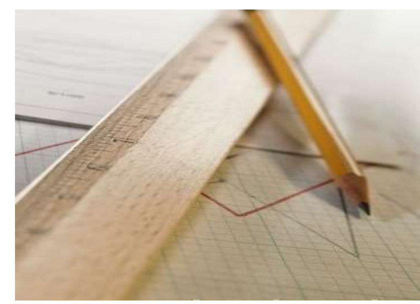
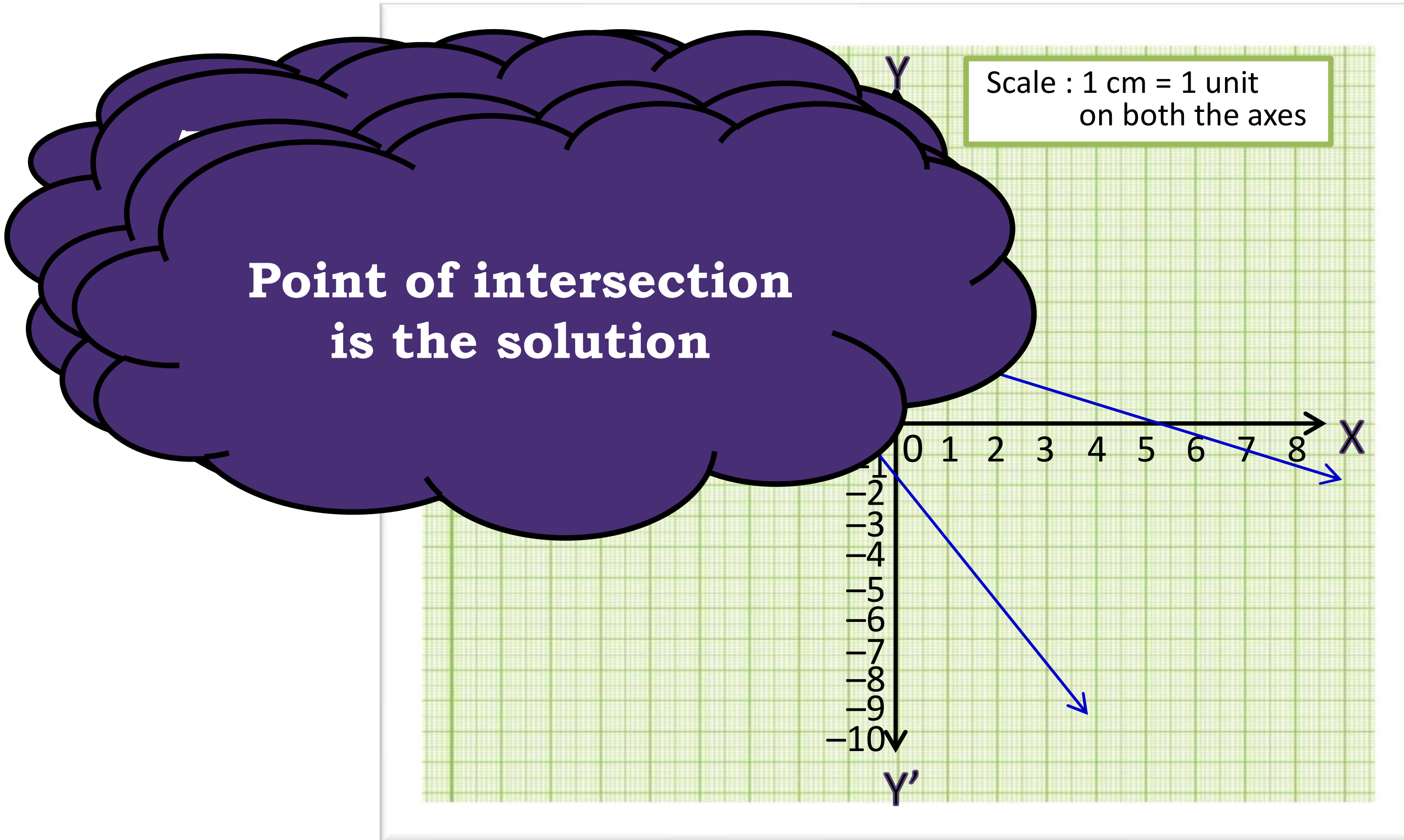
No. 25

