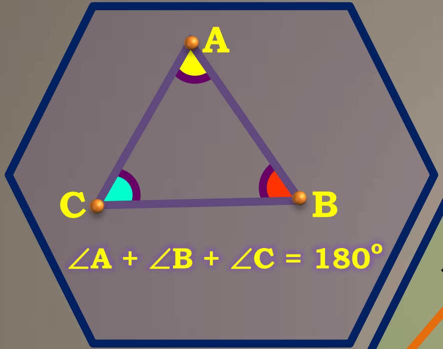
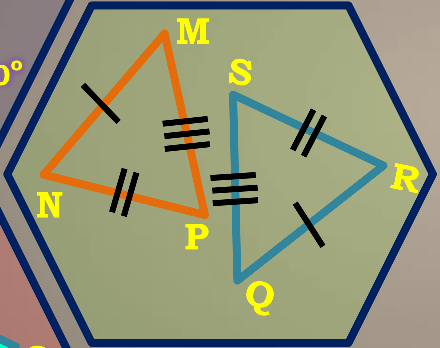
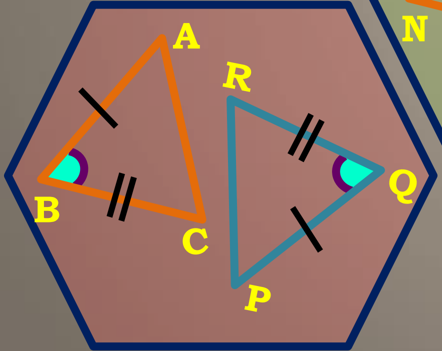
Lecture : 1

Triangles



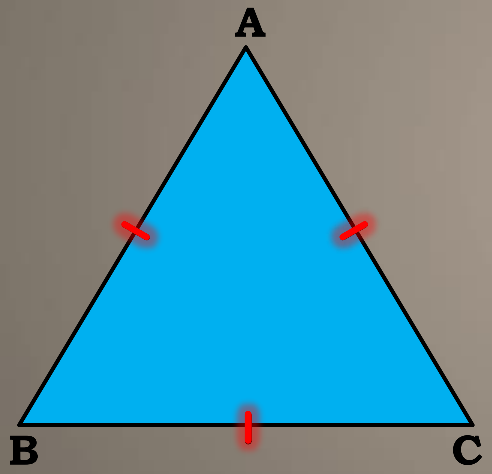




TYPES OF TRIANGLES

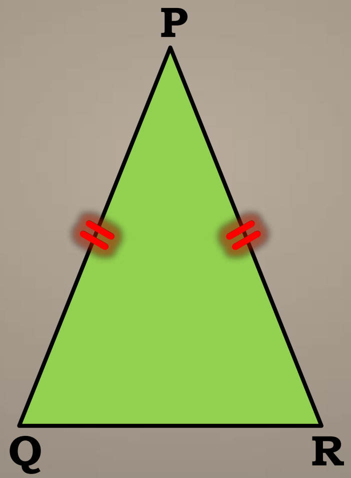
ON THE BASIS OF SIDES :

Equilateral triangle



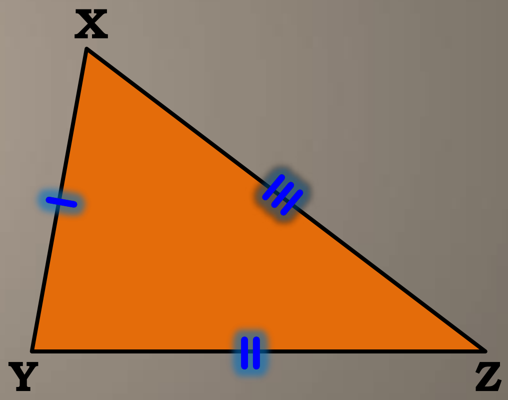
All three sides are equal

Isosceles triangle



Two sides are equal

Scalene triangle

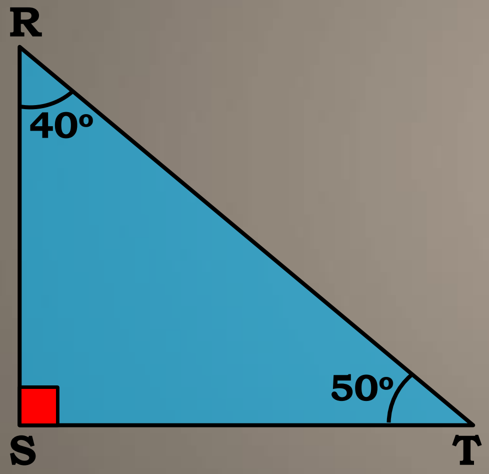


No two sides are equal

TYPES OF TRIANGLES

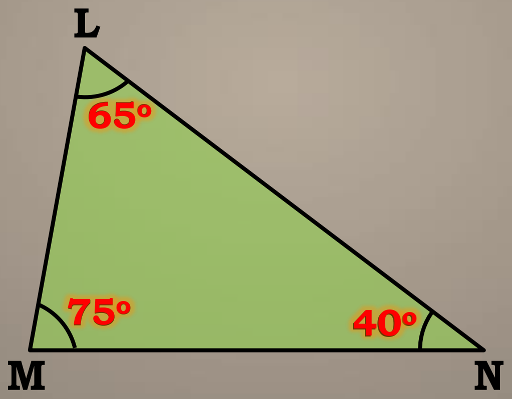
ON THE BASIS OF ANGLES :

Right angled triangle



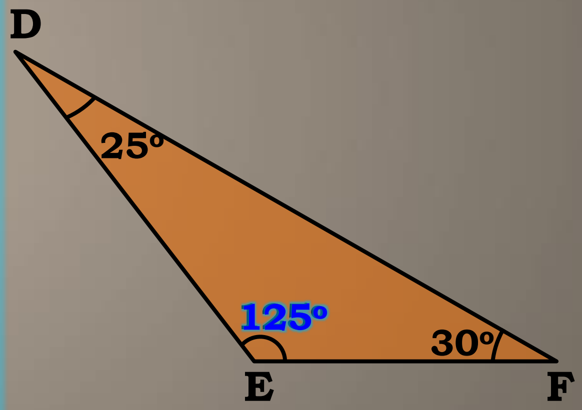
One angle is right angle

Acute angled triangle



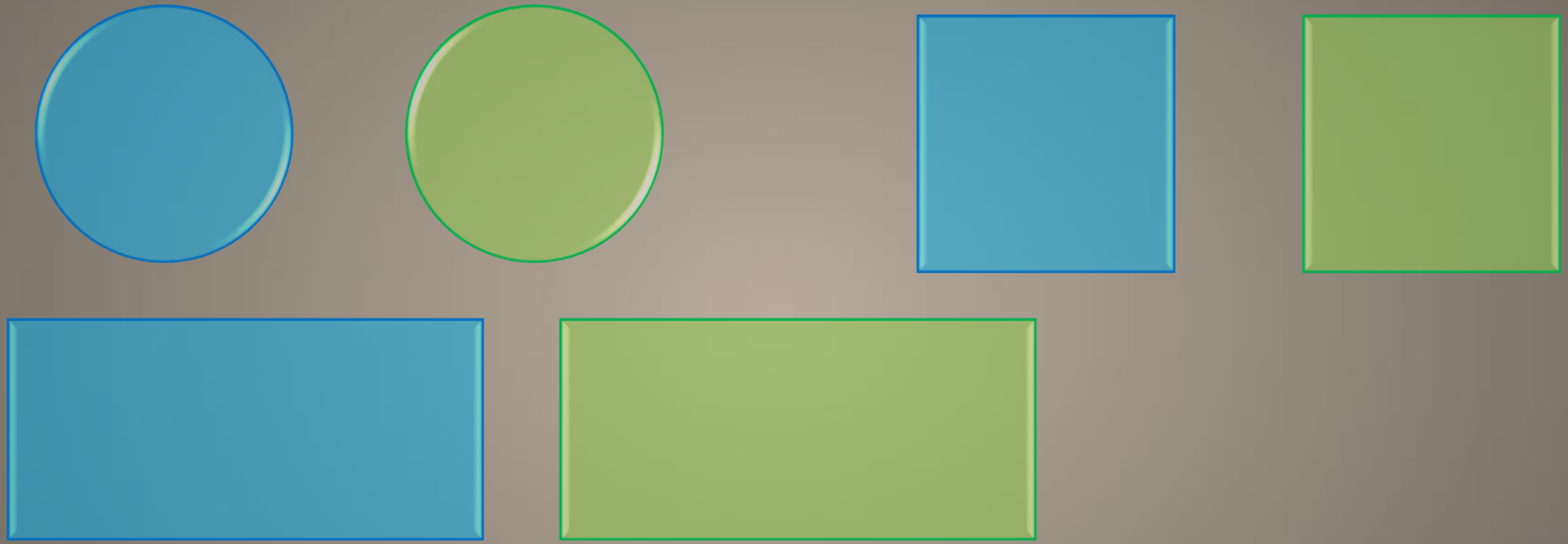
All three angles are acute

Obtuse angled triangle



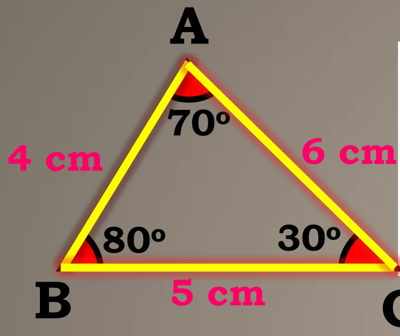
One angle is obtuse

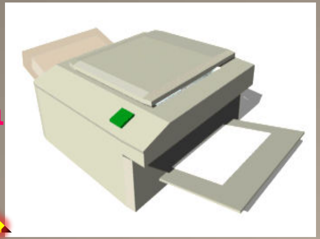
CONGRUENT FIGURES

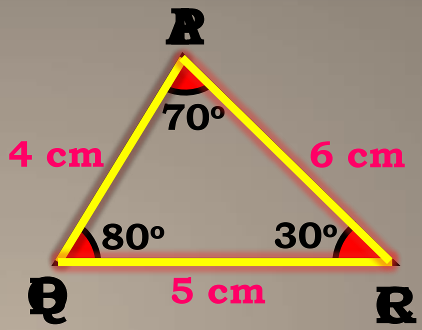


Figures having same shape and same size are congruent

∆ABC is congruent to ∆PQR







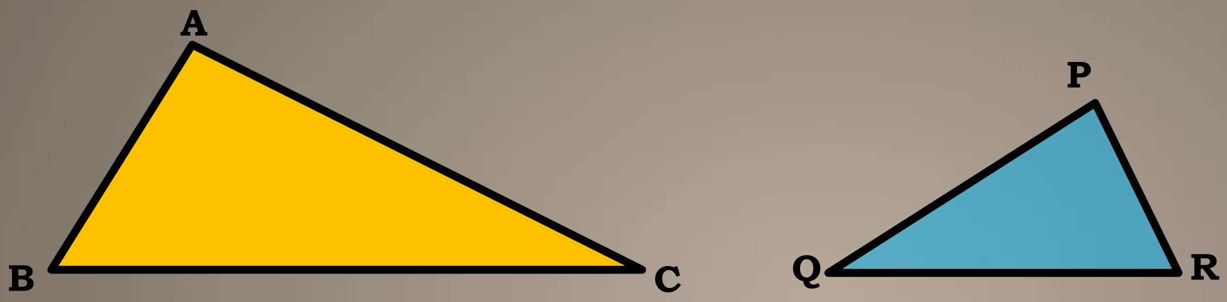
side AB ≅ side PQ, side BC ≅ side QR, side AC ≅ side PR

Corresponding sides of congruent triangles are congruent.

$\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$

Corresponding angles of congruent triangles are congruent.

Consider ΔABC and ΔPQR of any shape and size.



There are six different ways of placing one triangle over the other among the two sets of vertices of triangles.

They are :

$ABC \leftrightarrow PQR$

$ABC \leftrightarrow PQR$

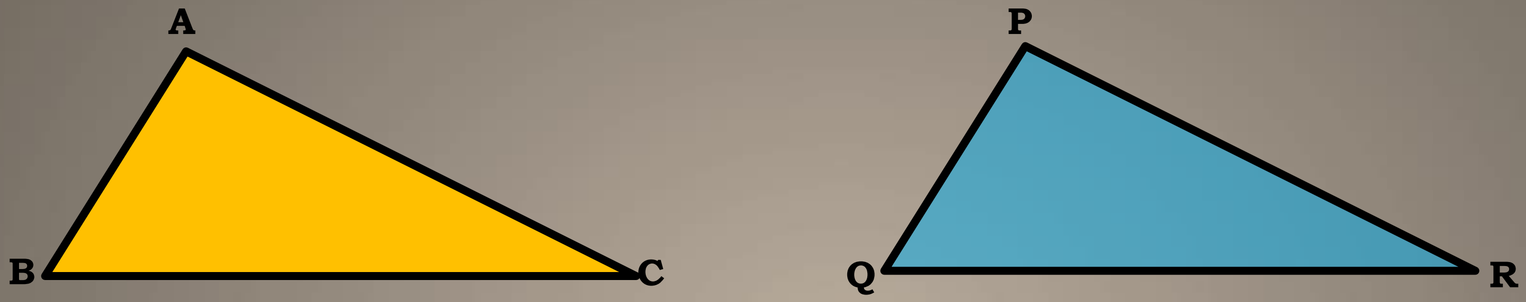
$ABC \leftrightarrow QPR$

$ABC \leftrightarrow QPR$

$ABC \leftrightarrow RPQ$

$ABC \leftrightarrow RPQ$

ΔABC and ΔPQR are congruent to each other.

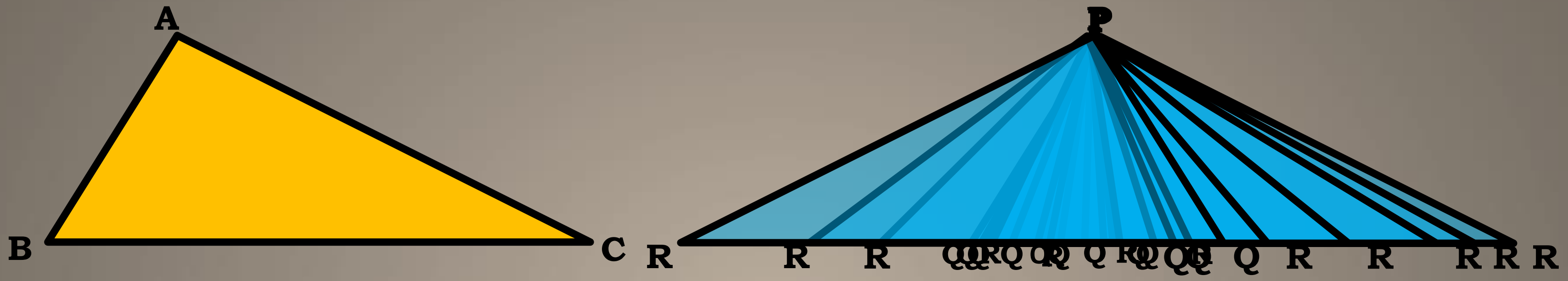


ABC ↔ QRP ✗

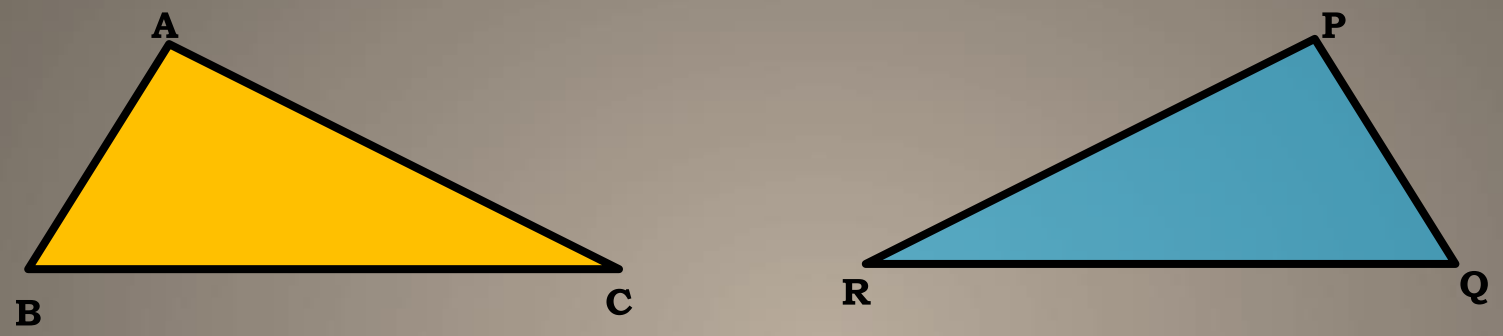
ABC ↔ RPQ ✗

ABC ↔ PQR ✓

∆ABC and ∆PQR are congruent to each other.



