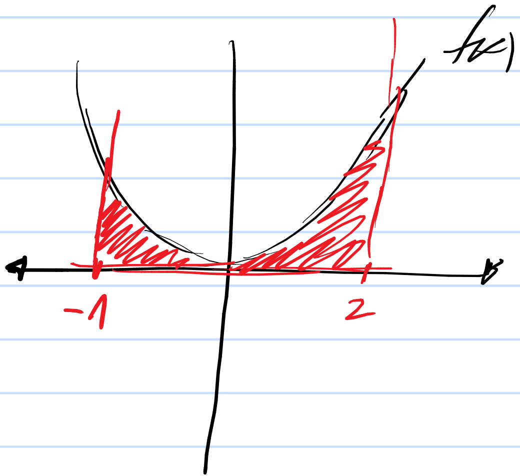
$\boxed{2015-2}$



$$\text{Area} = \int_{-1}^{2} f(x)\,dx = \int_{-1}^{2} x^2\,dx$$

$$= \frac{x^3}{3}\Big|_{-1}^{2} = \frac{(2)^3 - (-1)^3}{3}$$

$$= \frac{8-(-1)}{3} = \frac{9}{3} = 3$$

2016 - 1

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3+n^2+n}{n^4+n^3+n^2+n}$ $\qquad a_n = \dfrac{n^3+n^2+n}{n^4+n^3+n^2+n}$

$\displaystyle\sum_{n=1}^{\infty} a_n$ Con la serie $\displaystyle\sum_{n=1}^{\infty} b_n$ con $b_n = \dfrac{1}{n}$ $\quad\cdot\left(\dfrac{1}{n^4}\right)$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\dfrac{n^3+n^2+n}{n^4+n^3+n^2+n}}{\dfrac{1}{n}} = \lim_{n\to\infty} \frac{n^4+n^3+n^2}{n^4+n^3+n^2+n} \to \left(\frac{1}{n^4}\right)$$

$$= \lim_{n\to\infty} \frac{1+\frac{1}{n}+\frac{1}{n^2}}{1+\frac{1}{n}+\frac{1}{n^2}+\frac{1}{n^3}} = \frac{1}{1} = 1 \Rightarrow$$ Ambas series se comportan igual

$$\Rightarrow \sum_{n=1}^{\infty} a_n$$ diverge

b) $$\sum_{n=0}^{\infty} \frac{n^2}{2n^3+1}$$

$$a_n = \frac{n^2}{2n^3+1} \qquad \sum_{n=1}^{\infty} b_n \qquad b_n = \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n^2}{2n^3+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^3}{2n^3+1} = \frac{1}{2}$$

$\Rightarrow$ La serie diverge.

c) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n!}$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}} = \frac{\cancel{3^n}\cdot 3\cdot \cancel{n!}}{\underbrace{(n+1)!}_{(n+1)\cdot \cancel{n!}}\cdot \cancel{3^n}} = \frac{3}{(n+1)}$$

$$a_n = \frac{3^n}{n!}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{3}{n+1} = 0 \Rightarrow \;<1 \Rightarrow$$ la serie converge.

$\boxed{\text{Alternativas c)}}$

$\ln(1)=0$

d) $$\sum_{n=1}^{\infty} \frac{\ln(n)}{n+2} = \cancel{\frac{\ln(1)}{1+2}} + \sum_{n=2}^{\infty} \frac{\ln(n)}{n+2} = \sum_{n=3}^{\infty} \frac{\ln(n)}{n+2}$$

$+\frac{\ln(2)}{4}$ } cte

Diverge

diverge

Para $n \geq 3$ $\quad \ln(n) \geq 1$

$$\Rightarrow \underbrace{\sum_{n=3}^{\infty} \frac{\ln(n)}{n+2}}_{\text{diverge}} \geq \underbrace{\sum_{n=3}^{\infty} \frac{1}{n+2}}_{\text{diverge}}$$

$$\sum_{n=3}^{\infty} a_n \qquad a_n = \frac{1}{n+2}$$

$$\sum_{n=3}^{\infty} b_n \qquad \text{con } b_n = \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{n+2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{n+2} = 1$$