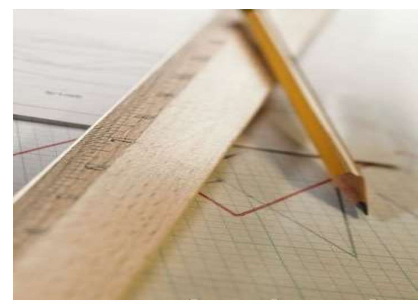
Graph Concept




Point of intersection
is the solution
Scale : 1 cm = 1 unit
on both the axes
Y
X
Y'
0 1 2 3 4 5 6 7 8
−2
−3
−4
−5
−6
−7
−8
−9
−10

No. 26

Graphical Method



(iv) **x + 2y = 6; 2x + y = 6**

Sol : x + 2y = 6

x	0	6
y	3	0
(x, y)	(0, 3)	(6, 0)

2x + y = 6

x	0	3
y	6	0
(x, y)	(0,6)	(3, 0)

When x = 0

When x = 0

2y = 6

2(0) + y = 6

$y = \frac{\cancel{6}^{3}}{\cancel{2}}$

y = 6

y = 3

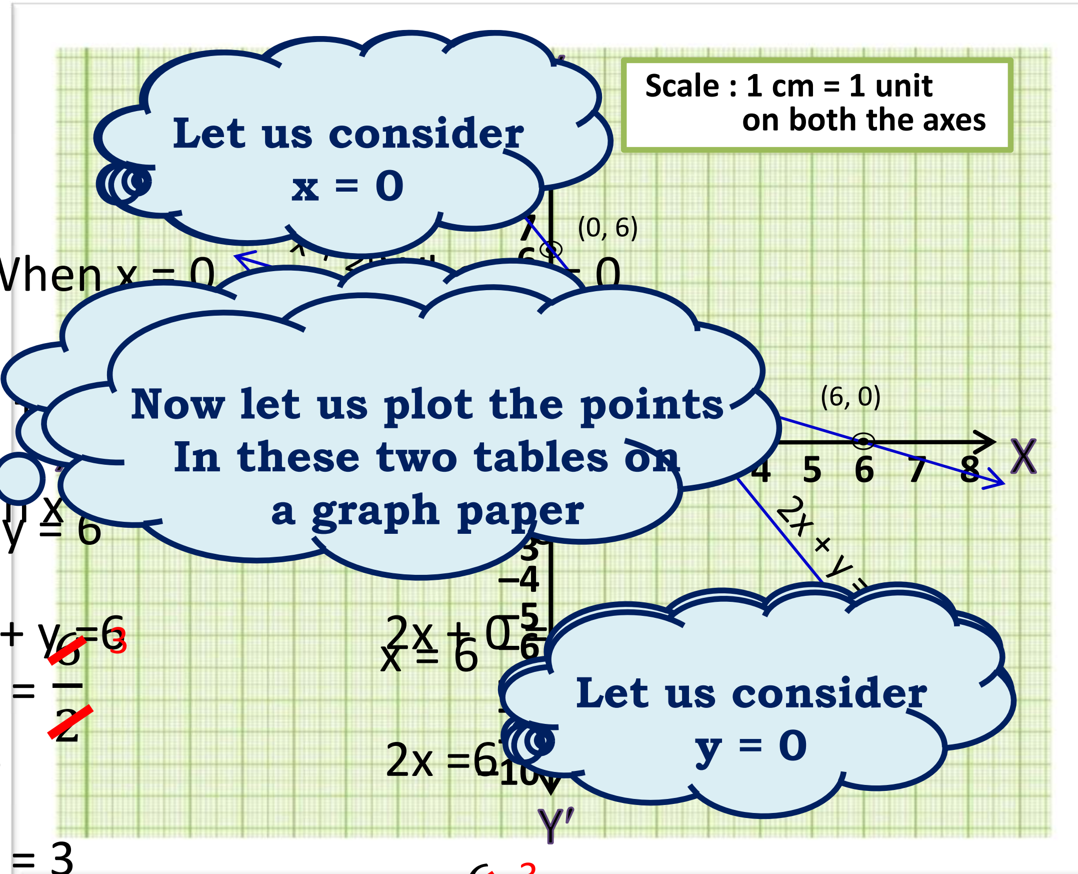
2x + 0 = 6

x = 6

2x = 6

$x = \frac{\cancel{6}^{3}}{\cancel{2}}$

