LECTURE_10

MODULE_33

Decimal expansion of rational numbers is terminating or non – terminating but repeating.

Example:

3.5, 7.272727..., 3.142857142857..., 25,

Terminating decimal expansion

$$3.5 = \frac{35}{10} = \frac{35}{2 \times 5}$$

$$47.1245 = \frac{471245}{10000} = \frac{471245}{2^4 \times 54}$$

$$0.222 = \frac{222}{1000} = \frac{222}{2^3 \times 53}$$

3.5 can be expressed as $\frac{P}{Q}$ i.e $\frac{35}{10}$

A decimal expansion that terminates when expressed in form of $\frac{p}{q}$, then prime factorisation of q is of the form 2^n5^m

Q.1 Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

Sol. (i) $\frac{13}{3125}$

$$\frac{13}{3125} = \frac{13}{5^5} = \frac{13}{2^0 \times 5^5}$$

Since denominator is in the form of 2^n5^m

$\therefore$ $\frac{13}{3125}$ has terminating decimal expansion.

(ii) $\frac{17}{8}$

$$\frac{17}{8} = \frac{17}{2^3} = \frac{17}{2^3 \times 5^0}$$

Since denominator is in the form of 2^n5^m

$\therefore$ $\frac{17}{8}$ has terminating decimal expansion.

Q.1 Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

Sol. (iii) $\frac{15}{1600}$

$$\frac{\cancel{15}^{3}}{\cancel{1600}_{320}} = \frac{3}{320} = \frac{3}{2^6 \times 5^1}$$

Since denominator is in the form of 2^n5^m

$\therefore$ $\frac{15}{1600}$ has terminating decimal expansion.

(iv) $\frac{77}{210}$

$$\frac{\cancel{77}^{11}}{\cancel{210}_{30}} = \frac{11}{30} = \frac{11}{2 \times 3 \times 5}$$

Since denominator is not in the form of 2^n5^m

$\therefore$ $\frac{77}{210}$ has non - terminating decimal expansion.

Q.1 Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

Sol. (v) $\frac{64}{455}$

$$\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$$

Since denominator is not in the form of 2^n5^m

$\therefore$ $\frac{64}{455}$ has non-terminating repeating decimal expansion.

(iv) $\frac{23}{2^3 5^2}$

$$\frac{23}{2^3 5^2}$$

Since denominator is in the form of 2^n5^m

$\therefore$ $\frac{23}{2^3 5^2}$ has terminating decimal expansion.

5	455
7	91
13	13
	1

MODULE_34

Q.2 **Write down the decimal expansions of the rational numbers which have terminating decimal expansions.**

Sol. (i) $\frac{13}{3125}$

$$\frac{13}{3125} = \frac{13 \times 2^5}{5^5 \times 2^5} = \frac{13 \times 2^5}{5^5 \times 2^5}$$

$$= \frac{416}{(10)^5} = 0.00416$$

$$\therefore \quad \frac{13}{3125} = 0.00416$$

(ii) $\frac{17}{8}$

$$\frac{17}{8} = \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 5^3}{2^3 \times 5^3}$$

$$= \frac{2125}{(10)^3} = 2.125$$

$$\therefore \quad \frac{17}{8} = 2.125$$

Q.2 Write down the decimal expansions of the rational numbers which have terminating decimal expansions.

Sol. (iii) $\frac{15}{1600}$

$$\frac{\cancel{15}^{\,3}}{\cancel{1600}_{\,320}} = \frac{3}{320} = \frac{3}{2^6 \times 5^1} = \frac{3 \times 5^5}{2^6 \times 5^1 \times 5^5}$$

$$= \frac{3 \times 3125}{2^6 \times 5^6} = \frac{9375}{10^6} = 0.009375$$

$$\therefore \quad \frac{15}{1600} = 0.009375$$

(iv) $\frac{23}{2^3 \times 5^2}$

$$\frac{23}{2^3 \times 5^2} = \frac{23 \times 5^1}{2^3 \times 5^2 \times 5^1} = \frac{23 \times 5}{2^3 \times 5^3} = \frac{115}{10^3} = 0.115$$

To express it as 10^m we need to multiply it by 5^5

To express it as 10^m we need to multiply it by 5^1

$$\therefore \quad \frac{23}{2^3 \times 5^2} = 0.115$$

Q.2 **Write down the decimal expansions of the rational numbers which have terminating decimal expansions.**

Sol. (v) $\frac{6}{15}$

$$\frac{6}{15} = \frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$$

$$\therefore \boxed{\frac{6}{15} = 0.4}$$

(vi) $\frac{35}{50}$

$$\frac{35}{50} = \frac{7}{10} = 0.7$$

$$\therefore \boxed{\frac{35}{50} = 0.7}$$

To express it as 10^m we need to multiply it by 2

Q.3 The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q ?

Sol. (i) 43.123456789

43.123456789

∴ 43.123456789 is a rational number and q will be in the form of 2^n5^m

(ii) 0.12012001200012000 0...

0.12012001200012000 0...

is a non-terminating and non-repeating decimal

∴ 0.12012001200012000 0... is not a rational number.

Decimal number is terminating

Hence prime factors of q i.e. denominator will be of the form 2^n5^m