

# 10

## MECHANICS OF MATERIALS

### MECHANICS OF MATERIALS-1

Where do stress concentrations occur?

- I. near the points of application of concentrated loads
- II. along the entire length of high distributed loads
- III. at discontinuities

(A) I and II      (B) I and III      (C) II and III      (D) I, II, and III

Stress concentrations occur under concentrated loads and at discontinuities, not under distributed loads.

The answer is (B).

### MECHANICS OF MATERIALS-2

What is the definition of normal strain,  $\epsilon$ ? ( $\delta$  is elongation, and  $L$  is the length of the specimen.)

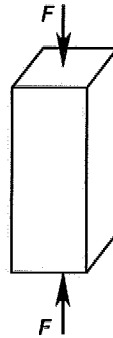
(A)  $\epsilon = \frac{L + \delta}{L}$       (B)  $\epsilon = \frac{L + \delta}{\delta}$       (C)  $\epsilon = \frac{\delta}{L + \delta}$       (D)  $\epsilon = \frac{\delta}{L}$

Strain is defined as elongation per unit length.

The answer is (D).

**MECHANICS OF MATERIALS-3**

The column shown has a cross-sectional area of  $13 \text{ m}^2$ . What can the approximate maximum load be if the compressive stress cannot exceed  $9.6 \text{ kPa}$ ?



- (A) 120 kN      (B) 122 kN      (C) 125 kN      (D) 130 kN

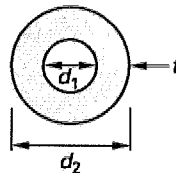
The equation for axial stress is

$$\begin{aligned}\sigma &= \frac{F}{A} \\ F &= \sigma A \\ &= \left( 9.6 \frac{\text{kN}}{\text{m}^2} \right) (13 \text{ m}^2) \\ &= 124.8 \text{ kN} \quad (125 \text{ kN})\end{aligned}$$

The answer is (C).

**MECHANICS OF MATERIALS-4**

A copper column of annular cross section has an outer diameter,  $d_2$ , of 5 m, and is subjected to an axial loading of 200 kN. The allowable compressive stress is  $14.4 \text{ kPa}$ . The wall thickness,  $t$ , should be most nearly



- (A) 0.5 m      (B) 0.8 m      (C) 1 m      (D) 2 m

For axial stress,

$$\sigma = \frac{F}{A}$$

Then,

$$\begin{aligned} A &= \frac{F}{\sigma} = \frac{\pi}{4}(d_2^2 - d_1^2) \\ d_1 &= \sqrt{d_2^2 - \frac{4F}{\pi\sigma}} \\ &= \sqrt{(5 \text{ m})^2 - \frac{(4)(200 \text{ kN})}{\pi \left(14.4 \frac{\text{kN}}{\text{m}^2}\right)}} \\ &= 2.7 \text{ m} \end{aligned}$$

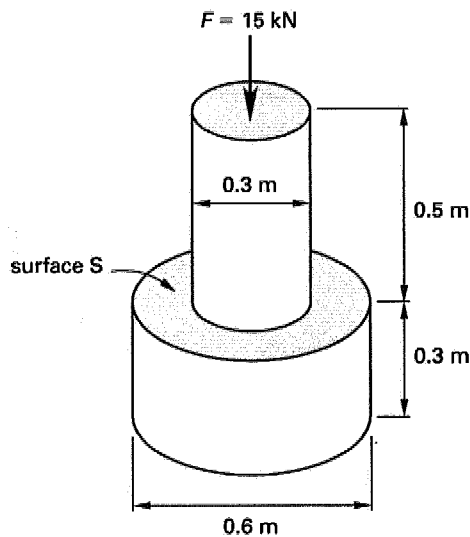
Therefore,

$$t = \frac{d_2 - d_1}{2} = \frac{5 \text{ m} - 2.7 \text{ m}}{2} = 1.15 \text{ m} \quad (1 \text{ m})$$

The answer is (C).

### MECHANICS OF MATERIALS-5

What is most nearly the stress at surface S of the cylindrical object shown? The specific weight of the material is  $\gamma = 76.9 \text{ kN/m}^3$ .



- (A) 100 kPa      (B) 150 kPa      (C) 200 kPa      (D) 250 kPa

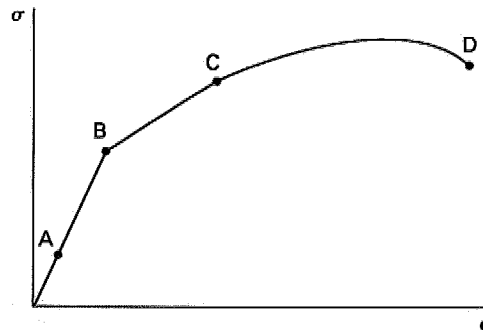
The stress at surface S is due to the weight of the material above it in addition to the force  $F$ . The total load is

$$\begin{aligned} F_{\text{total}} &= W + F = \gamma V + F \\ &= \left( 76.9 \frac{\text{kN}}{\text{m}^3} \right) \left( \frac{\pi}{4} \right) (0.3 \text{ m})^2 (0.5 \text{ m}) + 15 \text{ kN} \\ &= 17.72 \text{ kN} \\ \sigma &= \frac{F_{\text{total}}}{A} = \frac{17.72 \text{ kN}}{\left( \frac{\pi}{4} \right) (0.3 \text{ m})^2} \\ &= 250.7 \text{ kN/m}^2 \quad (250 \text{ kPa}) \end{aligned}$$

The answer is (D).

#### MECHANICS OF MATERIALS-6

Considering the stress-strain diagram for aluminum, which point is the fracture point?



- (A) A                      (B) B                      (C) C                      (D) D

Point D is where fracture occurs.

The answer is (D).

**MECHANICS OF MATERIALS-7**

In a stress-strain diagram, what is the correct term for the stress level at  $\epsilon = 0.2\%$  offset?

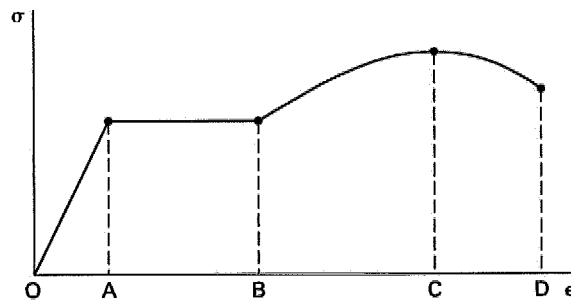
- (A) the elastic limit
- (B) the plastic limit
- (C) the offset rupture stress
- (D) the offset yield stress

This is known as the offset yield stress.

The answer is (D).

**MECHANICS OF MATERIALS-8**

Consider this stress-strain diagram for a carbon steel in tension. Determine the region of perfect plasticity or yielding.



- (A) O to A      (B) A to B      (C) B to C      (D) C to D

The plastic region is between points A and B. O to A is known as the linear region, B to C is where strain hardening occurs, and C to D is where reduction in area occurs.

The answer is (B).

**MECHANICS OF MATERIALS-9**

Under which type of loading does fatigue occur?

- (A) static load                      (B) plane load  
(C) high load                      (D) repeated load

Fatigue occurs under repeated loading cycles.

The answer is (D).

**MECHANICS OF MATERIALS-10**

A specimen is subjected to a load. When the load is removed, the strain disappears. From this information, which of the following can be deduced about this material?

- (A) It is elastic.  
(B) It is plastic.  
(C) It has a high modulus of elasticity.  
(D) It does not obey Hooke's law.

By definition, elasticity is the property of a material by which it returns to its original dimensions during unloading.

The answer is (A).

**MECHANICS OF MATERIALS-11**

Which of the following may be the Poisson ratio of a material?

- (A) 0.35                      (B) 0.52                      (C) 0.55                      (D) 0.60

The Poisson ratio must be in the range  $0 < \nu < 0.5$ . Option (A) is the only answer that satisfies this condition.

The answer is (A).

**MECHANICS OF MATERIALS-12**

A 2 m long aluminum bar (modulus of elasticity = 70 GPa) is subjected to a tensile stress of 175 MPa. Find the elongation.

- (A) 3.5 mm      (B) 5.0 mm      (C) 7.5 mm      (D) 9.0 mm

From Hooke's law,

$$\epsilon = \frac{\sigma}{E} = \frac{\delta}{L}$$

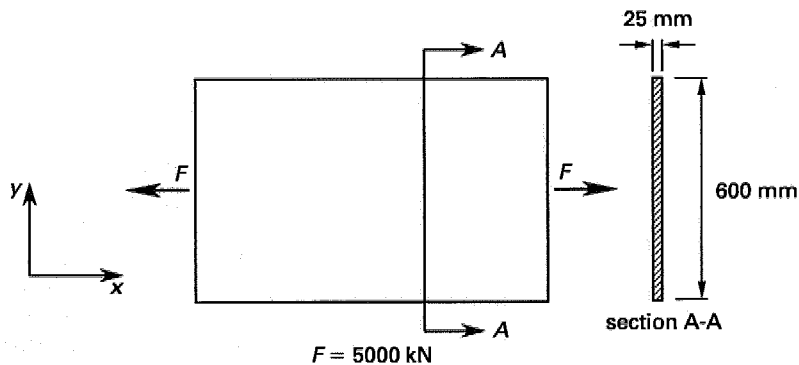
$$\delta = \frac{\sigma L}{E} = \frac{\left(175 \times 10^6 \frac{\text{N}}{\text{m}^2}\right) (2 \text{ m})}{70 \times 10^9 \frac{\text{N}}{\text{m}^2}}$$

$$= 0.005 \text{ m} \quad (5.0 \text{ mm})$$

The answer is (B).

**MECHANICS OF MATERIALS-13**

A 600 mm tall thin plate is placed in tension by a 5000 kN force as shown. What is the height ( $y$  direction) of the plate while tension is applied? The modulus of elasticity,  $E$ , is 200 GPa, and Poisson's ratio,  $\nu$ , is 0.3. Assume the load is distributed uniformly across the plate and the yield strength is not exceeded.



- (A) 599.7 mm      (B) 599.9 mm      (C) 600.2 mm      (D) 600.5 mm

The Poisson ratio is defined as the negative ratio of lateral strain,  $\epsilon_y$ , to axial strain,  $\epsilon_x$ . Using this and the equation for axial stress and strain,

$$\nu = -\frac{\epsilon_y}{\epsilon_x}$$

$$\epsilon_y = -\nu\epsilon_x \quad [I]$$

$$\epsilon_x = \frac{\sigma}{E} = \frac{F}{EA} \quad [II]$$

Combining equations I and II,

$$\begin{aligned} \epsilon_y &= -\frac{\nu F}{EA} = -\frac{(0.3)(5000 \text{ kN})}{\left(200 \times 10^6 \frac{\text{kN}}{\text{m}^2}\right)(0.015 \text{ m}^2)} \\ &= -0.0005 \end{aligned}$$

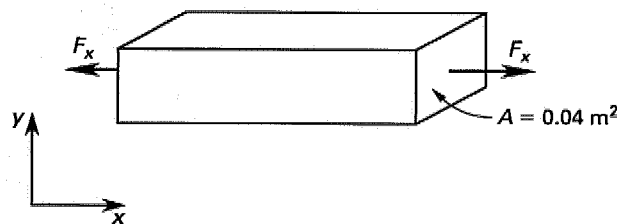
Therefore, the width while the plate is in tension is

$$\begin{aligned} w &= 600 \text{ mm} - \delta_y \\ &= 600 \text{ mm} - (0.0005)(600 \text{ mm}) \\ &= 599.7 \text{ mm} \end{aligned}$$

The answer is (A).

#### MECHANICS OF MATERIALS-14

What is most nearly the lateral strain,  $\epsilon_y$ , of the steel specimen shown if  $F_x = 3000 \text{ kN}$ ,  $E = 193 \text{ GPa}$ , and  $\nu = 0.29$ ?



- (A)  $-4 \times 10^{-4}$       (B)  $-1 \times 10^{-4}$       (C)  $1 \times 10^{-4}$       (D)  $4 \times 10^{-4}$

From Hooke's law and the equation for axial stress,

$$\begin{aligned}\epsilon_x &= \frac{\sigma_x}{E} = \frac{F_x}{EA} = \frac{3000 \text{ kN}}{\left(193 \times 10^6 \frac{\text{kN}}{\text{m}^2}\right) (0.04 \text{ m}^2)} \\ &= 3.89 \times 10^{-4}\end{aligned}$$

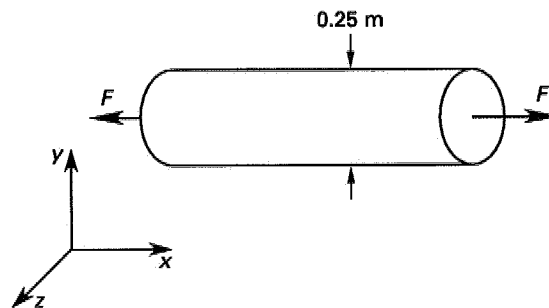
Use Poisson's ratio.

$$\begin{aligned}\epsilon_y &= -\nu\epsilon_x = -(0.29)(3.89 \times 10^{-4}) \\ &= -1.13 \times 10^{-4} \quad (-1 \times 10^{-4})\end{aligned}$$

The answer is (B).

