle APB = 80^\circ$$

\therefore The measure of $\angle APB = 80^\circ$



Exercise

Q. Two adjacent angles of a parallelogram are $(3x - 4)^\circ$ and $(3x + 16)^\circ$. find the value of 'x' and hence find the measure of each of its angles.

Sol:

We know that the sum of two adjacent angles of a parallelogram is 180°

$$\therefore \angle A + \angle B = 180^\circ$$

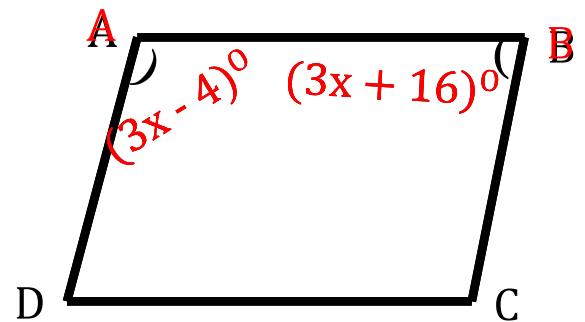
$$3x - 4 + 3x + 16 = 180$$
$$6x + 12 = 180$$

$$6x = 180^\circ - 12$$

$$6x = 168^\circ$$

$$\therefore x = \frac{168}{6}$$

$$x = 28^\circ$$



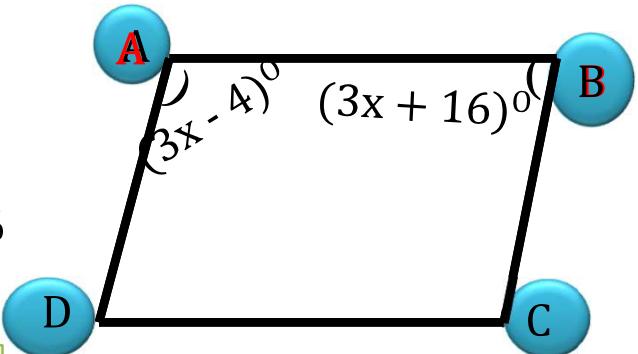
Exercise

Q. Two adjacent angles of a parallelogram are $(3x - 4)^\circ$ and $(3x + 16)^\circ$. find the value of x and hence find the measure of each of its angles.

Sol:

$$\begin{aligned}\therefore \angle A &= 3x - 4 \\ &= 3(28) - 4 \\ &= 80 \\ \angle A &= 80^\circ\end{aligned}$$

$$\begin{aligned}\therefore \angle B &= 3x - 16 \\ &= 3(28) - 16 \\ &= 100 \\ \angle B &= 100^\circ\end{aligned}$$



Opposite angles of a parallelogram are equal

$$\begin{aligned}\angle A &= \angle C \\ 80 &= \angle C \\ \therefore \angle C &= 80^\circ\end{aligned}$$

$$\begin{aligned}\angle B &= \angle D \\ 100 &= \angle D \\ \therefore \angle D &= 100^\circ\end{aligned}$$

Module 26

Exercise

Q. In the adjacent figure, $ABCD$ is a parallelogram and line segments AE and CF bisect the angles A and C respectively. Show that $AE \parallel CF$.

Sol: In triangles ADE and CBF ,

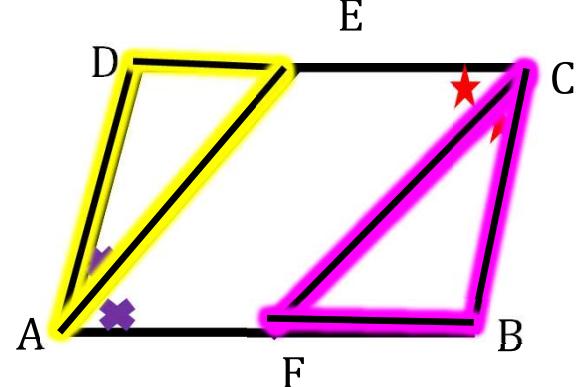
$$\therefore \text{We have } AD = BC, \dots(1)$$

$$\angle B = \angle D \quad \dots(2)$$

$$\because \angle A = \angle C$$

$$\therefore \frac{1}{2} \angle A = \frac{1}{2} \angle C$$

$$\therefore \angle DAE = \angle BCF \quad \dots(3)$$



❖ *Opposite sides are equal*

❖ *Opposite angles are equal*

Exercise

Q. In the adjacent figure, $ABCD$ is a parallelogram and line segments AE and CF bisect the angles A and C respectively. Show that $AE \parallel CF$.

Sol:

$$\triangle ADE \cong \triangle CBF \quad (\text{ASA test})$$

$$AE = CF \quad (\text{c.p.c.t})$$

$$DE = BF \quad \dots(4) \quad (\text{c.p.c.t})$$

$$CD = AB \quad \dots(5) \quad (\text{Opposite sides are equal})$$

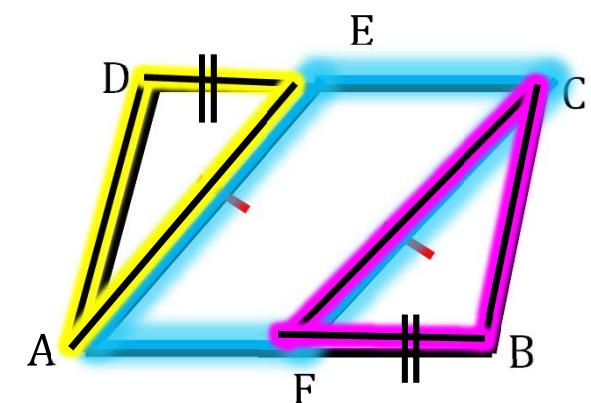
$$\therefore CD - DE = AB - BF \quad [\text{Subtracting (4) from (5)}]$$

$$\text{so, } CE = AF \quad [D-E-C, A-F-B]$$

\therefore AECF is a parallelogram.

