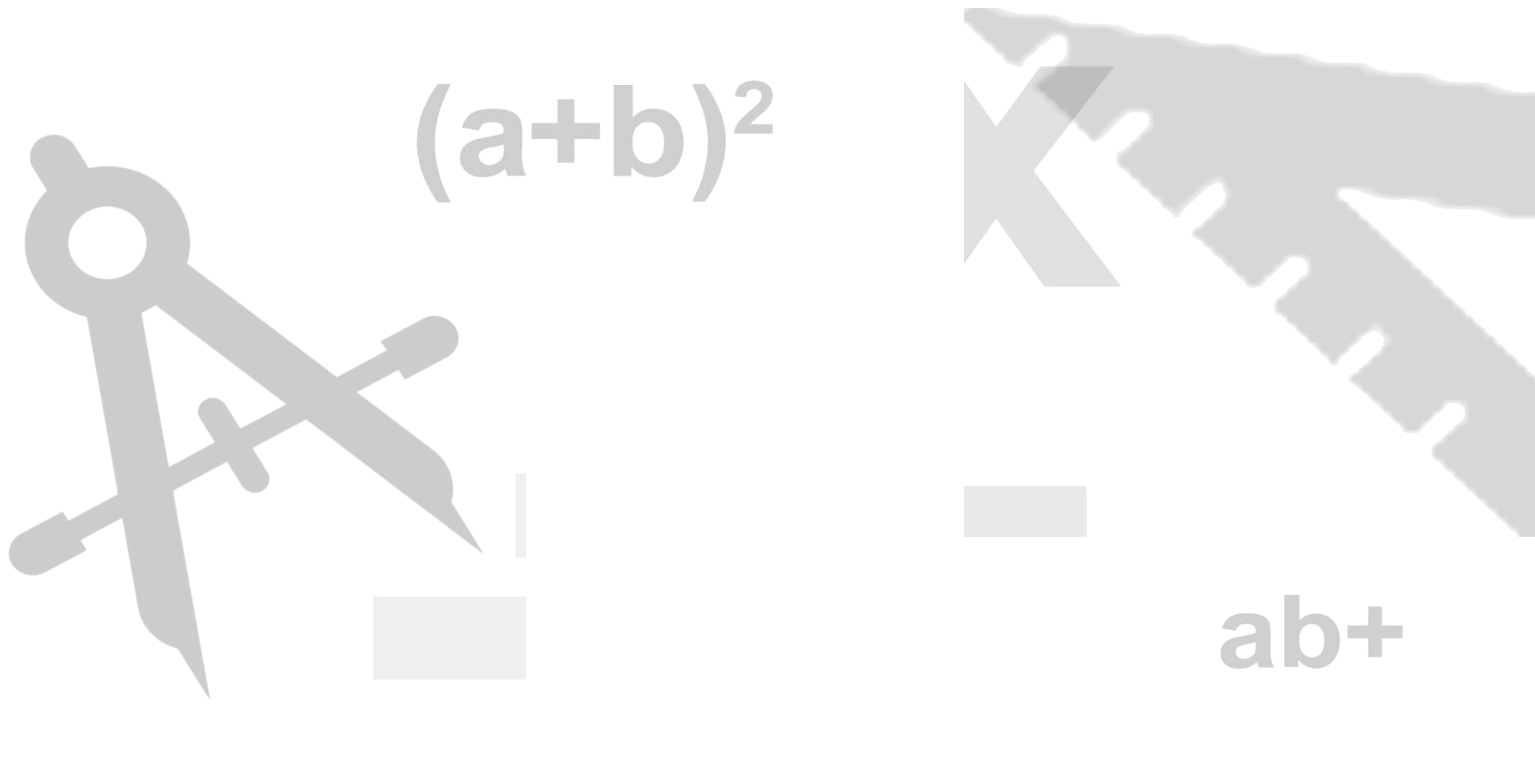
MATHS



Constructions

1. **To divide a line segment internally in a given ratio m: n**, where both m and n are positive integers, we follow the steps given below:

   Step 1: Draw a line segment AB of given length by using a ruler.