### MECHANICS OF MATERIALS-15

A steel specimen is subjected to a tensile force,  $F$ , of 2000 kN. If Poisson's ratio,  $\nu$ , is 0.29 and the modulus of elasticity,  $E$ , is 193 GPa, the dilatation,  $e$ , is most nearly



- (A)  $6.5 \times 10^{-5}$     (B)  $8.8 \times 10^{-5}$     (C)  $8.8 \times 10^{-4}$     (D)  $6.5 \times 10^{-4}$

Dilatation is defined as the sum of the strain in all three coordinate directions. In the axial  $z$  direction,

$$\epsilon_z = \frac{F}{EA} = \frac{2000 \text{ kN}}{\left(193 \times 10^6 \frac{\text{kN}}{\text{m}^2}\right) (0.049 \text{ m}^2)} = 2.1 \times 10^{-4}$$

From Poisson's ratio,

$$\begin{aligned}\nu &= -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z} \\ \epsilon_x &= \epsilon_y = -\nu\epsilon_z \\ &= -(0.29)(2.1 \times 10^{-4}) \\ &= -6.09 \times 10^{-5}\end{aligned}$$

Therefore,

$$\begin{aligned}e &= \epsilon_x + \epsilon_y + \epsilon_z \\ &= (2.1 \times 10^{-4}) + (2)(-6.09 \times 10^{-5}) \\ &= 8.82 \times 10^{-5} \quad (8.8 \times 10^{-5})\end{aligned}$$

The answer is (B).

### MECHANICS OF MATERIALS-16

Given a shear stress of  $\tau_{xy} = 35\,000$  kPa and a shear modulus of  $G = 75$  GPa, the shear strain is most nearly

- (A)  $2.5 \times 10^{-5}$  rad      (B)  $4.7 \times 10^{-4}$  rad  
(C)  $5.5 \times 10^{-4}$  rad      (D)  $8.3 \times 10^{-4}$  rad

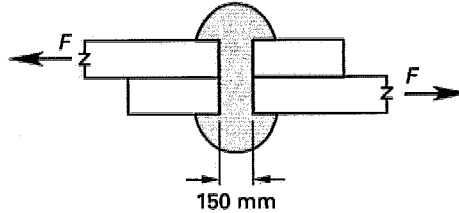
Hooke's law for shear gives

$$\begin{aligned}\gamma &= \frac{\tau_{xy}}{G} = \frac{35\,000 \frac{\text{kN}}{\text{m}^2}}{75 \times 10^6 \frac{\text{kN}}{\text{m}^2}} \\ &= 4.67 \times 10^{-4} \text{ rad} \quad (4.7 \times 10^{-4} \text{ rad})\end{aligned}$$

The answer is (B).

## MECHANICS OF MATERIALS-17

A 150 mm diameter rivet resists a shear force of  $V = 8$  kN. Find the average shear stress in the rivet.



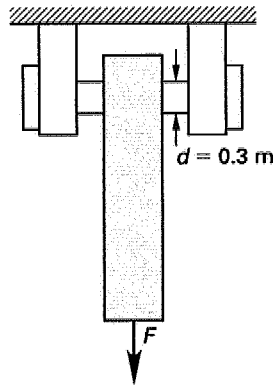
- (A) 230 kPa      (B) 370 kPa      (C) 430 kPa      (D) 450 kPa

$$\tau = \frac{V}{A} = \frac{8 \text{ kN}}{\left(\frac{\pi}{4}\right)(0.150 \text{ m})^2} = 452.7 \text{ kN/m}^2 \quad (450 \text{ kPa})$$

The answer is (D).

## MECHANICS OF MATERIALS-18

A steel bar carrying a 3000 kN load,  $F$ , is attached to a support by a round pin 0.3 m in diameter. What is most nearly the average shear stress in the pin?



- (A) 10 MPa      (B) 12 MPa      (C) 21 MPa      (D) 25 MPa

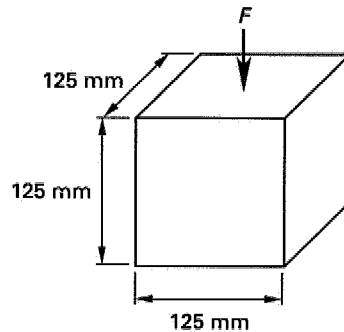
The pin will shear on two cross sections.

$$\tau = \frac{F}{2A} = \frac{3000 \text{ kN}}{(2) \left( \frac{\pi}{4} \right) (0.3 \text{ m})^2} = 21\,221 \text{ kN/m}^2 \quad (21 \text{ MPa})$$

The answer is (C).

### MECHANICS OF MATERIALS-19

What is most nearly the maximum allowable load,  $F$ , if the factor of safety is 1.5 and the compressive yield stress,  $\sigma_{\text{yield}}$ , is 20 670 kPa?



- (A) 220 kN      (B) 240 kN      (C) 300 kN      (D) 420 kN

$$\begin{aligned} \sigma_{\text{allowable}} &= \frac{\sigma_{\text{yield}}}{\text{SF}} \\ &= \frac{20\,670 \frac{\text{kN}}{\text{m}^2}}{1.5} = 13\,780 \text{ kN/m}^2 \end{aligned}$$

$$\begin{aligned} F &= \sigma_{\text{allowable}} A \\ &= \left( 13\,780 \frac{\text{kN}}{\text{m}^2} \right) (0.125 \text{ m})^2 \\ &= 215.3 \text{ kN} \quad (220 \text{ kN}) \end{aligned}$$

The answer is (A).

**MECHANICS OF MATERIALS-20**

The allowable tensile stress for a 6.25 mm diameter bolt with a thread length of 5.5 mm is 207 MPa. The allowable shear stress of the material is 103 MPa. Where and how will such a bolt be most likely to fail if placed in tension? (Assume threads are perfectly triangular and that the force is carried at the mean thread height.)

- (A) at the root diameter due to tension
- (B) at the threads due to shear
- (C) at the root diameter due to shear
- (D) at the threads due to tension

The bolt will most likely fail due to shearing of the threads or due to tensile failure of the bolt diameter.

$$\begin{aligned}F_{\text{allowable,thread}} &= \tau_{\text{allowable}}(\text{average shear area}) \\&= \tau_{\text{allowable}} \left( \frac{1}{2} \pi d h \right) \\&= \left( 103\,000 \frac{\text{kN}}{\text{m}^2} \right) \left( \frac{1}{2} \pi \right) (0.006\,25 \text{ m})(0.0055 \text{ m}) \\&= 5.56 \text{ kN} \\F_{\text{allowable,root}} &= \sigma_{\text{allowable}}(\text{root area}) \\&= \left( 207\,000 \frac{\text{kN}}{\text{m}^2} \right) \left( \frac{\pi}{4} \right) (0.006\,25 \text{ m})^2 \\&= 6.35 \text{ kN}\end{aligned}$$

The shear stress in the threads will exceed the allowable stress before the tensile load becomes excessive.

The answer is (B).

**MECHANICS OF MATERIALS-21**

Hexagonal nuts for 6.25 mm diameter bolts have a height of 5.5 mm. If the ultimate strength of the nut material in shear is 103 MPa, what is most nearly the maximum allowable shear force on the nut threads using a safety factor of 5?

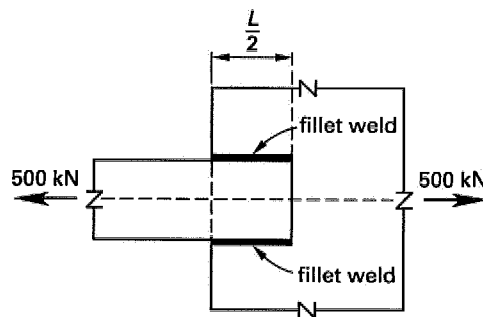
- (A) 0.72 kN
- (B) 0.8 kN
- (C) 1.0 kN
- (D) 1.1 kN

$$\begin{aligned}
 \tau_{\text{allowable}} &= \frac{\tau}{\text{SF}} \\
 &= \frac{103\,000 \frac{\text{kN}}{\text{m}^2}}{5} \\
 &= 20\,600 \text{ kN/m}^2 \\
 V &= \tau_{\text{allowable}} A = \tau_{\text{allowable}} \left( \frac{1}{2} \pi d h \right) \\
 &= \left( 20\,600 \frac{\text{kN}}{\text{m}^2} \right) \left( \frac{1}{2} \pi \right) (0.006\,25 \text{ m})(0.0055 \text{ m}) \\
 &= 1.11 \text{ kN} \quad (1.1 \text{ kN})
 \end{aligned}$$

The answer is (D).

### MECHANICS OF MATERIALS-22

Determine the total length,  $L$ , of the fillet weld for the lap joint shown. The weld has to resist a tension,  $F$ , of 500 kN. The effective throat for the weld,  $h$ , is 12 mm, and the allowable stress is 145 MPa.



- (A) 247 mm      (B) 252 mm      (C) 287 mm      (D) 312 mm

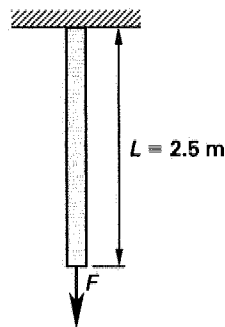
For a fillet weld, the average normal stress is

$$\begin{aligned}
 \sigma &= \frac{F}{hL} \\
 L &= \frac{F}{\sigma h} = \frac{500 \text{ kN}}{\left( 145\,000 \frac{\text{kN}}{\text{m}^2} \right) (0.012 \text{ m})} \\
 &= 0.287 \text{ m} \quad (287 \text{ mm})
 \end{aligned}$$

The answer is (C).

## MECHANICS OF MATERIALS-23

What is most nearly the elongation of the aluminum bar (cross section of 3 cm  $\times$  3 cm) shown in the figure when loaded to its yield point?  $E = 69$  GPa, and  $\sigma_{\text{yield}} = 255$  MPa. Neglect the weight of the bar.



- (A) 3.3 mm      (B) 9.3 mm      (C) 12 mm      (D) 15 mm

From Hooke's law, the axial strain is

$$\epsilon = \frac{\sigma}{E} = \frac{255 \times 10^6 \text{ Pa}}{69 \times 10^9 \text{ Pa}} = 0.0037$$

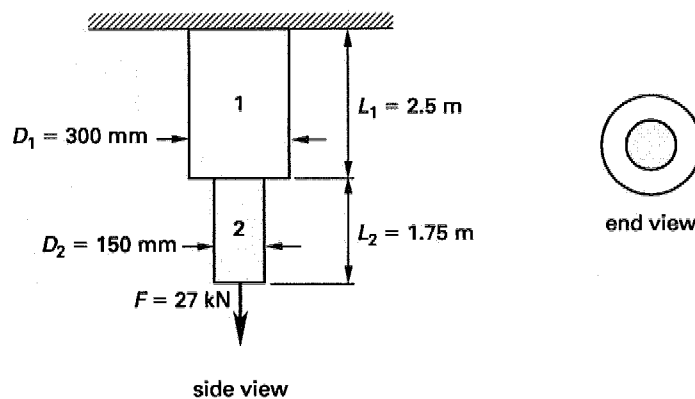
The total elongation is

$$\delta = \epsilon L = (0.0037)(2.5 \text{ m}) = 0.00925 \text{ m} \quad (9.3 \text{ mm})$$

The answer is (B).

## MECHANICS OF MATERIALS-24

What is most nearly the total elongation of the rod shown if  $E = 69$  GPa? Neglect bending.



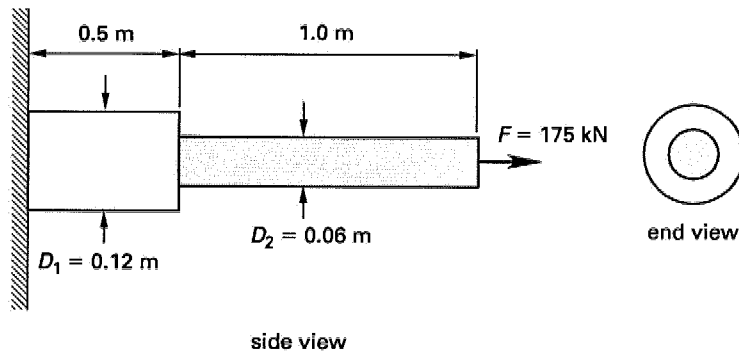
- (A) 0.01 mm      (B) 0.05 mm      (C) 0.2 mm      (D) 1.2 mm

$$\begin{aligned}
 \delta_{\text{total}} &= \frac{FL_1}{EA_1} + \frac{FL_2}{EA_2} = \frac{F}{E} \left( \frac{L_1}{A_1} + \frac{L_2}{A_2} \right) \\
 &= \frac{4F}{\pi E} \left( \frac{L_1}{D_1^2} + \frac{L_2}{D_2^2} \right) \\
 &= \left( \frac{(4)(27 \text{ kN})}{\pi \left( 69 \times 10^6 \frac{\text{kN}}{\text{m}^2} \right)} \right) \left( \frac{2.5 \text{ m}}{(0.3 \text{ m})^2} + \frac{1.75 \text{ m}}{(0.15 \text{ m})^2} \right) \\
 &= 5.26 \times 10^{-5} \text{ m} \quad (0.05 \text{ mm})
 \end{aligned}$$

The answer is (B).

### MECHANICS OF MATERIALS-25

What is most nearly the total elongation of this composite body under a force of 27 kN?  $E_1 = 70 \text{ GPa}$ , and  $E_2 = 100 \text{ GPa}$ .



