No. 18

To solve Equations with Variables in the denominator

1. $\frac{1}{x} + \frac{1}{y} = 8 \; ; \; \frac{4}{x} - \frac{2}{y} = 2$

$x^{-1} + y^{-1} = 8 \; ; \; 4x^{-1} - 2y^{-1} = 2$

Soln. Substituting $\frac{1}{x} = p$ and $\frac{1}{y} = q$

Degree

Equation (iii)

In either (i), (ii) or (iii)

was not considered.

q something...

Consider (

$p \; + \; \ldots$

$\therefore \; p = 8 \ldots$

Substituting (iii) in (ii)

$\therefore \; 4\,(8 - q) - 2q = 2$

$\therefore \; 32 - 4q - 2q = 2$

$\therefore \; 32 - 6q = 2$

$\therefore \; -6q = 2 - 32$

$-6q = -30$

$q = \cancel{\frac{30}{6}}\; 5$

$q = 5$

Substituting q = 5 in (iii), we get

$p = 8 - q$

$\therefore \; p = 8 - 5$

$\therefore \; p = 3$

Re substituting $p = \frac{1}{x}$

$p = \frac{1}{x}$

$3 = \frac{1}{x}$

$\therefore \; x = \frac{1}{3}$

Re substituting $q = \frac{1}{y}$

$q = \frac{1}{y}$

$\therefore \; 5 = \frac{1}{y}$

$\therefore \; y = \frac{1}{5}$

$\therefore \; x = \frac{1}{3} \; , \; y = \frac{1}{5}$

No. 19

(iii) $\frac{4}{x} + 3y = 14$; $\frac{3}{x} - 4y = 23$

$4 \times \frac{1}{x} + 3y = 14$; $3 \times \frac{1}{x} - 4y = 23$

Soln. Substituting $\frac{1}{x} = p$

$\therefore\ 4p + 3y = 14$ ... (i)

$\therefore\ 3p - 4y = 23$ ... (ii)

Consider (i),

$4p + 3y = 14$

$\therefore\ p = \frac{14 - 3y}{4}$ ... (iii)

Substituting (iii) in (ii)

$3\left(\frac{14 - 3y}{4}\right) - 4y = 23$

$\therefore\ \frac{42 - 9y}{4}$

$\therefore\ \frac{42 - 9y - 16y}{4}$

$\therefore\ 42 - 9y - 16y = 92$

$\therefore\ 42 - 25y$

Shift or

y = something...

We have to

In either (i) , (ii) or (iii)

Let us substitute y = -2 in (iii)

Degree is – 1

Substituting y = - 2 in (iii), we get

$p = \frac{14 - 3(-2)}{4}$

$\therefore\ p = \frac{14 + 6}{4}$

$\therefore\ p = \frac{\cancel{20}^{5}}{\cancel{4}}$

p = something...

$p = 5$

Re substituting $p = \frac{1}{x}$

$p = \frac{1}{x}$

$5 = \frac{1}{x}$

$\therefore\ x = \frac{1}{5}$

Solution

$x = \frac{1}{5}$

$y = -2$

ii). $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$; $\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

Soln. Substituting $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$

In this sum variables in the denominator under square root

Equation (iii)

Consider (i)

$2p + 3q = 2$

$\therefore 2p = 2 - 3q$

$\therefore \boxed{p = \frac{2 - 3q}{2}}$ ... (iii)

Substituting (iii) in (ii)

$\cancel{4}^{2}\left(\frac{2 - 3q}{\cancel{2}}\right) - 9q = -1$

$\therefore 4 - 6q - 9q = -1$

$\therefore -15q = -1 - 4$

By substituting

$\therefore -15q = -5$

Common Term / Common Denominator in both equations

Substituting $q = \frac{1}{3}$ in (iii), we get

$p = \frac{2 - \cancel{3}(1/\cancel{3})}{2}$

$\therefore p = \frac{2 - 1}{2}$

$\therefore \boxed{p = \frac{1}{2}}$

Re substituting $p = \frac{1}{\sqrt{x}}$

$\frac{1}{2} = \frac{1}{\sqrt{x}}$

$\sqrt{x} = 2$

In either (i), (ii) or (iii)

Squaring both sides

$\frac{1}{3} = \frac{1}{\sqrt{y}}$

$\therefore \sqrt{y} = 3$

Squaring both sides

$y = 9$

$\therefore \boxed{x = 4, \; y = 9}$

No. 20

To solve Equations with Variables and Numbers in the denominator

(i) $\dfrac{1}{2x} + \dfrac{1}{3y} = 2; \quad \dfrac{1}{3x} + \dfrac{1}{2y} = \dfrac{13}{6}$

Soln. $\dfrac{1}{2x} + \dfrac{1}{3y} = 2$

Multiplying throughout by 6

$$\overset{3}{\cancel{6}}\left(\frac{1}{\cancel{2}x}\right) + \overset{2}{\cancel{6}}\left(\frac{1}{\cancel{3}y}\right) = 2 \times 6$$

$$\frac{3}{x} + \frac{2}{y} = 12 \qquad \text{..... (i)}$$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$$

