Lecture_09

No. 58

Arithmetic Progressions

- **Word problems based on S_n formula**

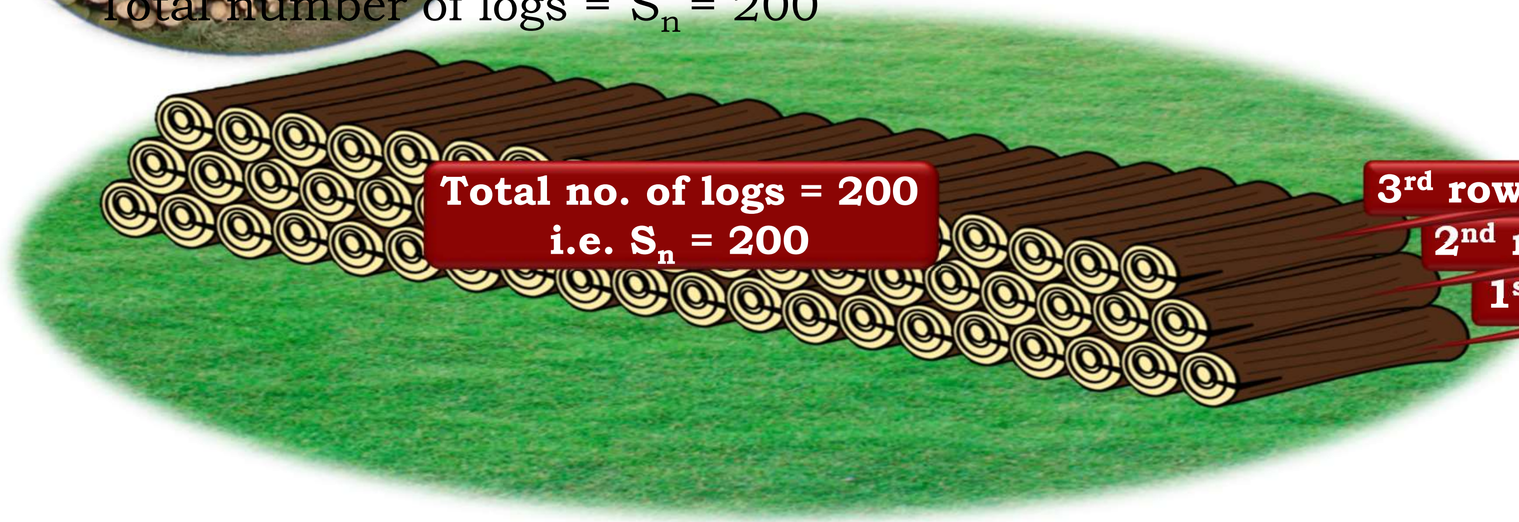
19) 200 logs are stacked in the following manner, 20 logs in the bottom row, 19 in next row, 18 in the next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

We need to find no. of rows i.e. value of 'n'

Sol: Number … logs placed in successive row starting … are as follows: 20, 19, 18, …

These numbers form an A.P. with $a = 20$ and $d = 19 - 20 = -1$

Total number of logs $= S_n = 200$



19) 200 logs are stacked in the following manner, 20 logs in the bottom row, 19 in next row, 18 in the next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

Sol: Number of logs placed in successive row starting from bottom are as follows: 20, 19, ...

These numbers form an A.P. wi... $- 20 = -1$

Total number of logs = $S_n = 200$

For given value of S_n, Lets use the formula

Substitute, $a = 20$, $d = -1$ & $S_n = 200$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore 200 = \frac{n}{2}[2(20) + (n-1)(-1)]$$

$$\therefore 200 \times 2 = n[40 - n + 1]$$

$$\therefore 400 = n[41 - n]$$

$$\therefore 400 = 41n - n^2$$

$$\therefore n^2 - 41n + 400 = 0$$

$$\therefore n^2 - 25n - 16n + 400 = 0$$

It's a quadratic equation, lets solve it by factorisation method

... $25) = 0$

400

$2 \times 2 \times 2 \times 2 \times 5 \times 5$

$-16 \quad -25$

$$\therefore (n - 25)(n - 16) = 0$$

$\therefore n - 25 = 0$ o...