- (A) 0.075 mm      (B) 0.73 mm      (C) 1.2 mm      (D) 3.0 mm

Total elongation is the elongation of section 1 plus the elongation of section 2.

$$\begin{aligned}
 \delta_{\text{total}} &= \delta_1 + \delta_2 = \frac{FL_1}{A_1 E_1} + \frac{FL_2}{A_2 E_2} \\
 &= \frac{(175 \text{ kN})(0.5 \text{ m})}{\left( \frac{\pi}{4} \right) (0.12 \text{ m})^2 \left( 70 \times 10^6 \frac{\text{kN}}{\text{m}^2} \right)} + \frac{(175 \text{ kN})(1.0 \text{ m})}{\left( \frac{\pi}{4} \right) (0.06 \text{ m})^2 \left( 100 \times 10^6 \frac{\text{kN}}{\text{m}^2} \right)} \\
 &= 7.29 \times 10^{-4} \text{ m} \quad (0.73 \text{ mm})
 \end{aligned}$$

The answer is (B).

**MECHANICS OF MATERIALS-26**

A 200 m cable is suspended vertically. At any point along the cable, the strain is proportional to the length of the cable below that point. If the strain at the top of the cable is 0.001, determine the total elongation of the cable.

- (A) 0.050 m      (B) 0.10 m      (C) 0.15 m      (D) 0.20 m

Since the strain is proportional to the cable length, it varies from 0 at the end to the maximum value of 0.001 at the supports. The average strain is

$$\epsilon_{\text{ave}} = \frac{\epsilon_{\text{max}}}{2} = \frac{0.001}{2} = 0.0005$$

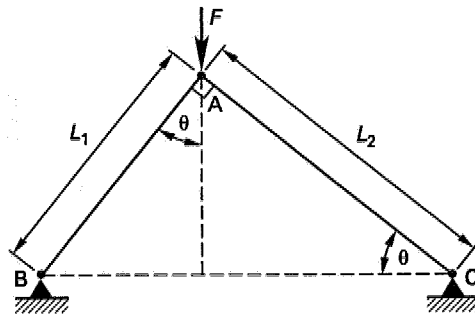
The total elongation is

$$\delta = \epsilon_{\text{ave}} L = (0.0005)(200 \text{ m}) = 0.10 \text{ m}$$

The answer is (B).

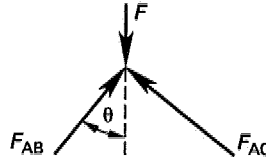
**MECHANICS OF MATERIALS-27**

The figure shows a two-member truss with a load  $F = 50\,000$  kN applied statically. Given that  $L_1 = 1.2$  m,  $L_2 = 1.5$  m, and each member's cross-sectional area,  $A$ , is  $4000 \text{ mm}^2$ , what is most nearly the elongation of member AB after  $F$  is applied? Use  $E = 200$  GPa.



- (A) -59 mm      (B) -48 mm      (C) -36 mm      (D) -23 mm

A free-body diagram of joint A gives



$$R_{AB} = F \cos \theta = F \frac{L_2}{\sqrt{L_1^2 + L_2^2}}$$

$$= \frac{(50\,000 \text{ kN})(1.5 \text{ m})}{\sqrt{(1.2 \text{ m})^2 + (1.5 \text{ m})^2}} = 39\,043 \text{ kN} \quad [\text{AB is in compression}]$$

$$F_{AB} = -R_{AB} = -39\,043 \text{ kN}$$

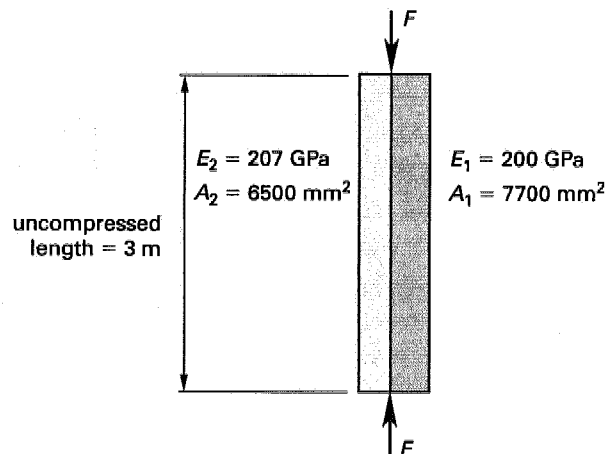
$$\delta_{AB} = \frac{F_{AB} L_1}{EA} = \frac{(-39\,043 \text{ kN})(1.2 \text{ m})}{\left(200 \times 10^6 \frac{\text{kN}}{\text{m}^2}\right)(0.004 \text{ m}^2)}$$

$$= -0.0586 \text{ m} \quad (-59 \text{ mm})$$

The answer is (A).

### MECHANICS OF MATERIALS-28

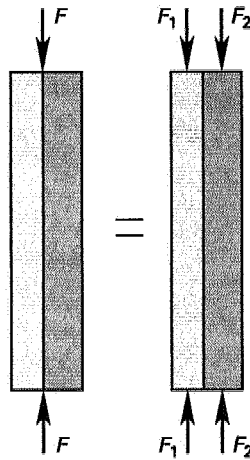
The two bars shown are perfectly bonded to a common face to form an assembly. The bars have moduli of elasticity and areas as given. If a force of  $F = 1300 \text{ kN}$  compresses the assembly, what is most nearly the reduction in length?



- (A) 1.2 mm      (B) 1.4 mm      (C) 1.5 mm      (D) 1.6 mm

From the principle of compatability, both bars are compressed the same length.

$$\begin{aligned}\epsilon_1 &= \frac{\sigma_1}{E_1} \\ &= \frac{F_1}{A_1 E_1} \\ \epsilon_2 &= \frac{\sigma_2}{E_2} \\ &= \frac{F_2}{A_2 E_2}\end{aligned}$$



Since  $\epsilon_1 = \epsilon_2$ ,

$$\begin{aligned}\frac{F_1}{A_1 E_1} &= \frac{F_2}{A_2 E_2} \\ F_1 &= \left( \frac{A_1 E_1}{A_2 E_2} \right) F_2\end{aligned}\quad [\text{I}]$$

From a force balance,

$$\begin{aligned}F_1 + F_2 &= 1300 \text{ kN} \\ F_1 &= 1300 \text{ kN} - F_2\end{aligned}\quad [\text{II}]$$

Combining equations I and II,

$$1300 \text{ kN} - F_2 = \left( \frac{A_1 E_1}{A_2 E_2} \right) F_2$$

$$1300 \text{ kN} = \left( 1 + \frac{A_1 E_1}{A_2 E_2} \right) F_2$$

$$F_2 = \frac{1300 \text{ kN}}{1 + \frac{A_1 E_1}{A_2 E_2}}$$

$$= \frac{1300 \text{ kN}}{1 + \frac{(7700 \text{ mm}^2)(200 \text{ GPa})}{(6500 \text{ mm}^2)(207 \text{ GPa})}}$$

$$= 606.2 \text{ kN}$$

$$\epsilon_2 = \frac{606.2 \text{ kN}}{(0.0065 \text{ m}^2) \left( 207 \times 10^6 \frac{\text{kN}}{\text{m}^2} \right)}$$

$$= 4.51 \times 10^{-4}$$

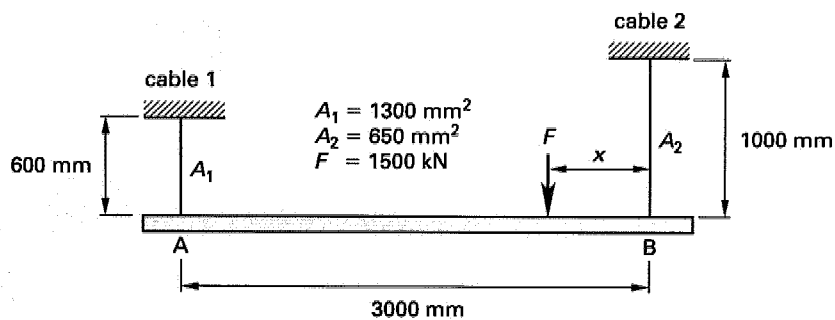
$$\delta = \epsilon L = (4.51 \times 10^{-4})(3 \text{ m})$$

$$= 0.00135 \text{ m} \quad (1.4 \text{ mm})$$

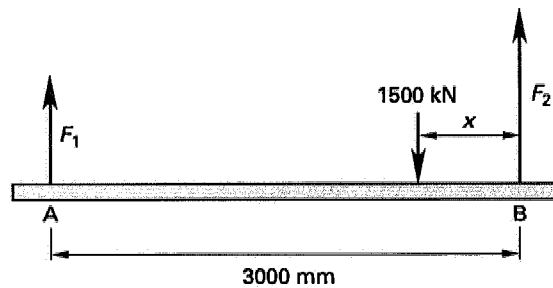
The answer is (B).

### MECHANICS OF MATERIALS-29

A rigid weightless bar is suspended horizontally by cables 1 and 2 as shown. The cross-sectional areas of the cables are given in the figure. The modulus of elasticity,  $E$ , is the same for both cables. If a concentrated load of  $F = 1500 \text{ kN}$  is applied between points A and B, what is most nearly the distance,  $x$ , for the bar to remain horizontal?



- (A) 1300 mm      (B) 1600 mm      (C) 1900 mm      (D) 2300 mm



From the free-body diagram, taking moments about point B gives

$$\begin{aligned}\sum M_B = 0 &= (1500 \text{ kN})x - (3000 \text{ mm})F_1 \\ (1500 \text{ kN})x &= (3000 \text{ mm})F_1 \\ x &= \left(2 \frac{\text{mm}}{\text{kN}}\right) F_1\end{aligned}\quad [\text{I}]$$

From a vertical force balance,

$$F_1 + F_2 = 1500 \text{ kN} \quad [\text{II}]$$

For the bar to remain horizontal, the deflection of cable 1 must equal the deflection of cable 2.

$$\begin{aligned}\delta_1 &= \delta_2 \\ \frac{F_1 L_1}{EA_1} &= \frac{F_2 L_2}{EA_2} \\ F_1 &= \frac{L_2 A_1}{L_1 A_2} F_2 = \frac{(1000 \text{ mm})(1300 \text{ mm}^2)}{(600 \text{ mm})(650 \text{ mm}^2)} F_2 \\ &= 3.33 F_2\end{aligned}\quad [\text{III}]$$

Solving equations II and III simultaneously,

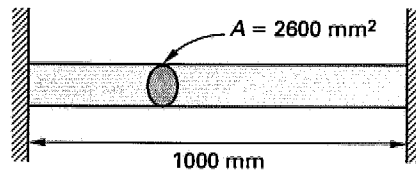
$$\begin{aligned}3.33 F_2 + F_2 &= 1500 \text{ kN} \\ 4.33 F_2 &= 1500 \text{ kN} \\ F_2 &= 346.4 \text{ kN} \\ F_1 &= 1153.6 \text{ kN}\end{aligned}$$

$$\begin{aligned}
 x &= \left(2 \frac{\text{mm}}{\text{kN}}\right) F \\
 &= \left(2 \frac{\text{mm}}{\text{kN}}\right) (1153.6 \text{ kN}) \\
 &= 2307 \text{ mm} \quad (2300 \text{ mm})
 \end{aligned}$$

The answer is (D).

### MECHANICS OF MATERIALS-30

A prismatic bar at  $10^\circ\text{C}$  is constrained in a rigid concrete wall at both ends. The bar is 1000 mm long and has a cross-sectional area of  $2600 \text{ mm}^2$ . What is most nearly the axial force in the bar if the temperature is raised to  $40^\circ\text{C}$ ?



$E$  = modulus of elasticity  
 $= 200 \text{ GPa}$   
 $\alpha$  = coefficient of thermal expansion  
 $= 9.4 \times 10^{-6}/^\circ\text{C}$

- (A) 116 kN      (B) 125 kN      (C) 134 kN      (D) 147 kN

Elongation due to temperature change is given by

$$\begin{aligned}
 \delta &= \alpha L (T_2 - T_1) \\
 &= \left(9.4 \times 10^{-6} \frac{1}{^\circ\text{C}}\right) (1000 \text{ mm}) (40^\circ\text{C} - 10^\circ\text{C}) \\
 &= 0.282 \text{ mm}
 \end{aligned}$$

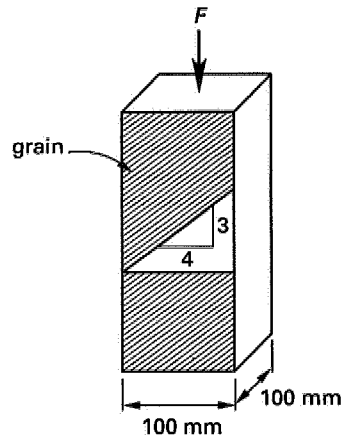
Elongation is

$$\begin{aligned}
 \delta &= \frac{FL}{EA} \\
 F &= \frac{\delta EA}{L} = \frac{(0.000282 \text{ m}) \left(200 \times 10^6 \frac{\text{kN}}{\text{m}^2}\right) (0.0026 \text{ m}^2)}{1 \text{ m}} \\
 &= 146.6 \text{ kN} \quad (147 \text{ kN})
 \end{aligned}$$

The answer is (D).

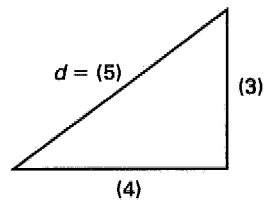
## MECHANICS OF MATERIALS-31

What is most nearly the maximum axial load,  $F$ , that can be applied to the wood post shown without exceeding a maximum shear stress of 1650 kPa parallel to the grain?