x = 3



∴ x = 2and y = 2 is the solution of the given simultaneous equations.

No. 27

Q. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x – axis, and shade the triangular region.

Sol : $x - y + 1 = 0$

x	0	-1
y	1	0
(x, y)	(0, 1)	(-1, 0)

When x = 0

$0 -$

$3x + 2y - 12 = 0$

x	0	4
y	6	0
(x, y)	(0, 6)	(4, 0)

When x = 0

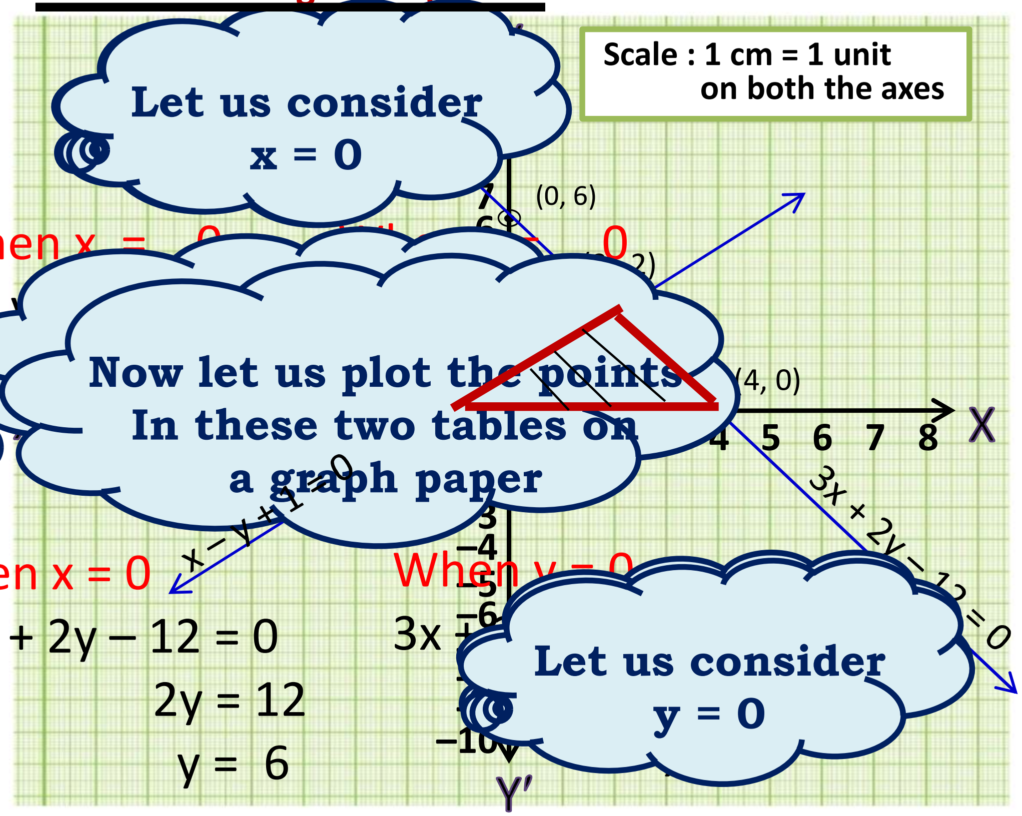
$3(0) + 2y - 12 = 0$

$2y = 12$

$y = 6$

When y = 0

$3x +$



$\therefore$ (3, 2), (4, 0) and (-1, 0) are the coordinates of the vertices of the triangle formed.

