Lecture 11

Module 36

Exercise 2.3

1. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Sol. Since two zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$

$\therefore$ $x - (2 + \sqrt{3})$ and $x - (2 - \sqrt{3})$

are the factors of the polynomials

$\therefore$ $[x - (2 + \sqrt{3})][x - (2 - \sqrt{3})]$

$= [(x - 2) - \sqrt{3}]\ [(x - 2) + \sqrt{3}]$

$= (x - 2)^2 - (\sqrt{3})^2$

$= x^2 - 4x + 4 - 3$

$= x^2 - 4x + 1$

i.e. $x^2 - 4x + 1$ factor of the given polynomial. Now we divide the given polynomial by $x^2 - 4x + 1$.

Exercise 2.3

1. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Sol. i.e. $x^2 - 4x + 1$ factor of the given polynomial. Now we divide the given polynomial by $x^2 - 4x + 1$.

$$
\begin{array}{r|rrrrr}
 & x^2 & -2x & -35 & & \\
\hline
x^2 - 4x + 1 & \cancel{x^4} & -6x^3 & -26x^2 & +138x & -35 \\
 & \cancel{x^4} & -4x^3 & +x^2 & & \\
 & (-) & (+) & (-) & & \\
\hline
 & & -\cancel{2x^3} & -27x^2 & +13\cancel{8x} & -35 \\
 & & -\cancel{2x^3} & +8x^2 & -\cancel{2x} & \\
 & & (+) & (-) & (+) & \\
\hline
 & & & -\cancel{35x^2} & +140x & \cancel{-35} \\
 & & & -\cancel{35x^2} & +140x & \cancel{-35} \\
 & & & (+) & (-) & (+) \\
\hline
 & & & & 0 &
\end{array}
$$

$\dfrac{\cancel{x^4}^{\,2}}{\cancel{x^2}} = x^2$ | $x^2(x^2 - 4x + 1)$
$= x^4 - 4x^3 + x^2$

$\dfrac{-2\cancel{x^3}}{\cancel{x^2}} = -2x$ | $-2x(x^2 - 4x + 1)$
$= -2x^3 + 8x^2 - 2x$

$\dfrac{-35\cancel{x^2}}{\cancel{x^2}} = -35$ | $-35(x^2 - 4x + 1)$
$= -35x^2 + 140x - 35$

Exercise 2.3

1. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Sol.

$$
\begin{array}{r|l}
 & \boxed{x^2 - 2x - 35} \\
\hline
\boxed{x^2 - 4x + 1} & \boxed{x^4 - 6x^3 - 26x^2 + 138x - 35} \\
 & x^4 - 4x^3 + x^2 \\
 & \underset{(-)}{} \quad \underset{(+)}{} \quad \underset{(-)}{} \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & \underset{(+)}{} \quad \underset{(-)}{} \quad \underset{(+)}{} \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & \underset{(+)}{} \quad \underset{(-)}{} \quad \underset{(+)}{} \\
\hline
 & \boxed{0}
\end{array}
$$

$\boxed{\text{Dividend}} = \boxed{\text{Divisor}} \times \boxed{\text{Quotient}} + \boxed{\text{Remainder}}$

So, $x^4 - 6x^3 - 26x^2 + 138x - 35$

$= (x^2 - 4x + 1)(x^2 - 2x - 35) + 0$

Now, $x^2 - 2x - 35$ $\quad a^2 + 2ab + b^2 = (a + b)^2$

$= (x + 1)^2$

$= (x - 7)(x + 5)$

$\therefore \quad x - 7 = 0$ and $x + 5 = 0$

$\therefore \quad x = 7$ and $x = -5$

Its zeroes are 7 and – 5

Therefore, the remaining zeroes of the given polynomial 7 and –5.

