11 DYNAMICS

DYNAMICS-1

How many degrees of freedom does a coin rolling on the ground have?

(A) one   (B) two   (C) three   (D) five

A coin has two translational degrees of freedom and one rotational degree of freedom.

> The answer is (C).

DYNAMICS-2

What is the definition of instantaneous velocity?

(A) $v = dx\ dt$ (B) $v = \int x\ dt$

(C) $v = \dfrac{dx}{dt}$ (D) $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta t}{\Delta x}$

By definition,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

> The answer is (C).

DYNAMICS-3

A car travels 100 km to city A in 2 h, then travels 200 km to city B in 3 h. What is the average speed of the car for the trip?

(A) 45 km/h   (B) 58 km/h   (C) 60 km/h   (D) 66 km/h

Average velocity is defined as total distance traveled over total time.

$$v_{ave} = \frac{\Delta x}{\Delta t} = \frac{100 \text{ km} + 200 \text{ km}}{2 \text{ h} + 3 \text{ h}}$$

$$= 60 \text{ km/h}$$

The answer is (C).

DYNAMICS-4

The position of a particle moving along the x-axis is given by $x(t) = t^2 - t + 8$, where x is in units of meters, and t is in seconds. Find the velocity of the particle when $t = 5$ s.

(A) 9.0 m/s   (B) 10 m/s   (C) 11 m/s   (D) 12 m/s

The velocity equation is the first derivative of the position equation with respect to time. Therefore,

$$v(t) = \frac{dx}{dt}$$

$$= \frac{d}{dt}(t^2 - t + 8)$$

$$= 2t - 1$$

$$v(5) = (2)(5) - 1 = 9.0 \text{ m/s}$$

The answer is (A).

DYNAMICS-5

If a particle's position is given by the expression $x(t) = 3.4t^3 - 5.4t$ m, what is most nearly the acceleration of the particle at $t = 5$ s?

(A) 1.0 m/s^2 (B) 3.4 m/s^2 (C) 18 m/s^2 (D) 100 m/s^2

The acceleration is found from the second derivative of the position equation. Therefore,

$$a(t) = \frac{d^2x}{dt^2}$$

$$= \frac{d^2}{dt^2}(3.4t^3 - 5.4t)$$

$$= \frac{d}{dt}(10.2t^2 - 5.4)$$

$$= 20.4t$$

$$a(5) = (20.4)(5) = 102 \text{ m/s}^2 \quad (100 \text{ m/s}^2)$$

The answer is (D).

DYNAMICS-6

A car starts from rest and moves with a constant acceleration of 6 m/s^2. What is the speed of the car after 4 s?

(A) 18 m/s   (B) 24 m/s   (C) 35 m/s   (D) 55 m/s

For uniformly accelerated motion,

$$v = v_0 + at = 0 + \left(6 \ \frac{\text{m}}{\text{s}^2}\right)(4 \text{ s})$$

$$= 24 \text{ m/s}$$

The answer is (B).

DYNAMICS-7

A car starts from rest and has a constant acceleration of 3 m/s^2. What is the average velocity during the first 10 s of motion?

(A) 12 m/s   (B) 13 m/s   (C) 14 m/s   (D) 15 m/s

The distance traveled by the car is

$$x = x_0 + \mathrm{v}_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \left(\frac{1}{2}\right)\left(3 \ \frac{m}{s^2}\right)(10 \text{ s})^2$$

$$= 150 \text{ m}$$

$$\mathrm{v_{ave}} = \frac{\Delta x}{\Delta t} = \frac{150 \text{ m}}{10 \text{ s}}$$

$$= 15 \text{ m/s}$$

The answer is (D).

DYNAMICS-8

A truck increases its speed uniformly from 13 km/h to 50 km/h in 25 s. What is most nearly the acceleration of the truck?

(A) 0.22 m/s^2 (B) 0.41 m/s^2 (C) 0.62 m/s^2 (D) 0.92 m/s^2

For uniformly accelerated rectilinear motion,

$$\mathrm{v} = \mathrm{v}_0 + at$$
$$at = \mathrm{v} - \mathrm{v}_0$$
$$a = \frac{\mathrm{v} - \mathrm{v}_0}{t} = \left(\frac{50 \ \frac{km}{h} - 13 \ \frac{km}{h}}{25 \text{ s}}\right)\left(\frac{1000 \ \frac{m}{km}}{3600 \ \frac{s}{h}}\right)$$

$$= 0.411 \text{ m/s}^2 \quad (0.41 \text{ m/s}^2)$$

The answer is (B).

DYNAMICS-9

A bicycle moves with a constant deceleration of -2 m/s^2. If the initial velocity of the bike is 4.0 m/s, how far does it travel in 3 s?

(A) 2.0 m (B) 2.5 m (C) 3.0 m (D) 4.0 m

For constant acceleration,

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$= 0 + \left(4.0\ \frac{m}{s}\right)(3\ s) + \left(\frac{1}{2}\right)\left(-2\ \frac{m}{s^2}\right)(3\ s)^2$$

$$= 3.0\ m$$

The answer is (C).

DYNAMICS-10

A ball is dropped from a height of 60 m above ground. How long does it take to hit the ground?

(A) 1.3 s (B) 2.1 s (C) 3.5 s (D) 5.5 s

The positive y direction is downward, and $y = 0$ at 60 m above ground. For uniformly accelerated motion,

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

$$60 = 0 + 0 + \left(\frac{1}{2}\right)\left(9.81\ \frac{m}{s^2}\right)t^2$$

$$t = 3.5\ s$$

The answer is (C).

DYNAMICS–11

A man driving a car at 65 km/h suddenly sees an object in the road 20 m ahead. Assuming an instantaneous reaction on the driver's part, what constant deceleration is required to stop the car in this distance?

(A) 7.1 m/s^2 (B) 7.5 m/s^2 (C) 8.0 m/s^2 (D) 8.1 m/s^2

For uniform deceleration, the velocity equation that is not a function of time is

$$v^2 = v_0^2 + 2a(x - x_0)$$

Using $v = 0$, $v_0 = 65$ km/h $= 18$ m/s, and $(x - x_0) = 20$ m,

$$0 = \left(18 \ \frac{m}{s}\right)^2 + 2a(20 \text{ m})$$

$$a = -\frac{\left(18 \ \frac{m}{s}\right)^2}{(2)(20 \text{ m})} = 8.1 \text{ m/s}^2$$

The answer is (D).

DYNAMICS–12

A ball is thrown vertically upward with an initial speed of 24 m/s. Most nearly how long will it take for the ball to return to the thrower?

(A) 2.3 s   (B) 2.6 s   (C) 4.1 s   (D) 4.9 s

At the apex of its flight, the ball has zero velocity and is at the midpoint of its flight time. If the total flight time is t_{total}, then the time elapsed at this point is $1/2 t_{total}$.

$$v = v_0 + at$$

Rearranging to solve for t_{total},

$$0 = 24 \ \frac{m}{s} + \left(-9.81 \ \frac{m}{s^2}\right) \tfrac{1}{2} t_{total}$$

$$t_{total} = (2) \left(\frac{24 \ \frac{m}{s}}{9.81 \ \frac{m}{s^2}}\right) = 4.893 \text{ s} \quad (4.9 \text{ s})$$

The answer is (D).

DYNAMICS-13

A projectile is launched upward from level ground at an angle of 60° from the horizontal. It has an initial velocity of 45 m/s. How long will it take before the projectile hits the ground?

(A) 4.1 s   (B) 5.8 s   (C) 7.9 s   (D) 9.5 s

The projectile will experience acceleration only in the y direction due to gravity. The y component of velocity is

$$v_{0y} = 45 \sin 60° = 39 \text{ m/s}$$

For uniform rectilinear motion with constant acceleration,

$$y = y_0 + v_{0y}t + \tfrac{1}{2}at^2$$

$$= 0 + \left(39 \ \frac{\text{m}}{\text{s}}\right)t + \left(\frac{1}{2}\right)\left(-9.81 \ \frac{\text{m}}{\text{s}^2}\right)t^2$$

$$= \left(39 \ \frac{\text{m}}{\text{s}}\right)t - \left(4.91 \ \frac{\text{m}}{\text{s}^2}\right)t^2$$

When the body is on the ground, $y = 0$.

$$0 = \left(39 \ \frac{\text{m}}{\text{s}}\right)t - \left(4.91 \ \frac{\text{m}}{\text{s}^2}\right)t^2 = t\left(39 \ \frac{\text{m}}{\text{s}} - 4.91 \ \frac{\text{m}}{\text{s}^2}t\right)$$

$$t = \frac{3.9 \ \frac{\text{m}}{\text{s}}}{4.9 \ \frac{\text{m}}{\text{s}^2}} = 7.94 \text{ s} \quad (7.9 \text{ s})$$
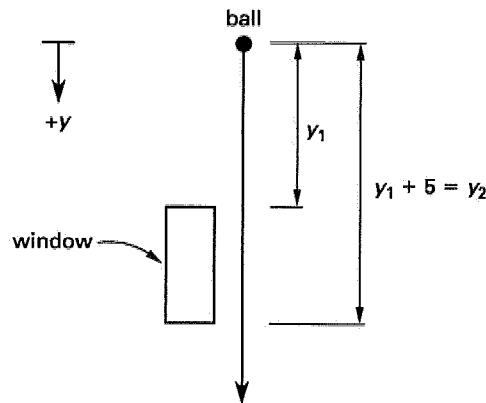
The answer is (C).

DYNAMICS-14

A man standing at a 5 m tall window watches a falling ball pass by the window in 0.3 s. From approximately how high above the top of the window was the ball released from a stationary position?

