MODULE - 1

CIRCLE

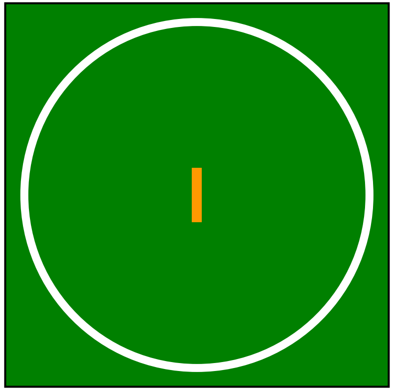
- **Introduction**

CIRCLE

Bangle

Ball

Coins



Boundary rope of a cricket ground

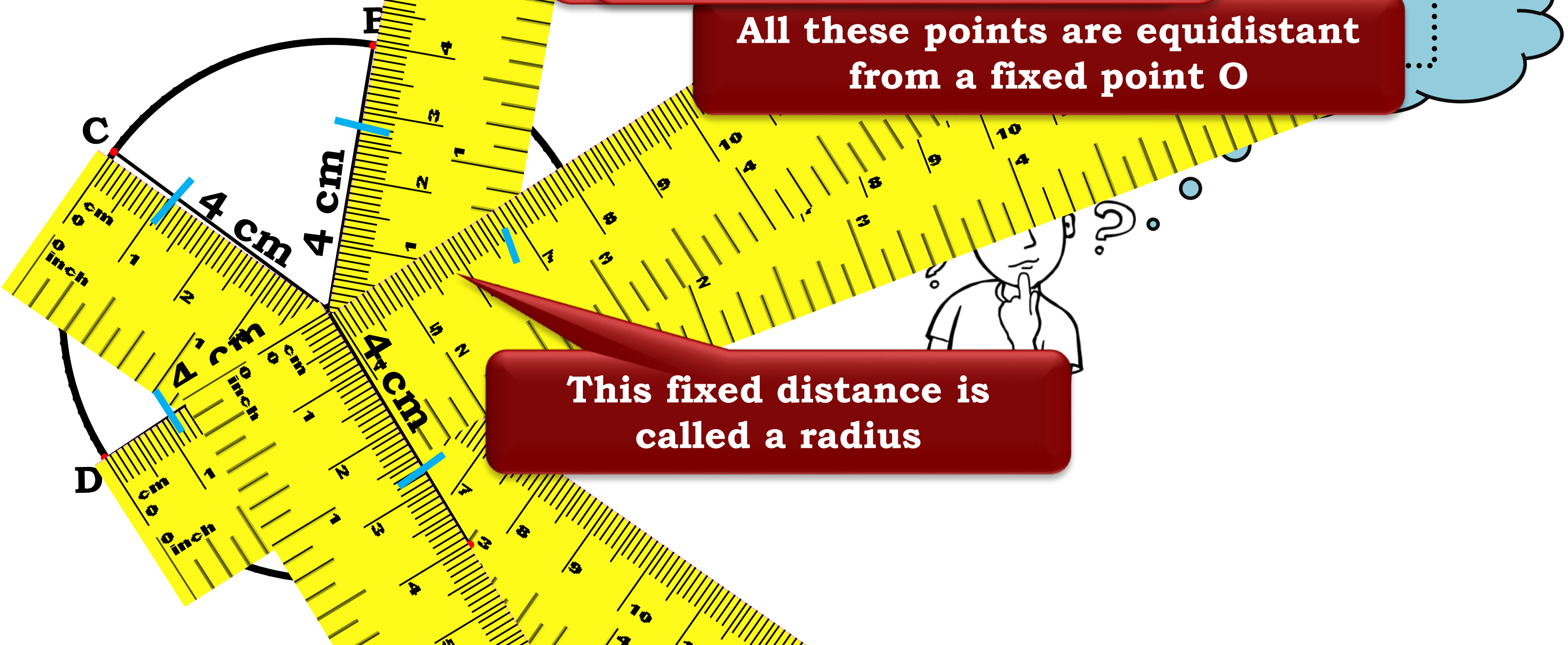
For this, let us understand the definition of circle

CIRCLE

Circle is the set o… t a fixed distance from a fixed po…

Each of these points here satisfy a condition
Now, let us understand..

All these points are equidistant from a fixed point O



This fixed distance is called a radius

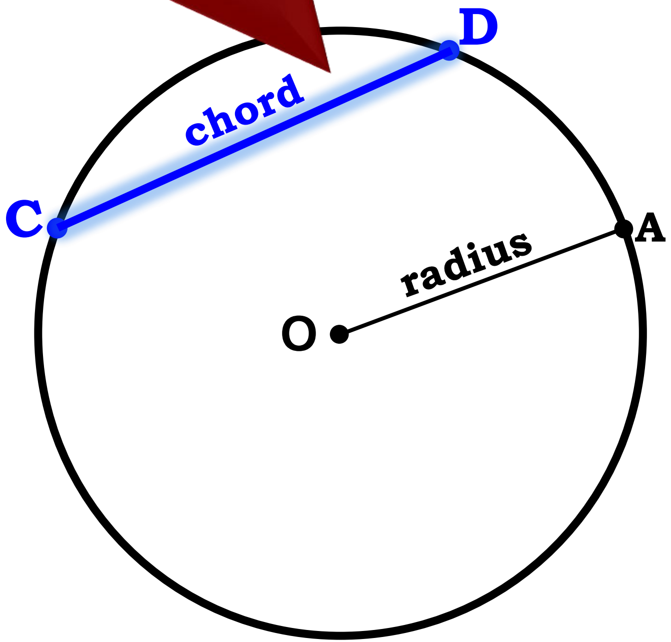
CIRCLE

A s … ircle is called a chord.

How many chords can … aw in a circle ?

Infinite

Such a segment is called a CHORD

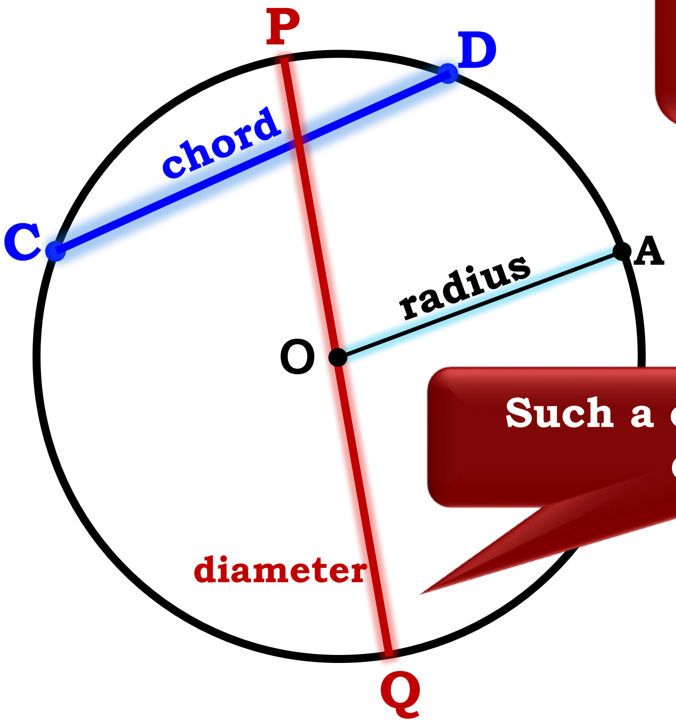


CIRCLE

A chord passing through the centre of the circle is called a diameter.

How many such diameters can we draw in a circle ?

Infinite



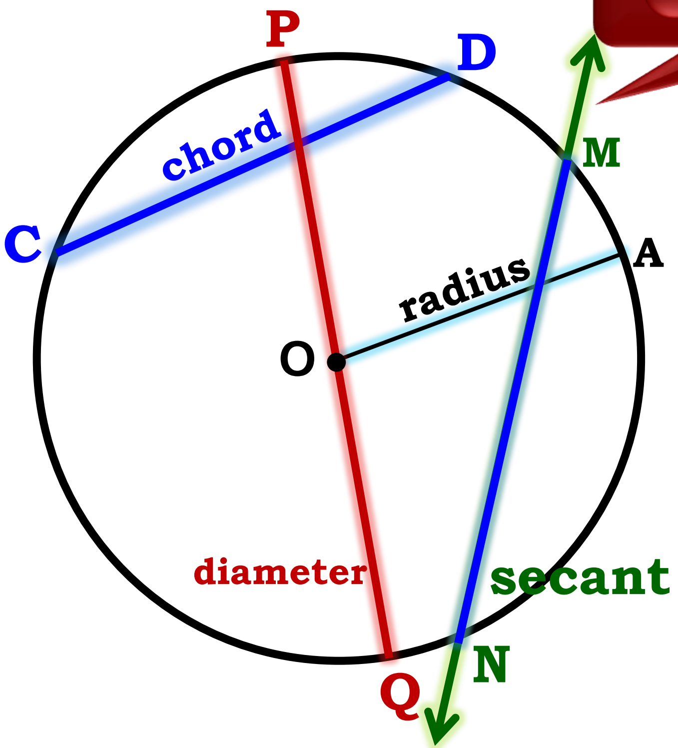
Such a chord is called a diameter

CIRCLE

A line intersecting a circle

Let us draw a line intersecting circle in two distinct points

s called a secant.



Note : A secant always contains a chord

MODULE - 2

CIRCLE

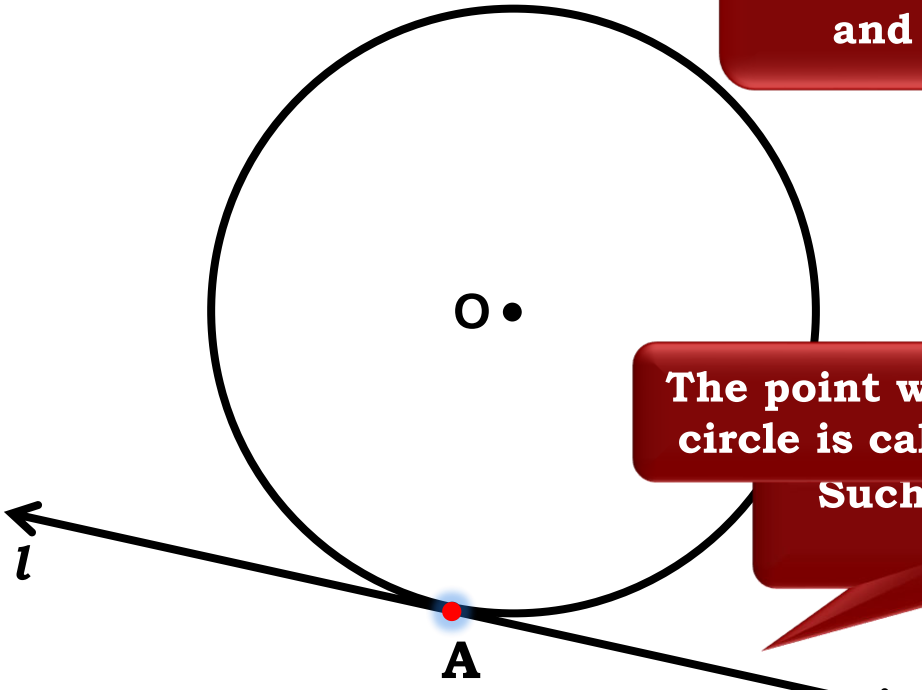
- Tangent

CIRCLE

A line in the plane of a circl… d only one point is called tangent.

Let us draw a line intersecting circle in one and only one point

Here, A is the point of contact



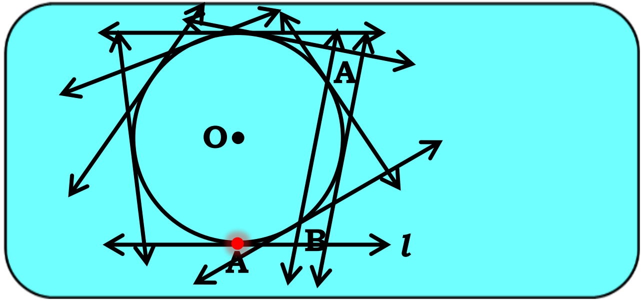
The point where the line touches the circle is called POINT OF CONTACT

Such a line is called TANGENT

Q. Answer the following :

1. How many tangents can a circle have?

Ans. **Circle can have infinite tangents.**



2. Fill in the blanks :

(i) A tangent to a circle intersects it in <u>One and only one</u> point (s).

(ii) A line intersecting a circle in two points is called a <u>secant</u>.

(iii) A circle can have <u>two</u> parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called <u>Point of contact</u>.

MODULE - 3

CIRCLE

- Theorem-Radius is perpendicular to the tangent
- Sums based on theorem

THEOREM

A tangent at any point ... licular to the radius, throu... ct.

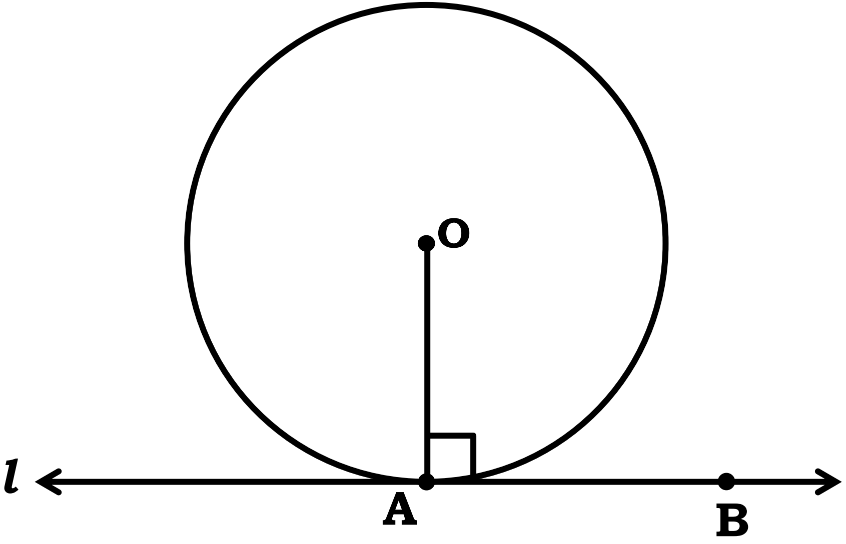
Let us take point B on the tangent

What is ∠AOB ?

