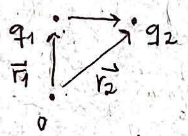
IX. ELECTROMAGNETISMO

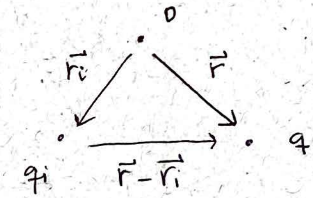
> Ley de Coulomb.



$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot q_1 q_2 \frac{\vec{r_2}-\vec{r_1}}{|\vec{r_2}-\vec{r_1}|^3} = k \cdot \frac{q_1 q_2}{r^2}$$

* Ppio superposición

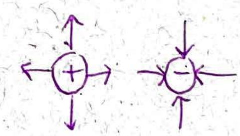
$$\vec{F_q} = \Sigma \vec{F_i} = \frac{1}{4\pi\varepsilon_0} \Sigma q q_i \frac{r-\vec{r_i}}{|\vec{r}-\vec{r_i}|^3}$$



> Campo eléctrico   prop. del espacio, causa de interacción de qs.

- carga puntual: $\vec{E} = \frac{\vec{F_0}}{q_0} \left[\frac{N}{C}\right]$ $\vec{E} = \frac{1}{4\pi\varepsilon_0} q_0 \cdot \frac{(\vec{r}-\vec{r_i})}{|\vec{r}-\vec{r_i}|^3}$

- Distribuciones de carga:
  - * lineal: $\vec{E} = k \int dl\, \lambda(\vec{r_i}) \frac{\vec{r}-\vec{r_i}}{|\vec{r}-\vec{r_i}|^3}$ $\to \lambda(\vec{r}) = dq(r)$ $\lambda\, dl = \lambda R\, d\theta.$
  - * Superficial: $\vec{E} = k \iint ds\, \sigma(\vec{r_i})$.
  - * Volumétrica: $\vec{E} = \iiint \rho\, dV(\vec{r_i})$.

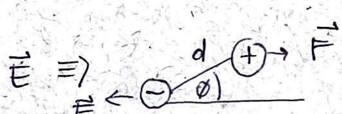


líneas campo.

$$\vec{E}(\text{disco}) = \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{|z|} - \frac{1}{\sqrt{R^2+z^2}}\right] ''$$

> Dipolo eléctrico

2 cargas = de signo opuesto



- Torsión: $\tau = \vec{p} \times \vec{E}$
  $|\tau| = qE \cdot d \operatorname{sen}\phi$ [N·m]

- Momento: $\vec{p} = q d \hat{k}$
  $\vec{p} = q(\vec{r_+} - \vec{r_-})$ [C·m].

$$\vec{E} = -\frac{k}{r^3}\left(\vec{p} - 3\hat{r}\vec{p}\hat{r}\right)$$

- Epot: $U = -\vec{p} \cdot \vec{E} = -pE\cos\phi$ [J]

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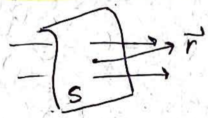
Flujo eléctrico



$$\Phi_E = \int_S \vec{E} \cdot d\vec{A} \quad \left[\frac{Nm^2}{C}\right]$$

* Puntual sobre esfera.
$\Phi_E = q_e/\varepsilon_0$.

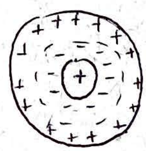
- Plana: $\Phi_E = E \cdot A$
- Inclinada: $\Phi_E = AE\cos\theta = \vec{E} \cdot \vec{A}$
- Cerrada: $\Phi_E = 0$.

Ley de Gauss

- Sup. cerrada. $\Phi_E = \int \vec{E} \cdot d\vec{\sigma} = \dfrac{Q_{enc}}{\varepsilon_0}$
- Lineal: $\vec{E} = \dfrac{\lambda}{2\pi r \varepsilon_0} \hat{r}$
- Esférica: $(R>r)$ $\vec{E} = \dfrac{Q_{enc}}{4\pi r^2}$ fuera, $(R<r)$ $\vec{E} = \dfrac{Q \cdot R}{4\pi\varepsilon_0 r^3}$ dentro
- Plano infinito: $\vec{E} \begin{cases} \sigma/2\varepsilon_0 \, \hat{j} & y>0 \\ -\sigma/2\varepsilon_0 \, \hat{j} & y<0 \end{cases}$
- Placas paralelas: $(\sigma; \sigma+)$ $\vec{E} = \sigma/\varepsilon_0$
- Cilindro: $\vec{E} = \dfrac{Q_{enc}}{2\pi r L \varepsilon_0}$.