(A) 8.2 m   (B) 9.6 m   (C) 12 m   (D) 21 m

The positive y direction is taken as downward, and the initial release point is y_0. Then,

$$y = y_0 + v_o t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}at^2$$
$$y_1 = \tfrac{1}{2}at_1^2$$
$$y_2 = \tfrac{1}{2}at_2^2$$

However, $y_2 = y_1 + 5$ m, and $t_2 = t_1 + 0.3$ s. Therefore,

$$y_1 + 5 \text{ m} = \tfrac{1}{2}a(t_1 + 0.3 \text{ s})^2$$
$$\tfrac{1}{2}at_1^2 + 5 \text{ m} = \tfrac{1}{2}a\left(t_1^2 + (2)(0.3 \text{ s})t_1 + (0.09 \text{ s}^2)\right)$$
$$= \tfrac{1}{2}at_1^2 + (0.3 \text{ s})at_1 + (0.045 \text{ s}^2)a$$

Rearrange and solve for t_1.

$$(0.3 \text{ s})at_1 = 5 \text{ m} - (0.045 \text{ s}^2)a$$
$$t_1 = \frac{5 \text{ m} - (0.045 \text{ s}^2)a}{(0.3 \text{ s})a}$$
$$= \frac{5 \text{ m} - (0.045 \text{ s}^2)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)}{(0.3 \text{ s})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)}$$
$$= 1.55 \text{ s}$$

Solving for y_1,

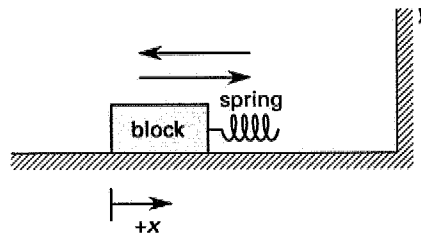$$y_1 = \tfrac{1}{2}at_1^2 = \left(\frac{1}{2}\right)\left(9.81\ \frac{m}{s^2}\right)(1.55\ s)^2$$

$$= 11.8\ m \quad (12\ m)$$

> The answer is (C).

DYNAMICS–15

A block with a spring attached to one end slides along a rough surface with an initial velocity of 7 m/s. After it slides 4 m, it impacts a wall for 0.1 s, and then slides 10 m in the opposite direction before coming to a stop. If the block's deceleration is assumed constant and the contraction of the spring is negligible, what is the average acceleration of the block during impact with the wall?

(A) -120 m/s^2 (B) -100 m/s^2 (C) -99 m/s^2 (D) -49 m/s^2

$a_{1\text{-}2}$	acceleration before impact	m/s^2
$a_{2\text{-}2'}$	acceleration during impact	m/s^2
$a_{2\text{-}3}$	acceleration after impact	m/s^2
$s_{1\text{-}2}$	distance traveled before impact	m
$s_{2\text{-}3}$	distance traveled after impact	m
v_1	initial velocity	m/s
v_2	velocity just before impact	m/s
v_2'	velocity after impact	m/s
v_3	final velocity	m/s

$$v_1 = 7\ m/s$$
$$v_3 = 0$$

Because Δx is small and energy is conserved, $v_2 = v_2'$.

$$v_2 = \sqrt{v_1^2 - 2a_{1\text{-}2}s_{1\text{-}2}}$$

$$= \sqrt{\left(7 \ \frac{m}{s}\right)^2 - 2(a_{1\text{-}2})(4 \ m)}$$

$$= \sqrt{49 \ \frac{m^2}{s^2} - 8a_{1\text{-}2}}$$

Alternatively,

$$a_{1\text{-}2} = \frac{49 \ \dfrac{m^2}{s^2} - v_2^2}{8 \ m}$$

$$v_3 = \sqrt{v_{2'}^2 - 2a_{2\text{-}3}s_{2\text{-}3}}$$

$$0 = \sqrt{v_{2'}^2 - 2a_{2\text{-}3}(10 \ m)}$$

Alternatively,

$$a_{2\text{-}3} = \frac{v_{2'}^2}{20}$$

But $a_{1\text{-}2} = a_{2\text{-}3}$, and $v_2 = v_{2'}$.

$$\frac{49 \ \dfrac{m^2}{s^2} - v_2^2}{8 \ m} = \frac{v_2^2}{20 \ m}$$

$$980 \ \frac{m^2}{s^2} - 20v_2^2 = 8v_2{}^2$$

$$v_2 = 5.9 \ m/s$$

Then, because of the direction change,

$$a_{2\text{-}2'} = \frac{v_{2'} - v_2}{\Delta t} = \frac{-5.9 \ \dfrac{m}{s} - 5.9 \ \dfrac{m}{s}}{0.10 \ s}$$

$$= -118 \ m/s^2 \quad (-120 \ m/s^2)$$

The answer is (A).

DYNAMICS-16

A car starting from rest moves with a constant acceleration of 15 km/h^2 for 1 h, then decelerates at a constant -7.5 km/h^2 until it comes to a stop. Most nearly how far has it traveled?

(A) 15 km   (B) 23 km   (C) 25 km   (D) 35 km

For constant acceleration,

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

During acceleration, $x_0 = 0$, $v_0 = 0$, $a = 15$ km/h^2, and $t = 1$ h. Then, the distance over which the car accelerates is

$$x_{\text{acc}} = \left(\frac{1}{2}\right)\left(15 \ \frac{\text{km}}{\text{h}^2}\right)(1 \ \text{h})^2 = 7.5 \ \text{km}$$

At the end of the hour, the car's velocity is

$$v = v_0 + at = \left(15 \ \frac{\text{km}}{\text{h}^2}\right)(1 \ \text{h})$$
$$= 15 \ \text{km/h}$$

During deceleration, $x_0 = 0$, $v_0 = 15$ km/h, and $a = -7.5$ km/h^2. The car has velocity $v = 0$ when it stops. Therefore,

$$v = v_0 + at$$
$$0 = 15 \ \frac{\text{km}}{\text{h}} + \left(-7.5 \ \frac{\text{km}}{\text{h}^2}\right)t$$
$$t = 2 \ \text{h}$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

The distance over which the car decelerates is

$$x_{\text{dec}} = \left(15 \ \frac{\text{km}}{\text{h}}\right)(2 \ \text{h}) + \left(\frac{1}{2}\right)\left(-7.5 \ \frac{\text{km}}{\text{h}^2}\right)(2 \ \text{h})^2 = 15 \ \text{km}$$
$$x_{\text{total}} = x_{\text{acc}} + x_{\text{dec}} = 7.5 \ \text{km} + 15 \ \text{km} = 22.5 \ \text{km} \quad (23 \ \text{km})$$

The answer is (B).

DYNAMICS-17

A train with a top speed of 75 km/h cannot accelerate or decelerate faster than 1.2 m/s². What is the minimum distance between two train stops in order for the train to be able to reach its top speed?

(A) 300 m   (B) 350 m   (C) 360 m   (D) 365 m

To travel the minimum distance, the train must accelerate from $v_0 = 0$ km/h to $v = 75$ km/h at a constant 1.2 m/s² and then decelerate at a constant 1.2 m/s² to $v = 0$ km/h. The train travels the same distance during acceleration as during deceleration, since the initial and final speeds are identical, as well as the magnitude of acceleration or deceleration. The following two equations apply for constant acceleration.

$$x = x_0 + v_0 t = \tfrac{1}{2}at^2$$
$$v = v_0 + at$$

During acceleration, $x_0 = 0$ m, $v_0 = 0$ km/h, $v = 75$ km/h = 20.8 m/s, and $a = 1.2$ m/s². Then,

$$t = \frac{v - v_0}{a} = \frac{20.8 \,\frac{m}{s} - 0 \,\frac{m}{s}}{1.2 \,\frac{m}{s^2}} = 17.3 \text{ s}$$

$$x = \left(\frac{1}{2}\right)\left(1.2 \,\frac{m}{s^2}\right)(17.3 \text{ s})^2 = 180 \text{ m}$$
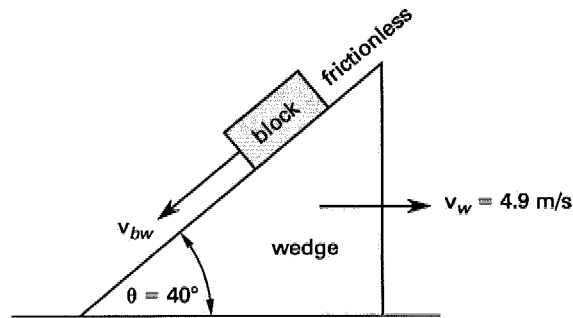
The minimum total distance is

$$x = (2)(180 \text{ m}) = 360 \text{ m}$$

The answer is (C).

DYNAMICS–18

A block with a mass of 150 kg slides down a frictionless wedge with a slope of 40°. The wedge is moving horizontally in the opposite direction at a constant velocity of 4.9 m/s. What is most nearly the absolute speed of the block 2 s after it is released from rest?

(A) 8.9 m/s   (B) 9.4 m/s   (C) 9.5 m/s   (D) 9.8 m/s

Let v_{bw} equal the velocity of the block relative to the wedge's slope. The component of gravitational force in this direction, F_{slope}, is $W \sin \theta$. Down the slope, relative to the wedge,

$$F_{slope} = W \sin \theta = ma_{slope}$$

$$a_{slope} = \frac{W \sin \theta}{m} = g \sin \theta$$

$$v_{bw} = v_0 + a_{slope}t = 0 + gt \sin \theta$$

$$= \left(9.81 \ \frac{m}{s^2} \right) (2 \ s) \sin 40°$$

$$= 12.6 \ m/s$$

The absolute velocity, v_b, can be found from a velocity triangle.

The law of cosines gives

$$v_b^2 = v_w^2 + v_{bw}^2 - 2v_w v_{bw} \cos\theta$$

$$= \left(4.9\ \frac{m}{s}\right)^2 + \left(12.6\ \frac{m}{s}\right)^2 - (2)\left(4.9\ \frac{m}{s}\right)\left(12.6\ \frac{m}{s}\right)\cos 40°$$

$$= 88.18\ m^2/s^2$$

$$v_b = 9.39\ m/s \quad (9.4\ m/s)$$

> The answer is (B).