$\boxed{2016-2}$

$$\int_a^b \frac{2+\sin(x)}{(x-a)^p}\,dx \qquad 0<a<b<\infty$$

$f(x)$

$$\boxed{\int_a^b \frac{1}{(x-a)^p}\,dx}$$

$g(x)$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{\frac{2+\sin(x)}{(x-a)^p}}{\frac{1}{(x-a)^p}} = \lim_{x\to a}\left[2+\sin(x)\right] = 2+\sin(a) > 0$$

$$\int_a^b \frac{1}{(x-a)^p}\,dx = \int_a^b (x-a)^{-p}\,dx =$$

$$= \lim_{t \to a^+} \int_t^b (x-a)^{-p}\,dx = \lim_{t \to a^+} \left[\frac{(x-a)^{-p+1}}{(-p+1)} \right]\Bigg|_{x=t}^{x=b}$$

$$= \lim_{t \to a^+} \left[\frac{(b-a)^{-p+1}}{(-p+1)} - \frac{(t-a)^{-p+1}}{(-p+1)} \right]$$

$$= \frac{(b-a)^{-p+1}}{(-p+1)} - \lim_{t \to a^+} \frac{(t-a)^{-p+1}}{(-p+1)}$$

diverge

converge

$$\lim_{t\to a^+} (t-a)^{-p+1} \to 0 \text{ eşitse } \to \infty$$

$$(-p+1)$$

$$\boxed{(-p+1) > 0} \qquad \boxed{1 > p} \qquad p \in (-\infty, 1)$$

$$\boxed{\text{Alternatif } 2)}$$

$\boxed{2017-1}$

a) $\sum_{n=0}^{\infty} a_n$ con $a_n = \frac{(n!)^2}{(2n)!}$ $\boxed{2n \text{ términos}}$

$$a_n = \frac{(n!)^2}{(2n)!} = \frac{\overbrace{n\cdot(n-1)\cdot\ldots\cdot 2\cdot 1}^{n!}\cdot\overbrace{\cancel{n}\cdot\cancel{(n-1)}\cdot\ldots\cdot\cancel{2}\cdot\cancel{1}}^{n!}}{2n\cdot(2n-1)\cdot\ldots\cdot\underbrace{(n+2)(n+1)}_{n \text{ términos}}\cdot\cancel{n}\cdot\cancel{(n-1)}\cdot\ldots\cdot\cancel{2}\cdot\cancel{1}}$$

$$= \frac{\overbrace{n}^{\le 1}\cdot\overbrace{(n-1)}^{\le 1}\cdot\ldots\cdot\overbrace{3}^{\le 1}\cdot 2\cdot 1}{2n\cdot(2n-1)\cdot\ldots\cdot(n+3)(n+2)(n+1)} \le \frac{2}{(n+2)(n+1)} = \frac{2}{n^2+3n+2}$$

$$a_n \le \frac{\cancel{3}\,2}{n^2+3n+2} \Rightarrow \sum_{n=0}^{\infty} a_n \le \sum_{n=0}^{\infty} \frac{2\,\cancel{3}}{n^2+3n+2}$$

Converge

Converge

$$b_n = \frac{\cancel{3}\,2}{n^2+3n+2}$$

$$c_n = \frac{1}{n^2}$$

$$\lim_{n\to\infty} \frac{b_n}{c_n} = \lim_{n\to\infty} \frac{\frac{\cancel{3}\,2}{n^2+3n+1}}{\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{2n^2}{n^2+3n+1} = 2$$

$$\sum b_n \quad \sum c_n$$

Alternativa e)

b) $\sum_{n=0}^{\infty} \frac{1}{4n+1}$ $\quad a_n = \frac{1}{4n+1}$ $\quad b_n = \frac{1}{n}$ $\quad \sum b_n$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{4n+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{4n+1} = \frac{1}{4}$$

$\Rightarrow$ Divergentă

g) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n+2}$ $\longrightarrow$ diverge.

d) $\displaystyle\sum_{n=2}^{\infty} \frac{n^3+4n}{n^4-8}$ a_n $\longrightarrow$ diverge

$b_n = \dfrac{1}{n}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\dfrac{n^3+4n}{n^4-8}}{\dfrac{1}{n}} = \lim_{n\to\infty} \frac{n^4+4n^2}{n^4-8} = 1$$