No. 28

Consistency
Inconsistency

UNIQUE SOLUTION
INTERSECTING LINES

$x + y + 3 = 0$ ..... (i)

$x - y + 1 = 0$ ..... (ii)

$x = 2$

$y = 1$

Consistent

From (i) , we get:

$a_1 = 1, \quad b_1 = 1, \quad c_1 = 3$

From (ii) , we get:

$a_2 = 1, \quad b_2 = -1, \quad c_2 = 1$

$$\frac{a_1}{a_2} = \frac{1}{1} \qquad \frac{b_1}{b_2} = \frac{1}{-1}$$

$$\boxed{\frac{a_1}{a_2} \neq \frac{b_1}{b_2}}$$

NO SOLUTION
PARALLEL LINES

$2x - y - 1 = 0$ ..... (i)

$2x - y - 4 = 0$ ..... (ii)

Su

$x =$

Which

From

$a_1 = 2,$

From (ii) ,

$a_2 = 2, \quad b_2$

$$\frac{a_1}{a_2} = 1 \qquad \frac{b_1}{b_2} = 1$$

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$$

INFINITE SOLUTIONS
OVERLAPPING LINES

$x - y - 2 = 0$ ..... (i)

$2x - 2y - 4 = 0$ ..... (ii)

sistent

e get:

$= -1, \quad c_1 = -2$

$c_2 = -4$

$c_1 \quad = \frac{1}{2}$

From (ii) get the values of a_2, b_2, c_2

Condition	Graphical Presentation	Algebraic Interpretation	Consistent or Inconsistent
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	**Intersecting Lines**	**Unique Solution**	**Consistent**
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	**Parallel Lines**	**No Solution**	**Inconsistent**
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	**Overlapping Lines**	**Infinite solutions**	**Consistent**

No. 29

Q. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident :

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Soln. $5x - 4y + 8 = 0$ … (i)

$7x + 6y - 9 = 0$ … (ii)

Comparing equation (i) with $a_1x + b_1y + c_1 = 0$

and equation (ii) with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 5$ $\quad b_1 = -4$ $\quad c_1 = 8$

$a_2 = 7$ $\quad b_2 = 6$ $\quad c_2 = -9$

$$\frac{a_1}{a_2} = \frac{5}{7} \quad \text{… (iii)}$$

$$\frac{b_1}{b_2} = \frac{-4}{6} \quad \text{… (iv)}$$

$$\frac{c_1}{c_2} = \frac{8}{-9} \quad \text{… (v)}$$

What we need to find

from (iii), (iv) and (v)

…s the …has to be in standard form

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Equations has unique solution (Consistent)
Intersecting lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Equations has no solution(Inconsistent)
Parallel lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Equations has infinite solutions (Consistent)
Coincident line

∴ The lines intersect each other.

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Soln. $9x + 3y + 12 = 0$ ... (i)

$18x + 6y + 24 = 0$ ... (ii)

Comparing equation (i) with $a_1x + b_1y + c_1 = 0$

and equation (ii) with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 9$ $\quad b_1 = 3$ $\quad c_1 = 12$

$a_2 = 18$ $\quad b_2 = 6$ $\quad c_2 = 24$

$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2} \quad \text{... (iii)}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2} \quad \text{... (iv)}$$

$$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2} \quad \text{... (v)}$$

from (iii), (iv) and (v)

$\frac{c_1}{c_2}$

...ident.