$\therefore n = 25$ o...

Lets find no. of ...

when, $n = 25$

$a_{25} = a + 24d = 20 + 24(-1) = 20 - 24 = -4$

That means in 25^{th} row there are -4 logs

Since, number of ... ve

$\therefore n \neq 25$

when, $n = 16$

Lets find no. of logs in 16^{th} row

... 16^{th} ... row there are 5 logs

$a_{16} = a + 15d = 20 + 15(-1) = 20 - 15 = 5$

$\therefore$ Number of rows in which 200 logs placed are 16 and 5 logs are placed in top row.

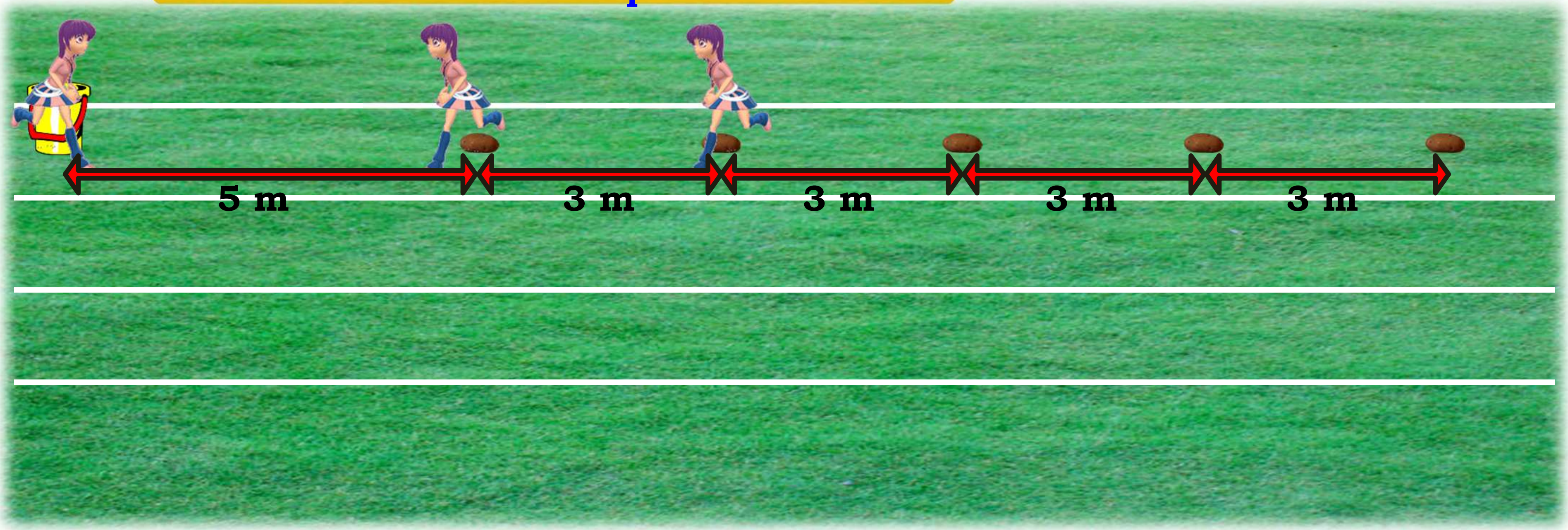
No. 59

Arithmetic Progressions

- **Word problems based on S_n formula**

Q.20] In a potato race, a bucket at the starting point, which is 5m from the first potato and the other potatoes are placed 3m apart in a straight line. There are ten potatoes in the line.

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?