Multiplying throughout by 6

$$\overset{2}{\cancel{6}}\left(\frac{1}{\cancel{3}x}\right) + \overset{3}{\cancel{6}}\left(\frac{1}{\cancel{2}y}\right) = \frac{13}{\cancel{6}} \times \cancel{6}$$

$$\frac{2}{x} + \frac{3}{y} = 13 \qquad \text{..... (ii)}$$

Substituting $\dfrac{1}{x} = p$ & $\dfrac{1}{y} = q$ in (i) and (ii)

In th

3p

$2p + 3q = 13$

Common Term / Common Denominator in both equations

Consider (iii)

$3p + 2q = 12$

$$\therefore p = \frac{12 - 2q}{3} \qquad \text{..... (v)}$$

Substituting (v) in (iv)

$$2\left[\frac{12 - 2q}{3}\right] + 3q = 13$$

$$\therefore \frac{24 - 4q}{3} + 3q = 13$$

$$\therefore \frac{24 - 4q + 9q}{3} = 13$$

$$\therefore 24 - 4q + 9q = 39$$

$$\therefore 24 + 5q = 39$$

$$\therefore 5q = 15$$

$$\therefore \boxed{q = 3}$$

Substituting q = 3 in (v),

$$p = \frac{12 - 2(3)}{3}$$

$$\therefore \boxed{p = 2}$$

Re substituting $p = \dfrac{1}{x}$ & $q = \dfrac{1}{y}$

$$2 = \frac{1}{x} \ \& \ 3 = \frac{1}{y}$$

$$\therefore \boxed{x = \frac{1}{2} \ \& \ y = \frac{1}{3}}$$

No. 21

To solve Equations with Binomial terms in the denominator

(iv) $\dfrac{5}{x-1} + \dfrac{1}{y-2} = 2$ $\dfrac{6}{x-1} - \dfrac{3}{y-2} = 1$

Soln. Substituting $\dfrac{1}{x-1} = p$ & $\dfrac{1}{y-2} = q$

$5p + q$ ...

What is the difference between this & the sums done before this ?

Binomial terms in denominator

Equation (iii)

By substituting ... How

In either (i), (ii) or (iii)

Common Term / Common Denominator in both equations

Consider (i),

$5p + q = 2$

$\therefore q = 2 - 5p$

Substituting (iii) in (ii)

$6p - 3(2 - 5p) = 1$

$\therefore 6p - 6 + 15p = 1$

$\therefore 21p = 1 + 6$

$\therefore 21p = 7$

$\therefore p = \dfrac{\cancel{7}^{\,1}}{\cancel{21}_{\,3}}$

$\therefore \boxed{p = \dfrac{1}{3}}$

Substituting $p = \dfrac{1}{3}$ in (iii), we get

$q = 2 - 5\left(\dfrac{1}{3}\right)$

$\therefore q = \dfrac{6 - 5}{3}$

$\therefore \boxed{q = \dfrac{1}{3}}$

Re substituting $p = \dfrac{1}{x-1}$

$\dfrac{1}{3} = \dfrac{1}{x-1}$

$\therefore x - 1 = 3$

$\therefore x = 3 + 1$

$\therefore x = 4$

Re substituting $q = \dfrac{1}{y-2}$

$\dfrac{1}{3} = \dfrac{1}{y-2}$

$\therefore y - 2 = 3$

$\therefore y = 5$

$\therefore \boxed{x = 4,\ y = 5}$

No. 22

(vii) $\dfrac{10}{x+y} + \dfrac{2}{x-y} = 4$ $\dfrac{15}{x+y} - \dfrac{5}{x-y} = -2$

Soln. Substituting $\dfrac{1}{x+y} = p$ & $\dfrac{1}{x-y} = q$

$10p + 2q = \dots$

$5p + q = 2$ ....(i)

Consider (i),

$5p + q = 2$

$\therefore q = 2 - 5p$ ....(iii)

Substituting (iii) in (ii)

$15p - 5(2 - 5p) = -2$

$\therefore 15p - 10 + 25p = -2$

$\therefore 40p = -2 + 10$

$\therefore 40p = 8$

$\therefore p = \dfrac{\cancel{8}^{1}}{\cancel{40}_{5}}$

$\therefore \boxed{p = \dfrac{1}{5}}$

Substituting $p = \dfrac{1}{5}$ in (iii), we get

$q = 2 - \cancel{5}\left(\dfrac{1}{\cancel{5}}\right)$

$\therefore q = \dots$

$\therefore \boxed{q = 1}$

Re substituting $p = \dfrac{1}{x+y}$

$\dfrac{1}{5} = \dfrac{1}{x+y}$

$\therefore x + y = 5$ ... (iv)

Re substituting $q = \dfrac{1}{x-y}$

$1 = \dfrac{1}{x-y}$

$\therefore x - y = 1$ ... (v)

Adding (iv) and (v)

$x + \cancel{y} = 5$

$x - \cancel{y} = 1$

$\therefore y = 2$

$\therefore \boxed{x = 4,\ y = 2}$

What is t... between this ... the sums done ... this ?

Binomial terms in ... den...

