# LECTURE_05

# MODULE_15

**Q.** If $a$ and $b$ are two odd positive integers such that $a > b$, then prove that one of the two numbers $\boxed{\dfrac{a+b}{2}}$ and $\dfrac{a-b}{2}$ is odd and the other is even.

**Sol.** Let $a = \boxed{2m+3}$ and $= \boxed{2m+1}$

[ where $m$ and $n$ are + ve integers ]

$$\therefore \quad \frac{a+b}{2} = \frac{(2m+3)+(2m+1)}{2}$$

$$= \frac{2m+2m+4}{2}$$

$$= \frac{4m+4}{2}$$

$$= \frac{\cancel{2}(2m+2)}{\cancel{2}}$$

$$\therefore \quad \frac{a+b}{2} = 2m+2 \qquad \text{[ which is even ]}$$

**Q.** If $a$ and $b$ are two odd positive integers such that $a > b$, then prove that one of the two numbers $\frac{a+b}{2}$ and $\boxed{\frac{a-b}{2}}$ is odd and the other is even.

**Sol.** Let $a = \boxed{2m+3}$ and $= \boxed{2m+1}$

[ where $m$ and $n$ are + ve integers ]

$\therefore \quad \frac{a+b}{2} = 2m + 2 \quad$ [ which is even ]

Now, $\frac{a-b}{2} = \frac{(2m+3) - (2m+1)}{2}$

$= \frac{\cancel{2m} + 3 - \cancel{2m} - 1}{2}$

$= \frac{3 - 1}{2}$

$= \frac{\cancel{2}^{1}}{\cancel{2}} = 1 \quad$ [ which is odd ]

$\therefore \quad \frac{a+b}{2} = 2m + 2$ and $\frac{a-b}{2} = 1$

$\therefore$ One of them is odd and the other is even

Hence Proved

# MODULE_16

**Q.** Express the HCF of 468 and 222 as $468x + 222y$ where $x, y$ are integers in two different ways.

**Sol.**

$$\begin{array}{r|l} 222 & 468 \; (2 \\ & -444 \\ \hline 24 & 222 \; (9 \\ & -216 \\ \hline 6 & 24 \; (4 \\ & -24 \\ \hline & 0 \end{array}$$

$$468 = 222 \times 2 + 24 \qquad \ldots\text{(i)}$$

$$222 = 24 \times 9 + 6 \qquad \ldots\text{(ii)}$$

$$24 = 6 \times 4 + 0 \qquad \ldots\text{(iii)}$$

$\therefore$ $\boxed{\text{HCF } (468, 222) = 6}$

DIVIDEND = DIVISOR × QUOTIENT + REMAINDER

**Q.** Express the HCF of 468 and 222 as $468x + 222y$ where $x, y$ are integers in two different ways.

**Sol.** To show :- $6 = 468x + 222y$

$468 = 222 \times 2 + 24$ ....(i)

$\boxed{222 = 24 \times 9 + 6}$ ....(ii)

$24 = 6 \times 4 + 0$ ....(iii)

$\therefore$ $\boxed{\text{HCF } (468, 222) = 6}$

$222 = (\underline{24 \times 9}) + 6$ ....from (ii)

$\therefore$ $6 = 222 - (24 \times 9)$

$\therefore$ $6 = 222 - 9 \times 24$

$\therefore$ $6 = 222 - 9 \times [468 - (222 \times 2)]$ ....from (i)

$\therefore$ $6 = 222 - 9 \times 468 + (9 \times 2 \times 222)$

$\therefore$ $6 = 222 - 9 \times 468 + 18 \times 222$

$\therefore$ $6 = \underline{222 + 18} \times 222 - 9 \times 468$

$\therefore$ $6 = 222(1 + 18) + (-9)\,468$

$\therefore$ $6 = 222(19) + 468(-9)$

$\therefore$ $6 = 468(-9) + 222(19)$

$= 468(x) + 222(y)$

where $x = -9$ and $y = 19$.

**Q.** Express the HCF of 468 and 222 as $468x + 222y$ where $x, y$ are integers in two different ways.

**Sol.** To show :- $6 = 468x + 222y$

$468 = 222 \times 2 + 24$ ....(i)

$222 = 24 \times 9 + 6$ ....(ii)

$24 = 6 \times 4 + 0$ ....(iii)

$\therefore$ $\boxed{\text{HCF }(468, 222) = 6}$

$\therefore$ $6 = 468(-9) + 222(19)$ ....(iv)

where $x = -9$ and $y = 19$.