To get this the equations has to be in standard form

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Equations has unique solution (Consistent) Intersecting lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Equations has no solution(Inconsisten) Parallel lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Equations has infinite solutions (Consistent) Coincident line

(iii) $6x - 3y + 10 = 0$

$2x - y - 9 = 0$

Soln. $6x - 3y + 10 = 0$ ... (i)

$2x - 1y - 9 = 0$ ... (ii)

Comparing equation (i) with $a_1x + b_1y + c_1 = 0$

and equation (ii) with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 6$ $b_1 = -3$ $c_1 = 10$

$a_2 = 2$ $b_2 = -1$ $c_2 = 9$

$$\frac{a_1}{a_2} = \frac{6}{2} = 3 \quad \text{... (iii)}$$

$$\frac{b_1}{b_2} = \frac{-3}{-1} = 3 \quad \text{... (iv)}$$

$$\frac{c_1}{c_2} = \frac{10}{9} \quad \text{... (v)}$$

from (iii), (iv) and (v)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

The lines are parallel to each other.

To get this the equations has to be in standard form

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Equations has unique solution (Consistent) **Intersecting lines**

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Equations has no solution (Inconsistent) **Parallel lines**

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Equations has infinite solutions (Consistent) **Coincident line**

No. 30

Q. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are Consistent, or inconsistent:

What we need to find

To get this the equations has to be in standard form

(i) $3x + 2y = 5$

$2x - 3y = 7$

Soln. $3x + 2y - 5 = 0$ ... (i)

$2x - 3y - 7 = 7$ ... (ii)

Comparing equation (i) with $a_1x + b_1y + c_1 = 0$

and equation (ii) with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 3$ $b_1 = 2$ $c_1 = -5$

$a_2 = 2$ $b_2 = -3$ $c_2 = -7$

$$\frac{a_1}{a_2} = \frac{3}{2} \quad \text{... (iii)}$$

$$\frac{b_1}{b_2} = \frac{2}{-3} = \frac{-2}{3} \quad \text{... (iv)}$$

$$\frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7} \quad \text{... (v)}$$

$\therefore$ The ... ting and has a com... solution.

The equati...s are consistent.

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Equations has unique solution (Consistent) Intersecting lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Equations has no solution (Inconsistent) Parallel lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Equations has infinite solutions (Consistent) Coincident line

(ii) $2x - 3y = 8$

$4x + y = 9$

Soln. $2x - 3y - 8 = 0$ … (i)

$4x - 6y - 9 = 0$ … (ii)

Comparing equation (i) with $a_1x + b_1y + c_1 = 0$

and equation (ii) with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 2$ $\quad b_1 = -3$ $\quad c_1 = -8$

$a_2 = 4$ $\quad b_2 = -6$ $\quad c_2 = -9$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2} \quad \text{… (iii)}$$

$$\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2} \quad \text{… (iv)}$$

$$\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9} \quad \text{… (v)}$$

from (iii), (iv) and (v)

To get this the equations has to be in standard form

∴ T… other … mon solution.

The linear equ…ns are inconsistent.

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Equations has unique solution (Consistent)

Intersecting lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Equations has no solution(Inconsistent)

Parallel lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Equations has infinite solutions (Consistent)

Coincident line

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$

Soln. $\therefore \frac{3}{2}x + \frac{5}{3}y - 7 = 0$ ...(i) $\therefore 9x - 10y - 14 = 0$ ...(ii)

To get this the equations has to be in standard form

Comparing equation (i) with $a_1x + b_1y + c_1 = 0$

and equation (ii) with $a_2x + b_2y + c_2 = 0$

We get, $a_1 = \frac{3}{2}$ $\quad b_1 = \frac{5}{3}$ $\quad c_1 = -7$

$a_2 = 9$ $\quad b_2 = -10$ $\quad c_2 = -14$

$$\therefore \frac{a_1}{a_2} = \frac{3/2}{9} = \frac{3}{18} = \frac{1}{6} \quad \text{... (iii)}$$

$$\therefore \frac{b_1}{b_2} = \frac{5/3}{-10} = \frac{5}{-30} = \frac{-1}{6} \quad \text{... (iv)}$$

$$\therefore \frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2} \quad \text{... (v)}$$