Equation (iii)

By substi...

How

In either (i), (ii) or (iii)

Common Term

Even after

We are getting 2 more equations....solve these equations to get the answer

No. 23

Q.] Solve the following pair of equations by reducing them to a pair of linear equations:

$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}; \quad \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$$

Sol : Substituting $\frac{1}{(3x+y)} = p$ & $\frac{1}{(3x-y)} = q$

Lets do the substitution for common value in denominator

$$p + q = \frac{3}{4}$$

Multiplying both sides by 4

$\therefore\ 4p + 4q = 3$ ...... (i)

$$\frac{p}{2} - \frac{q}{2} = -\frac{1}{8}$$

Multiplying both sides by 8

$\therefore\ 4p - 4q = -1$ ...... (ii)

Adding (i) and (ii)

$$\begin{aligned} 4p + \cancel{4q} &= 3 \\ 4p - \cancel{4q} &= -1 \\ \hline 8p &= 2 \end{aligned}$$

$\therefore\ p = \frac{1}{4}$

Substituting $p = \frac{1}{4}$ in eq^n (i)

$\therefore\ 4\left(\frac{1}{4}\right) + 4q = 3$

$\therefore\ 1 + 4q = 3$

$\therefore\ 4q = 3 - 1$

$\therefore\ 4q = 2$

$\therefore\ q = \frac{1}{2}$

Resubstituting the values of p and q

$$\frac{1}{(3x+y)} = \frac{1}{4}$$

$$\therefore \; 3x + y = 4 \; \ldots\ldots \; (iii)$$

$$\frac{1}{(3x-y)} = \frac{1}{2}$$

$$\therefore \; 3x - y = 2 \; \ldots\ldots \; (iv)$$

Adding (iii) and (iv)

$$\begin{array}{rcl} 3x + \cancel{y} & = & 4 \\ 3x - \cancel{y} & = & 2 \\ \hline 6x & = & 6 \end{array}$$

$$\therefore \; x = 1$$

Substituting x = 1 in (iii)

$$3(1) + y = 4$$

$$\therefore \; 3 + y = 4$$

$$\therefore \; y = 4 - 3$$

$$\therefore \; y = 1$$

$$\therefore \; \boxed{\text{Solution is } x = 1, \; y = 1}$$

$$\frac{1}{(3x+y)} = p \qquad \frac{1}{(3x-y)} = q$$

$$p = \frac{1}{4} \qquad q = \frac{1}{2}$$

No. 24

To solve Equations with "xy" terms

(vi) $\dfrac{7x - 2y}{xy} = 5 , \quad \dfrac{8x + 7y}{xy} = 15$

Soln. $\dfrac{7\cancel{x}}{\cancel{x}y} - \dfrac{2\cancel{y}}{x\cancel{y}} = 5 , \quad \dfrac{8\cancel{x}}{\cancel{x}y} + \dfrac{7\cancel{y}}{x\cancel{y}} = 15$

$\boxed{\dfrac{7}{y}} - \boxed{\dfrac{2}{x}} = 5 , \quad \dfrac{8}{\ } + \dfrac{7}{x} = 15 ,$

$\left(7 \times \boxed{\dfrac{1}{y}} - 2 \times \boxed{\dfrac{1}{x}} \right)$

$= q$

After solving it we get the final answer as

(vi) $6x + 3y = 6xy$; $2x + 4y = 5xy$

Soln.

$$\frac{\mathbf{6x + 3y}}{\mathbf{xy}} = \mathbf{6} \,, \quad \frac{\mathbf{2x + 4y}}{\mathbf{xy}} = \mathbf{5}$$

Now the sum is similar to the previous sum

Q.] Solve the following pair of equations by reducing them to a pair of linear equations: $6x + 3y = 6xy$; $2x + 4y = 5xy$

Sol : Divide both eq^n by xy

$$\frac{6}{y} + \frac{3}{x} = 6 \; ; \quad \frac{2}{y} + \frac{4}{x} = 5$$

Substituting $\frac{1}{y} = p$ & $\frac{1}{x} = q$

$6p + 3q = 6$ …… (i)

$2p + 4q = 5$ …… (ii)

Multiplying (ii) by 3, we get

$6p + 12q = 15$ …… (iii)

Subtracting (i) from (iii), we get

$$\begin{array}{rcrcr} \cancel{6p} & + & 12q & = & 15 \\ \cancel{6p} & + & 3q & = & 6 \\ (-) & & (-) & & (-) \\ \hline & & 9q & = & 9 \end{array}$$

$\therefore q = 1$

Lets do the substitution to reduce it to linear equation

Substituting q = 1 in (ii)

$2p + 4(1) = 5$

$\therefore 2p + 4 = 5$

$\therefore 2p = 5 - 4$

$\therefore 2p = 1$

$\therefore p = \frac{1}{2}$

Resubstituting the values of p and q

$\frac{1}{y} = \frac{1}{2}$ $\qquad$ $\frac{1}{x} = 1$

$\therefore y = 2$ $\qquad$ $\therefore x = 1$

$\boxed{\text{Solution is } x = 1, y = 2}$

Thank You