DYNAMICS-19

A stream flows at $v_s = 4.5$ km/h. At what angle, θ, upstream should a boat traveling at $v_b = 12$ km/h be launched in order to reach the shore directly opposite the launch point?

(A) 22°   (B) 24°   (C) 26°   (D) 28°

Draw a velocity triangle.

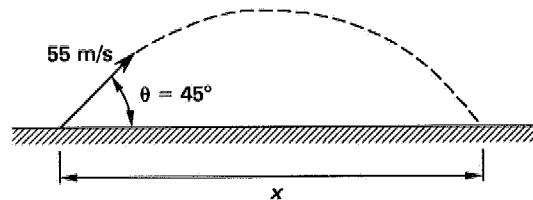$$\sin\theta = \frac{v_s}{v_b} = \frac{4.5\ \frac{km}{h}}{12\ \frac{km}{h}}$$

$$\theta = \sin^{-1}\left(\frac{4.5\ \frac{km}{h}}{12\ \frac{km}{h}}\right)$$

$$= 22°$$

The answer is (A).

DYNAMICS-20

An object is launched at 45° to the horizontal on level ground as shown. What is the range of the projectile if its initial velocity is 55 m/s? Neglect air resistance.

(A) 309 m　　　　(B) 617 m　　　　(C) 624 m　　　　(D) 680 m

Choosing the launch point as the origin of the x and y axes, $x_0 = y_0 = 0$. For uniform acceleration,

$$x = x_0 + v_{x_0}t + \tfrac{1}{2}a_x t^2$$

$$y = y_0 + v_{y_0}t + \tfrac{1}{2}a_y t^2$$

However, $a_x = 0$ m/s^2, and $a_y = -9.81$ m/s^2. Therefore,

$$x = \mathrm{v}_{x_0} t$$

$$y = \mathrm{v}_{y_0} t - \tfrac{1}{2} g t^2$$

$$\mathrm{v}_{x_0} = \mathrm{v}_0 \cos \theta$$

$$= \left(55 \ \frac{\mathrm{m}}{\mathrm{s}} \right) \cos 45°$$

$$= 38.9 \ \mathrm{m/s}$$

$$\mathrm{v}_{y_0} = \mathrm{v}_0 \sin \theta$$

$$= \left(55 \ \frac{\mathrm{m}}{\mathrm{s}} \right) \sin 45°$$

$$= 38.9 \ \mathrm{m/s}$$

$$y = 38.9 \ \frac{\mathrm{m}}{\mathrm{s}} t - 4.9 \ \frac{\mathrm{m}}{\mathrm{s}^2} t^2$$

When the projectile is on the ground, $y = 0$ m/s. Thus,

$$0 = t \left(38.9 \ \frac{\mathrm{m}}{\mathrm{s}} - 4.9 \ \frac{\mathrm{m}}{\mathrm{s}^2} t \right)$$

$$t = 7.94 \ \mathrm{s}$$

$$x = \left(38.9 \ \frac{\mathrm{m}}{\mathrm{s}} \right) t$$

$$= \left(38.9 \ \frac{\mathrm{m}}{\mathrm{s}} \right) (7.94 \ \mathrm{s})$$

$$= 308.9 \ \mathrm{m} \quad (309 \ \mathrm{m})$$

The answer is (A).

DYNAMICS–21

A projectile is fired with a velocity, v, perpendicular to a surface that is inclined at an angle, θ, with the horizontal. Determine the expression for the distance R to the point of impact.
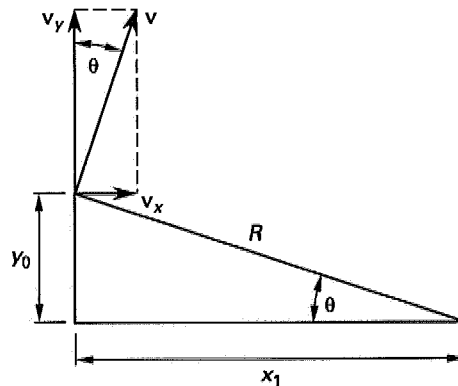

(A) $R = \dfrac{2v^2 \sin \theta}{g \cos^2 \theta}$ (B) $R = \dfrac{2v^2 \sin \theta}{g \cos \theta}$

(C) $R = \dfrac{2v \cos \theta}{g \sin \theta}$ (D) $R = \dfrac{2v \sin \theta}{g \cos \theta}$

Using the notation in the figure, $x_0 = 0$, $a_x = 0$, and $y_0 = R \sin \theta$. Therefore,

$$x = x_0 + v_x t + \tfrac{1}{2} a_x t^2$$
$$= v_x t$$
$$= v \sin \theta t$$
$$y = y_0 + v_y t + \tfrac{1}{2} a_y t^2$$
$$= R \sin \theta + v_y t + \tfrac{1}{2}(-g)t^2$$
$$= R \sin \theta + v \cos \theta t - \tfrac{1}{2} g t^2$$

At impact, let $t = t_1$, $x_1 = R\cos\theta$, and $y_1 = 0$. The two equations above give

$$R\cos\theta = v\sin\theta t_1$$

$$t_1 = \frac{R\cos\theta}{v\sin\theta} \qquad\qquad \text{[I]}$$

$$0 = R\sin\theta + v\cos\theta t_1 - \tfrac{1}{2}gt_1^2 \qquad\qquad \text{[II]}$$

Equations I and II give

$$0 = R\sin\theta + v\cos\theta\left(\frac{R\cos\theta}{v\sin\theta}\right) - \tfrac{1}{2}g\left(\frac{R\cos\theta}{v\sin\theta}\right)^2$$

$$0 = \sin^2\theta + \cos^2\theta - \tfrac{1}{2}g\left(\frac{R\cos^2\theta}{v^2\sin\theta}\right)$$

$$1 = \frac{g\cos^2\theta R}{2v^2\sin\theta}$$

$$R = \frac{2v^2\sin\theta}{g\cos^2\theta}$$

The answer is (A).

DYNAMICS-22

A cyclist on a circular track of radius $r = 240$ m is traveling at 8 m/s. His speed in the tangential direction (i.e., the direction of his travel) increases at the rate of 1 m/s^2. What is most nearly the cyclist's total acceleration?

(A) −0.9 m/s^2 (B) 0.7 m/s^2 (C) 0.9 m/s^2 (D) 1.0 m/s^2

The total acceleration is made up of tangential and normal components. The tangential component is given as 1 m/s^2. By definition, the normal component is
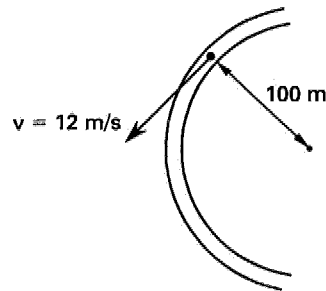
$$a_n = \frac{v^2}{r} = \frac{\left(8\ \frac{m}{s}\right)^2}{240\ m} = 0.27\ m/s^2$$

$$a = \sqrt{a_n^2 + a_t^2}$$

$$= \sqrt{\left(0.27\ \frac{m}{s^2}\right)^2 + \left(1\ \frac{m}{s^2}\right)^2}$$

$$= 1.04\ m/s^2 \quad (1.0\ m/s^2)$$

The answer is (D).

DYNAMICS-23

A motorcycle moves at a constant speed of v = 12 m/s around a curved road of radius $r = 100$ m. What is most nearly the magnitude and general direction of the motorcycle's acceleration?

(A) 1.1 m/s² away from the center of curvature
(B) 1.1 m/s² toward the center of curvature
(C) 1.4 m/s² away from the center of curvature
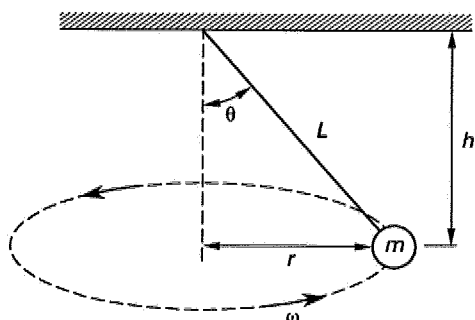(D) 1.4 m/s² toward the center of curvature

The normal acceleration, a_n, is

$$a_n = \frac{v^2}{r} = \frac{\left(12 \frac{m}{s}\right)^2}{100 \ m} = 1.44 \ m/s^2 \quad (1.4 \ m/s^2)$$

Since the velocity in the tangential direction is constant, $a_t = 0$. Thus, only the normal component of acceleration contributes to total acceleration, so $a = 1.44$ m/s². The normal component is always directed toward the center of curvature.

The answer is (D).

DYNAMICS-24

A pendulum of mass m and length L rotates about the vertical axis. If the angular velocity is ω, determine the expression for the height h.
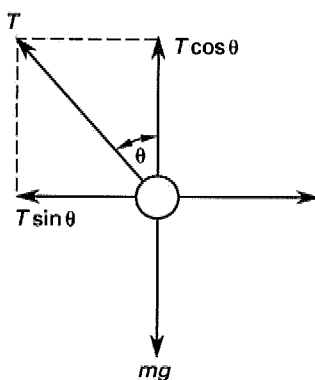

(A) $h = \dfrac{g\cos\theta}{\omega^2}$

(B) $h = \dfrac{g}{\omega^2}$

(C) $h = \dfrac{g}{\omega^2\cos\theta}$

(D) $h = \dfrac{gL\cos\theta}{\omega^2}$

A free-body diagram of the pendulum is

Since the pendulum undergoes uniform circular motion,

$$T\sin\theta = ma_n = mr\omega^2 \qquad\qquad \text{[I]}$$

Assuming the pendulum is not accelerating in the vertical direction, a force balance gives

$$T\cos\theta = mg \qquad\qquad \text{[II]}$$

Combining equations I and II,

$$\tan\theta = \frac{r\omega^2}{g}$$

However, $\tan\theta = r/h$. Therefore,

$$h = \frac{r}{\tan\theta} = \frac{g}{\omega^2}$$

The answer is (B).