$\therefore$ **From (iii),(iv) and (v)**

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\therefore$ **The two lines intersect each other and her a common solution**

$\therefore$ **The pair of linear equations are consistent.**

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Equations has unique solution (Consistent) **Intersecting lines**

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Equations has no solution (Inconsistent) **Parallel lines**

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Equations has infinite solutions (Consistent) **Coincident line**

No. 31

Q. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations are Consistent, or inconsistent:

$5x - 3y = 11$
$-10x + 6y = -22$

Soln. $5x - 3y - 11 = 0$ ... (i)

$-10x + 6y + 22 = 0$ ... (ii)

Comparing equation (i) with $a_1x + b_1y + c_1 = 0$

and equation (ii) with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 5$ $\quad b_1 = -3$ $\quad c_1 = -11$

$a_2 = -10$ $\quad b_2 = 6$ $\quad c_2 = 22$

$$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2} \quad \text{... (iii)}$$

$$\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2} \quad \text{... (iv)}$$

$$\frac{c_1}{c_2} = \frac{-11}{22} = \frac{-1}{2} \quad \text{... (v)}$$

To get this, the equations has to be in standard form

fr

a_2

$\therefore$ **The two lines coincide with each other and have infinite solutions.**

The pair of linear equations are consistent.

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Equations has unique solution (Consistent) **Intersecting lines**
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Equations has no solution (Inconsistent) **Parallel lines**
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Equations has infinite solutions (Consistent) **Coincident line**

Q. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations are Consistent, or inconsistent:

$$\frac{4}{3}x + 2y = 8$$
$$2x + 3y = 12$$

Soln.

$$\frac{4}{3}x + 2y - 8 = 0 \quad \dots (i)$$
$$2x + 3y - 12 = 0 \quad \dots (ii)$$

Comparing equation (i) with $a_1x + b_1y + c_1 = 0$

and equation (ii) with $a_2x + b_2y + c_2 = 0$

We get $a_1 = \frac{4}{3}$ $\quad b_1 = 2$ $\quad c_1 = -8$

$a_2 = 2$ $\quad b_2 = 3$ $\quad c_2 = -12$

$$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{4}{6} = \frac{2}{3} \quad \dots (iii)$$

$$\frac{b_1}{b_2} = \frac{2}{3} \quad \dots (iv)$$

$$\frac{c_1}{c_2} = \frac{-8}{-12} = \frac{8}{12} = \frac{2}{3} \quad \dots (v)$$

To get this the equations has to be in standard form $\frac{c_1}{c_2}$

$\therefore$ ... each othe...

The pair of linear equations are consistent.

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Equations has unique solution (Consistent)
Intersecting lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Equations has no solution(Inconsistent)
Parallel lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Equations has infinite solutions(Consistent)
Coincident line

No. 32

Q. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations are Consistent, or inconsistent:

$x - y = 8$

$3x - 3y = 16$

Soln. $1x - 1y - 8 = 0$ ... (i)

$3x - 3y - 16 = 0$ ... (ii)

Comparing equation (i) with $a_1x + b_1y + c_1 = 0$

and equation (ii) with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 1$ $b_1 = -1$ $c_1 = -8$

$a_2 = 3$ $b_2 = -3$ $c_2 = -16$

$$\frac{a_1}{a_2} = \frac{1}{3} \quad \text{... (iii)}$$

$$\frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3} \quad \text{... (iv)}$$

$$\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2} \quad \text{... (v)}$$

To get this the equations has to be in standard form

fr

$\frac{\quad}{a_2} \neq \frac{c_1}{c_2}$

$\therefore$ **The lines are parallel to each other.**
The pair of linear equations are inconsistent.

