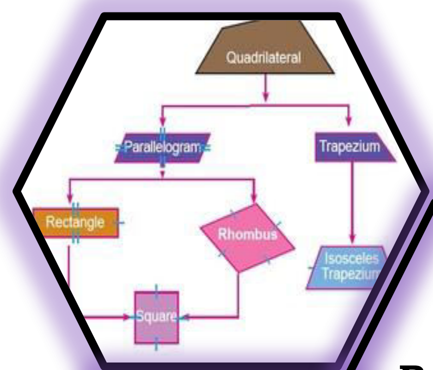
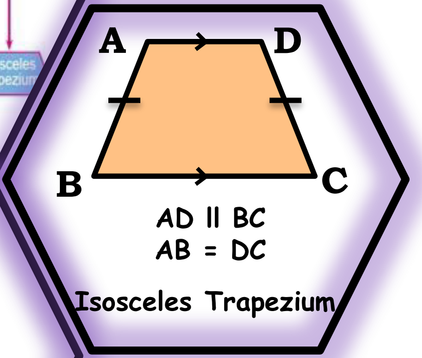
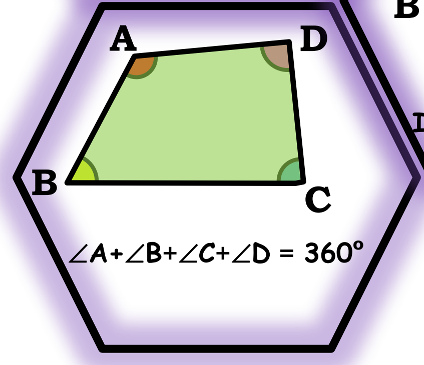
MODULE 1

QUADRILATERALS







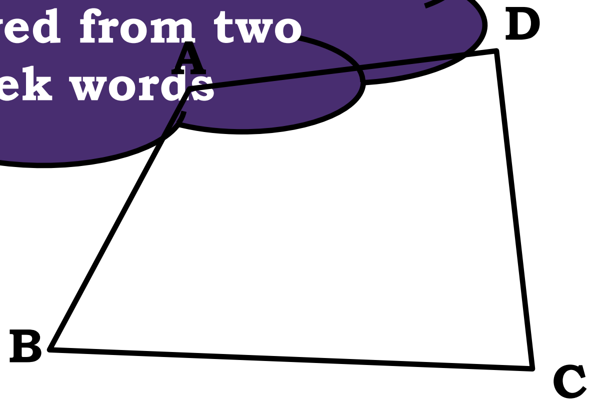
QUADRILATERALS

⇒ **FOUR**

⇒ **SIDES**

Definition : **Any four sided closed figure is called a Quadrilateral**

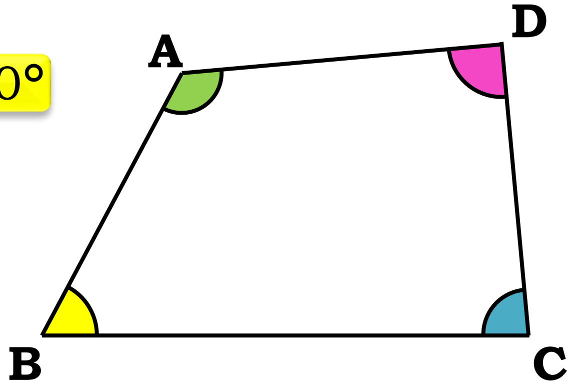
The word 'QUADRILATERAL' is derived from two Greek words



ANGLE SUM PROPERTY OF A QUADRILATERAL

❖ **Statement:**

Sum of all the angles of a quadrilateral is 360°



In □ABCD,

∠A + ∠B +

Name all the angles of □ABCD

∠A, ∠B, ∠C & ∠D

Q. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13.
Find all the angles of the quadrilateral.

Soln. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13.

Let the common multiple be x.

$\therefore$ Angles are $(3x)^\circ$, $(5x)^\circ$, $(9x)^\circ$ and $(13x)^\circ$.

$\therefore$ $3x + 5x + 9x + 13x = 360$ [Angle ... lateral]

$\therefore$ $30x = 360$

$\therefore$ $x = \dfrac{\cancel{360}^{12}}{\cancel{30}}$

$\therefore$ $\boxed{x = 12}$

$3x = 3 \times 12 = 36^\circ$

$5x = 5 \times 12 = 60^\circ$

$9x = 9 \times 12 = 108^\circ$

$13x = 13 \times 12 = 156^\circ$

The angles of the quadrilateral are 36°, 60°, 108° and 156°

$3x^\circ$, $5x^\circ$, $9x^\circ$ & $13x^\circ$

The angles are ?

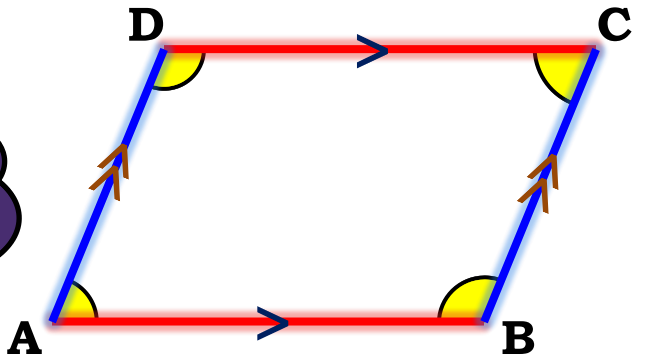
MODULE 2

Types of Quadrilaterals

- **PARALLELOGRAM**
- **RECTANGLE**
- **RHOMBUS**
- **SQUARE**
- **TRAPEZIUM**
- **KITE**

➤ PARALLELOGRAM

Definition : **A quadrilateral is called a Parallelogram if its opposite sides are parallel.**



□ABCD is a Parall

∴ AB || CD and
AD || BC

Similarly,
$\angle A + \angle B = 180^\circ$
and $\angle C + \angle D = 180^\circ$

Interior angles are supplementary

Adjacent angles of a paral ... plementary

PROPERTIES OF A PARALLELOGRAM

- **Opposite sides are equal**

  AB = CD

  AD = BC

- **Opposite a… l**

  ∠A = ∠C

  ∠B = ∠D

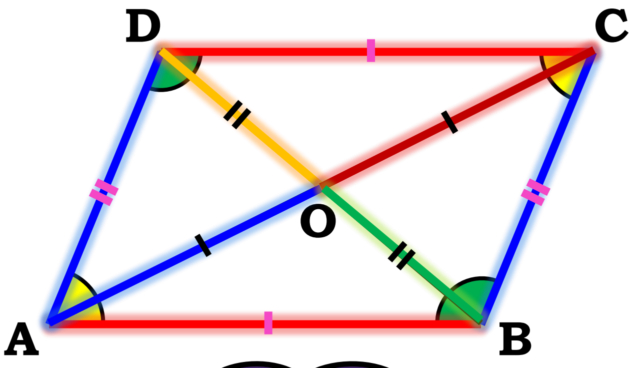
- **Diagonals bisect each oth…**

  OA = OC

  OB = OD

Name the pairs of opposite sides

Name the pairs of opposite angles



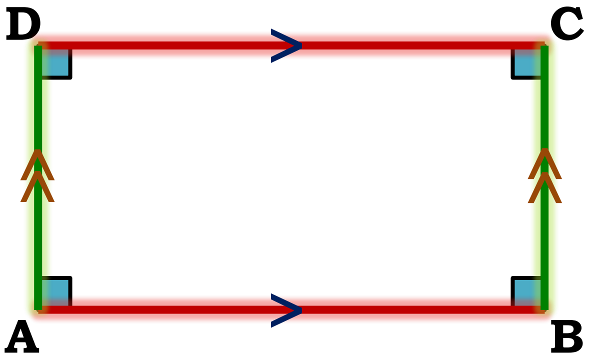
OA = OC &

OB = OD

MODULE 3

➢ Rectangle

Definition : A quadrilateral in which each angle is a right angle is called a Rectangle