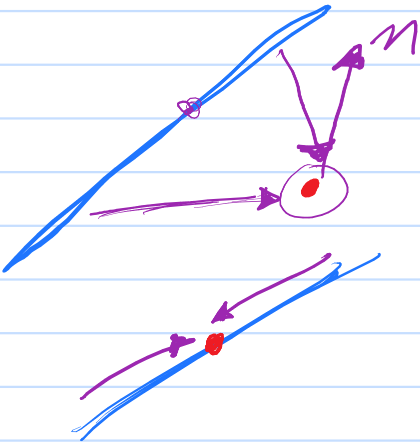
$\boxed{2017-2}$

$$\frac{x-2}{5}=\frac{y+5}{1}=\frac{z+1}{3}$$

$(7,-4,2)$

$x=7$
$y=-4$
$z=2$

$\boxed{\text{Alternativa d)}}$

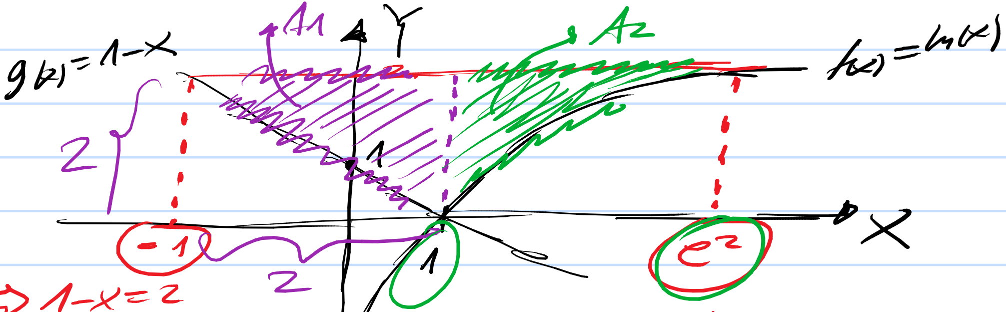


$$\frac{x-2}{5}=\frac{7-2}{5}=\frac{5}{5}=1$$

$$\frac{y+5}{1}=\frac{-4+5}{1}=\frac{1}{1}=1$$

$$\frac{z+1}{3}=\frac{2+1}{3}=\frac{3}{3}=1$$

$\boxed{2018-1}$



$g(x) = 2 \Rightarrow 1-x=2$

$\Rightarrow 1-2=x \Rightarrow x=-1$

$f(x)=2 \Rightarrow \ln(x)=2$

$e^2 = x$

$$A_1 = \int_{-1}^{1} [2-g(x)]\,dx = \int_{-1}^{1} (2-(1-x))\,dx = \int_{-1}^{1} (1+x)\,dx$$

$$= \left(x+\frac{x^2}{2}\right)\Big|_{x=-1}^{x=1} = \left(1+\frac{1}{2}\right) - \left(-1+\frac{1}{2}\right) =$$

$$1+\frac{1}{2}+1-\frac{1}{2} = 2 \Rightarrow \boxed{\text{Area}_1 = 2}$$

$$e^2+1$$

$$A_2 = \int_1^{e^2} (2-\ln(x))\,dx = \int_1^{e^2} (2-\ln(x))\,dx$$

$$= 2\cdot\int_1^{e^2} dx - \int_1^{e^2} \ln(x)\,dx = 2\cdot[e^2-1] - \int_1^{e^2} \ln(x)\,dx$$

$$\int \ln(x)\,dx = \ln(x)\cdot x - \underbrace{\int \frac{1}{x}\cdot x\,dx}_{\int dx = x} = \ln(x)\cdot x - x$$

$$= x\cdot[\ln(x)-1]$$

$f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$

$g'(x) = 1 \Rightarrow g(x) = x$

$$\int_1^{e^2} \ln(x)\,dx = x\cdot[\ln(x)-1]\Big|_{x=1}^{x=e^2} = e^2\cdot[\overbrace{\ln(e^2)}^{2}-1] - 1\cdot[\underbrace{\ln(1)-1}_{-1}] = e^2\cdot[1] - 1\cdot(-1) = e^2+1$$