ΔABC and ΔPQR are congruent to each other.



ABC $\leftrightarrow$ PQR ✓

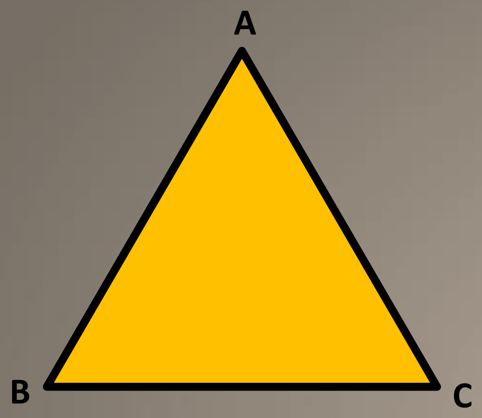
ABC $\leftrightarrow$ RQP ✗

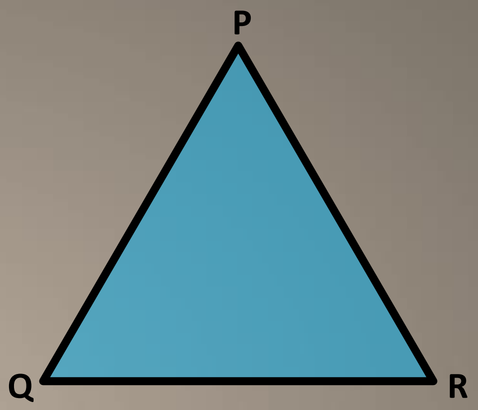
ABC $\leftrightarrow$ QPR ✗

ABC $\leftrightarrow$ PRQ ✗

We observed that ΔABC and ΔPQR are congruent for ABC $\leftrightarrow$ PQR
Therefore, we write it as $\Delta ABC \cong \Delta PQR$.

Consider two equilateral ΔABC and ΔPQR

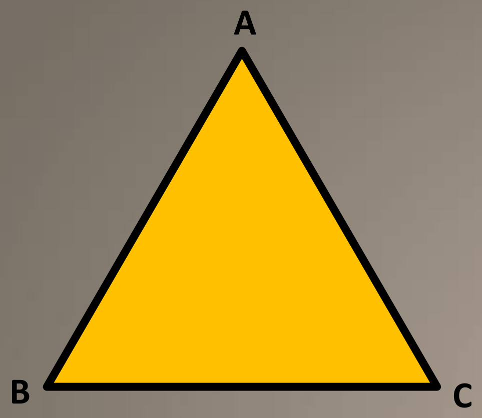


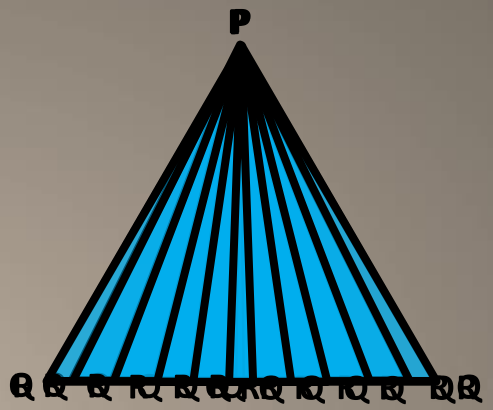


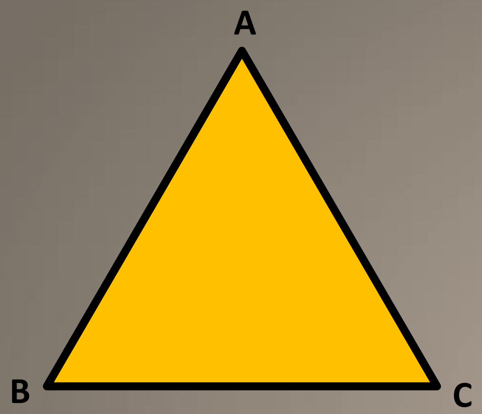
ABC ↔ PQR ✓

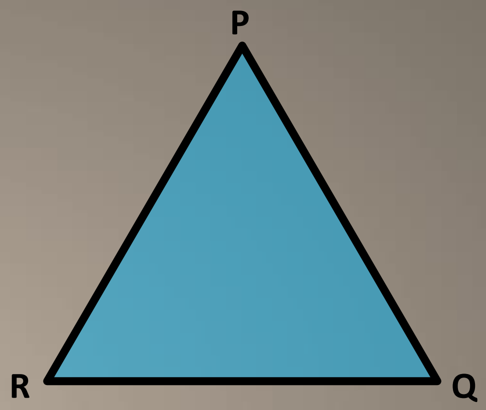
ABC ↔ QRP ✓

ABC ↔ RPQ ✓


A
B
C


P





ABC ↔ PRQ ✓

ABC ↔ RQP ✓

ABC ↔ QPR ✓

Two equilateral triangles of same size are congruent for all six correspondences.

SUFFICIENT CONDITIONS FOR CONGRUENCE OF TWO TRIANGLES

We know that, SIX conditions (i.e. 3 pairs of sides and 3 pairs of angles) are required to determine the congruency of two triangles.

Do we really need all the six conditions?

NO

SUFFICIENT CONDITIONS FOR CONGRUENCE OF TWO TRIANGLES

Out of six conditions, only THREE conditions (if properly chosen) are sufficient for determining the congruency of the two triangles.

When these three conditions are satisfied, then the other three automatically get satisfied, hence the triangles become congruent.

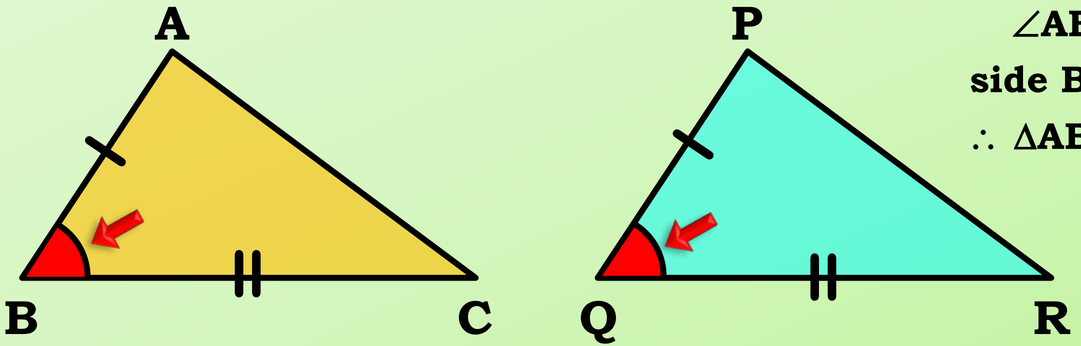
These sufficient conditions will be referred to as ‘TESTS OF CONGRUENCE OF TRIANGLES’

MODULE : 2

Criteria OF CONGRUENCY

1. SAS criteria

When two sides and included angle of one triangle are congruent to corresponding two sides and the included angle of the other triangle, then these two triangles are congruent.



In ΔABC and ΔPQR,

side $AB \cong$ side PQ [*Given*]

$\angle ABC \cong \angle PQR$ [*Given*]

side $BC \cong$ side QR [*Given*]

$\therefore\ \Delta ABC \cong \Delta PQR$ [*SAS test*]

Criteria OF CONGRUENCY

2. ASA criterion

When two angles and included side of one triangle are congruent to corresponding two angles and the included side of the other triangle, then these two triangles are congruent.

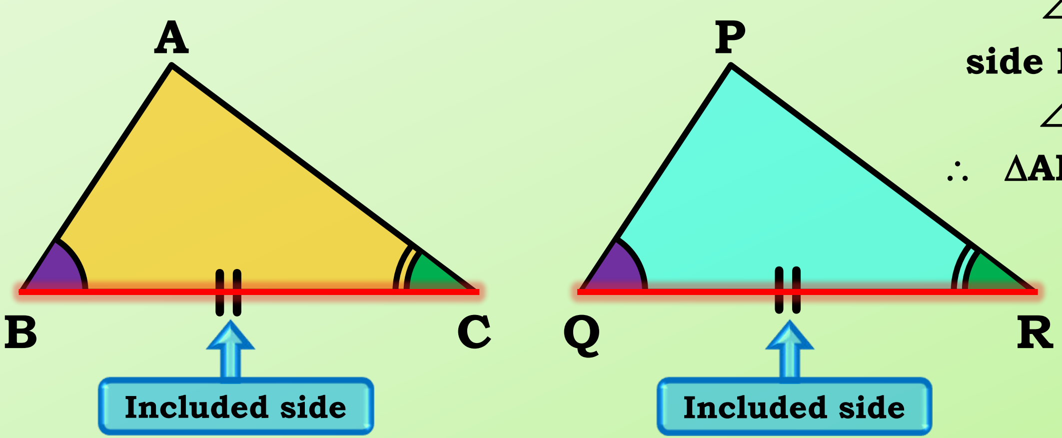
In ΔABC and ΔPQR,

$\angle B \cong \angle Q$ *[Given]*

side $BC \cong$ side QR *[Given]*

$\angle C \cong \angle R$ *[Given]*

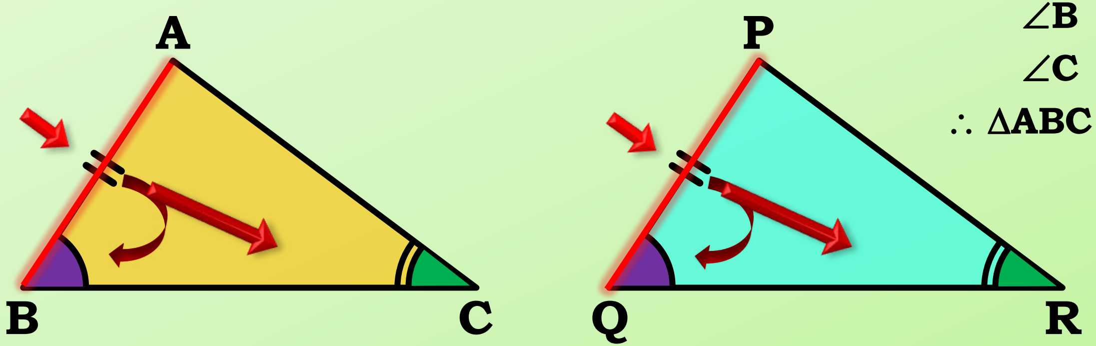
$\therefore \Delta ABC \cong \Delta PQR$ *[ASA test]*



Criteria OF CONGRUENCY

3. SAA criterion

When a side, an angle adjacent to it and the angle opposite to it of one triangle are respectively congruent to corresponding side, an angle adjacent to it and the angle opposite to it of another triangle, then the two triangles are congruent.



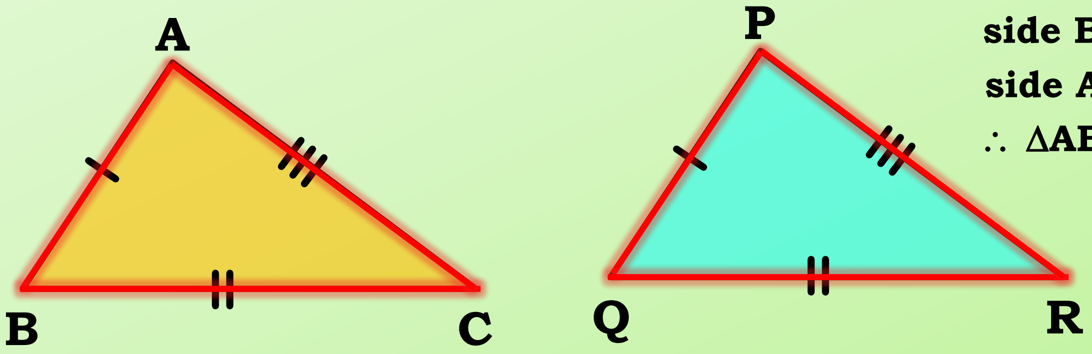
In ΔABC and ΔPQR,

side $AB \cong$ side PQ	[*Given*]
$\angle B \cong \angle Q$	[*Given*]
$\angle C \cong \angle R$	[*Given*]
$\therefore \Delta ABC \cong \Delta PQR$	[*SAA test*]

Criteria OF CONGRUENCY

4. SSS criterion

When three sides of one triangle are congruent with corresponding three sides of another triangle then the two triangles are congruent.



In ΔABC and ΔPQR,

side $AB \cong$ side PQ [*Given*]

side $BC \cong$ side QR [*Given*]

side $AC \cong$ side PR [*Given*]

$\therefore \Delta ABC \cong \Delta PQR$ [*SSS test*]

Criteria OF CONGRUENCY

5. RHS rule :

Two right angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the hypotenuse and corresponding side of the other triangle.

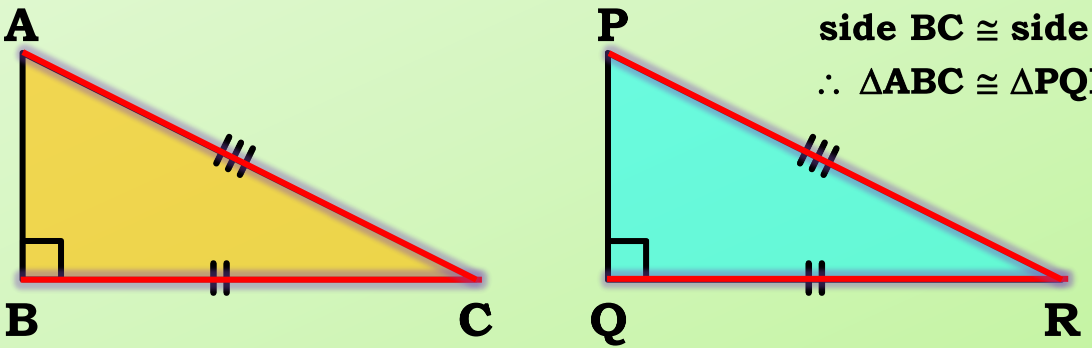
In ΔABC and ΔPQR,

$\angle B = \angle Q = 90^\circ$ *[Given]*

Hypotenuse $AC \cong$ hypotenuse PR *[Given]*

side $BC \cong$ side QR *[Given]*

$\therefore \Delta ABC \cong \Delta PQR$ *[Hypotenuse-side test]*



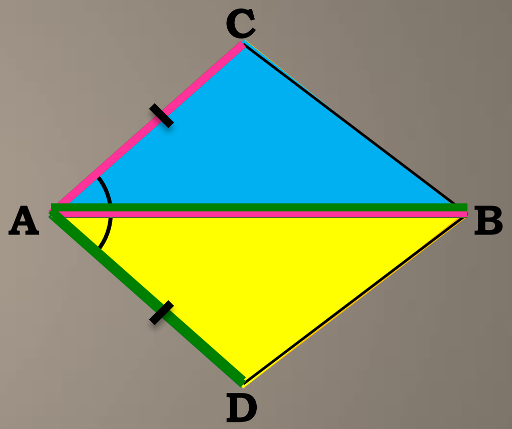
CRITERIA OF CONGRUENCY

1. SAS criterion
2. ASA criterion
3. AAS criterion
4. SSS criterion
5. RHS rule

MODULE : 3

Exercise 7.1 Q.1

Q. In quadrilateral ACBD, AC = AD and AB bisects $\angle A$. Show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD ?