AD || BC

□ABCD is a

$\therefore\ \angle A = \angle B = \angle C = \angle D = 90^\circ$

Both the pairs of opposite sides are parallel

So, □ABCD is also a parallelogram

Every Recta... ...arallelogram

Properties Of A Rectangle

- **Opposite sides are equal**

  AB = CD

  AD = BC

- **Diagonals** … **her**

  OA = OC

  OB = OD

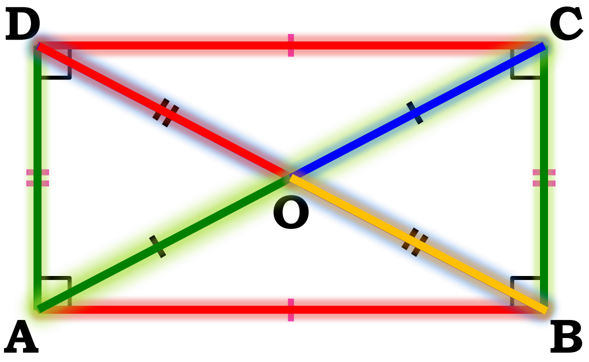
- **Diagonals are equal**

  AC = BD

Name the pairs of opposite sides

OA = OC &
OB = OD

AC = BD



MODULE 4

➢ RHOMBUS

Definition : A quadrilateral having **all sides equal** is called a **Rhombus**

Name all the sides of □ABCD

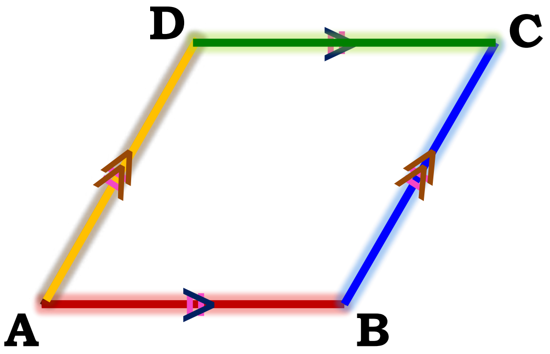
AB = BC = CD = AD

□ABCD is a Rhombus

∴ AB = BC = CD = AD

BC

□ABCD is also a parallelogram



Every Rhom ... rallelogram

PROPERTIES OF A RHOMBUS

- **Opposite angles are equal**

  $\angle A = \angle C$

  $\angle B = \angle D$

- **Diagonals** ... **er**

  $OA = OC$

  $OB = OD$

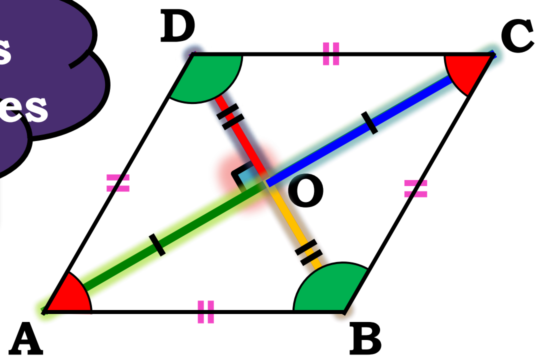
- **Diagonals are perpendicular to each other**

  $AC \perp BD$

Let
Rh

Name the pairs of opposite angles

$OA = OC$ &
$OB = OD$



MODULE 5

➤ SQUARE

Definition : A quadrilateral in which **each angle is a right angle** and **all sides are equal** is called a **Square**

AD || BC

□ABCD is a

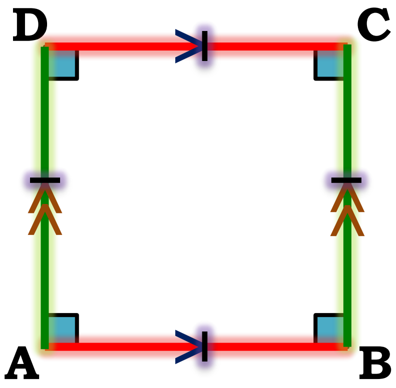
∴ $\angle A = \angle B = \angle C = \angle D$ … 0°

AB = BC = C…

Both the pairs of opposite sides are parallel

So, □ABCD is also a parallelogram

Every Squar… …llelogram



Properties Of A Square

- **Diagonals bisect each other**

  $OA = OC$

  $OB = OD$

- **Diagonals are perpendicular to each other**

  $AC \perp BD$

- **Diagonals are equal**

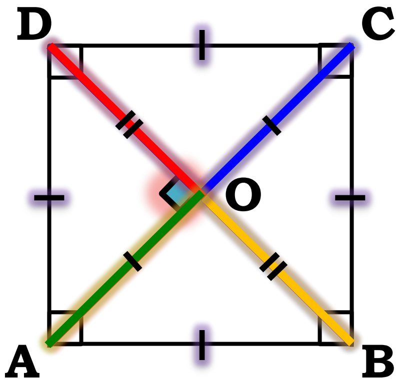
  $AC = BD$

Let us consider a Square ABCD

$OA = OC$ &

$OB = OD$

$AC = BD$



MODULE 6

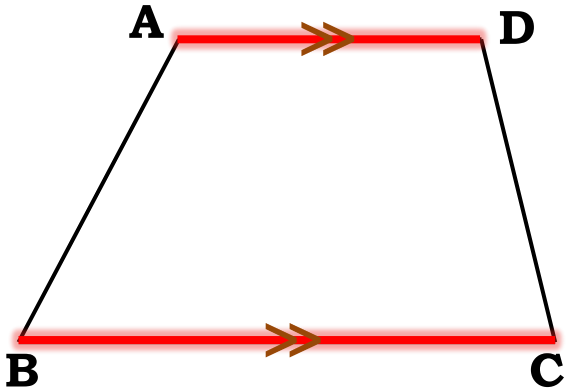
➢ TRAPEZIUM

Definition : A quadrilateral is said to be a trapezium, if only one pair of opposite sides is parallel.

AD || BC

□ABCD is a Trapezi

AD || BC



➢ ISOSCELES TRAPEZIUM

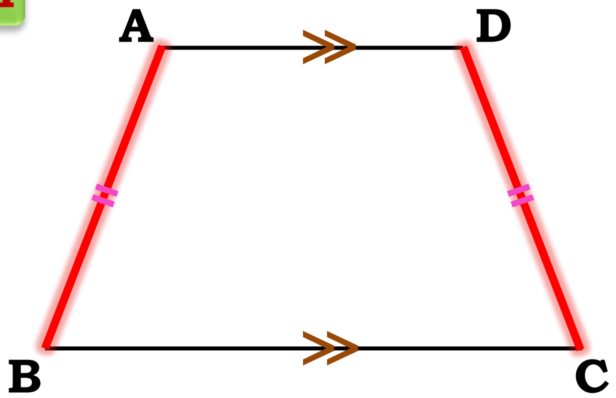
Definition : A Trapezium in which the non-parallel sides are equal, is called an isosceles trapezium

Which are
non-parallel sides ?

AB & DC

=

□ABCD is an Isosceles Trapezium



➤ KITE

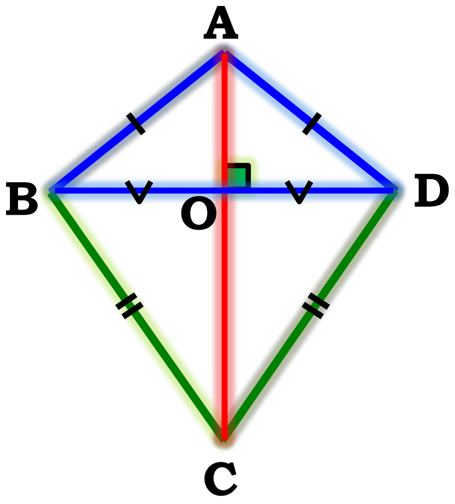
In □ABCD,

AB = AD

BC = DC

and AB ≠ BC

∴ **□ABCD is a Kite**



PROPERTY OF A KITE

➤ **Longer diagonal is the Perpendicular Bisector of the shorter diagonal.**

$AC \perp BD$

$OB = OD$

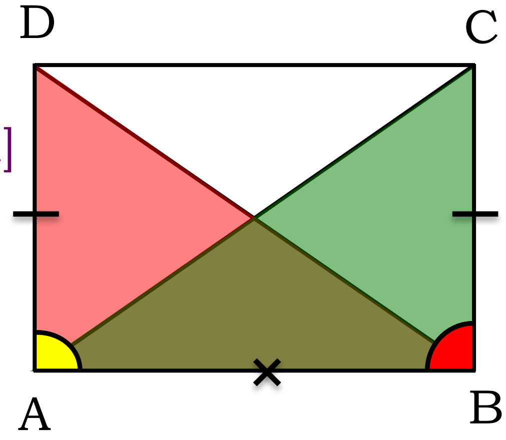
MODULE 7

Q. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Given : A parallelogram ABCD in which AC = BD.

HINT – Prove : ∠DAB = 90°

To prove : □ABCD is a rectangle.



Proof : In ΔDAB ...

AD ... llelogram]

be ... e

For proving
Angles congruent,
Prove triangles
congru...

∴ ΔDAB ≅ ΔC...

∴ ∠DAB = ...

ΔDAB

∠DAB + ∠CBA ... e? ...djacent angles are supplementary]

∴ ∠DAB + ∠DAB = 180° [from (i)]

proving ∠...B = 90°

Just Prove
∠DAB = ∠CBA

∴ 2 ∠DAB = 18...

