Lecture_07

No. 44

Arithmetic Progressions

- **Sums based on a_n and S_n formula**

Q.5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms of A.P and the common difference.

Sol: For an A.P $a = 5,\ a_n = 45,\ S_n = 400$

$$S_n = \frac{n}{2}[a + a_n]$$

$$\therefore\ 400 = \frac{n}{2}[5 + 45]$$

$$\therefore\ 400 = \frac{n}{\cancel{2}_{1}} \times \cancel{50}^{25}$$

$$\therefore\ 400 = n \times 25$$

$$\therefore\ \frac{400}{25} = n$$

$$\therefore\ n = 16$$

$$a_n = a + (n-1)d$$

$$\therefore\ 45 = 5 + (16-1)d$$

$$\therefore\ d = \frac{\cancel{40}^{8}}{\cancel{15}_{3}}$$

$$\therefore\ d = \frac{8}{3}$$

To find n

find d

For the given value of S_n formula

For given value of a_n. formula

For S_n substitute, $a = 5,\ a_n = 45\ \&\ S_n = 400$

For a_n substitute, $a = 5,\ a_n = 45\ \&\ n = 16$

6) The first and last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Sol: For given AP: $a = 17,$

We know that,

$$a_n = a + (n - 1)d$$

$$\therefore \quad 350 = 17 + (n - 1)(9)$$

$$\therefore \quad 350 - 17 = (n - 1)(9)$$

$$\therefore \quad 333 = (n - 1)(9)$$

$$\therefore \quad \frac{333}{9} = n - 1$$

$$\therefore \quad 37 = n - 1$$

$$\therefore \quad n = 38$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$\therefore S_{38} = \frac{38}{2}[17 + 350]$$

$= 19(367)$

$$\therefore \quad S_{38} = 6973$$

Substitute

a = 17,

We need

terms i.

We need to find their sum i.e. value of ‘S_n’

Lets use the formula

$\therefore$ There are 38 terms & their sum is 6973.

For S_n substitute,
$n = 38$, $a = 17$ & $a_n = 350$

No. 45

Arithmetic Progressions

- **Sums based on a_n and S_n formula**

3) In an AP.

iii) Given d = 5, S_9 = 7

For given value of S_9, Lets use the formula

Sol: For given AP:

$d = 5, \quad S_9 = 75$

We know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

For S_9 substitute, d = 5, S_9 = 75 & n = 9

$$\therefore S_9 = \frac{9}{2}[2a + (9-1)(5)]$$

$$\therefore 75 = \frac{9}{2}[2a + (8)(5)]$$

$$\therefore 150 = 9[2a + 40]$$

$$\therefore 150 = 18a + 360$$

$$\therefore 150 - 360 = 18a$$

$$\therefore -210 = 18a$$

$$\therefore a = \frac{-\cancel{210}^{35}}{\cancel{18}_{3}}$$

Now lets find a_9

$$a_9 = a + 8d$$

$$\therefore a_9 = \frac{-35}{3} + 8(5)$$

$$= \frac{-35}{3} + 40$$

$$= \frac{-35 + 120}{3}$$

$$\therefore a_9 = \frac{85}{3}$$

$$\therefore \boxed{a = \frac{-35}{3}, \quad a_9 = \frac{85}{3}}$$

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Arithmetic Progressions

- **Sums based on a_n and S_n formula**

3) In an AP.

iv) Given a = 2, d = 8, S_n = 9

Sol: For given AP:

$a = 2, \quad d = 8, \quad S_n = 90$

For given value o
Lets use the formula

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Substitute,
a = 2, d = 8 & S_n = 90

$$\therefore 90 = \frac{n}{2}[2(2) + (n - 1)(8)]$$

$$\therefore 90 \times 2 = n[4 + 8n - 8]$$

$$\therefore 180 = n[8n - 4]$$

$$\therefore 180 = 8n^2 - 4n$$

$$\therefore 0 = 8n^2 - 4n - 1$$

$$\therefore 8n^2 - 4n - 180 = 0$$

Dividing throughout by 4,

$$\therefore 2n^2 - n - 45 = 0$$

It's a quadratic equation,
lets solve it by
factorisation method

Take comm
first two

Take common from
last two terms

2 × 3 × 3 × 5

−10 +9

$$\therefore 2n^2 - 10n + 9n - 45 = 0$$

$$\therefore 2n(n - 5) + 9(n - 5) = 0$$

$$\therefore (n - 5)(2n + 9) = 0$$

$$\therefore n - 5 = 0 \quad \text{or} \quad 2n + 9 = 0$$

$$\therefore n = 5 \quad \text{or} \quad n = \frac{-9}{2}$$

Since 'n' cann

No

n = 5

Substitute,
a = 2, d = 8 & n = 5

$$a_n = a + (n - 1)d$$

$$= 2 + (5 - 1)(8)$$

3)

$$\therefore a_n = 34$$

$\therefore$ $\boxed{n = 5, \; a_n = 34}$

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Arithmetic Progressions

- **Sums based on a_n and S_n formula**

3) In an AP.

v) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n & d.