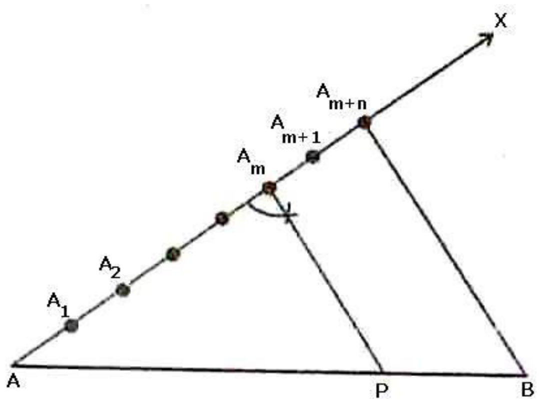
   Step 2: Draw any ray AX making an acute angle with AB.

   Step 3: Along AX mark off $(m + n)$ points A_1, A_2,………A_{m-1}, A_{m+1},………,A_{m+n}, such that $AA_1 = A_1A_2 = A_{m+n-1}A_{m+n}$.

   Step 4: Join BA_{m+n}

   Step 5: Through the point A_m, draw a line parallel to $A_{m+n}B$ by making an angle equal to $\angle AA_{m+n}B$ at A_m, intersecting AB at point P.

   The point P so obtained is the required point which divides AB internally in the ratio $m: n$.



Justification

In ΔABA_{m+n}, we observe that A_mP is parallel to $A_{m+n}B$. Therefore, by Basic Proportionality theorem, we have:

$$\frac{AA_m}{A_mA_{m+n}} = \frac{AP}{PB}$$

$$\Rightarrow \frac{AP}{PB} = \frac{m}{n} \qquad \left[\because \frac{AA_m}{A_mA_{m+n}} = \frac{m}{n}, \text{ by construction}\right]$$

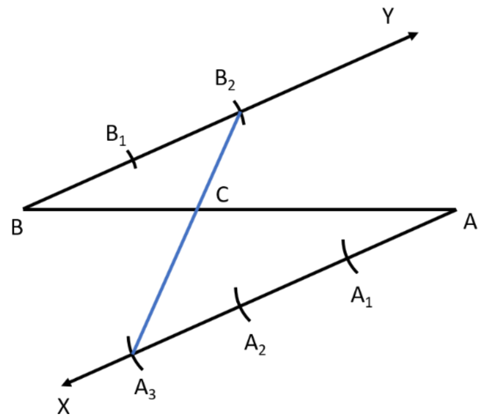
$$\Rightarrow AP : PB = m : n$$

Hence, P divides AB in the ratio $m: n$.

2. Alternative method to divide a line segment internally in a given ratio m: n

Example

Find the point C such that it divides BA in ratio 2:3



Steps of Construction :

1. Draw any ray XA making an acute angle with BA.
2. Draw a ray YB parallel to XA by making ∠YBA equal to ∠XAB.
3. Locate the points A_1, A_2, A_3 (m = 3) on AX and B_1, B_2 (n = 2) on BY such that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2$.
4. Join A_3B_2. Let it intersect AB at a point C

   Then BC : CA = 2:3

Justification

Here $\Delta BB_2C \sim \Delta AA_3C$ …AA test

$$\frac{BB_2}{AA_3} = \frac{BC}{AC} \text{...(c.p.s. t .)}$$

$$\frac{2}{3} = \frac{BC}{AC}$$

3. The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as a **scale factor**. The scale factor may be less or greater than 1.
4. If the scale factor is less than 1, then the new figure will be smaller in comparison to the given figure.
5. If the scale factor is greater than 1, then the new figure will be bigger in comparison to the given figure.

Construction of Triangle Similar to given Triangle

Consider a triangle *ABC*. Let us construct a triangle similar to ΔABC such that each of its sides is $\left(\frac{m}{n}\right)^{th}$ of the corresponding sides of ΔABC.

Steps of constructions when $m < n$:

Step 1: Construct the given triangle *ABC* by using the given data.

Step 2: Take any one of the three side of the given triangle as base. Let *AB* be the base of the given triangle.