90°

seg OA $\perp$ line l OR $\angle OAB = 90^\circ$

[Radius is perpendicular to the tangent]



Q. If TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, Find $\angle PTQ$.

Sol: In □OPTQ,

$\angle POQ = 110^\circ$

$\angle OPT = \angle OQT = 90$

We kn ... les

o

$\angle OQT = 90^\circ$

OQT

We know that, radius is perpendicular to the tangent

$\angle POQ + \angle OPT + \angle OQT + \angle PTQ = 360^\circ$

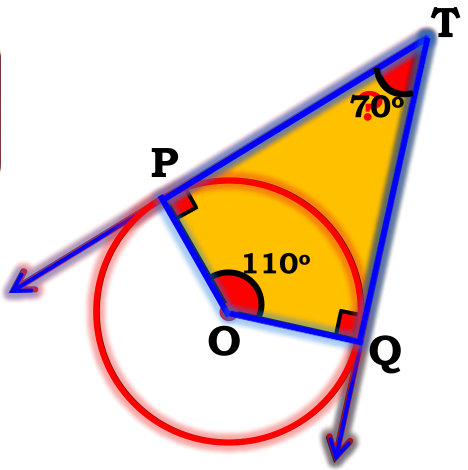
[Sum of all angles of quadrilateral is 360°]

$\therefore \quad 110 + 90 + 90 + \angle PTQ = 360$

$\therefore \quad 290 + \angle PTQ = 360$

$\therefore \quad \angle PTQ = 360 - 290$

$\therefore \quad \boxed{\angle PTQ = 70^\circ}$



Q. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Find length of PQ?

Sol. In ΔOPQ , $\angle OPQ = 9$

[Radius is perp ... gent]

I

Py (

$\therefore \angle OPQ = 90^\circ$

We know that, radius is perpendicular to the tangent

$$\boxed{OQ^2} = \boxed{OP^2} + \boxed{PQ^2}$$

$\therefore \quad 12^2 = 5^2 + PQ^2$

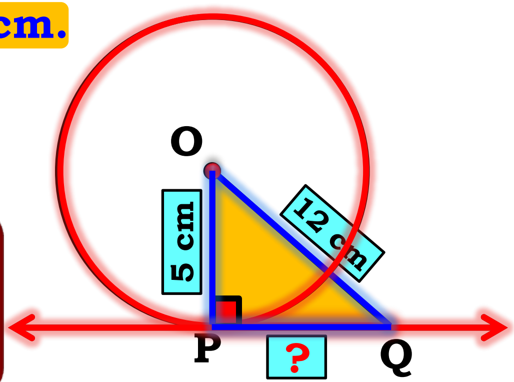
$\therefore \quad 144 = 25 + PQ^2$

$\therefore \quad 144 - 25 = PQ^2$

$\therefore \quad PQ^2 = 119$

$\therefore \quad PQ = \sqrt{119}$

$\therefore \quad$ Length of PQ is $\sqrt{119}$ cm.



MODULE - 4

CIRCLE

- **Sums based on Theorem – Radius is perpendicular to the tangent**

Q. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find radius of the circle

Sol: In ΔPOQ,

$\angle QPO = 90^\circ$ [Radius ... the tangent]

$\boxed{OQ^2} = \boxed{OP^2} + \boxed{PQ^2}$ [By P... m]

$\therefore \quad 25^2 = OP^2 + 24^2$

$\therefore \quad 625 = OP^2 + 576$

$\therefore \quad 625 - 576 = OP^2$

$\therefore \quad OP^2 = 49$

$\therefore \quad OP = \sqrt{49}$ [Taking square root]

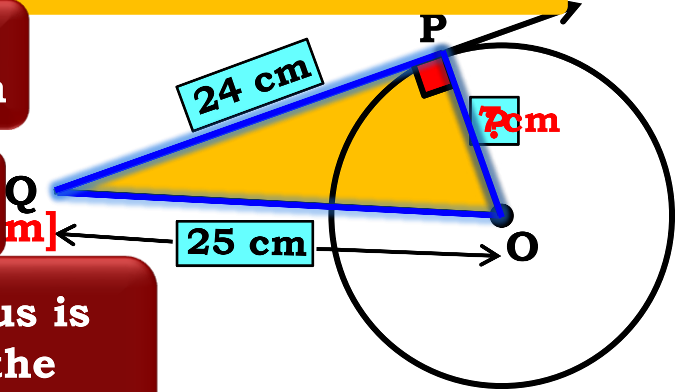
$\therefore \quad OP = 7$ cm

$\therefore$ **The radius of circle is 7cm.**

Consider ΔPOQ
Pythagoras theorem

$\therefore \angle QPO = 90^\circ$

We know that, radius is perpendicular to the tangent



Q. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Sol:

Observe $\angle OBA$

In $\triangle OBA$,

$\angle OBA = 90^\circ$

We know that, radius is perpendicular to the tangent

... r to

$\boxed{OA^2} = \boxed{AB^2} + \boxed{OB^2}$ [By Pythagoras theorem]

$\therefore \quad 5^2 = 4^2 + OB^2$

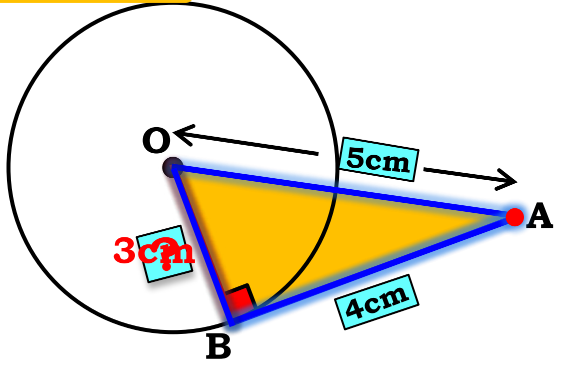
$\therefore \quad 25 = 16 + OB^2$

$\therefore \quad 25 - 16 = OB^2$

$\therefore \quad OB^2 = 9$

$\therefore \quad OB = 3$ [Taking square root]

$\therefore$ **The radius of circle is 3cm.**

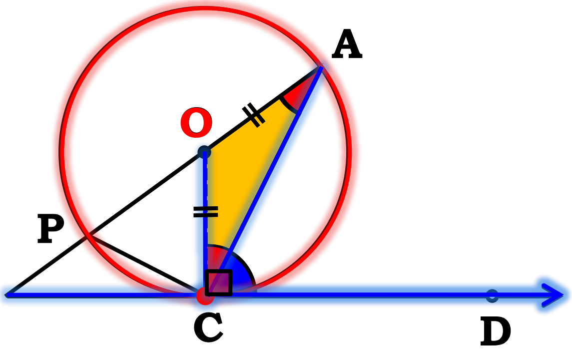


MODULE - 5

CIRCLE

- **Sums based on Theorem – Radius is perpendicular to the tangent**

Q. O is the centre of the circle and BCD is tangent to it at C. prove that $\angle BAC + \angle ACD = 90^\circ$.



Construction: Draw seg OC

Proof: In

$\angle OAC$ can be written as $\angle BAC$

$\therefore \angle OCD = 90^\circ$

$\angle OCD$ is made up of two angles $\angle OCA$ and $\angle ACD$

s OC

OC = OA [radii of

$\therefore$ $\angle OCA$ = $\angle$

$\therefore$ $\boxed{\angle OCA = \angle}$

$\boxed{\angle OCD} = 90^0$ [radius is perpendicular to the tangent]

$\therefore$ $\boxed{\angle OCA} + \angle ACD = 90^0$ [angle addition property]

$\therefore$ $\angle BAC + \angle ACD = 90^0$ [from (i)]

Q. Prove that the angle between the two tangents drawn from an external point to a ... he angle subtended by the li ... ts of contact at the centre.

$\therefore \angle OAP = \angle O\,P = 90^\circ$

Sum of angles is 180°

We know that, radius is perpendicular to the tangent

Given : A circle with ce ...
PA and PB are t ...
from an external point P to the circle.

To prove : $\angle APB + \angle AOB = 180^\circ$

Proof : $\boxed{\angle OAP} = \boxed{\angle OBP} \boxed{= 90^\circ}$ ...(i)

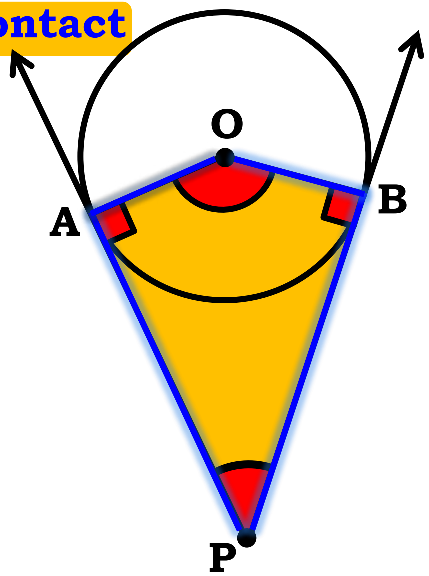
In □APBO,

$\boxed{\angle OAP} + \angle APB + \boxed{\angle OBP} + \angle AOB = 360^\circ$

$\therefore \quad 90 + \angle APB + 90 + \angle AOB = 360$ [from (i)]

$\therefore \quad \angle APB + \angle AOB = 360 - 180$

$\therefore \quad \angle APB + \angle AOB = 180^\circ$



MODULE - 6

CIRCLE

- **Sum based on Theorem – Radius is perpendicular to the tangent**

Q. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

AB is ... ler

Join O a...

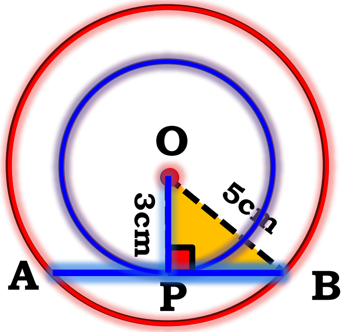
$\therefore \angle OPB = 90^\circ$

To Find : AB

Construction : Draw ...

Sol:

We know that, radius is perpendicular to the tangent



$\boxed{OP = 3cm}$ [radiu...

$\boxed{OB = 5cm}$ [radiu...

In ΔOPB,

$\angle OPB = 90^\circ$ [Radius is perpendicular to the tangent]

$\boxed{OB^2} = \boxed{OP^2} + \boxed{PB^2}$ [By Pythagoras theorem]

$\therefore \quad 5^2 = 3^2 + PB^2$

$\therefore \quad 25 = 9 + PB^2$

$\therefore \quad 25 - 9 = PB^2$

$\therefore \quad PB^2 = 16$

Q. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

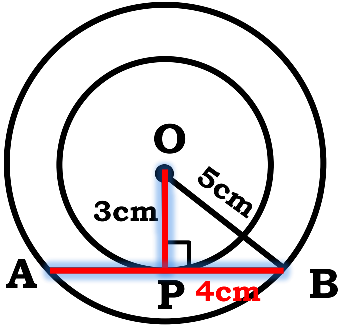
Sol:

$PB^2 = 16$

$\therefore \quad \boxed{PB = 4\text{ cm}}$ **[Taking squ**

OP ⊥ AB

Perpe ... om
the centre to the chord,
bisects the chord



$\boxed{PB} = \dfrac{1}{2}\ AB$ **[Perpendicular from centre of circle to the chord bisect the chord)**

$\therefore \quad 4 = \dfrac{1}{2} \times AB$

$\therefore \quad AB = 4 \times 2$

$\therefore \quad AB = 8\text{ cm}$

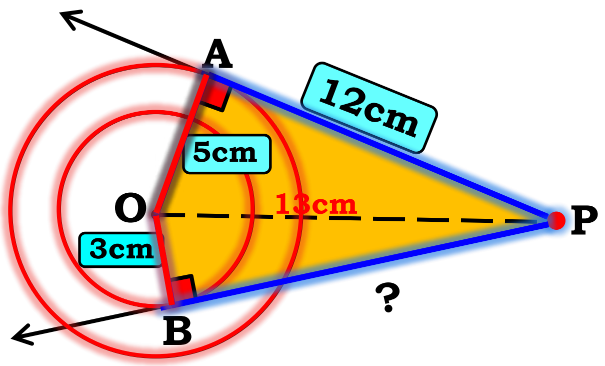
$\therefore$ **Length of the chord AB is 8 cm.**

MODULE - 7

CIRCLE

- **Sum based on Theorem – Radius is perpendicular to the tangent**

Q. There are two concentric circles with O centre of radii 5 cm and 3 cm. from an external point P, tangents PA and PB are drawn to these circles. If AP = 12cm, find the length of BP.



Sol.

