CFD张量公式

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张量公式

$$\nabla p = \begin{bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{bmatrix}$$

$$\nabla \cdot (\nabla p) = \nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}$$

$$\mathbf{U} \cdot \mathbf{V} = [u_1, u_2, u_3] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$$\mathbf{U}\mathbf{V} = \mathbf{U} \otimes \mathbf{V} = \begin{bmatrix} u_1 v_1 & u_1 v_2 & u_1 v_3 \\ u_2 v_1 & u_2 v_2 & u_2 v_3 \\ u_3 v_1 & u_3 v_2 & u_3 v_3 \end{bmatrix}$$

$$\mathbf{U} \times \mathbf{V} = \begin{bmatrix} u_2 v_3 - u_3 v_2 \\ u_3 v_1 - u_1 v_3 \\ u_1 v_2 - u_2 v_1 \end{bmatrix}$$

$$\nabla \cdot \mathbf{U} = \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z}$$

$$\nabla \mathbf{U} = \begin{bmatrix} \frac{\partial u_1}{\partial x} & \frac{\partial u_2}{\partial x} & \frac{\partial u_3}{\partial x} \\ \frac{\partial u_1}{\partial y} & \frac{\partial u_2}{\partial y} & \frac{\partial u_3}{\partial y} \\ \frac{\partial u_1}{\partial z} & \frac{\partial u_2}{\partial z} & \frac{\partial u_3}{\partial z} \end{bmatrix}$$

$$\nabla \cdot (\nabla \mathbf{U}) = \begin{bmatrix} \frac{\partial}{\partial x}\left(\frac{\partial u_1}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u_1}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial u_1}{\partial z}\right) \\ \frac{\partial}{\partial x}\left(\frac{\partial u_2}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u_2}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial u_2}{\partial z}\right) \\ \frac{\partial}{\partial x}\left(\frac{\partial u_3}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u_3}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial u_3}{\partial z}\right) \end{bmatrix}$$

$$\nabla \times \mathbf{U} = \begin{bmatrix} \frac{\partial u_3}{\partial y} - \frac{\partial u_2}{\partial z} \\ \frac{\partial u_1}{\partial z} - \frac{\partial u_3}{\partial x} \\ \frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y} \end{bmatrix}$$

$$\mathbf{U} + \mathbf{V} = \mathbf{V} + \mathbf{U}$$

$$\alpha \mathbf{U} = \mathbf{U} \alpha$$

$$\mathbf{U} \cdot \mathbf{V} = \mathbf{V} \cdot \mathbf{U}$$

$$\mathbf{U} \times \mathbf{V} = -\mathbf{V} \times \mathbf{U}$$

$$\mathbf{U} \times (\mathbf{V} \times \mathbf{W}) \neq (\mathbf{U} \times \mathbf{V}) \times \mathbf{W}$$

$$\nabla \cdot (\nabla \times \mathbf{U}) = 0$$

$$\nabla \times \nabla \alpha = 0$$

$$\nabla(\alpha p) = \alpha \nabla p + p \nabla \alpha$$

$$\nabla \cdot (\alpha \mathbf{U}) = \alpha \nabla \cdot \mathbf{U} + \mathbf{U} \cdot \nabla \alpha = \alpha \nabla \cdot \mathbf{U} + \nabla \alpha \cdot \mathbf{U}$$

$$\nabla \times (\alpha \mathbf{U}) = \alpha \nabla \times \mathbf{U} + (\nabla \alpha) \times \mathbf{U}$$

$$\nabla (\alpha \mathbf{U} \cdot \mathbf{V}) = \alpha \mathbf{U} \cdot \nabla \mathbf{V} + \mathbf{V} \cdot \nabla \alpha \mathbf{U}$$

$$\nabla \alpha \mathbf{U} = \alpha \nabla \mathbf{U} + \mathbf{U} \nabla \alpha$$

$$\nabla \cdot (\mathbf{U}\mathbf{U}) = \mathbf{U} \cdot \nabla \mathbf{U} + \mathbf{U} \nabla \cdot \mathbf{U}$$

$$\nabla \cdot (\alpha \tau) = \tau \cdot \nabla \alpha + \alpha \nabla \cdot \tau$$

$$\mathbf{tr}(\nabla \mathbf{U}) \mathbf{I} = \mathbf{tr}\left(\nabla \mathbf{U}^{\mathrm{T}}\right) \mathbf{I} = (\nabla \cdot \mathbf{U}) \mathbf{I}$$

$$\mathbf{tr}\left(\nabla \mathbf{U} + \nabla \mathbf{U}^{\mathrm{T}}\right) \mathbf{I} = 2 \mathbf{tr}(\nabla \mathbf{U}) \mathbf{I} = 2 (\nabla \cdot \mathbf{U}) \mathbf{I}$$

$$\nabla \cdot (\nabla \mathbf{U})^{T} = \nabla (\nabla \cdot \mathbf{U})$$

$$\nabla \cdot ((\nabla \cdot \mathbf{U}) \mathbf{I}) = \nabla (\nabla \cdot \mathbf{U})$$