- (A) 22 kN      (B) 33 kN      (C) 44 kN      (D) 57 kN

The length of the diagonal parallel to the grain,  $d$ , (part of a 3-4-5 triangle) is



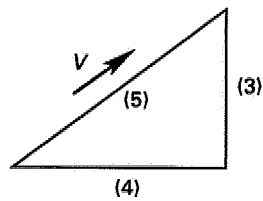
$$d = \left(\frac{5}{4}\right)(100 \text{ mm}) = 125 \text{ mm} \quad (0.125 \text{ m})$$

The area of the inclined plane is

$$A = (0.125 \text{ m})(0.100 \text{ m}) = 0.0125 \text{ m}^2$$

The total shear on the plane is

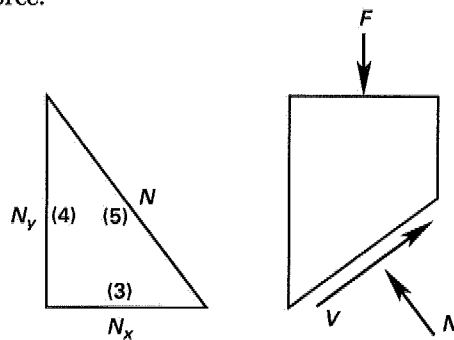
$$V = \tau A = \left(1650 \frac{\text{kN}}{\text{m}^2}\right)(0.0125 \text{ m}^2) = 20.63 \text{ kN}$$



The horizontal component of the shear is

$$V_x = \left(\frac{4}{5}\right) (20.63 \text{ kN}) = 16.5 \text{ kN}$$

Draw the free-body diagram of the upper section. Include the normal compressive force.



Balancing the  $x$ -components,

$$\sum F_x = 0 = V_x - N_x = 0$$

$$N_x = \frac{3}{5}N = V_x$$

$$\frac{3}{5}N = 16.5 \text{ kN}$$

$$N = \left(\frac{5}{3}\right) (16.5 \text{ kN}) = 27.5 \text{ kN}$$

Balancing the  $y$ -components,

$$\sum F_y = 0 = N_y - F = 0$$

$$F = N_y = \frac{4}{5}N = \left(\frac{4}{5}\right) (27.5 \text{ kN})$$

$$= 22 \text{ kN}$$

The answer is (A).

**MECHANICS OF MATERIALS-32**

The shear strain,  $\epsilon$ , along a shaft is

$$\epsilon = r \frac{d\phi}{dx}$$

$r$  is the radius from the shaft's centerline, and  $d\phi/dx$  is the change of the angle of twist with respect to the axis of the shaft. Which condition is NOT necessary for the above equation to be valid?

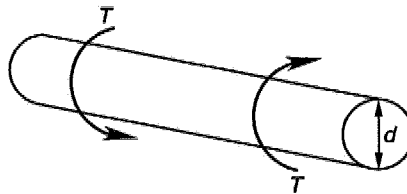
- (A) The area of interest must be free of connections and other load applications.
- (B) The material must be isotropic and homogeneous.
- (C) The loading must result in the stress being a torsional couple acting along the axis.
- (D)  $r$  must be the full radius of the shaft.

The equation may be evaluated for any value of  $r$ , giving the stress distribution over the shaft cross section.

The answer is (D).

**MECHANICS OF MATERIALS-33**

A 3 m diameter bar experiences a torque of 280 N·m. What is most nearly the maximum shear stress in the bar?



- (A) 2.2 Pa
- (B) 31 Pa
- (C) 42 Pa
- (D) 53 Pa

Maximum shear stress occurs at the outer surface.

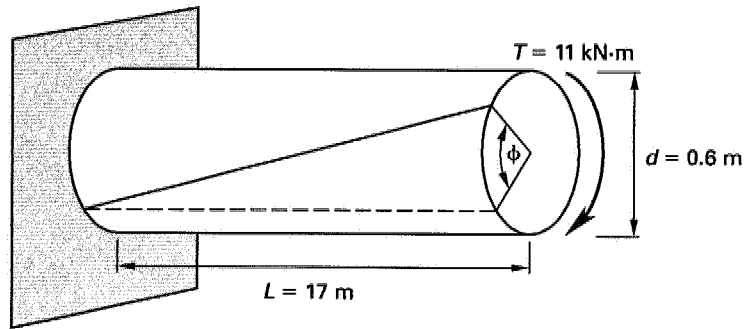
The equation for shear gives

$$\begin{aligned} \tau &= \frac{Tr}{J} = \frac{T \left( \frac{d}{2} \right)}{\frac{\pi}{32} d^4} = \frac{(280 \text{ N}\cdot\text{m}) \left( \frac{3 \text{ m}}{2} \right)}{\left( \frac{\pi}{32} \right) (3 \text{ m})^4} \\ &= 52.8 \text{ N/m}^2 \quad (53 \text{ Pa}) \end{aligned}$$

The answer is (D).

**MECHANICS OF MATERIALS-34**

What is most nearly the angle of twist,  $\phi$ , for the aluminum bar shown? The shear modulus of elasticity,  $G$ , is 26 GPa.



- (A)  $0.00055^\circ$       (B)  $0.0055^\circ$       (C)  $0.032^\circ$       (D)  $0.082^\circ$

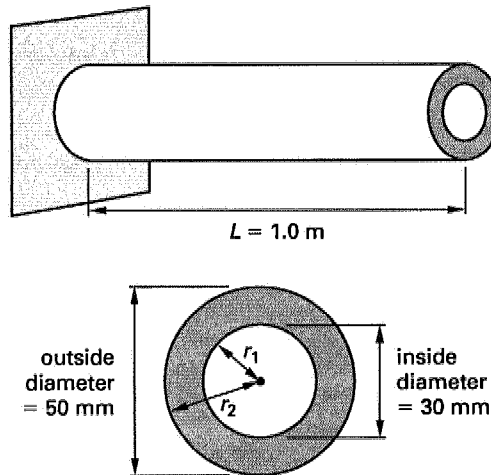
The angle of twist is given by

$$\begin{aligned}\phi &= \frac{TL}{GJ} = \frac{(11 \text{ kN}\cdot\text{m})(17 \text{ m})}{\left(26 \times 10^6 \frac{\text{kN}}{\text{m}^2}\right) \left(\frac{\pi}{32}\right) (0.6 \text{ m})^4} \\ &= (0.000565 \text{ rad}) \left(\frac{180^\circ}{\pi \text{ rad}}\right) \\ &= 0.032^\circ\end{aligned}$$

The answer is (C).

## MECHANICS OF MATERIALS-35

What torque,  $T$ , should be applied to the end of the steel shaft shown in order to produce a twist of  $1.5^\circ$ ? Use  $G = 80 \text{ GPa}$  for the shear modulus.



- (A)  $420 \text{ N}\cdot\text{m}$       (B)  $560 \text{ N}\cdot\text{m}$       (C)  $830 \text{ N}\cdot\text{m}$       (D)  $1100 \text{ N}\cdot\text{m}$

Converting the twist angle to radians and calculating the polar moment of inertia  $J$ ,

$$\phi = (1.5^\circ) \left( \frac{2\pi \text{ rad}}{360^\circ} \right) = 0.026 \text{ rad}$$

$$r_1 = 0.015 \text{ m}$$

$$r_2 = 0.025 \text{ m}$$

$$J = \frac{\pi}{2} (r_2^4 - r_1^4) = \left( \frac{\pi}{2} \right) ((0.025 \text{ m})^4 - (0.015 \text{ m})^4)$$

$$= 5.34 \times 10^{-7} \text{ m}^4$$

$$T = \frac{GJ}{L} \phi$$

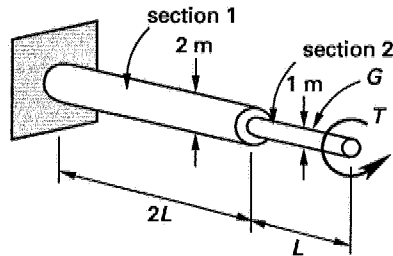
$$= \left( \frac{\left( 80 \times 10^9 \frac{\text{N}}{\text{m}^2} \right) (5.34 \times 10^{-7} \text{ m}^4)}{1 \text{ m}} \right) (0.026 \text{ rad})$$

$$= 1110 \text{ N}\cdot\text{m} \quad (1100 \text{ N}\cdot\text{m})$$

The answer is (D).

**MECHANICS OF MATERIALS-36**

Determine the maximum torque that can be applied to the shaft, given that the maximum angle of twist is 0.0225 rad. Neglect bending.



- (A)  $0.000625 \frac{\pi G}{L}$       (B)  $0.0500 \frac{\pi G}{L}$   
 (C)  $0.250 \frac{\pi G}{L}$       (D)  $0.525 \frac{\pi G}{L}$

The angle of twist is

$$\phi = \frac{TL}{GJ}$$

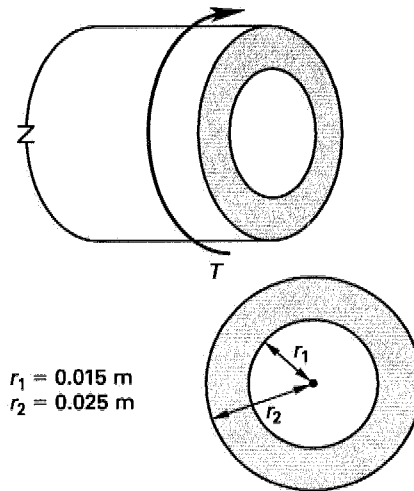
$J$  for a circular bar of diameter  $d$  is  $\frac{1}{2}\pi r^4 = \frac{1}{32}\pi d^4$ . The total angle of twist,  $\phi_{\text{total}}$ , is equal to the sum of the angles of twist for the two different sections. The torque is the same for both sections.

$$\begin{aligned}\phi_{\text{total}} &= \phi_1 + \phi_2 \\ &= \frac{T(2L)}{GJ_1} + \frac{TL}{GJ_2} \\ &= \left( \frac{32TL}{\pi G} \right) \left( \frac{2}{d_1^4} + \frac{1}{d_2^4} \right) \\ &= \left( \frac{32TL}{\pi G} \right) \left( \frac{2}{(2 \text{ m})^4} + \frac{1}{(1 \text{ m})^4} \right) \\ &= \frac{36TL}{\pi G} \\ T &= \frac{\pi G \phi_{\text{total}}}{36L} = \frac{\pi G (0.0225 \text{ rad})}{36L} \\ &= 0.000625 \frac{\pi G}{L}\end{aligned}$$

The answer is (A).

## MECHANICS OF MATERIALS-37

For the given shaft, what is most nearly the largest torque that can be applied if the shear stress is not to exceed 110 MPa?



- (A) 1700 N·m      (B) 1900 N·m      (C) 2400 N·m      (D) 3400 N·m

Since the shear stress is largest at the outer diameter, the maximum torque is found using this radius.

$$T_{\max} = \frac{\tau J}{r_2}$$

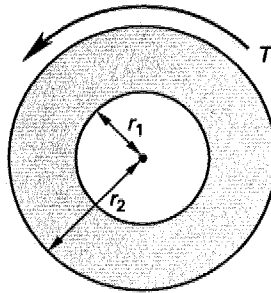
For an annular region,

$$\begin{aligned} J &= \frac{\pi}{2}(r_2^4 - r_1^4) = \left(\frac{\pi}{2}\right)((0.025 \text{ m})^4 - (0.015 \text{ m})^4) \\ &= 5.34 \times 10^{-7} \text{ m}^4 \\ T_{\max} &= \frac{(5.34 \times 10^{-7} \text{ m}^4) \left(110 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)}{0.025 \text{ m}} \\ &= 2350 \text{ N}\cdot\text{m} \quad (2400 \text{ N}\cdot\text{m}) \end{aligned}$$

The answer is (C).

**MECHANICS OF MATERIALS-38**

A hollow circular bar has an inner radius  $r_1$  and an outer radius  $r_2$ . If  $r_1 = r_2/2$ , most nearly what percentage of torque can the shaft carry in comparison with a solid shaft?



(A) 25%

(B) 55%

(C) 75%

(D) 95%

The equation for torsional stress is

$$\tau = \frac{Tr}{J}$$

$$T = \frac{\tau J}{r}$$

For the hollow shaft,

$$T_h = \frac{\tau \left( \frac{\pi}{2} \right) (r_2^4 - r_1^4)}{r_2}$$

For the solid shaft,

$$T_s = \frac{\tau \left( \frac{\pi}{2} \right) r_2^4}{r_2}$$

Therefore,

$$\begin{aligned}
 \frac{T_h}{T_s} &= \frac{\tau \left( \frac{\pi}{2} \right) \left( \frac{r_2^4 - r_1^4}{r_2} \right)}{\tau \left( \frac{\pi}{2} \right) \left( \frac{r_2^4}{r_2} \right)} \\
 &= \frac{r_2^4 - r_1^4}{r_2^4} \\
 &= \frac{r_2^4 - \left( \frac{r_2}{2} \right)^4}{r_2^4} \\
 &= \frac{r_2^4 - \frac{r_2^4}{16}}{r_2^4} \\
 &= \frac{\frac{15}{16} r_2^4}{r_2^4} \\
 &= \frac{15}{16} \\
 &= 0.94 \quad (95\%)
 \end{aligned}$$

The answer is (D).

### MECHANICS OF MATERIALS-39

What is the minimum solid shaft diameter that can be used for the rotor of a 4.5 kW motor operating at 3500 rpm, if the maximum shear stress for the shaft is 60 MPa?

