LECTURE_06

MODULE_18

Every composite number can be expressed as a product of primes which is unique

Fundamental Theorem Of Arithmetic (Prime Factorisation)

Find HCF and LCM of 180 and 54

$180 = \underline{2 \times 2} \times \underline{3 \times 3} \times 5$

$= 2^2 \times 3^2 \times 5$

$54 = 2 \times \underline{3 \times 3} \times 3$

$= 2 \times 3^3$

HCF $= 2 \times 3^2$

$= 2 \times 9$

$= 18$

LCM $= 2^2 \times 3^3 \times 5$

$= 4 \times 27 \times 5$

$= 540$

HCF × LCM $= 18 \times 540 = 9720$

PRODUCT OF NUMBERS $= 54 \times 180 = 9720$

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

HCF

Product of the least powers of common prime factors of the numbers

LCM

Product of the highest powers of all the prime factors of the numbers

MODULE_19

Exercise 1.2

Q.1 Express each number as a product of its prime factors:

(i) 140

Sol.

140

2	140
2	70
5	35
7	7
	1

$\therefore 140 = 2^2 \times 5 \times 7$

(ii) 156

Sol.

156

2	156
2	78
3	39
13	13
	1

$\therefore 156 = 2^2 \times 3 \times 13$

(iii) 3825

Sol.

3825

3	3825
3	1275
5	425
5	85
17	17
	1

$\therefore 3825 = 3^2 \times 5^2 \times 17$

Exercise 1.2

Q.1 Express each number as a product of its prime factors:

(iv) 5005

Sol.

5005

5	5005
7	1001
11	143
13	13
	1

$\therefore 5005 = 5 \times 7 \times 11 \times 13$

(v) 7429

Sol.

7429

17	7429
19	437
23	23
	1

$\therefore 7429 = 17 \times 19 \times 23$

Q. **Express each number as a product of its prime factors:**

(i) 1000

Sol.

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\therefore\ 1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$$
$$= 2^3 \times 5^3$$

Q. **Express each number as a product of its prime factors:**

(ii) 72

Sol.

2	72
2	36
2	18
3	9
3	3
	1

$$\therefore \quad 72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$= 2^3 \times 3^2$$

MODULE_20

Exercise 1.2

Q.2 Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = Product of the two numbers.

(ii) 510 and 12

Sol. $510 = 2 \times 3 \times 5 \times 17$

$12 = 2^2 \times 3$

2	510
3	255
5	85
17	17
	1

2	12
2	6
3	3
	1

HCF (510, 12) $= 2 \times 3 = 6$ (Product of common factors raised to least power)

LCM (510, 12) $= 2^2 \times 3 \times 5 \times 17 = 1020$ ...(Product of all the prime factors raised to highest power)

Verification :

LCM × HCF $= 1020 \times 6 = 6120$

Product of the two numbers $= 510 \times 12 = 6120$

∴ LCM × HCF = Product of the two numbers

Exercise 1.2

Q.2 Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = Product of the two numbers.

(iii) 336 and 54

Sol.

$336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3^3$

HCF (336, 54) = $2 \times 3 = 6$ (Product of common factors raised to least power)

LCM (336, 54) = $2^4 \times 3^3 \times 7 = 3024$ ...(Product of all the prime factors raised to highest power)

2	336
2	168
2	84
2	42
3	21
7	7
	1

2	54
3	27
3	9
3	3
	1

Verification :

LCM × HCF = 6 × 3024 = 18144

Product of the two numbers = 336 × 54 = 18144

∴ LCM × HCF = Product of the two numbers

MODULE_21

Exercise 1.2

Q.3 Find the LCM and HCF of the following integers by applying the prime factorisation method.

(ii) 17, 23 and 29

Sol.

$17 = 17 \times 1$

$23 = 23 \times 1$

$29 = 29 \times 1$

$\therefore$ HCF (17, 23, 29) = 1

(Product of common factors raised to least powers)

LCM (17, 23, 29) = $17 \times 23 \times 29$ = 11339

(Product of all the prime factors raised to highest powers)

Exercise 1.2

Q.3 Find the LCM and HCF of the following integers by applying the prime factorisation method.

(iii) 8, 9 and 25

Sol.

$8 = 2^3 \times 1$

$9 = 3^2 \times 1$

$25 = 5^2 \times 1$

$\therefore$ HCF (8, 9, 25) = 1

(Product of common factors raised to least powers)

LCM (8, 9, 25) = $2^3 \times 3^2 \times 5^2 = 1800$

(Product of all the prime factors raised to highest powers)

MODULE_22

Exercise 1.2

Q.3 Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

Sol.

$$12 = 2^2 \times 3$$
$$15 = 3 \times 5$$
$$21 = 3 \times 7$$

HCF $(12, 15, 21) = 3$ (Product of common factors raised to least power)

LCM $(12, 15, 21) = 3 \times 2^2 \times 5 \times 7 = 420$ (Product of all the prime factors raised to highest powers)

2	12
2	6
3	3
	1

3	21
7	7
	1

3	15
5	5
	1

Exercise 1.2

Q.4 Given that HCF (306, 657) = 9, find LCM (306, 657)

Sol. HCF (306, 657) = 9

Now, HCF (306, 657) × LCM (306, 657) = 306 × 657

We know the relation

HCF(a,b) × LCM(a,b) = a × b

$$\therefore \text{LCM}(306, 657) = \frac{\cancel{\overset{34}{\cancel{\overset{102}{306}}}} \times 657}{\cancel{\underset{\cancel{3}}{9}}}$$

$$= 34 \times 657$$

$$= 22338$$

Exercise 1.2

Q.7 There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point.

Sol. L.C.M of 18 and 12

$18 = 2 \times 3^2$

$12 = 2^2 \times 3$

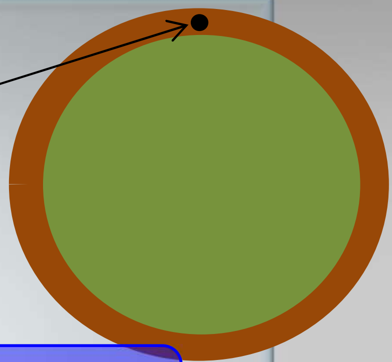
L.C.M $= 2^2 \times 3^2$

$= 36$

(Product of all the prime factors raised to highest powers)

Time taken to drive 1 round	
Sonia	Ravi
18 min	12 min
36 min	24 min
54 min	36 min

g point



In 36 minutes Sonia arrived at the starting point after making 2 rounds,

while in 36 minutes Ravi arrived at the same starting point after making 3 rounds.

Sonia takes 18 min to drive 1 round

That means, she will come to starting point after every 18 min

Ravi takes 12 min to drive 1 round

That means he will come to starting point after every 12 min

Hence they meet each other at the starting point after 36 minutes.

Therefore LCM of 18 & 12 will give us the time they meet again at starting point