MATHEMATICAL NOTATIONS

Basic set notations

$\mathbb{N}$	The set of natural numbers.
$\mathbb{R}$	The set of real numbers.
$\|S\|$	The size of a (finite) set S.
$\min S$	The smallest value found in set S.
$\max S$	The largest value found in set S.
$\forall$	The universal quantifier, used in statements such as "for all ...".
$\exists$	The existential quantifier, used in statements such as "there exists ...".
$x \in S$	Element x is a member of set S.
$V \backslash W$	The set V excluding elements that are also member of W.
$V \subseteq W$	Denotes that the set V is a subset of W, and possibly equal to W.
$V \subset W$	Denotes that V is a proper subset of W, i.e., $V \subseteq W$ and $V \neq W$.
$V \cap W$	The intersection of the two sets V and W.
$\bigcap_{i=1}^{n} V_i$	The intersection of n sets: $V_1 \cap V_2 \cap \cdots \cap V_n$
$V \cup W$	The union of the two sets V and W.
$\bigcup_{i=1}^{n} V_i$	The union of n sets: $V_1 \cup V_2 \cup \cdots \cup V_n$

General mathematical notations

$\lceil x \rceil$	The smallest natural number greater or equal to x.
$\lfloor x \rfloor$	The largest natural number smaller or equal to x.
$n!$	To be pronounced as n factorial: $n! \stackrel{\text{def}}{=} n \cdot (n-1) \cdot (n-2) \cdots 1$.
$n \gg k$	The fact that n is much larger than k.

$\sum$	Summation, such as $\sum_{i=1}^{n} x_i$, meaning $x_1 + x_2 + \cdots + x_n$.
Π	Multiplication, such as $\Pi_{i=1}^{n} x_i$, meaning $x_1 \times x_2 \times \cdots \times x_n$.
$[a_1, a_2, \ldots a_n]$	The (ordered) sequence of elements $a_1, a_2, \ldots, a_n$.
$x \leftarrow S$	x takes the value resulting from the expression S, pronounced as "x becomes S".
$f(x) \sim \mathcal{O}(g(x))$	$f(x)$ is bounded by $g(x)$: $\exists M \, \forall x > x_0 : \lvert f(x) \rvert < M \cdot \lvert g(x) \rvert$
$f(x) \sim \Omega(g(x)$	$f(x)$ is bounded from below by $g(x)$: $\exists M \, \forall x > x_0 : \lvert f(x) \rvert > M \cdot \lvert g(x) \rvert$. This also means that $g(x) \sim \mathcal{O}(f(x))$.
$f(x) \sim \Theta(g(x))$	$f(x)$ follows the same form as $g(x)$: $\exists M, M' \, \forall x > x_0 : M' \lvert g(x) \rvert < \lvert f(x) \rvert < M \lvert g(x) \rvert$.

General graph-theory notations

$G = (V, E)$	The undirected graph G with vertex set V and edge set E.
$\langle u, v \rangle$	The fact that vertex u and v are joined by an edge, that is, they are adjacent.
$\neg \langle u, v \rangle$	The fact that vertex u and v are *not* adjacent.
$D = (V, A)$	The directed graph D with vertex set V and arc set A.
$\langle \overrightarrow{u, v} \rangle$	The fact that vertex u and v are joined by an arc *from u to v*.
$G[V^*]$	The graph induced by the set of vertices $V^* \subseteq V(G)$.
$G[E^*]$	The graph induced by the set of edges $E^* \subseteq E(G)$.
$H \subseteq G$	H is a subgraph of G.
$G - v$	The graph induced by $V(G) \backslash \{v\}$.
$G - e$	The graph induced by $E(G) \backslash \{e\}$.
K_n	The complete graph on $n > 0$ vertices.
$K_{m,n}$	The complete bipartite graph with with two vertex sets of size m and n, respectively.
$\overline{G}$	The complement of graph G, i.e., the graph obtained from G by removing its edges and joining vertices that were nonadjacent in G.
$H_{k,n}$	A k-connected graph with n vertices and a minimal number of edges: a Harary graph.
$N(v)$	The set of neighbors of vertex v.
$N_{in}(v)$	The set of in-neighbors of vertex v.
$N_{out}(v)$	The set of out-neighbors of vertex v.

$\delta(v)$	The degree of vertex v, i.e., the number of incident edges.
$\delta_{in}(v)$	The indegree of vertex v, i.e., the number of incoming arcs at v.
$\delta_{out}(v)$	The outdegree of vertex v, i.e., the number of outgoing arcs from v.
$\Delta(G)$	The maximal degree of any vertex in graph G: $\max\{\delta(v) \mid v \in V(G)\}$.

Metrics on graphs

$d(u,v)$	The geodesic distance between vertex u and v. This is either a minimal-length (u,v)-path or a minimal-weight (u,v)-path..
$\epsilon(u)$	The eccentricity of vertex u: the maximum distance of u to any other vertex.
$\tau(G)$	The network transitivity of graph G: the ratio between the number of triangles and triples in G.
$c_C(u)$	The closeness of vertex u (in a graph G), measured as the reciproke of the total distance u has to the other vertices of G.
$c_B(u)$	The betweenness centrality of vertex u: the ratio of shortest paths between two vertices that go through u.
$c_E(u)$	The vertex centrality of u: the reciproke of its eccentricity.
$diam(G)$	The diameter of graph G: the length of the longest shortest path between any two vertices, i.e., the maximal eccentricity among the vertices of G.
$rad(G)$	The radius of graph G: the minimal eccentricity among its vertices.
$C(G)$	The center of graph G: the set of vertices for which the eccentricity is the same as the radius of G.
$cc(v)$	The clustering coefficient of vertex v.
$CC(G)$	The average clustering coefficient measured over all vertices of graph G.
$\omega(G)$	The number of components of graph G.
$\kappa(G)$	The size of a minimal vertex cut of graph G.
$\lambda(G)$	The size of a minimal edge cut of graph G.
$\chi'(G)$	The edge chromatic number of G: the minimal k for which graph G is k-edge colorable.
$\chi(G)$	The chromatic number of G: the minimal k for which graph G is k-vertex colorable.

Probabilities

$\mathbb{P}[\delta = k]$	The probability that the degree (of an arbitrarily chosen vertex) is equal to k.
$P[k]$	An abbreviation for $\mathbb{P}[\delta = k]$.
$\mathbb{E}[X]$	The expected value of the random variable X (often corresponding to the *mean*).

Special classes of graphs

$ER(n, p)$	The collection of Erdös-Rényi random graphs with n vertices and probability p that two distinct vertices are joined.
$WS(n, k, p)$	The collection of Watts-Strogatz random graphs with n vertices, initial vertex degree k and rewiring probability p.
$BA(n, n_0, m)$	The collection of Barabási-Albert random graphs with n vertices, n_0 initial vertices and a growth of m edges at each step.