S.V.C.

① $V = \frac{4}{3}\pi r^3$

$r = 20cm$

$\frac{dV}{dt} = -10\ cm^3/sec$

~~$\frac{dV}{dr} = \dots$~~ We want to know $\frac{dr}{dt}$.

$\frac{dV}{dr} \cdot \frac{dr}{dt} = \frac{dV}{dt}$ (Chain Rule)

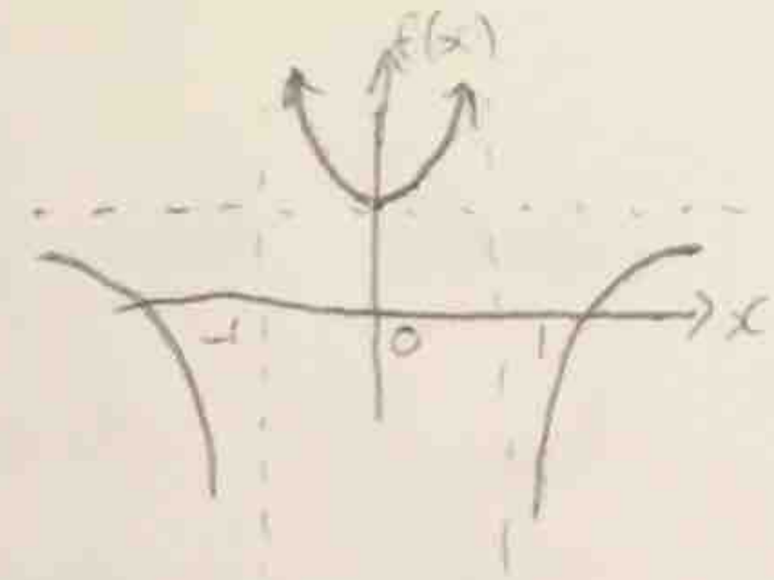
$3 \times \frac{4}{3}\pi r^2 \cdot \frac{dr}{dt} = -10$

$\frac{dr}{dt} = \frac{-10}{4\pi r^2}$

Since $r = 20$, $\frac{dr}{dt} = \frac{-10}{4\pi(20)^2} = -0.00199\ cm/s$

SVC

2a) Let $f(x) = \dfrac{1+x^2}{1-x^2}$



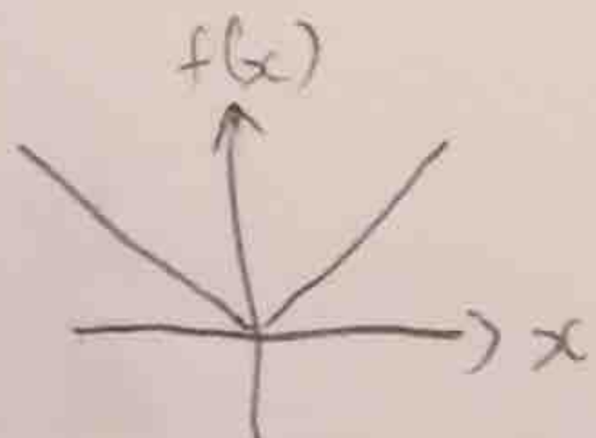
$f(x)$ is undefined when $1 - x^2 = 0$

ie $x = \pm 1$

$f(x)$ is ~~[illegible]~~ discontinuous if $\lim\limits_{x \to x_0} f(x) \neq f(x_0)$

Since $\lim\limits_{x \to -1} f(x)$ is undefined and $\lim\limits_{x \to 1} f(x)$ is also undefined, $f(x)$ is ~~[illegible]~~ discontinuous at $x = 1$ and $x = -1$.

b) Let $f(x) = |x| = \begin{cases} x & : x \geq 0 \\ -x & : x < 0 \end{cases}$



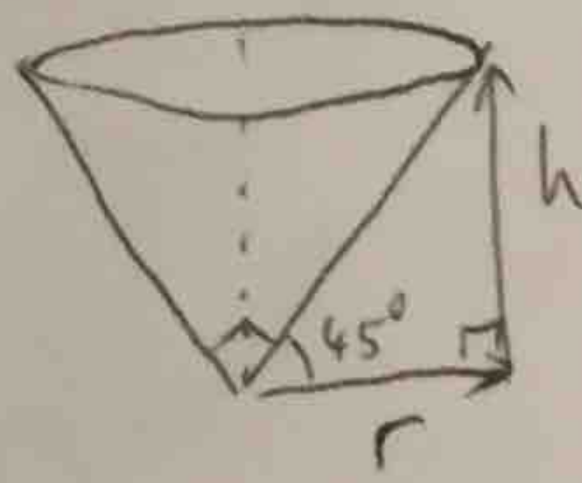
Then $\dfrac{d\,f(x)}{dx} = \begin{cases} 1 & : x \geq 0 \\ -1 & : x < 0 \end{cases}$

Since $\lim\limits_{x \to 0^+} \dfrac{d\,f(x)}{dx} = 1$ and $\lim\limits_{x \to 0^-} \dfrac{d f(x)}{dx} = -1$,

~~[illegible]~~ $\dfrac{d\,|x|}{dx}$ is ~~[illegible]~~ discontinuous when $x = 0$.