Proof: In ΔABC and ΔABD,

	$AC = AD$	[Given]
	$\angle CAB = \angle DAB$	[$\because$ AB bisects $\angle A$]
and	$AB = AB$	[Common]
$\therefore$	$\Delta ABC \cong \Delta ABD$	[SAS criterion]
$\therefore$	$BC = BD$	[c.p.c.t]

Exercise 7.1 Q.2

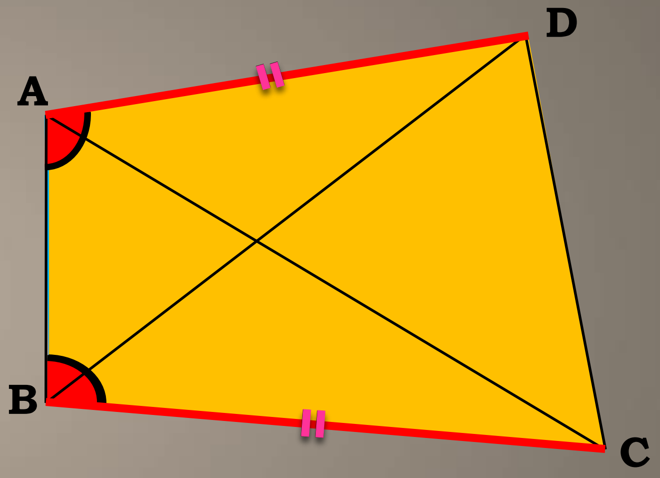
Q. ABCD is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$

Prove that

(i) $\Delta ABD \cong \Delta BAC$

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$



Proof: In ΔABD and ΔBAC,

AD	$=$	BC	[Given]
$\angle DAB$	$=$	$\angle CBA$	[Given]
BA	$=$	BA	[Common]
$\therefore \Delta ABD$	$\cong$	ΔBAC	[SAS criterion]
$\therefore BD$	$=$	AC	[c.p.c.t.]
$\therefore \angle ABD$	$=$	$\angle BAC$	[c.p.c.t.]

Thank You

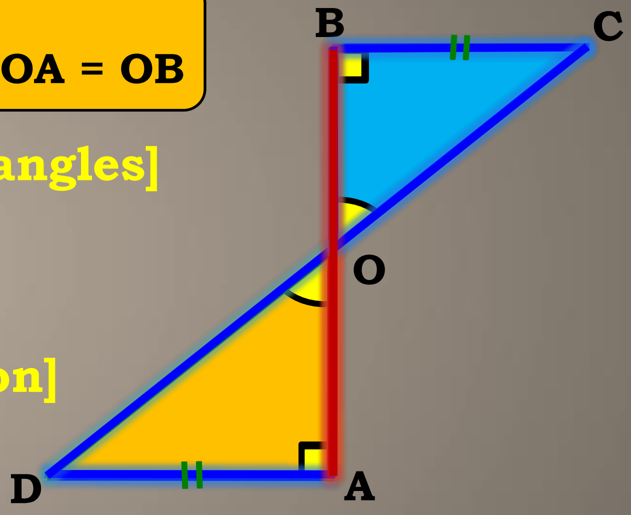
MODULE : 4

Exercise 7.1 Q.3

AD and BC are equal perpendicular to a line segment AB

Show that CD bisects AB.

Hint:
To prove: OA = OB



Proof: In ΔAOD and ΔBOC,

$\angle AOD = \angle BOC$ [Vertically opp. angles]

$\angle OAD = \angle OBC$ [Each 90°]

$AD = BC$ [given]

$\therefore \Delta AOD \cong \Delta BOC$ [AAS criterion]

$\therefore OA = OB$ [c.p.c.t.]

i.e., O is the mid - point AB.

$\therefore$ CD bisects AB.

Exercise 7.1 Q.4

l and m are two parallel lines intersected by another pair of parallel lines p and q. Show that $\Delta ABC \cong \Delta CDA$.

Proof: $l \parallel m$

AC is a transversal

$\therefore$ $\angle BCA = \angle DAC$ ...(i) [Alternate interior angles]

$p \parallel q$

AC is a transversal

$\therefore$ $\angle BAC = \angle DCA$ ...(ii) [Alternate interior angles]

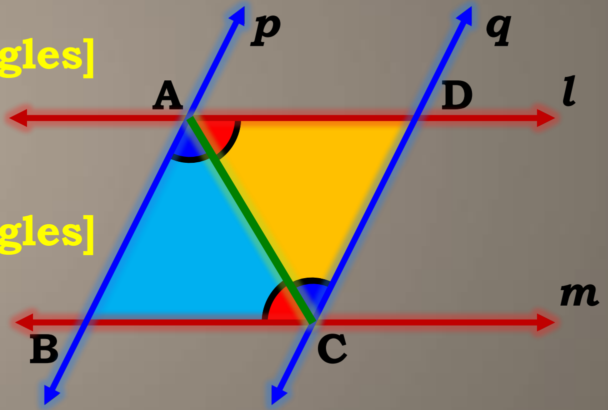
In ΔABC and ΔCDA,

$\angle BCA = \angle DAC$ [From (i)]

$AC = CA$ [Common side]

$\angle BAC = \angle DCA$ [From (ii)]

$\therefore$ $\Delta ABC \cong \Delta CDA$ [ASA criterion]



MODULE : 5

Solved Example.3

Line-segment AB is parallel to another line-segment CD.
O is the mid-point of AD.
Show that: (i) $\Delta AOB \cong \Delta DOC$
(ii) O is also the mid-point of BC.

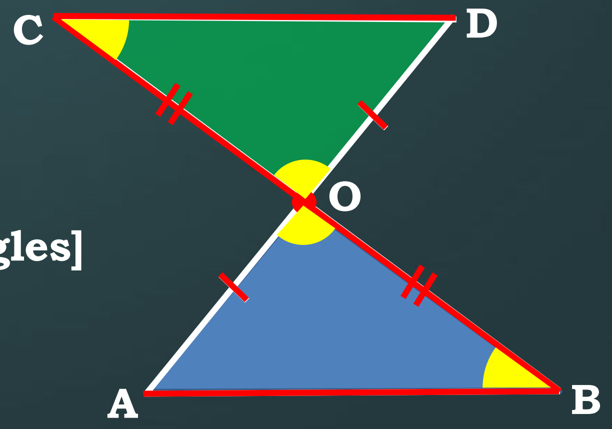
Sol. $AB \parallel CD$

On Transversal BC

$\therefore \angle ABC = \angle DCB$ [Alternate interior anges]

$\therefore \angle ABO = \angle DCO$ ...(i)

$AO = OD$ ...(ii)[Given]



In ΔAOB and ΔDOC,

$\angle ABO = \angle DCO$ [From (i)]

$\angle AOB = \angle DOC$ [Vertically opposite angles]

$AO = OD$ [From (ii)]

$\therefore$ $\boxed{\Delta AOB \cong \Delta DOC}$ [AAS rule]

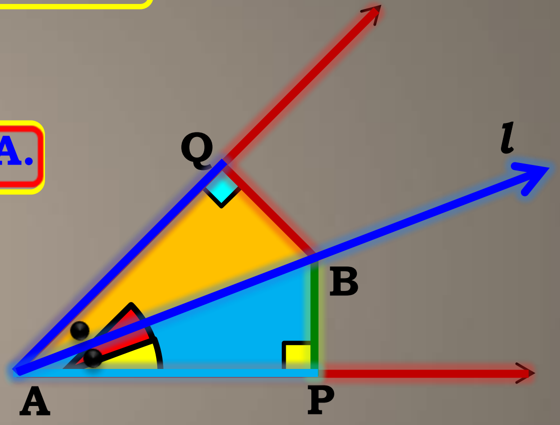
$\therefore OB = OC$ [CPCT]

O is the mid-point of BC.

MODULE : 6

Exercise 7.1-5

Q. Line l is the bisector of an angle A and B is any point on l.
BP and BQ are perpendiculars from B to the arms of $\angle A$
Show that :
(i) $\Delta APB \cong \Delta AQB$
(ii) BP = BQ or B is equidistant from the arms of $\angle A$.



$\angle PAB = \angle QAB$

Proof : In ΔAPB and ΔAQB,

$\angle APB = \angle AQB$ [$\because$ Each 90°]

$\angle PAB = \angle QAB$ [$\because$ AB bisects $\angle PAQ$]

$AB = AB$ [Common side]

$\therefore$ $\Delta APB \cong \Delta AQB$ [AAS congruence criterion]

$\therefore$ $BP = BQ$ [c.p.c.t.]

i.e., B is equidistant from the arms of $\angle A$

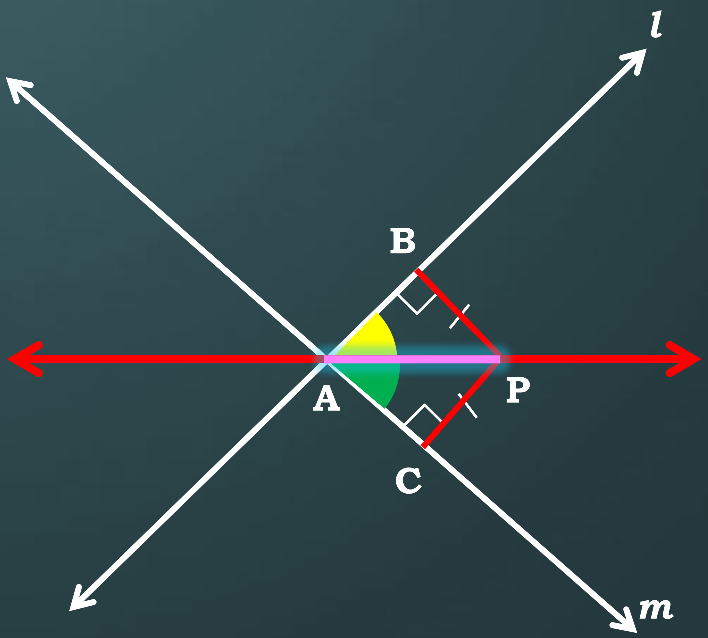
MODULE : 7

Solved Example. 8

P is a point equidistant from two lines l and m intersecting at point A. Show that the line AP bisects the angles between them.

Sol.

To Prove : $\angle PAB = \angle PAC$



$$\left.\begin{array}{c} PB \perp l \text{ and } PC \perp m. \\ PB = PC \\ \angle PBA = \angle PCA = 90^\circ \end{array}\right\} \quad \text{...(i) [Given]}$$

In ΔPAB and ΔPAC,

	$PB = PC$	[From (i)]
	$\angle PBA = \angle PCA$	[From (i)]
	$PA = PA$	[Common]
$\therefore$	$\Delta PAB \cong \Delta PAC$	[RHS rule]
$\therefore$	$\angle PAB = \angle PAC$	[CPCT]

Hence, line AP bisects angles between them

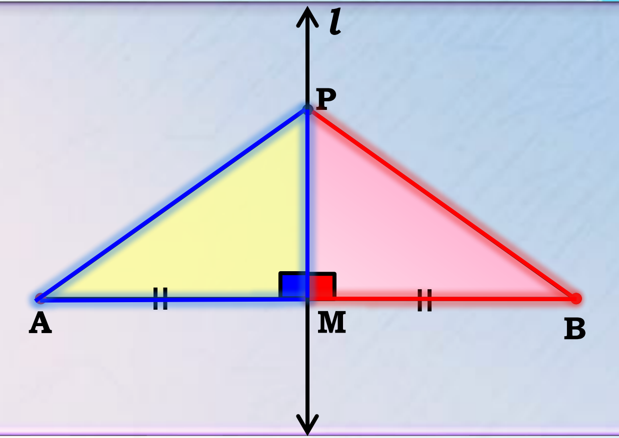
MODULE : 8

[Part – I]

Every point on the perpendicular bisector of a segment is equidistant from the end points of the segment.

Given : Line l is the perpendicular bisector of seg AB at point M

To prove : PA = PB



Proof :

In ΔPMA and ΔPMB

	Seg PM	$\cong$	Seg PM	*[Common side]*
	$\angle$PMA	$\cong$	$\angle$PMB	*[Each is a right angle]*
	Seg AM	$\cong$	Seg BM	*[Given]*
$\therefore$	ΔPMA	$\cong$	ΔPMB	*[SAS test of congruency]*
$\therefore$	Seg PA	$\cong$	Seg PB	*[c.s.c.t]*
$\therefore$	PA	=	PB	

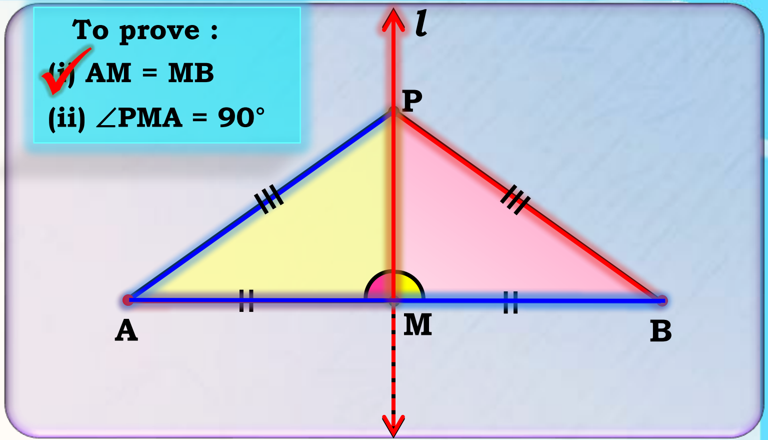
[Part – II]

...y point equidistant from the end points of a segment ...s on the perpendicular bisector of the segment.

Given : Point P is any point equidistant from the end points of seg AB. PA = PB

To prove: Point P is on the perpendicular bisector of seg AB.

Construction: Take midpoint M of seg AB and draw line PM.



Proof :

In ΔPAM and ΔPBM

	Seg PA	$\cong$	Seg PB	*[Given]*
	Seg AM	$\cong$	Seg BM	*[Construction]*
	Seg PM	$\cong$	Seg PM	*[Common side]*
$\therefore$	ΔPAM	$\cong$	ΔPBM	*[SSS test of congruency]*
$\therefore$	$\angle PMA$	$\cong$	$\angle PMB$ ... (i)	*[c.a.c.t]*
	$\angle PMA + \angle PMB$	$=$	$180°$	*[Angles in a linear pair]*
$\therefore$	$\angle PMA + \angle PMA$	$=$	180	*[From (i)]*
$\therefore$	$2\angle PMA$	$=$	180	

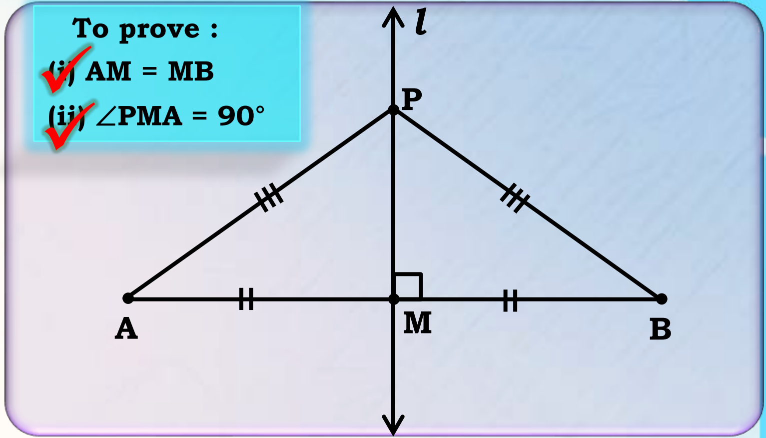
[Part – II]

Any point equidistant from the end points of a segment lies on the perpendicular bisector of the segment.

Given : Point P is any point equidistant from the end points of seg AB. PA = PB

To prove: Point P is on the perpendicular bisector of seg AB.

Construction: Take midpoint M of seg AB and draw line PM.



Proof :

$\therefore \quad 2\angle PMA = 180$

$\therefore \quad \angle PMA = \dfrac{180}{2}$

$\therefore \quad \angle PMA = 90^\circ$

$\therefore$ Seg PM $\perp$ Seg AB ... (ii)

Also, M is the midpoint of seg AB. ... (iii)

$\therefore$ Line PM is the perpendicular bisector of seg AB.

[From (ii) and (iii)]

$\therefore$ Point P is on the perpendicular bisector of seg AB.

Thank You

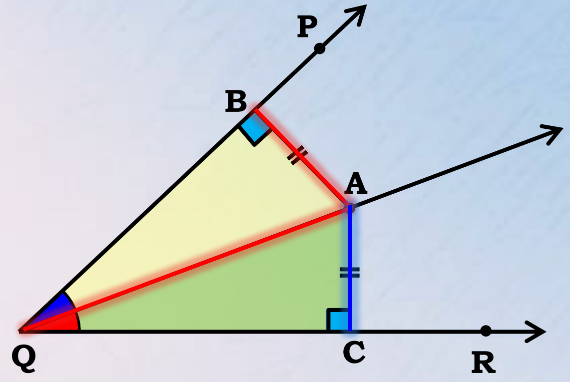
MODULE : 9

Angle Bisector Theorem – [Part – II]

Any point equidistant from sides of an angle is on the bisector of the angle.