- (A) 1.2 mm      (B) 2.1 mm      (C) 10 mm      (D) 20 mm

The relationship between the power,  $P$ , transmitted by a shaft and the torque,  $T$ , is

$$P = \frac{\pi n T}{30}$$

$n$  is in rpm,  $T$  is in N·m, and  $P$  is in W. Rearranging to solve for  $T$ ,

$$T = \frac{30P}{\pi n} = \frac{(30)(4500 \text{ W})}{\pi \left(3500 \frac{\text{rev}}{\text{min}}\right)} = 12.28 \text{ N}\cdot\text{m}$$

$$\tau_{\max} = \frac{Tr}{J} = \frac{Td}{2J}$$

$$J = \frac{Td}{2\tau_{\max}} = \frac{\pi d^4}{32}$$

Therefore,

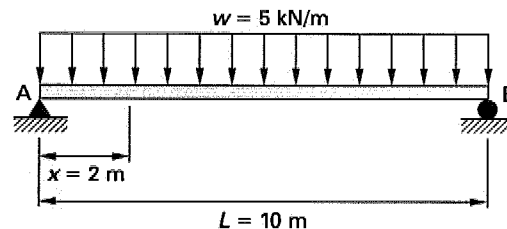
$$d = \left( \frac{32T}{2\pi\tau_{\max}} \right)^{1/3} = \left( \frac{(16)(12.28 \text{ N}\cdot\text{m})}{\pi \left(60 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)} \right)^{1/3}$$

$$= 0.0101 \text{ m} \quad (10 \text{ mm})$$

The answer is (C).

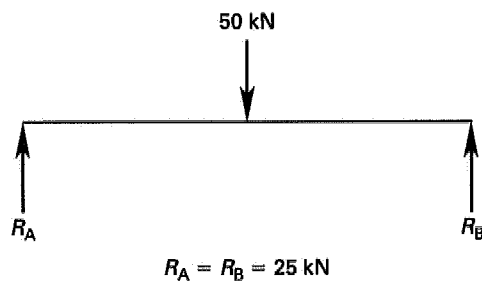
#### MECHANICS OF MATERIALS-40

A beam supports a distributed load,  $w$ , as shown. Find the shear force at  $x = 2 \text{ m}$  from the left end.

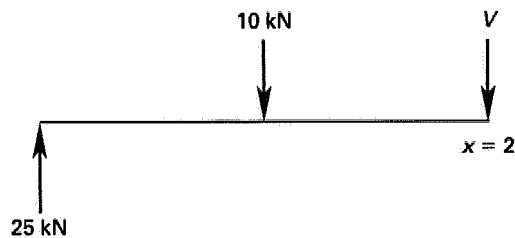


- (A) 11 kN      (B) 12 kN      (C) 13 kN      (D) 15 kN

The reactions at A and B are found by observation from symmetry to be  $R_A = R_B = 25 \text{ kN}$ .



Sectioning the beam at  $x = 2 \text{ m}$ , the free-body diagram with shear force is



$$\sum F_y = 0$$

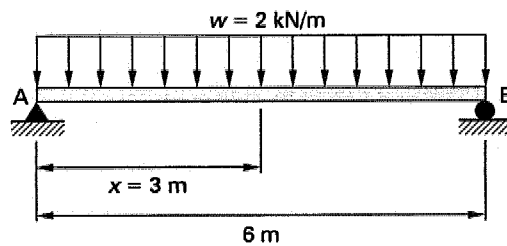
$$25 \text{ kN} - 10 \text{ kN} - V = 0 \text{ kN}$$

$$V = 25 \text{ kN} - 10 \text{ kN} = 15 \text{ kN}$$

The answer is (D).

#### MECHANICS OF MATERIALS-41

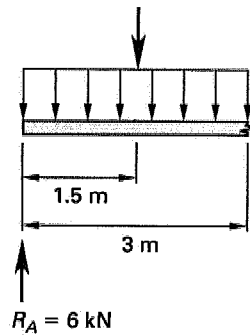
For the beam shown, find the bending moment,  $M$ , at  $x = 3 \text{ m}$ .



- (A) 4.5 kN·m      (B) 6.0 kN·m      (C) 7.5 kN·m      (D) 9.0 kN·m

By inspection from symmetry,  $R_A = R_B = 6 \text{ kN}$ . Sectioning the beam at  $x = 3 \text{ m}$  gives

$$F = \left( 2 \frac{\text{kN}}{\text{ft}} \right) (3 \text{ ft}) = 6 \text{ kN}$$



$$\sum M_{x=3} = 0$$

$$(6 \text{ kN})(3 \text{ m}) + (6 \text{ kN})(1.5 \text{ m}) + M = 0$$

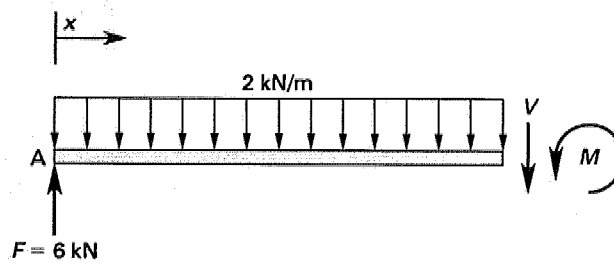
$$M = 18 \text{ kN}\cdot\text{m} - 9 \text{ kN}\cdot\text{m}$$

$$= 9 \text{ kN}\cdot\text{m}$$

The answer is (D).

#### MECHANICS OF MATERIALS-42

Find the expression for the bending moment as a function of distance from the left end,  $x$ , for the following beam.



(A)  $M = -x^3 + 2x$       (B)  $M = -x^2 + 1$

(C)  $M = -x^2 + 2x$       (D)  $M = x^3 - 2x^2$

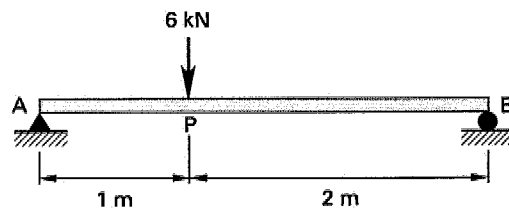
$$\sum M = -2x + (2x) \left( \frac{1}{2}x \right) + M = 0$$

$$M = -x^2 + 2x$$

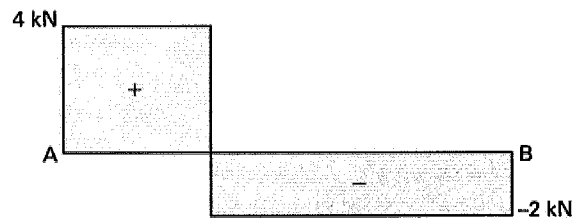
The answer is (C).

### MECHANICS OF MATERIALS—43

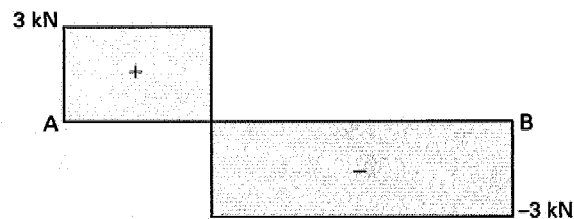
Which of the following is the shear force diagram for this beam?



(A)



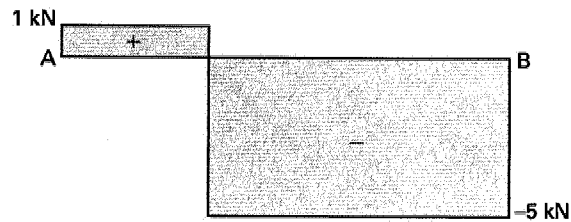
(B)



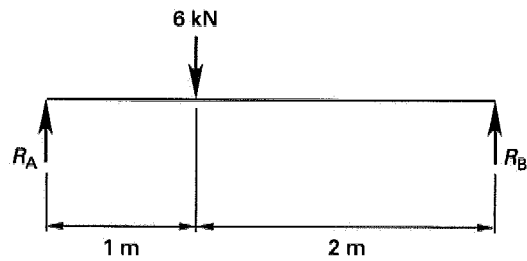
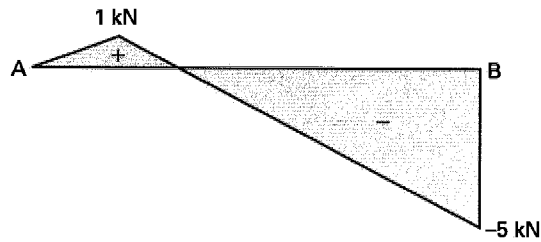
10-36

1001 SOLVED ENGINEERING FUNDAMENTALS PROBLEMS

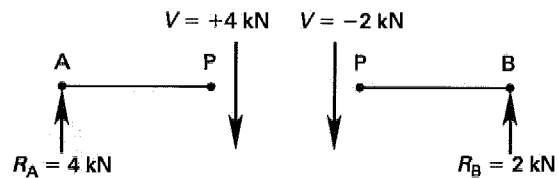
(C)



(D)



By observation, the reactions at points A and B are  $R_A = 4 \text{ kN}$  and  $R_B = 2 \text{ kN}$ . Draw free-body diagrams of the left and right sections of the beam.

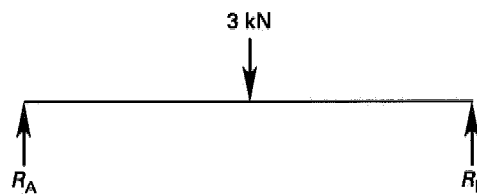
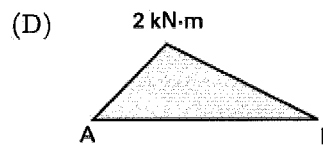
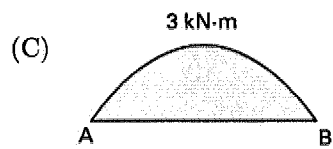
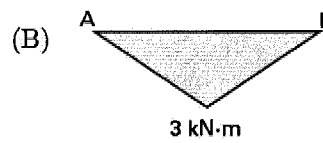
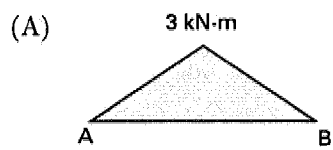
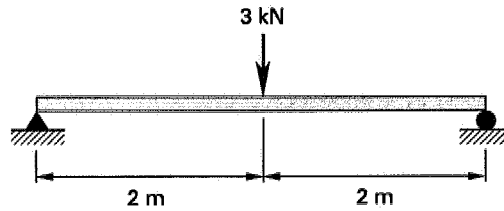


Thus,  $V = +4 \text{ kN}$  between points A and P, and  $V = -2 \text{ kN}$  between points P and B.

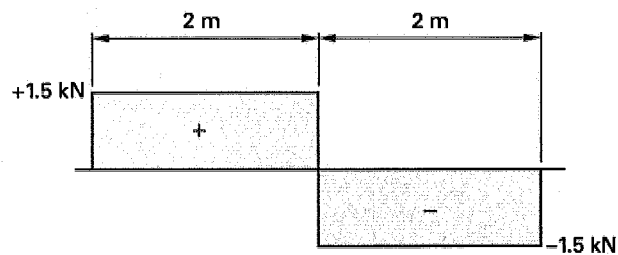
The answer is (A).

## MECHANICS OF MATERIALS-44

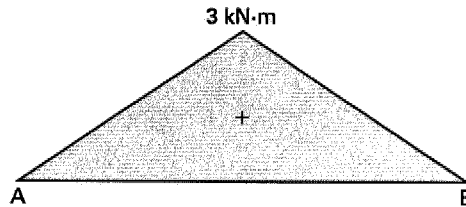
Which of the following is the bending moment diagram for this beam?



From the free-body diagram,  $R_A = R_B = 1.5$  kN. The shear force diagram is therefore,



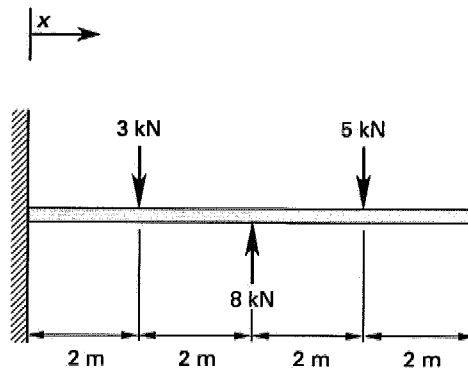
The bending moment increases linearly to  $(1.5 \text{ kN})(2 \text{ m}) = 3 \text{ kN}\cdot\text{m}$ , then decreases linearly back to  $0 \text{ kN}\cdot\text{m}$ .



The answer is (A).

### MECHANICS OF MATERIALS-45

The cantilever beam shown is loaded by three concentrated forces. What is the maximum shear force in the beam?



- (A) 1 kN      (B) 2 kN      (C) 3 kN      (D) 5 kN

Examining the shear force along the beam from left to right, for  $0 \text{ m} < x < 2 \text{ m}$ ,

$$V = 0 \text{ kN}$$

For  $2 \text{ m} < x < 4 \text{ m}$ ,

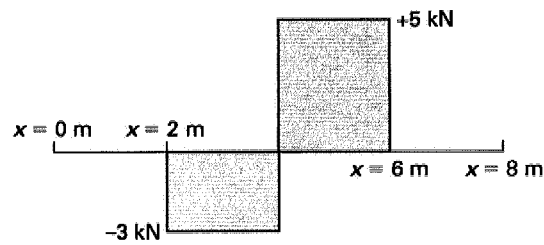
$$V = -3 \text{ kN}$$

For  $4 \text{ m} < x < 6 \text{ m}$ ,

$$V = 5 \text{ kN}$$

For  $6 \text{ m} < x < 8 \text{ m}$ ,

$$V = 0 \text{ kN}$$

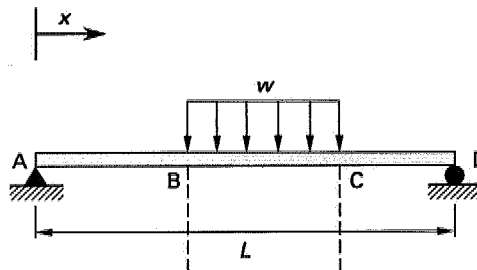


The maximum shear force is, therefore, 5 kN.