DYNAMICS-25

A 3 kg block is moving at a speed of 5 m/s. The force required to bring the block to a stop in 8×10^{-4} seconds is most nearly

(A) 10 kN   (B) 13 kN   (C) 15 kN   (D) 19 kN

Newton's second law gives

$$F = ma = m\frac{dv}{dt} = m\frac{\Delta v}{\Delta t}$$

$$= (3 \text{ kg})\left(\frac{5\ \frac{m}{s}}{8 \times 10^{-4}\ s}\right)\left(\frac{1}{1000\ \frac{N}{kN}}\right)$$

$$= 18.75 \text{ kN} \quad (19 \text{ kN})$$

The answer is (D).

DYNAMICS-26

A rope is used to tow an 800 kg car with free-rolling wheels over a smooth, level road. The rope will break if the tension exceeds 2000 N. What is the greatest acceleration that the car can reach without breaking the rope?

(A) 1.2 m/s²   (B) 2.5 m/s²   (C) 3.8 m/s²   (D) 4.5 m/s²

$$F_{\text{max}} = ma_{\text{max}}$$

$$2000 \text{ N} = (800 \text{ kg})a_{\text{max}}$$

Rearranging to solve for a_{max},

$$a_{max} = \frac{2000 \text{ N}}{800 \text{ kg}} = 2.5 \text{ m/s}^2$$

The answer is (B).

DYNAMICS-27

A force of 15 N acts on a 16 kg body for 2 s. If the body is initially at rest, how far is it displaced by the force?

(A) 1.1 m   (B) 1.5 m   (C) 1.9 m   (D) 2.1 m

The acceleration is found using Newton's second law.

$$a = \frac{F}{m} = \frac{15 \text{ N}}{16 \text{ kg}} = 0.94 \text{ m/s}^2$$

For a body undergoing constant acceleration, with $v_0 = 0$ m/s^2 and $t = 2$ s,

$$\Delta x = \tfrac{1}{2}at^2 = \left(\frac{1}{2}\right)\left(0.94 \ \frac{\text{m}}{\text{s}^2}\right)(2 \text{ s})^2$$
$$= 1.88 \text{ m}$$

The answer is (C).

DYNAMICS-28

A car of mass $m = 150$ kg accelerates in 10 s from rest at a constant rate to a speed of $v = 6$ m/s. What is the resultant force on the car due to this acceleration?

(A) 75 N   (B) 90 N   (C) 95 N   (D) 98 N

For constant acceleration,

$$v = v_0 + at$$

$$a = \frac{v - v_0}{t} = \frac{6 \, \frac{m}{s} - 0 \, \frac{m}{s}}{10 \, s}$$

$$= 0.6 \text{ m/s}^2$$

$$F = ma = (150 \text{ kg}) \left(0.6 \, \frac{m}{s^2}\right) = 90 \text{ N}$$

The answer is (B).

DYNAMICS-29

A man weighs himself twice in an elevator. When the elevator is at rest, he weighs 824 N; when the elevator starts moving upward, he weighs 932 N. Most nearly how fast is the elevator accelerating, assuming constant acceleration?

(A) 0.64 m/s^2 (B) 1.1 m/s^2 (C) 1.3 m/s^2 (D) 9.8 m/s^2

The mass of the man can be determined from his weight at rest.

$$W = mg$$

$$m = \frac{W}{g} = \frac{824 \text{ N}}{9.81 \, \frac{m}{s^2}}$$
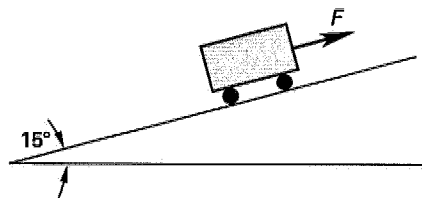
$$= 84.0 \text{ kg}$$

At constant acceleration,

$$F = ma$$

$$a = \frac{F}{m} = \frac{932 \text{ N} - 824 \text{ N}}{84.0 \text{ kg}}$$

$$= 1.29 \text{ m/s}^2 \quad (1.3 \text{ m/s}^2)$$

The answer is (C).

DYNAMICS-30

A truck weighing 1.4 kN moves up a slope of 15°. What is the force generated by the engine if the truck is accelerating at a rate of 3 m/s²? Assume the coefficient of friction is $\mu = 0.1$.
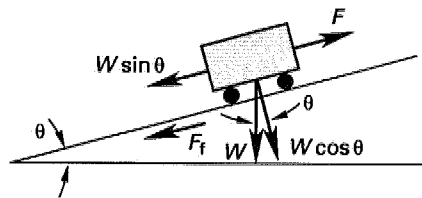

(A) 876 N   (B) 926 N   (C) 930 N   (D) 958 N

In the direction parallel to the slope, a force balance gives

$$\sum F_x = ma = F - (W \sin \theta + F_f)$$

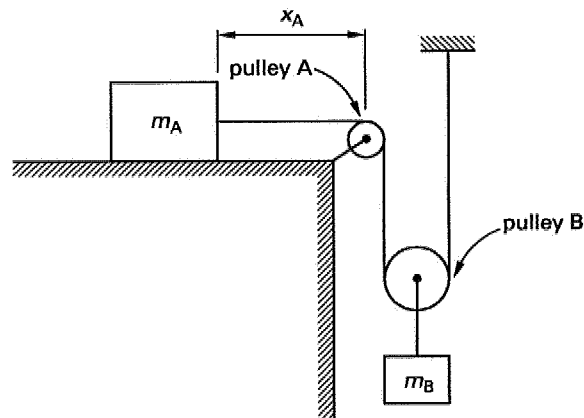F_f is the friction force, which is equal to $\mu N = \mu W \cos \theta$.

$F = W(\sin \theta + \mu \cos \theta) + ma$

$= (1400 \text{ N})(\sin 15° + 0.1 \cos 15°) + \left(\dfrac{1400 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} \right) \left(3 \frac{\text{m}}{\text{s}^2} \right)$

$= 925.7 \text{ N}$ (926 N)

The answer is (B).

DYNAMICS-31

In the illustration, the two pulleys and the horizontal surface are frictionless. The cord connecting the masses m_A and m_B is weightless. What is the ratio of the acceleration of mass A to the acceleration of mass B? Assume the system is released from rest.
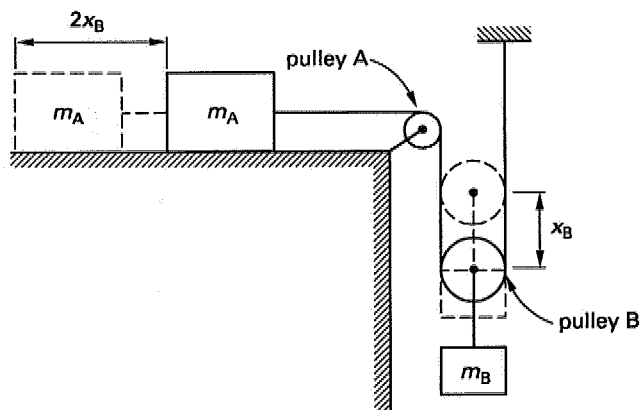

(A) 1/2   (B) 1   (C) 2   (D) m_A/m_B

Assuming the accelerations of both masses are constant, their respective displacement equations are

$$x_A = \tfrac{1}{2}a_A t^2$$

$$x_B = \tfrac{1}{2}a_B t^2$$

Taking the ratio of x_A to x_B,
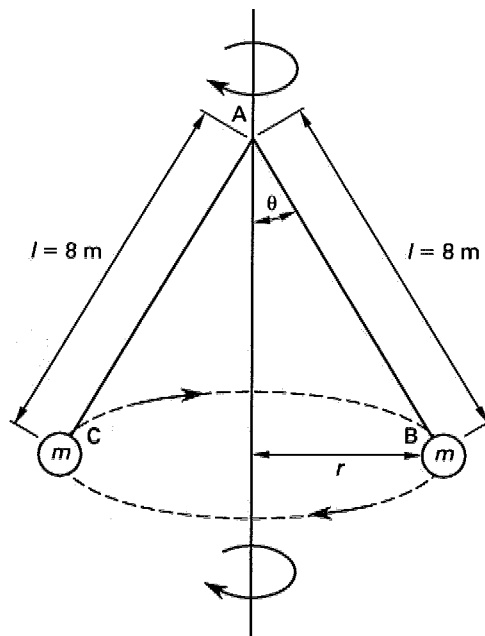
$$\frac{x_A}{x_B} = \frac{a_A}{a_B}$$

Since mass B is supported by a two-segment rope section and mass A is pulled by only one rope, the displacement of mass B is half the displacement of mass A. Therefore,

$$x_A = 2x_B$$

$$\frac{a_A}{a_B} = \frac{2x_B}{x_B}$$

$$= 2$$

The answer is (C).