$$A_2 = 2[e^2-1] - (e^2+1) = 2e^2 - 2 - e^2 - 1$$

$$= e^2 - 3 \qquad \boxed{A_2 = e^2 - 3} \quad \boxed{A_1 = 2}$$

$$A_1 + A_2 = e^2 - 3 + 2 = \boxed{e^2 - 1}$$

$\boxed{\text{Alternativa d)}}$

2018-2

a) $\int_1^\infty \sin^2\left(\frac{1}{x}\right) dx$ → converge

$\int_1^\infty \frac{1}{x^2} dx$ } converge

$f(x) = \sin^2\left(\frac{1}{x}\right)$

$g(x) = \frac{1}{x^2}$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{\sin^2\left(\frac{1}{x}\right)}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\left[\sin\left(\frac{1}{x}\right)\right]^2}{\left(\frac{1}{x}\right)^2}$$

$$= \lim_{x\to\infty} \left[\frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}\right]^2 = \left[\lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}\right]^2$$

$u = \frac{1}{x}$

$$\underset{\substack{x\to\infty \\ u\to 0^+}}{=} \left[\lim_{u\to 0^+} \frac{\sin(u)}{u}\right]^2 = [1]^2 = 1$$

b) $$\int_{-1}^{\infty} \frac{\sin^2\left(\frac{1}{x}\right)}{x^2}\,dx \le \int_{-1}^{\infty} \frac{dx}{x^2} \Big\} \text{converge}$$

≤ 1

Convergente $\Rightarrow$ Converge

$$\int^{\infty} \frac{dx}{x^p} \quad p>1 \quad p=\frac{1}{2}$$

c) $$\int_1^{\infty} \sqrt{\sin\left(\frac{1}{x}\right)}\,dx \qquad \int_1^{\infty} \frac{dx}{\sqrt{x}} \quad \text{diverge}$$

$f(x)$ $\qquad$ $g(x)$

$$\lim_{x\to\infty}\frac{f(x)}{g(x)} = \lim_{x\to\infty}\frac{\sqrt{\sin\left(\frac{1}{x}\right)}}{\frac{1}{\sqrt{x}}} = \lim_{x\to\infty}\frac{\sqrt{\sin\left(\frac{1}{x}\right)}}{\sqrt{\frac{1}{x}}}$$

$$= \lim_{x\to\infty}\sqrt{\frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}} = \sqrt{\lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}}$$

$u = \frac{1}{x}$

$x \to \infty,\ u \to 0^+$

$$= \sqrt{\lim_{u\to 0^+}\frac{\sin(u)}{u}} = \sqrt{1} = 1$$

Alternativa c)

d) $\displaystyle\int_1^\infty \frac{\sqrt{\sin\left(\frac{1}{x}\right)}}{x^2}\,dx \leq \int_1^\infty \frac{dx}{x^2}$ converge

converge

$\sin\left(\frac{1}{x}\right) \leq 1$

$\Rightarrow \sqrt{\sin\left(\frac{1}{x}\right)} \leq 1$

$\boxed{2019-1}$

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{2n+1} = \sum_{n=1}^{\infty} a_n$ ~~bude~~ $a_n = \dfrac{n-1}{2n+1}$

$\displaystyle\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n-1}{2n+1} = \frac{1}{2} \neq 0 \Rightarrow$ divergje

b) $a_n = \dfrac{\sqrt{n!}}{2^n}$ $\displaystyle\sum_{n=0}^{\infty} a_n$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\dfrac{\sqrt{(n+1)!}}{2^{n+1}}}{\dfrac{\sqrt{n!}}{2^n}} = \lim_{n\to\infty} \frac{\sqrt{(n+1)!}\cdot 2^n}{2^n\cdot 2\cdot\sqrt{n!}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{(n+1)\cdot n!}\cdot 2^n}{2^n\cdot 2\cdot\sqrt{n!}} = \lim_{n\to\infty} \frac{\sqrt{(n+1)}\cdot\cancel{\sqrt{n!}}\cdot\cancel{2^n}}{\cancel{2^n}\cdot 2\cdot\cancel{\sqrt{n!}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n+1}}{2} = \infty \Longrightarrow$$ La serie diverge.