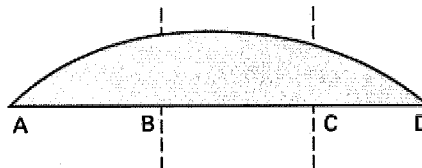
The answer is (D).

#### MECHANICS OF MATERIALS-46

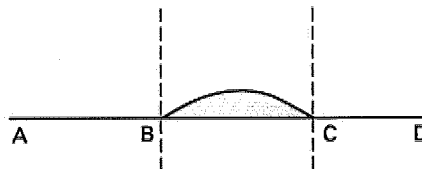
Which of the following bending moment diagrams corresponds to the simply supported beam shown? The beam is subjected to a distributed load,  $w$ , between points B and C.



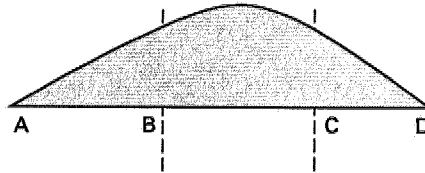
(A)



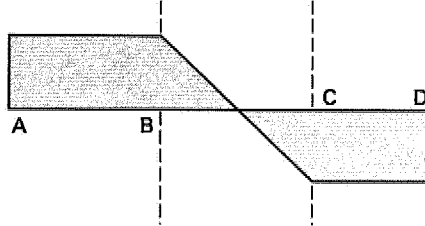
(B)



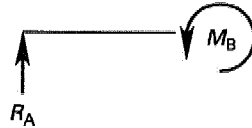
(C)



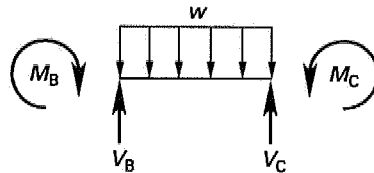
(D)



For sections AB and CD, the beam may be modeled as



$M(x)$  is linear with respect to  $x$ . For section BC, the beam is modeled as

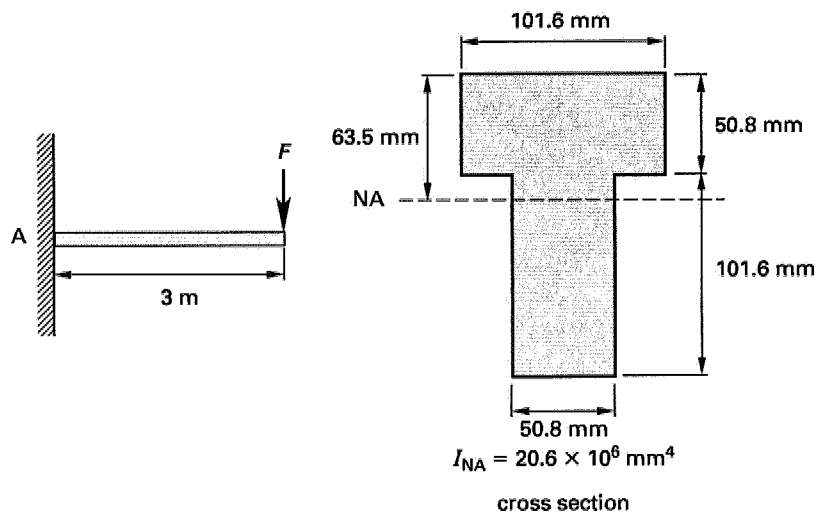


$M(x)$  is parabolic, reaching a maximum near or at the center.

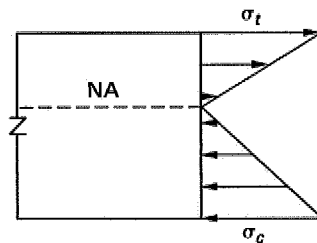
The answer is (C).

## MECHANICS OF MATERIALS-47

What is most nearly the maximum allowable load,  $F$ , on the cantilever? The maximum compressive stress is 7000 kPa, and the maximum tensile stress is 5500 kPa. The moment of inertia about the centroidal axis,  $I_{NA}$ , is  $20.6 \times 10^6 \text{ mm}^4$ .



- (A) 540 N      (B) 600 N      (C) 610 N      (D) 640 N



The maximum bending moment occurs at A, where  $M = 3F$ .

$$\sigma_{\max} = \frac{Mc}{I} = \frac{3Fc}{I}$$

$$F = \frac{\sigma_{\max} I}{3c}$$

$$I = (20.6 \times 10^6 \text{ mm}^4) \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right)^4$$

$$= 20.6 \times 10^{-6} \text{ m}^4$$

For compression,  $\sigma_{\text{allowable}} = 7000 \text{ kPa}$ .

$$c = 101.6 \text{ mm} + 50.8 \text{ mm} - 63.5 \text{ mm} = 88.9 \text{ mm}$$

$$F_{\text{allowable compression}} = \frac{\left(7000 \frac{\text{kN}}{\text{m}^2}\right) (20.6 \times 10^{-6} \text{ m}^4)}{(3 \text{ m})(0.0889 \text{ m})}$$

$$= 0.541 \text{ kN} \quad (541 \text{ N})$$

For tension,  $\sigma_{\text{allowable}} = 5500 \text{ kPa}$ , and  $c = 63.5 \text{ mm}$ .

$$F_{\text{allowable tension}} = \frac{\left(5500 \frac{\text{kN}}{\text{m}^2}\right) (20.6 \times 10^{-6} \text{ m}^4)}{(3 \text{ m})(0.0635 \text{ m})}$$

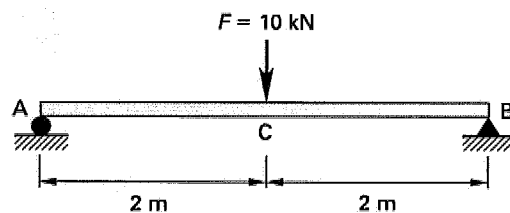
$$= 0.595 \text{ kN} \quad (600 \text{ N})$$

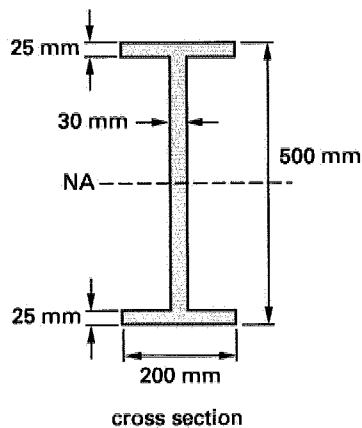
The maximum allowable load is 540 N.

The answer is (A).

### MECHANICS OF MATERIALS-48

A simply supported beam with the cross section shown supports a concentrated load,  $F = 10 \text{ kN}$ , at its center, C. What is most nearly the maximum bending stress in the beam?





- (A) 2300 kPa      (B) 3200 kPa      (C) 3800 kPa      (D) 4600 kPa

The reactions at A and B are  $R_A = R_B = 5 \text{ kN}$  by inspection from symmetry. Since the maximum bending moment occurs at C,

$$M_{\max} = (5 \text{ kN})(2 \text{ m}) = 10 \text{ kN}\cdot\text{m}$$

The moment of inertia about the neutral axis, NA, is the difference between the moments of inertia of an area measuring  $200 \text{ mm} \times 500 \text{ mm}$  and two areas measuring  $85 \text{ mm} \times 450 \text{ mm}$ .

$$\begin{aligned} I &= \frac{bh^3}{12} = \left(\frac{1}{12}\right)(200 \text{ mm})(500 \text{ mm})^3 - (2)\left(\frac{1}{12}\right)(85 \text{ mm})(450 \text{ mm})^3 \\ &= 792 \times 10^6 \text{ mm}^4 \end{aligned}$$

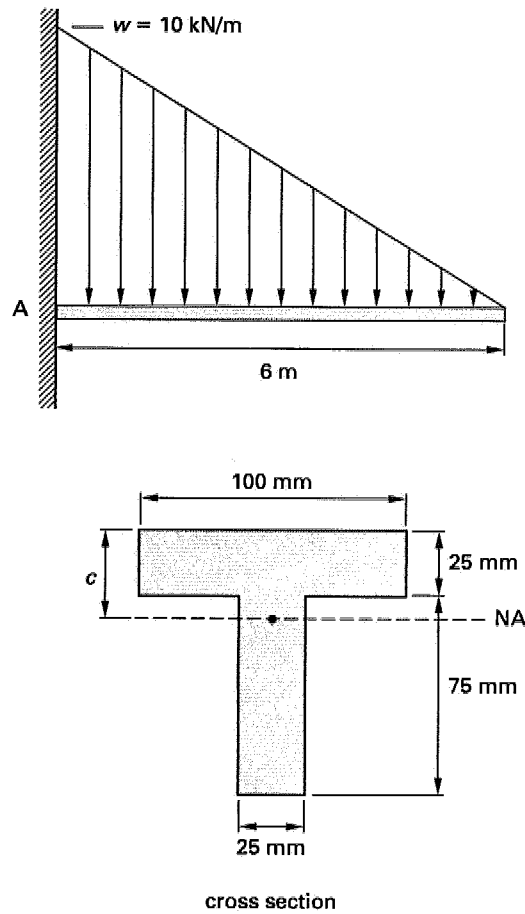
$$\text{Since } c = \left(\frac{1}{2}\right)(500 \text{ mm}) = 250 \text{ mm},$$

$$\begin{aligned} \sigma_{\max} &= \frac{Mc}{I} = \frac{(10 \text{ kN}\cdot\text{m})(0.25 \text{ m})}{792 \times 10^{-6} \text{ m}^4} \\ &= 3157 \text{ kN/m}^2 \quad (3160 \text{ kPa}) \end{aligned}$$

The answer is (B).

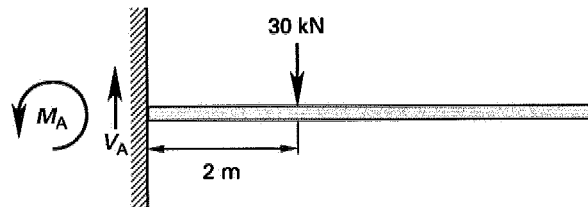
## MECHANICS OF MATERIALS-49

For the cantilever beam shown, what is the maximum tensile bending stress?



- (A) 230 MPa      (B) 320 MPa      (C) 480 MPa      (D) 550 MPa

The maximum moment occurs at point A and is a result of the distributed load  $w$ .  $w$  is equivalent to a concentrated load,  $W = (1/2)(6 \text{ m})(10 \text{ kN/m}) = 30 \text{ kN}$ , acting at a point  $(1/3)(6 \text{ m}) = 2 \text{ m}$  from point A. The equivalent loading diagram for the cantilever is as follows.



$$M_A = (30 \text{ kN})(2 \text{ m}) = 60 \text{ kN}\cdot\text{m}$$

The upper part of the beam will be under tension, with  $c$  equal to the distance between the neutral axis, NA, and the top edge of the beam.

$$\begin{aligned} c &= \frac{\sum A\bar{y}}{\sum A} = \frac{(25 \text{ mm})(100 \text{ mm})(12.5 \text{ mm}) + (25 \text{ mm})(75 \text{ mm})(62.5 \text{ mm})}{2500 \text{ mm}^2 + 1875 \text{ mm}^2} \\ &= 33.9 \text{ mm} \quad (0.0339 \text{ m}) \end{aligned}$$

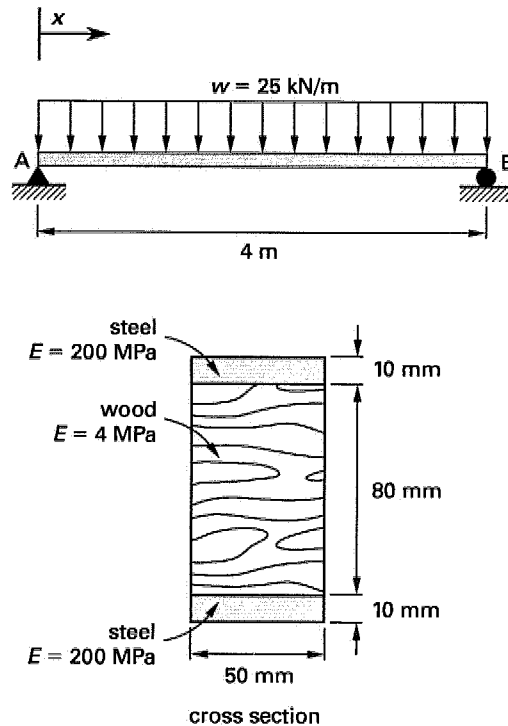
Use the parallel axis theorem to find  $I$ .

$$\begin{aligned} I_{NA} &= \frac{bh^3}{12} = \left(\frac{1}{12}\right)(100 \text{ mm})(25 \text{ mm})^3 \\ &\quad + (100 \text{ mm})(25 \text{ mm})(34 \text{ mm} - 12.5 \text{ mm})^2 \\ &\quad + \left(\frac{1}{12}\right)(25 \text{ mm})(75 \text{ mm})^3 + (75 \text{ mm})(25 \text{ mm})(62.5 \text{ mm} - 34 \text{ mm})^2 \\ &= 3.7 \times 10^6 \text{ mm}^4 \\ \sigma &= \frac{Mc}{I} = \frac{(60 \text{ kN}\cdot\text{m})(0.0339 \text{ m})}{(3.7 \times 10^6 \text{ mm}^4) \left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)^4} \\ &= 5.5 \times 10^5 \text{ kPa} \quad (550 \text{ MPa}) \end{aligned}$$

The answer is (D).

## MECHANICS OF MATERIALS-50

A composite beam made of steel and wood is subjected to a uniform distributed load,  $w$ . Determine the maximum compressive stress in the steel.