∴ ∠ DAB = 90°

... □ABCD is a rectangle.

Thank You

MODULE 8

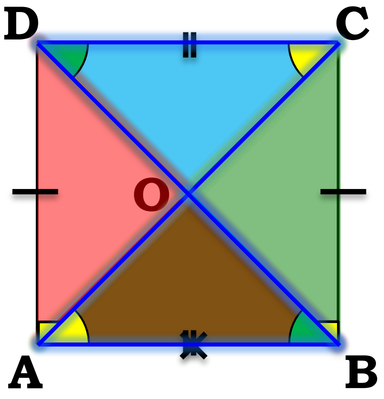
Q. Show that the diagonals of a square are equal and bisect each other at right angles.

To prove :

AC = BD

OA = OC & OB = OD

AC ⊥ BD



Proof : In ΔDAB and ΔCBA,

$AB = AB$ [common side]

$\angle DAB = \angle CBA$ [each is 90°]

$AD = BC$ [Sides of a square]

$\therefore \Delta DAB \cong \Delta CBA$ [SAS crit

$\therefore AC = BD$ [C.P.C

Consider ΔAOB and ΔCOD

In ΔAOB and ΔCOD

$\angle BAO = \angle DCO$ [Alternate interior angles]

$AB = CD$ [Sides of a square]

$\angle ABO = \angle CDO$ [Alternate interior angles]

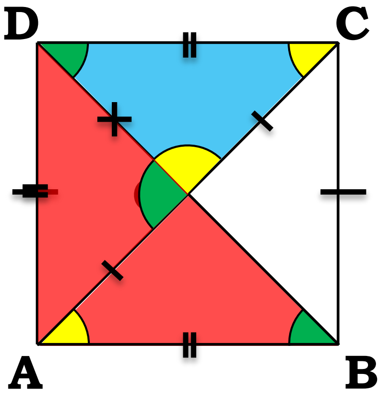
$\therefore \Delta AOB \cong \Delta AOD$ [ASA criterion]

MODULE 9

Q. Show that the diagonals of a square are equal and bisect each other at right angles.

To prove :

- ✓ **AC = BD**
- ✓ **OA = OC &**
- ✓ **OB = OD**
- ✓ **AC ⊥ BD**



Proof : $\Delta AOB \cong \Delta COD$ [ASA test]

$\therefore$ $OA = OC$ ...(i) [C.P.C.T.]

$OB = OD$ [C.P.C.T.]

Now, Consider ΔAOD and ΔCOD

In ΔAOD and ΔCOD

$AD = CD$ [sides of a square]

$OA = OC$ [From (i)]

$OD = OD$ [common side]

$\therefore$ $\Delta AOD \cong \Delta COD$ [SSS criterion]

$\angle AOD = \angle COD$ ...(ii) [C.P.C.T.]

$\angle AOD + \angle COD = 180^\circ$ [Linear pair]

$\angle AOD + \angle AOD = 180^\circ$ [From (ii)]

$\therefore$ $2\angle AOD = 180^\circ$

$\therefore$ $\angle AOD = 90^\circ$

$\therefore$ **AC ⊥ BD**

MODULE 10

Q. Diagonal AC of a parallelogram ABCD bisects ∠A. Show that (i) it bisects ∠C also (ii) ABCD is a rhombus.

Hint: To prove ∠DCA = ∠BCA

Proof : ∠DAC = ∠BAC ...(i) [Given]

□ABCD is a parallelogram

AD || BC and transversal A

∴ ∠DAC = ∠BCA ...(ii)]

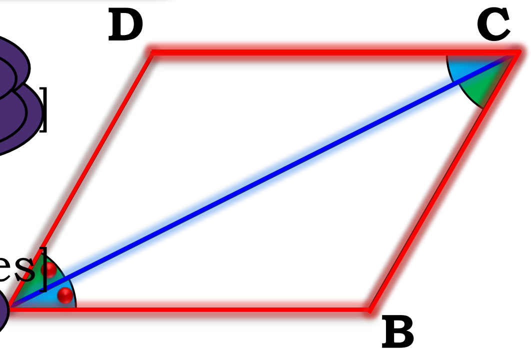
AB || DC and transversal A

∠BAC = ∠DCA ...(iii) [Alternate interior angles]

∴ ∠BCA = ∠DCA [fr

∠BAC = ∠DCA

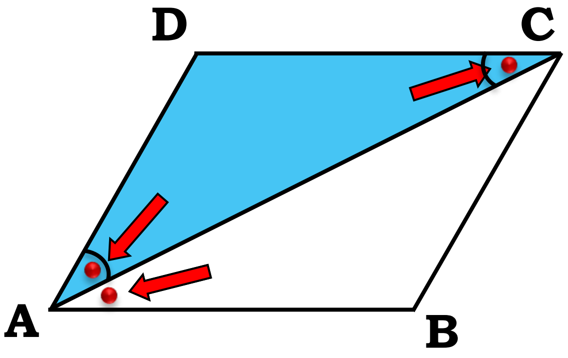
Alternate interior angles



Hence, AC bisects ∠C.

Q. Diagonal AC of a parallelogram ABCD bisects ∠A. Show that (i) it bisects ∠C also (ii) ABCD is a rhombus.

∠DAC = ∠BAC ...(i)
∠DCA = ∠BAC ...(iii)



Proof :

∠DAC = ∠BAC ...(i)

∠DCA = ∠BAC ...(iii)

In ΔADC

∠DAC = ∠DCA [from (i) and (iii)]

∴ DA = DC [Sides opp. to equal angles are equal]

∴ □ABCD is a rhombus. [A parallelogram is a rhombus if one pair of adjacent sides is equal]

MODULE 11

Q . □ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD resp.

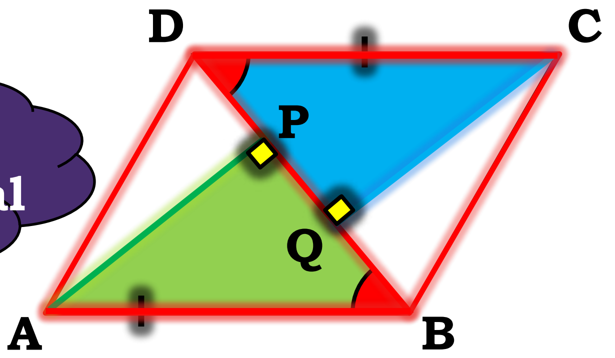
Show that :

(i) $\Delta APB \cong \Delta CQD$ **(ii) AP = CQ**

Proof.

In ΔAPB and ΔCQD,

AB || CD and BD is transversal



$\angle ABP = \angle CDQ$ [Alternate interior angles]

$\angle APB = \angle CQD$ [Each is 90^o]

$AB = CD$ [Opp. sides of a parallelogram]

$\therefore$ $\mathbf{\Delta APB \cong \Delta CQD}$ [By AAS criterion]

$\mathbf{AP = CQ}$ [C.P.C.T.]

MODULE 12

Q. In parallelogram ABCD, two points P and Q are taken on diagonal BD, such that DP = BQ (see figure). Show that :

(i) $\Delta APD \cong \Delta CQB$

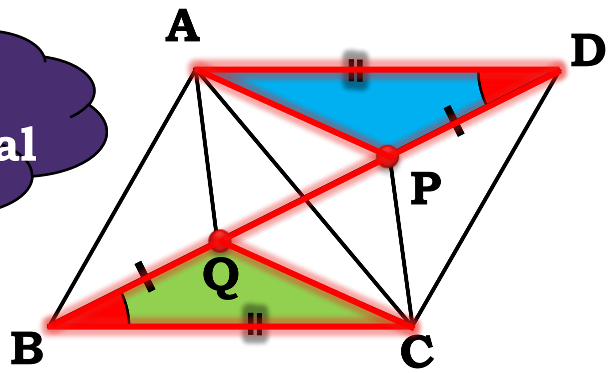
(ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$

(iv) AQ = CP

(v) APCQ is a parallelogram

AD || BC and BD is transversal



Proof.

(i) In ΔAPD and ΔCQB, we have

	$AD = BC$	[Opp. sides of a parallelogram]
	$\angle ADP = \angle CBQ$	[Alternate interior angles]
	$DP = BQ$	[Given]
$\therefore$	$\boldsymbol{\Delta APD \cong \Delta CQB}$	[SAS criterion]
$\therefore$	$\mathbf{AP = CQ}$ ...(i)	[C.P.C.T.]