In ΔOAP,

$\angle OAP = 90^\circ$ [radi… …t]

$OP^2 = OA^2 + AP$…

$\therefore \; OP^2 = 5^2 + 12$…

We know that, radius is perpendicular to the tangent

$\therefore \; OP^2 = 25 + 144$

$\therefore \; OP^2 = 169$

$\therefore \; OP = 13$ [Taking square-roots]

In ΔOBP,

$\angle OBP = 90^0$ [radius is perpendicular to tangent]

$OB^2 + BP^2 = OP^2$ [By Pythagoras theorem]

$\therefore \; 3^2 + BP^2 = 13^2$

$\therefore \; BP^2 = 169 - 9$

$\therefore \; BP^2 = 160$

$\therefore \; BP = \sqrt{160}$

$\therefore \; BP = \sqrt{16 \times 10}$

$\therefore \; \boxed{BP = 4\sqrt{10} \text{ cm}}$

Thank You

MODULE - 8

CIRCLE

- **Sums based on Theorem – Radius is perpendicular to the tangent**

Q. Prove that the tangents drawn at the ends of a diameter of a circle

We know that, radius is perpendicular to tang

$\angle OAP = 90^\circ$

serve $\angle OBR$

Observe $\angle OAP$

$\angle OBR = 90^\circ$

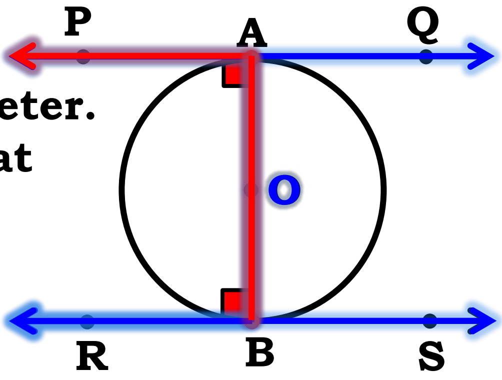
meter.

at

$\angle OAP$ written

$\angle OBR$ can be written as $\angle ABR$

A a



Prove that : li PQ || li RS

Proof : $\boxed{\angle OAP} = 90^\circ$ & $\boxed{\angle OBR} = 90^\circ$

[Radius is perpendicular to the tangent]

AB is the diameter of circle. [Given]

$\therefore$ Points A, O and B are collinear

$\angle BAP = 90^\circ$ ...(i) and $\angle ABR = 90^\circ$ ...(ii)

Adding (i) and (ii)

$\angle BAP + \angle ABR = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ Interior angles for PQ and RS on transversal AB are supplementary.

$\therefore$ line PQ || line RS [By interior angle test]

Q. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Given : A circle with centr ... t to the circle at T.

$PT \perp AB$

$\therefore \angle OTA = 90^\circ$

Consider point P such that $PT \perp AB$

Prove that : PT passe...

Proof : $OT \perp AB$ [Ra... the tangent]

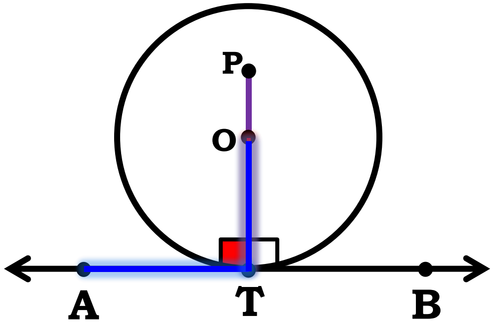
But $PT \perp AB$ [Given]

$\therefore$ OT and PT are perpendicular to same line AB at same point T on it.

$\therefore$ OT and PT are one and the same line.

[One and only one perpendicular can be drawn to a given line at a given point on it]

$\therefore$ PT passes through centre 'O'



MODULE - 9

CIRCLE

- **Theorem – The lengths of two tangents drawn from an external point to a circle are equal.**

THEOREM :

The lengths of ... **external point to a circ**...

Given : (i) A ci...

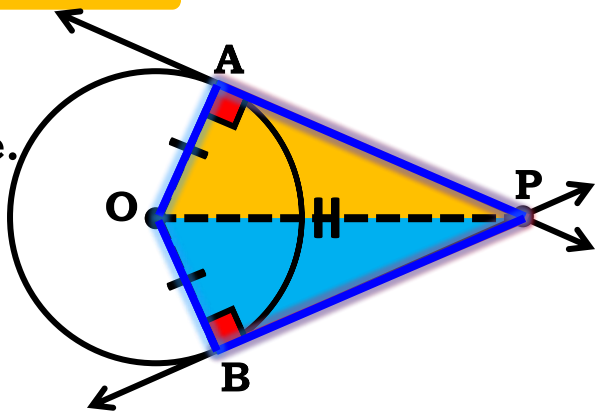
(ii) P is a ... rcle.

(iii) PA and ...

To prove : PA = PI...

∴ ∠OAP = ∠OBP = 90°

We know that, radius is perpendicular to the tangent



Construction : Draw OA, OB and OP

Proof : In ΔPAO and ΔPBO,

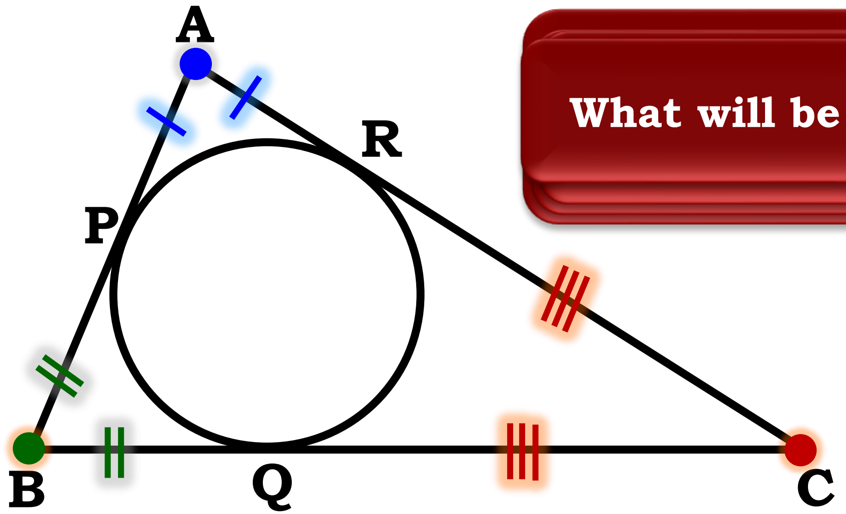
∠PAO = ∠PBO = 90° [Radius is perpendicular to the tangent]

Hypt. OP = Hypt. OP [Common side]

OA = OB [radii of the same circle]

∴ ΔPAO ≅ ΔPBO [RHS rule]

∴ PA = PB [c.p.c.t]



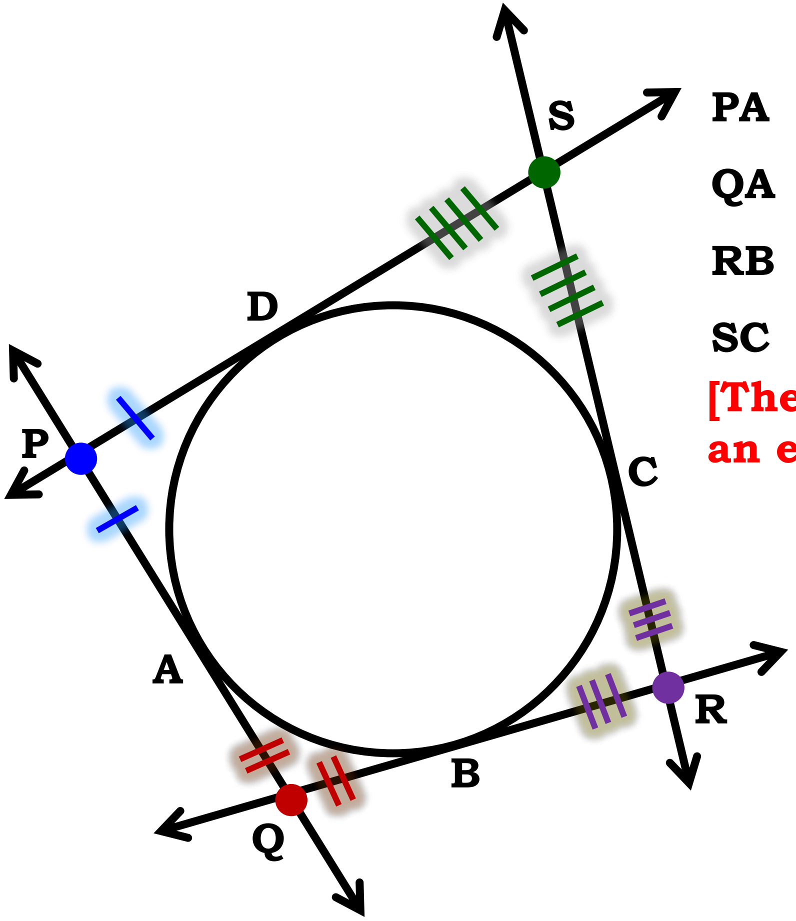
What will be the reason ?

AP = AR

BP = BQ

CQ = CR

[The lengths of two tangents drawn from an external point to a circle are equal]



PA = PD

QA = QB

RB = RC

SC = SD

Name the tangents from external point S

Let us consider point S

SC and SD

[The lengths of two tangents drawn from an external point to a circle are equal]

MODULE - 10

CIRCLE

- **Sum based on Theorem – The lengths of two tangents drawn from an external point to a circle are equal.**

Q. ABC is an isosceles in which AB = AC, circumscribed about a circle, as Show Prove that the base is bisected by the point of contact.

Prove that : BE = CE

Proof :

AD = AF ...(i)

BD = BE ...(ii)

CE = CF ...(iii)

[T
d
a

We know, tangents from an external point to a circle are equal in length.

CE and CF are tangents to the circle

AB = AC ...(iv)

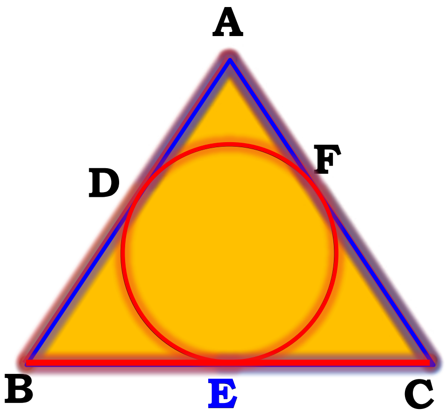
Subtracting (i) from (iv)

AB – AD = AC – AF

∴ BD = CF [A–D–B, A–F–C]

∴ BE = CE [from (ii) & (iii)]

∴ Base BC is bisected at E



MODULE - 11

CIRCLE

- Sum based on Theorems –
- Two tangents from an external point to a circle are equal and Radius is perpendicular to the tangent

Q. If tangents PA and PB from a point P to a circle with centre O are inclined to eac ... OA

We k ... les
of ... °

Observe ∠PBO

We know, radius is perpendicular to tangent

Sol: In □APBO,

$\boxed{\angle APB = 80^\circ}$

$\boxed{\angle PAO} = \boxed{\angle PBO = 90^\circ}$ ... cular

$\boxed{\angle AOB} + \boxed{\angle APB} + \boxed{\angle PAO} + \boxed{\angle PBO} = 360^\circ$

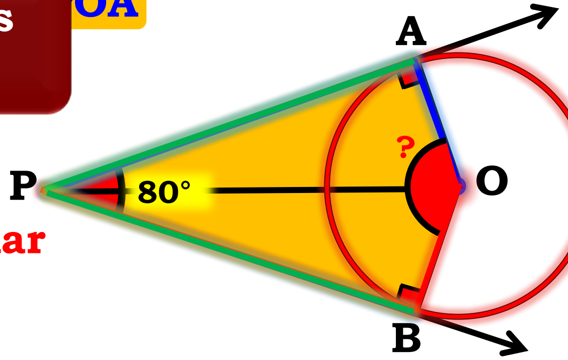
[Sum of all angles of quadrilateral is 360°]

$\therefore \quad \angle AOB + 80^\circ + 90^\circ + 90^\circ = 360^\circ$

$\therefore \quad \angle AOB + 260 = 360$

$\therefore \quad \angle AOB = 360 - 260$

$\therefore \quad \angle AOB = 100^\circ \quad ...(i)$

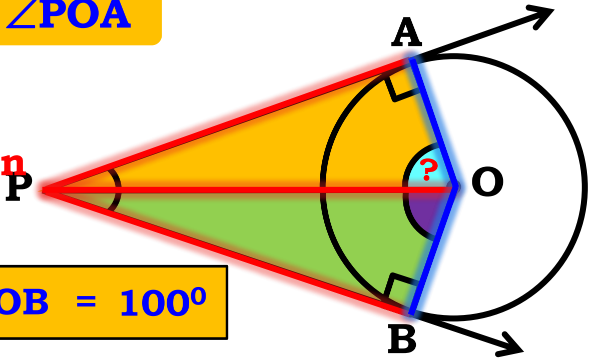


Q. If tangents PA and PB from a point P to a circle with centre O are inclined to e ind $\angle POA$

$\angle AOB$ is made up of two angles

$\angle POA$, $\angle POB$

Consider ΔAOP and ΔBOP



$\angle AOB = 100^0$

Sol: In ΔAOP and Δ

PA = PB [Len om an externa ual]

OA = OB [Radii

OP = OP [Common side]

$\therefore$ $\Delta POA \cong \Delta POB$ [SSS test]

$\therefore$ $\boxed{\angle POA = \angle POB}$ ...(ii) [c.p.c.t]

$\therefore$ $\angle AOB = \angle POA + \boxed{\angle POB}$ ...(iii) [Angle addition property]

