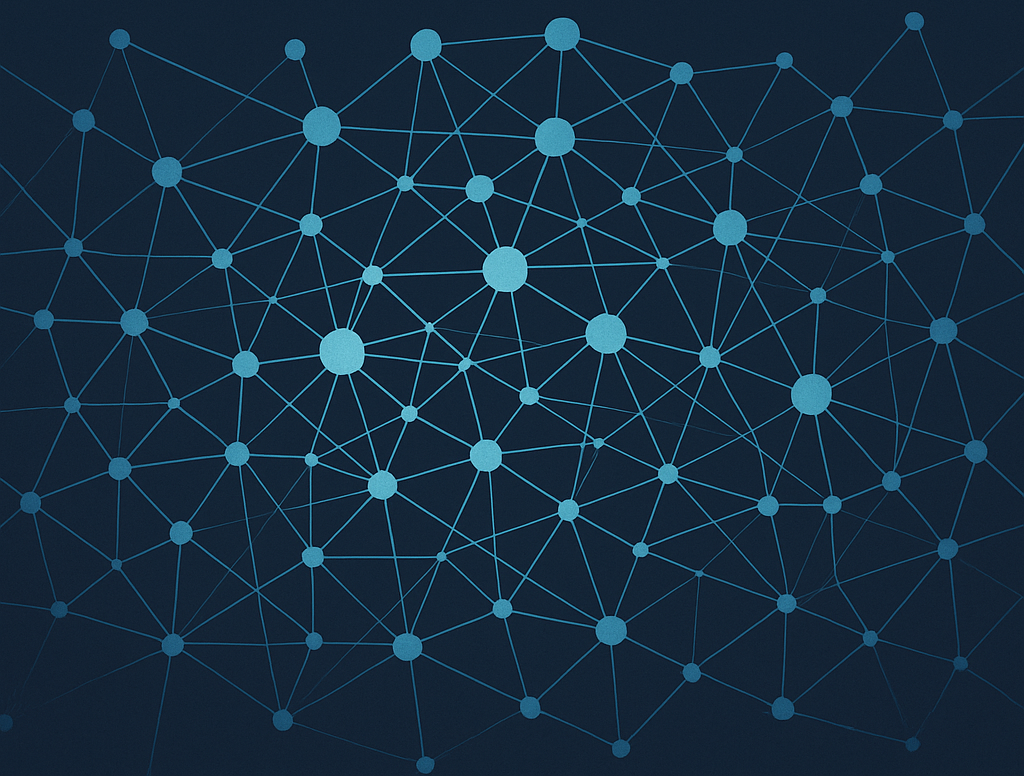
**Implementing, Training and Analysing Neural Networks**



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| --- | --- |
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Contents

[Part 1: Fractal Classification 4](#_Toc205077709)

[Step 1: Full3Net Architecture 4](#_Toc205077710)

[Code Snippet: 4](#_Toc205077711)

[Step 2: Finding Minimum Configurations for Convergence 4](#_Toc205077712)

[Formula for number of independent parameters required: 4](#_Toc205077713)

[10 hidden units in each layer: 5](#_Toc205077714)

[Changing the number of hidden units in each layer: 5](#_Toc205077715)

[Step 3: Full4Net Architecture: 8](#_Toc205077716)

[Code Snippet: 8](#_Toc205077717)

[Step 4: Minimum Convergence Configuration: 4 Layered Network 8](#_Toc205077718)

[Formula for number of independent parameters required: 8](#_Toc205077719)

[10 hidden units in each layer: 8](#_Toc205077720)

[Changing the number of hidden units in each layer: 10](#_Toc205077721)

[Step 5: DenseNet Architecture 15](#_Toc205077722)

[Objective: 15](#_Toc205077723)

[Architecture Description: 15](#_Toc205077724)

[Code Snippet: 15](#_Toc205077725)

[Step 6: Train the Dense Network: 16](#_Toc205077726)

[Varying the Number of Hidden Units: 16](#_Toc205077727)

[Step 7: 17](#_Toc205077728)

[A. Model Size vs. Training Behavior 17](#_Toc205077729)

[B. Qualitative Description of Layer Functions 19](#_Toc205077730)

[C. Overall Qualitative Decision Function 20](#_Toc205077731)

[Part 2: Encoder Networks: 21](#_Toc205077732)

[Dataset Design and Construction 21](#_Toc205077733)

[Result and Observations 22](#_Toc205077734)

[Explanation of the Horizontal Reflection 23](#_Toc205077735)

[Conclusion 23](#_Toc205077736)

Part 1: Fractal Classification

## Step 1: Full3Net Architecture

Implementation of a three-layer fully connected neural network (Full3Net) for classifying points in the below fractal pattern based on the provided dataset.

A diagram of a diamond shaped object

AI-generated content may be incorrect.

Code Snippet: for frac.py, Full3Net class:

import torch  
import torch.nn as nn  
  
class Full3Net(torch.nn.Module):  
 def \_\_init\_\_(self, hid):  
 super(Full3Net, self).\_\_init\_\_()  
 self.hid\_dim = hid  
  
 self.fc1 = nn.Linear(2, hid) # input (x, y) → hid  
 self.fc2 = nn.Linear(hid, hid) # hid → hid  
 self.fc3 = nn.Linear(hid, 1) # hid → output (1)  
  
 def forward(self, input):  
 self.hid1 = torch.tanh(self.fc1(input)) # first hidden layer  
 self.hid2 = torch.tanh(self.fc2(self.hid1)) # second hidden layer  
 output = torch.sigmoid(self.fc3(self.hid2)) # output layer  
 return output

## Step 2: Finding Minimum Configurations for Convergence

Formula for number of independent parameters required:

**Parameters in Layer 1 =** weights + biases = inputs x hidden units + hidden units

**Parameters in Layer 2 =** weights + biases = hidden units x hidden units + hidden units

**Parameters in Output Layer =** weights + biases = hidden units + outputs

Therefore, **total parameters** = (hidden units)2 + (inputs + 3) x (hidden units) + outputs

= hid\*\*2 + 5 hid + 1

10 hidden units in each layer:

Training has been started with **10 hidden units in each layer**. The resulting plot is as follows:

A graph showing a diamond shaped object with dots

AI-generated content may be incorrect.

Accuracy has been increasing up to 92.62% and loss decreasing, implying that the model is converging and successfully learning the fractal pattern.