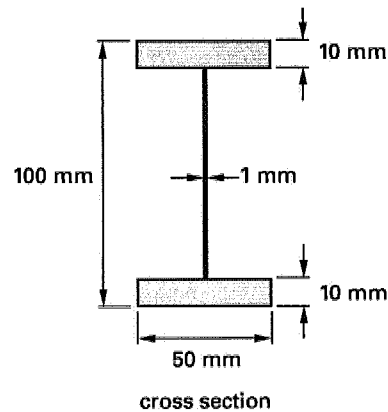
- (A) 620 MPa      (B) 850 MPa      (C) 1100 MPa      (D) 1200 MPa

The maximum moment is at the center of the beam, where  $x = 2 \text{ m}$ .

$$R_A = R_B = 50 \text{ kN} \quad [\text{by inspection}]$$

$$\begin{aligned} M_{\max} &= (50 \text{ kN})(2 \text{ m}) - \left(25 \frac{\text{kN}}{\text{m}}\right)(2 \text{ m})(1 \text{ m}) \\ &= 50 \text{ kN}\cdot\text{m} \end{aligned}$$

Since  $E_{\text{wood}}/E_{\text{steel}} = 4 \text{ MPa}/200 \text{ MPa} = 1/50$ , the wood is equivalent to a steel web 1 mm thick.



$$I = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(50 \text{ mm})(100 \text{ mm})^3 - (2)\left(\frac{1}{12}\right)(24.5 \text{ mm})(80 \text{ mm})^3$$

$$= 2.076 \times 10^6 \text{ mm}^4$$

$$\sigma = \frac{Mc}{I} = \frac{(50 \text{ kN}\cdot\text{m})(0.05 \text{ m})\left(\frac{1000 \text{ N}}{\text{kN}}\right)}{(2.076 \times 10^6 \text{ mm}^4)\left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)^4}$$

$$= 1200 \times 10^6 \text{ N/m}^2 \quad (1200 \text{ MPa})$$

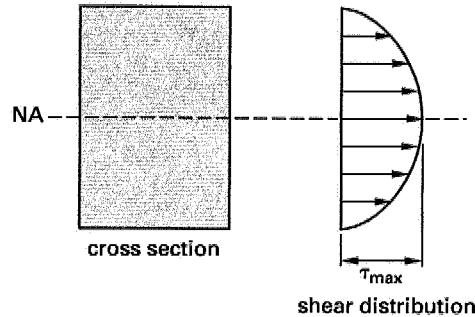
The answer is (D).

### MECHANICS OF MATERIALS-51

For a rectangular beam under transverse (bending) loading, where is the location of maximum shear stress?

- (A) at the top edge
- (B) at the bottom edge
- (C) at the neutral axis
- (D) at a location between the top edge and the neutral axis

The shear distribution is

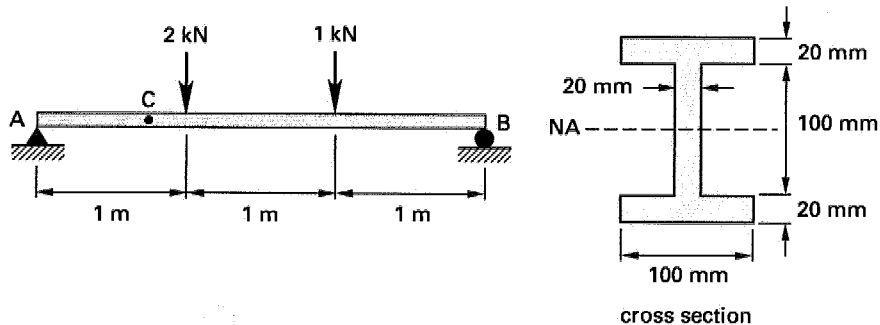


The maximum shear stress is at the neutral axis.

The answer is (C).

### MECHANICS OF MATERIALS-52

An I-beam is loaded as shown. What is most nearly the maximum shear stress,  $\tau$ , in the web at point C along the beam?



- (A) 160 kPa      (B) 370 kPa      (C) 400 kPa      (D) 750 kPa

The reaction at point A is found by taking the moment about point B.

$$\begin{aligned}\sum M_B &= 0 \\ &= -R_A(3 \text{ m}) + (2 \text{ kN})(2 \text{ m}) + (1 \text{ kN})(1 \text{ m}) \\ R_A &= 1.67 \text{ kN} \\ V_C &= R_A = 1.67 \text{ kN}\end{aligned}$$

The shear stress is given by  $\tau = VQ/It$ , where  $Q$  is the first moment of either the upper half or the lower half of the cross-sectional area with respect to the neutral axis.

$$Q = A'\bar{y} = (50 \text{ mm})(20 \text{ mm})(25 \text{ mm}) + (100 \text{ mm})(20 \text{ mm})(60 \text{ mm}) \\ = 145\,000 \text{ mm}^3$$

$$I = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(100 \text{ mm})(140 \text{ mm})^3 - (2)\left(\frac{1}{12}\right)(40 \text{ mm})(100 \text{ mm})^3 \\ = 16.2 \times 10^6 \text{ mm}^4$$

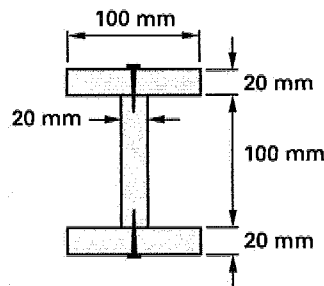
$\tau_{\max}$  occurs at the neutral axis. Thus,

$$\tau_{\max} = \frac{VQ}{It} = \frac{(1670 \text{ N})(145\,000 \text{ mm}^3)}{(16.2 \times 10^6 \text{ mm}^4)(20 \text{ mm})} \\ = 0.747 \text{ N/mm}^2 \quad (750 \text{ kPa})$$

The answer is (D).

### MECHANICS OF MATERIALS-53

An I-beam is made of three planks, each 20 mm × 100 mm in cross section, nailed together with a single row of nails on top and bottom as shown. If the longitudinal spacing between the nails is 25 mm, and the vertical shear force acting on the cross section is 600 N, what is most nearly the load in shear per nail,  $F$ ?



- (A) 56 N      (B) 76 N      (C) 110 N      (D) 160 N

The shear force per unit distance along the beam's axis is given by

$$f = \frac{VQ}{I}$$

For an I-beam,  $Q$  is the first moment of the upper flange area with respect to the  $z$ -axis.

$$Q = A\bar{y} = (60 \text{ mm})(100 \text{ mm})(20 \text{ mm}) = 120\,000 \text{ mm}^3$$

$$I = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(100 \text{ mm})(140 \text{ mm})^3 - (2)\left(\frac{1}{12}\right)(40 \text{ mm})(100 \text{ mm})^3$$

$$= 16.2 \times 10^6 \text{ mm}^4$$

$$f = \frac{(600 \text{ N})(120\,000 \text{ mm}^3)}{16.2 \times 10^6 \text{ mm}^4} = 4.44 \text{ N/mm}$$

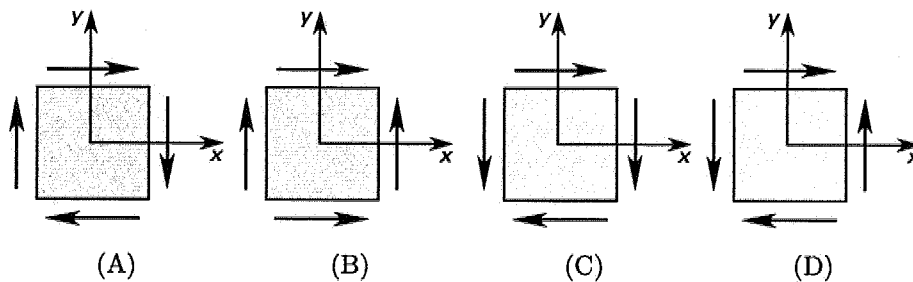
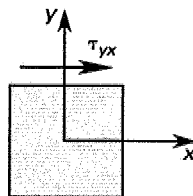
The load capacity of the nails per unit length is  $F/L$ . Therefore,

$$F = Lf = (25 \text{ mm})\left(4.44 \frac{\text{N}}{\text{mm}}\right) = 111 \text{ N} \quad (110 \text{ N})$$

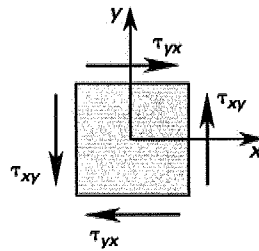
The answer is (C).

#### MECHANICS OF MATERIALS-54

Considering the orientation of shear force  $\tau_{yx}$  in the illustration, find the direction of the shear stress on the other three sides of the stress element.



For static equilibrium, the shear stresses on opposite faces of an element must be equal in magnitude and opposite in direction. Also, the shear stresses on adjoining faces must not produce rotation of the element.



The answer is (D).

#### MECHANICS OF MATERIALS-55

If the principal stresses on a body are  $\sigma_1 = 400$  kPa,  $\sigma_2 = -700$  kPa, and  $\sigma_3 = 600$  kPa, what is the maximum shear stress?

- (A) 150 kPa      (B) 250 kPa      (C) 550 kPa      (D) 650 kPa

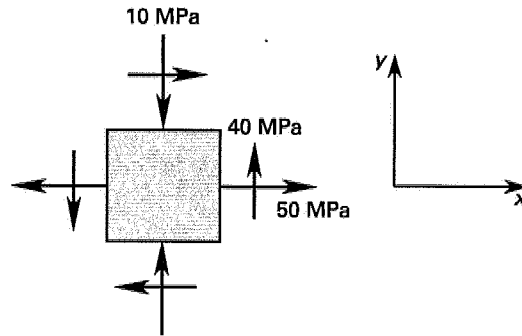
The maximum shear stress is equal to one-half of the difference between the principal stresses. Comparing the three combinations, the maximum shear stress is given by the difference between  $\sigma_2$  and  $\sigma_3$ .

$$\tau_{\max} = \left| \frac{\sigma_2 - \sigma_3}{2} \right| = \left| \frac{-700 \text{ kPa} - 600 \text{ kPa}}{2} \right| = 650 \text{ kPa}$$

The answer is (D).

**MECHANICS OF MATERIALS-56**

For the element of plane stress shown, find the principal stresses.



- (A)  $\sigma_{\max} = 35 \text{ MPa}$ ,  $\sigma_{\min} = -25 \text{ MPa}$   
 (B)  $\sigma_{\max} = 45 \text{ MPa}$ ,  $\sigma_{\min} = 55 \text{ MPa}$   
 (C)  $\sigma_{\max} = 70 \text{ MPa}$ ,  $\sigma_{\min} = -30 \text{ MPa}$   
 (D)  $\sigma_{\max} = 85 \text{ MPa}$ ,  $\sigma_{\min} = 15 \text{ MPa}$

The stresses on the element are

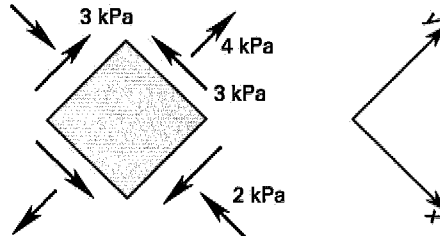
$$\sigma_x = 50 \text{ MPa} \quad \sigma_y = -10 \text{ MPa} \quad \tau_{xy} = 40 \text{ MPa}$$

$$\begin{aligned} \sigma_{\max, \min} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{50 \text{ MPa} - 10 \text{ MPa}}{2} \pm \sqrt{\left(\frac{50 \text{ MPa} + 10 \text{ MPa}}{2}\right)^2 + (40 \text{ MPa})^2} \\ &= 20 \text{ MPa} \pm 50 \text{ MPa} \\ &= 70 \text{ MPa or } -30 \text{ MPa} \end{aligned}$$

The answer is (C).

## MECHANICS OF MATERIALS-57

What are the principal (maximum and minimum) stresses of the stress element shown?



- (A)  $\sigma_{\max} = 1.16 \text{ kPa}$ ,  $\sigma_{\min} = -6.16 \text{ kPa}$   
 (B)  $\sigma_{\max} = 2.00 \text{ kPa}$ ,  $\sigma_{\min} = -4.00 \text{ kPa}$   
 (C)  $\sigma_{\max} = 3.24 \text{ kPa}$ ,  $\sigma_{\min} = -5.24 \text{ kPa}$   
 (D)  $\sigma_{\max} = 5.24 \text{ kPa}$ ,  $\sigma_{\min} = -3.24 \text{ kPa}$

The stresses on the element are

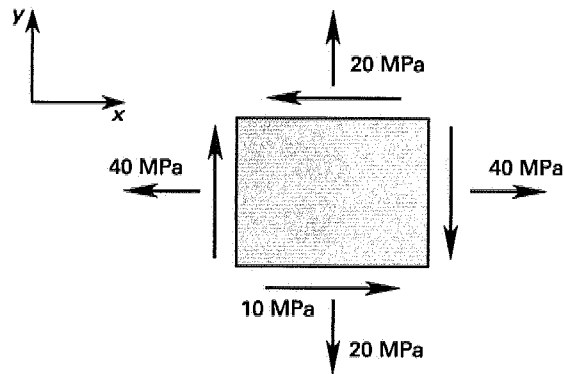
$$\sigma_x = -2 \text{ kPa} \quad \sigma_y = +4 \text{ kPa} \quad \tau_{xy} = -3 \text{ kPa}$$

$$\begin{aligned} \sigma_{\max, \min} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{-2 \text{ kPa} + 4 \text{ kPa}}{2} \pm \sqrt{\left(\frac{-2 \text{ kPa} - 4 \text{ kPa}}{2}\right)^2 + (-3 \text{ kPa})^2} \\ &= 5.24 \text{ kPa or } -3.24 \text{ kPa} \end{aligned}$$

The answer is (D).

