

Example exam “Graph Theory”. Grade = $1 + 9 \cdot (\text{points}/\text{max points})$, Pass ≥ 5.5

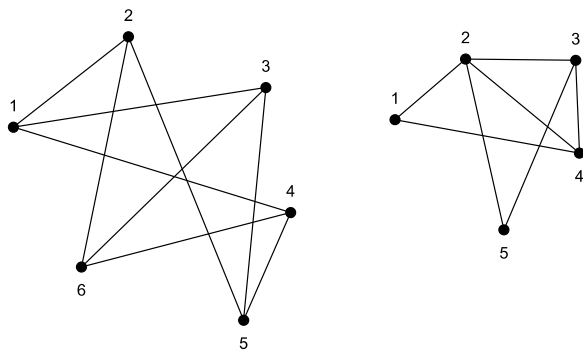
1) Model the following situations as (possibly weighted, possibly directed) graphs. Draw each graph, and give the corresponding adjacency matrices.

a) Ada and Bertrand are friends. Ada is also friends with Cecilia and David. Bertrand, Cecilia and Évariste are all friends of each other. [5]

b) It is well-known that in the Netherlands, there is a 2-lane highway from Amsterdam to Breda, another 2-lane highway from Amsterdam to Cappele aan den IJssel, a 3-lane highway from Breda to Dordrecht, a 1-lane road from Breda to Ede and another one from Dordrecht to Ede, and a 5-lane superhighway from Cappele aan den IJssel to Ede. [5]

2) Show that two graphs with the same degree sequence need not be isomorphic. [10]

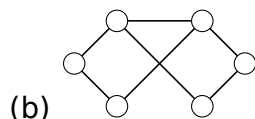
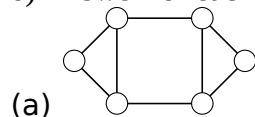
3) Draw the complement of the following two graphs: [10]



4) Show that it is not possible to have a group of seven people such that each person in the group knows exactly three other people in the group. [10]

5) Find the fewest vertices needed to construct a complete graph with at least 1000 edges. [10]

6) Answer for each of these graphs: Is it planar? Is it bipartite? [10]



7) Show that in a graph with n vertices, the length of a path cannot exceed $(n - 1)$ and the length of a cycle cannot exceed n [10]

8) Based on algorithm 3.1 (p.62) for the reachability of vertices, adapt it to the apply to the connectivity of an an *undirected* graph. [10]

9) Give an example of a simple, undirected graph G for which :

$$CC(G) \neq \rho(G)$$

Consider the case that all vertices of G have at least degree 2. [10]

Hints:

$$CC(G) \stackrel{\text{def}}{=} \frac{1}{|V|} \sum_{v \in V} cc(v) \quad (6.9)$$

$$\rho \stackrel{\text{def}}{=} m / \binom{n}{2} \quad (6.15)$$

10) Many social media companies hope to grow large quickly, and nudge you to make as many “friends” as possible to stimulate a so-called power-law dynamic in the network: the friends of people with lots of friends also have lots of friends. Explain what graphical method you would use to show such a dynamic at work. [10]