SVC.

3a)



Volume of a cone $= \frac{1}{3}\pi r^2 h$

Since the vertex is $90°$, $r = h$

$\Rightarrow V = \frac{1}{3}\pi h^3$

We know $\frac{dV}{dt} = 3 \text{ cm}^3/\text{sec}$

$h = 2\text{cm}$

We want to know $\frac{dh}{dt}$

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt} \quad \text{(Chain Rule)}$$

$$3 = 3 \times \frac{1}{3}\pi h^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{3}{\pi h^2}$$

$\therefore$ At $h = 2\text{cm}$, $\frac{dh}{dt} = \frac{3}{\pi(2)^2} = \frac{3}{4\pi} = 0.239 \text{ cm}/\text{sec}$

SVC

3b) We want to know $\frac{dh}{dt}$ of the water in the cone due to evaporation. I am going to assume that ~~the~~ the rate of evaporation of water is directly proportional to the surface area of a body of water that is exposed to air. This is a reasonable assumption given that you can reasonably imagine that it would take much longer for a tall glass of water to evaporate compared to the same volume of water dispersed onto a towel which is then hung up to dry; clearly the towel dries faster because the surface area of water exposed to air is much higher.

$\therefore$ Assume $\frac{dV}{dt} = c \times$ surface area of water exposed to air cm^3/sec.

where c is some constant.

For a cone, the surface area of water exposed to air is πr^2

$$\therefore \frac{dv}{dt} = c\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt} \quad \text{(Chain Rule)}$$

$$c\pi r^2 = 3 \times \frac{1}{3} \pi h^2 \frac{dh}{dt}$$

Since the vertex is a right angle, $r = h$

ie $\quad c\pi h^2 = \pi h^2 \frac{dh}{dt}$

$$\Rightarrow \frac{dh}{dt} = c$$

$\therefore$ The water falls at a constant rate.

SUC

(4) $\int_0^1 \frac{x \cancel{dx}}{(1+3x^2)^{\frac{1}{2}}} \, dx$

<u>Guess</u> $y = (1+3x^2)^{\frac{1}{2}}$.

then $\frac{dy}{dx} = \frac{1}{2} \times \frac{6x}{(1+3x^2)^{1/2}} = \frac{3x}{(1+3x^2)^{1/2}}$

$\rightarrow$ ~~the~~ y is 3 times too large.

$\therefore \int_0^1 \frac{x}{(1+3x^2)^{\frac{1}{2}}} \, dx = \left. \frac{1}{3}(1+3x^2)^{\frac{1}{2}} + c \right]_0^1$

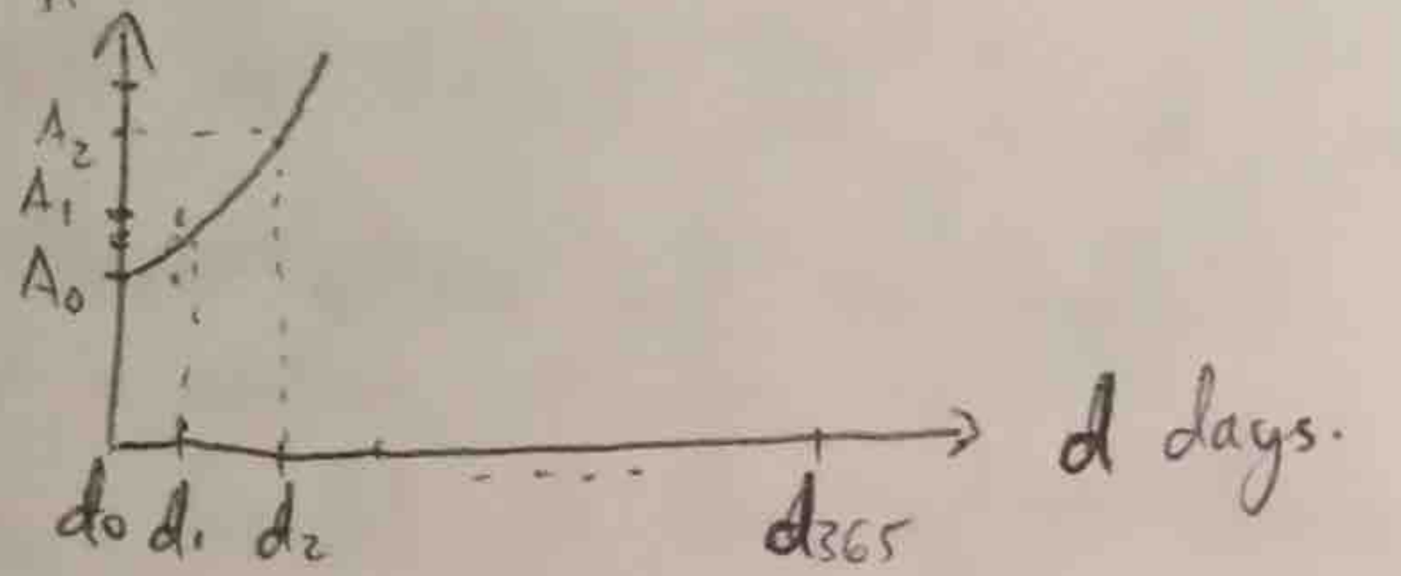
$= \frac{1}{3}(1+3(1)^2)^{\frac{1}{2}} + \cancel{c} - \left(\frac{1}{3}(1+3(0)^2)^{\frac{1}{2}} + \cancel{c}\right)$

$= \frac{2}{3} - \frac{1}{3}$

$= \frac{1}{3}$

S.U.C.