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Equations has unique solution (Consistent) **Intersecting lines**
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Equations has no solution(Inconsistent) **Parallel lines**
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Equations has infinite solutions(Consistent) **Coincident line**

Q. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations are Consistent, or inconsistent:

$2x - 2y - 2 = 0$
$4x - 4y - 5 = 0$

To get this the equations has to be in standard form

Soln. $2x - 2y - 2 = 0$ … (i)

$4x - 4y - 5 = 0$ … (ii)

Comparing equation (i) with $a_1x + b_1y + c_1 = 0$

and equation (ii) with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 2$ $\quad b_1 = -2$ $\quad c_1 = -2$

$a_2 = 4$ $\quad b_2 = -4$ $\quad c_2 = -5$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2} \quad \text{… (iii)}$$

$$\frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2} \quad \text{… (iv)}$$

$$\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5} \quad \text{… (v)}$$

The pa… equations are inconsistent.

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Equations has unique solution (Consistent) Intersecting lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Equations has no solution (Inconsistent) Parallel lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Equations has infinite solutions (Consistent) Coincident line

No. 33

Q. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

$2x + y - 6 = 0$

$4x + 2y - 4 = 0$

Soln. $2x + 1y - 6 = 0$ ... (i)

$4x - 2y - 4 = 0$ ... (ii)

Comparing equation (i) with $a_1x + b_1y + c_1 = 0$

and equation (ii) with $a_2x + b_2y + c_2 = 0$

$a_1 = 2$ $\quad b_1 = 1$ $\quad c_1 = -6$

$a_2 = 4$ $\quad b_2 = -2$ $\quad c_2 = -4$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2} \quad \text{... (iii)}$$

$$\frac{b_1}{b_2} = \frac{1}{-2} \quad \text{... (iv)}$$

$$\frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2} \quad \text{... (v)}$$

From (iii), (iv) and (v)

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore$ The lines are consistent

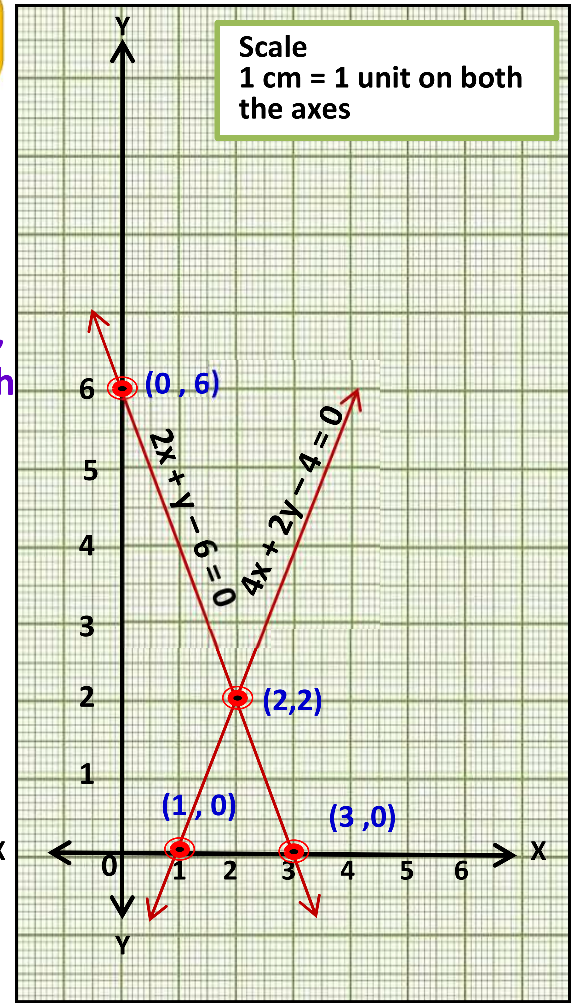
Now to represent graphically, we find two solutions of each equation

$2x + y - 6 = 0$

x	0	3
y	6	0
(x, y)	(0, 6)	(3, 0)

$4x - 2y - 4 = 0$

x	2	1
y	2	0
(x, y)	(2, 2)	(1, 0)



$\therefore$ $\boxed{\text{(2, 2) is the common solution for both the equation.}}$

Thank You