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QUADRATIC EQUATIONS

- Finding Value of k when roots are equal

Find the **values of k** for each of the following quadratic equations, so that they have two equal roots.

i) $2x^2 + kx + 3 = 0$

Sol: $2x^2 + kx + 3 = 0$

On comparing with

we get; $a = 2$, $b = k$

Since, roots of the quadratic equation are real and equal.

$$\therefore b^2 - 4ac = 0$$

$$\therefore (k)^2 - 4(2)(3) = 0$$

$$\therefore k^2 - 24 = 0$$

$$\therefore k^2 = 24$$

$$\therefore k = \pm\sqrt{4 \times 6}$$

$$\therefore \boxed{k = \pm 2\sqrt{6}}$$

So that we have two equal roots

Is it in a standard form
find the values of a, b & c
Yes

ii) $kx(x - 2) + 6 = 0$

Sol: $kx(x - 2) + 6 = 0$

$$kx^2 - 2kx + 6 = 0$$

On comparing with

we get; $a = k$, $b = -2k$

Since, roots of the quadratic equation are real and equal.

$$\therefore b^2 - 4ac = 0$$

$$\therefore (-2k)^2 - 4(k)(6) = 0$$

$$\therefore 4k^2 - 24k = 0$$

$$\therefore 4k(k - 6) = 0$$

$$\therefore 4k = 0 \quad \text{or} \quad k - 6 = 0$$

$$\therefore k = 0 \quad \text{or} \quad k = 6$$

But $k \neq 0$ ($\because a \neq 0$)

$$\therefore \boxed{k = 6}$$

So that we have two equal roots

find the values of a, b & c

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QUADRATIC EQUATIONS

- Finding Value of k when roots are equal

Find the value of k for which given equation has real and equal roots.

iii) $(k - 12)x^2 + 2(k - 12)x + 2 = 0$

Sol: The given quadratic equation is

$$(k - 12)x^2 + 2(k - 12)x + 2 = 0$$

Comparing with $ax^2 + bx + c = 0$

We have $a = k - 12$

\therefore The roots of given equation are real and equal.

$$\therefore b^2 - 4ac = 0$$

$$\therefore [2(k - 12)]^2 - 4(k - 12)(2) = 0$$

$$\therefore 4(k - 12)^2 - 8(k - 12) = 0$$

The given information is $b^2 - 4ac = 0$

So we need to find the values of a, b & c

For that the equation should be in a standard form

Is it in a standard form?

Yes

\therefore Dividing throughout by 4,

$$(k - 12)^2 - 2(k - 12) = 0$$

$$(k - 12)(k - 12 - 2) = 0$$

$$(k - 12)(k - 14) = 0$$

Now that in $ax^2 + bx + c = 0$
 $a \neq 0$

$$k - 12 = 0$$

$$k - 12 \neq 0$$

$$\therefore k - 14 = 0$$

$$\therefore k = 14$$

The value of k is 14.

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QUADRATIC EQUATIONS

- Finding Value of k when roots are equal

Find the value of k for which given equation has real and equal roots.

iv) $k^2x^2 - 2(k-1)x + 4 = 0$

Sol: The given quadratic equation is

$$k^2x^2 - 2(k-1)x + 4 = 0$$

Comparing with $ax^2 + bx + c = 0$

We have $a = k^2$, $b = -2(k-1)$, $c = 4$

\therefore The roots of given equation are real and equal.

$$\therefore b^2 - 4ac = 0$$

$$\therefore [-2(k-1)]^2 - 4(k^2)(4) = 0$$

$$\therefore 4(k-1)^2 - 16k^2 = 0$$

\therefore Dividing throughout by 4, we get

$$(k-1)^2 - 4k^2 = 0$$

$$\therefore (k-1)^2 - (2k)^2 = 0$$

$$\therefore (k-1+2k)(k-1-2k) = 0$$

$$\therefore 3k-1 = 0 \text{ or } -k-1 = 0$$

$$3k = 1 \text{ or } -k = 1$$

$$k = \frac{1}{3} \text{ or } k = -1$$

The given information is $b^2 - 4ac = 0$

Is it in a standard form?

