No. 22

QUADRATIC EQUATIONS

- **Solving Quadratic Equations Using Formula Method**

3] Formula Method

General Form of a Quadratic Equation is

$$ax^2 + bx + c = 0$$

Formula to find value of 'x'

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the following by using formula method

i) $3q^2 = 2q + 8$

Sol : $3q^2 = 2q + 8$

Standard form

$\therefore\ 3q^2 - 2q - 8 = 0$

On comparing with $ax^2 + bx + c = 0$,

we get, $a = 3$, $b = -2$ & $c = -8$

$$b^2 - 4ac = (-2)^2 - 4(3)(-8)$$

$$= 4 + 96$$

$$= 100$$

$$\therefore q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore q = \frac{-(-2) \pm \sqrt{100}}{2(3)}$$

$$\therefore q = \frac{2 \pm 10}{6}$$

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To use formula we should have value of a, b & c

$aq^2 + bq + c = 0$

$\therefore q = \frac{2 + 10}{6}$ or $q = \frac{2 - 10}{6}$

$\therefore q = \frac{12}{6}$ or $q = \frac{-8}{6}$

$\therefore q = 2$ or $q = \frac{-4}{3}$

$3q^2 - 2q - 8 = 0$

$\therefore$ **The roots of the given quadratic equations are 2 and $-\frac{4}{3}$**

No. 23

QUADRATIC EQUATIONS

- **Solving Quadratic Equations Using Formula Method Contd...**

Solve the following by using formula method

ii) $6m^2 - 4m = 3$

Sol : $6m^2 - 4m = 3$

$\boxed{6}m^2 \boxed{-4}m \boxed{-3} = 0$

Standard form

On comparing with $\boxed{a}m^2 + \boxed{b}m + \boxed{c} = 0$, we get

$a = 6,\ b = -4,\ c = -3$

$b^2 - 4ac = (-4)^2 - (4)(6)(-3)$

$= 16 + 72$

$= 88$

$$\therefore m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore m = \frac{-(-4) \pm \sqrt{88}}{2(6)}$$

88 cannot be brought out of the root

$$\therefore m = \frac{4 \pm \sqrt{4 \times 22}}{12}$$

$$\therefore m = \frac{4 \pm 2\sqrt{22}}{12}$$

Can … ancel 2 … ?

$$\therefore m = \frac{2(2 \pm \sqrt{22})}{12}$$

$$\frac{2 \pm \sqrt{22}}{6}$$

Let us simplify it

$$\frac{\sqrt{22}}{6} \quad \text{or} \quad m = \frac{2 - \sqrt{22}}{6}$$

…f the given quadratic … are $\frac{2 + \sqrt{22}}{6}$ and $\frac{2 - \sqrt{22}}{6}$

Solve the following by using formula method

iii) $3y^2 + 7y + 4 = 0$

Sol : $\boxed{3}y^2 + \boxed{7}y + \boxed{4} = 0$

On comparing with $\boxed{a}y^2 + \boxed{b}y + \boxed{c} = 0$, we get

$$a = 3, b = 7, c = 4$$

$$b^2 - 4ac = (7)^2 - (4)(3)(4)$$

$$= 49 - 48$$

$$= 1$$

$$\therefore y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore y = \frac{-7 \pm \sqrt{1}}{2(3)}$$

$$\therefore y = \frac{-7 \pm 1}{6}$$

$$\therefore y = \frac{-7+1}{6} \quad \text{or} \quad y = \frac{-7-1}{6}$$

$$\therefore y = \frac{-6}{6} \quad \text{or} \quad y = -\frac{8}{6}$$

$$\therefore y = -1 \quad \text{or} \quad y = -\frac{4}{3}$$

$\therefore$ **The roots of the given quadratic equations are - 1 and - $\frac{4}{3}$**

No. 24

QUADRATIC EQUATIONS

- **Solving Quadratic Equations Using Formula Method Contd…**

Solve the following by using formula method

iv) $2x^2 + \frac{x-1}{5} = 0$

Sol: $2x^2 + \frac{x-1}{5} = 0$

Multiplying throughout by 5,we get

$\boxed{10}x^2 + \boxed{x}\boxed{-1} = 0$

On comparing with $\boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$,
we get

$a = 10, b = 1, c = -1$

$b^2 - 4ac = (1)^2 - (4)(10)(-1)$

$= 1 + 40$

$= 41$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-1 \pm \sqrt{41}}{2(10)}$

$\therefore x = \frac{-1 \pm \sqrt{41}}{20}$

$\therefore x = \frac{-1 + \sqrt{41}}{20}$ or $x = \frac{-1 - \sqrt{41}}{20}$

$\therefore$ **The roots of the given quadratic equations are** $\frac{-1 + \sqrt{41}}{20}$ **and** $\frac{-1 - \sqrt{41}}{20}$

No. 25

QUADRATIC EQUATIONS

- **Solving Quadratic Equations Using Formula Method**

Q.) Find the roots of the following quadratic equations given, by applying the quadratic formula.