Given : A is a point in the interior of $\angle PQR$.
Seg $AB \perp$ ray PQ, seg $AC \perp$ ray QR, AB = AC

To prove: Ray QA is the bisector of $\angle PQR$



Proof :

In ΔABQ and ΔACQ,

$\angle ABQ = \angle ACQ = 90^\circ$ *[Given]*

Hypotenuse AQ $\cong$ Hypotenuse AQ *[Common side]*

seg AB $\cong$ seg AC *[Given]*

$\Delta ABQ \cong \Delta ACQ$ *[Hypotenuse-Side test]*

$\angle BQA \cong \angle CQA$ *[c.a.c.t]*

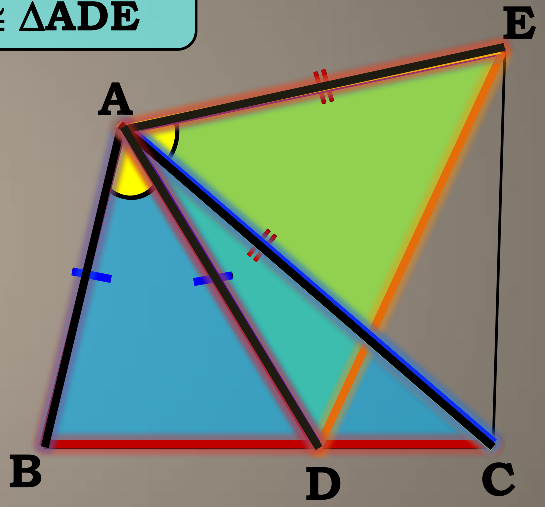
$\therefore$ Ray QA is the bisector of $\angle PQR$

MODULE : 10

Exercise 7.1-6

AC = AE, AB = AD and ∠BAD = ∠EAC.
Show that BC = DE.

Hint :
To Prove : $\Delta ABC \cong \Delta ADE$



Proof :

$\angle BAD = \angle EAC$

$\therefore$ $+ \angle DAC =$ $+ \angle DAC$

$\therefore$ $\boxed{\angle BAC = \angle DAE}$ ...(i)

In ΔABC and ΔADE,

$AC = AE$ [Given]

$\angle BAC = \angle DAE$ [From (i)]

$AB = AD$ [Given]

$\therefore$ $\Delta ABC \cong \Delta ADE$ [S.A.S. criterion]

$\therefore$ $\boxed{BC = DE}$ [c.p.c.t.]

MODULE : 11

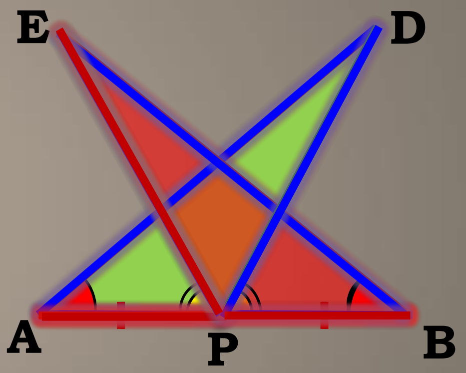
Exercise 7.1-7

AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$

Show that : (i) $\Delta DAP \cong \Delta EBP$

(ii) AD = BE.

AP = BP



Proof :

$\angle BAD = \angle ABE$

$\therefore \angle PAD = \angle PBE$ ...(i)

$\angle EPA = \angle DPB$

$\therefore \angle EPA + \angle DPE = \angle DPB + \angle DPE$

$\therefore \angle DPA = \angle EPB$ ...(ii)

In ΔEBP and ΔDAP

$\angle EPB = \angle DPA$ [From (ii)]

$BP = AP$ [Given]

$\angle EBP = \angle DAP$ [From (i)]

$\therefore \Delta DAP \cong \Delta EBP$ [ASA criterion]

$\therefore AD = BE$ [c.p.c.t]

MODULE : 12

Exercise 7.1-8

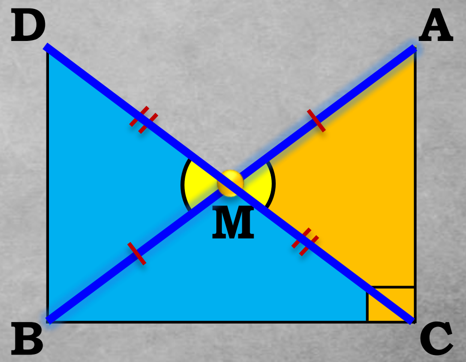
In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B.

Show that :

$AM = BM$

(i) $\Delta AMC \cong \Delta BMD$ (ii) $\angle DBC$ is a right angle

(iii) $\Delta DBC \cong \Delta ACB$ (iv) $CM = \frac{1}{2} AB$.



Proof : In ΔAMC and ΔBMD,

$AM = BM$ [M is the mid-point of AB]

$\angle AMC = \angle BMD$ [Vertically opposite angles]

$CM = MD$ [Given]

$\therefore$ $\Delta AMC \cong \Delta BMD$ [SAS criterion]

$\therefore$ $\angle ACM = \angle BDM$ ...(i) [c.p.c.t.]

Exercise 7.1-8

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B.

Show that : $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle (iii) $\Delta DBC \cong \Delta ACB$ (iv) $CM = \frac{1}{2}AB$.

Proof : $\angle ACM = \angle BDM$ ...(i) [c.p.c.t.]

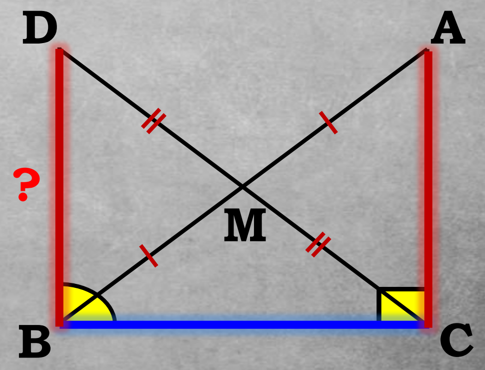
$\therefore \quad BD \parallel CA$

$\angle DBC + \angle ACB = 180°$ [co-interior angles]

$\angle DBC + 90° = 180°$ [$\because \angle ACB = 90°$]

$\therefore \quad \angle DBC = 180° - 90°$

$\therefore \quad \boxed{\angle DBC = 90°}$



Exercise 7.1-8

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B.

Show that : $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle (iii) $\Delta DBC \cong \Delta ACB$ (iv) $CM = \frac{1}{2} AB$.

In ΔDBC and ΔACB,

$BD = CA$ [c.p.c.t.]

$\angle DBC = \angle ACB$ [Each 90^0]

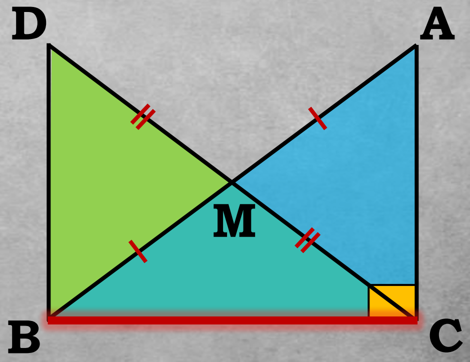
$BC = BC$ [Common side]

$\therefore \Delta DBC \cong \Delta ACB$ [S.A.S criterion]

$\therefore CD = AB$ [c.p.c.t.]

$\therefore \frac{1}{2} CD = \frac{1}{2} AB$

$\therefore \boxed{CM = \frac{1}{2} AB}$



Thank You

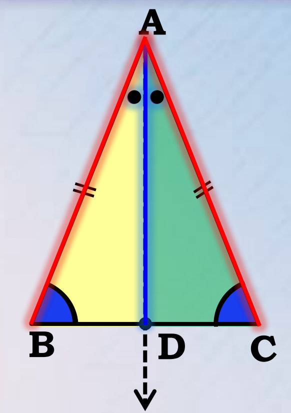
MODULE : 13

If two sides of a triangle are congruent, then the angles opposite to them are congruent.

Given : In ΔABC, side $AB \cong$ side AC

To prove : $\angle B \cong \angle C$

Construction : Draw ray AD, the bisector of $\angle BAC$ intersecting side BC at point D.



Proof :

In ΔABD and ΔACD,

	side $AB \cong$ side AC	*[Given]*
	$\angle BAD \cong \angle CAD$	*[Construction]*
	side $AD \cong$ side AD	*[Common side]*
$\therefore$	$\Delta ABD \cong \Delta ACD$	*[SAS test of congruency]*
$\therefore$	$\angle B \cong \angle C$	*[c.a.c.t]*

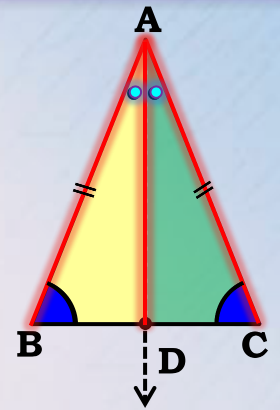
MODULE : 14

If two angles of a triangle are congruent, then the sides opposite to them are congruent

Given : In ΔABC, $\angle B \cong \angle C$

To prove : side $AB \cong$ side AC

Construction : Draw ray AD, the bisector of $\angle BAC$ intersecting side BC at point D.



Proof :

In ΔABD and ΔACD,

	side $AD \cong$ side AD	*[Common side]*
	$\angle BAD \cong \angle CAD$	*[Construction]*
	$\angle B \cong \angle C$	*[Given]*
$\therefore$	$\Delta ABD \cong \Delta ACD$	*[SAA test of congruency]*
$\therefore$	side $AB \cong$ side AC	*[c.s.c.t]*

MODULE : 15

Exercise 7.2-Q.1

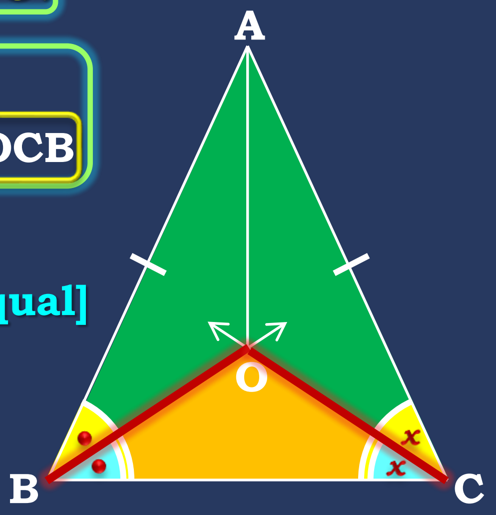
In an isosceles triangle ABC, with AB = AC, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O.

Show that : (i) OB = OC

(ii) AO bisects $\angle A$

Hint :

To prove : $\angle OBC = \angle OCB$



Sol. In ΔABC

AB = AC [Given]

$\therefore$ $\angle ABC = \angle ACB$ [Angles opposite to equal sides are equal]

$2\angle OBC = 2\angle OCB$ [BO and CO are bisectors of $\angle B$ and $\angle C$ respectively]

$\therefore$ $\angle OBC = \angle OCB$ ...(i)

In ΔOBC

$\boxed{\angle OBC = \angle OCB}$ [From (i)]

$\therefore$ OB = OC ...(ii) [Sides opposite to equal angles are equal]

Exercise 7.2-Q.1

In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O.

Show that : (ii) AO bisects ∠A

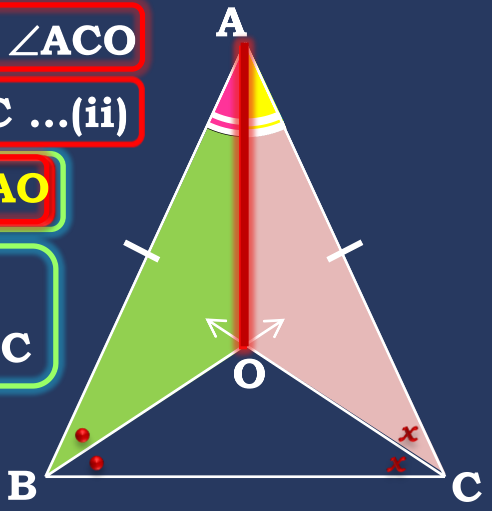
$\angle ABO = \angle ACO$

$OB = OC$ ...(ii)

To prove : $\angle BAO = \angle CAO$

Hint :

To prove : $\Delta AOB \cong \Delta AOC$



Sol. In ΔAOB and ΔAOC

	$AB = AC$	[Given]
	$OB = OC$	[From (ii)]
	$AO = AO$	[Common side]
$\therefore$	$\Delta AOB \cong \Delta AOC$	[S.S.S. criterion]
$\therefore$	$\angle BAO = \angle CAO$	[c.p.c.t.]

$\therefore$ $\boxed{\text{AO bisects } \angle A}$

MODULE : 16

Exercise 7.2-Q.2

In ΔABC, AD is the perpendicular bisector of BC
Show that Δ ABC is an isosceles triangle in which AB = AC.

BD = CD

Hint :
To prove : $\Delta ADB \cong \Delta ADC$

Sol. In ΔABD and ΔACD,

$BD = CD$ [Given]

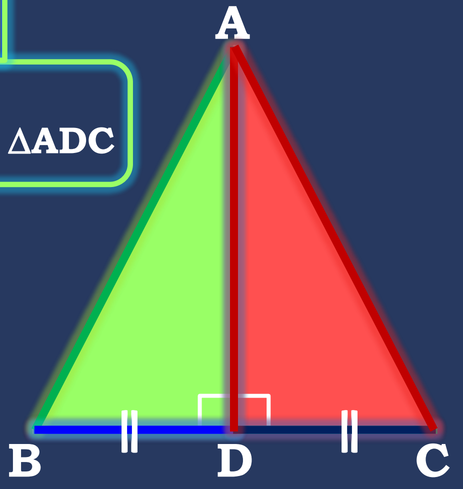
$\angle ADB = \angle ADC$ [$\because$ $AD \perp BC$]

$AD = AD$ [common side]

$\therefore$ $\Delta ABD \cong \Delta ACD$ [SAS criterion]

$\therefore$ $AB = AC$ [c.p.c.t.]

$\therefore$ ΔABC is isosceles.

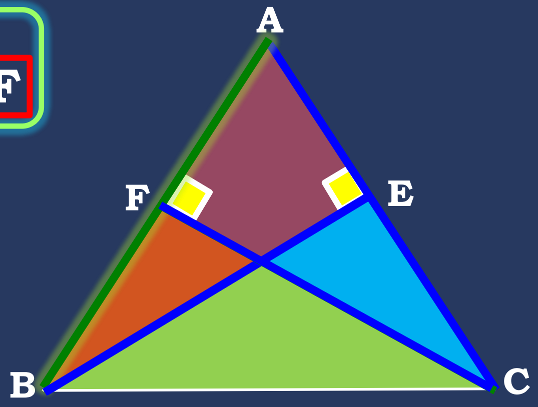


MODULE : 17

Exercise 7.2-Q.3

ABC is an isosceles triangle in which altitudes BE and CF are drawn to sides AC and AB respectively. Show that altitudes are equal.

To prove : BE = CF



Sol.

In ΔABE and ΔACF

$\angle AEB = \angle AFC$ [∵ Each 90°]

$\angle A = \angle A$ [Common angle]

$AB = AC$ [Given]

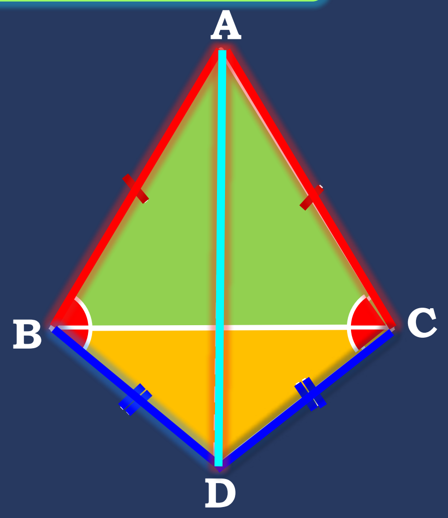
$\therefore$ $\boxed{\Delta ABE \cong \Delta ACF}$ [AAS criterion]

$\therefore$ $\boxed{BE = CF}$ [c.p.c.t.]

MODULE : 18

Exercise 7.2-Q.5

ABC and DBC are two isosceles triangles on the same base BC Show that $\angle ABD = \angle ACD$.



Proof : Join AD

In ΔABD and ΔACD

$AB = AC$

$BD = CD$

$AD = AD$ [Common side]

$\therefore$ $\Delta ABD \cong \Delta ACD$ [SSS criterion]

$\therefore$ $\boxed{\angle ABD = \angle ACD}$ [c.p.c.t.]