c) $$\sum_{n=0}^{\infty} \frac{e^n}{n! \cdot \left(\sqrt{n+1} - \sqrt{n}\right)}$$

$$a_n = \frac{e^n}{n! \cdot \left(\sqrt{n+1} - \sqrt{n}\right)}$$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{e^{n+1}}{(n+1)! \cdot \left(\sqrt{n+2} - \sqrt{n+1}\right)}}{\dfrac{e^n}{n! \cdot \left(\sqrt{n+1} - \sqrt{n}\right)}} = \frac{\cancel{e^n} \cdot e \cdot \cancel{n!} \left(\sqrt{n+1} - \sqrt{n}\right)}{\cancel{e^n} \cdot (n+1)! \cdot \left(\sqrt{n+2} - \sqrt{n+1}\right)}$$

$$(n+1) \cdot \cancel{n!}$$

$$= \frac{e}{(n+1)} \cdot \left(\frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n+2} - \sqrt{n+1}} \right)$$

$$(\sqrt{n+1})^2 - (\sqrt{n})^2 = (n+1) - n = 1$$

$$\lim_{n\to\infty} \left(\frac{e}{n+1} \right) \cdot \left(\frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n+2} - \sqrt{n+1}} \right)$$

$$= \lim_{n\to\infty} \left[\frac{e}{(n+1)} \cdot \left(\frac{\cancel{\sqrt{n+1} - \sqrt{n}}}{\cancel{\sqrt{n+2} - \sqrt{n+1}}} \right) \cdot \left(\frac{\cancel{\sqrt{n+1} + \sqrt{n}}}{\sqrt{n+1} + \sqrt{n}} \right) \cdot \left(\frac{\sqrt{n+2} + \sqrt{n+1}}{\cancel{\sqrt{n+2} + \sqrt{n+1}}} \right) \right]$$

$$1 \qquad 1$$

$$(\sqrt{n+2})^2 - (\sqrt{n+1})^2 = n+2 - (n+1) = 1$$

$$= \lim_{n\to\infty} \frac{e}{(n+1)} \left(\frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+1}+\sqrt{n}}\right)$$

$$1 \qquad \frac{1}{\sqrt{n}} \qquad \frac{1}{\sqrt{n}}$$

$$= \left[\underbrace{\lim_{n\to\infty}\left(\frac{e}{n+1}\right)}_{\to 0}\right]\cdot\left[\lim_{n\to\infty}\frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+1}+\sqrt{n}}\right]$$

$$0$$

$$\lim_{n\to\infty}\frac{\sqrt{1+\frac{2}{n}}+\sqrt{1+\frac{1}{n}}}{\sqrt{1+\frac{1}{n}}+\cancel{\ }\,1} = \frac{\sqrt{1}+\sqrt{1}}{\sqrt{1}+1}$$

$$= \frac{2}{2} = 1$$

$$= 0$$

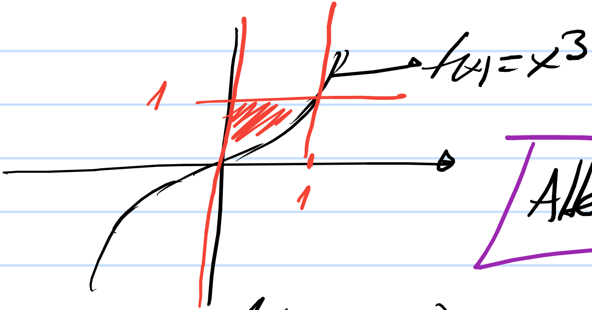
la serie converge.

$\boxed{\text{Attenzione c)}}$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n}{4n-1}$ $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cdot a_n$

$a_n = \dfrac{n}{4n-1}$ $\displaystyle\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{4n-1} = \frac{1}{4} \Longrightarrow$ DIVERGE.

$\boxed{2019-2}$



$\boxed{\text{Alternativa d)}}$

$$\text{Area} = \int_0^1 (1-f(x))\,dx = \int_0^1 (1-x^3)\,dx =$$

$$\left(x - \frac{x^4}{4}\right)\Bigg|_{x=0}^{x=1} = 1 - \frac{1}{4} = \frac{4}{4} - \frac{1}{4} = \boxed{\frac{3}{4}}$$