DYNAMICS-32

A simplified model of a carousel is illustrated. The 8 m long arms AB and AC attach the seats B and C, each with a mass of 200 kg, to a vertical rotating shaft. What is the maximum angle of tilt, θ, for the seats, if the carousel operates at 12 rpm?
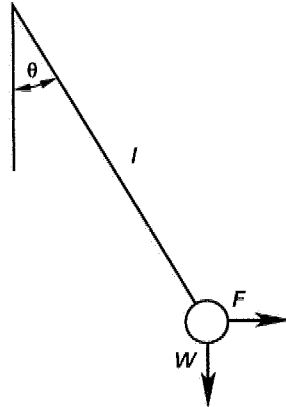

(A) 39°   (B) 40°   (C) 45°   (D) 51°

The free-body diagram is

The angular velocity, ω, of the carousel is

$$\omega = \frac{\left(12\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)}{60\ \frac{\text{s}}{\text{min}}}$$

$$= 1.257\ \text{rad/s}$$

The rotational force, F, expressed in terms of θ is

$$F = ma = mr\omega^2$$

$$= ml\sin\theta\omega^2$$

$$= (200\ \text{kg})(8\ \text{m})\sin\theta\left(1.257\ \frac{\text{rad}}{\text{s}}\right)^2$$

$$= (2528\ \text{N})\sin\theta$$

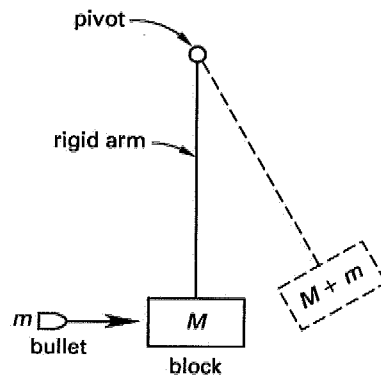From the free-body diagram,

$$\tan\theta = \frac{F}{W} = \frac{(2528\ \text{N})\sin\theta}{(200\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$= 1.288\sin\theta$$

$$\cos\theta = \frac{1}{1.288}$$

$$\theta = \cos^{-1}\left(\frac{1}{1.288}\right)$$

$$= 39.07°\quad(39°)$$

The answer is (A).

DYNAMICS-33

In the ballistic pendulum shown, a bullet of mass m is fired into a block of mass M that can swing freely. Which of the following is true for the system during the swing motion after impact?
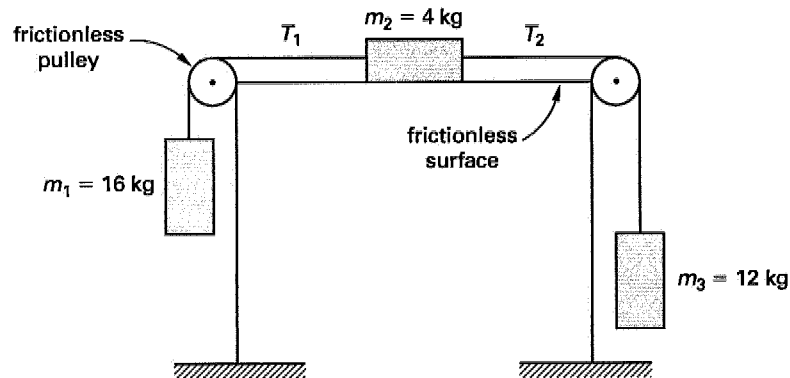

(A) Both mechanical energy and momentum are conserved.
(B) Mechanical energy is conserved; momentum is not conserved.
(C) Momentum is conserved; mechanical energy is not conserved.
(D) Neither mechanical energy nor momentum is conserved.

Momentum is not conserved since an external force, gravity, acts on the bullet-block mass. Only mechanical energy is conserved.
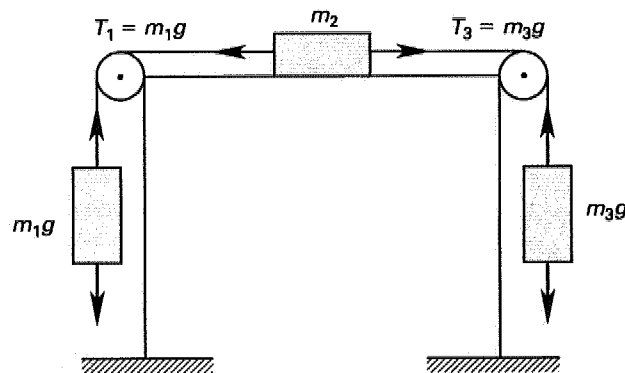
The answer is (B).

DYNAMICS-34

Three masses are attached by a weightless cord as shown. If mass m_2 is exactly halfway between the other masses and is located at the center of the flat surface when the masses are released, what is most nearly its initial acceleration? Assume there is no friction in the system and that the pulleys have no mass.

frictionless pulley T_1 $m_2 = 4\ \text{kg}$ T_2

frictionless surface

$m_1 = 16\ \text{kg}$

$m_3 = 12\ \text{kg}$

(A) 1.0 m/s²   (B) 1.2 m/s²   (C) 9.8 m/s²   (D) 12 m/s²

$T_1 = m_1 g$ m_2 $T_3 = m_3 g$

$m_1 g$ $m_3 g$

Since $m_1 > m_3$, m_1 will move downward and m_2 will be displaced to the left. All masses contribute to the inertia of the system.

$$T_1 - T_3 = a_2 m_{\text{total}} = a_2(m_1 + m_2 + m_3)$$

T_1 and T_3 are the tensions in the cord due to the masses m_1 and m_3.
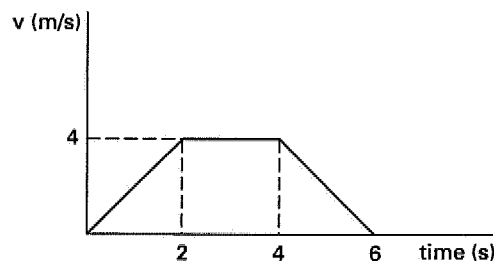
$$T_1 = m_1 g$$

$$T_3 = m_3 g$$

$$a_2 = g \left(\frac{m_1 - m_3}{m_1 + m_2 + m_3} \right)$$

$$= \left(9.81 \ \frac{m}{s^2} \right) \left(\frac{16 \ \text{kg} - 12 \ \text{kg}}{16 \ \text{kg} + 4 \ \text{kg} + 12 \ \text{kg}} \right)$$

$$= 1.23 \ \text{m/s}^2 \quad (1.2 \ \text{m/s}^2)$$

The answer is (B).

DYNAMICS-35

The maximum capacity (occupant load) of an elevator is 1000 N. The elevator starts from rest, and its velocity varies with time as shown in the graph. What is most nearly the maximum additional tension in the elevator cable due to the occupants at full capacity? Neglect the mass of the elevator.

(A) 960 N   (B) 1000 N   (C) 1200 N   (D) 1400 N

The maximum tension occurs during the period of maximum acceleration. This occurs for $0 \ \text{s} < t < 2 \ \text{s}$, with acceleration, a, equal to $v/t = 4 \ \text{m/s} / 2 \ \text{s} = 2 \ \text{m/s}^2$. The mass of the occupants is $m = 1000 \ \text{N}/9.81 \ \text{m/s}^2$. During this time,

$$\sum F = ma = T - W$$

$$T = W + ma = 1000 \ \text{N} + \left(\frac{1000 \ \text{N}}{9.81 \ \frac{m}{s^2}} \right) \left(2 \ \frac{m}{s^2} \right)$$

$$= 1204 \ \text{N} \quad (1200 \ \text{N})$$

The answer is (C).

DYNAMICS-36

What is most nearly the kinetic energy of a 3924 N motorcycle traveling at 40 km/h?

(A) 11 100 J   (B) 12 300 J   (C) 23 600 J   (D) 24 600 J

$$m = \frac{W}{g} = \frac{3924 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} = 400 \text{ kg}$$

$$v = \left(40 \frac{\text{km}}{\text{h}}\right) \left(\frac{1000 \frac{\text{m}}{\text{km}}}{3600 \frac{\text{s}}{\text{h}}}\right) = 11.1 \text{ m/s}$$
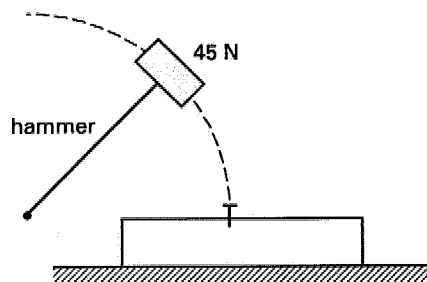
The kinetic energy is

$$E_k = \tfrac{1}{2}mv^2 = \left(\frac{1}{2}\right)(400 \text{ kg})\left(11.1 \frac{\text{m}}{\text{s}}\right)^2$$

$$= 24\,640 \text{ J} \quad (24\,600 \text{ J})$$

The answer is (D).

DYNAMICS-37

A lead hammer weighs 45 N. In one swing of the hammer, a nail is driven 1.5 cm into a wood block. The velocity of the hammer's head at impact is 4.5 m/s. What is most nearly the average resistance of the wood block?

(A) 3090 N   (B) 3100 N   (C) 3920 N   (D) 4090 N

Because energy is conserved, the kinetic energy of the hammer before impact is equal to the work done by the resistance force of the wood block. $m = 45 \text{ N}/9.81 \text{ m/s}^2 = 4.59 \text{ kg}$, $v = 4.5 \text{ m/s}$, and $x_{nail} = 0.015 \text{ m}$.

$$\tfrac{1}{2}mv^2 = Fx$$

$$F = \frac{\tfrac{1}{2}mv^2}{x} = \frac{\left(\dfrac{1}{2}\right)(4.59 \text{ kg})\left(4.5 \dfrac{\text{m}}{\text{s}}\right)^2}{0.015 \text{ m}}$$

$$= 3098 \text{ N} \quad (3100 \text{ N})$$

The answer is (B).

DYNAMICS-38

An automobile uses 74.6 kW to maintain a uniform speed of 96 km/h. What is the thrust force provided by the engine?

(A) 0.87 kN   (B) 2.8 kN   (C) 3.2 kN   (D) 5.6 kN

Power is defined as work done per unit time, which, for a linear system, is equivalent to force times velocity. Therefore,

$$P = Fv$$

$$F = \frac{P}{v}$$

$$= \frac{(74.6 \text{ kW})\left(1000 \dfrac{\text{W}}{\text{kW}}\right)}{\left(96 \dfrac{\text{km}}{\text{h}}\right)\left(1000 \dfrac{\text{m}}{\text{km}}\right)\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right)}$$

$$= 2797.5 \text{ N} \quad (2.8 \text{ kN})$$

The answer is (B).