[A quadrilateral is a parallelogram
if opposite sides are equal]

Hence, $AE \parallel CF$



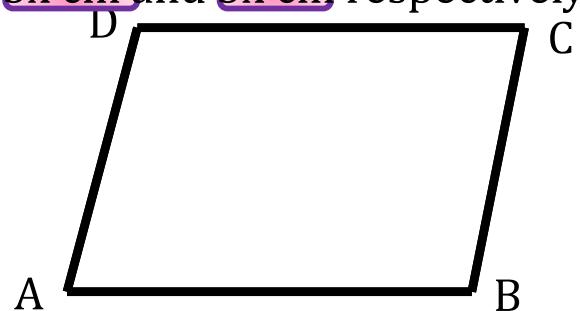
Exercise

Q. Two sides of a parallelogram are in the ratio 5 : 3. If its perimeter is 64 cm, find the lengths of its sides.

Sol:

Let the lengths of two sides of the parallelogram be $5x$ cm and $3x$ cm respectively.

$$\begin{aligned}\text{perimeter} &= 2(l+b) \text{cm} \\ &= 2(5x + 3x) \text{cm} \\ &= 2 \times (8x) \\ &= 16x.\end{aligned}$$



But perimeter = 64

$$\therefore 16x = 64$$

$$\therefore x = \frac{64}{16}$$

~~4~~
1

$$x = 4^\circ$$

But perimeter is 64

$$= 20 \text{ cm}$$

$$\therefore \text{Other side} = (3 \times 4) \text{cm}$$

$$= 12 \text{ cm}$$

Module 27

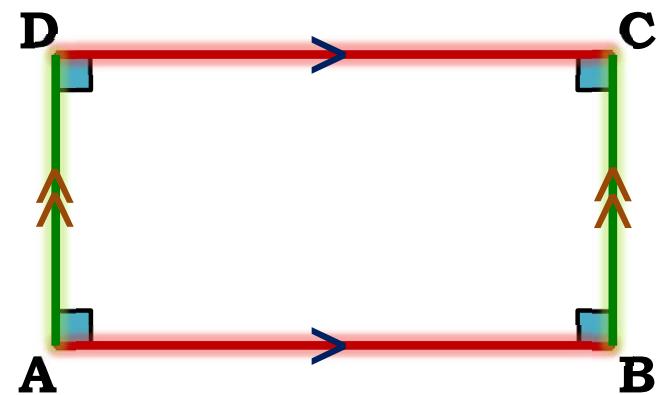
➤ RECTANGLE

Definition : A quadrilateral in which each angle is a right angle is called a Rectangle

$\square ABCD$ is a rectangle
 $\therefore \angle A = \angle B = \angle C = \angle D = 90^\circ$

Both the pairs of opposite sides are parallel
So $\square ABCD$ is also a parallelogram

Every Rectangle is a parallelogram



PROPERTIES OF A RECTANGLE

- **Opposite sides are equal**

$$AB = CD$$

$$AD = BC$$

- **Diagonals are equal**

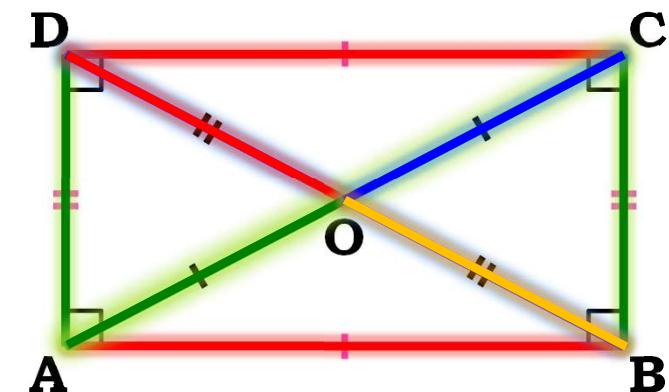
$$OA = OC$$

$$OB = OD$$

Name the pairs
of opposite sides

$$OA = OC \text{ &}$$

$$OB = OD$$



- **Diagonals are equal**

$$AC = BD$$

$$AC = BD$$

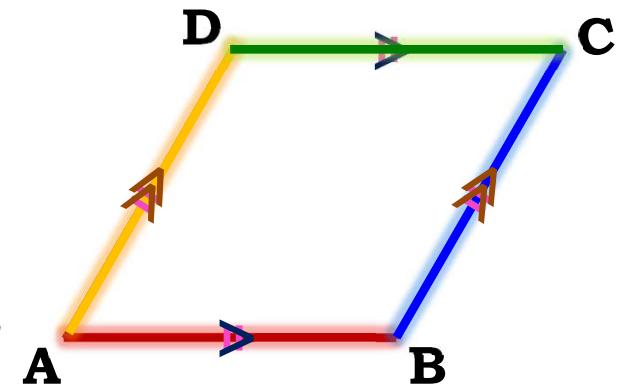
➤ RHOMBUS

Definition : A quadrilateral having all sides equal is called a Rhombus

Name all the sides of $\square ABCD$
 $AB = BC = CD = AD$
 $\therefore AB = BC = CD = AD$