**Total parameters =** hid\*\*2 + 5 hid + 1 = 10\*\*2 + 5\*10 + 1 = 151

Changing the number of hidden units in each layer: Following plots show the fractal classification with 30, 20, 8, 7 and 6 hidden units in each layer.

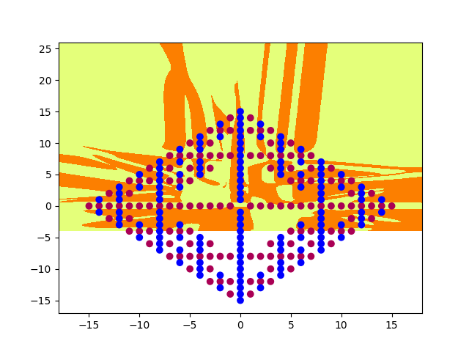
**30 Hidden Units:** **Total parameters =** hid\*\*2 + 5 hid + 1 = 30\*\*2 + 5\*30 + 1 = 1051

A graph showing a diamond shaped object

AI-generated content may be incorrect.

Accuracy reached 100% after 103700 to 123700 epochs and loss has constantly decreased.

**20 Hidden Units: Total parameters =** hid\*\*2 + 5 hid + 1 = 20\*\*2 + 5\*20 + 1 = 501



Accuracy has been increasing (up to 99.62%) and loss decreasing till 200000 epochs, implying that the model is converging and successfully learning the fractal pattern.

**8 Hidden Units: Total parameters =** hid\*\*2 + 5 hid + 1 = 8\*\*2 + 5\*8 + 1 = 105

Accuracy has been increasing up to 83% and loss decreasing till 200000 epochs. Though, some fluctuations were observed, pertaining to probable local minima. This implies that the model is still converging and successfully learning the fractal pattern. This is the minimal configuration for the model to converge and learn the fractal pattern.

**A graph showing a diamond shaped object with dots

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**7 Hidden Units: Total parameters =** hid\*\*2 + 5 hid + 1 = 7\*\*2 + 5\*7 + 1 = 85

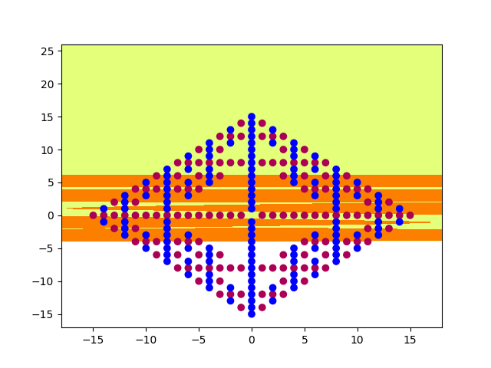
Accuracy has been very slowly increasing to 76% and loss very slowly decreasing after 35000 epochs. After this the learning practically stops, loss and accuracy both keep fluctuating.

A graph showing a triangle with dots

AI-generated content may be incorrect.

**6 Hidden Units: Total parameters =** hid\*\*2 + 5 hid + 1 = 6\*\*2 + 5\*6 + 1 = 67

Accuracy stagnates and below 74% and loss stops decreasing with increasing number of epochs. This implies that the model has stopped converging and learning the fractal pattern.



**4 Hidden Units: Total parameters =** hid\*\*2 + 5 \* hid + 1 = 37

Learning stops at ~20000epochs with accuracy of 65%.

A graph of a graph showing a diamond shaped object

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**Varying Learning Rate to 0.001 with 8 Hidden Units:** Learning Rate has been decreased to 0.001 to observe if the convergence get better, 8 hidden units. Accuracy went up to 84.36% and loss stagnated. The model performance does not increase (rather gets slightly degraded) by decreasing the learning rate from 0.01 to 0.001. Convergence remains similar.

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**Varying Initial Weight Scale to 0.1 with 8 Hidden Units (Learning Rate to 0.01):** Initial weight scale has been changed to 0.1 to observe if the convergence get better. Learning Rate to = 0.01. With 8 neurons per hidden layer and Learning Rate = 0.01, varying the Initial Weight Scale to 0.1, accuracy increases slightly to 86.15% and loss stagnates with increasing epochs. Therefor the model converges and performance gets slightly better with initial weight scale = 0.1

A graph showing a diamond shaped object with blue dots

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## Step 3: Full4Net Architecture:

Implementation of a 4-layer fully connected neural network (Full3Net) for classifying points in the above fractal pattern based on provided dataset.

Code Snippet: appended to frac.py for Full4Net class:

class Full4Net(nn.Module):  
 def \_\_init\_\_(self, hid):  
 super(Full4Net, self).\_\_init\_\_()  
 self.fc1 = nn.Linear(2, hid) # Input to Hidden Layer 1  
 self.fc2 = nn.Linear(hid, hid) # Hidden Layer 1 to Hidden Layer 2  
 self.fc3 = nn.Linear(hid, hid) # Hidden Layer 2 to Hidden Layer 3  
 self.fc4 = nn.Linear(hid, 1) # Hidden Layer 3 to Output

def forward(self, x):  
 # Hidden layers with tanh activation  
 self.hid1 = torch.tanh(self.fc1(x))  
 self.hid2 = torch.tanh(self.fc2(self.hid1))  
 self.hid3 = torch.tanh(self.fc3(self.hid2))  
 # Output layer with sigmoid activation  
 out = torch.sigmoid(self.fc4(self.hid3))  
 return out

## Step 4: Minimum Convergence Configuration: 4 Layered Network

Formula for number of independent parameters required:

**Parameters in Layer 1 =** weights + biases = inputs x hidden units + hidden units

**Parameters in Layer 2 =** weights + biases = hidden units x hidden units + hidden units

**Parameters in Layer 3 =** weights + biases = hidden units x hidden units + hidden units

**Parameters in Output Layer =** weights + biases = hidden units + outputs

Therefore, **total parameters** = 2 x (hidden units)2 + (inputs + 4) x (hidden units) + outputs

= 2 \* hid \*\*2 + 6 \* hid + 1

10 hidden units in each layer: **Total parameters =** 2 \* hid\*\*2 + 6 \* hid + 1 = 261

Training has started with **10 hidden units in each layer**. The resulting plot is as follows:

A graph showing a triangle with dots

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Output layer combines all features to assign a probability that a point belongs to the fractal set.