Q. In parallelogram ABCD, two points P and Q are taken on diagonal BD, such that DP = BQ (see figure). Show that :

(i) $\Delta APD \cong \Delta CQB$

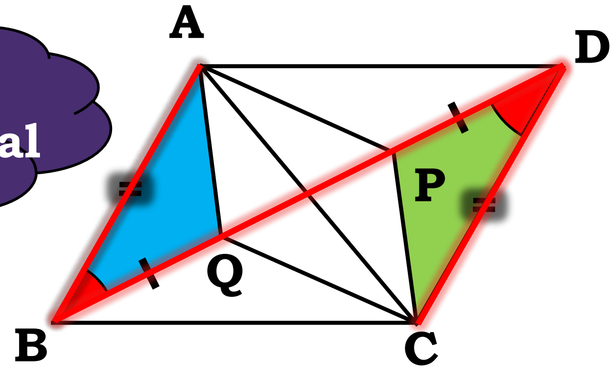
(ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$

(iv) AQ = CP

(v) APCQ is a parallelogram

AB || CD and
BD is transversal



Proof.

(i) In ΔAPD and ΔCQB, we have

	AB = CD	[Opp. sides of a parallelogram]
	$\angle ABQ = \angle CDP$	[Alternate interior angles]
	BQ = DP	[Given]
$\therefore$	$\boldsymbol{\Delta AQB \cong \Delta CPD}$	[S.A.S. criterion]
$\therefore$	**AQ = CP** ...(ii)	[C.P.C.T.]

Q. In parallelogram ABCD, two points P and Q are taken on diagonal BD, such that DP = BQ (see figure). Show that :

(i) $\Delta APD \cong \Delta CQB$

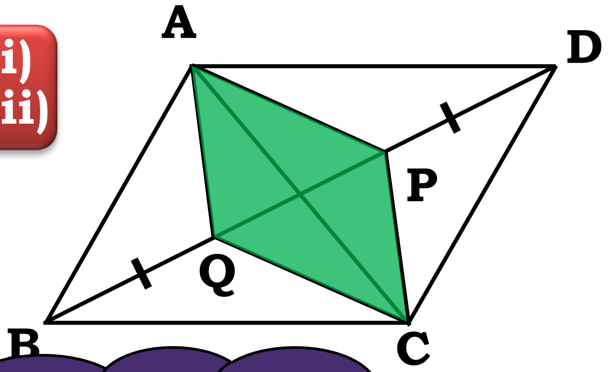
(ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$

(iv) AQ = CP

(v) APCQ is a parallelogram

AP = CQ ...(i)
AQ = CP ...(ii)



Proof.

In □APCQ,

AP = CQ [from (i)]

AQ = CP [from (ii)]

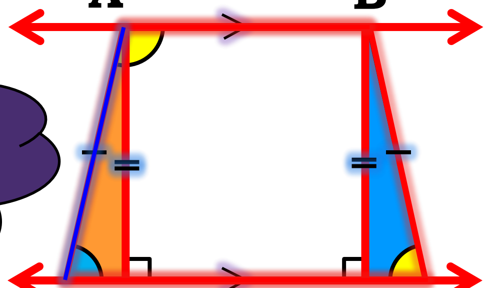
A quad. is parallelogram if opposite sides are equal

∴ □APCQ is a parallelogram [A quadrilateral in which opposite sides are equal is a parallelogram]

Thank You

MODULE 13

Q . ABCD is a trapezium in which AB || CD and AD = BC
Show that : (i) $\angle C = \angle D$ (ii) $\angle A = \angle B$
(iii) $\Delta ABC \cong \Delta BAD$ (iv) diagonal AC = diagonal BD



Construction : Draw $AM \perp DC$ & $BN \perp DC$.

Proof : In ΔAMD and ΔBNC

$AD = BC$ [G

$\angle AMD = \angle BNC$ [I

$AM = BN$ [D

pa

$\therefore \Delta AMD \cong \Delta BNC$ [R.

$\angle C = \angle D$ ....(i)

Interior angles are supplementary

D

same

lin

al

ΔBNC

For th

will

draw perpendiculars from A and B on side DC

(ii) □ABCD is a trapezium

AB || CD on transversal AD,

$\angle A + \angle D = 180°$ ....(ii) [Interior angles are supplementary]

Similarly,

$\angle B + \angle C = 180°$ ....(iii) [Interior angles are supplementary]

$\angle A + \angle D = \angle B + \angle C$

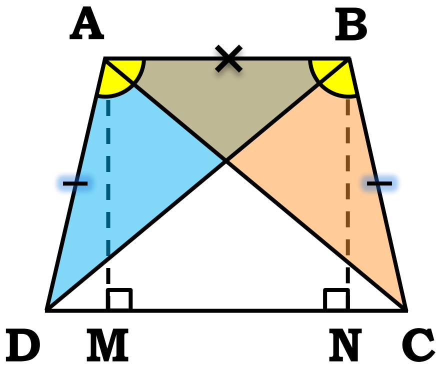
$\angle A + \cancel{\angle D} = \angle B + \cancel{\angle D}$

$\therefore \angle A = \angle B$

Q . ABCD is a trapezium in which AB || CD and AD = BC

Show that : (i) $\angle C = \angle D$ (ii) $\angle A = \angle B$

(iii) $\Delta ABC \cong \Delta BAD$ (iv) diagonal AC = diagonal BD



Proof : In ΔABC and ΔBAD

$AB = AB$ [Common side]

$\angle A = \angle B$ [Proved]

$AD = BC$ [Given]

$\therefore \Delta ABC \cong \Delta BAD$ [S.A.S. criterion]

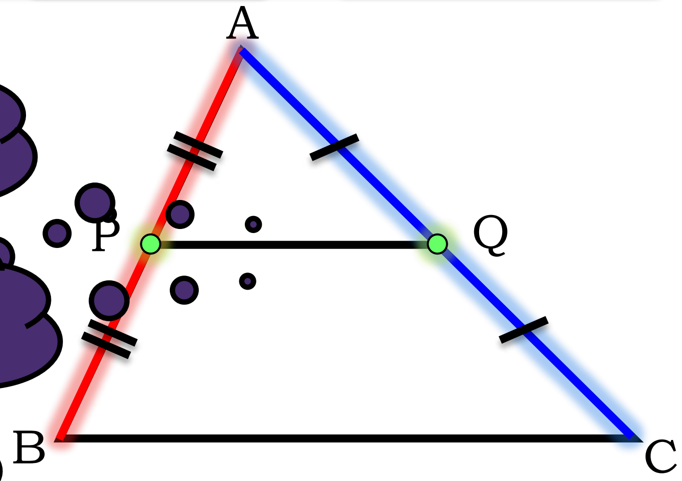
$\therefore AC = BD$ [C.P.C.T.]

$\therefore$ diagonal AC = diagonal BD

MODULE 14

MIDPOINT THEOREM

In a triangle, the line segment joining the midpoints of any two sides is parallel to third side and is half of it.



In ΔABC,

Points P and Q are the midpoints of sides AB and AC respectively.

$\therefore$ seg PQ || side BC

and

seg PS = ½ side BC

PQ || BC

PQ = ½ BC

Let us see the presentation

CONVERSE OF MIDPOINT THEOREM

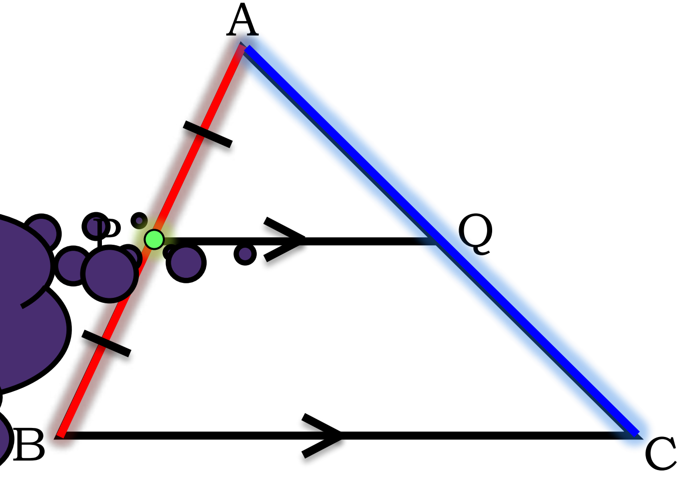
If a line drawn through the midpoint of one side of a triangle is parallel to the second side, then it bisects the third side.