$\square ABCD$ is also a parallelogram

Every Rhombus is a Parallelogram



PROPERTIES OF A RHOMBUS

- Opposite angles are equal

$$\angle A = \angle C$$

$$\angle B = \angle D$$

Name the pairs
of opposite angles

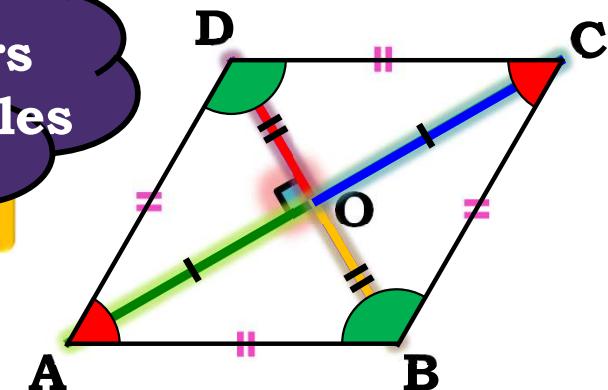
Let
Rho

- Diagonals

$$OA = OC$$

$$OB = OD$$

$$OA = OC \text{ &} \\ OB = OD$$



- Diagonals are perpendicular to each other

$$AC \perp BD$$

➤ SQUARE

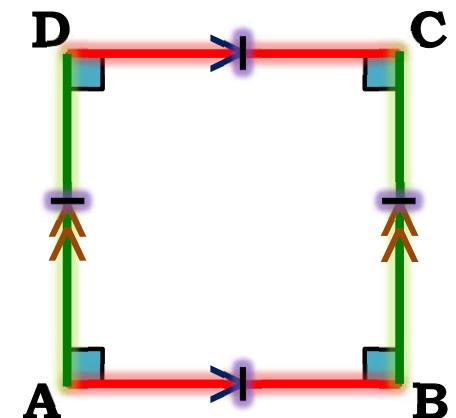
Definition : A quadrilateral in which each angle is a right angle and all sides are equal is called a Square

$\square ABCD$ is a
 $\therefore \angle A = \angle B = \angle C = \angle D = 90^\circ$

$AB = BC = CD = DA$

Both the pairs of
opposite sides are
parallel

Every Square is a Parallelogram



PROPERTIES OF A SQUARE

- **Diagonals bisect each other**

$$OA = OC$$

$$OB = OD$$

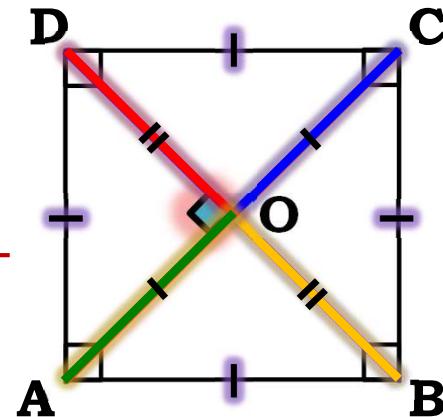
- **Diagonals are perpendicular to each other**

$$AC \perp BD$$

Let us consider a Square ABCD

$$OA = OC &$$

$$OB = OD$$



- **Diagonals are equal**

$$AC = BD$$

$$AC = BD$$

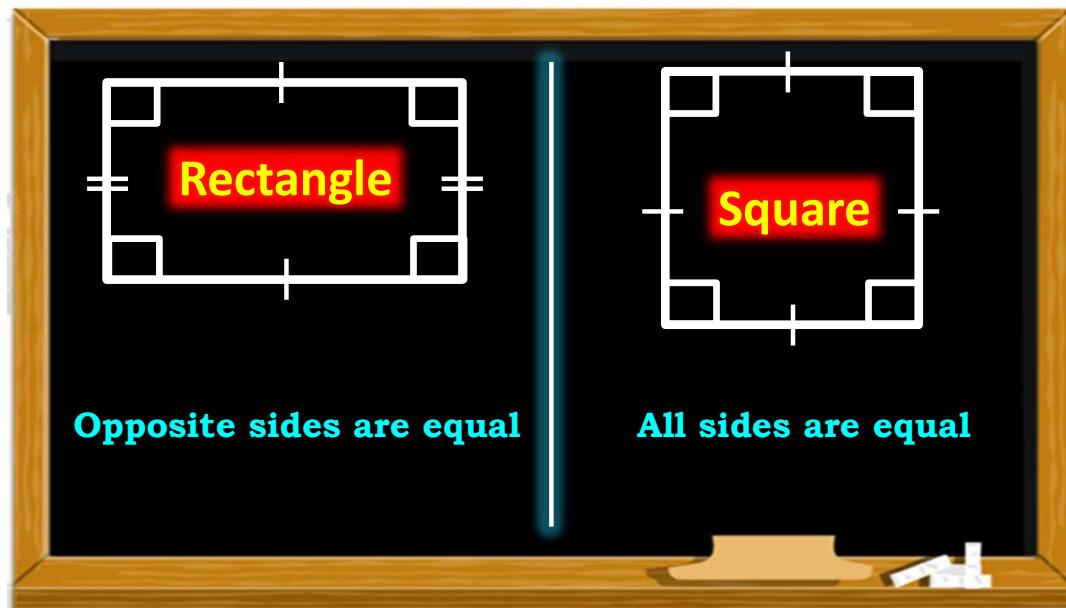
Module 28

Q. State whether true or false:

a) All rectangles are squares.

Sol. False

All Squares are Rectangles but,
all Rectangles are not Squares

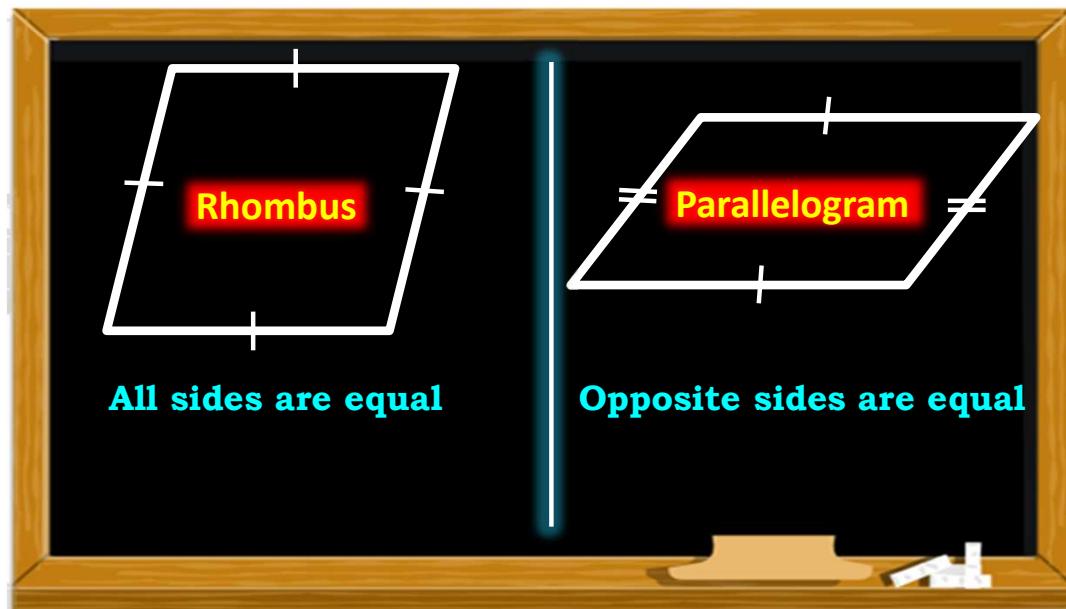


Q. State whether true or false:

b) All rhombuses are parallelograms

Sol. True

Since, Rhombus possesses all properties of Parallelogram.