Accuracy has been increasing up to 91.92% and loss decreasing, implying that the model is converging and successfully learning the fractal pattern. The model’s performance remains more or less same with the addition of one extra hidden layer, slightly degraded due to overfitting.

**Outputs from Hidden Layer-1 of 4 Layer Network with 10 Hidden Neurons in Each Layer:** Applies nonlinear transformation to the 2-D input. The tanh activation maps the raw coordinates into a richer representation, effectively carving the input plane into broad nonlinear regions.

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AI-generated content may be incorrect.A graph showing a triangle with a point in the center

AI-generated content may be incorrect.A graph showing a diamond with dots

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**Outputs from Hidden Layer-2 of 4 Layer Network with 10 Hidden Neurons in Each Layer:** Refines these regions further, learning more localized curved boundaries by recombining activations from h1.

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AI-generated content may be incorrect.A graph of a function

AI-generated content may be incorrect.A graph of a graph

AI-generated content may be incorrect.A graph of a graph of a triangle with dots

AI-generated content may be incorrect.A graph of a graph showing a triangle with dots

AI-generated content may be incorrect.A graph of a graph showing a triangle with dots

AI-generated content may be incorrect.A graph of a point in a triangle

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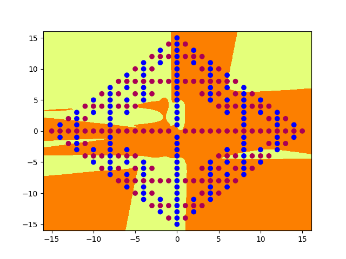
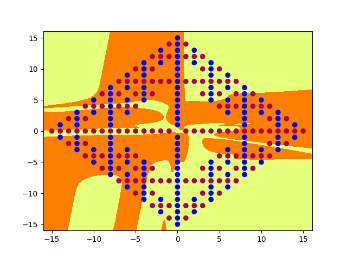
A graph of a graph

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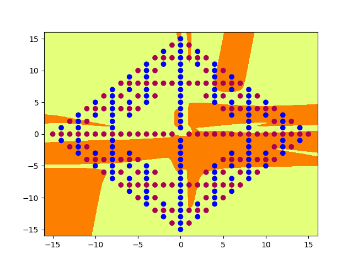
AI-generated content may be incorrect.

**Outputs from Hidden Layer-3 of 4 Layer Network with 10 Hidden Neurons in Each Layer:** Adds additional nonlinearity and complexity, enabling the network to represent finer details of the fractal pattern. At this stage, neurons often act as detectors for specific “patches” or structures in the fractal.

A graph of a function

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AI-generated content may be incorrect.A graph of a triangle with dots

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Changing the number of hidden units in each layer: Following plots show the fractal classification with 20, 8, 7 and 6 hidden units in each layer.

**20 Hidden Units:** **Total parameters =** 2\*hid\*\*2 + 6 \* hid + 1 = 2\*20\*\*2 + 6\*20 + 1 = 921

A graph showing a triangle with dots

AI-generated content may be incorrect.

Accuracy reached 100% after 92400 to 96400 epochs and loss has constantly decreased. Performance slightly increased from 3 layer to 4 layer network with 20 hidden units per layer.

Outputs from each hidden layer of a 4 Layer Network with 20 Hidden Neurons have been omitted as the number of images is (20\*4 = 80) too big.

**8 Hidden Units: Total parameters =** 2\*hid\*\*2 + 6 \* hid + 1 = 2\*8\*\*2 + 6\*8 + 1 = 177

A graph of a diamond shaped object with dots

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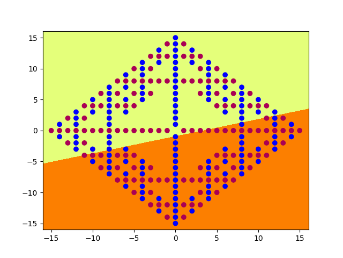
With moderate fluctuations, accuracy increased to 83% and loss decreased till 200000 epochs. No further improvement from the previous 3-layer network.

**Outputs from Hidden Layer-1 of 4 Layer Network with 8 Hidden Neurons in Each Layer:**

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AI-generated content may be incorrect.A graph of a graph

AI-generated content may be incorrect.A graph of a graph showing a triangle with dots

AI-generated content may be incorrect.A graph of a graph

AI-generated content may be incorrect.A graph of a triangle with dots

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**Outputs from Hidden Layer-2 of 4 Layer Network with 8 Hidden Neurons in Each Layer:**

A graph of a graph of a triangle with dots

AI-generated content may be incorrect.A graph of a graph

AI-generated content may be incorrect.A graph of a pyramid

AI-generated content may be incorrect.A graph of a graph showing a diamond shape

AI-generated content may be incorrect.A graph of a graph showing a diamond shape

AI-generated content may be incorrect.A graph of a pyramid

AI-generated content may be incorrect.A graph of a pyramid

AI-generated content may be incorrect.A graph of a graph

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**Outputs from Hidden Layer-3 of 4 Layer Network with 8 Hidden Neurons in Each Layer:**

A graph of a graph showing a diamond with blue dots

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AI-generated content may be incorrect.A graph of a graph showing a diamond with dots

AI-generated content may be incorrect.A graph of a graph showing a diamond with blue dots

AI-generated content may be incorrect.A graph of a graph showing a diamond shape

AI-generated content may be incorrect.A graph of a diagram

AI-generated content may be incorrect.A graph of a graph

AI-generated content may be incorrect.A graph of a graph showing a diamond with blue dots

AI-generated content may be incorrect.

**7 Hidden Units: Total parameters =** 2\*hid\*\*2 + 6\*hid + 1 = 2\*7\*\*2 + 6\*7 + 1 = 141

Accuracy has been very slowly increasing to 75.46% and loss very slowly decreasing.

A graph of a graph showing a diamond shaped object

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The outputs of each hidden layers are omitted as outputs of the 4 layer networks with 8 (above) units and 6 (below) units have been shown.

**6 Hidden Units: Total parameters =** 2\*hid\*\*2 + 6\*hid + 1 = 2\*6\*\*2 + 6\*6 + 1 = 109

Accuracy stagnates below 72.6% and loss stops decreasing with increasing number of epochs. This implies that the model has stopped converging and learning the fractal pattern.

A graph showing a diamond shape with dots

AI-generated content may be incorrect.