(i) $2x^2 - 7x + 3 = 0$

Sol: $\boxed{2}x^2 \boxed{-7}x \boxed{+3} = 0$

Is it in a Standard form ?

Yes

On comparing with $\boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$ we get

$a = 2, \ b = -7, \ c = 3$

$$b^2 - 4ac = (-7)^2 - (4)(2)(3)$$
$$= 49 - 24$$
$$= 25$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-(-7) \pm \sqrt{25}}{2\ (2)}$$

$$= \frac{7 \pm \sqrt{25}}{4}$$

$$\therefore \quad x = \frac{7 \pm 5}{4}$$

$$\therefore \quad x = \frac{7+5}{4} \quad \text{or} \quad x = \frac{7-5}{4}$$

$$\therefore \quad x = \frac{\cancel{12}^{\,3}}{\cancel{4}} \quad \text{or} \quad x = \frac{\cancel{2}^{\,1}}{\cancel{4}_{\,2}}$$

$$\therefore \quad x = 3 \quad \text{or} \quad x = \frac{1}{2}$$

$\therefore$ **The roots of the given quadratic equation are 3 and $\frac{1}{2}$.**

Q.) Find the roots of the following quadratic equations given, by applying the quadratic formula.

(ii) $2x^2 + x - 4 = 0$

Sol: $\boxed{2}x^2 \boxed{-1}x \boxed{-4} = 0$

Is it in a Standard form ?

Yes

On comparing with $\boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$, we get

$a = 2, \; b = 1, \; c = -4$

$$b^2 - 4ac = (1)^2 - (4)(2)(-4)$$
$$= 1 + 32$$
$$= 33$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-(1) \pm \sqrt{33}}{2\,(2)}$$

$$\frac{1 \pm \sqrt{33}}{4}$$

$$\therefore \quad x = \frac{-1 + \sqrt{33}}{4} \quad \text{or} \quad x = \frac{-1 - \sqrt{33}}{4}$$

$\therefore$ **The roots of the given quadratic equation are $\frac{-1 + \sqrt{33}}{4}$ and $\frac{-1 - \sqrt{33}}{4}$.**

No. 26

QUADRATIC EQUATIONS

- **Solving Quadratic Equations Using Formula Method**

Q.) Find the roots of the following quadratic equations given, by applying the quadratic formula.

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Sol: $\boxed{4}x^2 + \boxed{4\sqrt{3}}x + \boxed{3} = 0$

Is it in a Standard form ?

Yes

On comparing with $\boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$, we get

$a = 4,\ b = 4\sqrt{3},\ c = 3$

$b^2 - 4ac = (4\sqrt{3})^2 - (4)(4)(3)$

$= (4)^2 \times (\sqrt{3})^2$
$= 16 \times 3$
$= 48$

$= 48 - 48$

$= 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-(4\sqrt{3}) \pm \sqrt{0}}{2\,(4)}$$

$$\frac{4\sqrt{3} \pm 0}{8}$$

$$\therefore \quad x = \frac{4\sqrt{3} + 0}{8} \quad \text{or} \quad x = \frac{-4\sqrt{3} - 0}{8}$$

$$\therefore \quad x = \frac{-\cancel{4}^{1}\sqrt{3}}{\cancel{8}_{2}} \quad \text{or} \quad x = \frac{-\cancel{4}^{1}\sqrt{3}}{\cancel{8}_{2}}$$

$$x = \frac{-\sqrt{3}}{2} \quad \text{or} \quad x = \frac{-\sqrt{3}}{2}$$

$\therefore$ **The roots of the given quadratic equation are $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$.**

Q.) Find the roots of the following quadratic equations given, by applying the quadratic formula.

(iv) $\mathbf{2x^2 + x + 4 = 0}$

Sol: $\boxed{2}x^2 \boxed{+1}x \boxed{+4} = 0$

Is it in a Standard form ?

Yes

On comparing with $\boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$, we get

$a = 2, \; b = 1, \; c = 4$

$b^2 - 4ac = (1)^2 - (4)(2)(4)$

$= 1 - 32$

$= -31$

Is –31 greater than 0 ?

–31 is less than 0

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-(1) \pm \sqrt{-31}}{2\,(2)}$$

$$\frac{1 \pm \sqrt{-31}}{4}$$

$b^2 - 4ac < 0$

$\therefore$ Roots of the equation $2x^2 + x + 4 = 0$ are not real nos. i.e. roots do not exist.

given quadratic equation has no real roots.

