MATHS

$(a+b)^2$

ab+

Arithmetic Progressions

1. What is a Sequence?

- A **sequence** is an arrangement of numbers in a definite order according to some rule.
- The various numbers occurring in a sequence are called its **terms**.
- We denote the terms of a sequence by a_1, a_2, a_3…etc. Here, the subscripts denote the positions of the terms in the sequence.
- In general, the number at the n^{th} place is called the n^{th} term of the sequence and is denoted by a_n. The n^{th} term is also called the **general term** of the sequence.
- A sequence having a finite number of terms is called a **finite sequence**.
- A sequence which do not have a last term and which extends indefinitely is known as an **infinite sequence**.

2. Arithmetic Progression:

- An **arithmetic progression** is a list of numbers in which each term is obtained by adding a fixed number to the preceding term, except the first term.
- Each of the numbers of the sequence is called a **term** of an Arithmetic Progression. The fixed number is called the **common difference**. This common difference could be a positive number, a negative number or even zero.

3. General form and general term (n^{th} term) of an A.P:

- The **general form of an A.P.** is a, a + d, a + 2d, a + 3d….., where 'a' is the first term and 'd' is the common difference.
- The **general term(n^{th} term)** of an A.P is given by $a_n = a + (n - 1)d$, where 'a' is the first term and 'd' is the common difference.

- If the A.P a, a + d, a + 2d, …….., *l* is reversed to *l*, *l – d*, *l – 2d*, ….., a then the common difference changes to negative of the common difference of the original sequence.
- To find the **n^{th} term from the end**, we consider this AP backward such that the last term becomes the first term.
  l, $(l - d)$, $(l - 2d)$ ……
  The general term of this AP is given by $a_n = \ell + (n - 1)(-d)$

4. Algorithm to determine whether a sequence is an AP or not:

When we are given an algebraic formula for the general term of the sequence:

Step 1: Obtain a_n.
Step 2: Replace n by (n + 1) in a_n to get a_{n+1}
Step 3: Calculate $a_{n+1} – a_n$
Step 4: Check the value of $a_{n+1} – a_n$.
If $a_{n+1} – a_n$ is independent of n, then the given sequence is an A.P. Otherwise, it is not an A.P.

OR

A list of numbers a_1, a_2, a_3…… is an A.P, if the differences $a_2 – a_1$, $a_3 – a_2$, $a_4 – a_3$ … give the same value, i.e., $a_{k+1} – a_k$ is same for all different values of k.

5. Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient.

Number of terms	Terms	Common difference
3	a – d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

It should be noted that in case of an odd number of terms, the middle term is 'a' and the common difference is 'd' while in case of an even number of terms the middle terms are a – d, a + d and the common differences is 2d.

6. Arithmetic mean:

If three number a, b, c (in order) are in A.P. Then,
b – a = c – b = common difference
⇒ 2b = a + c

Thus a, b and c are in A.P., if and only if 2b = a + c. In this case, b is called the **Arithmetic mean** of a and c.

7. Sum of n terms of an A.P:

- **Sum of n terms of an A.P.** is given by
  $$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
  where 'a' is the first term, 'd' is the common difference and 'n' is the total number of terms.

- **Sum of n terms of an A.P.** is also given by:
  $$S_n = \frac{n}{2}[a + \ell]$$
  where 'a' is the first term and 'λ' is the last term.
- **Sum of first n natural numbers** is given by $\frac{n(n+1)}{2}$.

8. The n[th] term of an A.P is the difference of the sum to first n terms and the sum to first (n -1) terms of it. That is, $a_n = S_n - S_{n-1}$