DYNAMICS-39

A 580 N man is standing on the top of a building 40 m above the ground. What is his potential energy relative to the ground?
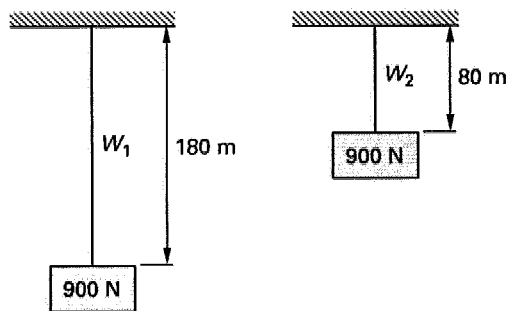
(A) 10 kJ   (B) 12 kJ   (C) 20 kJ   (D) 23 kJ

$$E_p = Wy = (580 \text{ N})(40 \text{ m}) = 23\,200 \text{ J}$$

The answer is (D).

DYNAMICS-40

A 900 N object is initially suspended on a 180 m long cable. The object is then raised 100 m. If the cable weighs 16 N/m, how much work is done?

(A) 100 000 J   (B) 298 000 J   (C) 320 000 J   (D) 398 000 J

The weight of the extended cable for the two situations is

$$W_1 = (180 \text{ m}) \left(16 \ \frac{\text{N}}{\text{m}} \right) = 2880 \text{ N}$$

$$W_2 = (80 \text{ m}) \left(16 \ \frac{\text{N}}{\text{m}} \right) = 1280 \text{ N}$$

These weights may be considered to be concentrated at the midpoints of the extended cables. Choosing the datum to be at the top of the cable, and using the work-energy principle, the work done is equal to the difference in potential energies of the two situations.

$$E_p = (\text{weight})(\text{distance}) + W_1 L$$

$$E_{p1} = (900 \text{ N})(-180 \text{ m}) + (2880 \text{ N}) \left(\frac{-180 \text{ m}}{2} \right) = -421\,200 \text{ J}$$
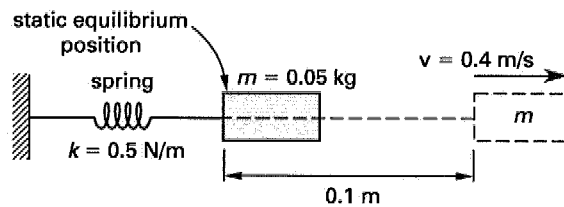
$$E_{p2} = (900 \text{ N})(-80 \text{ m}) + (1280 \text{ N}) \left(\frac{-80 \text{ m}}{2} \right) = -123\,200 \text{ J}$$

$$W = E_{p2} - E_{p1} = -123\,200 \text{ J} - (-421\,200 \text{ J})$$
$$= 298\,000 \text{ J}$$

The answer is (B).

DYNAMICS-41

A 0.05 kg mass attached to a spring (spring constant, $k = 0.5$ N/m) is accelerated to a velocity of 0.4 m/s. What is the total energy for the body in the following diagram? Neglect the spring mass.

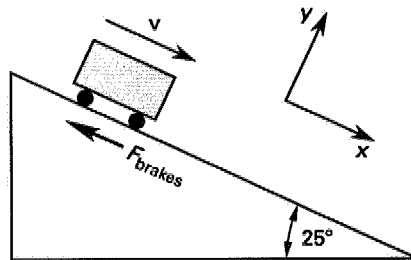(A) 0.0025 J   (B) 0.0040 J   (C) 0.0065 J   (D) 0.0092 J

The total energy is the sum of the kinetic and potential energies.

$$E = E_k + E_p$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$
$$= \left(\frac{1}{2} \right)(0.05 \text{ kg}) \left(0.4 \, \frac{\text{m}}{\text{s}} \right)^2 + \left(\frac{1}{2} \right) \left(0.5 \, \frac{\text{N}}{\text{m}} \right)(0.1 \text{ m})^2$$
$$= 0.0065 \text{ J}$$

The answer is (C).

DYNAMICS-42

A 1000 kg car is traveling down a 25° slope. At the instant that the speed is 13 m/s, the driver applies the brakes. What constant force parallel to the road must be generated by the brakes if the car is to stop in 90 m?

(A) 1290 N   (B) 2900 N   (C) 5080 N   (D) 8630 N

The change in energy is equal to the work done by the brakes. The change in velocity squared is

$$v^2 - v_0^2 = 0 - \left(13 \; \frac{m}{s}\right)^2 = -169 \; m^2/s^2$$

The change in elevation of the car is

$$h - h_0 = 0 - (90 \; m)\sin 25° = -38 \; m$$

$$\Delta E_k + \Delta E_p = Fx$$

$$\tfrac{1}{2}m(v^2 - v_0^2) + mg(h - h_0) = Fx$$

$$\left(\frac{1}{2}\right)(1000 \; kg)\left(-169 \; \frac{m^2}{s^2}\right)$$

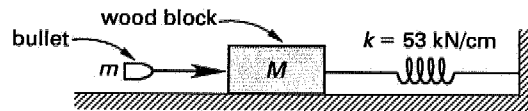$$+ (1000 \; kg)\left(9.81 \; \frac{m}{s^2}\right)(-38 \; m) = F(90 \; m)$$

$$F = -5080 \; N$$

The answer is (C).

DYNAMICS-43

A bullet of mass 100 g is fired at a wooden block resting on a horizontal surface. A spring with stiffness $k = 53$ kN/cm resists the motion of the block. If the maximum displacement of the block produced by the impact of the bullet is 3.4 cm, what is most nearly the velocity of the bullet at impact? Assume there are no losses at impact, and the spring has no mass.

(A) 250 km/h   (B) 450 km/s   (C) 630 km/h   (D) 890 km/h

Due to the conservation of energy, the kinetic energy of the bullet before impact is equal to the potential energy of the spring-mass-bullet system at maximum compression.

$$E_{k,\text{bullet}} = E_{p,\text{system}}$$

$$\tfrac{1}{2}m_{\text{bullet}}\text{v}^2 = \tfrac{1}{2}kx^2$$

$$\text{v} = \sqrt{\frac{kx^2}{m_{\text{bullet}}}}$$

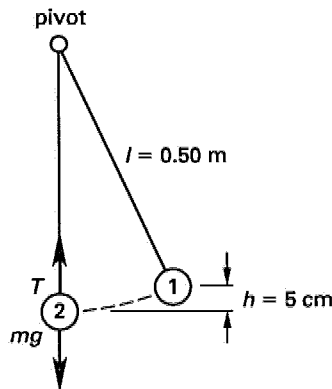$$= \frac{\sqrt{\dfrac{\left(53\,000\ \dfrac{\text{N}}{\text{cm}}\right)(3.4\ \text{cm})^2\left(\dfrac{1\ \text{m}}{100\ \text{cm}}\right)}{0.1\ \text{kg}}}\left(60\ \dfrac{\text{s}}{\text{min}}\right)\left(60\ \dfrac{\text{min}}{\text{h}}\right)}{1000\ \dfrac{\text{m}}{\text{km}}}$$

$$= 891\ \text{km/h}\quad (890\ \text{km/h})$$

The answer is (D).

DYNAMICS-44

A simple pendulum consists of a 100 g mass attached to a weightless cord. If the mass is moved laterally such that $h = 5$ cm and then released, what is the maximum tension in the cord, T?

(A) 1.08 N   (B) 1.12 N   (C) 1.18 N   (D) 1.25 N

The maximum tension will occur when the pendulum is at its lowest point, position 2 in the figure. The force balance in the vertical y direction gives

$$ma_y = T - mg$$
$$T = ma_y + mg$$
$$= \frac{mv^2}{l} + mg \qquad \text{[I]}$$

From the conservation of energy,

$$E_{p1} = E_{k2}$$
$$mgh = \tfrac{1}{2}mv^2$$
$$v = \sqrt{2gh} \qquad \text{[II]}$$

Equations I and II give

$$T_{max} = \frac{m\left(\sqrt{2gh}\right)^2}{l} + mg$$
$$= mg\left(\frac{2h + l}{l}\right)$$
$$= (100 \text{ g})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{(2)(5 \text{ cm}) + 50 \text{ cm}}{50 \text{ cm}}\right)\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right)$$
$$= 1.177 \text{ kg·m/s}^2 \quad (1.18 \text{ N})$$

The answer is (C).

DYNAMICS-45

A stationary passenger car of a train is set into motion by the impact of a moving locomotive. What is the impulse delivered to the car if it has a velocity of 11 m/s immediately after the collision? The weight of the car is 56.8 kN.

(A) 45.5 kN·s   (B) 57.5 kN·s   (C) 63.7 kN·s   (D) 64.1 kN·s

From the impulse-momentum principle,

$$\text{Imp} = \Delta mv$$

$$mv_1 + \text{Imp} = mv_2$$

$$\text{Imp} = m(v_2 - v_1)$$

$$= \left(\frac{56.8 \text{ kN}}{9.81 \frac{\text{m}}{\text{s}^2}} \right) \left(11 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}} \right)$$

$$= 63.7 \text{ kN·s}$$

The answer is (C).

DYNAMICS-46

Which of the following statements is FALSE?
(A) The time rate of change of the angular momentum about a fixed point is equal to the total moment of the external forces acting on the system about the point.
(B) The coefficient of restitution can be less than zero.
(C) The frictional force always acts to resist motion.
(D) Momentum is conserved during elastic collisions.

The coefficient of restitution is defined as the ratio of the impulses corresponding to the period of restitution and to the period of deformation of a body, respectively. Its value is always between 0 and 1.

The answer is (B).

DYNAMICS-47

Two identical balls hit head-on in a perfectly elastic collision. Given that the initial velocity of one ball is 0.85 m/s and the initial velocity of the other is −0.53 m/s, what is the relative velocity of each ball after the collision?