No. 27

QUADRATIC EQUATIONS

- Solving Quadratic Equations Using Formula Method

Q.) Find the roots of the following equations:

(i) $\mathbf{x - \dfrac{1}{x} = 3,\ x \neq 0}$

Sol: $x - \dfrac{1}{x} = 3$

Multiplying throughout by x, we get

$\therefore \quad x^2 - 1 = 3x$

$\therefore \boxed{1}x^2 \boxed{-\,3}x \boxed{-\,1} = 0$

Is it in a Standard form ?

Yes

On comparing with $\boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$, we get

$a = 1,\ b = -3,\ c = -1$

$b^2 - 4ac = (-3)^2 - (4)(1)(-1)$

$= 9 + 4$

$= 13$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-(-3) \pm \sqrt{13}}{2\,(1)}$$

$$\therefore \quad x = \frac{3 \pm \sqrt{13}}{2}$$

$$x = \frac{3 + \sqrt{13}}{2} \quad \text{or} \quad x = \frac{3 - \sqrt{13}}{2}$$

$\therefore$ **The roots of the given quadratic equation are** $\mathbf{\dfrac{3 + \sqrt{13}}{2}}$ **and** $\mathbf{\dfrac{3 - \sqrt{13}}{2}}$**.**

Q) Find the roots of the following equations:

(ii) $\dfrac{1}{x+4} - \dfrac{1}{x-7} = \dfrac{11}{30};\ x \neq -4, 7$

Sol:

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\frac{(x-7)-(x+4)}{(x+4)\,(x-7)} = \frac{11}{30}$$

$$\therefore \quad \frac{x-7-x-4}{(x+4)\,(x-7)} = \frac{11}{30}$$

$$\therefore \quad \frac{-11}{x^2-3x-28} = \frac{11}{3}$$

$$\therefore \quad -330 = 1$$

$$\therefore \quad 11x^2 - 33x - 308 + 33$$

$$\therefore \quad 11x^2 - 33x + 22 = 0$$

$$\therefore \quad \boxed{1}x^2 \boxed{-3}x + \boxed{2} = 0$$

On comparing with $\boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$,
we get $a = 1,\ b = -3,\ c = 2$

Dividing throughout by 11

n ?

Yes

$$b^2 - 4ac = (-3)^2 - (4)(1)(2)$$
$$= 9 - 8$$
$$= 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-(-3) \pm \sqrt{1}}{2\,(1)}$$

$$\therefore \quad x = \frac{3 \pm \sqrt{1}}{2}$$

$$x = \frac{3+1}{2} \quad \text{or} \quad x = \frac{3-1}{2}$$

$$\therefore \quad x = \frac{\cancel{4}^{\,2}}{\cancel{2}} \quad \text{or} \quad x = \frac{\cancel{2}^{\,1}}{\cancel{2}}$$

$$x = 2 \quad \text{or} \quad x = 1$$

$\therefore$ **The roots of the given quadratic equation are 1 and 2.**

No. 28

QUADRATIC EQUATIONS

. Understanding nature of roots of a Quadratic Equation

Solve the following by using formula method

v) $3y^2 + 9y + 4 = 0$

Sol: $\boxed{3}y^2 + \boxed{9}y + \boxed{4} = 0$

On comparing with

$\boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$, we get

$a = 3, b = 9, c = 4$

$b^2 - 4ac = (9)^2 - 4\,(3)\,(4)$

$= 81 - 48$

$= 33$

$b^2 - 4ac > 0$

Hence roots of the quadratic equation are real and unequal.

$\therefore y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \dfrac{-9 \pm \sqrt{33}}{2\,(3)}$

$\therefore y = \dfrac{-9 \pm \sqrt{33}}{6}$

$\therefore y = \dfrac{-9 + \sqrt{33}}{6}$ or $y = \dfrac{-9 - \sqrt{33}}{6}$

$\therefore$ The roots of the given quadratic equations are $\dfrac{-9 + \sqrt{33}}{6}$ and $\dfrac{-9 - \sqrt{33}}{6}$

Is it in a Standard form ?

Yes

Is 33 greater than 0 ?

Yes

vi) $y^2 + 6y + \ldots$

$\ldots y \ldots$

$\ldots$ari$\ldots$

$\ldots x^2 + \boxed{b}x + \boxed{c} = 0$, w$\ldots$

$\ldots = 6, c = 9$

$\ldots 4ac = (6)^2 - 4\,(1)\,(9)$

$= 36 - 36$

$\ldots 4ac$

equation$\ldots$

$\therefore y = \dfrac{-6 \ldots}{\ldots}$

$\therefore y = \dfrac{-6 \pm 0}{2}$

$\therefore y = \dfrac{-6 + 0}{2}$ or $y = \dfrac{-6 - 0}{2}$

$\therefore y = -3$ or $y = -3$

$\therefore$ The roots of the given quadratic equations is -3

Is it in a Standard form ?