$6 = 222 \times 19 + 468 \times (-9)$ ....from (iv)

$= 222 \times 19 + 468 \times (-9) + [(468 \times 222) - (468 \times 222)]$

$= \underline{222 \times 19} + \underline{468 \times (-9) + (468 \times 222)} - (\underline{468 \times 222})$

$= (\underline{222} \times 19) - (468 \times \underline{222}) + [\underline{468} \times (-9)] + (\underline{468} \times 222)$

$= 222\,[19 - 468] + 468\,[(-9) + 222]$

$= 222(-449) + 468(213)$

$= 468\,x + 222\,y$ where $x = 213$ and $y = 449.$

# MODULE_17

**Q.** If d is the $\boxed{\text{HCF of 56 \& 72,}}$ find $x$, $y$ satisfying d = $56x + 72y$. Also, show that $x$ and $y$ are not unique.

**Sol.** Applying Euclid's division lemma to 56 to 72,

$$72 = \boxed{56 \times 1} + 16 \quad \text{...... (i)}$$

$\because$ Remainder is 16

So, we consider the divisor 56 and remainder 16 and apply division lemma.

$$56 = \boxed{16 \times 3} + 8 \quad \text{...... (ii)}$$

$\because$ Remainder is 8

So, we consider the divisor 16 and remainder 8 and apply division lemma.

$$16 = 8 \times 2 + 0$$

The remainder at this stage is 0.

$\therefore$ Last divisor 8 is HCF of 56 and 72

From (ii), we get

$$8 = 56 - 16 \times 3$$

$$8 = 56 - (72 - 56 \times 1) \times 3 \quad \text{... from (i)}$$

$$8 = 56 - 72 \times 3 + 56 \times 3$$

$$56\overline{)\,72\,}(1$$
$$\underline{-\,56}$$
$$16$$

$$16\overline{)\,56\,}(3$$
$$\underline{-\,48}$$
$$8$$

From (i) we get
$16 = 72 - 56 \times 1$

$$8\overline{)\,16\,}(2$$
$$\underline{-\,16}$$
$$0$$

**Q.** If d is the HCF of 56 & 72, find $x$, $y$ satisfying d = $56x + 72y$. Also, show that $x$ and $y$ are not unique.

**Sol.** Applying Euclid's division lemma to 56 to 72,

$$72 = 56 \times 1 + 16 \quad \ldots\ldots \text{(i)}$$

$\because$ Remainder is 16

So, we consider the divisor 56 and remainder 16 and apply division lemma.

$$56 = 16 \times 3 + 8 \quad \ldots\ldots \text{(ii)}$$

$\because$ Remainder is 8

So, we consider the divisor 16 and remainder 8 and apply division lemma.

$$16 = 8 \times 2 + 0$$

The remainder at this stage is 0.

$\therefore$ Last divisor 8 is HCF of 56 and 72

From (ii), we get

$$8 = 56 - 16 \times 3$$

$$8 = 56 - (72 - 56 \times 1) \times 3 \quad \ldots \text{from (i)}$$

$$8 = \boxed{56} - 72 \times 3 + \boxed{56 \times 3}$$

$$8 = 56 \times 4 - 72 \times 3$$

$$8 = \boxed{56 \times 4 + 72 \times (-3)}$$

Comparing with $56x + 72y$

$\therefore$ $x = 4$ & $y = -3$

Now,

$$8 = \boxed{56 \times 4} + \boxed{72 \times (-3)} + \boxed{56 \times 72} \boxed{- 56 \times 72}$$

$$8 = \boxed{56 \times 4 + 56 \times 72} \boxed{+ 72 \times (-3) - 56 \times 72}$$

$$8 = 56\boxed{(4 + 72)} + 72\boxed{(-3 - 56)}$$

$$8 = \boxed{56 \times 76 + 72 \times (-59)}$$

Comparing with $56x + 72y$

$\therefore$ $x = 76$ & $y = -59$

Hence, $x$ and $y$ are not unique