(A) 0.85 m/s and −0.53 m/s
(B) 1.2 m/s and −0.72 m/s
(C) 1.2 m/s and −5.1 m/s
(D) 1.8 m/s and −0.98 m/s

Let v_1 and v_2 be the velocities of balls 1 and 2, respectively, after the collision. The conservation of momentum equation is

$$mv_{01} + mv_{02} = mv_1 + mv_2$$
$$0.85 \ \frac{m}{s} + \left(-0.53 \ \frac{m}{s}\right) = v_1 + v_2$$
$$v_1 + v_2 = 0.32 \ \text{m/s} \qquad [\text{I}]$$

Since kinetic energy is conserved,

$$\tfrac{1}{2}mv_{01}^2 + \tfrac{1}{2}mv_{02}^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}mv_2^2$$
$$\left(0.85 \ \frac{m}{s}\right)^2 + \left(-0.53 \ \frac{m}{s}\right)^2 = v_1^2 + v_2^2$$
$$v_1^2 + v_2^2 = 1 \ \text{m}^2/\text{s}^2 \qquad [\text{II}]$$

Combining Eqs. I and II,

$$v_2^2 - 0.32v_2 - 0.4488 = 0$$

$$v_2 = \frac{0.32 \ \frac{m}{s} \pm \sqrt{\left(-0.32 \ \frac{m}{s}\right)^2 - (4)(1)\left(-0.4488 \ \frac{m^2}{s^2}\right)}}{2}$$

$$= 0.85 \ \text{m/s or} -0.53 \ \text{m/s} \quad [\text{negative value not used}]$$

$$v_1 = 0.32 - v_2 = 0.32 \ \frac{m}{s} - 0.85 \ \frac{m}{s}$$

$$= -0.53 \ \text{m/s}$$

The answer is (A).

DYNAMICS-48

A steel ball weighing 490 N strikes a stationary wooden ball weighing 490 N. If the steel ball has a velocity of 5.1 m/s at impact, what is its velocity immediately after impact? Assume the collision is central and perfectly elastic.

(A) −5 m/s   (B) −2 m/s   (C) 0 m/s   (D) 5 m/s

Since the balls have the same weight, they have equal mass. Denoting the instances before and after the collision by the subscripts 1 and 2, respectively, $v_{s1} = 5.1$ m/s and $v_{w1} = 0$. Conservation of momentum gives

$$m_s v_{s1} + m_w v_{w1} = m_s v_{s2} + m_w v_{w2}$$

$$v_{s2} + v_{w2} = v_{s1} = 5.1 \text{ m/s} \qquad \text{[I]}$$

Conservation of energy gives

$$\tfrac{1}{2} m_s v_{s1}^2 + \tfrac{1}{2} m_w v_{w1}^2 = \tfrac{1}{2} m_s v_{s2}^2 + \tfrac{1}{2} m_w v_{w2}^2$$

$$v_{s2}^2 + v_{w2}^2 = v_{s1}^2 = \left(5.1 \ \frac{\text{m}}{\text{s}}\right)^2$$

$$= 26.01 \text{ m}^2/\text{s}^2 \qquad \text{[II]}$$

Solving Eqs. I and II simultaneously,
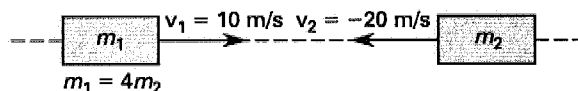
$$v_{s2}^2 + \left(26.01 \ \frac{\text{m}^2}{\text{s}^2} - 10.2 v_{s2} + v_{s2}^2\right) = 26.01 \text{ m}^2/\text{s}^2$$

$$2 v_{s2}^2 - 10.2 v_{s2} = 0$$

$$v_{s2}^2 - 5.1 \ \frac{\text{m}}{\text{s}} v_{s2} = 0 \ \frac{\text{m}}{\text{s}^2}$$

$$v_{s2} = 0 \text{ m/s}, \ 5.1 \text{ m/s}$$

If $v_{s2} = 5.1$ m/s, then $v_{w2} = 0$ m/s, and no change has occurred during the collision. This is physically impossible, so $v_{s2} = 0$ m/s.

The answer is (C).

DYNAMICS-49

Two masses collide in a perfectly inelastic collision. Given the data in the illustration, find the velocity and direction of motion of the resulting combined mass.

$$v_1 = 10 \text{ m/s} \quad v_2 = -20 \text{ m/s}$$
$$m_1 = 4m_2$$

(A) The mass is stationary.
(B) 4 m/s to the right
(C) 5 m/s to the left
(D) 10 m/s to the right

Let the positive direction of motion be to the right. Let m_3 be the resultant combined mass moving at velocity v_3 after the collision. Since momentum is conserved,

$$m_1 v_1 + m_2 v_2 = m_3 v_3$$

However, $m_3 = m_1 + m_2 = 4m_2 + m_2 = 5m_2$. Therefore,

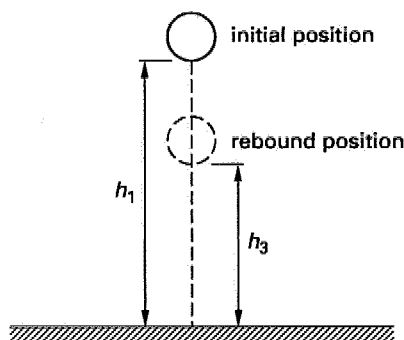$$4m_2 \left(10 \ \frac{m}{s}\right) + m_2 \left(-20 \ \frac{m}{s}\right) = 5m_2 v_3$$

$$40m_2 - 20m_2 = 5m_2 v_3$$

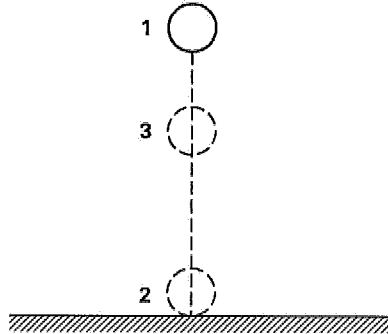$$v_3 = 4 \text{ m/s to the right}$$

The answer is (B).

DYNAMICS-50

A ball is dropped onto a solid floor from an initial height, h_0. If the coefficient of restitution, e, is 0.90, how high will the ball rebound?

initial position

rebound position

h_1

h_3

(A) $0.45h_1$ (B) $0.81h_1$ (C) $0.85h_1$ (D) $0.90h_1$

The subscripts 1, 2, and 3 denote the positions shown.

Conservation of energy gives, before impact,

$$E_{1,\text{total}} = E_{2,\text{total}}$$

Since the kinetic energy at position 1 and the potential energy at position 2 are zero,

$$mgh_1 = \tfrac{1}{2}mv_2^2$$
$$v_2 = \sqrt{2gh_1}$$

After impact, the kinetic energy at position 3 is zero.

$$\tfrac{1}{2}mv_2^2 = mgh_3$$
$$v_2 = \sqrt{2gh_3}$$

By definition, the coefficient of restitution is

$$e = \frac{v_{\text{ball}} - v_{\text{floor}}}{v_{1,\text{floor}} - v_{1,\text{ball}}} = -\frac{v}{v_1}$$
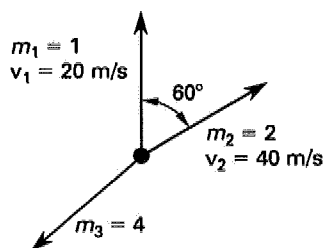$$= -\frac{\sqrt{2gh_3}}{\sqrt{2gh_1}} = -\sqrt{\frac{h_3}{h_1}}$$
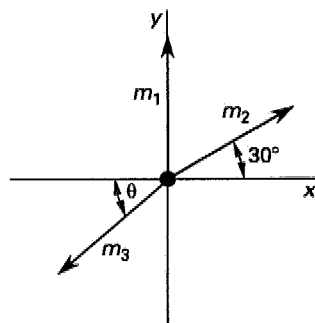$$h_3 = e^2 h_1 = (0.9)^2 h_1$$
$$= 0.81 h_1$$

The answer is (B).

DYNAMICS-51

A mass suspended in space explodes into three pieces whose masses, initial velocities, and directions are given in the illustration. All motion is within a single plane. Find the velocity of m_3.

(A) 20 m/s   (B) 23 m/s   (C) 35 m/s   (D) 40 m/s

Defining the x and y axes as shown, conservation of momentum for the x direction gives

$$m_2 v_2 \cos 30° + m_3 v_3 \cos \theta = 0$$

$$2v_2 \cos 30° + 4v_3 \cos \theta = 0$$

$$(2) \left(40 \ \frac{m}{s} \right) \cos 30° = -4v_3 \cos \theta$$

$$20 \ \frac{m}{s} \cos 30° = -v_3 \cos \theta$$

$$v_3 = -\frac{17.32 \ \dfrac{m}{s}}{\cos \theta} \qquad \text{[I]}$$

For the y direction,

$$m_1 v_1 + m_2 v_2 \sin 30° + m_3 v_3 \sin \theta = 0$$

$$m_1 \left(20 \; \frac{m}{s} \right) + 2m_1 \left(40 \; \frac{m}{s} \right) \sin 30° = -4m_1 v_3 \sin \theta$$

$$-4v_3 \sin \theta = 60 \; \frac{m}{s}$$

$$v_3 = -\frac{60 \; \frac{m}{s}}{4 \sin \theta} \qquad \text{[II]}$$

Equations I and II give

$$\tan \theta = \frac{60 \; \frac{m}{s}}{\left(17.32 \; \frac{m}{s} \right)(4)}$$

$$\theta = 40.9°$$

$$v_3 = \frac{-17.32 \; \frac{m}{s}}{-\cos 40.9°}$$

$$= 22.9 \text{ m/s}$$

The answer is (B).
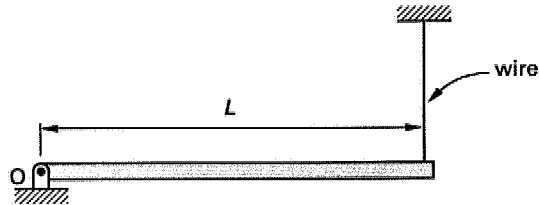
DYNAMICS–52

Which of the following statements is FALSE?