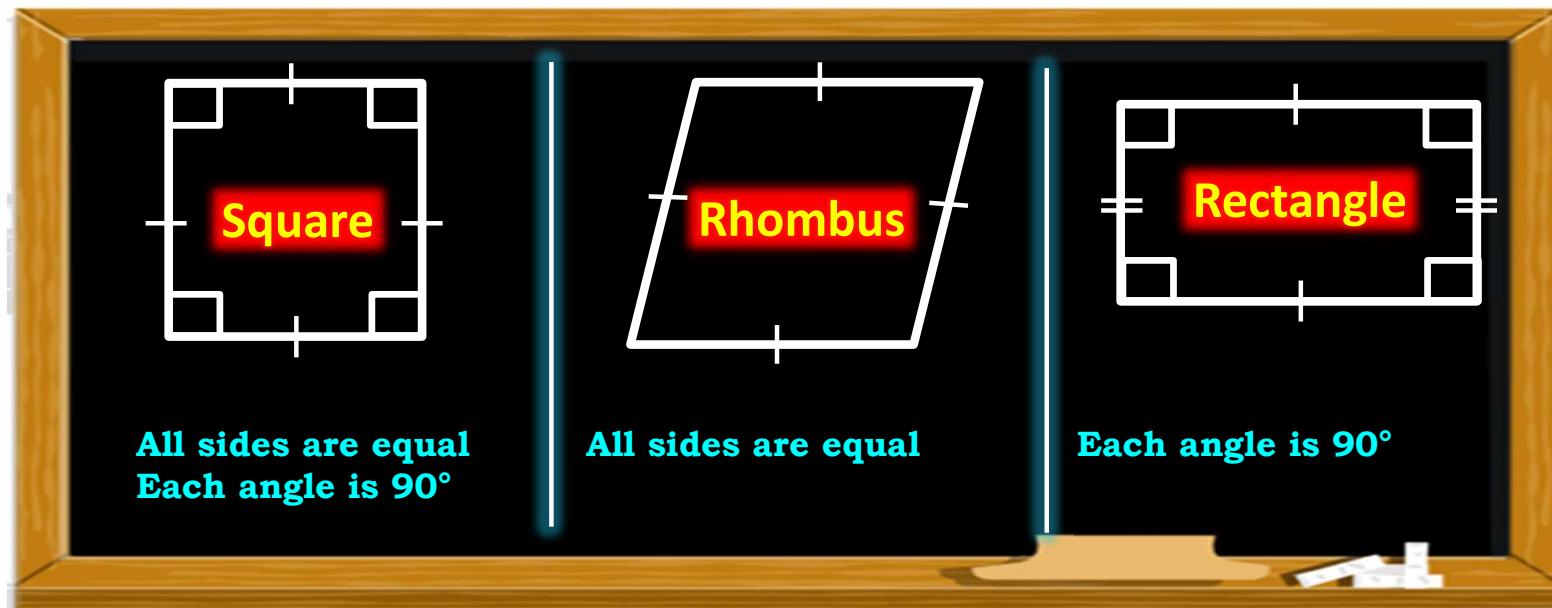


Q. State whether true or false:

c) All squares are rhombuses and also rectangles.

Sol. True

A Square possesses all the properties of Rhombus and Rectangle.

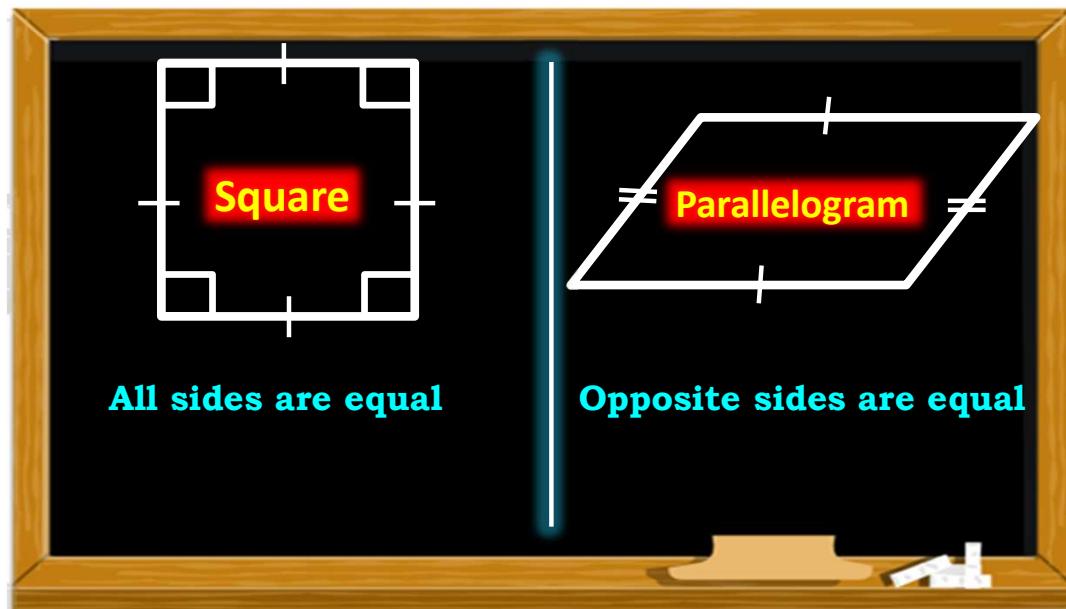


Q. State whether true or false:

d) All squares are not parallelograms.