Step 3: At one end, say *A*, of base *AB*. Construct an acute angle $\angle BAX$ below the base *AB*.

Step 4: Along *AX* mark off *n* points $A_1, A_2, A_3, \ldots\ldots\ldots, A_n$ such that

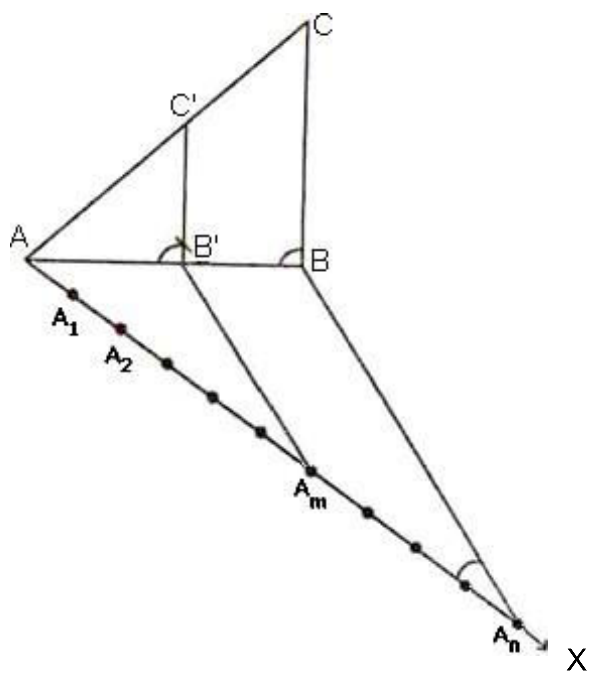
$$AA_1 = A_1A_2 = \ldots\ldots\ldots = A_{n-1}A_n$$

Step 5: Join A_nB

Step 6: Draw A_mB' parallel to A_nB which meets *AB* at *B*'.

Step 7: From *B*' draw $B'C' \parallel BC$ meeting *AC* at *C*'.

Triangle *AB*'*C*' is the required triangle each of whose sides is $\left(\frac{m}{n}\right)^{th}$ of the corresponding side of ΔABC.



Justification

Since $A_m B' \| A_n B$. Therefore

$$\frac{AB'}{B'B} = \frac{AA_m}{A_m A_n}$$

[by basic proportionality theorem]

$$\Rightarrow \frac{AB'}{B'B} = \frac{m}{n-m}$$

$$\Rightarrow \frac{B'B}{AB'} = \frac{n-m}{m}$$

Now $\frac{AB}{AB'} = \frac{AB' + B'B}{AB'}$

$$\Rightarrow \frac{AB}{AB'} = 1 + \frac{B'B}{AB'} = 1 + \frac{n-m}{m} = \frac{n}{m}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{m}{n}$$

In triangles ABC and $AB'C'$, we have

$$\angle BAC = \angle B'AC'$$

and $\angle ABC = \angle AB'C'$

So, by AA similarity criterion, we have

$$\Delta AB'C' \sim \Delta ABC$$

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{m}{n}$$

Steps of construction when m > n:

Step 1: Construct the given triangle by using the given data.

Step 2: Take any one of the three sides of the given triangle and consider it as the base. Let *AB* be the base of the given triangle.

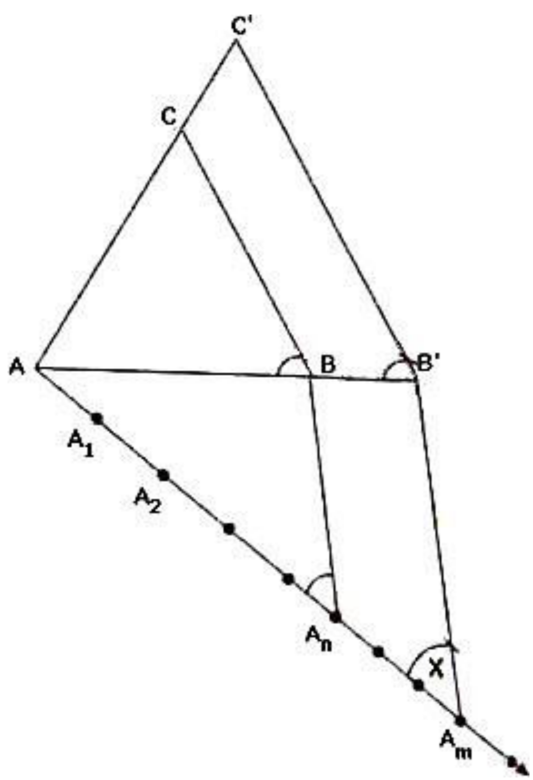
Step 3: At one end, say *A,* of base *AB*. Construct an acute angle $\angle BAX$ below base AB i.e., on the opposite side of the vertex *C*.