Yes

Here, b^2-4ac = 0

$\ldots\sqrt{3}x + 16 = 0$

$\ldots\boxed{\ldots 3}x + \boxed{16} = 0$

$\ldots$ng with

$\boxed{a}\ldots + \boxed{c} = 0$, we get

$a \ldots 5\sqrt{3}, c = 16$

$b^2 - 4ac = \left(5\sqrt{3}\right)^2 - 4(2)(16)$

$\ldots 28$

$\ldots$ ac < 0

$\ldots = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\ldots$ots of the quadratic equation are not real

$\therefore x = \dfrac{-5\sqrt{3} \pm \sqrt{-53}}{2\,(2)}$

But $\sqrt{-53}$ is not a real number

Is it in a Standard form ?

Yes

-53 is less than 0

No. 29

QUADRATIC EQUATIONS

- **Finding the nature of roots and finding roots if they are real**

Q) Find the nature of roots of the following quadratic equations. If the real roots exist, find them.

ii) $3x^2 - 4\sqrt{3}\,x + 4 = 0$

Sol: $\boxed{3}x^2 \boxed{-4\sqrt{3}}\,x + \boxed{4} = $

On comparing with $\boxed{a}x$

we get; $a = 3$, $b = -4$

$b^2 - 4ac = (-4\sqrt{3})^2$

$= 16 \times 3$

$= 48 - 48$

$= 0$

$b^2 - 4ac = 0$

$\therefore$ The two roots are real and equal

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4\sqrt{3}) \pm 0}{2 \times 3}$$

$x = \frac{4\sqrt{3} + 0}{6}$ or $x = \frac{4\sqrt{3} - 0}{6}$

$\therefore x = \frac{\cancel{4}^{\,2}\sqrt{3}}{\cancel{6}_{\,3}}$ or $x = \frac{\cancel{4}^{\,2}\sqrt{3}}{\cancel{6}_{\,3}}$

$\therefore x = \frac{2\sqrt{3}}{3}$ or $x = \frac{2\sqrt{3}}{3}$

$\therefore x = \frac{2}{\sqrt{3}}$ or $x = \frac{2}{\sqrt{3}}$

$\therefore$ **The roots of the equation are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.**

Is it in a Standard form ?

For that, equation should be in the ... form

$= (4)^2 \times (\sqrt{3})^2$
$= 16 \times 3$
$= 48$

No. 30

QUADRATIC EQUATIONS

- Discriminant
- Sums Based on Nature Of Roots

DISCRIMINANT

- If $b^2 - 4ac > 0$ → Two distinct real roots
- If $b^2 - 4ac = 0$ → real roots
- If $b^2 - 4ac < 0$ → No real roots

Discriminant is the value of $b^2 - 4ac$

Find the nature of roots of the following quadratic equations. If the real roots exist, find them.

i) $2x^2 - 3x + 5 = 0$

Sol: $\boxed{2}x^2 \boxed{-3}x + \boxed{5} =$

On comparing wit

we get; $a = 2$, $b = -3$, $c = 5$

$b^2 - 4ac = (-3)^2 - 4$

$= 9 - 40$

$= -31$

$b^2 - 4ac < 0$

Hence roots of the quadratic equation are not real

Means we have

t the value

For that equation

the

orm

Is it in a standard form ?

Yes

Find the nature of roots of the following quadratic equations. If the real roots exist, find them.

ii) $2x^2 - 6x + 3 = 0$

$\boxed{2}x^2 \boxed{-6}x + \boxed{3} = 0$

On comparing with $\boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$,

we get; $a = 2$, b ...

$b^2 - 4ac = (-6)^2 -$ ...

$= 36 - 2$ ...

$= 12$

$b^2 - 4ac > 0$

Is it in a standard form ?

Yes

Hence roots of the quadratic equation are real and distinct.

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{12}}{2(2)}$$

$$= \frac{6 \pm 2\sqrt{3}}{4}$$

$$= \frac{2\,(3 \pm \sqrt{3})}{4}$$

$$= \frac{3 \pm \sqrt{3}}{2}$$

$$\therefore x = \frac{3 + \sqrt{3}}{2} \quad \text{or} \quad x = \frac{3 - \sqrt{3}}{2}$$

$\therefore$ The roots of the given quadratic equations are $\frac{3+\sqrt{3}}{2}$ and $\frac{3-\sqrt{3}}{2}$

Thank You