In ΔABC,
Points P is the midpoint of side AB
and seg PQ || side BC
∴ Point Q is the midpoint of side BC

PQ bisects side AC

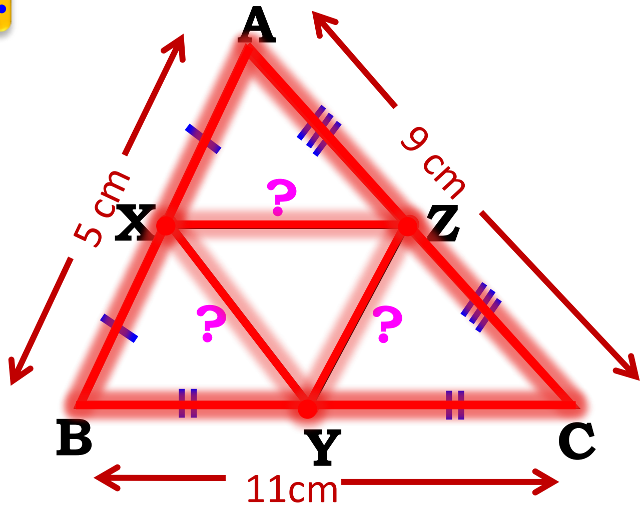
i.e. Q is the midpoint

Let us see the presentation



MODULE 15

Q. In ΔABC, X, Y and Z are midpoints of side AB, side BC and side AC respectively. AB = 5 cm, AC = 9 cm and BC = 11 cm. Find (i) XY, (ii) YZ (iii) XZ.



Sol.

In ΔABC,

X and Y are the midpoints of AB and BC resp.

$\therefore$ XY = ½ AC [midpoint theorem]

$\therefore$ XY = ½ × 9

$\therefore$ **XY = 4.5 cm**

Y and Z are the midpoints of BC and AC resp.

$\therefore$ YZ = ½ AB [midpoi

$\therefore$ YZ = ½ × 5

$\therefore$ **YZ = 2.5 cm**

Which theorem can we apply here ?

midpoint theorem

X and Z are the midpoints of AB and AC resp.

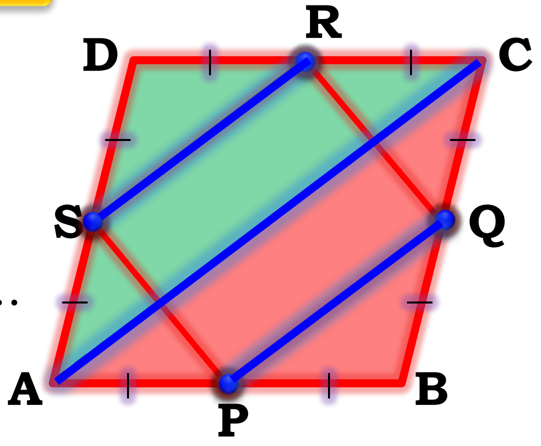
$\therefore$ XZ = ½ BC [midpoint theorem]

$\therefore$ XZ = ½ × 11

$\therefore$ **XZ = 5.5 cm**

MODULE 16

Q. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that : □PQRS is a rectangle



Construction : Draw diagonal AC.

Proof :

In ΔABC,

P and Q are the

$\therefore$ PQ || AC ...(i)

PQ $= \frac{1}{2}$ AC ...(ii)

[Midpoint theorem]

∴ By midpoint theorem

PQ || AC

And PQ $= \frac{1}{2}$ AC

In ΔADC,

S and R are the mid-point

$\therefore$ SR || AC ...(iii)

SR $= \frac{1}{2}$ AC ...(iv)

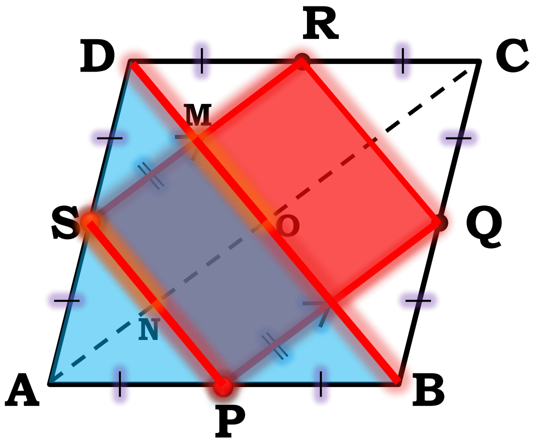
∴ By midpoint theorem

SR || AC

and SR $= \frac{1}{2}$ AC

Q. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.
Show that : □PQRS is a rectangle

Proof :



$\therefore$ PQ || SR [from (i) and (iii)]

PQ = SR [from (ii) and (iv)]

□PQRS is a parallelogram.

[A quadrilateral is a parallelogram, if a pair of opposite sides is parallel and equal]

PQ || AC ...(i)

PQ = $\frac{1}{2}$AC ...(ii)

SR || AC ...(iii)

SR = $\frac{1}{2}$AC ...(iv)

In ΔADB,

S and P are th resp.

$\therefore$ By midpoint theorem

SP || DB

And SP = $\frac{1}{2}$DB

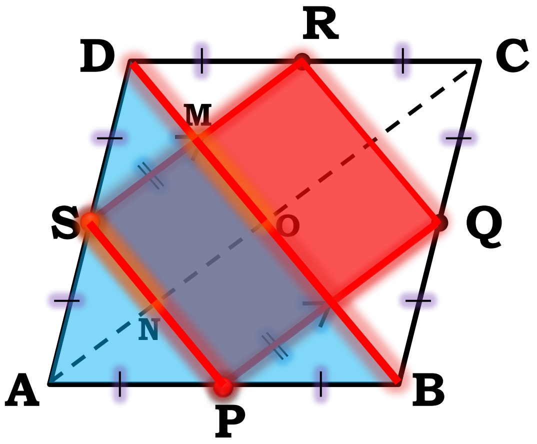
$\therefore$ SP || DB [

$\therefore$ SN || MO

MODULE 17

Q. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.
Show that : □PQRS is a rectangle

Proof :



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PQ = $\frac{1}{2}$AC ...(ii)

SR || AC ...(iii)

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In ΔADB,

S and P are th ... resp.

$\therefore$ By midpoint theorem

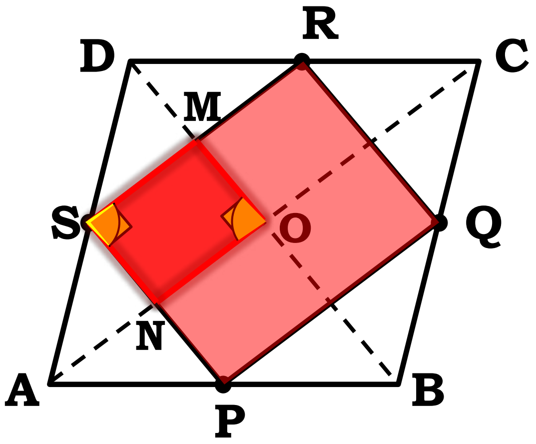
SP || DB

And SP = $\frac{1}{2}$DB

$\therefore$ SP || DB [

$\therefore$ SN || MO

Q. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.
Show that : □PQRS is a rectangle



Proof :

SN || MO

SM || NO [since SR ‖ AC]

□SMON is a parallelogram [By definition]

$\angle MON = \angle MSN$ [Opp. angles are equal]

$\angle MON = 90^{o}$ [Diagonals of a Rhombus are perpendicular to each other]

$\therefore$ $\angle MSN = 90^{o}$

i.e. $\angle S = 90^{o}$

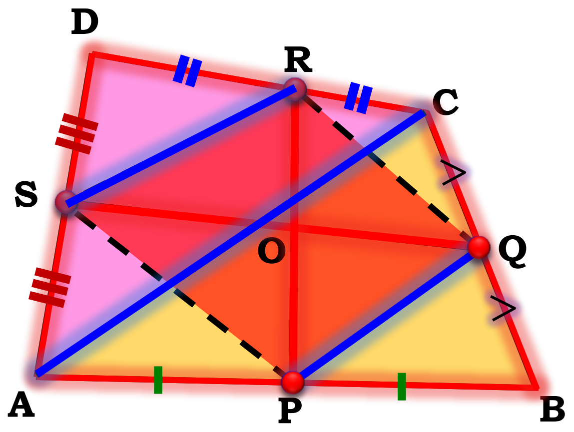
$\therefore$ □PQRS is a rectangle. [If one angle of a parallelogram is a right angle, then it is rectangle]

Thank You

MODULE 18

Q. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

To prove : OP = OR, OQ = OS



Proof :

In ABC,

P and Q are th

$\therefore$ PQ || AC ...

PQ $= \dfrac{1}{2}$ AC ...(ii)

$\therefore$ By midpoint theorem
PQ || AC
And PQ $= \dfrac{1}{2}$ AC

In ΔADC,

S and R are th ... sp.