Sol. False

Since, squares have all the properties of parallelogram.

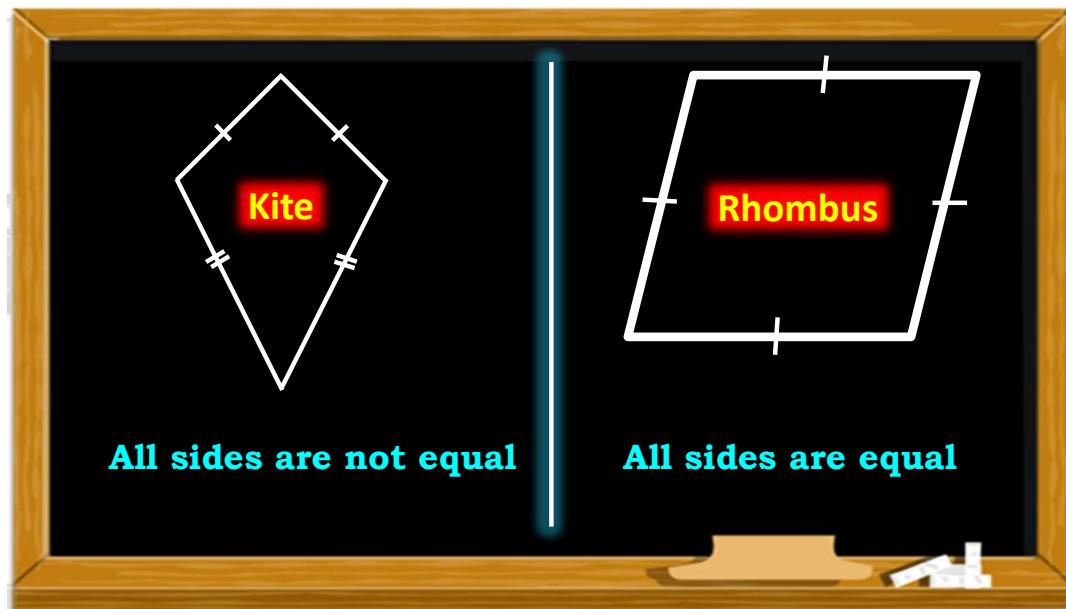


Q. State whether true or false:

e) All kites are rhombuses.

Sol. False

Since, Kites do not have equal sides.

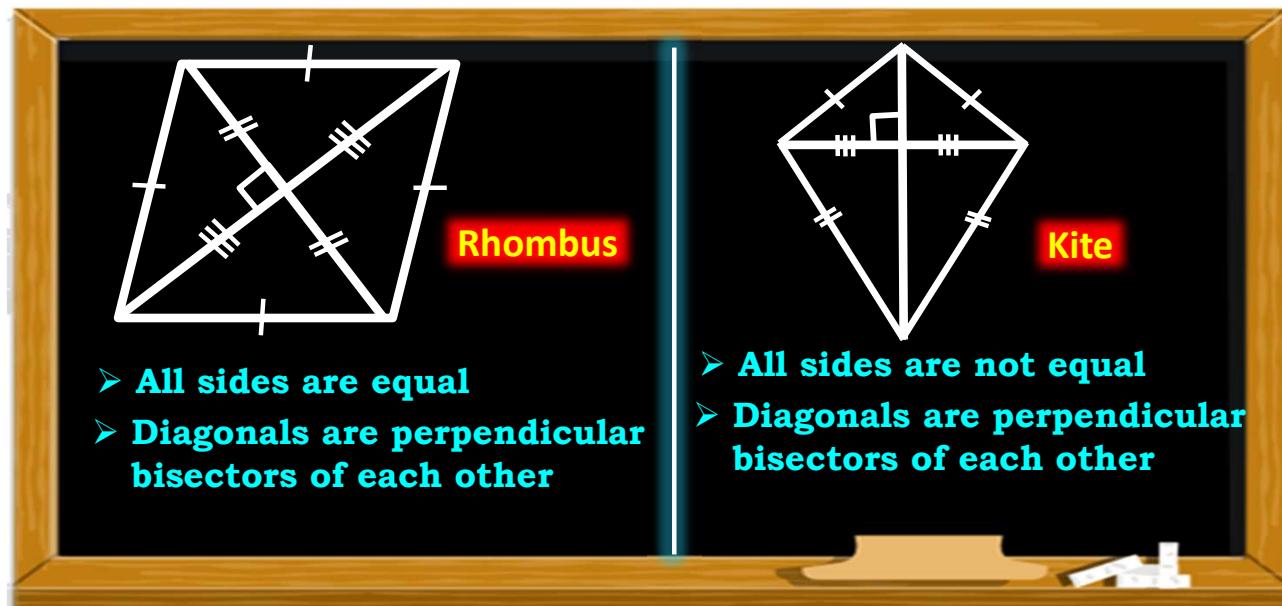


Q. State whether true or false:

f) All rhombuses are kites

Sol. True

Since, all rhombuses have equal sides and diagonals bisect each other.