$$\nabla \cdot \boldsymbol{\tau} = \begin{bmatrix} \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \end{bmatrix}$$

$$\boldsymbol{\tau} : \boldsymbol{\tau} = \tau_{11}\tau_{11} + \tau_{12}\tau_{21} + \tau_{13}\tau_{31} + \tau_{21}\tau_{12} + \tau_{22}\tau_{22} + \tau_{23}\tau_{32} + \tau_{31}\tau_{13} + \tau_{32}\tau_{23} + \tau_{33}\tau_{33}$$

$$|\boldsymbol{\tau}| = \sqrt{\boldsymbol{\tau} : \boldsymbol{\tau}}$$

$$|\boldsymbol{\tau}|^2 = \boldsymbol{\tau} : \boldsymbol{\tau}$$

$$|\nabla \nabla \mathbf{U}|^2 = \left(\frac{\partial u_i}{\partial x_j \partial x_k}\right)\left(\frac{\partial u_i}{\partial x_j \partial x_k}\right) = |\nabla \nabla u_1|^2 + |\nabla \nabla u_2|^2 + |\nabla \nabla u_3|^2$$

$$= \begin{vmatrix} \frac{\partial u_1}{\partial x \partial x}, \frac{\partial u_1}{\partial x \partial y}, \frac{\partial u_1}{\partial x \partial z} \\ \frac{\partial u_1}{\partial y \partial x}, \frac{\partial u_1}{\partial y \partial y}, \frac{\partial u_1}{\partial y \partial z} \\ \frac{\partial u_1}{\partial z \partial x}, \frac{\partial u_1}{\partial z \partial y}, \frac{\partial u_1}{\partial z \partial z} \end{vmatrix}^2 + \begin{vmatrix} \frac{\partial u_2}{\partial x \partial x}, \frac{\partial u_2}{\partial x \partial y}, \frac{\partial u_2}{\partial x \partial z} \\ \frac{\partial u_2}{\partial y \partial x}, \frac{\partial u_2}{\partial y \partial y}, \frac{\partial u_2}{\partial y \partial z} \\ \frac{\partial u_2}{\partial z \partial x}, \frac{\partial u_2}{\partial z \partial y}, \frac{\partial u_2}{\partial z \partial z} \end{vmatrix}^2 + \begin{vmatrix} \frac{\partial u_3}{\partial x \partial x}, \frac{\partial u_3}{\partial x \partial y}, \frac{\partial u_3}{\partial x \partial z} \\ \frac{\partial u_3}{\partial y \partial x}, \frac{\partial u_3}{\partial y \partial y}, \frac{\partial u_3}{\partial y \partial z} \\ \frac{\partial u_3}{\partial z \partial x}, \frac{\partial u_3}{\partial z \partial y}, \frac{\partial u_3}{\partial z \partial z} \end{vmatrix}^2$$

OpenFOAM基本运算

$\boldsymbol{\tau}$为 `tensor` 二阶张量，$\mathbf{U}, \mathbf{V}$为 `vector` 矢量，$\boldsymbol{a}, \boldsymbol{b}$为 `scalar` 标量。

$$\texttt{dev(tau)} = \boldsymbol{\tau} - \frac{1}{3} \mathbf{tr}(\boldsymbol{\tau}) \mathbf{I}$$

$$\texttt{dev2(tau)} = \boldsymbol{\tau} - \frac{2}{3} \mathbf{tr}(\boldsymbol{\tau}) \mathbf{I}$$

$$\texttt{symm(gradU)} = \frac{\nabla \mathbf{U} + \nabla \mathbf{U}^T}{2}$$

$$\texttt{twoSymm(gradU)} = \nabla \mathbf{U} + \nabla \mathbf{U}^T$$

$$\texttt{dev(twoSymm(gradU))} = \nabla \mathbf{U} + \nabla \mathbf{U}^T - \frac{1}{3} \mathbf{tr}\left(\nabla \mathbf{U} + \nabla \mathbf{U}^T\right) \mathbf{I} = \nabla \mathbf{U} + \nabla \mathbf{U}^T - \frac{2}{3} (\nabla \cdot \mathbf{U}) \mathbf{I}$$

$$\texttt{dev(symm(gradU))} = \frac{\nabla \mathbf{U} + \nabla \mathbf{U}^T}{2} - \frac{1}{3} \mathbf{tr}\left(\frac{\nabla \mathbf{U} + \nabla \mathbf{U}^T}{2}\right) \mathbf{I} = \frac{\nabla \mathbf{U} + \nabla \mathbf{U}^T}{2} - \frac{1}{3} (\nabla \cdot \mathbf{U}) \mathbf{I}$$