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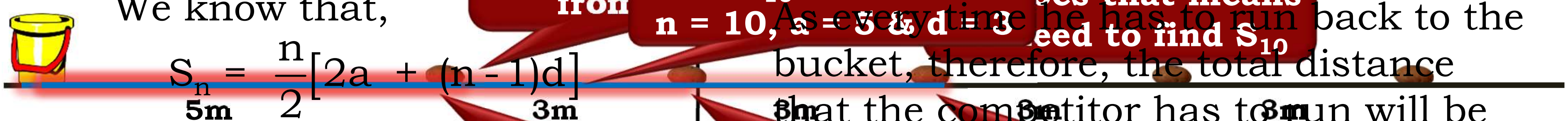
Sol: The distance of potatoes are as follows: 5, 8, 11, ...

These numbers form an A.P. with $a = 5$ and $d = 8 - 5 = 3$

We know that,



$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{10} = \frac{10}{2}[2(5) + (10-1)(3)]$$

$$= 5\,[10 + (9)(3)]$$

$$= 5\,[10 + 27]$$

$$= 5\,[37]$$

$$\therefore S_{10} = 185$$

As every time he has to run back to the bucket, therefore, the total distance that the competitor has to run will be two times of it.

$\therefore$ Total distance that the competitor will run

$= 2 \times 185$

$\boxed{= 370\text{m}}$

For S_{10} substitute, $n = 10$, $a = 5$ & $d = 3$

Distance of third potato from bucket

No. 60

Arithmetic Progressions

- **Additional sums based on concepts of AP**

Q.1] The 24^{th} t … s 10^{th} term. Show that its 72^{nd} term is 4 …

$a_{24} = a + 2$ …

$a_{10} = a + 9d$

Sol: $a_{24} = 2\,(a_{10})$

$\therefore\ a + 23d = 2\,(a + 9d)$

$\therefore\ a + 23d = 2a + 18d$

$\therefore\ 23d - 18d = 2a - a$

$\therefore\ 5d = a \quad \ldots.(i)$

Lets check, what is given?

Show that $a_{72} = 4\,(a_{15})$

Lets simplify LHS & RHS

$a_{72} = a + 71d$

LHS,

$a_{72} = a + 71d$

$= 5d + 71d$

$\boxed{= 76d}$

$a_{15} = a + 14d$

RHS,

$4(a_{15}) = 4(a + 14d)$

$= 4(5d + 14d)$

$= 4(29d)$

$\boxed{= 76d}$

$\therefore\ \boxed{a_{72} = 4\,(a_{15})}$

Q.2] The sum of first n terms of an AP is $5n^2 + 3n$. If its mth term is 168, find the value of m. Also, find the 20th term of this AP.

Sol:

$S_n = 5n^2 + 3n$

$\therefore S_1 = 5(1)^2 + 3(1)$

$= 5 + 3$

$= 8$

$\therefore S_1 = a_1 = 8$

$\therefore S_2 = 5(2)^2 + 3(2)$

$= 5(4) + 6$

$= 20 + 6$

$= 26$

$a_2 = S_2 - S_1$

$= 26 - 8$

$= 18$

Find S_2 by putting n = 2

S_1 means sum of first term i.e. first term itself

$d = a_2 - a_1$

$= 18 - 8$

$= 10$

$a_m = 168$

$\therefore a + (m - 1)d = 168$

$a_m = a + (m - 1)d$

$\therefore 8 + (m - 1)10 = 168$

$\therefore (m - 1)10 = 160$

$\therefore m - 1 = 16$

$\therefore$ **m = 17**

$a_{20} = a + 19d$

$= 8 + 19(10)$

$= 8 + 190$

$\mathbf{a_{20} = 198}$

No. 61

Arithmetic Progressions

- **Additional sums based on concepts of AP**

Q.3] The sum of first m terms of an AP is same as the sum of its n terms, show that the sum of its (m + n) term is zero. $(m - n \neq 0)$