$\therefore$ $\boxed{\angle AOB} = \angle POA + \angle POA$

$\therefore$ $100 = 2\angle POA$

$\therefore$ $\boxed{\angle POA = 50^o}$

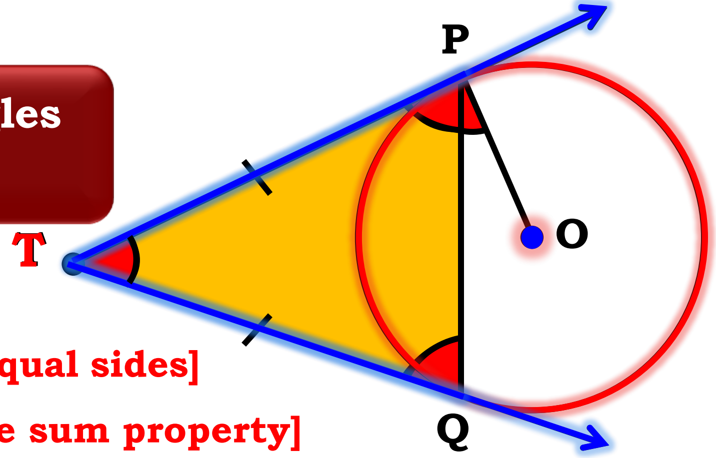
MODULE - 12

CIRCLE

- **Sum based on Theorems –**
- **Two tangents from an external point to a circle are equal and Radius is perpendicular to the tangent**

Q. Two tangents TP and TQ are drawn to a circle with centre O from an external point T.

Prove that $\angle PTQ = 2\angle OPQ$



Proof :

We ... angles ... 30⁰

Consider ΔPTQ

In ΔPTQ,

$PT = TQ$ [... drawn from an external point to a circle are equal]

$\therefore \boxed{\angle TPQ = \angle TQP}$ ...(i) [Angles opposite to equal sides]

$\angle PTQ + \angle TPQ + \boxed{\angle TQP} = 180^\circ$ [Angle sum property]

$\therefore \angle PTQ + \boxed{\angle TPQ + \angle TPQ} = 180^\circ$ [from (i)]

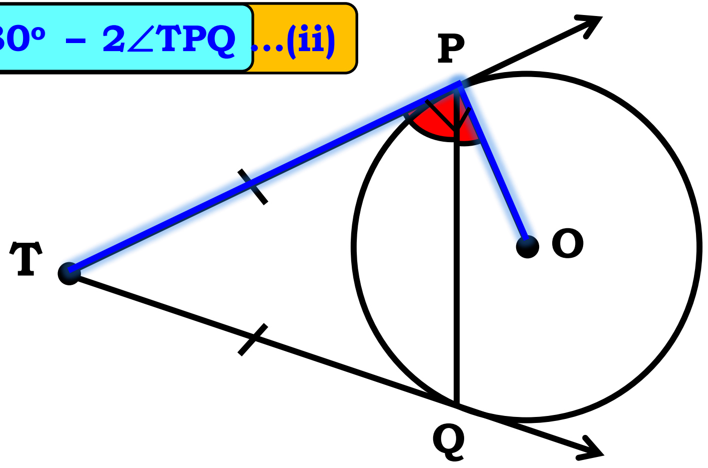
$\therefore \angle PTQ + 2\angle TPQ = 180^\circ$

$\therefore \angle PTQ = 180^\circ - 2\angle TPQ$ ...(ii)

Q. Two tangents TP and TQ are drawn to a circle with centre O from an external point T.

Prove that $\angle PTQ = 2\angle OPQ$

$\angle PTQ = 180^\circ - 2\angle TPQ$ ...(ii)



Proof :

$\angle OPT = 90^\circ$ [Radius is perpendicular to tangent]

$\angle OPQ + \angle TPQ = \angle O$ $\angle OPT = 90^0$

$\therefore$ $\angle OPQ$

$\angle OPQ$ is a part of $\angle OPT$

$\therefore$

$\therefore$ $2\angle OPQ = 180^\circ - 2\angle TPQ$ ...(iii)

$\angle OPT$ is made up of two angles

$\angle TPQ$

Let us multiply throughout by 2

$\therefore$ n (ii) and (iii)]

MODULE - 13

CIRCLE

- **Sum based on Theorem – The lengths of two tangents drawn from an external point to a circle are equal.**

Q. A ... dra... ircle.

BC

We know, tangents drawn from external point are equal

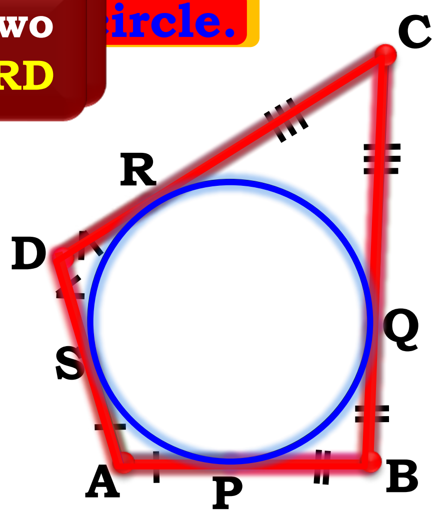
CD is made up of two segments CR and RD

To prove : $\boxed{AB}\boxed{+}\boxed{CD}$ = AD + BC

Proof :

AP	=	AS	...(i)
PB	=	BQ	...(ii)
CR	=	CQ	...(iii)
RD	=	DS	...(iv)

(Tangents drawn from external point to a circle are equal)



Adding (i), (ii), (iii) & (iv)

$$\underline{AP + PB} + \underline{CR + RD} = \underline{AS} + \underline{BQ + CQ} + \underline{DS}$$

$$\therefore \quad (\underline{AP + PB}) + (\underline{CR + RD}) = (\underline{AS + DS}) + (\underline{BQ + CQ})$$

$$\therefore \quad AB + CD = AD + BC$$

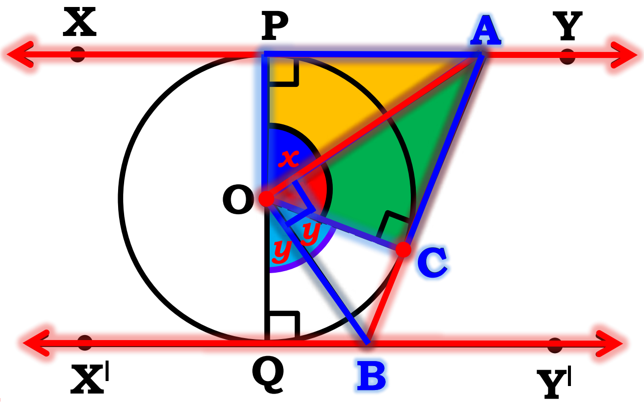
MODULE - 14

CIRCLE

- Sum based on Theorems –
- Two tangents from an external point
  to a circle are equal and
  Radius is perpendicular to the tangent

Q. In fig. XY and X'Y' … ents to a circle with centre O and … int of contact C intersecting … Prove that ∠A…

We know that, radius is perpendicular to tangent



Construction : Draw OC

Proof : In ΔOPA and ΔOCA,

$OP = OC$ [radii of same circle]

$OA = OA$ [common side]

$AP = AC$ [length of the tangents drawn from an external point to the circle are equal]

$\therefore \Delta OPA \cong \Delta OCA$ [by SSS congruency]

$\therefore \angle POA = \angle COA$ [c.p.c.t.]

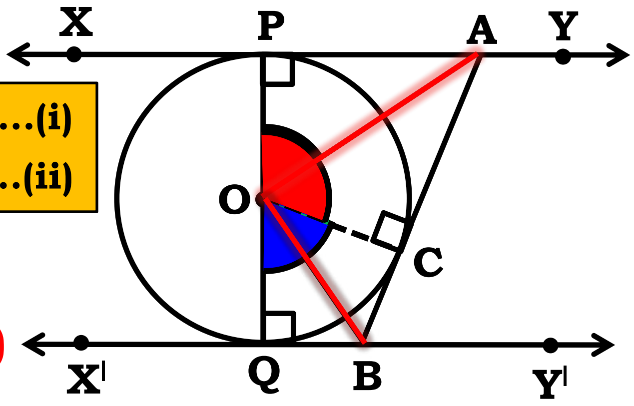
Let $\angle POA = \angle COA = x$ …(i)

Similarly, we can get

$\angle QOB = \angle COB = y$ …(ii)

Q. In fig. XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B.

Prove that ∠AOB = 90°.



$\angle POA = \angle COA = x$ ...(i)

$\angle QOB = \angle COB = y$ ..(ii)

Proof :

$\angle POC + \angle QOC = 180^\circ$

$\therefore \angle POA + \angle COA + \angle QOB + \angle COB = 180^\circ$

∠AOB is made up of two angles ∠AOC and ∠BOC

$\therefore$ ... $y = 180$ [From (i) and (ii)]

$\therefore$ ... $2y = 180$

$\therefore$ ... $y = 90^\circ$ ...(iii)

∠Q... f two

an... ∠COB

Observe ∠AOB

...gle addition property]

$\therefore \angle AOB$ = ... [From (iii)]

∴ Their sum is 180°

$\therefore \angle$ ... [From (iii)]

Thank You

MODULE - 15

CIRCLE

- Sum based on Theorem – The lengths of two tangents drawn from an external point to a circle are equal.

Q. PA and PB are tangents from an external point P to a circle with circle O. LN touches the circle at M

prove that PL + LM =

MN and NB are tangents from external point N

To Prove : PL + LM = P

Proof :

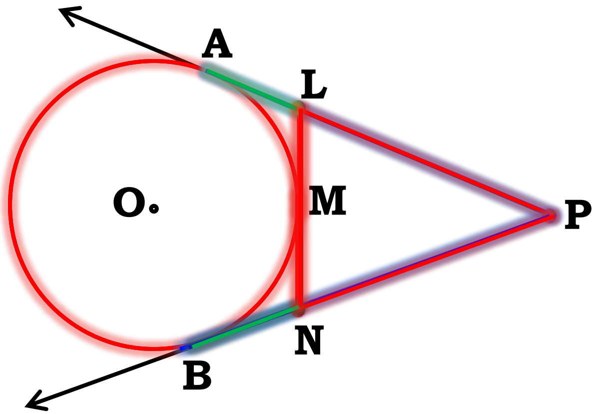
$$\left.\begin{array}{ll} PA = PB & \dots(i) \\ \boxed{LA = LM} & \dots(ii) \\ \boxed{MN = NB} & \dots(iii) \end{array}\right\}$$ (Length of two tangents from external point to circle are equal)

$\boxed{PA} = \boxed{PB}$ [From (i)]

∴ B-N-P]

We know, tangents drawn from an external point to a circle are equal in length.

∴ (ii) and (iii)]



MODULE - 16

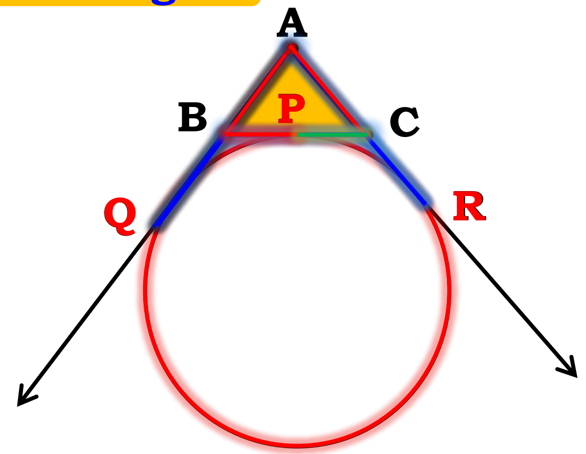
CIRCLE

- **Sum based on Theorem – The lengths of two tangents drawn from an external point to a circle are equal.**

Q. A circle touching the side BC of ΔABC at P and touching AB and AC produced at Q and R respectively.

Prove that AQ = $\frac{1}{2}$ (Perimeter of ΔABC)

Sum of all sides



Proof :

AQ = AR ...(i)

BP = BQ ...(ii)

CP = CR ...(iii)

[The tangents from an external point to a circle are equal in length.]

Now,

+ BC + AC

+ BP + CP + AC

+ BQ + CR + AC [From (ii) and (iii)]

AR

Q [From (i)]

What can you say about AQ and AR? tangents to the circle

We know, tangents from an external point to a circle are equal in length.

∴

∴ AQ = $\frac{1}{2}$ (Perimeter of ΔABC)

MODULE - 17

CIRCLE

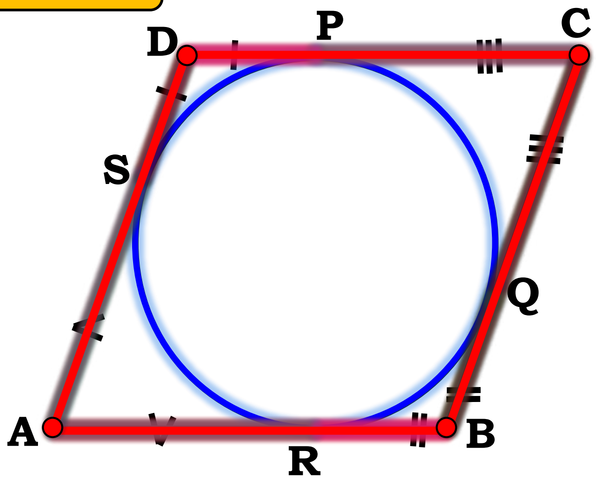
- **Sum based on Theorem – The lengths of two tangents drawn from an external point to a circle are equal.**

We know, tangents drawn from external point are equal

Q. Pr nscribing a circle is a rhombus.

To nbus.

