LECTURE_08

MODULE_26

Q. Find the largest number that will divide 445 , 572 and 699 leaving remainders 4, 5 and 6 respectively.

Sol. The required number when divides 445 , 572 and 699 leaves remainder 4 , 5 and 6.

∴ The numbers completely divisible by required number are

$445 - 4 = 441$,

$572 - 5 = 567$ and

$699 - 6 = 693$

The required number will be HCF of 441, 567 and 693

$441 = 3^2 \times 7^2$

$567 = 3^4 \times 7$

$693 = 3^2 \times 7 \times 11$

∴ HCF of 441, 567 and 693 = $3^2 \times 7 = 63$

3	441
3	147
7	49
7	7
	1

3	693
3	231
7	77
11	11
	1

3	567
3	189
3	63
3	21
7	7
	1

∴ The largest number that divides 445 , 572 and 699 leaves remainder 4, 5 and 6 is 63

MODULE_27

Q. Find the smallest number which when increased by 17 is exactly divisible by both 520 and 468.

Sol. The least number divisible by both 520 and 468 is the Least Common Multiple of 520 and 468 (L.C.M)

$$520 = \underline{2 \times 2 \times 2} \times \underline{5} \times \underline{13} = 2^3 \times 5 \times 13$$

$$468 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{13} = 2^2 \times 3^2 \times 13$$

Clearly, L.C.M. $= 2^3 \times 3^2 \times 5 \times 13 = 4680$

So, least number divisible by both 520 and 468 is 4680.

Let the number which when increased by 17 gives the least number divisible by 520 and 468 (i.e. LCM of the two i.e. 4680) be x

So, $x + 17 = 4680$

$x = 4680 - 17$

$x = 4663$

Therefore, 4663 is the required answer.

2	520
2	260
2	130
5	65
13	13
	1

2	468
2	234
3	117
3	39
13	13
	1

MODULE_28

$$\frac{16}{36} = \frac{4}{9}$$

Co-prime numbers *two numbers having no common factor other than 1 are co-prime numbers.*

If a and b are co-prime numbers then they have no common factor other than 1

Example:

12 & 17, 21 & 22, 33 & 40,

Let p be a prime number,

If p divides a^2 , then p divides a

Example:

If 2 divides $(8)^2$ then 2 divides 8

If 7 divides $(35)^2$ then 7 divides 35

Exercise 1.3

Q.1 Prove that $\sqrt{5}$ is irrational.

Proof

Let us assume that $\sqrt{5}$ is a rational number.

$\therefore$ There exist co-prime integers a and b, ($b \neq 0$) such that,

$$\sqrt{5} = \frac{a}{b}$$

$$\sqrt{5}\,b = a$$

squaring both sides,

$$5b^2 = a^2 \quad \dots (1)$$

$\therefore$ 5 divides a^2 $\Rightarrow$ 5 divides a ... (2)

Let $a = 5c$ where c is some integer

substituting this value of a in (1)

$$5b^2 = (5c)^2$$

$$5b^2 = 25c^2$$

$$\therefore \quad b^2 = 5c^2$$

5 divides b^2 $\Rightarrow$ 5 divides b ... (4)

From (3) and (5), we get,

a and b both have common factor 5.

This contradicts the fact that a and b are co-prime.

Our assumption that $\sqrt{5}$ is a rational number is wrong.

$\therefore$ $\sqrt{5}$ is an irrational number.

We will prove it by Contradiction method

Rational number $= \frac{a}{b}$, ($b \neq 0$)

& a, b are co-prime integers

If 5 divides 15

That means 5 × integer = 15

Dividing both sides by 5

5 divides a & b both

5 is a factor of a... 3

After equation 2

5 is a factor of b....5

After equation 4

MODULE_29

Exercise 1.3

Q.2 Prove that $3 + 2\sqrt{5}$ is irrational.

Proof

Let us assume that $3 + 2\sqrt{5}$ is a rational number.

$\therefore$ There exist co-prime integers a and b ($b \neq 0$) such that,

$$3 + 2\sqrt{5} = \frac{a}{b}$$

Rational number $= \frac{a}{b}$, $(b \neq 0)$

& a, b are co-prime integer

$$\therefore \quad 2\sqrt{5} = \frac{a}{b} - 3$$

$$\therefore \quad 2\sqrt{5} = \frac{a - 3b}{b}$$

$$\therefore \quad \sqrt{5} = \frac{a - 3b}{2b}$$

We will prove it by Contradiction method

Since a and b are integers,

$\therefore \quad \frac{a - 3b}{2b}$ is rational $\Rightarrow$ $\sqrt{5}$ is also rational

This also implies That $\sqrt{5}$ is rational

This contradicts the fact that $\sqrt{5}$ is irrational.

$\therefore$ Our assumption that $3 + 2\sqrt{5}$ is a rational number is wrong.

$\therefore$ $\boxed{3 + 2\sqrt{5} \text{ is irrational.}}$

Exercise 1.3

Q.3 Prove that the following are irrationals.

(i) $\frac{1}{\sqrt{2}}$

Proof

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Let us assume that $\frac{\sqrt{2}}{2}$ is rational.

$\therefore$ There exist co-prime integers a and b $(b \neq 0)$ such that,

$$\therefore \quad \frac{\sqrt{2}}{2} = \frac{a}{b}$$

$$\therefore \quad \sqrt{2} = \frac{2a}{b}$$

Since a and b are integers,

$\therefore$ $\frac{2a}{b}$ is rational $\Rightarrow$ $\sqrt{2}$ is also rational,

but this contradicts the fact that $\sqrt{2}$ is irrational.

$\therefore$ Our assumption that $\frac{\sqrt{2}}{2}$ i.e. $\frac{1}{\sqrt{2}}$ is rational is wrong.

$\therefore$ $\boxed{\frac{1}{\sqrt{2}} \text{ is an irrational number}}$

Multiply both numerator & denominator by $\sqrt{2}$

We will first rationalise the denominator

2a is integer

b is integer & b $\neq$ 0

Now we will prove by Contradiction method

Arrange this equation in such a way that we get only $\sqrt{2}$ in L.H.S

This also implies That $\sqrt{2}$ is rational