**Outputs from Hidden Layer-1 of 4 Layer Network with 6 Hidden Neurons in Each Layer:**

**A graph of a triangle with dots

AI-generated content may be incorrect.A graph of a graph showing a triangle with dots

AI-generated content may be incorrect.A graph of a graph showing a pyramid

AI-generated content may be incorrect.A graph of a triangle with dots

AI-generated content may be incorrect.A graph showing a diamond with dots

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AI-generated content may be incorrect.**

**Outputs from Hidden Layer-2 of 4 Layer Network with 6 Hidden Neurons in Each Layer:**

**A graph of a graph

AI-generated content may be incorrect.A graph of a graph of a diamond

AI-generated content may be incorrect.A graph of a triangle with dots

AI-generated content may be incorrect.A graph of a triangle with dots

AI-generated content may be incorrect.A graph of a graph of a triangle with dots

AI-generated content may be incorrect.A graph of a graph

AI-generated content may be incorrect.**

**Outputs from Hidden Layer-3 of 4 Layer Network with 6 Hidden Neurons in Each Layer:**

**A graph of a graph

AI-generated content may be incorrect.A graph of a graph showing a pyramid

AI-generated content may be incorrect.A graph of a graph showing a pyramid

AI-generated content may be incorrect.A graph of a graph

AI-generated content may be incorrect.A graph of a graph of a line graph

AI-generated content may be incorrect.A graph showing a diamond with dots

AI-generated content may be incorrect.**

**4 Hidden Units: Total parameters =** 2\*hid\*\*2 + 6\*hid + 1 = 57

Learning does not practically happen with an increasing number of epochs. Accuracy fluctuates constantly between 63 to 67%.

A graph showing a diamond shape

AI-generated content may be incorrect.

**Outputs from Hidden Layer-1 of 4 Layer Network with 4 Hidden Neurons in Each Layer:**

A graph showing a triangle with dots

AI-generated content may be incorrect.A graph of a graph showing a triangle with dots

AI-generated content may be incorrect.A graph of a pyramid

AI-generated content may be incorrect.A graph of a graph with dots

AI-generated content may be incorrect.

**Outputs from Hidden Layer-2 of 4 Layer Network with 4 Hidden Neurons in Each Layer:**

A graph of a pyramid

AI-generated content may be incorrect.A graph of a diagram

AI-generated content may be incorrect.A graph of a triangle with dots

AI-generated content may be incorrect.A graph of a graph of a triangle with dots

AI-generated content may be incorrect.

**Outputs from Hidden Layer-3 of 4 Layer Network with 4 Hidden Neurons in Each Layer:**

A graph of a pyramid

AI-generated content may be incorrect.A graph of a graph showing a diamond with blue dots

AI-generated content may be incorrect.A graph of a triangle with blue dots

AI-generated content may be incorrect.A graph of a graph showing a triangle with dots

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## Step 5: DenseNet Architecture

Objective:  
To implement a 3-layer densely connected neural network (DenseNet) that extends the Full3Net model by including shortcut (dense) connections. These connections allow each hidden layer and the output layer to receive inputs not only from the immediately preceding layer but also directly from earlier layers and the raw input.

Architecture Description:

* **Input Layer:** 2 input nodes representing the (x, y) coordinates.
* **First Hidden Layer (h1):** **hid** nodes, each using the **tanh** activation:

h1 = tanh ( b1 + W10 x )

* **Second Hidden Layer (h2):** **hid** nodes with **tanh** activation, receiving inputs from both the original input **x** and the first hidden layer **h1**​:

h2 = tanh ( b2 + W20 x + W21 h1 )

* **Output Layer:** A single node with **sigmoid** activation. It receives contributions from the input, the first hidden layer, and the second hidden layer:

out= σ ( bout + W30 x + W31 h1 + W32 h2)

Code Snippet:

class DenseNet(nn.Module):  
 def \_\_init\_\_(self, hid: int):  
 super().\_\_init\_\_()  
 self.hid = hid  
 # x ∈ R^2  
 self.fc1 = nn.Linear(2, hid) # W10, b1  
 self.fc2 = nn.Linear(2 + hid, hid) # [W20 | W21], b2  
 self.fc3 = nn.Linear(2 + hid + hid, 1) # [W30 | W31 | W32], b\_out  
  
 def forward(self, x: torch.Tensor) -> torch.Tensor:  
 # h1  
 self.hid1 = torch.tanh(self.fc1(x))  
 # h2 uses both x and h1  
 h2\_in = torch.cat([x, self.hid1], dim=1)  
 self.hid2 = torch.tanh(self.fc2(h2\_in))  
 # out uses x, h1, h2  
 out\_in = torch.cat([x, self.hid1, self.hid2], dim=1)  
 out = torch.sigmoid(self.fc3(out\_in))  
 return out

## Step 6: Train the Dense Network:

### Varying the Number of Hidden Units:

**20 Hidden Units:** If the above DenseNet code is run for 20 hidden units, the model converges very fast with a 100% accuracy from as low as 8800 to 18800 epochs. This is a considerable improvement from the Fully connected 3-layer network above. Resulting plot is shown below.

A graph showing a triangle with dots

AI-generated content may be incorrect.

**8 hidden units:** Model converges consistently with decreasing loss and increasing accuracy up to 88.34%. It shows a considerable improvement from the Fully connected 3-layer network.

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AI-generated content may be incorrect.  
**6 hidden units:** The model still converges with consistent loss and accuracy up to 75% at about 50000 epochs. This improvement is due to the Dense Network connections architecture. After 50000 epochs there is no considerable learning.

A graph of a triangle with dots

AI-generated content may be incorrect.

**4 hidden units:** The model hardly shows any decrease in loss below and accuracy is always below 64.7% right from the initial few epochs till end. There is no considerable learning.