5 $A(t) = A_0 e^{rt}$



Let d_i be the i^{th} number of days and A_i be the amount on the i^{th} day.

$$A_0 = A(d_0) = A_0 e^{rd_0} = A_0 e^{r\times 0} = A_0 \times 1 = A_0.$$

$$A_1 = A(d_1) = A_0 e^{rd_1} + \frac{k}{365} = A_0 e^{\frac{r}{365}} + \frac{k}{365}$$

$$A_2 = A(d_2) = A_1 e^{rd_2} + \frac{k}{365} = \left(A_0 e^{\frac{r}{365}} + \frac{k}{365}\right) e^{\frac{r}{365}} + \frac{k}{365}$$

$$= A_0 e^{\frac{r}{365}} e^{\frac{r}{365}} + \frac{k}{365} e^{\frac{r}{365}} + \frac{k}{365}$$

$$= A_0 \left(e^{\frac{r}{365}}\right)^2 + \frac{k}{365} e^{\frac{r}{365}} + \frac{k}{365}$$

$\vdots$

$$A_{365} = A_0 \left(e^{\frac{r}{365}}\right)^{365} + \frac{k}{365}\left(e^{\frac{r}{365}}\right)^{364} + \cdots + \frac{k}{365} e^{\frac{r}{365}} + \frac{k}{365}$$

$$= A_0 e^{r} + \frac{k}{365}\left(e^{\frac{364r}{365}} + e^{\frac{363}{365}r} + \cdots e^{\frac{r}{365}} + 1\right)$$

$$= A_0 e^{r} + \frac{k}{365} \sum_{i=0}^{364} \left(e^{\frac{r}{365}}\right)^i$$

$$= A_0 e^{r} + k \sum_{i=0}^{364} \left(e^{\frac{r}{365}}\right)^i \Delta t \text{ where } \Delta t = \frac{1}{365}.$$

$$\boxed{\text{As } \Delta t \to 0,\ A_{365} = A_0 e^{r} + k \int_0^{364} e^{\frac{r}{365}}\, dt}$$

(6) The volume of a thin cylindrical shell is given by $dV = 2\pi r h \, dx$ where

dV = volume of the thin shell.
r = radius of the cylinder
h = height of the cylinder
dx = thickness of the shell.

The height h is given by $y = e^x$ and $r = x$.

$$\therefore \frac{dv}{dx} = 2\pi x e^x$$

The volume of the solid is $\int_0^1 \frac{dv}{dx} \, dx$

$$= \int_0^1 2\pi x e^x \, dx$$

$$= 2\pi \int_0^1 x e^x \, dx.$$

Integrating xe^x by parts we get

$$V = 2\pi \times \left(xe^x \cancel{+} - \int_0^1 1 \times e^x \, dx \right)$$

$$= 2\pi \times \left(xe^x \cancel{+} - e^x \right]_0^1)$$

$$= 2\pi \left(1e^1 \cancel{+} - e^1 - (0e^0 \cancel{+} - e^0) \right)$$

$$= 2\pi (\quad - 0 - (0 - 1))$$

$$\therefore \boxed{V = 2\pi \text{ units}^3.}$$

(8) For every dollar she raises the price, ~~they~~ 10 less people attend.

$\therefore \quad R(x) = (1200 - 10x)(50 + x)$

$= 60000 + 1200x - 500x - 10x^2$

$= 60000 + 700x - 10x^2.$

Revenue will be maximised when $R'(x) = 0$.

ie $\quad R'(x) = 700 - 100x = 0$

$\Rightarrow x = \cancel{\$}7$

If the promoter raises the price by \$7, she will attain the maximum revenue. The revenue will be.

$R(7) = 60000 + 700(7) - 10(7^2)$

$= 60000 + 4900 - 490$

$= \$64410.$

(9) Revenue $= pq$

$p = 300 - 0.02q$ $\therefore$ Revenue $= (300 - 0.02q)q$

$= 300q - 0.02q^2$.

Cost $= 9000 + 30q$

Profit $=$ Revenue $-$ Costs

$= 300q - 0.02q^2 - (9000 + 30q)$

$P = -9000 + 270q - 0.02q^2$

Profit is maximised when $\frac{dP}{dq} = 0$

ie $\frac{dP}{dq} = 270 - 0.04q = 0$

$\Rightarrow q = \frac{270}{0.04}$

$= 6750.$

$\therefore$ When $q = 6750$, Profit $= -9000 + 270(6750) - 0.02(6750)^2$

$= -9000 + 1822500 - 911250$

$= \$902{,}250$