Lecture_05

No. 32

ARITHMETIC PROGRESSIONS

- **Sums based on 'a_n' Formula**

15) For what value of n, are the nth terms of two APs: 63, 65, 67,... and 3, 10, 17,... equal?

Sol: For first AP: 63, 65, 67,...

$a = 63, \quad d = 65 - 63 = 2$

We know that,

$a_n = a + (n - 1)d$

$= 63 + (n - 1)(2)$

$= 63 + 2n - 2$

$a_n = 61 + 2n \quad .....(i)$

For second AP: 3, 10, 17,...

$a = 3, \quad d = 10 - 3 = 7$

We know that,

$A_n = a + (n - 1)d$

$= 3 + (n - 1)(7)$

$= 3 + 7n - 7$

$A_n = -4 + 7n \quad .....(ii)$

We need to find 'n' Such that, $a_n = A_n$

$\therefore 61 + 2n = -4 + 7n$

$\therefore 61 + 4 = 7n - 2n$

$\therefore 65 = 5n$

That means, $a_{13} = A_{13}$

$\therefore n = 13$

$\therefore$ **For n = 13, nth terms of two APs are equal.**

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ARITHMETIC PROGRESSIONS

- **Word problems based on 'a_n' Formula**

Q.19) Subba Rao started work in 1995 at an annual salary of Rs.5000 and received an increment of Rs.200 each year. In which year did his income reach Rs.7000 ?

Sol:

Rise in salary

Year 1995
Annual salary Rs.5000 — a_1

+ Rs. — a_2

Year 1996
Annual salary Rs.5200

+ Rs. — a_3

Year 1997
Annual salary Rs.5400

⋮

Year ?
Annual salary Rs.7000 — a_n

...ount of ...nual salaries forms an AP

We need to find n when $a_n = 7000$

Q.19) Subba Rao started work in 1995 at an annual salary of Rs.5000 and received an increment of Rs.200 each year. In which year did his income reach Rs.7000 ?

We need to find n when $a_n = 7000$

Sol: The salaries of each year after 1995 are

5000, 5200, 5400, ..., 7000

$\therefore$ The incomes that Subba Rao obtained in various years form an A.P with first term (a) = 5000 and common difference(d) = 200

Let after n^{th} year, his salary will be Rs. 7000

Now $a_n = 7000$

$\therefore \quad a_n = a + (n-1)d$

$\therefore \quad 7000 = 5000 + (n-1)\,200$

$\therefore \quad 7000 - 5000 = (n-1)\,200$

$\therefore \quad 2000 = (n-1)\,200$

$\therefore \quad \frac{2000}{200} = (n-1)$

$\therefore \quad 10 = n - 1$

$\therefore \quad n = 10 + 1$

$\therefore \quad n = 11$

Therefore, in 11th year, his salary will be Rs.7000.

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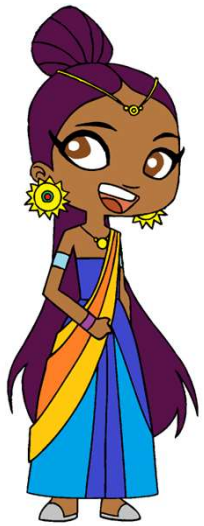
ARITHMETIC PROGRESSIONS

- **Word problems based on 'a_n' Formula**

Q.20) Ramkali saved Rs.5 in the first week of a year and then increased her weekly savings by Rs.1.75. If in the nth week, her weekly savings become Rs. 20.75, find n.

Sol:



a_1

Rs.5

First week of

+

a_2

Rs.6.75

Second week

+

a_3

Rs.9.50

Third week of a year

⋮

a_n

Rs.20.75

n^{th} week of a year

We need to find n when $a_n = 20.75$

Amount of weekly savings forms an AP

Q.20) Ramkali saved Rs.5 in the first week of a year and then increased her weekly savings by Rs.1.75. If in the nth week, her weekly savings become Rs.20.75, find n.

We need to find n when $a_n = 20.75$

Sol: The weekly savings of Ramkali are as follows:

5, 6.75, 9.50, ..., 20.75

Amount of her weekly savings form an AP

$\therefore$ $a = 5, \quad d = 1.75, \quad a_n = 20.75$

$\therefore$ $a_n = a + (n - 1)\, d$

$\therefore$ $20.75 = 5 + (n - 1)\, 1.75$

$\therefore$ $15.75 = (n - 1)\, 1.75$

$\therefore$ $(n - 1) = \dfrac{15.75}{1.75}$

$\therefore$ $(n - 1) = 9$

$\therefore$ $n = 9 + 1$

$\therefore$ $n = 10$

$\dfrac{15.75}{1.75} = \dfrac{1575}{175} = 9$

Hence, n is 10

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Arithmetic Progressions

- **Introduction of S_n**

S_n = Sum of first n terms

3 + 5 + 7 + 9 ..

$S_1 = 3$

$S_2 = 8$

$S_3 = 15$

$S_4 = 24$

$S_{37} = ?$

Sum of first term

Sum of first two terms

Sum of first three terms

Sum of first four terms

By using a formula of first 37 terms?

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Lets simplify this formula

$$= \frac{n}{2}[a + a + (n-1)d]$$

$$\frac{n}{2}[a + a_n]$$

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Arithmetic Progressions

- **Sums based on S_n formula**

1) Find the sum of the following AP.