**MECHANICS OF MATERIALS-58**

What is most nearly the maximum principal stress of the element shown?



- (A) 30 MPa      (B) 34 MPa      (C) 40 MPa      (D) 44 MPa

The stresses on the element are

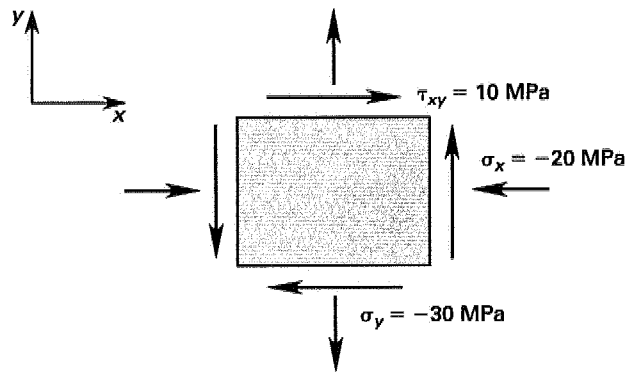
$$\sigma_x = 40 \text{ MPa} \quad \sigma_y = 20 \text{ MPa} \quad \tau_{xy} = -10 \text{ MPa}$$

$$\begin{aligned} \sigma_{\max} &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{40 \text{ MPa} + 20 \text{ MPa}}{2} + \sqrt{\left(\frac{40 \text{ MPa} - 20 \text{ MPa}}{2}\right)^2 + (-10 \text{ MPa})^2} \\ &= 44.1 \text{ MPa} \quad (44 \text{ MPa}) \end{aligned}$$

The answer is (D).

## MECHANICS OF MATERIALS-59

For the following stress element, what is most nearly the maximum shear stress?



- (A) 10 MPa      (B) 11 MPa      (C) 14 MPa      (D) 27 MPa

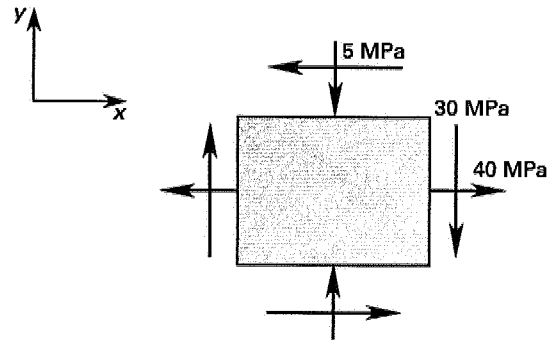
The maximum shear stress is

$$\begin{aligned}\tau_{\max} &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{\left(\frac{-20 \text{ MPa} - (-30 \text{ MPa})}{2}\right)^2 + (10 \text{ MPa})^2} \\ &= 26.9 \text{ MPa}\end{aligned}$$

The answer is (D).

**MECHANICS OF MATERIALS-60**

For the state of plane stress shown, what are the inclination angles of the principal planes?



- (A)  $32.5^\circ$  and  $122^\circ$
- (B)  $25.5^\circ$  and  $115^\circ$
- (C)  $-26.5^\circ$  and  $-117^\circ$
- (D)  $-11.5^\circ$  and  $-102^\circ$

The stresses on the element are

$$\sigma_x = 40 \text{ MPa} \quad \sigma_y = -5 \text{ MPa} \quad \tau_{xy} = -30 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-30 \text{ MPa})}{40 \text{ MPa} - (-5 \text{ MPa})}$$

$$= -1.33$$

$$2\theta_p = -53.0^\circ \text{ or } -233^\circ$$

$$\theta_p = -26.5^\circ \text{ or } -117^\circ$$

The answer is (C).

## MECHANICS OF MATERIALS-61

A steel ( $\sigma_{\text{yield}} = 200 \text{ MPa}$ ) pressure tank is designed to hold pressures up to 7 MPa. The tank is cylindrical with a diameter of 1 m. If the longitudinal stress must be less than 20% of the yield stress of the steel, what is the necessary wall thickness,  $t$ ?

- (A) 22 mm      (B) 44 mm      (C) 88 mm      (D) 120 mm

For a thin-walled cylinder of diameter  $d$  containing a pressure  $p$ ,

$$\begin{aligned}\sigma_{\text{long}} &= \frac{pd}{4t} = (1 - 0.2)\sigma_{\text{yield}} \\ t &= \frac{pd}{0.8\sigma_{\text{yield}}} = \frac{\left(7 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)(1 \text{ m})}{(0.8)\left(200 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)} \\ &= 0.044 \text{ m} \quad (44 \text{ mm})\end{aligned}$$

The answer is (B).

## MECHANICS OF MATERIALS-62

In designing a cylindrical pressure tank 1 m in diameter, a factor of safety of 2.5 is used. The cylinder is made of steel ( $\sigma_{\text{yield}} = 200 \text{ MPa}$ ), and will contain pressures up to 7 MPa. What is the required wall thickness,  $t$ , based on circumferential stress considerations?

- (A) 22 mm      (B) 44 mm      (C) 88 mm      (D) 120 mm

For a thin-walled cylinder of diameter  $d$  containing a pressure  $p$ ,

$$\begin{aligned}\sigma_{\text{circumferential}} &= \frac{pd}{2t} = \frac{\sigma_{\text{yield}}}{2.5} \\ t &= \frac{1.25pd}{\sigma_{\text{yield}}} = \frac{(1.25)\left(7 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)(1 \text{ m})}{200 \times 10^6 \frac{\text{N}}{\text{m}^2}} \\ &= 0.044 \text{ m} \quad (44 \text{ mm})\end{aligned}$$

The answer is (B).

**MECHANICS OF MATERIALS-63**

What is most nearly the maximum principal strain at a point where  $\epsilon_x = 1500 \mu\text{m}$ ,  $\epsilon_y = -750 \mu\text{m}$ , and  $\epsilon_{xy} = 1000 \mu\text{m}$ ?

- (A)  $1160 \mu\text{m}$       (B)  $1490 \mu\text{m}$       (C)  $1610 \mu\text{m}$       (D)  $1830 \mu\text{m}$

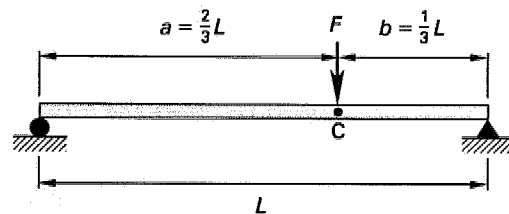
The equation for principal strain gives

$$\begin{aligned}\epsilon_{\max, \min} &= \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\epsilon_{xy}}{2}\right)^2} \\ &= \frac{1500 \mu\text{m} - 750 \mu\text{m}}{2} \pm \sqrt{\left(\frac{1500 \mu\text{m} + 750 \mu\text{m}}{2}\right)^2 + \left(\frac{1000 \mu\text{m}}{2}\right)^2} \\ &= 375 \mu\text{m} \pm 1231 \mu\text{m} \\ \epsilon_{\max} &= 1606 \mu\text{m} \quad (1610 \mu\text{m})\end{aligned}$$

The answer is (C).

**MECHANICS OF MATERIALS-64**

A beam of length  $L$  carries a concentrated load,  $F$ , at point C. Determine the deflection at point C in terms of  $F$ ,  $L$ ,  $E$ , and  $I$ , where  $E$  is the modulus of elasticity, and  $I$  is the moment of inertia.



- (A)  $\frac{2FL^3}{243EI}$       (B)  $\frac{4FL^3}{243EI}$       (C)  $\frac{FL^3}{27EI}$       (D)  $\frac{FL^3}{9EI}$

The equation for bending moment in the beam is

$$EI \frac{d^2\delta}{dx^2} = -M$$

Computing  $M$  for the different beam sections,

$$EI \frac{d^2 \delta}{dx^2} = -\frac{Fbx}{L} \quad (0 \leq x \leq a)$$

$$EI \frac{d^2 \delta}{dx^2} = -\frac{Fbx}{L} + F(x-a) \quad (a \leq x \leq L)$$

Integrating each equation twice gives

$$EI \delta = -\frac{Fbx^3}{6L} + C_1x + C_3 \quad (0 \leq x \leq a)$$

$$EI \delta = -\frac{Fbx^3}{6L} + \frac{F(x-a)^3}{6} + C_2x + C_4 \quad (a \leq x \leq L)$$

The constants are determined by the following conditions: (1) at  $x = a$ , the slopes  $d\delta/dx$  and deflections  $\delta$  are equal; (2) at  $x = 0$  and  $x = L$ , the deflection  $\delta = 0$ . These conditions give

$$C_1 = C_2 = \frac{Fb(L^2 - b^2)}{6L}$$

$$C_3 = C_4 = 0$$

Evaluating the equation for  $(0 \leq x \leq a)$  at  $x = a = \frac{2}{3}L$  and  $b = \frac{1}{3}L$ ,

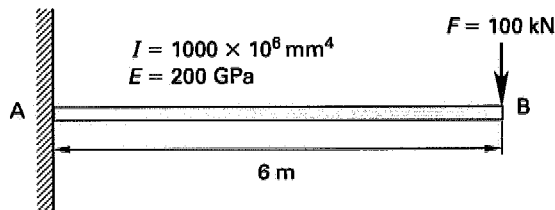
$$EI \delta = \left( \frac{F(\frac{1}{3}L)(\frac{2}{3}L)}{6L} \right) \left( L^2 - \frac{L^2}{9} - \frac{4L^2}{9} \right)$$

$$\delta = \frac{4FL^3}{243EI}$$

The answer is (B).

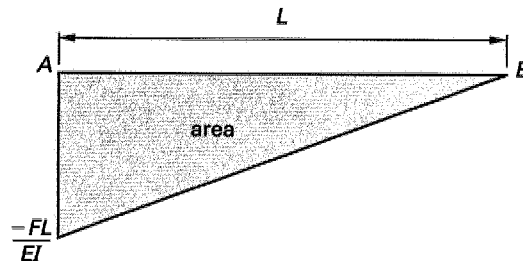
### MECHANICS OF MATERIALS-65

What is the deflection at point B for the beam shown?



- (A) 17 mm      (B) 25 mm      (C) 36 mm      (D) 48 mm

Using the moment-area method, the  $M/EI$  diagram is



$$A = \frac{1}{2}L \left( \frac{-FL}{EI} \right) = -\frac{FL^2}{2EI}$$

The first moment with respect to point A is

$$Q = A\left(\frac{2}{3}L\right) = -\left(\frac{FL^2}{2EI}\right)\left(\frac{2}{3}L\right) = -\frac{FL^3}{3EI}$$

The deflection is

$$y = -Q = \frac{FL^3}{3EI} = \frac{(100 \text{ kN})(6 \text{ m})^3}{(3) \left( 200 \times 10^6 \frac{\text{kN}}{\text{m}^2} \right) (1000 \times 10^6 \text{ mm}^4) \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right)^4}$$

$$= 0.036 \text{ m} \quad (36 \text{ mm})$$

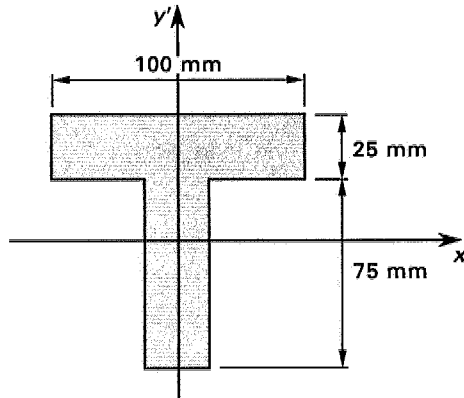
The answer is (C).

### MECHANICS OF MATERIALS-66

What is the Euler buckling load for a 10 m long steel column pinned at both ends and with the given properties and cross section?

$$I_{x'x'} = 3.70 \times 10^6 \text{ mm}^4$$

$$E = 200 \text{ GPa}$$



- (A) 15 kN      (B) 24 kN      (C) 43 kN      (D) 73 kN

$x'x'$  and  $y'y'$  are centroidal axes.

$$I_{x'x'} = 3.70 \times 10^6 \text{ mm}^4$$

$I_{y'y'}$  is computed by applying the equation for  $I$  ( $bh^3/12$ ) about the centroidal axis of a rectangle. For this cross section,  $b_1 = 25 \text{ mm}$ ,  $h_1 = 100 \text{ mm}$ ,  $b_2 = 75 \text{ mm}$ , and  $h_2 = 25 \text{ mm}$ .

$$\begin{aligned} I_{y'y'} &= \left( \left( \frac{1}{12} \right) (25 \text{ mm})(100 \text{ mm})^3 + \left( \frac{1}{12} \right) (75 \text{ mm})(25 \text{ mm})^3 \right) \\ &\quad \times \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right)^4 \\ &= 2.18 \times 10^{-6} \text{ m}^4 \end{aligned}$$

The Euler buckling load,  $P_{\text{cr}}$ , is

$$P_{\text{cr}} = \frac{\pi^2 EI}{L^2}$$

$I$  is the minimum  $I$  value.  $I_{y'y'}$  is less than  $I_{x'x'}$ .

$$\begin{aligned} P_{\text{cr}} &= \frac{\pi^2 \left( 200 \times 10^6 \frac{\text{kN}}{\text{m}^2} \right) (2.18 \times 10^{-6} \text{ m}^4)}{(10 \text{ m})^2} \\ &= 43 \text{ kN} \end{aligned}$$

The answer is (C).