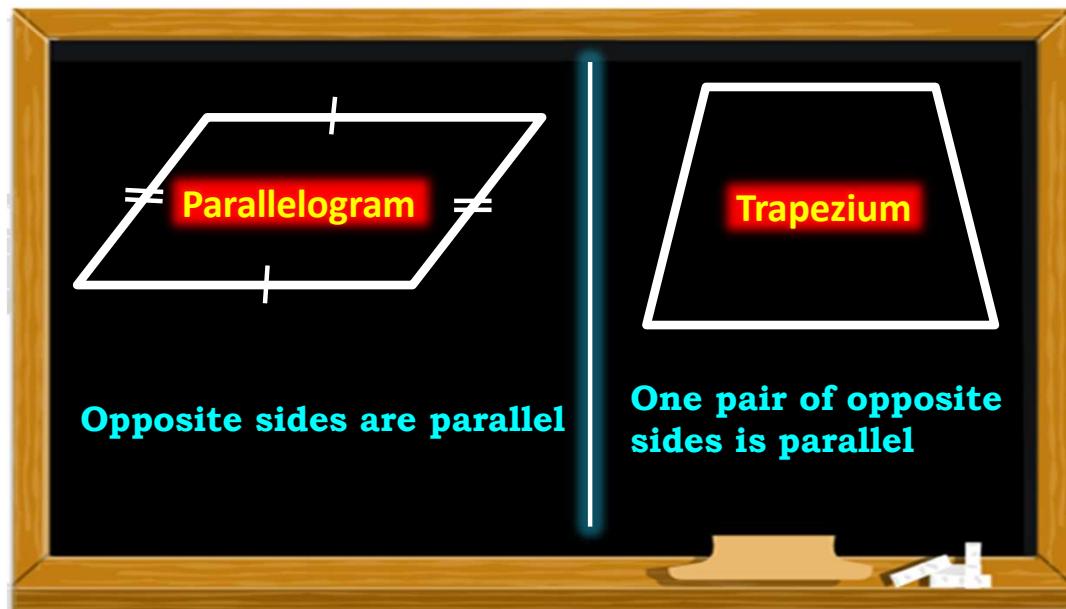


Q. State whether true or false:

g) All parallelograms are trapeziums.

Sol. True

Since, trapezium has one pair of opposite sides parallel.

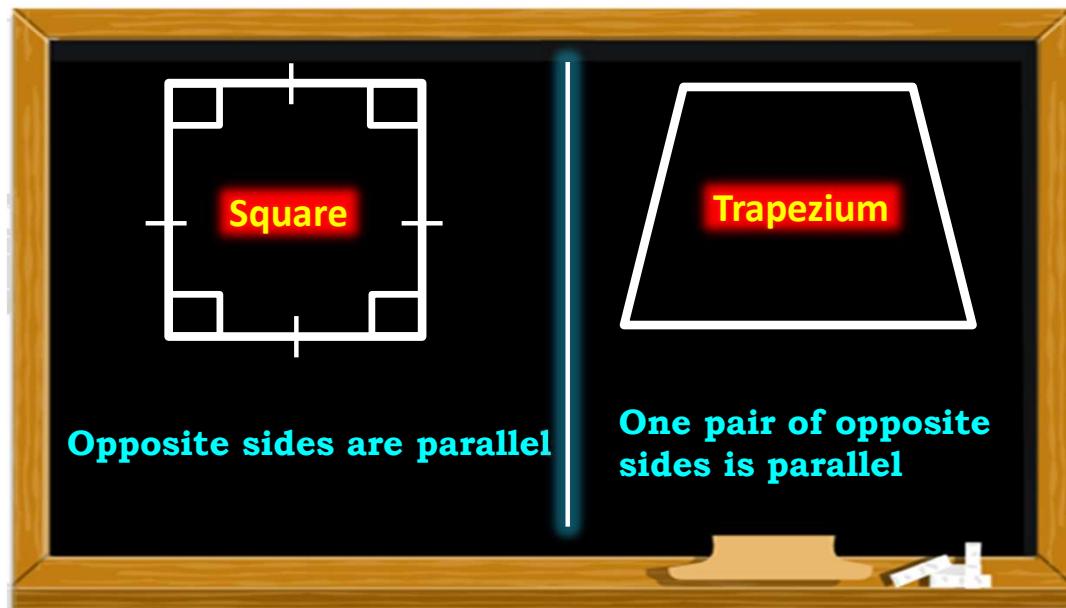


Q. State whether true or false:

h) All squares are trapeziums.

Sol. True

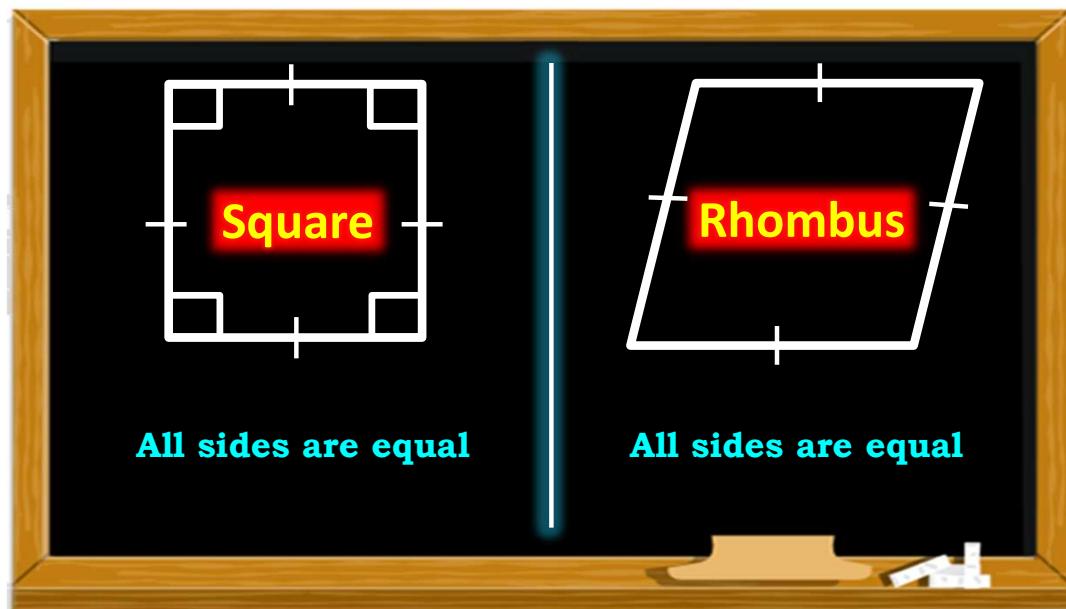
Since, trapezium has one pair of opposite sides parallel.