(i) 2, 7, 12, ..., to 10 terms

Sol: For given AP: 2, 7, 12, ...

$a = 2, \quad d = 7 - 2 = 5$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

We need to find S_{10}

For S_{10} substitute, n = 10, a = 2 & d = 5

$$\therefore S_{10} = \frac{10}{2}[2(2) + (10-1)(5)]$$

$$= 5\ [4 + (9)(5)]$$

$$= 5\ [4 + 45]$$

$$= 5\ [49]$$

$$\therefore \boxed{S_{10} = 245}$$

1) Find the sum of the following AP.

(ii) 0.6, 1.7, 2.8, ..., to 100 terms

Sol: For given AP: 0.6, 1.7, 2.8,...

$a = 0.6,\ d = 1.7 - 0.6 = 1.1$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

We need to find S_{100}

For S_{100} substitute, n = 100, a = 0.6 & d = 1.1

$$\therefore S_{100} = \frac{100}{2}[2(0.6) + (100-1)(1.1)]$$

$$= 50\ [1.2 + (99)(1.1)]$$

$$= 50\ [1.2 + 108.9]$$

$$= 50\ [110.1]$$

$$\therefore \mathbf{S_{100} = 5505}$$

1. Find the sum of the following A.P's:

(iii) – 37, – 33, – 29, ……., to 12 terms

Sol: For this A.P – 37, – 33, – 29

$a = -37$

$d = -33 - (-37)$

$= -33 + 37$

$= 4$

We need to find S_{12}

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

For S_{12} substitute n = 12, a = – 37 and d = 4

$$S_{12} = \frac{12}{2}[\,2(-37) + (12-1)\,4]$$

$$= 6[-74 + 44]$$

$$= 6(-30)$$

$$= -180$$

$\therefore$ **$S_{12} = -180$**

1. Find the sum of the following A.P's:

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ... **to 11 terms**

Sol: For given AP : $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$

To find S_{11}

$a = \frac{1}{15}$, $d = \frac{1}{12} - \frac{1}{15} = \frac{5}{6}$

For S_{11} substitute n = 11, $a = \frac{1}{15}$ and $d = \frac{1}{60}$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + (10)\frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$$

$$= \frac{11}{2}\left[\frac{2 \times 2 + 1 \times 5}{30}\right]$$

$$= \frac{11}{2} \times \frac{3}{30_{10}}$$

$$S_{11} = \frac{33}{20}$$

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Arithmetic Progressions

- **Sums based on S_n formula**

3) In an AP.

(vii) Given $l = 28$, $S_n = 144$, and there are total 9 terms. Find a.

Sol: $l = a_n = 28,$

l means last term i.e. a_n

Number of terms are given

For given value of S_n, Lets use the formula

$$S_n = \frac{n}{2}[a + a_n]$$

$$\therefore 144 = \frac{9}{2}[a + 28]$$

Substitute, Value of S_n, an & n

$$\therefore 288 = 9[a + 28]$$

$$\therefore 32 = a + 28$$

$$\therefore a = 32 - 28$$

$$\therefore \boxed{a = 4}$$

3) In an AP.

(x) Given a = 3, n = 8, S_n = 192, find d.

Sol: a = 3, n = 8, S_n = 192

For given value of S_n Let's use the formula

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$$\therefore\ 192 = \frac{8}{2}[2(3) + (8-1)\,d]$$

$$\therefore\ 192 = 4\,[6 + 7d]$$

$$\therefore\ \frac{192}{4} = 6 + 7d$$

$$\therefore\ 48 = 6 + 7d$$

$$\therefore\ 48 - 6 = 7d$$

$$\therefore\ 42 = 7d$$

$$\therefore\ d = \frac{42}{7}$$

$$\therefore\ \boxed{d = 6}$$

No. 38

Arithmetic Progressions

- **Sums based on S_n formula**

4) How many terms of the AP: 9, 17, 25,... give a sum of 636 ?

Sol: For given AP: 9, 17, 25,...

$a = 9, \quad d = 17 - 9 = 8,$ $S_n =$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore 636 = \frac{n}{2}[2(9) + (n-1)(8)]$$

$$\therefore 636 \times 2 = n[18 + 8n - 8]$$

$$\therefore 1272 = n[8n + 10]$$

$$\therefore 1272 = 8n^2 + 10n$$

$$\therefore 0 = 8n^2 + 10n - 1272$$

$$\therefore 8n^2 + 10n - 1272 = 0$$

Dividing throughout by 2, we get

$$\therefore 4n^2 + 5n - 636 = 0$$

$$\therefore 4n^2 + 53n - 48n - 636 = 0$$

$$\therefore n(4n + 53) - 12(4n + 53) = 0$$

$$\therefore (4n + 53)(n - 12) = 0$$

$$\therefore 4n + 53 = 0 \quad \text{or} \quad n - 12 = 0$$

$$\therefore 4n = -53 \quad \text{or} \quad n = 12$$

$$\therefore n = \frac{-53}{4} \quad \text{or} \quad n = 12$$

As 'n' cannot be negative

$\therefore n = 12$

We need to find no. of terms i.e. value of 'n'

For giv... e the formula

Take comm... first two

Take common from last two terms

Substitute, $a = 9, d = 8$ & $S_n = 636$

$2 \times 2 \times 2 \times 2 \times 3 \times 53$

$+53 \quad -48$

It's a quadratic equation, lets solve it by factorisation method

Thank You