$$\texttt{tr(tau)} = \tau_{xx} + \tau_{yy} + \tau_{zz}$$

$$\texttt{sph(tau)} = \frac{1}{3} (\tau_{xx} + \tau_{yy} + \tau_{zz})$$

$$\texttt{skew(gradU)} = \frac{\nabla \mathbf{U} - \nabla \mathbf{U}^T}{2}$$

$$\texttt{magSqrGradGrad(U)} = |\nabla \nabla \mathbf{U}|^2$$

$$\texttt{det(tau)} = |\boldsymbol{\tau}|$$

$$\texttt{innerSqr(tau)} = \tau\cdot\tau = \begin{bmatrix} \tau_{xx}\tau_{xx}+\tau_{xy}\tau_{xy}+\tau_{xz}\tau_{xz}, \tau_{xx}\tau_{xy}+\tau_{xy}\tau_{yy}+\tau_{xz}\tau_{yz}, \tau_{xx}\tau_{xz}+\tau_{xy}\tau_{yz}+\tau_{xz}\tau_{zz} \\ \tau_{xx}\tau_{xy}+\tau_{xy}\tau_{yy}+\tau_{xz}\tau_{yz}, \tau_{xy}\tau_{xy}+\tau_{yy}\tau_{yy}+\tau_{yz}\tau_{yz}, \tau_{xy}\tau_{xz}+\tau_{yy}\tau_{yz}+\tau_{yz}\tau_{zz} \\ \tau_{xx}\tau_{xz}+\tau_{xy}\tau_{yz}+\tau_{xz}\tau_{zz}, \tau_{xy}\tau_{xz}+\tau_{yy}\tau_{yz}+\tau_{yz}\tau_{zz}, \tau_{xz}\tau_{xz}+\tau_{yz}\tau_{yz}+\tau_{zz}\tau_{zz} \end{bmatrix}$$

$$\texttt{cof(tau)} = \begin{bmatrix} \tau_{yy}\tau_{zz}-\tau_{zy}\tau_{yz} & \tau_{zx}\tau_{yz}-\tau_{yx}\tau_{zz} & \tau_{yx}\tau_{zy}-\tau_{yy}\tau_{zx} \\ \tau_{xz}\tau_{zy}-\tau_{xy}\tau_{zz} & \tau_{xx}\tau_{zz}-\tau_{xz}\tau_{zx} & \tau_{xy}\tau_{zx}-\tau_{xx}\tau_{zy} \\ \tau_{xy}\tau_{yz}-\tau_{xz}\tau_{yy} & \tau_{yx}\tau_{xz}-\tau_{xx}\tau_{yz} & \tau_{xx}\tau_{yy}-\tau_{yx}\tau_{xy} \end{bmatrix}$$

$$\texttt{inv(tau)} = \tau^{-1}$$

$$\texttt{invariantI(tau)} = \mathrm{tr}(\tau)$$

$$\texttt{invariantII(tau)} = \tau_{xx}\tau_{yy}+\tau_{yy}\tau_{zz}+\tau_{xx}\tau_{zz}-\tau_{xy}\tau_{yx}-\tau_{yz}\tau_{zy}-\tau_{xz}\tau_{zx}$$

$$\texttt{invariantIII(tau)} = \det(\tau)$$

$$\texttt{tau.T()} = \tau^T$$

$$\texttt{U \& V} = \mathbf{U}\cdot\mathbf{V}$$

$$\texttt{U ^ V} = \mathbf{U}\times\mathbf{V}$$

$$\texttt{U * V} = \mathbf{U}\mathbf{V}$$

$$\texttt{tau \& tau} = \tau\cdot\tau$$

$$\texttt{tau \&\& tau} = \tau:\tau$$

$$\texttt{sign(a)}\ \mathrm{sgn}(a)$$

$$\texttt{log(a)} = \ln(a)$$

$$\texttt{log10(a)} = \log(a)$$

流体单位

力 $\mathbf{F}\,\frac{\mathrm{kg\cdot m}}{\mathrm{s^2}}$

压力 $p\,\frac{\mathrm{kg}}{\mathrm{ms^2}}$

湍流动能 $k\,\frac{\mathrm{m^2}}{\mathrm{s^2}}$

湍流动能耗散率 $\varepsilon\,\frac{\mathrm{m^2}}{\mathrm{s^3}}$

湍流频率 $\omega\,\frac{1}{\mathrm{s}}$

运动粘度 $\nu\,\frac{\mathrm{m^2}}{\mathrm{s}}$

动力粘度 $\mu\,\frac{\mathrm{kg}}{\mathrm{m\cdot s}}$

加速度 $\mathbf{A}\,\frac{\mathrm{m}}{\mathrm{s^2}}$

动量$m\mathbf{U}\ \frac{\mathrm{kg\cdot m}}{\mathrm{s}}$

动量密度$\rho\mathbf{U}\ \frac{\mathrm{kg}}{\mathrm{m^2s}}$

比气体常数$R\ \frac{\mathrm{m^2}}{\mathrm{s^2T}}$

定容比热容$C_v\ \frac{\mathrm{m^2}}{\mathrm{s^2T}}$

东岳流体 2014 - 2020

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