Hint : prove : AB = BC



Proof :

AR	=	AS	...(i)
BR	=	BQ	...(ii)
CP	=	CQ	...(iii)
PD	=	DS	...(iv)

[Length of the tangents drawn from an external point to a circle are equal]

Adding (i), (ii), (iii) & (iv)

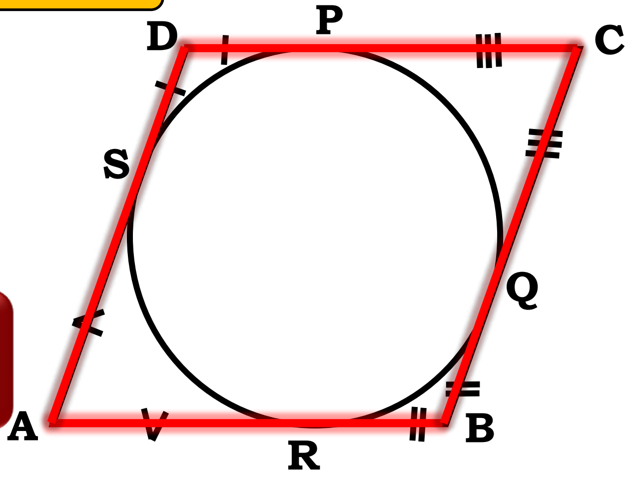
$$(AR + BR) + (CP + PD) = (AS + SD) + (BQ + CQ)$$

$\therefore$ $AB + CD = AD + BC$ ...(v) [A-R-B, B-Q-C, C-P-D, A-S-D]

Q. Prove : Parallelogram circumscribing a circle is a rhombus.

To prove : □ABCD is a rhombus.

Hint : prove : AB = BC



Proof :

$\boxed{AB} + \boxed{CD} = \boxed{AD} + \boxed{BC}$ ...(v)

But, $\boxed{AB = CD}$ ...(vi)

$\boxed{AD = BC}$ ...(vii)

[Opposite sides of a parallelogram are equal]

In a parallelogram opposite sides are equal

$AB + AB = BC + BC$

$\therefore \cancel{2}AB = \cancel{2}BC$

$\therefore AB = BC$

∴ A pair of adjacent sides in a parallelogram is equal

∴ □ABCD is a rhombus. (A parallelogram is a rhombus if a pair of adjacent sides is equal)

MODULE - 18

CIRCLE

- **Sum based on Theorem – The lengths of two tangents drawn from an external point to a circle are equal.**

Q. A circle is inscribed in a ΔABC having sides 8 cm, 10 cm and 12 cm. Find AD, BE and CF.

Sol:

$$\left.\begin{aligned} AD &= AF = x \\ BD &= BE = y \\ CE &= CF = z \end{aligned}\right\}$$ [Tangents drawn from an external point to a circle are equal length]

Now,

$\boxed{AB} = 12$ cm

$\therefore \boxed{AD} + \boxed{DB} = 12$

$\therefore \boxed{x + y} = 12$ ...(i)

$\therefore \boxed{y + z} = 8$ ...(ii)

$\therefore \boxed{z + x} = 10$ ...(iii)

Adding (i), (ii) and (iii)

$+ 10$

$\therefore$

$\therefore$

$\therefore \quad x + y + z = 15$

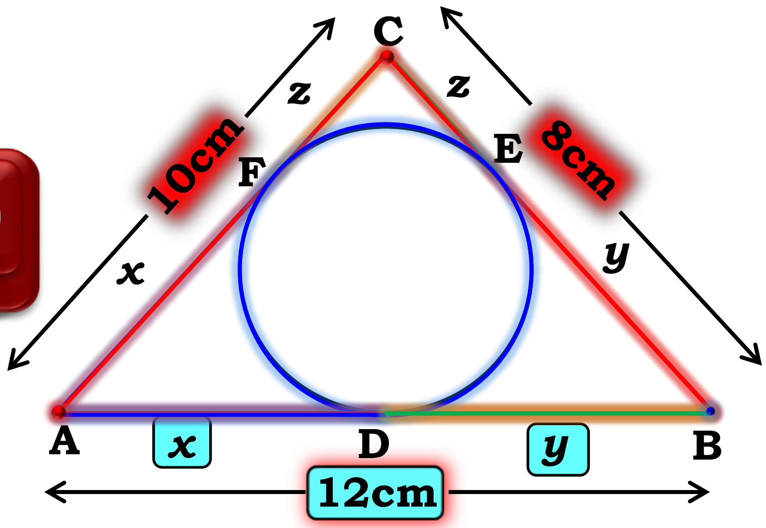
Let u ... ii)

Similarly,

CF + AF = 10cm

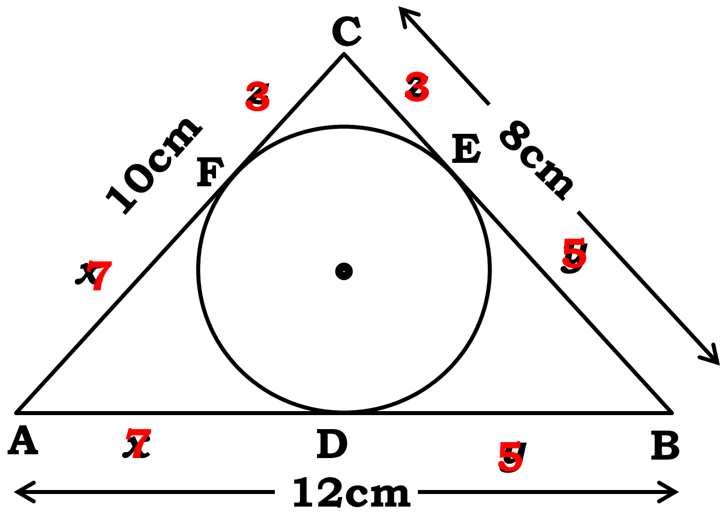
$\therefore z + x = 10$cm

We know that, tangents from an external point to a circle are equal in length



Q. A circle is inscribed in a ΔABC having sides 8 cm, 10 cm and 12 cm. Find AD, BE and CF.

$\boxed{x + y = 12}$, $\boxed{y + z = 8}$, and $\boxed{z + x = 10}$



Sol:

$$\boxed{x + y} + z = 15$$

$$\therefore \quad 12 + z = 15$$

$$\therefore \quad z = 3$$

$$\therefore \quad CF = 3\text{cm}$$

$$x + \boxed{y + z} = 15$$

$$\therefore \quad x + 8 = 15$$

$$\therefore \quad x = 7$$

$$\therefore \quad AD = 7\text{cm}$$

$$\boxed{x} + y \boxed{+ z} = 15$$

$$\therefore \quad y + 10 = 15$$

$$\therefore \quad y = 5$$

$$\therefore \quad BE = 5\text{cm}$$

Hence, AD = 7 cm, BE = 5 cm and CF = 3 cm.

MODULE - 19

CIRCLE

- **Sum based on Theorem – The lengths of two tangents drawn from an external point to a circle are equal.**

Q. ABC is a right triangle right-angled at A such that AB = 6 cm and AC = 8 cm. Find the radius of its incircle.

Sol. Seg OL $\perp$ side AB

Seg OM $\perp$ side BC

Seg ON $\perp$ side AC

[Radius is perpendicular to the tangent]

We know that, Radius is perpe t

Consider ΔABC

In ΔABC, $\angle A = 9$

m $\angle CAB = 90^\circ$

$\therefore CB^2 = CA^2 + AB^2$ [By Pythagoras theorem]

$\therefore CB^2 = 8^2 + 6^2$

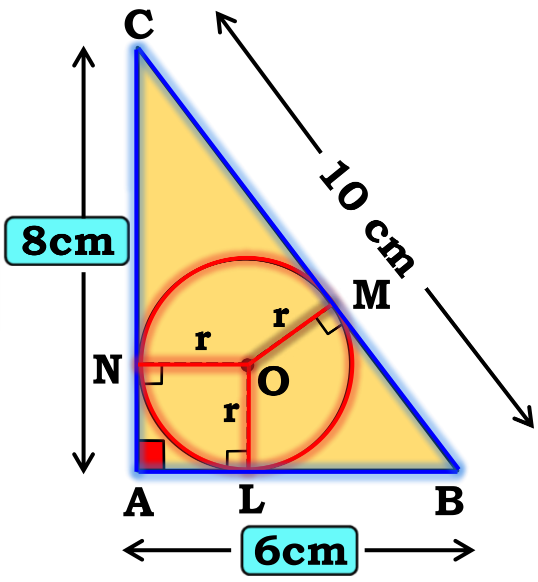
$\therefore CB^2 = 64 + 36$

$\therefore CB^2 = 100$

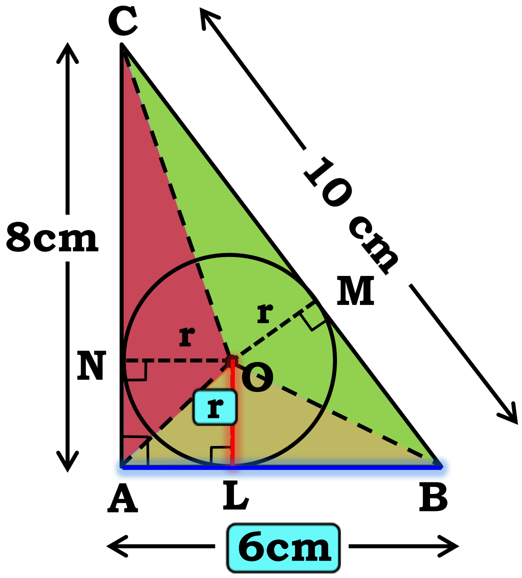
$\therefore CB = 10\text{cm}$ ...(i) [Taking square roots]

Let the radius of the circle be r

OL = OM = ON = r ...(ii) [Radii of the same circle]



Q. ABC is a right triangle right-angled at B such that AB = 6 cm and AC = 8 cm. Find the radius of its incircle.



Sol.

$$\text{ar}(\Delta AOB) = \frac{1}{2} \times \boxed{AB} \times \boxed{OL}$$

$$\therefore \boxed{\text{ar}(\Delta AOB)} = \frac{1}{2} \times 6 \times r \boxed{= 3r\ \text{cm}^2}$$

Simi

ar (Δ

Area

ar (ΔBOC) cm

ΔABC is made up of 3 triangles

ΔAOB, ΔBOC, ΔAOC

Also,

$$\text{ar}(\Delta ABC) = \frac{1}{2} \times 8 \times 6 = 24\ \text{cm}^2$$

$$\boxed{\text{ar}(\Delta AOB)} + \boxed{\text{ar}(\Delta BOC)} + \boxed{\text{ar}(\Delta AOC)} = \text{ar}(\Delta ABC)$$ [Area Addition Property]

$$\therefore 3r + 5r + 4r = 24$$

$$\therefore 12r = 24$$

$$\therefore \boxed{r = 2}$$

MODULE - 20

CIRCLE

- **Sum based on Theorems –**
- **Two tangents from an external point to a circle are equal and Radius is perpendicular to the tangent**

Sum of angles is 180°

Q. Prove ... f a quadrilateral circumscribing a circle, subtend supplementary angles at the centre of the circle.

Whenever, we see centre and point of contact we always draw radius

To prove : (i) ...

(ii) $\angle$ ...

Construction : ...

Proof : In ΔAPO and ΔASO

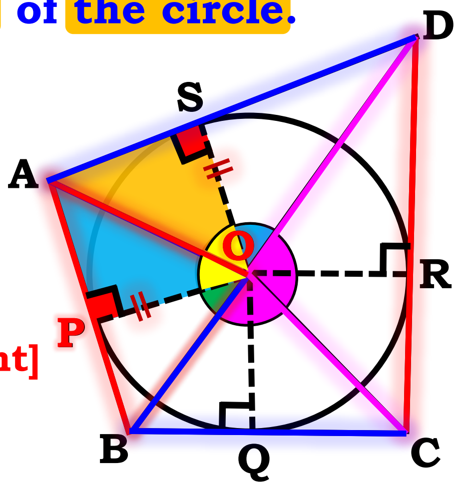
$\angle APO = \angle ASO = 90^\circ$ [Radius is perpendicular to tangent]

$AO = AO$ [common side]

$OP = OS$ [radius of the same circle]

$\therefore \quad \Delta APO \cong \Delta ASO$ [RHS rule]

$\therefore \quad \angle AOP = \angle AOS$ [c.p.c.t.]



Q. Prove the opposite sides ... **a circle, subtend supplem**... **the circle.**

To prove : (i) $\angle AOB + \angle COD = 180^\circ$

(ii) $\angle AOD + \angle BOC = 180^\circ$

Sol:

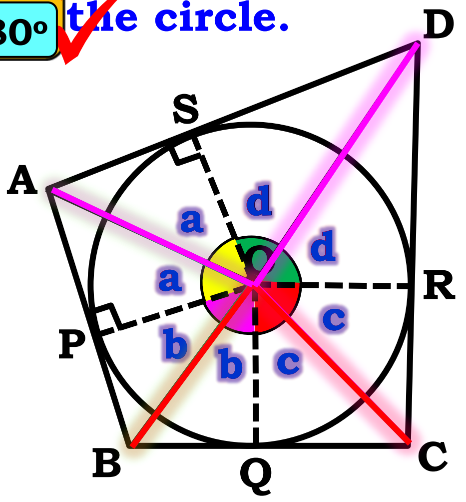
Let, $\angle AOP = \angle AO$...

Similarly, $\angle BOP = \angle BO$...

$\angle COQ = \angle CO$...