A graph of a graph showing a triangle with dots

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## Step 7:

### Model Size vs. Training Behavior

**Parameter formulas (with brief derivations)**

1. **Fully connected 3-layer (Full3Net)**  
   **Formula:**   params = hid^2 + 5 \* hid + 1  
   **Derivation:**

* Input to H1: weights = 2 \* hid, biases = hid => 3 \* hid
* H1 to H2: weights = hid \* hid, biases = hid => hid^2 + hid
* H2 to Out: weights = hid, biases = 1
* **Sum =** hid^2 + 5 \* hid + 1

1. **Fully connected 4-layer (Full4Net)  
   Formula:**   params = 2 \* hid^2 + 6 \* hid + 1

**Derivation:**

* Input to H1: weights = 2 \* hid, biases = hid => 3 \* hid
* H1 to H2: weights = hid \* hid, biases = hid => hid^2 + hid
* H2 to H3: weights = hid \* hid, biases = hid => hid^2 + hid
* H3 to Out: weights = hid, biases = 1
* **Sum =** 2 \* hid^2 + 6 \* hid + 1

1. **DenseNet (3-layer with shortcuts)**  
   **Formula:**   params = hid^2 + 8 \* hid + 3

**Derivation:**

* h1 = tanh ( b1 + W10 \* x ) => 2 \* hid + hid = 3\*hid
* h2=tanh (b2 + W20\*x + W21\*h1) => W20(2\*hid) + W21(hid^2) + b2(hid) = hid^2 + 3\*hid
* out=σ(bo+W30\*x+W31\*h1+W32\*h2) => W30(2) + W31(hid) + W32(hid) + + b0(1) = 2\*hid + 3
* **Sum** = hid^2 + 8\*hid+3

**Reliable convergence happens at:**

* **Full3Net (hid = 30)**  
  Therefore, Number of independent Parameters = 30^2 + 5\*30 +1 = 1051
* **Full4Net (hid = 20)**   
  Therefore, Number of independent Parameters = 2 \* 20^2 + 6 \* 20 + 1 = 921
* **DenseNet (hid = 20)**   
  Therefore, Number of independent Parameters = 20^2 + 8 \* 20 + 3 = 563

**Note:** Although Full4Net is deeper and has more parameters, DenseNet achieves the smallest parameter count among the successful settings due to its efficient dense connections.

**Approximate training epochs observed**

* **Full3Net**
  + **hid = 30:** stabilizes ~**100,000** epochs (accuracy 100%).
  + **hid = 8, 10, 20:** continues learning up to **200,000** epochs with fluctuations (accuracy improving, loss decreasing).
  + **hid = 7:** stalls learning at ~**35,000** epochs.
  + **hid = 4:** stalls learning at ~**20,000** epochs.
* **Full4Net:** Adding depth alone (Full4Net) helped compared with small Full3Net
  + **hid = 20:** stabilizes ~**90,000** epochs (accuracy 100%).
  + **hid = 4:** highly fluctuating; no stable convergence observed.
* **DenseNet:** converged fastest (after fewest epochs) at comparable accuracy, despite fewer parameters than Full3Net and Full4Net at the successful settings. **Dense shortcuts** provided stronger gradient flow and feature reuse, markedly reducing epochs to convergence.
  + **hid = 20:** stabilizes **~10,000 epochs** (accuracy 100%)**.**
  + **hid = 6:** stalls learning at ~**50,000** epochs.
* **Capacity threshold matters**: below certain hid values (e.g., Full3Net at 7, DenseNet at 6), training stagnates—indicating insufficient model capacity to represent the fractal decision boundary.

### Qualitative Description of Layer Functions

**Full4Net**

* **First hidden layer (h1):** Applies a nonlinear transformation to the 2-D input (x, y). The tanh activation maps the raw coordinates into a richer representation, effectively carving the input plane into broad nonlinear regions.
* **Second hidden layer (h2):** Refines these regions further, learning more localized curved boundaries by recombining activations from h1.
* **Third hidden layer (h3):** Adds additional nonlinearity and complexity, enabling the network to represent finer details of the fractal pattern. At this stage, neurons often act as detectors for specific “patches” or structures in the fractal.
* **Output layer:** Combines all features to assign a probability that a point belongs to the fractal set.

Overall, the depth of Full4Net allows a hierarchical build-up: from broad shapes (layer 1) → intermediate motifs (layer 2) → detailed fragments (layer 3).

**DenseNet**

* **First hidden layer (h1):** Similar to Full4Net’s h1, it maps the 2-D input into nonlinear basis functions.
* **Second hidden layer (h2):** Receives both the raw input and h1. This means it can simultaneously capture direct input features (like global axes or symmetries in the fractal) and higher-order combinations from h1, leading to more efficient feature reuse.
* **Output layer:** Integrates information from all sources (x, h1, h2). This dense connectivity allows the output to directly exploit both low-level (raw input, broad shapes) and high-level (refined hidden activations) features.

As a result, DenseNet’s functions are less strictly hierarchical and more **feature-sharing**. Earlier representations continue to influence later layers, leading to faster training and smoother decision boundaries.

This qualitative comparison above, emphasizes the difference:

* **Full4Net** builds up complexity in stages, with each layer depending primarily on the previous one.
* **DenseNet** blends information from all levels, making it more efficient at representing complex fractal boundaries with fewer epochs.

### Overall Qualitative Decision Function

**Fully connected 3-layer (Full3Net)**

* **Shape/complexity:** Can learn a nonlinear boundary, but with limited width it often shows **coarser partitions** and **fragmented islands** in the fractal—evidence of capacity limits.
* **Artifacts:** More susceptible to **underfitting** (missed fine detail) or **wavy/rippled** boundaries when it struggles to reconcile global and local structure using only two hidden transforms.
* **Takeaway:** Adequate for broad structure; fine fractal detail requires larger width or many epochs.

**Fully connected 4-layer (Full4Net)**

* **Shape/complexity:** Added depth enables a **more hierarchical composition** of features, so the output boundary is typically **finer and more articulated** than Full3Net at similar width.
* **Artifacts:** Can still show **training fluctuations**; without shortcuts, information must pass layer-by-layer, which can slow optimization and sometimes yield **over-sharp local features** or uneven refinement.
* **Takeaway:** Better at representing intricate boundaries than Full3Net, but may need careful tuning (epochs, initialization of weights, learning rate).

**DenseNet (3-layer with shortcuts)**

* **Shape/complexity:** Dense skip connections let the output depend **directly on input and earlier features**, producing a boundary that is **smooth where appropriate** yet **captures fine detail earlier** in training.
* **Optimization effect:** Shortcuts aid gradient flow and **feature reuse**, so the learned function often looks **cleaner and more stable** at lower width and fewer epochs.
* **Takeaway:** For comparable capacity, DenseNet tends to achieve a **more accurate and less noisy** approximation of the fractal decision set.