Step 4: Along *AX* mark off *m* (large of m and n) points $A_1, A_2, A_3, \ldots\ldots\ldots A_m$ of *AX* such that $AA_1 = A_1A_2 = \ldots\ldots\ldots = A_{m-1}A_m$.

Step 5: Join A_nB to *B* and draw a line through A_m parallel to A_nB, intersecting the extended line segment *AB* at *B*'.

Step 6: Draw a line through *B*' parallel to *BC* intersecting the extended line segment *AC* at *C*'.

Step 7: $\Delta AB'C'$ so obtained is the required triangle.



Justification

Consider triangle *ABC* and *AB' C'*. We have:

$$\angle BAC = \angle B'AC'$$

$$\angle ABC = \angle AB'C'$$

So, by *AA* similarity criterion,

$$\Delta ABC \sim \Delta AB'C'$$

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$$

$$In\ \Delta A A_m B',\ A_n B \parallel A_m B'.$$

$$\therefore \frac{AB}{BB'} = \frac{AA_n}{A_n A_m}$$

$$\Rightarrow \frac{BB'}{AB} = \frac{A_n A_m}{AA_n}$$

$$\Rightarrow \frac{BB'}{AB} = \frac{m-n}{n}$$

$$\Rightarrow \frac{AB' - AB}{AB} = \frac{m-n}{n}$$

$$\Rightarrow \frac{AB'}{AB} - 1 = \frac{m-n}{n}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{m}{n}$$

From (i) and (ii), we have

$$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{m}{n}$$

The tangent to a circle is a line that intersects the circle at exactly one point.

Tangent to a circle is perpendicular to the radius through the point of contact.

Construction of Triangle to a Circle from a point outside the Circle

Construction of a tangent to a circle from a point outside the circle, when its centre is known

The steps of constructions are as follows:

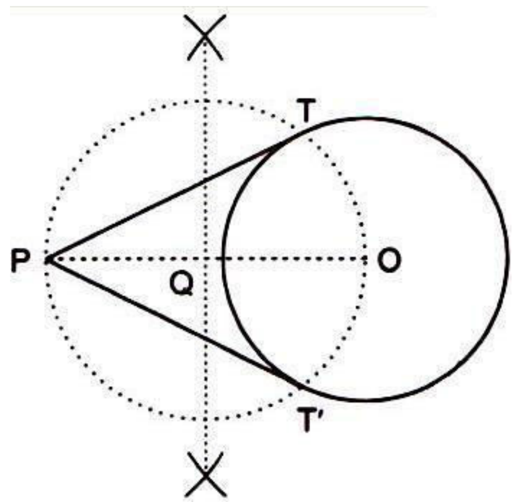
Step 1: Join the centre O of the circle to the point P.

Step 2: Draw perpendicular bisector of OP intersecting OP at Q.

Step 3: With Q as centre and radius OQ, draw a circle. This circle has OP as its diameter.

Step 4: Let this circle intersect the first circle at two points T and T'. Join PT and PT'.

PT and PT' are the two tangents to the given circle from the point P.



Justification

Join OT and OT'

It can be seen that $\angle PTO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$\therefore \angle PTO = 90°$

$\Rightarrow OT \perp PT$

Since OT is the radius of the circle, PT has to be a tangent of the circle. Similarly, PT' is a tangent of the circle.