$\therefore$ SR || AC

SR $= \dfrac{1}{2}$ AC

$\therefore$ By midpoint theorem
SR || AC
And SR $= \dfrac{1}{2}$ AC

Now, draw seg AC

parallelogram

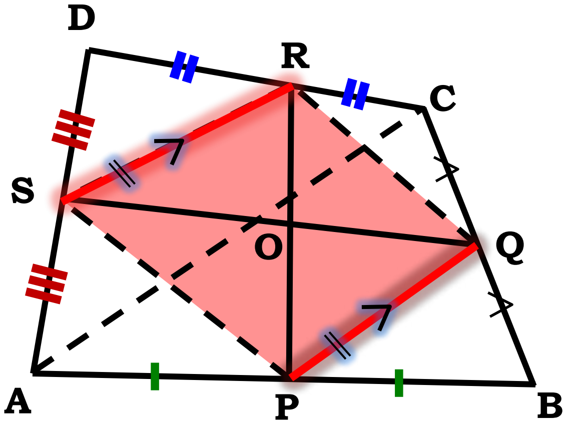
Q. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

$\therefore$ PQ || SR [from (i) and (iii)]

PQ = SR [from (ii) and (iv)]

□PQRS is a parallelogram.

[A quadrilateral is a parallelogram, if a pair of opposite sides is parallel and equal]



$\therefore$ Diagonals PR and QS of a parallelogram PQRS bisect each other

$\therefore$ OP = OR and OQ = OS

Thus, the line segments joining the mid-points Of quadrilateral bisect each other

PQ || AC ...(i)

$PQ = \frac{1}{2}AC$ ...(ii)

SR || AC ...(iii)

$SR = \frac{1}{2}AC$ ...(iv)

MODULE 19

Q. In the figure, point X is the midpoint of side BC. Seg XZ is drawn parallel to side AB. Seg YZ || seg AX.

Prove that YC = $\frac{1}{4}$ BC

Proof :

In ABC,

X is the midpoint of BC

XZ || AB

} [Given]

$\therefore$ Z is the midp ... erse ... oint ... m]

and XZ || AB

Now, let us write 'XC' in terms of BC

In AXC,

Z is the midp

ZY || AX

$\therefore$ Y is the midpo ... of ... theorem]

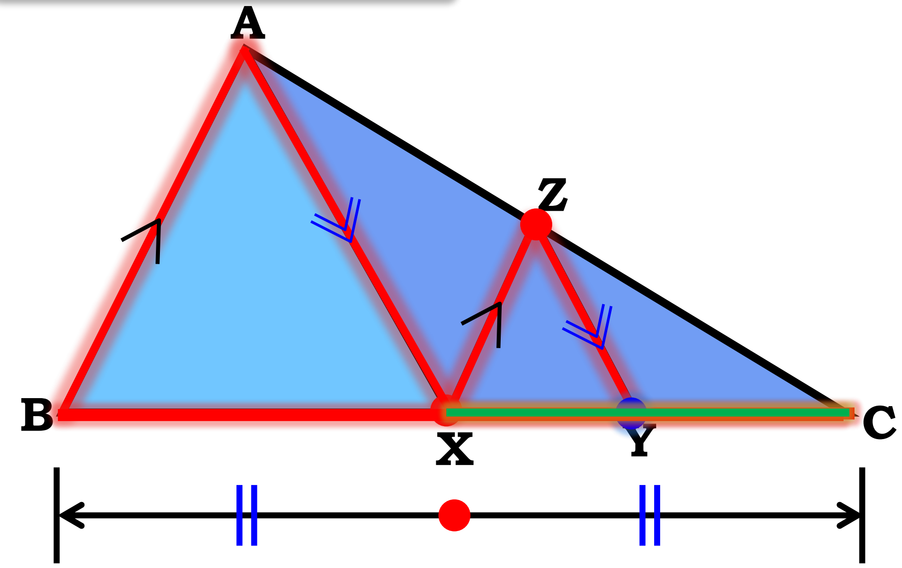
and

Y is

i.e XC = $\frac{1}{2}$ BC

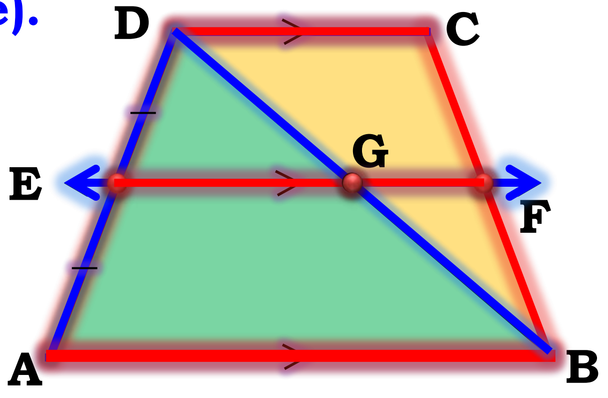
YC = $\frac{1}{2}$ XC

YC = $\frac{1}{2} \times \frac{1}{2}$ BC $\quad\therefore$ YC = $\frac{1}{4}$ BC



MODULE 20

Q. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see figure). Show that F is the mid-point of BC.



Proof : In ΔDAB

E is the midp

EG || AB

$\therefore$ G is midpoint

of

theorem]

$\therefore$ By converse of midpoint theorem G is midpoint of BD

EF || AB

AB || DC

[Given]

$\therefore$ EF || DC

$\therefore$ GF || DC

In ΔBDC

G is midpoint of DB

GF || DC

$\therefore$ F is midpoint of BC

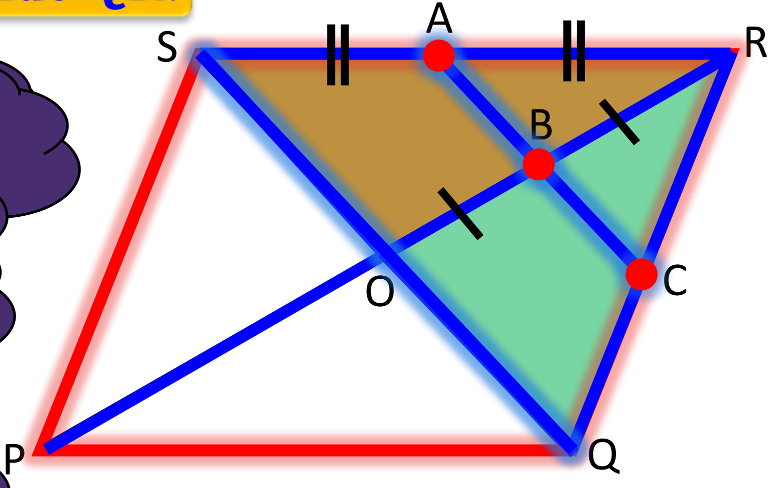
[By converse of mid-point theorem]

MODULE 21

Q. PQRS is a parallelogram in which A is the mid-point of side SR and point B is a point on diagonal PR such that $RB = \frac{1}{4}PR$. Also, AB when produced meets QR at C. Prove that point C is the midpoint of side QR.

Hint:

Prove : AC || SQ



Proof :

$RB = \frac{1}{4}\boxed{PR}$

Diagonals of parallelogram bisect each other

$\boxed{PR = 2(OR)}$

$RB = \frac{1}{\cancel{4}_{2}} \times \cancel{2}(OR)$

$RB = \frac{1}{2}OR$

$\therefore$ B is midpoint of seg OR

In ΔRSO

A and B are midpoints of SR and OR respectively

AB || SO [Midpoint theorem]

$\therefore$ AC || SQ

Now consider ΔRSQ

In ΔRSQ

A is midpoint of seg RS and AC || AQ

$\therefore$ By conver... theore... point ... as ... midpoint of QR

C is midpoint of side QR

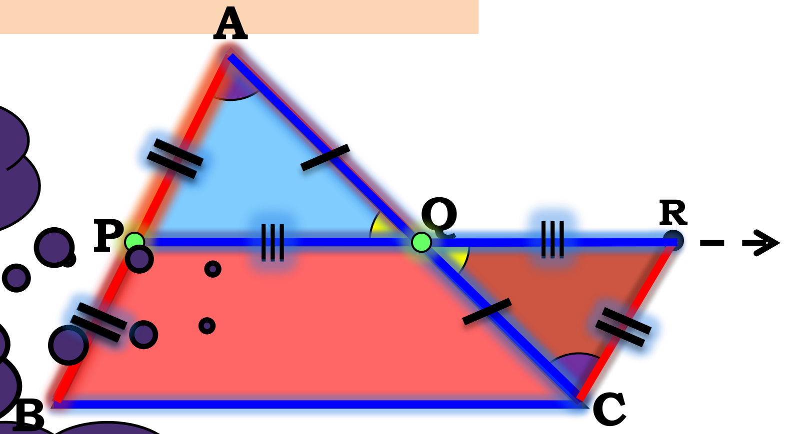
[Converse of midpoint theorem]

Thank You

MODULE 22

MIDPOINT THEOREM

In a triangle, the line segment joining midpoints of any two sides is parallel to third side and is half of it.



Given : In ΔABC, P and Q are midpoints of AB and AC resp.

To prove : (i) PQ || BC

Construction : T...

that P-Q...

Proof : In ΔAQP and ΔCQR

AQ ...