**Big-picture summary**

* With enough width/depth, all three are universal approximators; differences emerge **under practical limits** (your chosen hid, training time).
* **Full3Net:** coarser/global patterns first; fine details come late or not at all.
* **Full4Net:** deeper hierarchy adds detail, but optimization is still strictly sequential.
* **DenseNet:** shortcut-aided function that blends global (input) and intermediate features at the output, typically yielding the **best qualitative boundary** at the same or lower parameter count and epochs.

**Caveat:** Exact visuals depend on initialization, learning rate, and stochasticity; the above trends match the observed convergence/epoch behavior.

Part 2: Encoder Networks: Australia Dataset (aus26)

## Dataset Design and Construction

The objective of this task was to create a **26 × 20 tensor** (aus26) in *encoder.py* such that, when passed as the target to encoder\_main.py, the trained encoder network would produce a stylized representation of Australia.

The design process began by superimposing a 10×10 coordinate grid over a geographical map of Australia (see diagram below). This grid was used to identify and mark 20 key feature points representing notable geographical anchors, plus six fixed anchors at the four corners and midpoints of the left and right borders of the grid.

A map of australia with green squares

AI-generated content may be incorrect.

Each point’s coordinates on the grid were mapped to binary activations for the network’s 20 output units, corresponding to the “half-space” on one side of a learned linear decision boundary (i.e., vertical or horizontal line segments drawn in the hidden-unit space). The first 10 output units encode boundaries along the x-axis, while the next 10 encode boundaries along the y-axis.

A 1 indicates the point lies on the “positive” side of the respective decision boundary, while 0 indicates the “negative” side.

The final tensor (aus26) below was manually constructed to match the positional relationships observed in *Diagram.png*. This ensured the placement of dots and decision lines closely matched the stylized map.

aus26 = torch.Tensor([  
 [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0], # bottom-left anchor  
 [0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,1,1], # top-left anchor  
 [1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0], # bottom-right anchor  
 [1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1], # top-right anchor  
 [0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,0,0,0,0], # mid-left edge anchor (0, 6)  
 [1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0], # mid-right edge anchor (10, 5)  
 [1,1,1,1,0,0,0,0,0,0,1,1,1,1,1,1,1,1,1,0], # WA (4, 9) #1  
 [1,1,1,1,1,0,0,0,0,0,1,1,1,1,1,1,1,1,1,0], # NT (5,9) #2  
 [1,1,1,1,1,0,0,0,0,0,1,1,1,1,1,1,1,1,0,0], # NT (5,8) #3  
 [1,1,1,1,1,1,0,0,0,0,1,1,1,1,1,1,1,1,0,0], # NT (6,8) #4  
 [1,1,1,1,1,1,1,0,0,0,1,1,1,1,1,1,1,1,1,0], # QLD (7,9) #5  
 [1,1,1,1,1,1,1,1,0,0,1,1,1,1,1,1,1,1,0,0], # QLD (8,8) #6  
 [1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,0,0,0], # QLD (9,7) #7  
 [1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,0,0,0,0], # QLD (9,6) #8  
 [1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,0,0,0,0,0], # NSW (9,5) #9  
 [1,1,1,1,1,1,1,1,0,0,1,1,1,1,0,0,0,0,0,0], # NSW (8,4) #10  
 [1,1,1,1,1,1,1,0,0,0,1,1,1,1,0,0,0,0,0,0], # VIC (7,4) #11  
 [1,1,1,1,1,1,0,0,0,0,1,1,1,1,1,0,0,0,0,0], # SA (6,5) #12  
 [1,1,1,1,1,0,0,0,0,0,1,1,1,1,1,0,0,0,0,0], # SA (5,5) #13  
 [1,1,1,1,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0], # WA (4,4) #14  
 [1,1,1,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0], # WA (3,4) #15  
 [1,1,0,0,0,0,0,0,0,0,1,1,1,1,1,0,0,0,0,0], # WA (2,5) #16  
 [1,1,0,0,0,0,0,0,0,0,1,1,1,1,1,1,0,0,0,0], # WA (2,6) #17  
 [1,1,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,0,0,0], # WA (2,7) #18  
 [1,1,1,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,0,0], # WA (3,8) #19  
 [1,1,1,1,1,1,1,1,0,0,1,1,0,0,0,0,0,0,0,0] # TAS (8,2) #20  
])

## Result and Observations

The following command produced the output shown below.

python3 encoder\_main.py --target aus26 .

A graph of lines and dots

AI-generated content may be incorrect.

## Explanation of the Horizontal Reflection

The resulting plot matched the target configuration in terms of dot placement and decision-line arrangement but appeared as a horizontal mirror image of the original geographical layout. The horizontal reflection occurs due to the way the encoder network maps input coordinates to the 2D hidden space. Specifically:

1. Encoding transformation: The learned input-to-hidden weight matrix can apply an axis inversion if the sign of one of its weight vectors is reversed during training.
2. Symmetry of the target pattern: Because the target tensor only encodes *relative half-space relationships* with respect to vertical and horizontal boundaries, the network can achieve the same loss minimization whether reflects the image or not — as long as the relative positions between points and boundaries are preserved.
3. No explicit constraint on orientation: The training objective minimizes reconstruction error, not absolute geographical orientation. This means the solution space includes both the correct orientation and any flipped/mirrored versions that yield identical half-space classifications.

In effect, the encoder has found an equally valid solution by reflecting the pattern horizontally, swapping the east and west halves of the map. This is analogous to a PCA or autoencoder projection where the principal components may be flipped without changing their explanatory power.

## Conclusion

The aus26 dataset successfully demonstrated how careful manual encoding of target outputs can guide an encoder network to produce structured, meaningful 2D representations. The observed horizontal reflection highlights the inherent symmetry in such learned feature mappings, underlining the importance of incorporating orientation constraints if absolute spatial accuracy is required.

# Part 3 – Japanese Character Recognition

This part of the assignment focuses on the recognition of handwritten Japanese Hiragana characters using neural networks. The dataset used is *Kuzushiji-MNIST (KMNIST)*, a modern benchmark designed to reflect the challenges of recognizing cursive Japanese characters from historical texts. It consists of 70,000 grayscale images (28×28 pixels) across 10 classes of Hiragana. The task involves implementing and evaluating three neural network architectures of increasing complexity—a linear model, a fully connected model, and a convolutional neural network—comparing their performance in terms of classification accuracy, parameter count, and confusion patterns.