Thank You

MODULE : 19

Exercise 7.2-7

ABC is a right angled triangle in which $\angle A = 90°$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Sol. In ΔABC,

$AB = AC$ [Given]

$\therefore \angle B = \angle C$ ...(i) [Angles opposite to equal sides are equal]

Let $\angle B = \angle C = x$

$\angle A + \angle B + \angle C = 180°$ [Angle - sum property of a triangle]

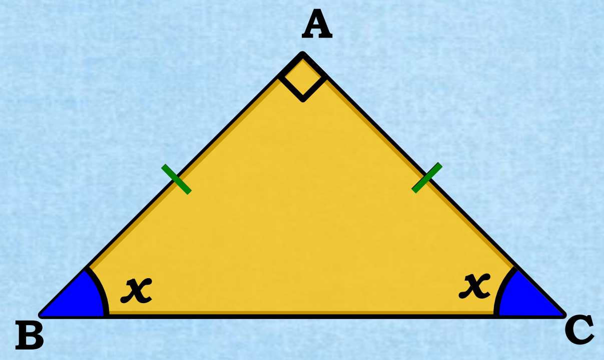
$\therefore 90 + x + x = 180$

$\therefore 2x = 180 - 90$

$\therefore 2x = 90$ ... (ii)

$\therefore \boxed{x = 45°}$

$\therefore \boxed{\angle B = \angle C = 45°}$



Exercise 7.2-Q.8

Show that the angles of an equilateral triangle are 60° each.

Proof : $AB = AC = BC$.

Now, $AB = AC$

$\therefore \quad \boxed{\angle B = \angle C \quad \ldots(i)}$ [Angles opposite to equal sides are equal]

Also, $CB = CA$

$\therefore \quad \boxed{\angle B = \angle A \quad ..(ii)}$ [Angles opposite to equal sides are equal]

$\therefore \quad \boxed{\angle A = \angle B = \angle C} \quad \ldots(iii)$ [from (i) and (ii)]

Also, $\angle A + \angle B + \angle C = 180°$ [Angle - sum property of a triangle]

$\therefore \quad \underline{\angle A + \angle A + \angle A} = 180°$ [From (iii)]

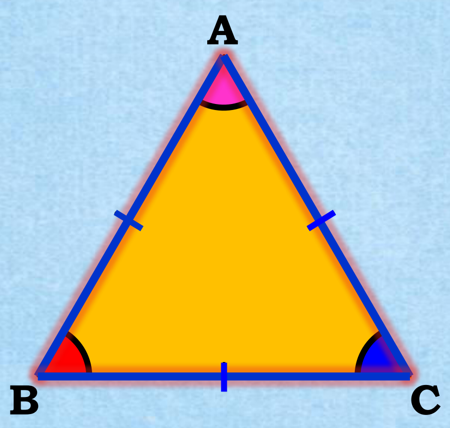
$\therefore \quad 3\angle A = 180°$

$\therefore \quad \angle A = \dfrac{\cancel{180}^{\,60}}{\cancel{3}}$

$\therefore \quad \angle A = 60°$

$\angle A = \angle B = \angle C = 60°$

$\therefore$ Thus, each angle of an equilateral triangle is 60°

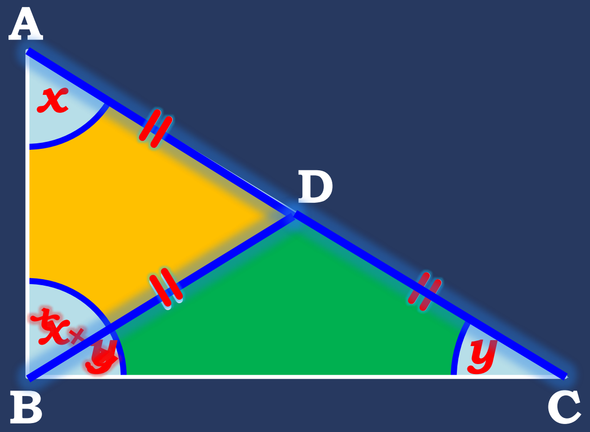


MODULE : 20

Exercise 7.2-Q.6

AD = DC = BD

Prove that ∠ABC is right angle.



Proof :

To prove : $\angle ABC = 90^\circ$

In ΔADB,

$DA = DB$ [Given]

$\angle DAB = \angle DBA$

Let $\angle DAB = \boxed{\angle ABD = x}$

In ΔDBC,

$DB = DC$ [Given]

$\angle DBC = \angle DCB$

Let $\boxed{\angle DBC} = \angle DCB = \boxed{y}$

$\angle ABC = \boxed{\angle ABD} + \boxed{\angle DBC}$

$\boxed{m\angle ABC = x + y}$ ...(i)

In ΔABC,

$\boxed{\angle BAC} + \boxed{\angle ABC} + \boxed{\angle ACB} = 180^\circ$

$\therefore\ x + x + y + y = 180$

$\therefore\ 2x + 2y = 180$

$\therefore\ 2(x + y) = 180$

$\therefore\ \boxed{x + y = 90}$

$\therefore\ \boxed{\angle ABC = 90^\circ}$ [From (i)]

MODULE : 21

Exercise 7.3-Q.5

ABC is an isosceles triangle with AB = AC. Draw AP $\perp$ BC to show that $\angle B = \angle C$.

AB = AC

Proof: In ΔABP and ΔACP,

$\angle APB = \angle APC$ [$\because$ Each 90°]

AB = AC [Given]

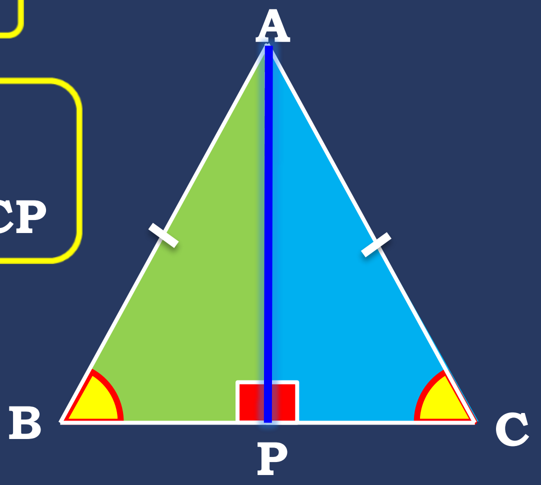
and AP = AP [common side]

$\therefore$ By R.H.S criterion of congruence,

$\Delta ABP \cong \Delta ACP$

$\boxed{\angle B = \angle C}$ [c.p.c.t.]

Hint:
To prove-
$\Delta ABP \cong \Delta ACP$



MODULE : 22

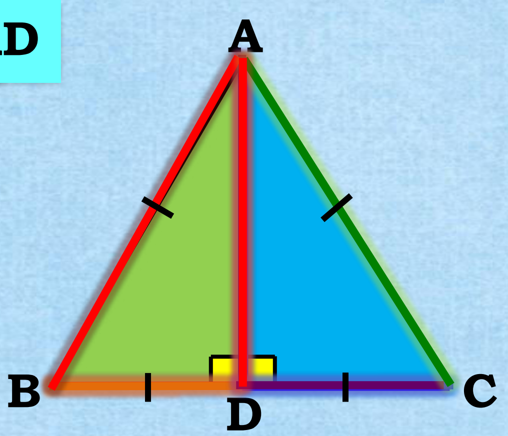
Exercise 7.3-2

AD is an altitude of an isosceles triangle ABC in which AB = AC.
Show that

(i) AD bisects BC

(ii) AD bisects $\angle$A.

To prove : $\angle BAD = \angle CAD$



Proof : In ΔADB and ΔADC,

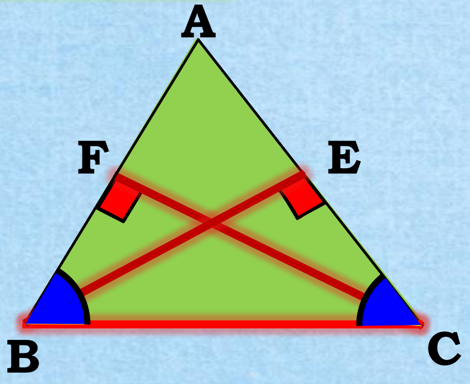
	$\angle ADB = \angle ADC$	[∵ Each 90°]
	Hyp. AB = Hyp. AC	[Given]
	AD = AD	[common side]
∴	$\Delta ADB \cong \Delta ADC$	[R.H.S. congruence criterion]
∴	BD = CD	[c.p.c.t.]
∴	$\angle BAD = \angle CAD$	[c.p.c.t.]

MODULE : 23

Exercise 7.3 - 4

BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Hint: To prove: AB = AC



Sol. In ΔBCF and ΔCBE,

$\angle BFC = \angle CEB$ [$\because$ Each 90°]

$BC = BC$ [Common side]

$FC = EB$ [Given]

$\Delta BCF \cong \Delta CBE$ [R.H.S. criterion]

$\therefore \angle B = \angle C$ ...(i) [c.p.c.t.]

Now, in ΔABC

$\therefore \angle B = \angle C$ [From (i)]

$\therefore$ $\boxed{AB = AC}$ [$\because$ sides opposite to equal $\angle$s of a Δ are equal]

$\therefore$ ΔABC is an isosceles triangle.

MODULE : 24

Exercise 7.3-Q.1

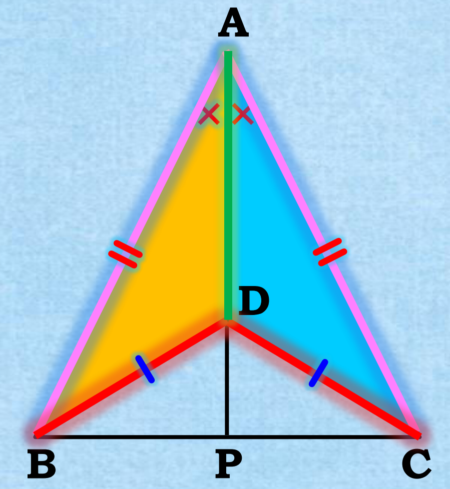
ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that :

(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisect $\angle A$ as well as $\angle D$.

(iv) AP is the perpendicular bisector of BC.



Sol. In ΔABD and ΔACD,

$AB = AC$ (Given)

$BD = DC$ (Given)

$AD = AD$ (common side)

$\therefore$ $\boxed{\Delta ABD \cong \Delta ACD}$ (By SSS criterion of congruence)

$\angle BAD = \angle CAD$ ...(i) (c.p.c.t.)

Exercise 7.3-Q.1

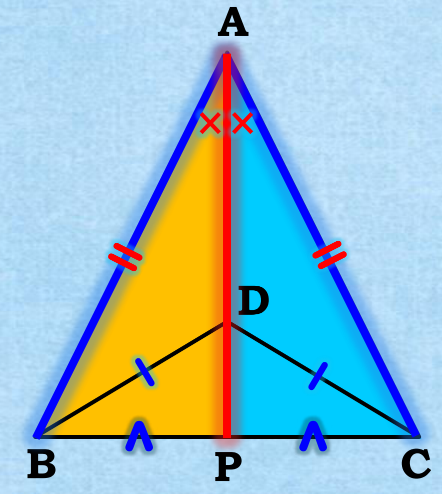
DABC and DDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC.
If AD is extended to intersect BC at P, show that :

(i) DABD ≅ DACD

(ii) DABP ≅ DACP

(iii) AP bisect ÐA as well as ÐD.

(iv) AP is the perpendicular bisector of BC.



Sol. In ΔABP and ΔACP,

$\angle BAP = \angle CAP$ [From (i)]

$AB = AC$ [Given]

$AP = AP$ [common side]

$\therefore$ $\boxed{\Delta ABP \cong \Delta ACP}$ [By SAS criterion of congruence]

$BP = CP$ ...(ii) [c.p.c.t.]

$\angle APB = \angle APC$ ...(iii) [c.p.c.t.]

Exercise 7.3-Q.1

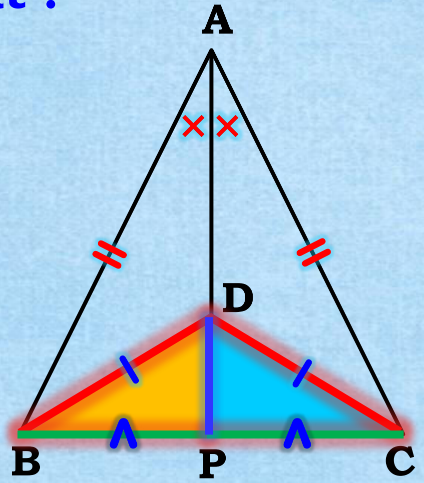
DABC and DDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC.
If AD is extended to intersect BC at P, show that :

✓ (i) DABD ≅ DACD

✓ (ii) DABP ≅ DACP

✓ (iii) AP bisect ∠A as well as ∠D.

(iv) AP is the perpendicular bisector of BC.



Sol. In ΔBDP and ΔCDP,

BD = CD		[Given]
BP = CP		[From (ii)]
DP = DP		[common side]
∴	$\Delta BDP \cong \Delta CDP$	[By SSS criterion of congruence]
	$\angle BDP = \angle CDP$	[c.p.c.t.]

∴ AP bisect ∠BDC

∴ AP bisect ∠A as well as ∠D

Exercise 7.3-Q.1

ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that :

(i) $\Delta ABD \cong \Delta ACD$

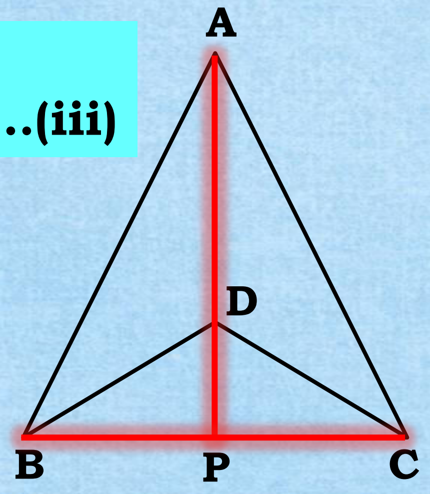
(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisect $\angle A$ as well as $\angle D$.

(iv) AP is the perpendicular bisector of BC.

BP = PC ...(ii)

$\angle APB = \angle APC$ ...(iii)



Sol. Since AP stands on BC

$\angle APB + \angle APC = 180°$ [linear pair]

$\therefore \angle APB + \angle APB = 180°$ [From (iii)]

$\therefore 2\angle APB = 180°$

$\therefore \angle APB = 90°$

BP = PC [From (ii)]

$\therefore$ AP is perpendicular bisector of BC.

Exercise 7.3-Q.1

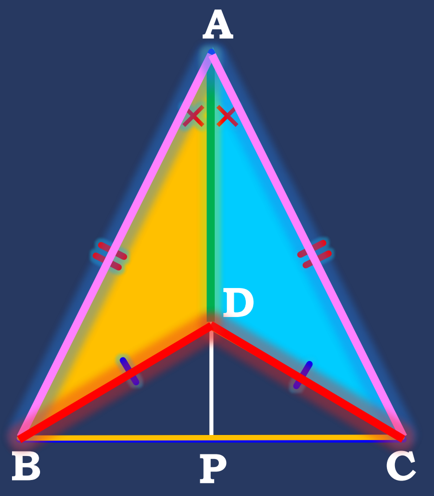
ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that :

(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisect $\angle A$ as well as $\angle D$.

(iv) AP is the perpendicular bisector of BC.



Sol. In ΔABD and ΔACD,

AB = AC (Given)

BD = DC (Given)

AD = AD (common side)

$\therefore$ $\boxed{\Delta ABD \cong \Delta ACD}$ (By SSS criterion of congruence)

$\angle BAD = \angle CAD$ ...(i) (c.p.c.t.)

Exercise 7.3-Q.1

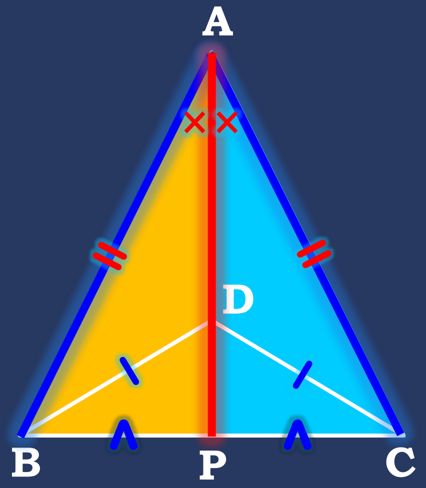
ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that :

(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisect $\angle A$ as well as $\angle D$.

(iv) AP is the perpendicular bisector of BC.