$\angle$AQP ...

PQ = QR

ΔAQP $\cong$ ΔCQR [S...

$\angle$PAQ $\cong$ $\angle$RCQ [C.P.C.T.]

AB || CR

PB || CR ...(i) [A-P-B]

AP = CR ...(ii) [C.P.C.T.]

But, AP = PB...(iii) [Given]

$\therefore$ PB = CR [from (ii), (iii)]

C is midpoint of AB

$\therefore$ AC = BC = $\frac{1}{2}$ AB ?

For proving ... Construction ... requ...

PQ || BC

PQ = ½ BC

For that we need to prove ☐PBCR is a parallelogram

... make PQ as of a 'segment' such that PQ is half of it.

NO

☐PBCR is a parallelogram.

[A quad. is a parallelogram if one pair of opp. sides is parallel and equal]

$\therefore$ PR || BC

$\therefore$ PQ || BC [P-Q-R]

PQ = ½ PR [by construction]

But, PR = BC [opp. sides of || gm]

$\therefore$ PQ = ½ BC.

...t.)

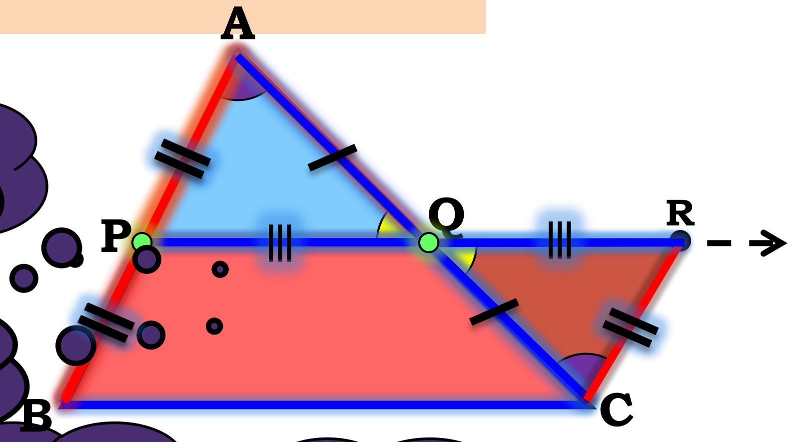
...BC,

... to prove PR || BC

MODULE 23

MIDPOINT THEOREM

In a triangle, the line segment joining midpoints of any two sides is parallel to third side and is half of it.



Given : In ΔABC, P and Q are midpoints of AB and AC resp.

To prove : (i) PQ ll BC

Construction : T...

that P-Q...

Proof: In ΔAQP and ΔCQR

AQ ...

∠AQP ...

PQ = QR

ΔAQP ≅ ΔCQR [S...]

∠PAQ ≅ ∠RCQ [C.P.C.T.]

AB ll CR [Alt. angle...]

C is midpoint of ...

PB ll CR ...(i) [A-P-B]

∴ AC = BC = $\frac{1}{2}$ AB ?

AP = CR ...(ii) [C.P.C.T.]

But, AP = PB...(iii) [Given]

∴PB = CR [from (ii), (iii)]

PQ || BC

For proving ... Construction ... requ...

PQ = ½ BC

For that we need to prove □PBCR is a parallelogram

...take PQ as ... of a 'segment' such that PQ is half of it.

NO

□PBCR is a parallelogram.

[A quad. is a parallelogram if one pair of opp. sides is parallel and equal]

∴ PR ll BC

∴ PQ ll BC [P-Q-R]

PQ = ½ PR [by construction]

But, PR = BC [opp. sides of ||gm]

∴ PQ = ½ BC.

...t.)

...BC,

...d to prove PR ll BC

MODULE 24

CONVERSE OF MIDPOINT THEOREM

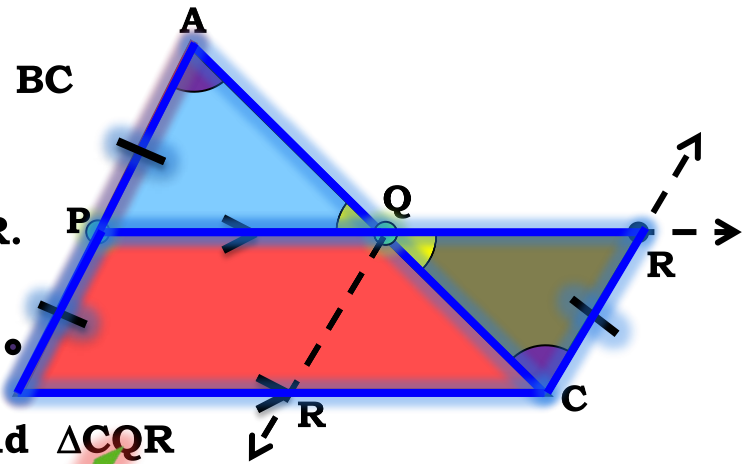
If a line drawn through the midpoint of one side of a triangle is parallel to second side, then it bisects the third side.

Given : In ΔABC, P is midpoint of AB and PQ || BC

To prove : Q is midpoi...

Construction : Dr... through C an... R.

Proof :



Through 'Q' draw

F...

Let...

Let us consider ΔAQP and ΔCQR

PQ || BC

$\therefore$ PR || BC

AB || RC [co...

PB || RC

$\therefore$ □PBCR is a pa...

$\therefore$ PB = RC [Op...

But, PB = AP [g...

AP = RC ...(i)

... ΔAQP and ΔCQR

[from (i)]

[vertically opp. angles]

[Alternate angles]

[SAA criterion]

[C.P.C.T.]

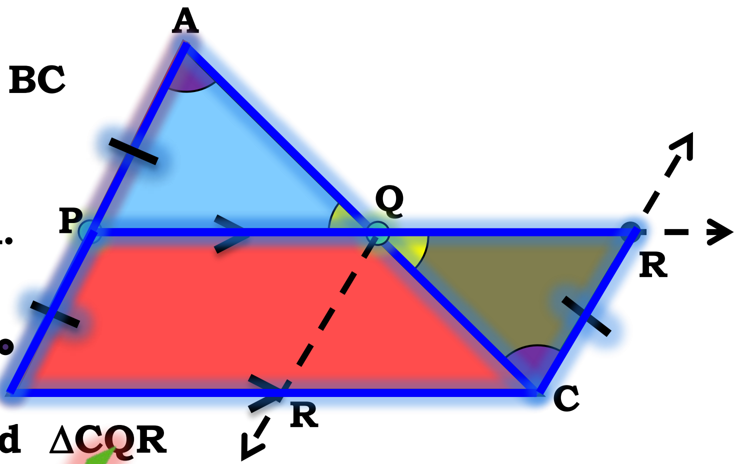
... Q is midpoint of AC

Draw line parallel to AB through C and let it interest PQ produced at R.

MODULE 25

CONVERSE OF MIDPOINT THEOREM

If a line drawn through the midpoint of one side of a triangle is parallel to second side, then it bisects the third side.



Given : In ΔABC, P is midpoint of AB and PQ || BC

To prove : Q is midpoi…

Construction : Dr…

through C an… R.

Proof :

F…

Through 'Q' draw …

Let…

Let us consider ΔAQP and ΔCQR

PQ || BC

$\therefore$ PR || B…

AB || RC [c…

PB || RC

$\therefore$ □PBCR is a pa…

$\therefore$ PB = RC [Op…

But, PB = AP [g…

AP = RC …(i)

PC…

Draw line parallel to AB through C and let it interest PQ produced at R.

…AQP and ΔCQR

[from (i)]

[vertically opp. angles]

[Alternate angles]

[SAA criterion]

[C.P.C.T.]

…Q is midpoint of AC

MODULE 26

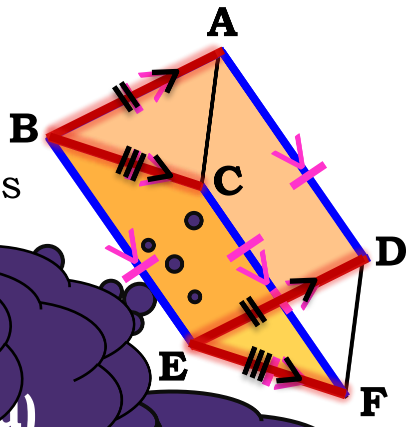
Q. In △ABC and △DEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to D, E and F resp.

Show that : (i) □ABED is a parallelogram

Proof : (i) In □ABED,

AB = DE &
AB || DE } (Given)

∴ □ABED is a parallelogram. [One pair of opposite sides is parallel and equal.]