Module 37

Exercise 2.3

1. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$ which satisfy the division algorithm and

(i) $\deg p(x) = \deg q(x)$ (ii) $\deg q(x) = \deg r(x)$

(iii) $\deg r(x) = 0$

Sol. (i) $\deg p(x) = \deg q(x)$

$p(x) = 2x^2 - 2x + 14, \quad g(x) = 2$

$q(x) = x^2 - x + 7, \quad r(x) = 0$

Dividend = Divisor × Quotient + Remainder

$$
\begin{array}{r|l}
 & x^2 - x + 7 \\
\hline
2 & 2x^2 - 2x + 14 \\
 & -2ge^2 \\
\hline
 & \quad -2x + 14 \\
 & \quad -2x \\
 & \quad + \\
\hline
 & \qquad 14 \\
 & \quad (-)\ 14 \\
\hline
 & \qquad 0
\end{array}
$$

Exercise 2.3

1. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$ which satisfy the division algorithm and

(i) $\deg p(x) = \deg q(x)$ (ii) $\deg q(x) = \deg r(x)$

(iii) $\deg r(x) = 0$

Sol. (i) $\deg p(x) = \deg q(x)$

$p(x) = 2x^2 - 2x + 14, \quad g(x) = 2$

$q(x) = x^2 - x + 7, \quad r(x) = 0$

(ii) $\boxed{\deg q(x) = \deg r(x)}$

$p(x) = x^3 + x^2 + x + 1, \; g(x) = x^2 - 1$

$q(x) = x + 1, \quad r(x) = 2x + 2$

$$
\begin{array}{r|l}
 & \boxed{x^{1} + 1} \\
\hline
\boxed{x^2 - 1} & \boxed{x^3 + x^2 + x + 1} \\
 & x^3 \qquad - x \\
 & (-) \qquad (+) \\
\hline
 & \qquad x^2 + 2x + 1 \\
 & \qquad x^2 \qquad - 1 \\
 & \qquad (-) \qquad (+) \\
\hline
 & \qquad \boxed{2x^{1} + 2}
\end{array}
$$

Exercise 2.3

1. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$ which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$ (ii) deg $q(x)$ = deg $r(x)$

(iii) deg $r(x) = 0$

Sol.

(ii) deg $q(x)$ = deg $r(x)$

$p(x) = x^3 + x^2 + x + 1$, $g(x) = x^2 - 1$

$q(x) = x + 1$, $r(x) = 2x + 2$

(iii) $\boxed{\text{deg } r(x) = 0}$

$p(x) = x^3 + 2x^2 + x + 2$, $q(x) = x^2 + 1$

$g(x) = x + 2$, $r(x) = 0$

$$
\begin{array}{r|l}
 & \boxed{x^2 + 1} \\
\boxed{x + 2} & \boxed{x^3 + 2x^2 + x + 2} \\
 & \underline{\overset{(-)}{x^3} + \overset{(-)}{2x^2}} \\
 & \qquad\quad x + 2 \\
 & \qquad\quad \underline{\overset{(-)}{x} + \overset{(-)}{2}} \\
 & \qquad\qquad \boxed{0}
\end{array}
$$

Module 38

Exercise 2.3

1. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $ax + b$, the find a and b.

Sol. Dividend $= 6x^4 + 8x^3 + 17x^2 + 21x + 7$

Divisor $= 3x^2 + 4x + 1$

$$
\begin{array}{r|l}
 & 2x^2 + 5 \\
\hline
3x^2 + 4x + 1 & \cancel{6x^4} + \cancel{8x^3} + 17x^2 + 21x + 7 \\
 & \cancel{6x^4} + \cancel{8x^3} + 2x^2 \\
 & (-) \quad (-) \quad (-) \\
\hline
 & \cancel{15x^2} + 21x + 7 \\
 & \cancel{15x^2} + 20x + 5 \\
 & (-) \quad (-) \quad (-) \\
\hline
 & x + 2
\end{array}
$$

$$\frac{\cancel{6x^4}^{\,2}}{\cancel{3x^2}} = 2x^2 \qquad 2x^2(3x^2 + 4x + 1) = 6x^4 + 8x^3 + 2x^2$$

$$\frac{\cancel{15x^2}^{\,5}}{\cancel{3x^2}} = 5 \qquad 5(3x^2 + 4x + 1) = 15x^2 + 20x + 5$$

Here, Remainder comes out to be, $x + 2$

But, it is given that, Remainder $= ax + b$,

$\therefore\ ax + b = 1x + 2$

$\therefore\ \boxed{a = 1 \text{ and } b = 2}$

Module 39

Exercise 2.3

1. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Sol.

