Number Systems

1. Numbers 1, 2, 3…….∞, which are used for counting are called **natural numbers**. The collection of natural numbers is denoted by **N.** Therefore, N = {1, 2, 3, 4, 5, ...}.

2. When 0 is included with the natural numbers, then the new collection of numbers called is called **whole number**. The collection of whole numbers is denoted by **W**. Therefore, W = {0, 1, 2, 3, 4, 5, ...}.

3. The negative of natural numbers, 0 and the natural number together constitutes **integers**. The collection of integers is denoted by **I**. Therefore, I = {…, -3, -2, -1, 0, 1, 2, 3, …}.

4. The numbers which can be represented in the form of p/q, where $q \neq 0$ and p and q are integers are called **rational numbers**. Rational numbers are denoted by ***Q*.** If p and q are co-prime then the rational number is in its simplest form.

5. All-natural numbers, whole numbers and integer are rational number.

6. **Equivalent rational numbers** (or fractions) have same (equal) values when written in the simplest form.

7. **Rational number between two numbers** x and $y = \frac{x+y}{2}$**.**

8. There are infinitely many rational numbers between any two given rational numbers.

9. The numbers which are not of the form of $p/\ q$, where $q \neq 0$ and p and q are integers are called **irrational numbers**. For example: $\sqrt{2}, \sqrt{7}, \pi$, etc.

10. Rational and irrational numbers together constitute are called **real numbers**. The collection of real numbers is denoted by ***R*.**

11. **Irrational number between two numbers** x and y

$$= \begin{cases} \sqrt{xy}\text{, if } x \text{ and } y \text{ both are irrational numbers} \\ \sqrt{xy}\text{, if } x \text{ is rational number and } y \text{ is irrational number} \\ \sqrt{xy}\text{, if } x \times y \text{ is not a perfect square and } x,\ y \text{ both are rational numbers} \end{cases}$$

12. **Terminating fractions** are the fractions which leaves remainder 0 on division.

13. **Recurring fractions** are the fractions which never leave a remainder 0 on division.

14. The decimal expansion of **rational** number is **either terminating or non-terminating recurring**. Also, a number whose decimal expansion is terminating or non-terminating recurring is rational.

15. The decimal expansion of an **irrational** number is **non-terminating non-recurring**. Also, a number whose decimal expansion is non-terminating non-recurring is irrational.

16. Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.

17. The process of visualization of numbers on the number line through a magnifying glass is known as the process of **successive magnification.** This technique is used to represent a real number with non-terminating recurring decimal expansion.

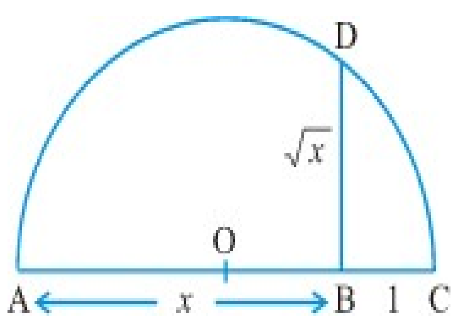
18. Irrational numbers like $\sqrt{2}, \sqrt{3}, \sqrt{5} \ldots \sqrt{n}$, for any positive integer *n* can be represented on number line by using Pythagoras theorem.

19. If a > 0 is a real number, then $\sqrt{a}$ = b means b^2 = a and b > 0.

20. For any positive real number x, we have:

$$x = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2$$

21. For every positive real number x, $\sqrt{x}$ can be represented by a point on the number line using the following steps:
    i. Obtain the positive real number, say x.
    ii. Draw a line and mark a point A on it.
    iii. Mark a point B on the line such that AB = x units.
    iv. From B, mark a distance of 1 unit on extended AB and name the new point as C.
    v. Find the mid-point of AC and name that point as O.
    vi. Draw a semi-circle with centre O and radius OC.
    vii. Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D.
    viii. Length BD is equal to $\sqrt{x}$.



22. **Properties of irrational numbers**:
    i. The sum, difference, product and quotient of two irrational numbers need not always be an irrational number.
    ii. Negative of an irrational number is an irrational number.
    iii. Sum of a rational and an irrational number is irrational.
    iv. Product and quotient of a non-zero rational and irrational number is always irrational.

23. Let a > 0 be a real number and n be a positive integer. Then $\sqrt[n]{a}$ = b, if b^n = a and b > 0.

The symbol '$\sqrt{\ }$' is called the **radical sign**.

24. For real numbers a > 0 and b > 0:
    i. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
    ii. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
    iii. $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$
    iv. $(\sqrt{a} + \sqrt{b})(\sqrt{c} - \sqrt{d}) = \sqrt{ac} + \sqrt{bc} - \sqrt{ad} - \sqrt{bd}$
    v. $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$
    vi. $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$

25. The process of removing the radical sign from the denominator of an expression to convert it to an equivalent expression whose denominator is a rational number is called **rationalising the denominator**.

26. The multiplicating factor used for rationalising the denominator is called the **rationalising factor**.

27. If a and b are positive real numbers, then
    i. Rationalising factor of $\frac{1}{\sqrt{a}}$ is $\sqrt{a}$
    ii. Rationalising factor of $\frac{1}{a \pm \sqrt{b}}$ is $a \mp \sqrt{b}$
    iii. Rationalising factor of $\frac{1}{\sqrt{a} \pm \sqrt{b}}$ is $\sqrt{a} \mp \sqrt{b}$

28. The **exponent** is the number of times the base is multiplied by itself.

29. In the exponential representation a^m, a is called the **base** and m is called the **exponent or power**.

30. **Laws of exponents**: If a, b are positive real numbers and m, n are rational numbers, then
    i. $a^m \times a^n = a^{m+n}$
    ii. $a^m \div a^n = a^{m-n}$
    iii. $\left(a^m\right)^n = a^{mn}$
    iv. $a^{-n} = \frac{1}{a^n}$
    v. $(ab)^n = a^n b^n$
    vi. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
    vii. $a^{m/n} = \left(a^m\right)^{1/n} = \left(a^{1/n}\right)^m$ or $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
    viii. $a^0 = 1$