Conductores

$\vec{E}$ dentro $= 0$. Carga neta en superficie.
Campo perpendicular. $\vec{E} = \sigma/\varepsilon_0$.



Divergencia

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho(\vec{r})}{\varepsilon_0} = k \int \vec{\nabla} \left(\frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} \right) \rho(r')\, d^3r'$$

- Integral de línea

$$L = \int_C \vec{E} \cdot d\vec{\ell} = kq \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$$

Curva cerrada: $L = 0$.

Rotor: $\vec{\nabla} \times \vec{E} = 0$.

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> **E pot eléctrica** : trabajo necesario para cargar capacitor.

F conservativa. $W_{ab} = U_a - U_b = -\Delta U$.

- Campo uniforme : $U = q_0 E \cdot y$
- Cargas puntuales : $U = \frac{1}{4\pi\varepsilon_0} \frac{q q_0}{r}$
- Dist. carga : $U = \frac{q_0}{4\pi\varepsilon_0} \sum_i \frac{q_i}{|r - r_i|}$

$* \ U_T = \frac{1}{2} \sum_j q_j \cdot V_j$

$V_j = \sum \frac{k q_i}{|r_i - r_j|}$

$U_T = \frac{1}{4\pi\varepsilon_0} \sum_{i<j} \frac{q_i q_j}{r_{ij}}$

> **Potencial eléctrico** : $U = q_0 V$ $\quad V = E \cdot d$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$ $\quad V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}')\, d^3r'}{|\vec{r} - \vec{r}_i|}$

$\frac{W_{ab}}{q_0} = V_a - V_b$ $\quad V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$ $\quad$ O : $q/4\pi\varepsilon R$

* Equipotenciales : V cte.
conductores con cargas en reposo.

Cable : $V = E \cdot L$

líneas campo $\perp$ curvas equipot.

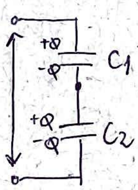
> **Capacitancia**

Capacitor: conductores separados por vacío o aislante.
+Q, −Q. $\quad C = Q / V_{ab}$ (C/V).

$E \rightarrow V \rightarrow C$
Gauss $-\int E\,dl$ $\quad Q/V$

- Placas paralelas : $C = A\varepsilon_0 / d$ $\quad A \gg d$.
- Corazas esféricas : $C = 4\pi\varepsilon_0 \left(\frac{r_a r_b}{r_b - r_a} \right)$; $r_a < r_b$.
- Cond. cilíndricos : $C = \frac{2\pi\varepsilon_0 L}{\ln(r_b/r_a)}$, $r_a < r_b$.

* En serie :

Vab


$\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$

$C_{eq} = \frac{Q}{V}$

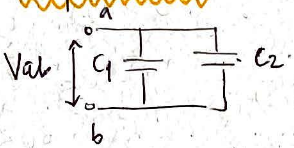
$V_1 = Q/C_1$

$V_2 = Q/C_2$

$V_{ab} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

cargas en placas tienen la misma magnitud. (V, C)

* En paralelo:



$$C_{eq} = \sum_i C_i$$

$$Q_1 = C_1 \cdot V$$

$$Q_2 = C_2 \cdot V$$

$$Q = Q_1 + Q_2 = (C_1 + C_2) V$$

Diferencia de potencial es la misma en cada capacitor. (Q, C) V_{ab}.

> Energía almacenada:

$$U = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV \quad (J)$$

$$U = \frac{\varepsilon_0}{2} \int E^2 dV = \int u \, dV \; ; \quad u = \frac{\varepsilon_0}{2} E^2 \left(\frac{E}{V}\right)\left(\frac{J}{m^3}\right)$$

$$\vec{F} = -\vec{\nabla} \cdot \vec{U} \quad (+ \text{ si ag externo})$$

> Dieléctricos

Almacenar + carga y E.
Material entre conductores.

$$K = \frac{C_K}{C_0} \qquad \varepsilon = k\varepsilon_0.$$

$$C_K = \frac{k\varepsilon_0 A}{d} = \frac{A\varepsilon}{d} \qquad u = \frac{1}{2} k\varepsilon_0 E^2 = \frac{1}{2} \varepsilon E^2$$

- Gauss

$$\int kE \, dA = \frac{Q_{enc\,libre}}{\varepsilon_0}$$

- Polarización

$$\vec{p} = \alpha \vec{E}$$

$$\sigma_p = \vec{P}(\vec{r}') \cdot \hat{n}$$

$$\rho_p = -\vec{\nabla} \cdot \vec{P}(\vec{r})$$

- Desplazamiento.