Divisor $= \boxed{x^2 - 2x + k}$

Dividend $= \boxed{x^4 - 6x^3 + 16x^2 - 25x + 10}$

Remainder $= \boxed{x + a}$

$x^4 - 6x^3 + 16x^2 - 25x + 10$

$= (x^2 - 2x^2 + k) \times \text{Quotient} + \underline{(x + a)}$

$x^4 - 6x^3 + 16x^2 - 25x + 10 - (x + a)$

$= (x^2 - 2x^2 + k) \times \text{Quotient}$

$x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = (x^2 - 2x^2 + k) \times \text{Quotient}$

$$\frac{x^4 - 6x^3 + 16x^2 - 26x + 10 - a}{x^2 - 2x + k} = \text{Quotient}$$

If the polynomial $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ is divided by $x^2 - 2x + k$, remainder

DIVIDEND = DIVISOR × QUOTIENT + REMAINDER

Exercise 2.3

1. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Sol. If the polynomial $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ is divided by $x^2 - 2x + k$, remainder will be zero.

$$x^2 - 4x + (8 - k)$$

$$x^2 - 2x + k \,\big)\, x^4 - 6x^3 + 16x^2 - 26x + 10 - a$$

$$\cancel{x^4} - 2x^3 + kx^2$$

$$(-)\quad (+)\quad (-)$$

$$-\cancel{4x^3} + (16 - k)x^2 - 26x + 10 - a$$

$$-\cancel{4x^3} + 8x^2 - 4kx$$

$$(+)\quad (-)\quad (+)$$

$$(8 - \cancel{k)x^2} + (26 - 4k)x + 10 - a$$

$$(8 - \cancel{k)x^2} - (16 - 2k)x + 8k - k^2$$

$$(-)\quad (+)\quad (-)$$

$$-(10 - 2k)x + (10 - 9) - (8k - k^2)$$

$$(16 - k)x^2 - 8x^2$$
$$= (16 - k - 8)\, x^2$$
$$= (8 - k)\, x^2$$

$\dfrac{x^{\cancel{4}\,2}}{\cancel{x^2}} = x^2$	$x^2(x^2 - 2x + k)$ $= x^4 - 2x^3 + kx^2$
$\dfrac{-4x^{\cancel{3}}}{\cancel{x^2}} = -4x$	$-4x(x^2 - 2x + k)$ $= -4x^3 + 8x^2 - 4kx$
$\dfrac{(8 - k)\cancel{x^2}}{\cancel{x^2}}$ $= 8 - k$	$8 - k(x^2 - 2x + k)$ $= 8x^2 - 16x - 8k - kx^2 + 2kx - k^2$ $= 8x^2 - kx^2 - 16x + 2kx + (8k - k^2)$ $= (8x^2 - kx^2) - (16x - 2kx) + (8k - k^2)$

Exercise 2.3

1. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Sol. On comparing,

$-10 + 2k = 0$

$\Rightarrow 2k = 10$

$\Rightarrow k = 5$

and

$10 - a - 8k + k^2 = 0$

$\Rightarrow 10 - a - 8 \times 5 + 5^2 = 0$ [As $k = 5$]

$\Rightarrow 10 - a - 40 + 25 = 0$

$\Rightarrow -a - 5 = 0$

$\Rightarrow a = -5$

Hence, $k = 5$ and $a = -5$