$\angle DOR = \angle DOS = d^\circ$ ...(iv)

We know, sum of all angles at a point is 360°



$\angle AOB + \angle BOC + \angle COD + \angle AOD = 360^\circ$

[Sum of all angles at a point is 360°]

$\therefore$ $a + b + b +$ ... 360°

It can also be written as

$a + d + b + c = 180^\circ$

$\therefore$ ... 360°

$\therefore$ $2\ (a + b + c$ ... $= 360^\circ$

$\therefore$ $a + b + c + d = 180^\circ$ ...(v)

$\therefore$ $(a + b) + (c + d) = 180^\circ$

$\therefore$ $\angle AOB + \angle COD = 180^\circ$

$\therefore$ $(a + d) + (b + c) = 180^\circ$ [from (v)]

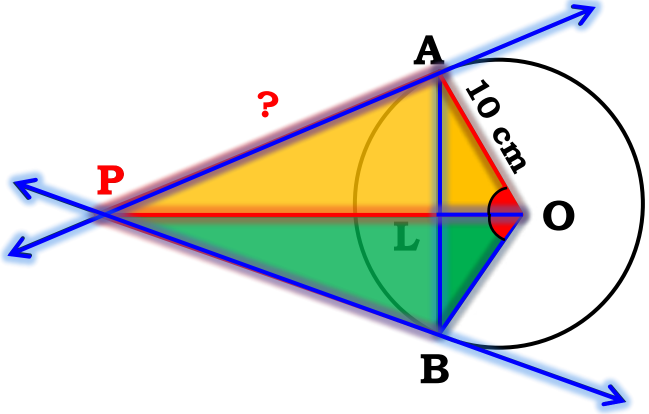
$\therefore$ $\angle AOD + \angle BOC = 180^\circ$

MODULE - 21

CIRCLE

- Sum based on Theorems –
- Two tangents from an external point
  to a circle are equal and
  Radius is perpendicular to the tangent

Q. AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.



Sol. In ΔAOP and ΔBOP

$OA = OB$ (radii of same circle)

$OP = OP$ (common side)

$PA = PB$ [Tange ternal point t ength]

Consider ΔAOL and ΔBOL

$\therefore \Delta AOP \cong \Delta BOP$ (SSS test)

$\therefore \angle AOP = \angle BOP$ (c.p.c.t.)

$\therefore \boxed{\angle AOL = \angle BOL}$ ...(i) (P-L-O)

In ΔAOL and ΔBOL

$OA = OB$ (radii of same circle)

$\angle AOL = \angle BOL$ [from (i)]

$OL = OL$ (common side)

$\therefore \Delta AOL \cong \Delta BOL$ (SAS test)

Q. AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.

Sol. $\therefore$ $\angle OLA = \angle OLB$ (c.p.c.t.)

Le

We know that, Perpendicular drawn from the centre to the chord, bisects the chord

r pair)

$\therefore$

$\therefore$

$\therefore$ $x = 90^0$

$\therefore$ $OL \perp AB$

$\therefore$ $AL = LB = \frac{1}{2}\boxed{AB}$ (Perpendicular from the centre to the chord, bisects the chord)

$\therefore$ $AL = LB = \frac{1}{\cancel{2}} \times \overset{8}{\cancel{16}}$

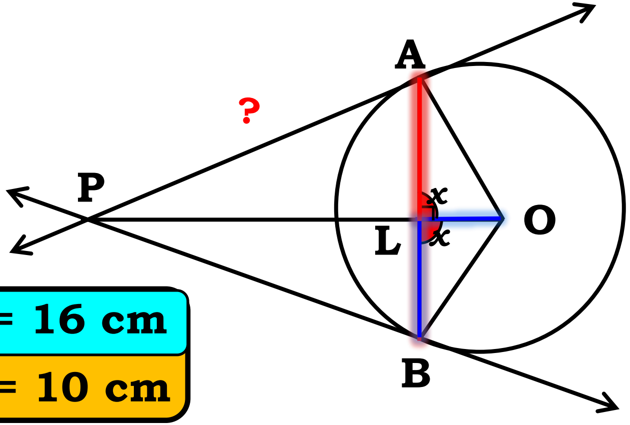
$\therefore$ $AL = LB = 8$ cm

$\angle OLA$ and $\angle OLB$ are what type of angle?

Linear pair

AB = 16 cm

AO = 10 cm



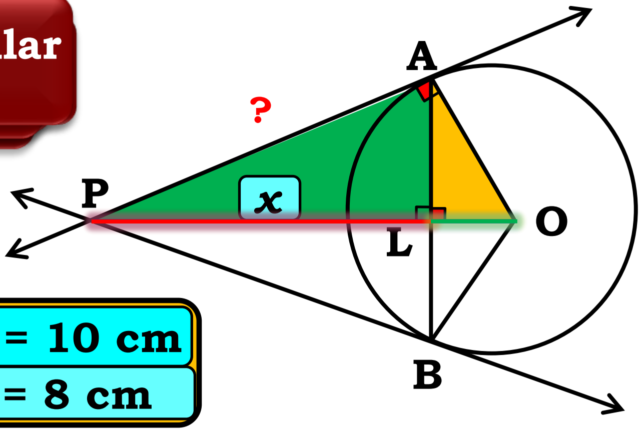
MODULE - 22

CIRCLE

- **Sum based on Theorems –**
- **Two tangents from an external point to a circle are equal and Radius is perpendicular to the tangent**

Q. AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.

Radius is perpendicular to tangent



AO = 10 cm
AL = 8 cm

Sol. In ΔOLA,

$\angle OLA = 90^0$

$OA^2 = AL^2 + OL^2$ (by Pythagoras theorem)

$\therefore \quad 10^2 = 8^2 + OL^2$

$\therefore \quad OL^2 = 100 - 64$

$\therefore \quad OL^2 = 36$

$\therefore \quad \boxed{OL = 6cm}$

$PO = PL + OL$

$\boxed{PO = x + 6}$

In ΔPAO,

$\angle PAO = 90^0$

$PA^2 + OA^2 = OP^2$ (by

$(a + b)^2 = a^2 + 2ab + b^2$

$\therefore \quad PA^2 + 10^2 = (x + 6)^2$

$\therefore \quad PA^2 = x^2 + 12x + 36 - 100$

$\therefore \quad PA^2 = x^2 + 12x - 64 \quad \ldots(ii)$

Q. AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.

$$\boxed{PA^2 = x^2 + 12x - 64 \quad ..(ii)}$$

Sol. In ΔPLA,

$\angle PLA = 90^0$

$PA^2 = \boxed{AL^2} + \boxed{PL^2}$ (by Pythagoras theorem)

$\therefore \quad PA^2 = 8^2 + x^2$

$\therefore \quad \boxed{PA^2 = 64 + x^2 \quad ...(iii)}$

From (ii) and (iii),

$\cancel{x^2} + 12x - 64 = 64 + \cancel{x^2}$

$\therefore \quad 12x = 64 + 64$

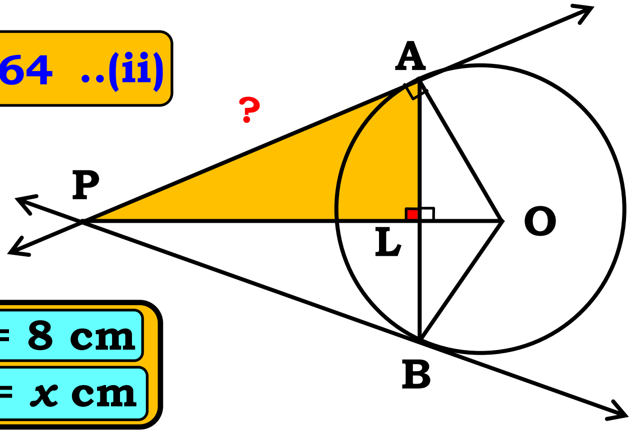
$\therefore$

Now, let us consider ΔPLA

32

$\therefore \quad x = \frac{\quad}{\cancel{12}_{3}}$

$\therefore \quad x = \frac{32}{3}$



$\boxed{AL = 8 \text{ cm}}$

$\boxed{PL = x \text{ cm}}$

Q. AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.

Sol. $\therefore \quad \boxed{x = \frac{32}{3}}$

Substituting value in (iii)

$$PA^2 = 64 + \boxed{x^2}$$

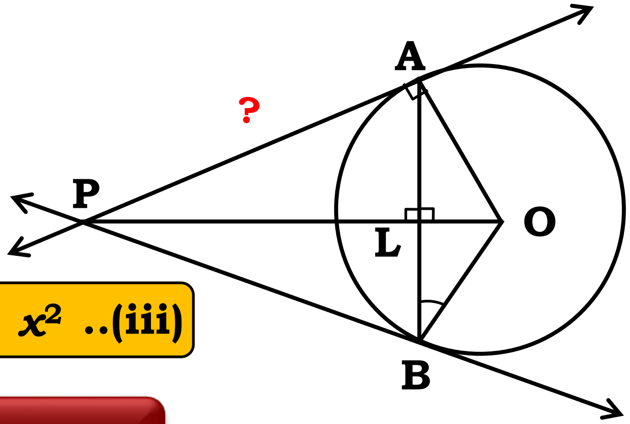
$$\therefore \quad PA^2 = 64 + \left(\frac{32}{3}\right)^2$$

$$= 64 + \frac{1024}{9}$$

$$= \frac{576 + 1024}{9}$$

$$\therefore \quad PA^2 = \frac{1600}{9}$$

$$\therefore \quad \boxed{PA = \frac{40}{3} \text{ cm}}$$



$PA^2 = 64 + x^2$ ..(iii)

Taking square root

Thank You

MODULE - 23

CIRCLE

- Sum based on Theorems –
- Two tangents from an external point
  to a circle are equal and
  Radius is perpendicular to the tangent

Q. A triangle ABC is drawn to circumscribe a circle of radius 4cm such th … to which BC is divided by t … ngths 8cm and 6cm respect … C.

Dra

pe

Consider ΔOBC

Area of triangle = ½ × base × height

Sol:

$CD = CQ = 6$ cm
$BD = BR = 8$ cm
$AQ = AR = x$ cm

[Tangents from an external point to a circle are equal in length]

$$A(\Delta OBC) = \frac{1}{2} \times \boxed{BC} \times \boxed{OD}$$

$$= \frac{1}{\cancel{2}} \times 14 \times \cancel{4}^{2}$$

Wh … of

Consider point A

we can draw radius

$$\therefore \boxed{A(\Delta OBC) = 28cm^2}$$

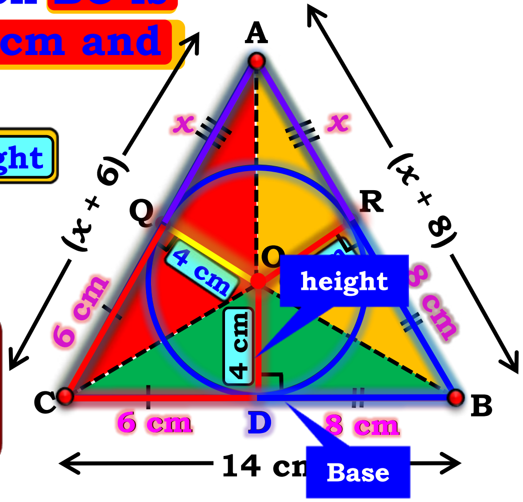
We know, tangents from an external point to a circle are equal in length.

$$\therefore \boxed{A(\Delta OAC) = 2(x + 6)cm^2}$$

$$A(\Delta OAB) = \frac{1}{2} \times \boxed{AB} \times \boxed{OR}$$

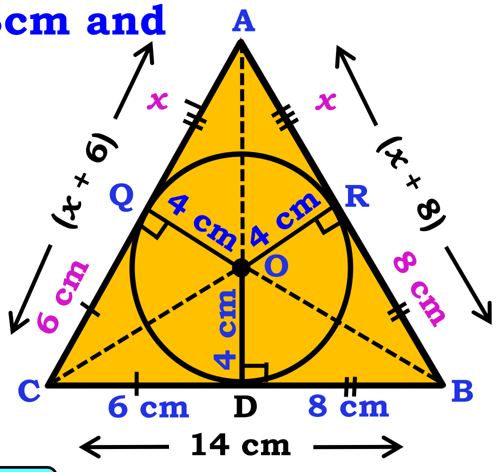
$$= \frac{1}{\cancel{2}} \times (x + 8) \times \cancel{4}^{2}$$

$$\boxed{A(\Delta OAB) = 2(x + 8)cm^2}$$



Q. A triangle AB… …ircle of radius 4cm such th… …which BC is divided by t… …ths 8cm and 6cm respect…

Now let us apply heron's formula to find area of ΔABC



Sol: $A(\Delta ABC) = \boxed{A(\Delta OBC)} + \boxed{A(\Delta OAC)} + \boxed{A(\Delta OAB)}$

$= 28 + 2(x + 6) + 2(x + 8)$

$= \underline{28} + \underline{2x} + \underline{12} + \underline{2x} + \underline{16}$

$= 56 + 4x$

$A(\Delta ABC) = 4\ (14 + x)$ ...(i)

$A(\Delta OBC) = 28\text{cm}^2$

$A(\Delta OAC) = 2(x + 6)\text{cm}^2$

$A(\Delta OAB) = 2(x + 8)\text{cm}^2$

For ΔABC,

Let $\boxed{a = AC = x + 6}$, $\boxed{b = AB = x + 8}$, and $\boxed{c = BC = 14}$.

Semi-perimeter of ΔABC (s) $= \dfrac{\boxed{a} + \boxed{b} + \boxed{c}}{2}$

$= \dfrac{x + 6 + x + 8 + 14}{2}$

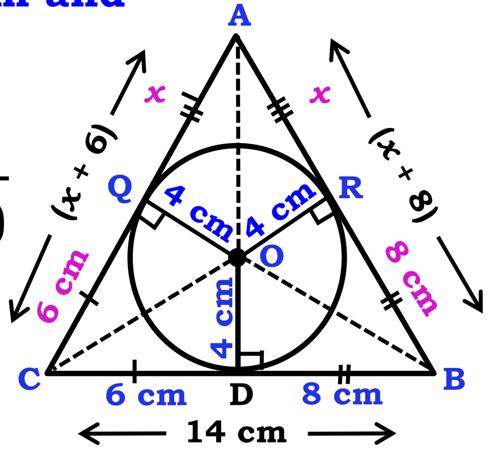
$= \dfrac{(2x + 28)}{2} = \dfrac{2\ (x + 14)}{2} = (x + 14)$ cm

MODULE - 24

CIRCLE

- Sum based on Theorems –
- Two tangents from an external point to a circle are equal and Radius is perpendicular to the tangent

Q. A triangle ABC is drawn to circumscribe a circle of radius 4cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8cm and 6cm respectively. **Find the sides AB and AC.**



Sol: $A(\Delta ABC) = \sqrt{\boxed{s}(s - \boxed{a})(s - \boxed{b})(s - \boxed{c})}$

$= \sqrt{(x + 14)(x + 14 \ldots \cancel{14} - 14)}$

$= \sqrt{(x + 14)(\cancel{x} + 1 \ldots}$

What is heron's formula to find area of triangle?