Q. Identify all the quadrilaterals that have.

a) Four sides of equal length

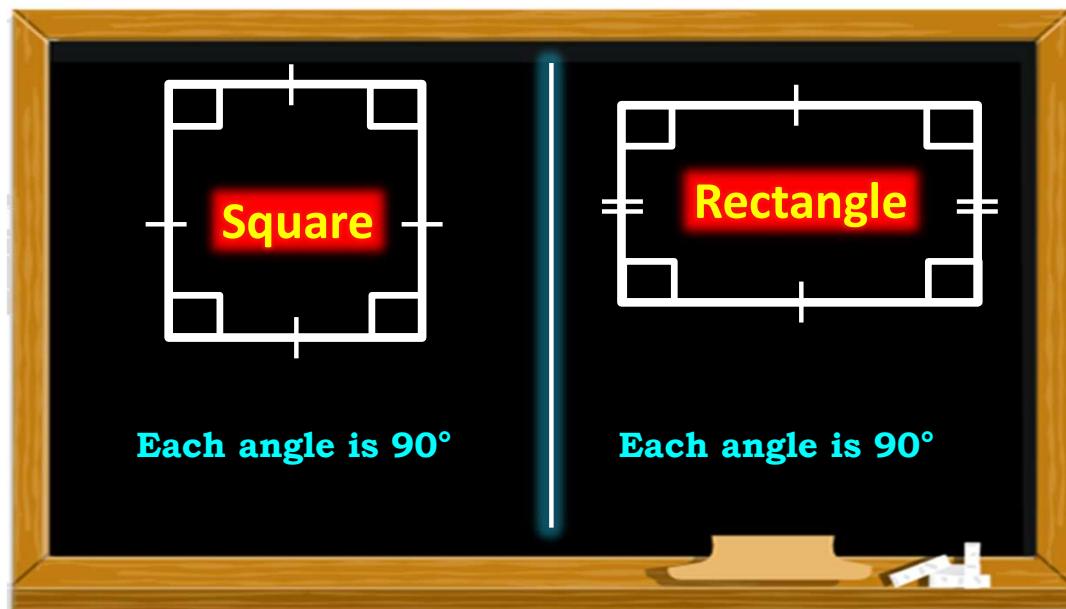
Sol. Square and Rhombus have four sides of equal length.



Q. Identify all the quadrilaterals that have.

b) Four right angles

Sol. Square and Rectangle have four right angles.



Q. Explain why a square is a quadrilateral.

Any four sided closed figure is a quadrilateral.

(a)

A square is a quadrilateral, because it has four sides
Quadrilateral having opposite sides parallel

(b)

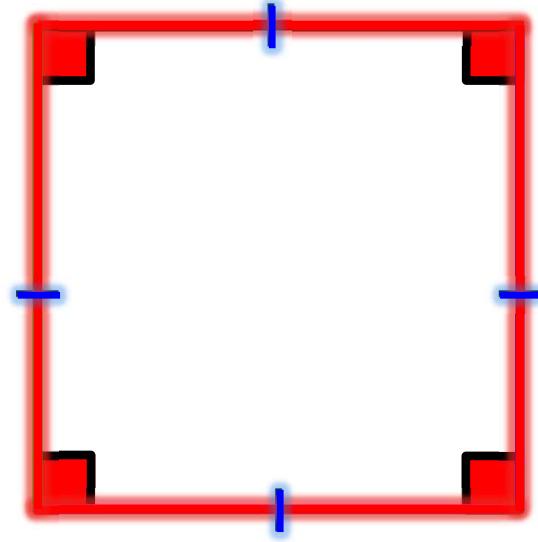
A square is a parallelogram, because it has opposite sides parallel
Quadrilateral having all sides equal

(c)

A square is a rhombus, because it has all four sides equal
Quadrilateral with each angle 90°

(d)

A square is a rectangle, because it has each angle 90°

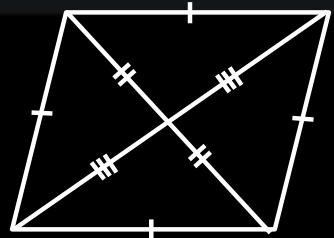


Module 29

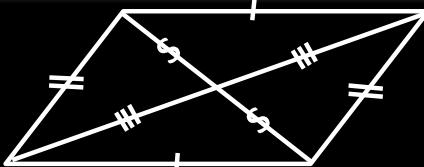
Q. Name the quadrilateral whose diagonals:

a) bisect each other

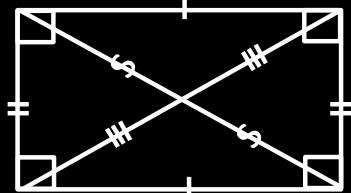
Sol. Rhombus, Parallelogram, Rectangle and Square are the quadrilaterals whose diagonals bisects each other.



