LECTURE_09

MODULE_30

Exercise 1.3

Q.3 Prove that the following are irrationals.

(ii) $7\sqrt{5}$

Sol : Let us assume that $7\sqrt{5}$ is rational.

$\therefore$ There exist co-prime integers a and b ($b \neq 0$) such that,

Lets prove this by contradiction

$$7\sqrt{5} = \frac{a}{b}$$

$$\therefore \sqrt{5} = \boxed{\frac{a}{7b}}$$

Integer
Integer

It should be in form of a/b

Since a and b are integers, it implies that 7b is also integer

$\therefore$ $\frac{a}{7b}$ is rational it implies that $\sqrt{5}$ is also rational,

But this contradicts the fact that $\sqrt{5}$ is irrational.

$\therefore$ Our assumption that $7\sqrt{5}$ is rational is wrong.

$\therefore$ $7\sqrt{5}$ is irrational.

Exercise 1.3

Q.3 Prove that the following are irrationals.

(iii) $6 + \sqrt{2}$

Sol : Let us assume that $6 + \sqrt{2}$ is rational.

$\therefore$ There exist co-prime integers a and b ($b \neq 0$) such that,

$$6 + \sqrt{2} = \frac{a}{b}$$

Lets prove this by contradiction

$$\therefore \quad \sqrt{2} = \frac{a}{b} - 6$$

It should be in form of a/b

$$\therefore \quad \sqrt{2} = \boxed{\frac{a - 6b}{b}}$$

$\frac{\text{Integer}}{\text{Integer}}$

Since a and b are integers, it implies that a – 6b is also integer

$\therefore$ $\frac{a - 6b}{b}$ is rational which implies that $\sqrt{2}$ is also rational,

But this contradicts the fact that $\sqrt{2}$ is irrational.

$\therefore$ Our assumption that $6 + \sqrt{2}$ is rational is wrong.

$\therefore$ **$6 + \sqrt{2}$ is irrational.**

MODULE_31

Q. Prove that $\sqrt{2} + \sqrt{5}$ is an irrational number.

Sol. Let us assume that $\sqrt{2} + \sqrt{5}$ is a rational number.

So, there exist co-prime integers a and b such that

$$\sqrt{2} + \sqrt{5} = \frac{a}{b} \quad (\text{where } b \neq 0)$$

Squaring both sides, we get

$$\therefore \quad [\sqrt{2} + \sqrt{5}]^2 = \left[\frac{a}{b}\right]^2$$

$$\therefore \quad \underline{2 + 5} + 2\sqrt{10} = \frac{a^2}{b^2}$$

$$\therefore \quad 7 + 2\sqrt{10} = \frac{a^2}{b^2}$$

$$\therefore \quad 2\sqrt{10} = \frac{a^2}{b^2} - \frac{7}{1}$$

$$\therefore \quad 2\sqrt{10} = \frac{a^2 - 7b^2}{b^2}$$

$$\therefore \quad \sqrt{10} = \frac{a^2 - 7b^2}{2b^2}$$

We know, $(a + b)^2 = (a^2 + b^2 + 2ab)$

Q. Prove that $\sqrt{2} + \sqrt{5}$ is an irrational number.

Sol. Let us assume that $\sqrt{2} + \sqrt{5}$ is a rational number.

So, there exist co-prime integers a and b such that

$$\sqrt{2} + \sqrt{5} = \frac{a}{b} \qquad (\text{where } b \neq 0)$$

$$\therefore \qquad \sqrt{10} = \frac{a^2 - 7b^2}{2b^2}$$

Here, $\dfrac{a^2 - 7b^2}{2b^2}$ is an rational number

This implies, $\sqrt{10}$ is also rational number.

But, we know that $\sqrt{10}$ is an irrational number

Therefore, there is a contradiction and our assumption is wrong

$\therefore$ $\sqrt{2} + \sqrt{5}$ is not a rational number.

$\therefore$ $\sqrt{2} + \sqrt{5}$ is an irrational number.

Hence Proved.

MODULE_32

Q. Prove that $\sqrt[3]{5}$ is an irrational number.

Sol. Let us assume $\sqrt[3]{5}$ is a rational number

So, there exist co-prime integers a and b where $b \neq 0$

such that $\sqrt[3]{5} = \dfrac{a}{b}$

Cubing on both sides, we get

$$\therefore \quad \boxed{[\sqrt[3]{5}\,]^3} = \left[\frac{a}{b}\right]^3$$

$$5 = \frac{a^3}{b^3} \Rightarrow \boxed{b^3 = \frac{a^3}{5}} \quad \ldots (1)$$

$\therefore$ 5 divides $a^3 \Rightarrow$ 5 divides a

$\therefore$ 5 is a factor of a ... (2)

Let $a = 5c$

$$\therefore \quad b^3 = \frac{(5c)^3}{5} \quad \ldots\text{from (1)}$$

$$\therefore \quad b^3 = \frac{\cancel{5} \times 5 \times 5c^3}{\cancel{5}}$$

$$\therefore \quad \frac{b^3}{5} = 5c^3$$

$\therefore$ 5 divides $b^3 \Rightarrow$ 5 divides 5.

$\therefore$ 5 is a factor of b ... (3)

From (2) and (3)

5 is a common factor of a and b

$\therefore$ a and b are not co-prime

$\therefore$ our assumption is wrong

$\therefore$ $\sqrt[3]{5}$ is an irrational number.