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_L \qquad \vec{P} = (\varepsilon - \varepsilon_0) \vec{E}.$$

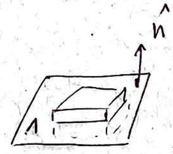
$$\oint \vec{D} \, dS = Q_{libre\,enc.}$$

- Condición de borde

$$E_{\perp 1} - E_{\perp 2} = \sigma / \varepsilon_0 \qquad D_{\perp 1} - D_{\perp 2} = Q_{enc\,libre}$$

$$E_{\parallel 1} = E_{\parallel 2} \qquad \varepsilon_1 E_{\perp 1} - \varepsilon_2 E_{\perp 2} = \sigma_{libre}$$

$$V_1 = V_2$$



Corriente eléctrica

cargas libre (+)

$I = \frac{dQ}{dt} = n|q||\vec{v}_d| \cdot A \quad [A]$ n: concentración partículas.

$\vec{J} = \frac{I}{A_T} = nq\vec{v}_d \left[\frac{A}{m^2}\right]$ $I = \int_S \vec{J} d\vec{s}$

Resistividad

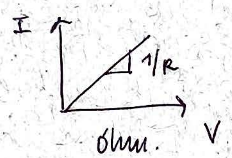
$\vec{\rho} = \frac{\vec{E}}{\vec{J}} \quad [\Omega \cdot m]$ $+ \vec{\rho}$ + campo necesario para causar $\vec{J}$.

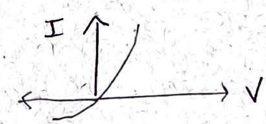
- Conductividad $= \frac{1}{\vec{\rho}}$ $\rho(T) = \rho_0 [1 + \alpha (T - T_0)]$

- Resistencia

$R = \frac{\rho L}{A} \quad [\Omega]$ * cilindro hueco: $\frac{\rho}{2\pi L} \ln\left(\frac{b}{a}\right)$

LEY DE OHM: $V = IR$.
(ρ cte)





* Serie: $R_{eq} = \Sigma R_i$
* Paralelo: $R_{eq} = \Sigma \, 1/R_i$

FEM (ε)

$V_{ab} = \varepsilon - Ir$

$I = \frac{\varepsilon}{R + r}$ $\varepsilon_{ab} = -\int_b^a \vec{E} d\vec{l}$ r: resistencia interna.

$-a+$ pot.

Potencia

$P = V \cdot I = I^2 R = \frac{V^2}{R}$

Salida fuente: $P = \underset{\text{rap}}{\varepsilon I} - \underset{\text{disip.}}{I^2 r}$ Entrada: $P = \varepsilon + I^2 R$.

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→ Circuitos corriente directa

- **Resistencias:**

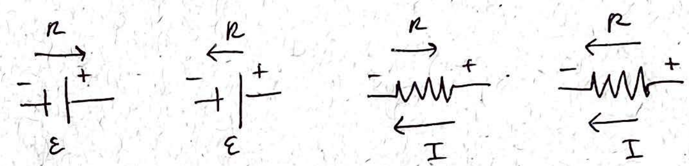
  * Serie: $R_{eq} = \sum_i R_i$ — misma I

  Paralelo: $R_{eq} = \left[\sum \frac{1}{R_i}\right]^{-1}$ — mismo V

- **Kirchhoff:**

  Uniones: $\sum I = 0$

  Espiras: $\sum V = 0$



$+\varepsilon \qquad -\varepsilon \qquad +IR \qquad -IR$

- **R-C:**

  $q = C\varepsilon\,(1-e^{-t/RC}) = Q_f\,(1-e^{-t/RC})$ carga

  $i = \frac{\varepsilon}{R}\,(I_0) \cdot e^{-t/RC}$

  $q = q_0\, e^{-t/RC}$ descarga

→ Campos y fuerzas magnéticas

- **Fuerza Lorentz:** $\vec{F} = q\vec{v}_\perp \times \vec{B}$, $d\vec{F} = I\,d\vec{l} \times \vec{B}$

  $R = \frac{v \cdot m}{qB}$ $\qquad \phi_B = \int \vec{B} \cdot d\vec{A}$ (flujo)

- **Biot-Savart:**

  $\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int_c \frac{d\vec{l} \times (\vec{r}_p - \vec{r}\,')}{|\vec{r}_p - \vec{r}\,'|^3}$ [T] $\qquad \mu_0 = 4\pi \cdot 10^{-7}\ \frac{N}{A^2}$

  Carga puntual: $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$

  * Espira: $\vec{B}(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+x^2)^{3/2}}\,\hat{k} \cdot N$

  * Alambre ∞: $\vec{B}(r) = \frac{\mu_0 I}{2\pi r}$ $(r > R)$ $\qquad \vec{B}(r) = \frac{\mu_0 I r}{2\pi r^2}$ $(r < R)$

  * F. A//: $\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$

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> ley de Ampere.