Sol. In ΔABP and ΔACP,

$\angle BAP = \angle CAP$ [From (i)]

$AB = AC$ [Given]

$AP = AP$ [common side]

$\therefore$ $\boxed{\Delta ABP \cong \Delta ACP}$ [By SAS criterion of congruence]

$\angle BAD = \angle CAD$ ...(i)

$BP = CP$ ...(ii) [c.p.c.t.]

$\angle APB = \angle APC$ ...(iii) [c.p.c.t.]

Exercise 7.3-Q.1

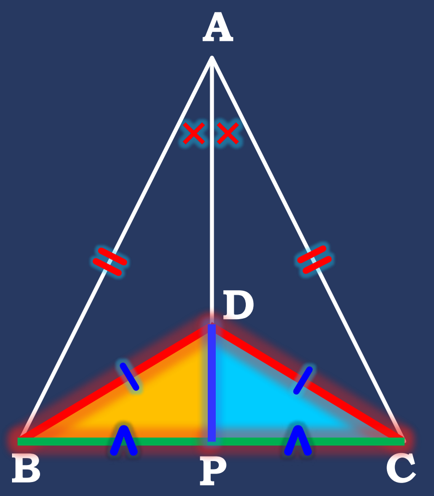
ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that :

✓ (i) $\Delta ABD \cong \Delta ACD$

✓ (ii) $\Delta ABP \cong \Delta ACP$

✓ (iii) AP bisect $\angle A$ as well as $\angle D$.

(iv) AP is the perpendicular bisector of BC.



Sol. In ΔBDP and ΔCDP,

$BD = CD$ [Given]

$BP = CP$ [From (ii)]

$DP = DP$ [common side]

$\therefore \Delta BDP \cong \Delta CDP$ [By SSS criterion of congruence]

$\angle BDP = \angle CDP$ [c.p.c.t.]

$\therefore$ AP bisect $\angle BDC$

$\therefore$ $\boxed{\text{AP bisect } \angle A \text{ as well as } \angle D}$

BP = CP...(ii)

Exercise 7.3-Q.1

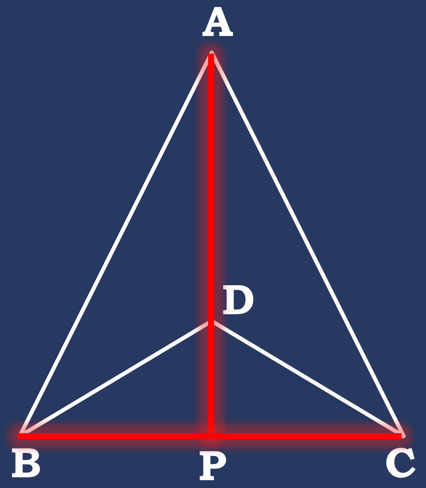
ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that :

(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisect $\angle A$ as well as $\angle D$.

(iv) AP is the perpendicular bisector of BC.



$\angle APB = \angle APC$...(iii)

$BP = CP$...(ii)

Sol. Since AP stands on BC

$\angle APB + \angle APC = 180^\circ$ [linear pair]

$\therefore \angle APB + \angle APB = 180^\circ$ [From (iii)]

$\therefore 2\angle APB = 180^\circ$

$\therefore \angle APB = 90^\circ$

$BP = PC$ [From (ii)]

$\therefore$ AP is perpendicular bisector of BC.

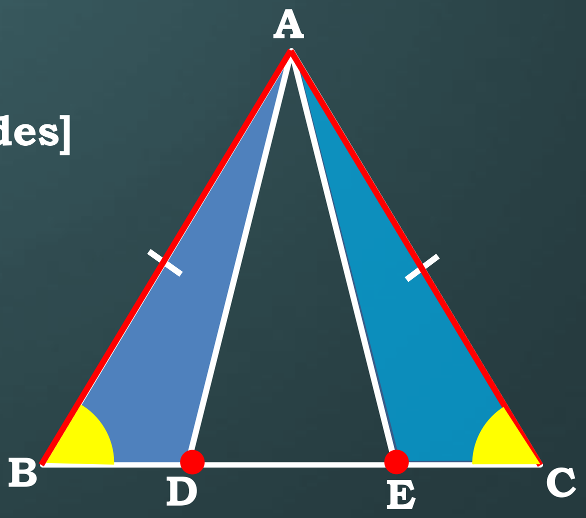
Thank You

MODULE : 25

Solved Example.6

In an isosceles tringle ABC with AB = AC, D and E are points on BC such that BE = CD . Show that AD = AE.

Sol.



AB = AC ...(i) [Given]

$\therefore$ $\angle B = \angle C$ ...(ii) [Angles opposite to equal sides]

BE = CD ...(iii) [Given]

$\therefore$ BE – DE = CD – DE

BD = CE ...(iv)

In ΔABD and ΔACE,

AB = AC ...[From (i)]

$\angle B = \angle C$ ...[From (ii)]

BD = CE ...[From (iv)]

$\therefore$ $\Delta ABD \cong \Delta ACE$ [SAS Criterion of congruency]

$\boxed{AD = AE}$

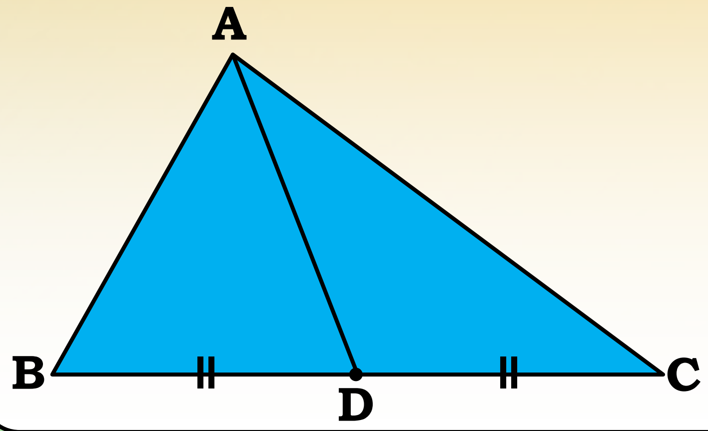
MODULE : 26

MEDIAN OF A TRIANGLE

The segment joining a vertex and the midpoint of the side opposite to it is called a Median of the triangle.

Point D is the midpoint of side BC.

$\therefore$ Seg AD is a median of ΔABC.



Exercise 7.3-Q.3

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR

Show that :

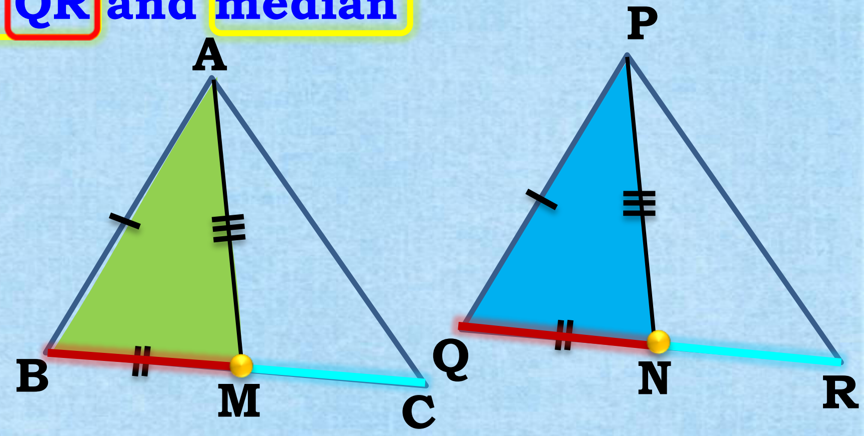
(i) $\Delta ABM \cong \Delta PQN$

(ii) $\Delta ABC \cong \Delta PQR$

AB = PQ

BC = QR

AM = PN



Proof :

$BM = CM = \frac{1}{2} BC$ ...(i) [$\because$ AM is the median]

$QN = RN = \frac{1}{2} QR$ ...(ii) [$\because$ PN is the median]

BC = QR ...(iii)

$\therefore$ BM = QN ...(iv) [From (i), (ii) and (iii)]

In ΔABM and ΔPQN,

AB = PQ [Given]

AM = PN [Given]

BM = QN [From (iv)]

$\therefore$ $\Delta ABM \cong \Delta PQN$ [SSS criterion]

Exercise 7.3-Q.3

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR

Show that :

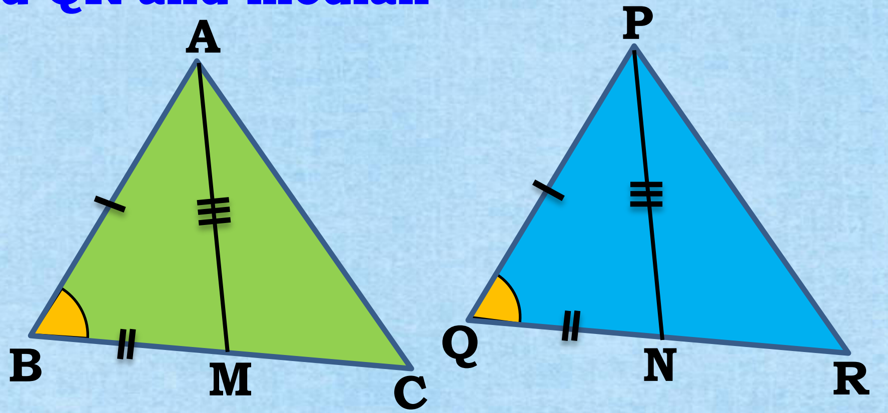
(i) $\Delta ABM \cong \Delta PQN$

(ii) $\Delta ABC \cong \Delta PQR$

$AB = PQ$

$BC = QR$

$\Delta ABM \cong \Delta PQN$



Proof :

$\Delta ABM \cong \Delta PQN$ [Proved]

$\therefore$ $\boxed{\angle B = \angle Q}$ ...(v) [c.p.c.t.]

In ΔABC and ΔPQR,

$AB = PQ$ [Given]

$\angle B = \angle Q$ [From (v)]

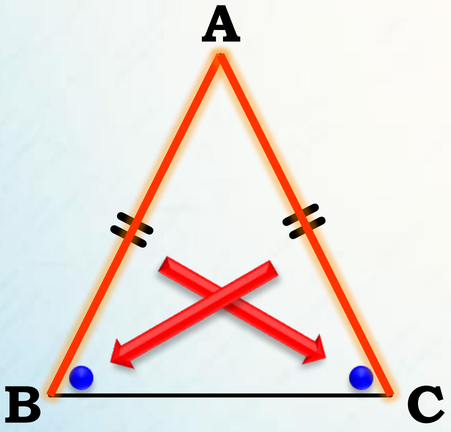
$BC = QR$ [Given]

$\therefore$ $\boxed{\Delta ABC \cong \Delta PQR}$ [SSS criterion]

MODULE : 27

THEOREM

If two sides of a triangle are equal, then the angles opposite to them are also equal.



In $\triangle ABC$,

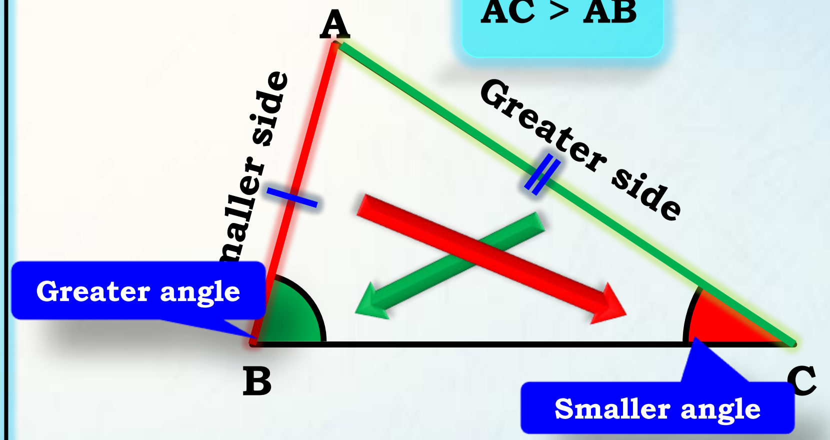
side $AB \cong$ side AC

$\therefore \quad \angle B \cong \angle C$

THEOREM

If two sides of a triangle are not equal, then the angle opposite to the greater side is greater.

$AC > AB$



In $\triangle ABC$,

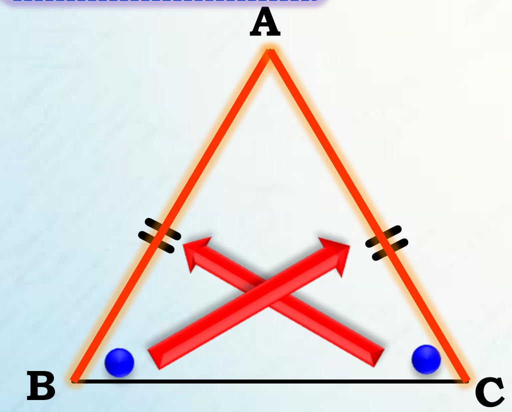
$AC > AB$

$\therefore \quad \angle B > \angle C$

MODULE : 28

THEOREM

If two angles of a triangle are equal, then the sides opposite to them are also equal.



In ΔABC,

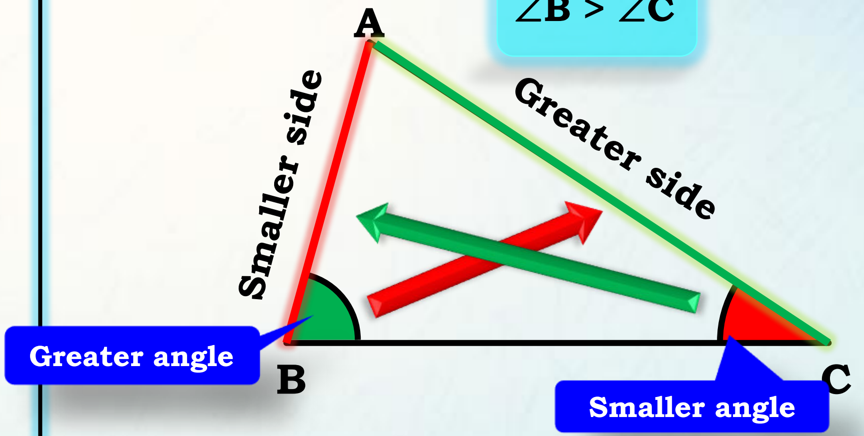
$\angle B \cong \angle C$

$\therefore$ side $AB \cong$ side AC

THEOREM

If two angles of a triangle are not equal, then the side opposite to the greater angle is greater.

$\angle B > \angle C$



In ΔABC,

$\angle B > \angle C$

$\therefore$ $AC > AB$

MODULE : 29

THEOREM

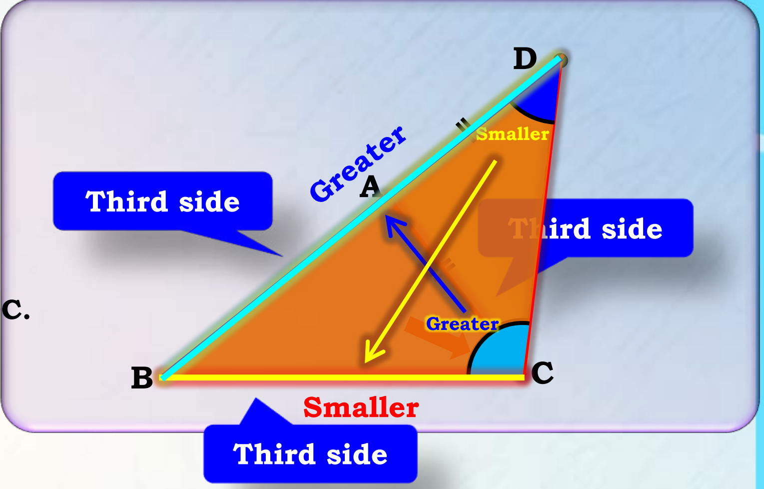
The sum of any two sides of a triangle is greater than the third side.

Given : ΔABC is any triangle.

To prove: $AB + AC > BC$, $AB + BC > AC$

$BC + AC > AB$

Construction: Take a point D on ray BA such that AD = AC. Draw seg DC.