(A) Kinematics is the study of the effects of motion, while kinetics is the study of the causes of motion.

(B) The radius of gyration for a mass of uniform thickness is identical to the radius of gyration for a planar area of the same shape.

(C) Angular momentum for rigid bodies may be regarded as the product of angular velocity and inertia.

(D) The acceleration of any point within a homogenous body rotating with a constant angular velocity is proportional to the distance of that point to the center of mass.

A body rotating at a constant angular velocity has no angular acceleration.

The answer is (D).

DYNAMICS-53

A uniform beam of weight W is supported by a pin joint and a wire. What will be the angular acceleration, α, at the instant that the wire is cut?

(A) $\dfrac{g}{L}$ (B) $\dfrac{3g}{2L}$ (C) $\dfrac{2g}{L}$ (D) $\dfrac{Wg}{L}$

The only force on the beam is its weight acting at a distance of $L/2$ from point O. Taking the moment about O,
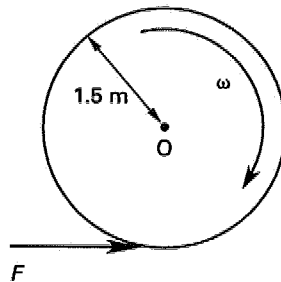
$$W\left(\frac{L}{2}\right) = I_O \alpha$$

For a slender beam rotating about its end,

$$I_O = \frac{1}{3}mL^2$$

$$\alpha = \frac{WL}{2I_O} = \frac{WL}{2\left(\frac{1}{3}mL^2\right)}$$

$$= \frac{3g}{2L}$$

The answer is (B).

DYNAMICS-54

A thin circular disk of mass 25 kg and radius 1.5 m is spinning about its axis with an angular velocity of $\omega = 1800$ rpm. It takes 2.5 min to stop the motion by applying a constant force, F, to the edge of the disk. The force required is most nearly

(A) 7.2 N   (B) 16 N   (C) 24 N   (D) 32 N

The relationship between the retarding moment, Fr, and the deceleration is

$$Fr = -I_O \alpha$$

Designating the positive rotational direction as counterclockwise, $\omega = -1800$ rpm. Therefore,

$$F = -\frac{I_O \alpha}{r} = -\frac{\frac{1}{2}mr^2 \alpha}{r}$$

$$= -\frac{1}{2}mr\frac{\Delta \omega}{\Delta t}$$

$$= \left(-\frac{1}{2}\right)(25 \text{ kg})(1.5 \text{ m})\left(\left(\frac{-1800 \frac{\text{rev}}{\text{min}}}{2.5 \text{ min}}\right)(2\pi)\left(\frac{1 \text{ min}^2}{3600 \text{ s}^2}\right)\right)$$

$$= 23.6 \text{ N} (24 \text{ N})$$

The answer is (C).

DYNAMICS-55

A mass, m, of 0.025 kg is hanging from a spring whose spring constant, k, is 0.44 N/m. If the mass is pulled down and released, what is the period of oscillation?

(A) 0.50 s (B) 1.2 s (C) 1.5 s (D) 2.1 s

By definition, the period T is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025 \text{ kg}}{0.44 \frac{\text{N}}{\text{m}}}}$$

$$= 1.5 \text{ s}$$

The answer is (C).

DYNAMICS-56

A body hangs from an ideal spring. What is the frequency of oscillation of the body if its mass, m, is 0.015 kg, and k is 0.5 N/m?

(A) 0.51 Hz (B) 0.66 Hz (C) 0.78 Hz (D) 0.92 Hz

By definition, the frequency, f, is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{0.5 \frac{\text{N}}{\text{m}}}{0.015 \text{ kg}}}$$

$$= 0.92 \text{ Hz}$$

The answer is (D).

DYNAMICS-57

What is the natural frequency, ω, of an oscillating body whose period of oscillation is 1.8 s?

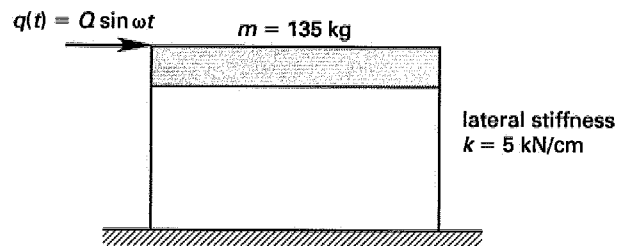(A) 1.8 rad/s (B) 2.7 rad/s (C) 3.5 rad/s (D) 4.2 rad/s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.8 \text{ s}}$$
$$= 3.5 \text{ rad/s}$$

The answer is (C).

DYNAMICS-58

A one-story frame is subjected to a sinusoidal forcing function $q(t) = Q \sin \omega t$ at the transom. What is most nearly the frequency of $q(t)$, in hertz, if the frame is in resonance with the force?

$q(t) = Q \sin \omega t$ $m = 135$ kg

lateral stiffness
$k = 5$ kN/cm

(A) 2.6 Hz   (B) 2.9 Hz   (C) 3.6 Hz   (D) 9.7 Hz

Resonance occurs when the forced frequency, ω, equals the natural frequency, ω_n.
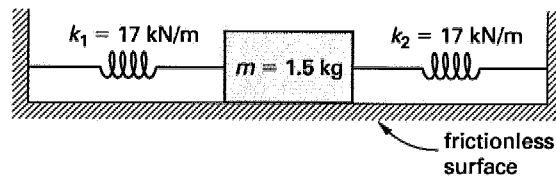
$$m = 135 \text{ kg}$$

$$k = \left(5000 \ \frac{\text{N}}{\text{cm}}\right) \left(100 \ \frac{\text{cm}}{\text{m}}\right) = 500\,000 \text{ N/m}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{500\,000 \ \frac{\text{N}}{\text{m}}}{135 \text{ kg}}}$$

$$= 9.69 \text{ Hz} \quad (9.7 \text{ Hz})$$

The answer is (D).

DYNAMICS-59

In the mass-spring system shown, the mass, m, is displaced 0.09 m to the right of the equilibrium position and then released. Find the maximum velocity of m.

$k_1 = 17$ kN/m $m = 1.5$ kg $k_2 = 17$ kN/m

frictionless surface

(A) 0.3 m/s   (B) 5 m/s   (C) 8 m/s   (D) 14 m/s

The kinetic energy before the mass is released is zero. The maximum velocity will occur when the mass returns to the point of static equilibrium, where the deflection is zero and, hence, the potential energy equals zero. Therefore, since the total energy of the system is constant,

$$E_{p,1} = E_{k,2}$$

$$\tfrac{1}{2}k_1 x_1^2 + \tfrac{1}{2}k_2 x_2^2 = \tfrac{1}{2}mv^2$$
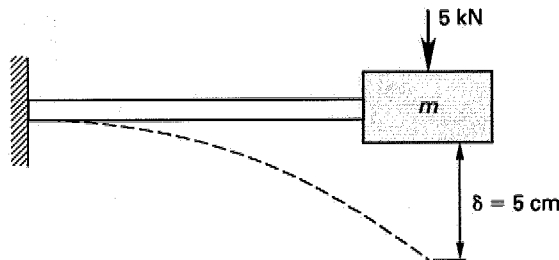
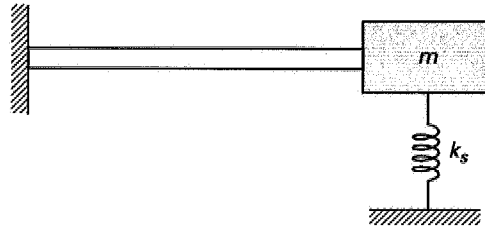The displacement of each spring is

$$x = 0.09 \text{ m}$$

$$v = \sqrt{\frac{k_1 x_1^2 + k_2 x_2^2}{m}}$$

$$= \sqrt{\frac{\left(\left(17 \, \dfrac{\text{kN}}{\text{m}}\right)(0.09 \text{ m})^2 + \left(17 \, \dfrac{\text{kN}}{\text{m}}\right)(0.09 \text{ m})^2\right)\left(1000 \, \dfrac{\text{N}}{\text{kN}}\right)}{1.5 \text{ kg}}}$$

$$= 13.5 \text{ m/s} \quad (14 \text{ m/s})$$

The answer is (D).

DYNAMICS-60
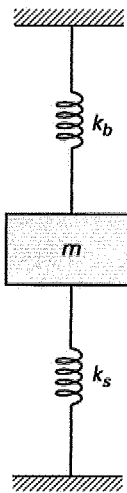
A cantilever beam with an end mass, $m = 7000$ kg, deflects 5 cm when a force of 5 kN is applied at the end. The beam is subsequently mounted on a spring of stiffness, $k_s = 1.5$ kN/cm. What is most nearly the natural frequency of the mass-beam-spring system?

(A) 1.5 rad/s   (B) 3.1 rad/s   (C) 6.0 rad/s   (D) 6.3 rad/s

A cantilever with an end mass m can be modeled as follows.

$$k_b = \frac{5000 \text{ N}}{5 \text{ cm}} = 1000 \text{ N/cm}$$

For this model, both springs undergo the same deflection. Hence,

$$k = k_b + k_s = 1000 \ \frac{\text{N}}{\text{cm}} + 1500 \ \frac{\text{N}}{\text{cm}}$$
$$= 2500 \text{ N/cm}$$

The natural frequency is, therefore,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\left(2500 \ \dfrac{\text{N}}{\text{cm}}\right)\left(100 \ \dfrac{\text{cm}}{\text{m}}\right)}{7000 \text{ kg}}}$$
$$= 5.98 \text{ rad/s} \quad (6.0 \text{ rad/s})$$

The answer is (C).