$\int_S \vec{B} \cdot d\vec{S} = 0$ (FLUJO) $\Rightarrow \int_C \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{enc}$

- Superficie: $\vec{B} = \frac{\mu_0 K}{2} \hat{k}$
- Solenoide ∞ : $\vec{B} = \mu_0 n I \hat{k}$ $\quad n = \frac{N}{L}$
- Toroide: $\vec{B} = \frac{\mu_0 N I}{2\pi r} (-\hat{\theta})$

(*) Momento dipolar: $m = I \int dS$

Torque: $\vec{\tau} = \vec{\mu} \times \vec{B} = IAB \operatorname{sen}\phi$

Epot dipolo: $U = -\vec{\mu} \cdot \vec{B}$

> Conducción electro-magnética $\quad \varepsilon = I \cdot R$

campo mg que varía en tpo actúa como fuente de elec. y vv.

$\vec{B}$ cte y cond. mov o $\vec{B}$ variable $\Rightarrow$ flujo cambia.

- ley de Faraday.

$$\varepsilon = -N \cdot \frac{d\Phi_B}{dt} \; [V]$$

Fem mov: $\varepsilon = vBL$ (varilla)

$$\varepsilon = \int (\vec{v} \times \vec{B})\, dl$$

- ley de Lenz

Dirección cualquier efecto IEM es la que se opone a la causa del efecto. $\int \vec{E} \cdot d\vec{l} = -\frac{\Phi_B}{dt}$

- Maxwell

$\int E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$; $\int B \cdot d\vec{A} = 0$; $\int B \cdot d\vec{l} = \dots$

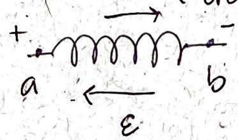
> Inductancia (oposición)

- Mutua: $\varepsilon_2 = -M \frac{di_1}{dt}$; $\varepsilon_1 = -M \frac{di_2}{dt}$ $\quad M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$ [H]

∘ Autoinductancia

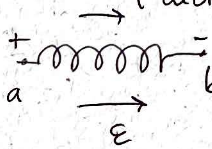
$L = \frac{N \Phi_B}{i}$ [H] $\varepsilon = -L\frac{di}{dt}$ * Cable: $L = \frac{\mu_0 L}{2\pi} \cdot \ln\left(\frac{r_2}{r_1}\right)$

Inductores. i creciente



$V_{ab} = L\frac{di}{dt} > 0$

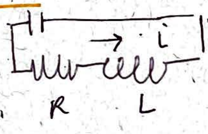
i decreciente



$V_{ab} = L\frac{di}{dt} < 0.$

$V_{alm} = \frac{1}{2} \cdot L I^2$ $\mu = \frac{B^2}{2\mu_0}$

∘ R-L

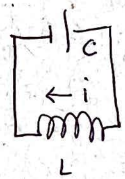


$i = \frac{\varepsilon}{R}(1 - e^{-(R/L)t})$

↳ $\varepsilon - ir - L\frac{di}{dt} = 0$

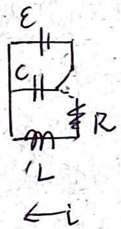
∘ L-C

$-L\frac{di}{dt} - \frac{q}{c} = 0$ $w = \sqrt{\frac{1}{LC}}$



$q = Q\cos(wt + \phi)$

∘ L-R-C



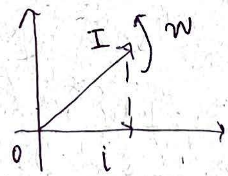
$iR - L\frac{di}{dt} - \frac{q}{c} = 0$ $w' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

> Corriente alterna

∘ Fasores:

$v = V\cos(wt)$

$i = I\cos(wt)$



∘ Reactancia

Resistor

$V_R = IR$

Inductor

$X_L = wL$

$V_L = I X_L$

Capacitor

$X_C = 1/wC$

$V_C = I \cdot X_C$

$i = \cos(wt)$

ϕ: ángulo fase

R (0), I (90°)

$Z = V/I$

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- L-R-C

$V = I \cdot Z$ $\quad Z = \sqrt{R^2 + (X_L - X_C)^2}$ $\quad \tan\phi = \frac{X_L - X_C}{R}$

- Potencia

$P_{med} = \frac{1}{2} VI \cos\phi$

- Resonancia

$X_L = X_C$, $\omega_0 = 1/\sqrt{LC}$

- Transformadores

$\frac{V_2}{V_1} = \frac{N_2}{N_1}$ $\quad V_1 I_1 = V_2 I_2$