Proof :

In ΔACD, $\boxed{AC = AD}$ *[Construction]*

$\therefore \angle ACD = \angle ADC$ *[Isosceles triangle theorem]*

$\therefore \boxed{\angle ACD + \angle ACB} > \angle ADC$

$\therefore \angle BCD > \angle ADC$ ...(i)

Adding $\angle ACB$ to LHS, we get

In ΔBDC,

$\angle BCD > \angle ADC$ *[From (i)]*

$\therefore \boxed{BD} > BC$ *[In a triangle, side opposite to greater angle is greater]*

$\therefore BA + \boxed{AD} > BC$ *[$\because$ BD = BA + AD]*

$\therefore BA + AC > BC$ *[$\because$ AD = AC]*

THEOREM

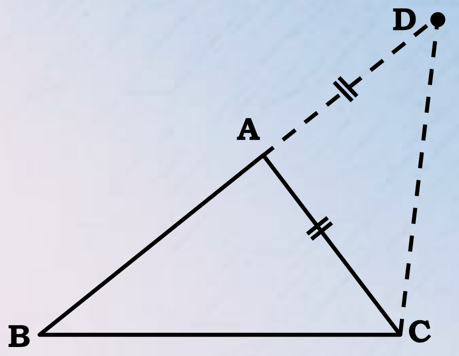
The sum of any two sides of a triangle is greater than the third side.

Given : ΔABC is any triangle.

To prove: $AB + AC > BC$, $\boxed{AB + BC > AC}$

$\boxed{BC + AC > AB}$

Construction: Take a point D on ray BA such that AD = AC. Draw seg DC.



Proof :

$BA + AC > BC$

Similarly we can prove that

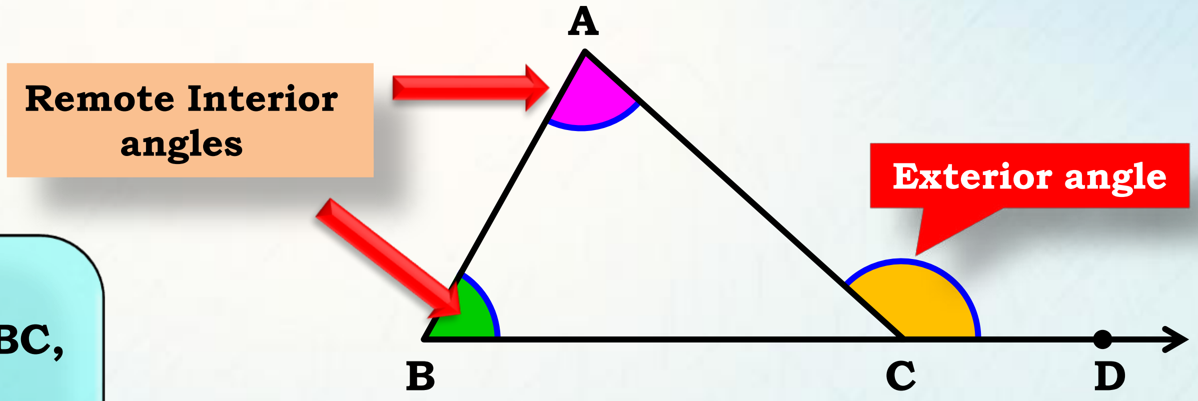
$AB + BC > AC$ and

$BC + AC > AB$

MODULE : 30

PROPERTY OF AN EXTERIOR ANGLE OF A TRIANGLE

An exterior angle of a triangle is always greater than each of its remote interior angles



$\angle ACD$ is an exterior angle of ΔABC,

$\angle ACD > \angle A$

and

$\angle ACD > \angle B$

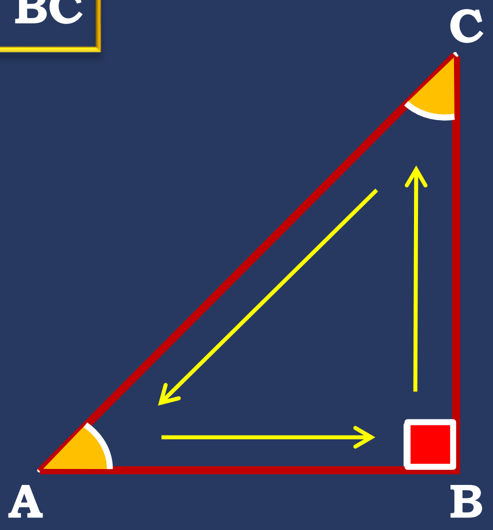
Thank You

MODULE : 31

Exercise 7.4-Q.1

Show that in a right angled triangle, the hypotenuse is the longest side.

Show - $AC > AB$ & $AC > BC$



Proof:

In ΔABC, $\angle B = 90°$

$\angle ABC + \angle BCA + \angle CAB = 180°$

$\therefore \quad 90° + \angle BCA + \angle CAB = 180°$

$\therefore \quad \angle BCA + \angle CAB = 90°$

$\therefore$ $\angle BCA$ and $\angle CAB$ are acute angles

$\therefore$ $\angle BCA < \angle ABC$ and $\angle CAB < \angle ABC$

$\therefore \quad AC > AB$ and $AC > BC$ [$\because$ side opposite to greater angle is larger]

$\therefore$ In a right triangle, the hypotenuse is the longest side.

MODULE : 32

Exercise 7.4-Q.6

Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Given : Let P be any point outside line l , PM $\perp$ line l

To prove : PM is the shortest of all line segments

Construction : Take a point N on line l, join PN

Proof :

In ΔPMN, $\angle P + \angle M + \angle N = 180^\circ$ (sum of angles of a triangle)

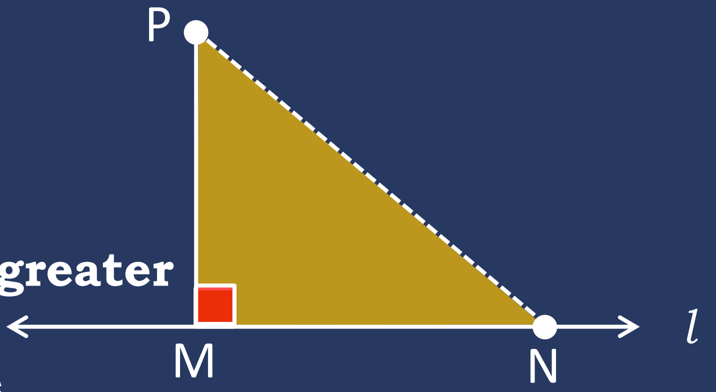
$\therefore \quad \angle P + 90^\circ + \angle N = 180^\circ$

$\therefore \quad \angle P + \angle N = 180^\circ - 90^\circ = 90^\circ$

$\therefore \quad \angle N < 90^\circ$

$\therefore \quad \angle N < M$

$\therefore \quad PM < PN$ (side opposite to greater angle is greater)

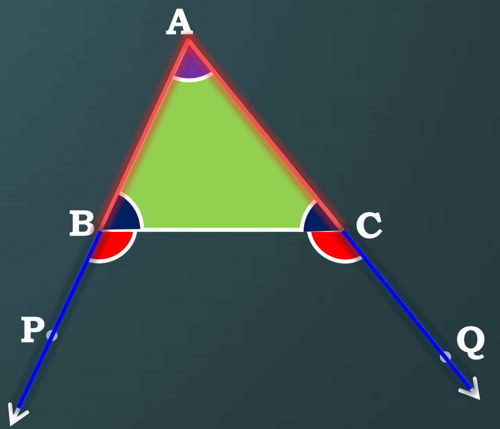


Hence, PM is the shortest of all line segments

MODULE : 33

Exercise 7.4.Q.2

Sides AB and AC of ΔABC are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.



Sol. $\angle PBC < \angle QCB$

i.e. $\boxed{\angle QCB > \angle PBC}$ ...(i)

$\angle QCB$ is an exterior angle of ΔABC,

$\boxed{\angle QCB = \angle ABC + \angle A}$ ...(ii) [Exterior angle is equal to sum of two interior opposite angles]

$\angle PBC$ is an exterior angle of ΔABC,

$\boxed{\angle PBC = \angle ACB + \angle A}$ ...(iii) [Exterior angle is equal to sum of two interior opposite angles]

$\angle ABC + \cancel{\angle A} > \angle ACB + \cancel{\angle A}$ [From (i) (ii) and (iii)]

$\angle ABC > \angle ACB$ ...(iv)

In ΔABC,

$\angle ABC > \angle ACB$ [From (iv)]

$\therefore$ $\boxed{AC > AB}$

[In a triangle, side opposite to greater angle is greater]

MODULE : 34

Exercise 7.4.Q.3

$\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.

Sol. In ΔOAB

$\angle B < \angle A$

$\therefore \quad OA < OB \quad ...(i)$

[In a triangle, side opposite to greater angle is greater]

In ΔOCD

$\angle C < \angle D$

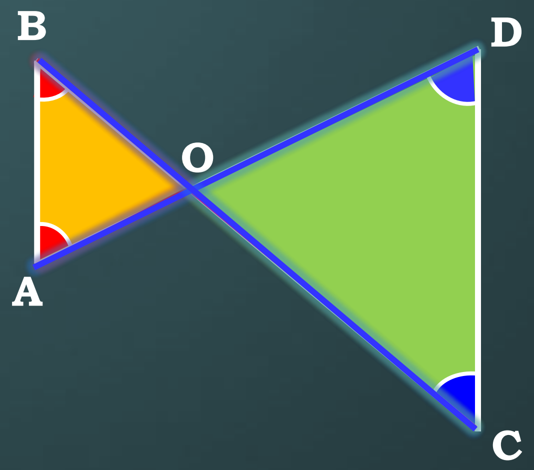
$\therefore \quad OD < OC \quad ...(ii)$

[In a triangle, side opposite to greater angle is greater]

Adding (i) and (ii)

$AO + OD < BO + OC$

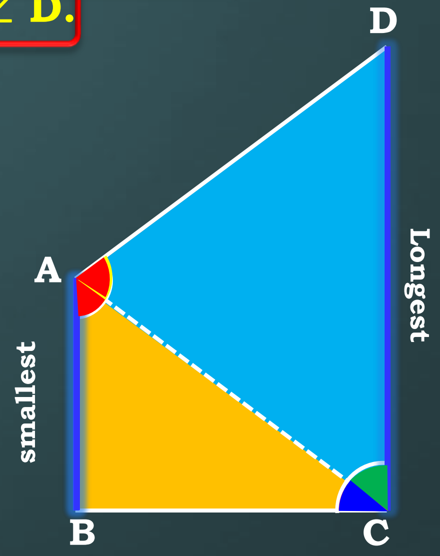
$\therefore \quad AD < BC$



MODULE : 35

Exercise 7.4.Q.4

AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD. Show that $\angle A > \angle C$ and $\angle B > \angle D$.



Sol. Draw AC

In Δ ABC,

$BC > AB$

$\therefore$ $\angle BAC > \angle ACB$ ...(i) [$\because$ Angle opposite to greater side is greater]

In Δ ACD,

$CD > AD$

$\therefore$ $\angle DAC > \angle ACD$ ...(ii) [$\because$ Angle opposite to greater side is greater]

Adding (i) and (ii), we get

$\angle BAC + \angle DAC > \angle ACB + \angle ACD$

$\angle BAD > \angle BCD$

i.e. $\boxed{\angle A > \angle C}$

Exercise 7.4.Q.4

AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD . Show that $\angle A > \angle C$ and $\angle B > \angle D$.

Sol. Draw BD

In ΔABD,

$AD > AB$

$\therefore$ $\angle ABD > \angle ADB$ ...(iii) [$\because$ Angle opposite to greater side is greater]

In ΔBDC,

$CD > BC$

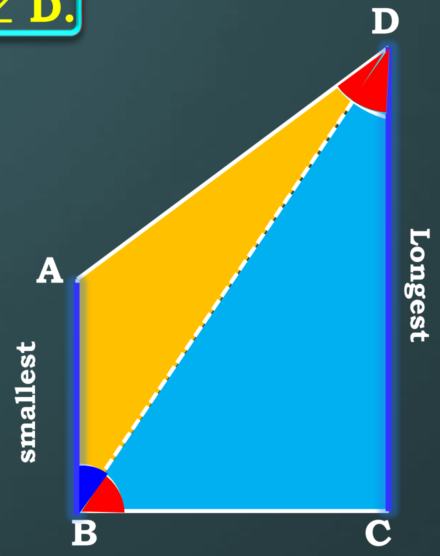
$\therefore$ $\angle DBC > \angle BDC$ ...(iv) [$\because$ Angle opposite to greater side is greater]

Adding (iii) and (iv), we get

$$\angle ABD + \angle DBC > \angle ADB + \angle BDC$$

$$\angle ABC > \angle ADC$$

i.e. $\boxed{\angle B > \angle D}$

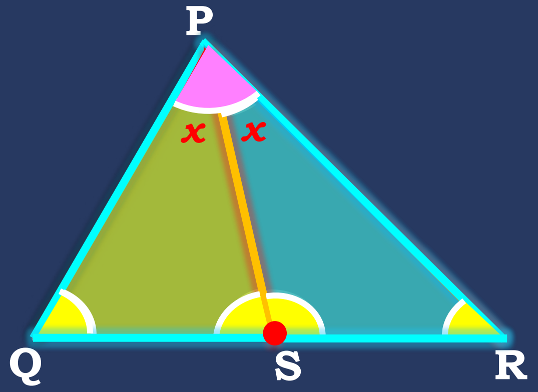


Thank You

MODULE : 36

Exercise 7.4-Q.5

PR > PQ and PS bisects $\angle QPR$.
Prove that $\angle PSR > \angle PSQ$.



Sol. In ΔPQR, $PR > PQ$

$\therefore \quad \angle PQR > \angle PRQ$ ...(i) [Angles opposite to greater side its greater]

$\therefore \quad \boxed{\angle PQS > \angle PRS}$ ...(ii) [From(i)]

Let, $\angle QPS = \angle RPS = x$ [PS bisect $\angle QPR$]

$\angle PSR = \angle PQS + x$ [Exterior angle of triangle its equal to sum interior opposite angles]

$\boxed{\angle PQS = \angle PSR - x}$ ...(iii)

$\therefore \quad \angle PSQ = \angle PRS + x$

$\therefore \quad \boxed{\angle PRS = \angle PSQ - x}$ ...(iv)

$\therefore \quad \angle PSR - \cancel{x} > \angle PSQ - \cancel{x}$ [From(ii),(iii) and (iv)]

$\therefore \quad \boxed{\angle PSR > \angle PSQ}$

MODULE : 37

Extra Example.9

D is a point on side BC of ΔABC such that AD = AC. Show that: AB > AD.

Sol.

In ΔDAC,

$\boxed{AD = AC}$ ... (i) [Given]

$\therefore$ $\boxed{\angle ADC = \angle ACD}$ ... (ii) [Angle opposite to equal sides]

$\therefore$ $\angle ADC$ is an exterior angle for ΔABD.

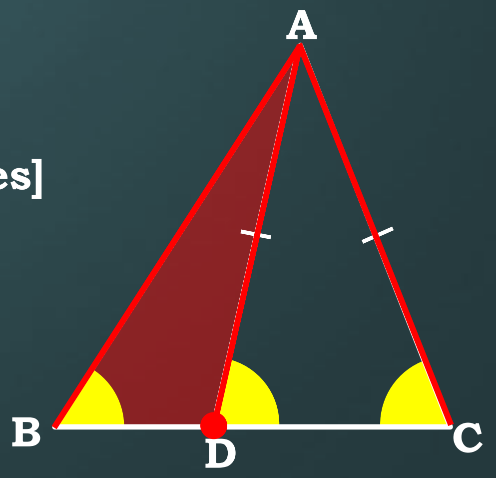
$\therefore$ $\boxed{\angle ADC > \angle ABD}$ ... (iii)

$\therefore$ $\angle ACD > \angle ABD$ From (ii) and (iii)

$\therefore$ $\angle ACB > \angle ABC$

$\therefore$ $\boxed{AB > AC}$ ... (iv) [side opposite to larger angle in ΔABC]

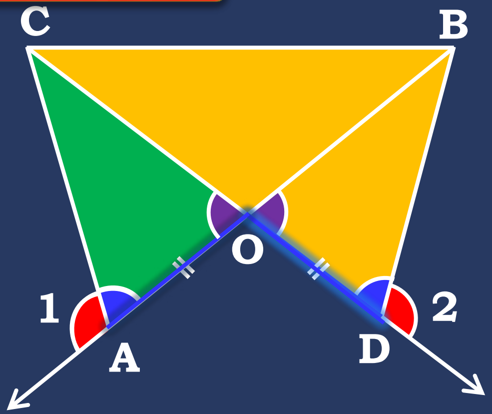
$\therefore$ $\boxed{AB > AD}$ [From (i) & (iv)]



MODULE : 38

If $OA = OD$, $\angle 1 = \angle 2$. Prove : ΔOCB is an isosceles triangle.