Sol: For an A.P $a = 8$ $a_n = 62$ $S_n = 210$

$$S_n = \frac{n}{2}[a + a_n]$$

$$\therefore 210 = \frac{n}{2}[8 + 62]$$

$$\therefore 210 = \frac{n}{\cancel{2}_1} \times \cancel{70}^{35}$$

$$\therefore 210 = n \times 35$$

$$\therefore \frac{210}{35} = n$$

$$\therefore n = 6$$

$$a_n = a + (n - 1)d$$

$$\therefore 62 = 8 + (6 - 1)d$$

$\therefore$

$$\therefore d = \frac{54}{5}$$

$$\boxed{\therefore n = 6 \text{ and } d = \frac{54}{5}}$$

To find n

For given value of a_n.

For S_n substitute,
$a = 8$, $a_n = 62$ & $S_n = 210$

For a_n substitute,
$a = 8$, $a_n = 62$ & $n = 6$

No. 48

Arithmetic Progressions

- **Sums based on a_n and S_n formula**

3) In an AP.

vi) Given d = 2, a_n =

Sol: For given AP:

$d = 2, \quad a_n = 4, \quad S_n = -14$

We know that,

$a_n = a + (n-1)d$

$\therefore 4 = a + (n-1)(2)$

$\therefore 4 = a + 2n - 2$

$\therefore 4 + 2 = a + 2n$

$\therefore 6 = a + 2n$

$\therefore a = \boxed{6 - 2n}$ ....

$S_n = \frac{n}{2}[a + a_n]$

$\therefore -14 = \frac{n}{2}[6 - 2n + 4]$

$\therefore -14 \times 2 = n(10 - 2n)$

... $= 10n - 2n^2$

... $- 28 = 0$

...throughout by 2, we get

$\therefore n^2 - 5n - 14 = 0$

$\therefore n^2 - 7n + 2n - 14 = 0$

$\therefore n(n-7) + 2(n-7) = 0$

$\therefore (n-7)(n+2) = 0$

... $= 0$

... $= 0$

$n = 7$ ... 2

As n cannot be negative, $n \neq -2$

$\boxed{\therefore n = 7}$

$\therefore a = 6 - 2(7)$

$\therefore a = 6 - 14$

$\boxed{\therefore a = -8}$

14

– 7 + 2

For given value of a_n, Lets use the formula

...ue of S_n, ...formula

Substitute, $d = 2$ & $a_n = 4$

We don't know the value of n or a

Substitute, $a_n = 4$ & $S_n = -14$

Substitute, value of n in equation (i)

No. 49

Arithmetic Progressions

- **Sums based on a_n and S_n formula**

3) In an AP.

(ix) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

Sol: $a_3 = 15$, $S_{10} = 125$

$a_3 = a + 2d$

$\therefore 15 = a + 2d$

$\therefore a + 2d = 15$

$S_n = \frac{n}{2}[2a + (n - 1) d]$

$S_{10} = \frac{10}{2}[2a + (10$

$125 = 5 (2a + 9d)$

$\therefore \frac{125}{5} = 2a + 9d$

$\therefore 25 = 2a + 9d$

$\therefore 2a + 9d = 25 \;....(ii)$

Multiplying (i) by 2

$\therefore 2a + 4d = 30 \;...(iii)$

Subtracting (iii) from (ii)

$= 30$

(-) (-) (-)

$5d = -5$

5

5

1

Substituting value of d in (i)

$a + 2(-1) = 15$

$\therefore a - 2 = 15$

$a = 15 + 2$

$a = 17$

$a_{10} = a + 9d$

$\therefore a_{10} = 17 + 9(-1)$

$\therefore a_{10} = 17 - 9$

$\therefore a_{10} = 8$

$\therefore$ **d = –1, a_{10} = 8**

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I

For given value o

Substitute n = 10

Lets find value of a_{10}

We will make coefficient of variable 'a' same

Same coefficient and same sign

No. 50

Arithmetic Progressions

- **Sums based on a_n and S_n formula**

Q.7. Find the sum of first 22 terms of an AP in which d = 7 and 22^{nd} term is 149.

Substitute,
$d = 7$ & $a_{22} = 149$

Sol: For an A.P : $d = 7$, a_{22}

$$a_{22} = a + 21d$$

For given value of a_{22}

$$\therefore \quad 149 = a + 21 \times 7$$

$$\therefore \quad 149 = a + 147$$

$$\therefore \quad 149 - 147 = a$$

$$\therefore \quad a = 2$$

Substitute,
$n = 22$, $a = 2$ & $a_{22} = 149$

$$S_n = \frac{n}{2}[a + a_n]$$

$$\therefore \quad S_{22} = \frac{22}{2}[2 + 149]$$

$$= 11\,(151)$$

$$\boxed{S_{22} = 1661}$$

Q.8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol: For an A.P : $a_2 = 14$

$d = a_3 - a_2 = 18 - 1$

$a_2 = a + d$

$\therefore \quad 14 = a + 4$

$\therefore \quad 14 - 4 = a$

$\therefore \quad a = 10$

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$$S_{51} = \frac{51}{2}[2(10) + (51-1)\,4]$$

$$= \frac{51}{2}[\,20 + (50)\,4]$$

$$= \frac{51}{2}[20 + 200]$$

$$= \frac{51}{2} \times 220$$

$$= 51 \times 110$$

$$= 5610$$

$\therefore$ **Sum of first 51 terms is 5610**

To find S_{51}

For given value of a_2

To find S_{51} we need to find the value of a & d

Lets find S_{51}

Substitute n = 51, a = 10 & d = 4

Thank You