## Step 1 – Linear Model (NetLin)

To establish a baseline, a linear classifier NetLin was implemented, consisting of a single fully connected layer that maps the flattened 28×28 grayscale image to the 10 output classes, followed by a log softmax activation. This model is equivalent to multinomial logistic regression and does not include any hidden layers or non-linear transformations. The model was trained for 10 epochs using the training portion of the Kuzushiji-MNIST dataset, with the negative log-likelihood (NLL) loss function and stochastic gradient descent. As expected, the model's performance plateaued around 70% accuracy on the test set.

### Final Test Accuracy

Accuracy: 69.53% (6953 / 10000)

### Confusion Matrix

Each row corresponds to the true label and each column to the predicted label. The labels correspond to the 10 Hiragana characters in the order:  
0="o", 1="ki", 2="su", 3="tsu", 4="na", 5="ha", 6="ma", 7="ya", 8="re", 9="wo".

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **0** | 764 | 5 | 8 | 14 | 30 | 62 | 2 | 63 | 31 | 21 |
| **1** | 7 | 669 | 106 | 17 | 28 | 24 | 59 | 14 | 26 | 50 |
| **2** | 8 | 62 | 690 | 25 | 27 | 21 | 48 | 35 | 46 | 38 |
| **3** | 5 | 35 | 60 | 755 | 15 | 57 | 14 | 18 | 28 | 13 |
| **4** | 62 | 52 | 78 | 22 | 624 | 18 | 32 | 37 | 20 | 55 |
| **5** | 8 | 28 | 125 | 17 | 19 | 725 | 27 | 8 | 33 | 10 |
| **6** | 5 | 21 | 148 | 10 | 25 | 24 | 722 | 21 | 10 | 14 |
| **7** | 16 | 32 | 26 | 12 | 82 | 17 | 53 | 625 | 89 | 48 |
| **8** | 11 | 37 | 93 | 42 | 8 | 30 | 46 | 7 | 702 | 24 |
| **9** | 8 | 50 | 91 | 4 | 51 | 30 | 19 | 31 | 39 | 677 |

### Observations

* The linear model was able to achieve basic discrimination among character classes.
* Misclassifications were common between visually similar classes such as:
  + "su" vs "ma" or "ha"
  + "na" vs "re"
  + "ki" vs "su"
* These confusions reflect the difficulty of linearly separating characters that have subtle stroke differences, especially when handwritten and down sampled to 28×28 resolution.
* Baseline result will serve as a point of comparison for deeper networks in following steps.

## Step 2 – Fully Connected Network (NetFull)

In this step, a fully connected two-layer neural network, NetFull was implemented, to classify the 10 classes of the Kuzushiji-MNIST dataset. The network consists of a hidden layer with a tunable number of units using the tanh activation function, followed by an output layer with log softmax. Training was done using the negative log-likelihood loss function for 10 epochs.  
  
To determine the optimal number of hidden units, the network was tested with multiple values in multiples of 10 (e.g. like 30, 60, 80, 90, 100, 110, 120, 140 and 160). Goal was to achieve at least 84% classification accuracy on the test set. The best result was obtained with 160 hidden units.

### Final Test Accuracy

Accuracy: 84.68% (8468 / 10000) with 160 hidden units.

### Confusion Matrix

Each row corresponds to the true label and each column to the predicted label. The labels correspond to the 10 Hiragana characters in the order:  
0="o", 1="ki", 2="su", 3="tsu", 4="na", 5="ha", 6="ma", 7="ya", 8="re", 9="wo".

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **0** | 857 | 1 | 2 | 4 | 24 | 26 | 4 | 42 | 31 | 9 |
| **1** | 7 | 820 | 37 | 2 | 16 | 9 | 62 | 5 | 20 | 22 |
| **2** | 8 | 20 | 820 | 50 | 8 | 21 | 25 | 13 | 20 | 15 |
| **3** | 4 | 6 | 29 | 925 | 2 | 15 | 5 | 2 | 7 | 5 |
| **4** | 43 | 29 | 18 | 7 | 810 | 6 | 27 | 20 | 25 | 15 |
| **5** | 9 | 11 | 68 | 12 | 12 | 840 | 27 | 1 | 15 | 5 |
| **6** | 3 | 14 | 42 | 9 | 17 | 5 | 897 | 8 | 1 | 4 |
| **7** | 19 | 10 | 16 | 4 | 26 | 11 | 34 | 818 | 29 | 33 |
| **8** | 11 | 25 | 25 | 51 | 4 | 8 | 29 | 3 | 837 | 7 |
| **9** | 3 | 18 | 44 | 8 | 29 | 6 | 20 | 17 | 11 | 844 |

### Parameter Count

Each input image in the Kuzushiji-MNIST dataset is 28 × 28 pixels = 784 input pixels.

In the hidden layer using **H hidden units**, we have 784 X H weights and H biases = (784+1) X H

In the output layer with 10 output classes, we have 10 X H weights and 10 biases = (H+1) X 10

Therefore, in total, for a fully connected feed forward neural network with H hidden units, we have:

(784+1) X H + (H+1) X 10 = **795 X H + 10 parameters**.

Therefore, the network with **160 hidden units** has 795 X 160 +10 = **127,210 parameters.**

While the one with 90 hidden units has 795 X 90 +10 = 71,560 parameters.

### Observations

* The network achieved over 84% accuracy with hidden layer sizes ≥ 90.
* Increasing hidden units improves model capacity, up to a point where gains become marginal.
* Misclassifications decreased for characters with more distinct features, such as “tsu” and “ha”, but some confusion remained between visually similar pairs such as “su” vs “ma”, and “na” vs “re”.
* This model performs significantly better than the linear model, validating the importance of non-linear representation.

## Step 3 – Convolutional Neural Network (NetConv)

## Network Architecture and Training Setup

In this step, a Convolutional Neural Network (CNN) named NetConv was implemented using PyTorch. The model architecture includes two convolutional layers followed by a fully connected (linear) layer, with ReLU as the activation function throughout, and a final output layer using LogSoftmax. Max pooling was applied to reduce spatial dimensions and increase robustness. The network was trained using the Stochastic Gradient Descent (SGD) optimizer with a learning rate of 0.01 and momentum of 0.9 for 10 epochs. Input images were normalized using transforms.Normalize((0.5,), (0.5,)).

### Final Test Accuracy

The network achieved a final test accuracy of 93.19% after 10 training epochs.