$\sqrt{s(s - a)(s - b)(s - c)}$

$= \sqrt{(x + 14)(8)(6)(x)}$

$\boxed{A(\Delta ABC) = 4(14 + x) \ldots (i)}$

$\boxed{A(\Delta ABC) = \sqrt{(x + 14)(x)(48)}}$

$\boxed{a = x + 6,}$ $\boxed{b = x + 8,}$ $\boxed{c = 14,}$ $\boxed{s = x + 14}$

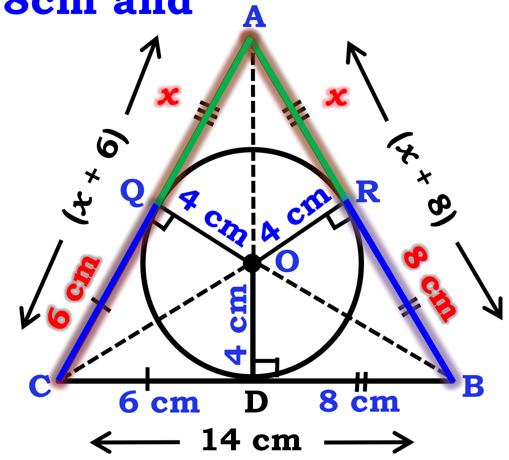
$\therefore \quad \sqrt{48(x)(x + 14)} = 4(14 + x)$ [from (i) and (ii)]

$\therefore \quad 48 \times x \times (x + 14) = 4^2(14 + x)^2$ [squaring both sides]

$\therefore \quad \overset{3}{\cancel{48}} \times x \times \cancel{(x + 14)} = \cancel{16}\cancel{(14 + x)}^2$

$\therefore \quad 3x = 14 + x$

Q. A triangle ABC is drawn to circumscribe a circle of radius 4cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8cm and 6cm respectively. **Find the sides AB and AC.**



Sol:

$$3x = 14 + x$$

$$\therefore \quad 3x - x = 14$$

$$\therefore \quad 2x = 14$$

$$\therefore \quad \boxed{x = 7}$$

$$AC = \boxed{AQ} + \boxed{CQ}$$
$$= \boxed{x} + 6$$
$$= 7 + 6$$
$$\therefore \quad \boxed{AC = 13 \text{ cm}}$$

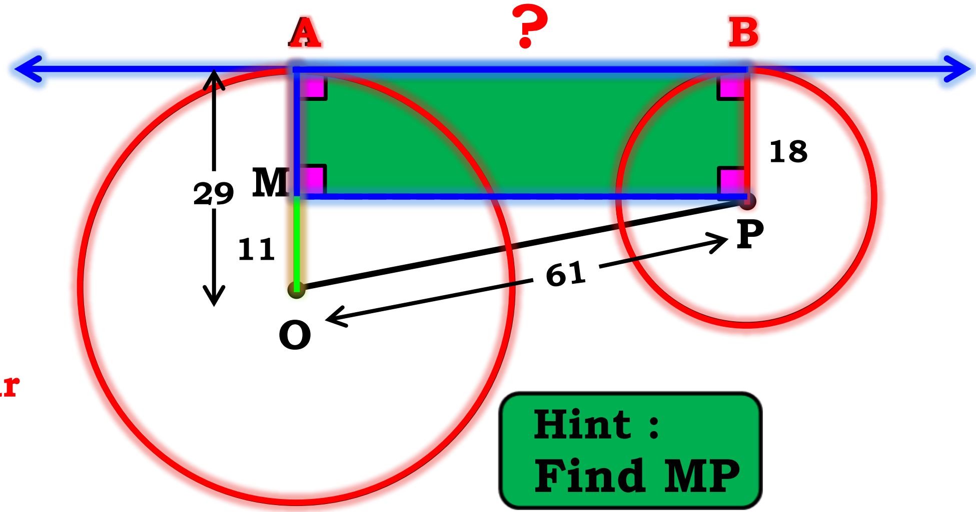
$$AB = \boxed{AR} + \boxed{BR}$$
$$= \boxed{x} + 8$$
$$= 7 + 8$$
$$\therefore \quad \boxed{AB = 15 \text{ cm}}$$

MODULE - 25

CIRCLE

- **Sum based on theorem-**
  Radius is perpendicular to the tangent

Q. In the adjoining figure, line AB is tangent to both the circles touching at A and B. OA = 29, BP = 18, OP = 61 then find AB.



Hint : Find MP

Construction: Draw seg PM ⊥ seg OA

Sol. In □PBAM,

$\angle PBA = 90^\circ$ } [Radius is perpendicular
$\angle BAM = 90^\circ$ } to the tangent]

$\angle PMA = 90^\circ$ [Construction]

$\therefore \angle MPB = 90^\circ$ [Remaining Angle]

$\therefore$ □PBAM is a rectangle [By definition]

We know, opposite sides of a rectangle are equal

$\therefore$ PB = AM = 18 units ...(i) }
$\therefore$ AB = MP ...(ii) } [Opposite sides of a rectangle]

AB = MP

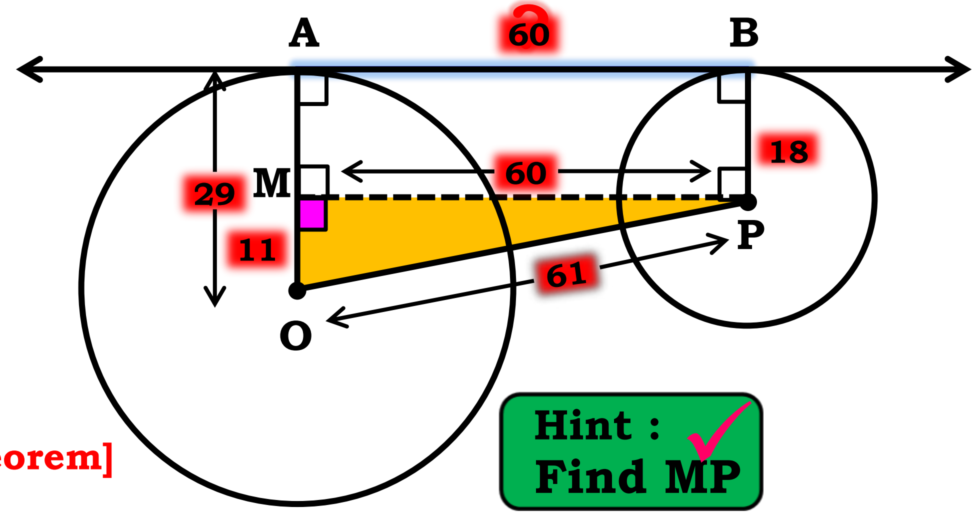
$\boxed{OA} = OM + \boxed{AM}$ [∵ A-M-O]

$\therefore$ 29 = OM + 18

$\therefore$ OM = 29 − 18

$\therefore$ OM = 11 units

Q. In the adjoining figure, line AB is tangent to both the circles touching at A and B. OA = 29, BP = 18, OP = 61 then find AB.



Hint : Find MP

Sol. OM = 11 units

In ΔPMO,

$\angle PMO = 90^\circ$

$\therefore \quad \boxed{OP^2} = \boxed{OM^2} + PM^2$ [Pythagoras theorem]

$\therefore \quad 61^2 = 11^2 + PM^2$

$\therefore \quad 3721 = 121 + PM^2$

$\therefore \quad PM^2 = 3721 - 121$

$\therefore \quad PM^2 = 3600$

Taking square roots

$\therefore \quad PM = 60$ units

But, AB = PM [Opposite sides of a rectangle]

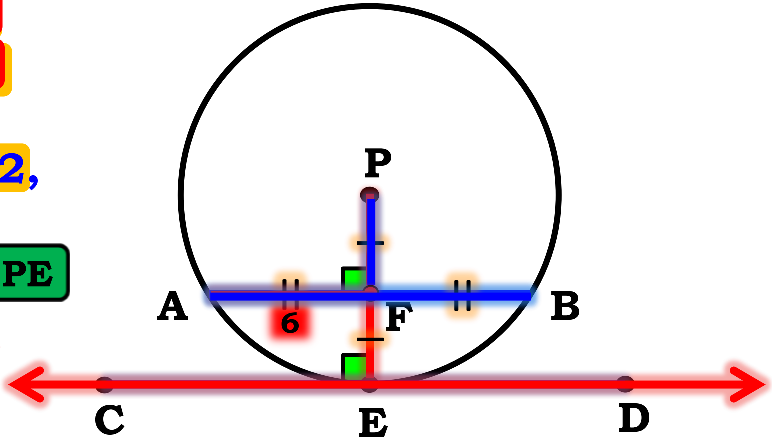
$\therefore \quad \boxed{AB = 60 \text{ units}}$

MODULE - 26

CIRCLE

- **Sum based on theorem-**
  Radius is perpendicular to the tangent

Q. In a circle with centre P, a chord AB is parallel to a tangent and intersects the radius drawn from a point of contact at the midpoint of the radius. If AB = 12, find the radius of the circle.



To find : PE

Sol. $\angle PEC = 90^\circ$ ...(i) [Radius is perpendicular to the tangent]

We know that, radius is perpendicular to the tangent

$\therefore$ ...wn

...to

$\therefore$ ...g angles]

$\therefore$ ...i)]

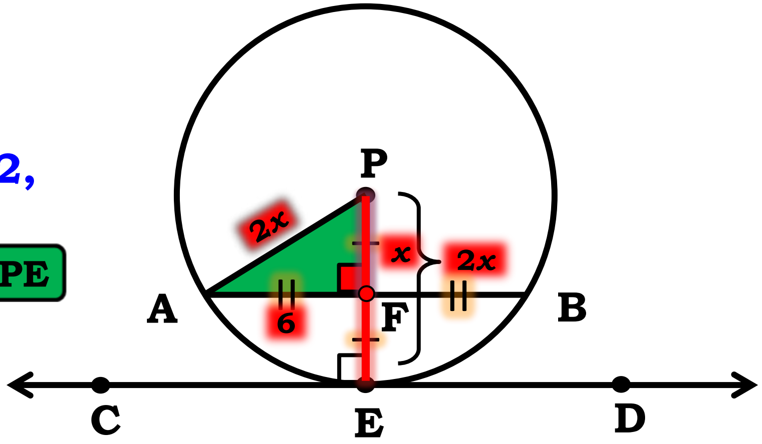
$\therefore$ $PF \perp AB$ [From (iii)]

$\therefore$ $AF = \frac{1}{2} \times \boxed{AB}$ [The perpendicular drawn from the centre of the circle to the chord bisects the chord]

$\therefore$ $AF = \frac{1}{\cancel{2}} \times \cancel{12}^{6}$

$\therefore$ $AF = 6$ units

Q. In a circle with centre P, a chord AB is parallel to a tangent and intersects the radius drawn from a point of contact at the midpoint of the radius. If AB = 12, find the radius of the circle.



To find : PE

Sol. $\boxed{AF = 6 \text{ units}}$

Let the radius of the circle be $2x$ units

$\therefore \boxed{PA} = \boxed{PE = 2x \text{ units}}$ [Radii of same circle]

$\therefore PF = \frac{1}{2} \times \boxed{PE}$ [$\because$ F is the midpoint of PE]

$\therefore PF = \frac{1}{\cancel{2}} \times \cancel{2}x$

$\therefore \boxed{PF = x \text{ units}}$

Consider ΔPFA

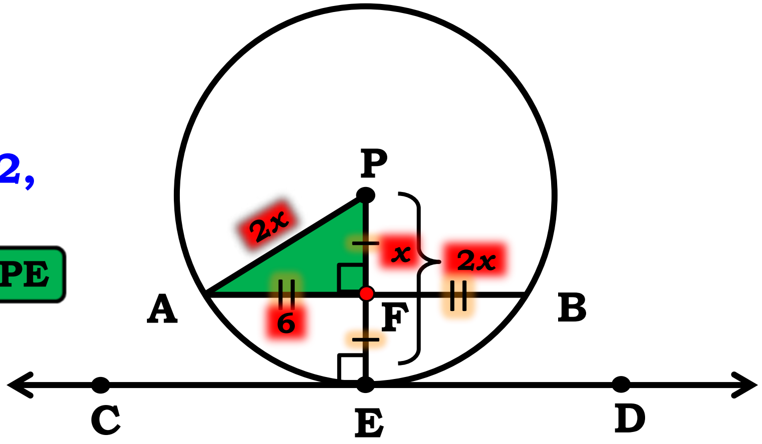
In ΔPFA, $\angle PFA = 90^\circ$

$\boxed{PA^2} = \boxed{PF^2} + \boxed{AF^2}$ [By Pythagoras theorem]

$\therefore (2x)^2 = x^2 + 6^2$

$\therefore 4x^2 = x^2 + 36$

Q. In a circle with centre P, a chord AB is parallel to a tangent and intersects the radius drawn from a point of contact at the midpoint of the radius. If AB = 12, find the radius of the circle.



To find : PE

$\boxed{PA = PE = 2x \text{ units}}$

Sol.