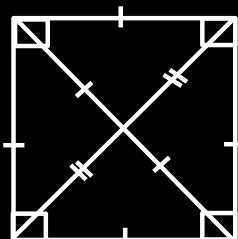
Rhombus



Parallelogram



Rectangle

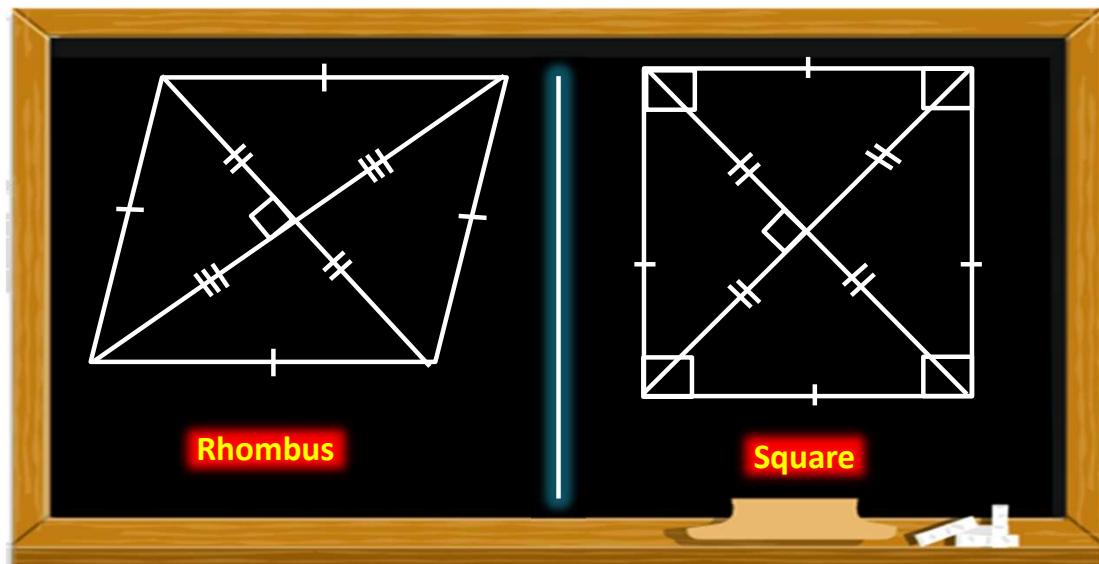


Square

Q. Name the quadrilateral whose diagonals:

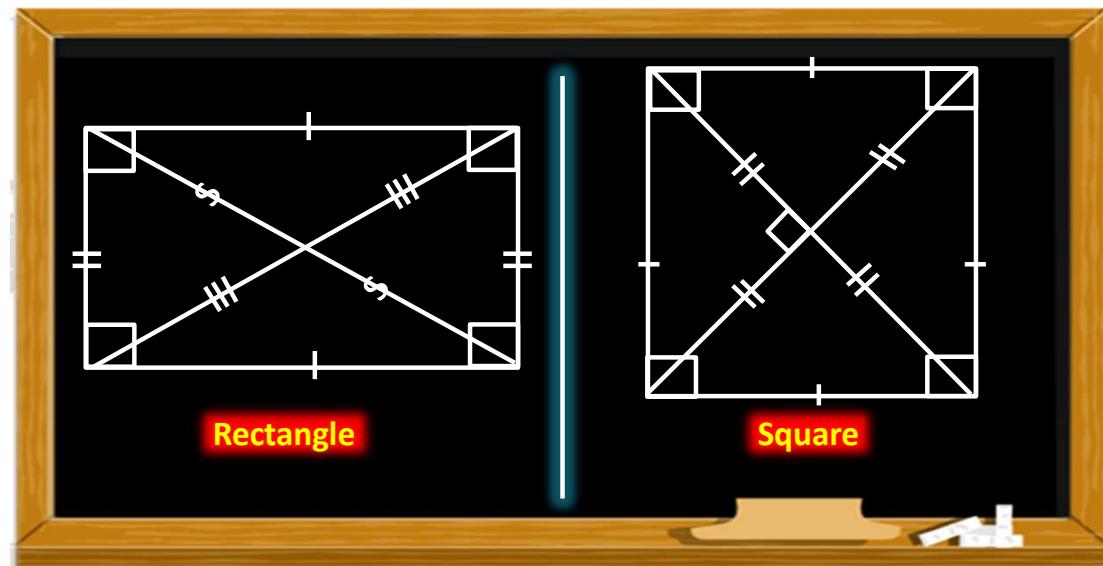
b) are perpendicular bisectors of each other

Sol. Rhombus and Square are the quadrilaterals whose diagonals are perpendicular bisectors of each other.



Q. Name the quadrilateral whose diagonals:
c) are equal.

Sol. Rectangle and Square have diagonals equal.

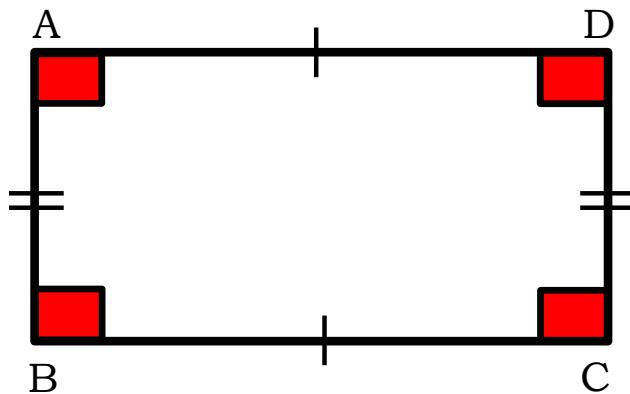


Q. Explain why a rectangle is a convex quadrilateral.

Sol. In a rectangle, measure of each angle is 90°

\therefore A rectangle is a convex quadrilateral

Each angle of a polygon
is less than 180°



Q. ABC is a right-angled triangle and O is the mid-point of the side opposite to the right angle.

Explain why O is equidistant from A, B and C.

Sol. $AO = OC$

$OB = OD$

Diagonals of $\square ABCD$ bisects each other

$\therefore \square ABCD$ is a Parallelogram

$$\angle B = 90^\circ$$

$\therefore \square ABCD$ is a rectangle

Diagonal of rectangle bisect each other

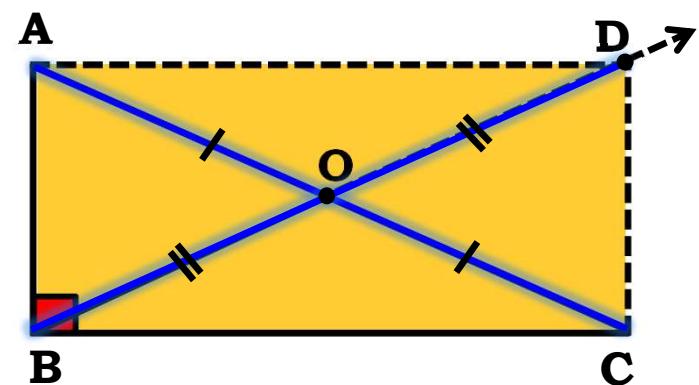
$$OA = OC = \frac{1}{2} AC \quad \dots(i)$$

Ex' Draw AD and DC int D

A1
 $\therefore AC = DC$ $\therefore BO = OD$ $\dots(ii)$

$$\therefore OA = OC = OB = OD \quad [\text{from (i) (ii) and (iii)}]$$

$\therefore O$ is equidistant from A, B and C



To show : $OA = OB = OC$

*Thank
You*