Test set: Average loss: 0.3071, Accuracy: 9319/10000 (93%)

### Confusion Matrix

Each row corresponds to the true label and each column to the predicted label. The labels correspond to the 10 Hiragana characters in the order:  
0="o", 1="ki", 2="su", 3="tsu", 4="na", 5="ha", 6="ma", 7="ya", 8="re", 9="wo".

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **0** | 935 | 6 | 1 | 1 | 29 | 5 | 1 | 11 | 6 | 5 |
| **1** | 3 | 906 | 8 | 1 | 10 | 1 | 45 | 10 | 8 | 8 |
| **2** | 6 | 6 | 907 | 30 | 12 | 9 | 10 | 3 | 4 | 13 |
| **3** | 1 | 0 | 12 | 967 | 3 | 4 | 4 | 4 | 1 | 4 |
| **4** | 17 | 7 | 4 | 12 | 925 | 0 | 15 | 5 | 13 | 2 |
| **5** | 3 | 8 | 44 | 6 | 4 | 895 | 24 | 3 | 4 | 9 |
| **6** | 2 | 2 | 15 | 5 | 7 | 0 | 963 | 4 | 1 | 1 |
| **7** | 3 | 3 | 3 | 0 | 6 | 2 | 10 | 952 | 8 | 13 |
| **8** | 2 | 7 | 10 | 7 | 8 | 3 | 11 | 1 | 950 | 1 |
| **9** | 11 | 1 | 14 | 4 | 14 | 0 | 13 | 8 | 16 | 919 |

### Parameter Count

The total number of independent parameters in the network was calculated by summing the weights and biases of all layers:

* **First Convolutional Layer (Conv1):**
  + **Input channels** = 1, **Output channels** = 32, **Kernel size** = 3×3
  + Each filter has: 1 × 3 × 3 = 9 weights
  + Total weights: 32 × 9 = **288**
  + Total biases: 32
  + **Total Conv1 parameters** = 288 + 32 = **320**
* **Second Convolutional Layer (Conv2):**
* **Input channels** = 32, **Output channels** = 64, **Kernel size** = 3×3
* Each filter: 32 × 3 × 3 = 288 weights
* Total weights: 64 × 288 = **18,432**
* Total biases: 64
* **Total Conv2 parameters** = 18,432 + 64 = **18,496**
* **Fully Connected Layer (FC1):**
  + Input: 64 channels × 14 × 14 = 12,544
  + Output: 128
  + Weights: 12,544 × 128 = **1,605,632**
  + Biases: 128
  + **Total FC1 parameters** = 1,605,632 + 128 = **1,605,760**
* **Output Layer:**
  + Input: 128, Output: 10
  + Weights: 128 × 10 = 1,280
  + Biases: 10
  + **Total output layer parameters** = 1,280 + 10 = **1,290**

**Total Parameters** = 320 + 18,496 + 1,605,760 + 1, 290 = **1,625,866**

## Step 4 – Comparative Discussion of Models

### Relative Accuracy of the Three Models

The accuracy achieved by the three networks—NetLin, NetFull, and NetConv—demonstrates a clear correlation between model complexity and classification performance on the Kuzushiji-MNIST dataset.

* **NetLin (Linear Model):** Achieved approximately 79% test accuracy. This model treats the input image as a flat vector and performs a simple weighted sum followed by log softmax. The absence of any non-linearity limits its capacity to model complex relationships among pixel values.
* **NetFull (Fully Connected Neural Network):** Achieved around 84% accuracy. This model adds one hidden layer with a tanh activation, allowing the network to capture non-linear relationships. However, since it still operates on flattened pixel values and ignores spatial structure, it is limited compared to CNNs.
* **NetConv (Convolutional Neural Network):** Achieved a consistent 93%+ accuracy. The model leverages convolutional layers, ReLU activations, and max pooling to capture local spatial patterns and hierarchical features. Its superior performance is due to its ability to extract translation-invariant and hierarchical features from the input images.

### Number of Independent Parameters in Each Model

The number of trainable parameters in a model is a key indicator of its complexity and capacity:

* **NetLin:** Linear Model
* Input layer: 28×28 = 784 pixels
* Output layer: 784 × 10 weights + 10 biases = **7,850** parameters
* **NetFull (with 160 hidden units):** Fully Connected Neural Network
* Input to hidden layer: 784 × 160 = 125,440
* Hidden layer biases: 160
* Hidden to output layer: 160 × 10 = 1,600
* Output layer biases: 10
* Total = 125,440 + 160 + 1,600 + 10 = **127,210** parameters
* **NetConv:** Convolutional Neural Network
* Conv1: 320
* Conv2: 18,496
* Fully Connected Layer: 1,605,760
* Output Layer: 1290
* Total = 320 + 18,496 + 1,605,760 + 1,290 = **1,625,866** parameters

**Convolutional Neural Network** (NetConv) can achieve higher accuracy with relatively fewer parameters, due to parameter sharing and local connectivity inherent in convolutional layers.

## 3. Confusion Matrix Analysis

Analyzing the confusion matrices of the three models reveals insights into common misclassifications:

**NetLin (Linear Model):**

Struggles to distinguish visually similar characters due to its limited capacity to capture patterns. Common confusions included:

* 'na' (**な**) mistaken as 'wo' (**を**)
* 'su' (**す**) mistaken as 'tsu' (つ)
* 're' (**れ**) mistaken as 'tsu' (つ)

Linear models fail to model spatial features, leading to overlap in classification boundaries.  
**NetFull (Fully Connected Neural Network):**

Improved differentiation with non-linear activations. Still showed significant misclassifications in cases like:

* 'ha' (**は**) vs 'su' (**す**)
* 'ya' (**や**) vs 're' (**れ**)
* 'wo' (**を**)vs 'na' (**な**)

Flattened inputs restrict spatial understanding, causing ambiguity in similar shapes.

**NetConv (Convolutional Neural Network):**  
Demonstrated the cleanest confusion matrix, with only a few minor confusions:

* 'na' (**な**) vs 'wo' (**を**): Similar looping strokes may mislead filters.
* 'su' (**す**)vs 'tsu' (つ) : Slight stroke variations between these can overlap when handwritten.
* 'ha' (**は**) vs 'su' (**す**): Shared horizontal and vertical line features.

These errors are expected given character similarity and handwriting variations.

Overall, convolutional models are far more robust in preserving spatial features, explaining their superior clarity in the confusion matrix.