(ii) □BEFC is a parallelogram

In □BEFC,

&

BC || EF

(Given)

∴

(iii) AD ||

Now, AD || BE and AD = BE

and, CF || BE and CF = BE

AD || CF and AD = CF

A quadrilateral is a parallelogram, if a pair of opposite sides is parallel and equal

In □

pa

∴ CF || BE

CF = BE

d)

am

gram

en

BC || EF

Given

A quadrilateral is a parallelogram, if a pair of opposite sides is parallel and equal

MODULE 27

Q. In ΔABC and ΔDEF, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to D, E and F resp. Show that :

$AD \parallel CF$ and $AD = CF$
(proved)

(iv) □ACFD is a parallelogram

(iv) In □ACFD

$AD = CF$

$AD \parallel CF$

$\therefore$ □ACFD is a parallelogram. [One pair of opposite sides is parall...

(v) AC = DF

Since, □ACFD is parallelogra...

$\therefore$ $AC = DF$ ...| gm ACFD]

(vi) $\Delta ABC \cong \Delta DEF$

In Δ ABC and ΔDEF,

$AB = DE$

$BC = EF$

$CA = FD$

$\therefore$ $\Delta ABC \cong \Delta DEF$ [By SS...

$\therefore$ AC = DF

In

AD = CF

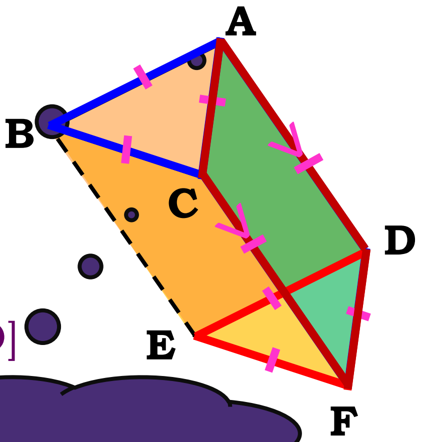
Let us consider parallelogram ACFD

...arallelogram

...a

...elogram, if a pair of ...site sides is parallel and equal

CA = FD

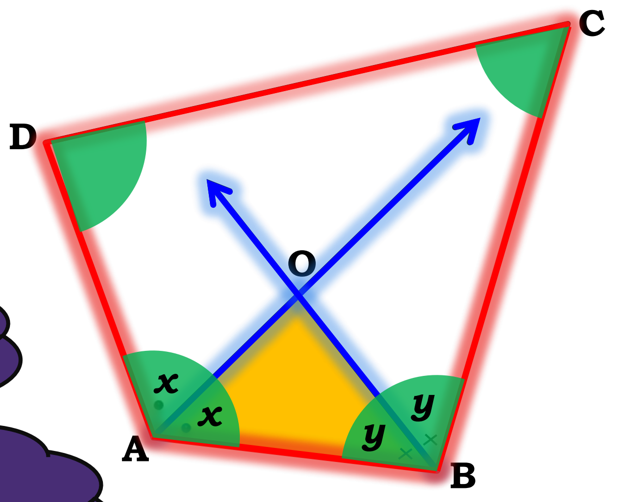


Thank You

MODULE 28

Q. In a □ABCD, AO & BO are the bisector of ∠A and ∠B resp.

Prove : $\angle AOB = \frac{1}{2}(\angle C + \angle D)$



Proof: $\angle DAO = \angle OAB$ [AO is an angle bisector]

let $\angle DAO = \angle OAB = x$ …(i)

$\angle COB = \angle OBA$ [BO … ector]

let $\angle COB = \angle OBA = y$ …(ii)

In ΔAOB

$\angle OAB + \angle OBA + \angle AO$…

$\therefore \quad x + y + \angle AOB = 180$…

$\therefore \quad \angle AOB = 180 - x - y$

$\therefore \quad \angle AOB = 180 - (x + y)$

In □ABCD

$\angle A + \angle B + \angle C + \angle D = 360$

$\therefore \quad 2x + 2y + \angle C + \angle D = 360$

$\therefore \quad 2(x + y) = 360 - (\angle C + \angle D)$

$\therefore \quad x + y = 180 - \frac{1}{2}(\angle C + \angle D)$ …(iv)

…B $= 180 - [180 - \frac{1}{2}(\angle C + \angle D)]$

$\angle AOB = \cancel{180} - \cancel{180} + \frac{1}{2}(\angle C + \angle D)$

$\angle AOB = \frac{1}{2}(\angle C + \angle D)$

∠COB = ∠OBA

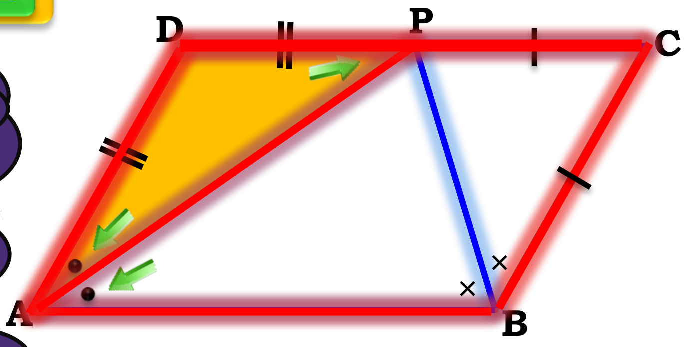
We know that, sum of all an… tr…

Dividing throughout by 2

…um … es of a quadrilateral is 360°

MODULE 29

Q. ☐ABCD is parallelogram. If the bisectors of $\angle A$ and $\angle B$ meet at P,
Prove that AD = DP, PC = BC and DC = 2AD



Proof : AP is a bisector of $\angle$DA

$\therefore$ $\angle DAP = \angle PAB$ …(i)

☐ABCD is parallelogra

$\therefore$ AB || DC

On transversal AP

$\therefore$ $\angle PAB = \angle DPA$ …(ii)

In ΔDAP

$\therefore$ $\angle DAP = \angle DPA$

$\therefore$ AD = DP …(i

equal angles]

Similarly,

PC = BC …(iv)

DC = DP + PC [D-P-C]

= AD + BC [From (iii) and (iv)]

= AD + AD

$\therefore$ DC = 2AD

☐ABCD is a parallelogram

DC is made up of two segments (DP and PC)

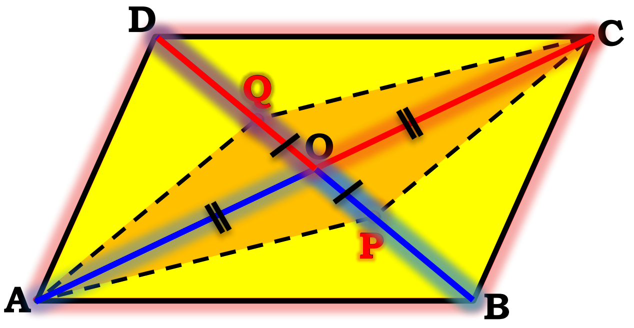
On transversal AP

$\angle PAB = \angle DPA$

Opposite sides of parallelogram

MODULE 30

Q. P and Q are the points of trisection of the diagonal BD of a parallelogram ABCD
Prove that □APCQ is a parallelogram.



Proof .

DQ = QP = PB ...(i)

□ABCD is parallelogram

OA = OC ...(ii)
OD = OB ...(iii)
[Diagonals bisect each other]

DQ + QO = OP + PB [D – Q – O, O –

~~DQ~~ + QO = OP + ~~DQ~~ [From (i)]

QO = OP ...(iv)

OA = OC

OD = OB

In □APCQ

OA = OC [From (ii)]

QO = OP [From (iv)]

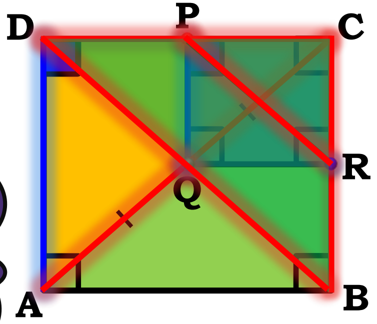
∴ □APCQ is a parallelogram. [A quadrilateral is a parallelogram if diagonals bisect each other]

MODULE 31

Q. □ABCD & □PQRC are rectangles and Q is the midpoint of AC prove that

(i) DP = PC (ii) $PR = \frac{1}{2} AC$

Hint : prove QP ll AD



Proof : $\angle CPQ = \angle CDA = 90^\circ$ [Angles

$\therefore$ QP || AD [corre

In ΔADC

Q is midpoint of AC

QP || AD

$\therefore$ P is midpoint of D

Similarly, we can p

In ΔBCD

P and R are midpoints of I

$\therefore PR = \frac{1}{2} DB$

But DB = AC [Diagonals of rectangle]

$\therefore PR = \frac{1}{2} AC$

Considering

transversal DC

Diagonals of

rectangle are

equal

W

In a tria

joining

sides is parallel to third

side and half of it.

$\therefore PR = \frac{1}{2} DB$

(corresponding

angles test)

Thank You