$4x^2 = x^2 + 36$

$\therefore \quad 4x^2 - x^2 = 36$

$\therefore \quad 3x^2 = 36$

$\therefore \quad x^2 = 12$

$\therefore \quad x = \sqrt{12}$ [Taking square roots]

$\therefore \quad x = \sqrt{4 \times 3}$

$\therefore \quad \boxed{x = 2\sqrt{3}}$

$PA = PE = 2\boxed{x}$

$= 2 \times 2\sqrt{3}$

$\therefore \quad PA = PE = 4\sqrt{3}$

$\therefore$ **Radius of the circle is $4\sqrt{3}$ units.**

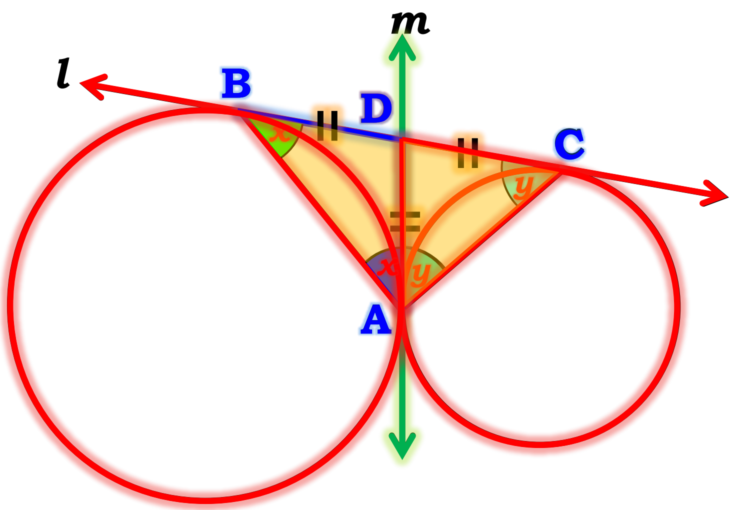
MODULE - 27

CIRCLE

- **Sum based on Theorem – The lengths of two tangents drawn from an external point to a circle are equal.**

Q. Point A is a common point of contact of two externally touching circles and line l is a common tangent to both the circles touching at B and C. Line m is another common tangent at A and it intersects BC at D. Prove : (i) $\angle BAC = 90^\circ$

(ii) Point D is the midpoint of seg BC.



Proof. In ΔBDA,

We know, length of two tangents drawn from an external point to a circle are equal

Let, $\angle DBA = \angle DAB = x^\circ$

We know, angles opposite to equal sides are equal

In ΔDAC,

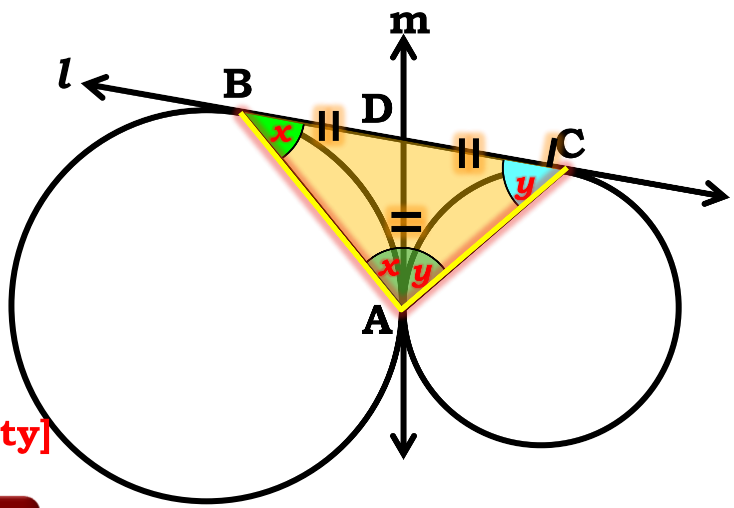
$DC = DA$ ...(iii) [The

external point to a circle are equal]

$\therefore \quad \angle DCA = \angle DAC$

Let, $\angle DAC = \angle DCA = y^\circ$ ...(iv)

Q. Point A is a common point of contact of two externally touching circles and line l is a common tangent to both the circles touching at B and C. Line m is another common tangent at A and it intersects BC at D. **Prove : (i) $\angle BAC = 90^\circ$**
(ii) Point D is the midpoint of seg BC.



$\angle DBA = \angle DAB = x$ ...(ii)
$\angle DCA = \angle DAC = y$ ...(iv)

Proof.

$\angle BAC = \angle DAB + \angle DAC$ [Angle Addition property]

$\therefore \angle BAC = (x + y)^\circ$ ...(v) [From (ii) and (iv)]

Observe $\angle BAC$

In ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180^\circ$ [Angle sum property]

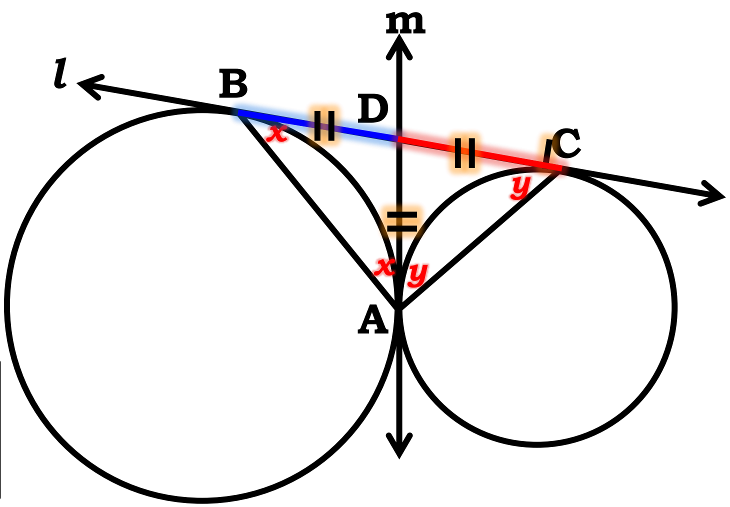
$\therefore x + y + x + y = 180$ [From (ii), (iv), (v)]

$\therefore 2x + 2y = 180$

$\therefore x + y = 90$

$\therefore \angle BAC = 90^\circ$ [From (v)]

Q. Point A is a common point of contact of two externally touching circles and line l is a common tangent to both the circles touching at B and C. Line m is another common tangent at A and it intersects BC at D. Prove : (i) $\angle BAC = 90^\circ$
(ii) Point D is the midpoint of seg BC.



Proof.

DB = DA	...(i)
DC = DA	...(iii)

DB = $\boxed{\text{DA}}$ [from (i)]

DC = $\boxed{\text{DA}}$ [from (iii)]

$\therefore$ DB = DC [from (i) & (iii)]

$\therefore$ D is the midpoint of seg BC.

MODULE - 28

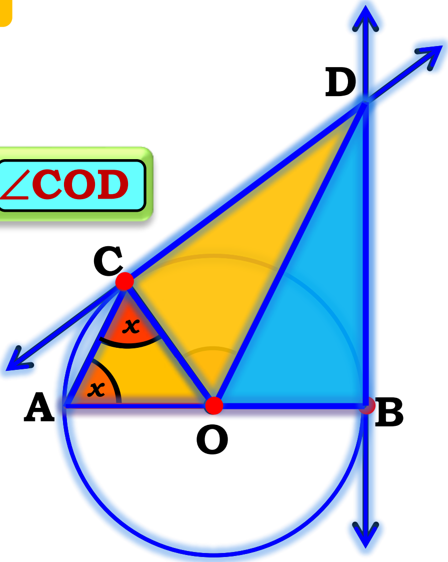
CIRCLE

- Sum based on Theorems –
- Two tangents from an external point
  to a circle are equal and
  Radius is perpendicular to the tangent

Q. O is the centre and seg AB is a diameter. At the point C on th… Line BD is a tan… Sho… lel

Now consider ΔCOD and ΔBOD

Hint: $\angle ACO = \angle COD$



Construction : Draw OC.

Proof : In ΔOAC,

$OA = OC$ [Radii of same circle]

$\therefore \angle OAC = \angle OCA$ [Angles opp. to equal sides are equal]

Let, $\angle OAC = \angle OCA = x$ ...(i)

In ΔOCD and ΔOBD,

$OC = OB$ [Radii of same circle]

$CD = BD$ [The length of the two tangents from an external point to a circle are equal]

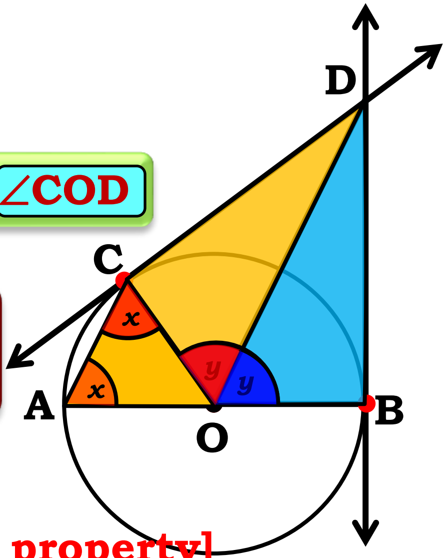
$OD = OD$ [Common side]

$\therefore \Delta OCD \cong \Delta OBD$ [By SSS test of congruence]

Q. O is the centre and seg AB is a diameter. At the point C
on the ci... e Line BD is a
tangent...
Show th...

Now, for ΔACO,
$\angle$COB is an exterior angle

Hint: $\angle$ACO = $\boxed{\angle\text{COD}}$



Proof : $\Delta OCD \cong \Delta OBD$ **[By SSS test of congruence]**

$\therefore$ $\angle COD = \angle BOD$ [c...

$\angle$COB is made up of 2 angles $\angle$COD and $\angle$BOD

Let, $\boxed{\angle COD = \angle BOD = \ldots}$

$\angle OAC + \angle OCA = \boxed{\angle COB}$

$\therefore$ $\boxed{\angle OAC + \angle OCA} = \boxed{\angle COD + \angle BOD}$ **[Angle addition property]**

$\therefore$ $x + x = y + y$

$\therefore$ $\cancel{2}x = \cancel{2}y$

$\therefore$ $x = y$

$\therefore$ $\angle ACO = \angle COD$

$\boxed{\angle OAC = \angle OCA = x \quad \ldots(i)}$

$\therefore$ OD $\parallel$ AC **[Alternate angles test]**

MODULE - 29

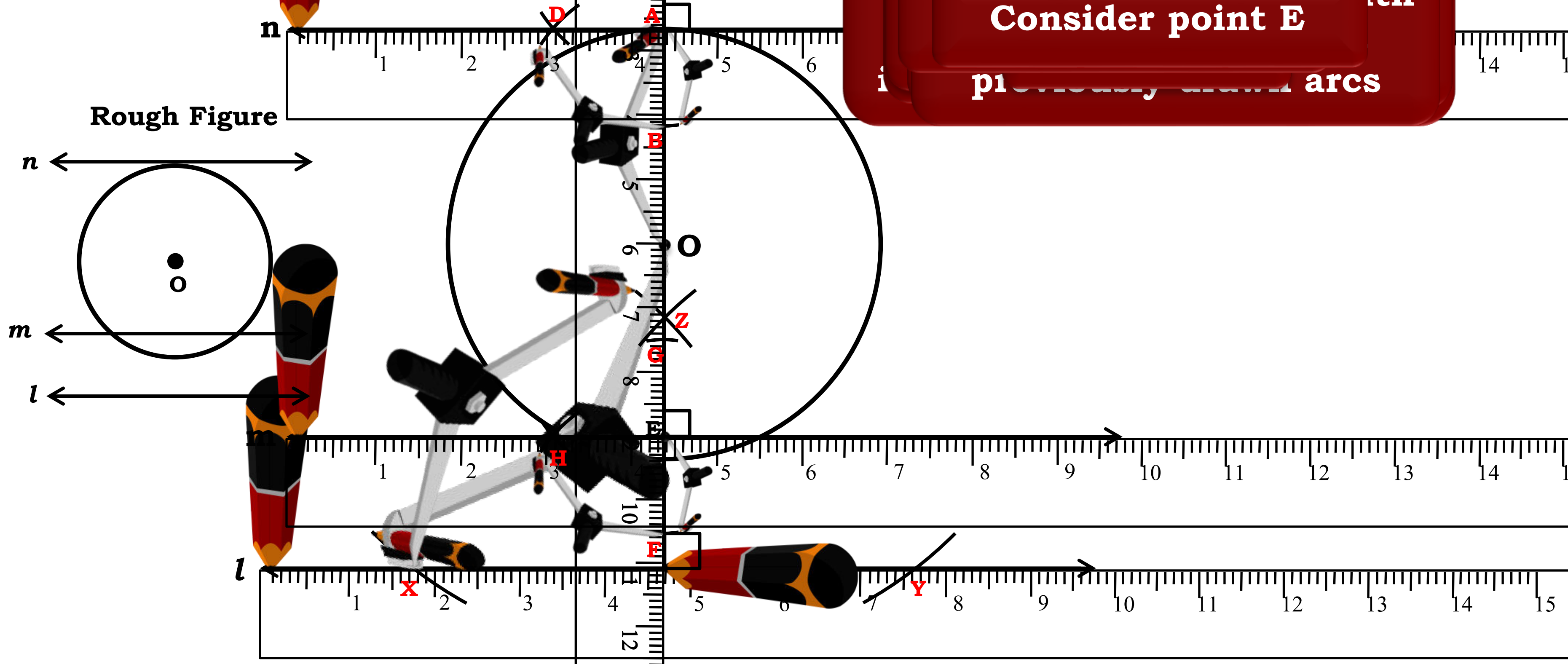
CIRCLE

- **Sum based on constructing tangent and secant**

Q. Draw a circle and two lines parallel to a given line such that one is tangent and the other, a secant to the circle.

Consider point E

Rough Figure



Thank You