Sol:

$$S_m = S_n$$

$$\frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$$

$m[2a + \ldots$

$2am + m \ldots md \ldots$

$2am - 2an + m^2d \ldots md + nd = 0$

$2a(m - n) + \ldots - d(m - n) = 0$

$2a(m - n) + d(m + n)(m - n) - d(m - n) = 0$

$(m - n)[2a + d(m + n) - d] = 0$

$2a + d(m + n) - d = 0$

$2a + dm + dn - d = 0 \quad \ldots.(i)$

For S_m, replace n by m

We need to show that: $S_{(m+n)} = 0$

Take 2a common

By using $a^2 - b^2 = (a - b)(a + b)$

Bring all terms to LHS

We need '0' in RHS

How to get (m – n) in this term?

Take (m – n) common

...mmon

...1)d]

$$S_{(m+n)} = \frac{m+n}{2}[2a + (m+n-1)d]$$

$$= \frac{m+n}{2}[2a + dm + dn - d]$$

$$= \frac{m+n}{2}[0] \quad \ldots(\text{From i})$$

$$S_{(m+n)} = 0$$

The sum of $(m + n)^{th}$ term is 0

No. 62

Arithmetic Progressions

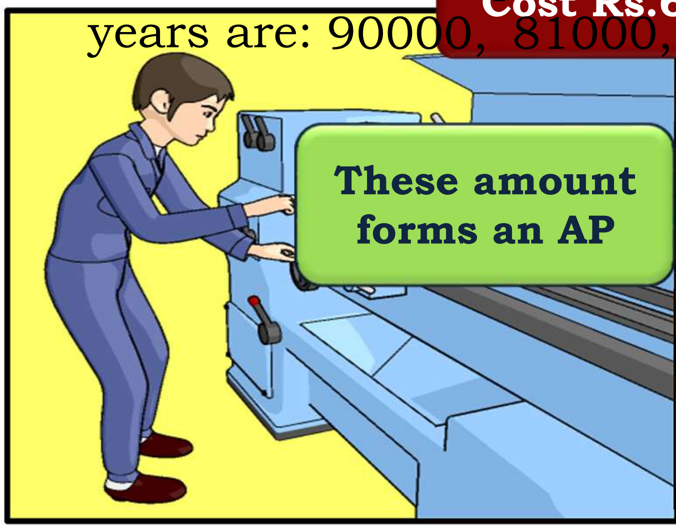
- **Additional sums based on concepts of AP**

Q.4] A piece of equipment cost a certain factory Rs.6,00,000. If it depreciates in value, 15% the first year, 13.5% the next year, 12% the third year, and so on. What will be its value at the end of 10 years, all percentage applying to the

Decrease in value

Sol: The amount of depreciation for successive years are: 90000, 81000, 72000, ...

Cost Rs.6,00,000



These amount forms an AP

End of first year – 15% of Rs.6,00,000

$\frac{15}{100} \times 600000 = 90000$

End of second year – 13.5% of Rs.6,00,000

$\frac{13.5}{100} \times 600000 = 81000$

End of third year – 12% of Rs.6,00,000

$\frac{12}{100} \times 600000 = 72000$

Q.4] A piece of equipment cost a certain factory Rs.6,00,000. If it depreciates in value, 15% the first, 13.5% the next year, 12% the third year, and so on. What will be its value at the end of 10 years, all percentage applying to the original cost?

We need to find: **Total depreciation value of equipment in 10 years**

Find S_{10} for list of depreciation value

Sol: The amount of depreciation for successive years are: 90000, 81000, 72000, ...

Which forms an A.P. with a = 90000

and d = 81000 – 90000 = – 9000

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{10} = \frac{10}{2}[2(90000) + (10-1)(-9000)]$$

$$= 5 [180000 + 9(-9000)]$$

$$= 5 [180000 - 81000]$$

$$= 5 [99000]$$

$$= 495000$$

In 10 years value of equipment depreciates by Rs.4,95,000

$\therefore$ **Value of equipment at the end of 10 years will be** = 600000 – 495000

= **Rs.105000**

Thank You