So we need to find values of a, b & c

Yes

For that the equation should be in a standard form

or $k = -1$

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QUADRATIC EQUATIONS

- Finding Value of m when there is only one root

Q. Find m , if the quadratic equation $(m - 1)x^2 - 2(m - 1)x + 1 = 0$ has only one root.

Sol : $(m - 1)x^2 - 2(m - 1)x + 1 = 0$

On comparing with $ax^2 + bx + c = 0$

$a = m - 1$, $b = -2(m - 1)$, $c = 1$

Since the given quadratic equation has only one root

$$\therefore b^2 - 4ac = 0$$

$$\therefore [-2(m - 1)]^2 - 4(m - 1)(1) = 0$$

$$\therefore 4(m - 1)^2 - 4(m - 1) = 0$$

Dividing throughout by 4, we get

$$\therefore (m - 1)^2 - (m - 1) = 0$$

$$\therefore (m - 1)^2 - m + 1 = 0$$

$$\therefore m^2 - 2m + 1 - m + 1 = 0$$

$$\therefore m^2 - 3m + 2 = 0$$

Means the roots are real

General Quadratic Equation:-
 $(a-b)^2 = a^2 - 2ab + b^2$
 where a, b and c are real numbers and $a \neq 0$

Now the co-efficient of x^2 must not be 0

For the given information
 So $(m-1) = (1-1) = 0$ to find the values of a, b & c

For that the equation should be in a standard form

of m is 2

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QUADRATIC EQUATIONS

- **Proof based on concept of equal roots**

If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$.

Sol. $(b - c)x^2 + (c - a)x + (a - b) = 0$
 On comparing with $Ax^2 + Bx + C = 0$, we get
 $A = (b - c)$, $B = (c - a)$, $C = (a - b)$

Since, given equation has equal roots

$$\therefore B^2 - 4AC = 0$$

$$\therefore (c - a)^2 - 4(b - c)(a - b) = 0$$

$$\therefore c^2 - 2ac + a^2 - 4(ab - b^2 - ac + bc) = 0$$

$$\therefore c^2 - 2ac + a^2 - 4ab + 4b^2 + 4ac - 4bc = 0$$

$$\therefore c^2 + 2ac + a^2 - 4ab + 4b^2 - 4bc = 0$$

$$\therefore a^2 + 2ac + c^2 - 4b(a + c) = 0$$

$$\therefore (a + c)^2 + (2b)^2 - 4b(a + c) = 0$$

$$\therefore [(a + c) - (2b)]^2 = 0$$

Taking square roots on both sides

$$\therefore (a + c) - (2b) = 0$$

$$\therefore a + c = 2b$$

Hence proved

Means

We know

$$a^2 + 2ac + c^2 = (a + c)^2$$

$$\text{in the } 4b^2 = (2b)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2 \quad \text{ie terms}$$

$$x = (a + c) \quad \text{form using}$$

$$y = 2b \quad \text{abets A, B, C}$$

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QUADRATIC EQUATIONS

- **Word Problem based on Mango Grove**

1. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800m^2 ? If so, find its length and breadth.

Sol. Let the breadth of mango grove = x m

\therefore Length of mango grove = $2x$ m

\therefore According to the given condition,

Area of rectangular mango grove = 800 m^2

$$(2x)(x) = 800$$

$$\therefore 2x^2 = 800$$

$$\therefore x^2 = 400$$

$$\therefore x = \pm 20$$

$$\therefore x = 20 \text{ or } x = -20$$

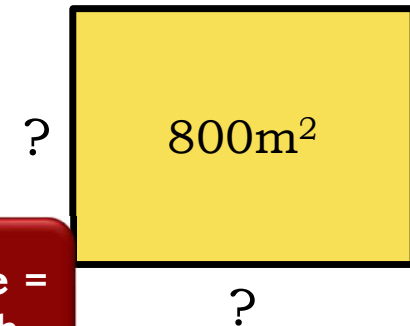
But breadth cannot be negative,

So neglecting $x = -20$, we get $\therefore x = 20$

$$\text{and } 2x = 2 \times 20 = 40$$

Yes it is possible to design a rectangular mango grove whose length is twice its breadth and the area is 800m^2

Breadth of mango grove is 20 m and length of mango grove is 40m.



**Area of a rectangle =
Length \times Breadth**

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QUADRATIC EQUATIONS

- **Word problem based on Age**

2. Is the following situation possible ? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Sol. Let first friend's age = x years

\therefore Second friend's age = $(20 - x)$ years

\therefore Four years ago,

First friend's age = $(x - 4)$ years

Second friend's age = $(20 - x - 4)$ years
= $(16 - x)$ years

According to the given condition,

$$(x - 4)(16 - x) = 48$$

$$\therefore 16x - x^2 - 64 + 4x = 48$$

$$\therefore -x^2 + 20x - 64 - 48 = 0$$

$$\therefore -x^2 + 20x - 112 = 0$$

$$\therefore 1x^2 - 20x + 112 = 0$$

On comp Calculation $bx + c = 0$,

1st F

We don't get two factors in such a way that by adding we get middle number 20

$$\therefore b^2 -$$

$$\therefore \text{The}$$

Let us do the prime factorization of 112

Since it is not possible to factorise the equation. Let us use the formula method

Thank You