Hint : Prove OC = OB



Sol.

$\boxed{\angle 1 + \angle CAO = \boxed{180^\circ}}$ ...(i) [Linear pair]

$\boxed{\angle 2 + \angle BDO = \boxed{180^\circ}}$ ...(ii) [Linear pair]

$\cancel{\angle 1} + \angle CAO = \cancel{\angle 2} + \angle BDO$ [From (i) and (ii)]

$\therefore$ $\boxed{\angle CAO = \angle BDO}$ ...(iii) [$\because \angle 1 = \angle 2$]

In ΔCAO and ΔBDO,

$\angle CAO = \angle BDO$ [From (iii)]

$AO = OD$ [Given]

$\angle COA = \angle BOD$ [Vertically opposite angles]

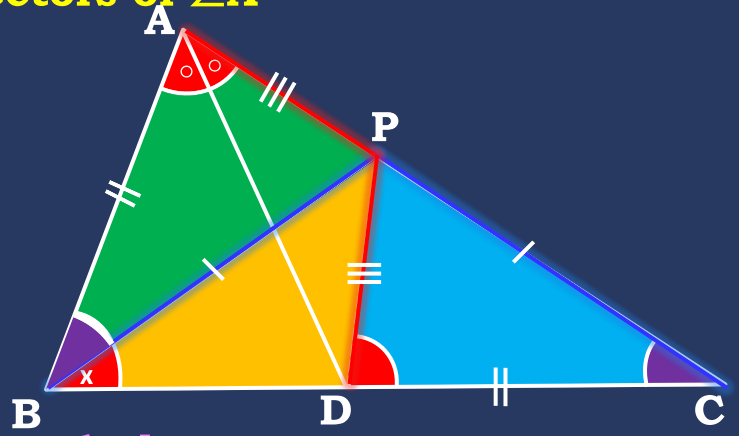
$\therefore$ $\Delta CAO \cong \Delta BOD$ [By ASA test]

$\therefore$ $OC = OB$ [c.p.c.t]

$\therefore$ $\boxed{\Delta OBC \text{ is an isosceles triangle.}}$

Exercise 7.3-Q.

$\boxed{\angle B = 2\angle C,}$ AB = DC, AD and BP are bisectors of $\angle A$ and $\angle B$ respectively. Find: $\angle BAC$



Sol. $\angle B = 2\angle C$

$\therefore \quad \cancel{2}\angle PBC = \cancel{2}\angle C$

$\therefore \quad \boxed{\angle PBC = \angle C}$ ...(i)

In ΔPBC

$\angle PBC = \angle C$ [From (i)]

$\therefore \quad \boxed{PB = PC}$ ...(ii) [sides opposite to equal angles]

In ΔABP and ΔDCP

$PB = PC$ [From (ii)]

$\angle ABP = \angle C$ [Given]

$AB = DC$ [Given]

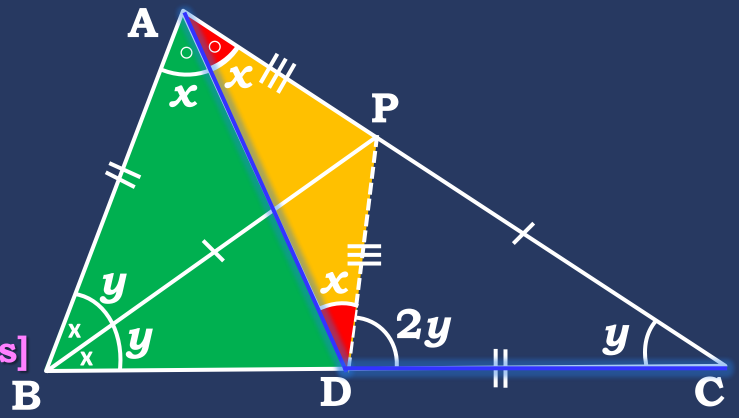
$\therefore \Delta ABP \cong \Delta DCP$ [SAS criterion]

$\therefore \quad AP = DP$ ...(iii) [c.p.c.t.]

$\therefore \angle BAP = \angle PDC$ ...(iv) [c.p.c.t.]

Exercise 7.3-Q.

$\angle B = 2\angle C$, $AB = DC$, AD and BP are bisectors of $\angle A$ and $\angle B$ respectively. Find: $\angle BAC$



Sol. $\boxed{AP = DP}$ ...(iii)

$\angle BAP = \angle PDC$ ...(iv)

In ΔAPD

$AP = DP$ [From (iii)]

$\therefore \angle PAD = \angle PDA$ [Angle opposite to equal sides]

Let $\angle PAD = \boxed{\angle PDA} = \boxed{\angle BAD = \boxed{x}}$

Let $\angle ABP = \angle PBC = \angle C = y$

$\therefore \boxed{\angle ABC = 2y}$

$\angle ADC = \boxed{\angle BAD} + \boxed{\angle ABD}$

$\boxed{\angle ADC = (x + 2y)}$

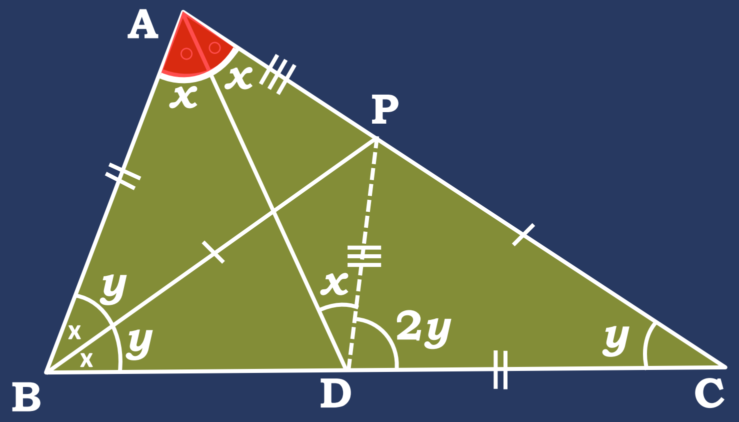
Also, $\boxed{\angle ADC} = \boxed{\angle ADP} + \angle PDC$

$\therefore x + 2y = x + \angle PDC$

$\therefore \angle PDC = 2y$ ...(v)

Exercise 7.3-Q.

$\angle B = 2\angle C$, AB = DC, AD and BP are bisectors of $\angle A$ and $\angle B$ respectively. Find: $\angle BAC$



Sol.

$\boxed{\angle BAP = \angle PDC}$ ...(iv)

$\boxed{\angle PDC = 2y}$ ...(v)

$\therefore \boxed{\angle BAP} = 2y$ [From (iv) and (v)]

$\therefore 2x = 2y$

$\therefore \boxed{x = y}$

In ΔABC

$\angle BAC + \angle ABC + \angle C = 180^\circ$

$\therefore 2x + \boxed{2y + y} = 180$

$\therefore 2x + 2x + x = 180$

$\therefore 5x = 180$

$\therefore x = 36$

$\angle BAC = 2x$

$= 2(36)$

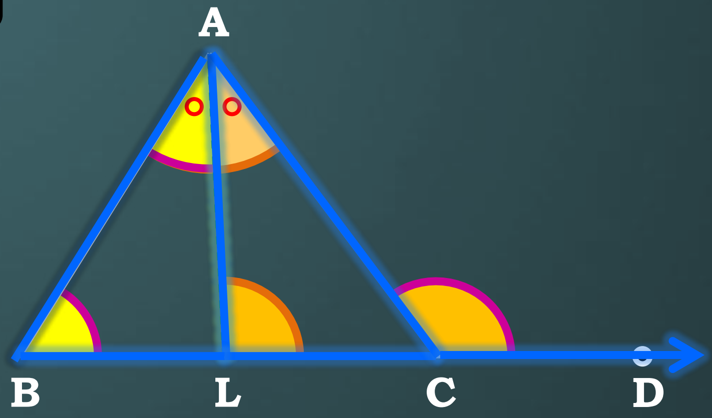
$= 72^\circ$

$\therefore \boxed{\angle BAC = 72^\circ}$

Exercise

PS 2—12

Q. The side BC of ΔABC is produced to D. The bisector of $\angle A$ meets BC at L. Prove that $\angle ABC + \angle ACD = 2\ \angle ALC$



Proof :

$\therefore\ \angle BAL = \angle LAC$ ...(i)

[Ray AL bisects $\angle BAC$]

$\angle ACD$ is an exterior angle of ΔALC,

$\therefore\ \angle ACD = \angle LAC + \angle ALC$

$\therefore\ \angle ACD - \angle LAC = \angle ALC$ ...(ii)

$\angle ALC$ is an exterior angle of ΔABL

$\therefore\ \angle ALC = \angle ABL + \angle BAL$

$\therefore\ \angle ABL + \angle BAL = \angle ALC$ ...(iii)

Adding (ii) and (iii),

$\therefore\ \angle ACD - \angle LAC + \angle ABL + \angle BAL = 2\ \angle ALC$

$\therefore\ \angle ACD - \cancel{\angle BAL} + \angle ABL + \cancel{\angle BAL} = 2\ \angle ALC$

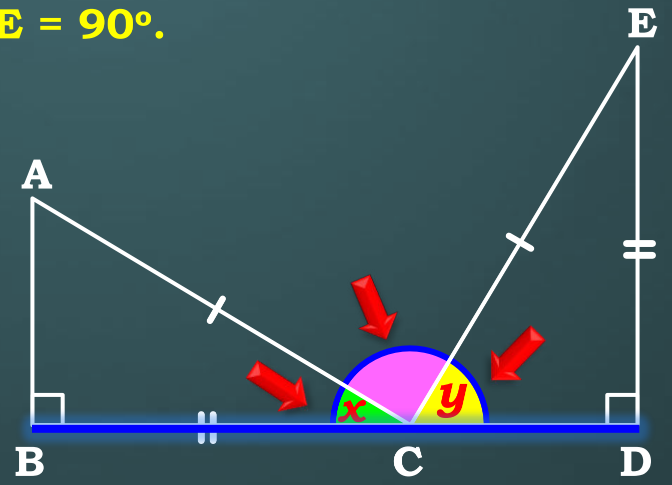
[From (i)]

$\therefore\ \angle ABL + \angle ACD = 2\ \angle ALC$

$\therefore\ \angle ABC + \angle ACD = 2\ \angle ALC$

Exercise

Q. In the adjoining figure, $\angle ABC = \angle CDE = 90^\circ$.
$AC = CE$, $BC = ED$.
Show that :
(a) $\Delta ABC \cong \Delta CDE$
(b) $\angle BAC = \angle ECD$
(c) $\boxed{\angle ACE = 90^\circ}$



Proof :

$\boxed{x + y} = 90$ ...(i)

$\boxed{\angle ACB} + \angle ACE + \boxed{\angle ECD} = 180^\circ$ [Angles in a straight line]

$\therefore \quad x + \angle ACE + y = 180$

$\therefore \quad \boxed{x + y} + \angle ACE = 180$

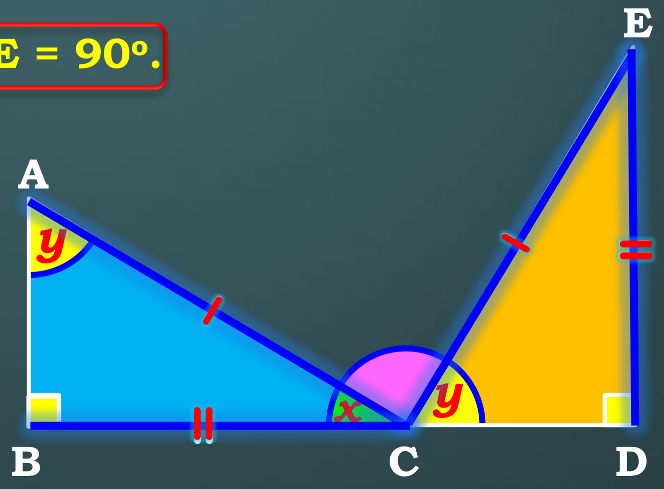
$\therefore \quad 90 + \angle ACE = 180$ [From (i)]

$\therefore \quad \angle ACE = 180 - 90$

$\therefore \quad \boxed{\angle ACE = 90^\circ}$

Exercise

Q. In the adjoining figure, $\angle ABC = \angle CDE = 90^\circ$. $AC = CE$, $BC = ED$.
Show that :
(a) $\Delta ABC \cong \Delta CDE$
(b) $\angle BAC = \angle ECD$
(c) $\angle ACE = 90^\circ$



Proof :

In ΔABC and ΔCDE,

$\angle ABC = \angle CDE = 90^\circ$ [Given]

Hypt. AC = hypt. CE [Given]

$BC = ED$ [Given]

$\therefore \Delta ABC \cong \Delta CDE$ [RHS rule]

$\therefore \angle BAC = \angle ECD$ [c.p.c.t.]

Let $\angle BAC = \angle ECD = y$

Let $\angle ACB = x$

In ΔABC,

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

[Sum of all angles of a triangle is 180°]

$\therefore y + 90 + x = 180$

$\therefore x + y = 180 - 90$

$\therefore x + y = 90$ ...(i)

If two sides of a triangle are equal, then the angles opposite to them are equal

Given : In ΔABC,
$AB = AC$

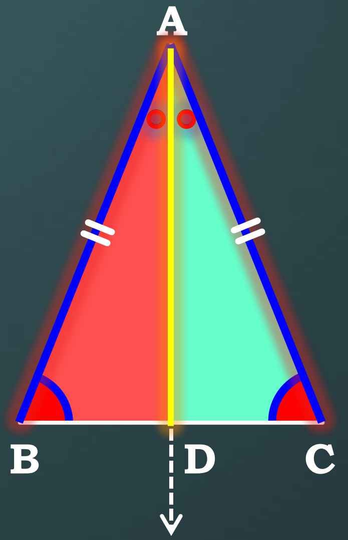
To prove: $\boxed{\angle B = \angle C}$

Construction: Draw ray AD, the angle bisector of $\angle BAC$ intersecting BC at point D.

Proof :

In ΔABD and ΔACD,

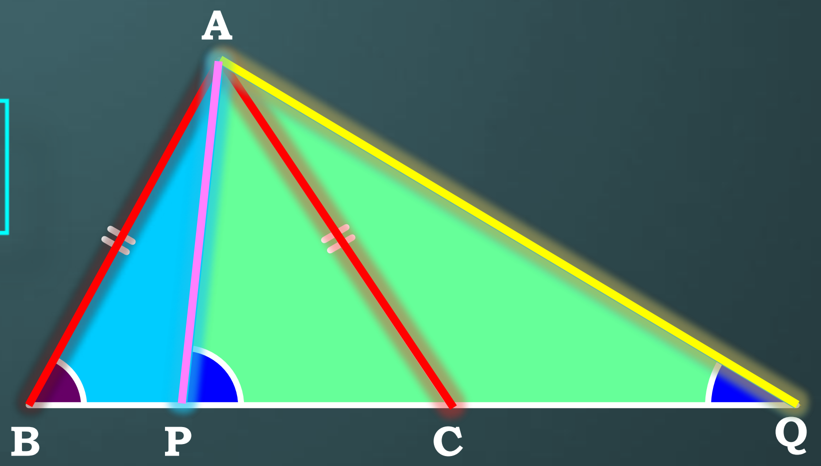
	$AB = AC$	[Given]
	$\angle BAD = \angle CAD$	[Construction]
	$AD = AD$	[Common side]
$\therefore$	$\Delta ABD \cong \Delta ACD$	[SAS criterion]
$\therefore$	$\boxed{\angle B = \angle C}$	[c.p.c.t.]



Extra Example

In the adjoining figure,
AB = AC is given.
To prove : AP < AQ

Hint : Prove : $\angle APQ > \angle Q$



Proof : AQ > AP

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle B = \angle ACB$ ...(i)

[Angles opposite to equal sides are equal]

$\angle APQ$ is an exterior angle of ΔABP

$\therefore \angle APQ > \angle B$ ...(ii) [Exterior angle is greater then its two interior opposite angles]

$\angle ACB$ is an exterior angle of ΔACQ

$\therefore \angle ACB > \angle Q$

$\therefore \angle B > \angle Q$ ...(iii) [From (i)]

In ΔAPQ,

$\therefore \angle APQ > \angle Q$ [From (ii) and (iii)]

$\therefore AQ > AP$

[Side opposite to greater angle is greater]

$\therefore AP < AQ$

Thank You