$\boxed{2015-2}$

P_0 $\quad r$

$$\boxed{P(t) = P_0 \cdot (1+r)^t}$$

t está en años.

$$P(t^*) = 3P_0 \Rightarrow \cancel{P_0} \cdot (1+r)^{t^*} = 3\cancel{P_0}$$

$$\Rightarrow (1+r)^{t^*} = 3 \Rightarrow \ln\left((1+r)^{t^*}\right) = \ln(3)$$

$$t^* \cdot \ln(1+r) = \ln(3) \Rightarrow t^* = \frac{\ln(3)}{\ln(1+r)} = \frac{\ln(3)}{\ln(1.02)} \Rightarrow \boxed{t^* = 55{,}48}$$

$\boxed{2016-1}$

$$\frac{dx}{dt} = 3x(t) - 2y(t)$$

$$\vec{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$$\frac{dy}{dt} = 2x(t) - 2y(t)$$

$$\Rightarrow \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \qquad \boxed{A = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}}$$

$$\boxed{\vec{x}'(t) = A\vec{x}(t)} \qquad \boxed{\vec{x}(t) = e^{\lambda t} \cdot \vec{v}} \Rightarrow \boxed{\vec{x}'(t) = e^{\lambda t} \cdot \lambda \cdot \vec{v}}$$

$$e^{\lambda t} \lambda \cdot \vec{v} = A \cdot e^{\lambda t} \vec{v} \Rightarrow \boxed{A \cdot \vec{v} = \lambda \vec{v}}$$

(i) $\det(A-\lambda I)=0$ $\qquad A=\begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}$

$$A-\lambda I=\begin{bmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{bmatrix}$$

$$\det(A-\lambda I)=(3-\lambda)(-2-\lambda)-\underbrace{(-2)\cdot 2}_{-4}=(\lambda-3)(\lambda+2)+4$$

$$=\lambda^2-\lambda-6+4=\boxed{\lambda^2-\lambda-2=0}$$

$a=1$
$b=-1$
$c=-2$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1-4\cdot(-2)}}{2} = \frac{1 \pm \sqrt{1+8}}{2}$$

$$= \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2} \Rightarrow \boxed{\lambda_1 = 2} \quad \boxed{\lambda_2 = -1}$$

$$\boxed{\lambda_1 = 2} \quad (A - \lambda I)\cdot \vec{v} = 0 \qquad A - 2I = \begin{bmatrix} 3-2 & -2 \\ 2 & -2-2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}$$

$$\det(\underbrace{A - \lambda I}_{A-\lambda I}) = 0$$

$$\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}}_{\vec{v}} = 0 \Rightarrow v_1 - 2v_2 = 0 \Rightarrow \boxed{v_1 = 2v_2}$$

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 2v_2 \\ v_2 \end{pmatrix} = v_2 \boxed{\begin{pmatrix} 2 \\ 1 \end{pmatrix}} \qquad \boxed{\vec{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}}$$

$$\boxed{\lambda_2 = -1} \qquad A - \lambda I = A + I = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 2 & -1 \end{bmatrix}$$

$\vec{v}$

$$A+I = \begin{bmatrix} 4 & -2 \\ 2 & -1 \end{bmatrix}$$

$(A+I)\cdot\vec{v} = 0$ $\qquad \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$ $\qquad 4v_1 - 2v_2 = 0$

$\Rightarrow 2v_1 = v_2$

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ 2v_1 \end{pmatrix} = v_1 \boxed{\begin{pmatrix} 1 \\ 2 \end{pmatrix}} \qquad \boxed{\vec{v_2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

$$\boxed{\lambda_1 = 2} \quad \boxed{\vec{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}} \qquad \boxed{\lambda_2 = -1} \quad \boxed{\vec{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

$$\vec{x}(t) = c_1 \cdot e^{2t} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 \cdot e^{-t} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} c_1 \cdot e^{2t} \cdot 2 \\ c_1 \cdot e^{2t} \end{pmatrix} + \begin{pmatrix} c_2 \cdot e^{-t} \\ 2c_2 \cdot e^{-t} \end{pmatrix} = \boxed{\begin{pmatrix} 2c_1 \cdot e^{2t} + c_2 \cdot e^{-t} \\ c_1 \cdot e^{2t} + 2c_2 \cdot e^{-t} \end{pmatrix}}$$

$$\vec{x}(0) = \boxed{\begin{pmatrix} 2c_1 + c_2 \\ c_1 + 2c_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}} \qquad \begin{matrix} x(0) = 1 \\ y(0) = 5 \end{matrix}$$

$$2c_1 + c_2 = 1 \Rightarrow \boxed{c_2 = 1 - 2c_1} \quad (*) \quad c_2 = 1 - 2(-1)$$

$$= 1 + 2$$

$$= 3$$

$$c_1 + 2c_2 = 5 \Rightarrow c_1 + 2 \cdot [1 - 2c_1] = 5$$

$$\Rightarrow c_1 + 2 - 4c_1 = 5 \Rightarrow -3c_1 + 2 = 5 \Rightarrow 2 - 5 = 3c_1$$

$$\Rightarrow 3c_1 = -3 \Rightarrow \boxed{c_1 = -1} \qquad \boxed{c_2 = 3}$$

$$\boxed{c_1 = -1} \qquad \boxed{c_2 = 3}$$

$$\vec{x}(t) = \begin{pmatrix} 2c_1 e^{2t} + c_2 \cdot e^{-t} \\ c_1 \cdot e^{2t} + 2c_2 \cdot e^{-t} \end{pmatrix} = \begin{pmatrix} -2 \cdot e^{2t} + 3e^{-t} \\ -e^{2t} + 6e^{-t} \end{pmatrix}$$

$$\boxed{\vec{x}(t) = \begin{pmatrix} -2e^{2t} + 3e^{-t} \\ -e^{2t} + 6e^{-t} \end{pmatrix}} \Rightarrow \boxed{\text{Allenstms a)}}$$

$\boxed{2016-2}$

$$y'' - 2y' + 2y = 0$$

$$\boxed{y(x) = e^{rx}} \Rightarrow \boxed{y'(x) = r \cdot e^{rx}} \Rightarrow \boxed{y''(x) = r^2 \cdot e^{rx}}$$

$$\Rightarrow r^2 \cdot e^{rx} - 2 \cdot r \cdot e^{rx} + 2 \cdot e^{rx} = 0$$

$$\Rightarrow (r^2 - 2r + 2) \cdot \underline{e^{rx}} = 0 \Rightarrow \boxed{r^2 - 2r + 2 = 0}$$

$$r^2-2r+2=0 \qquad \boxed{a=1} \quad \boxed{b=-2} \quad \boxed{c=2}$$

$$r_{1,2}=\frac{2\pm\sqrt{4-4\cdot 2}}{2}=\frac{2\pm\sqrt{4-8}}{2}=\frac{2\pm\sqrt{-4}}{2}$$

$$=\frac{2\pm 2\sqrt{-1}}{2}=1\pm\sqrt{-1}=1\pm i \;\Big\}\; a\pm bi$$

$y_1(x)=e^{a\cdot x}\cdot\cos(bx)$

$y_2(x)=e^{ax}\cdot\sin(bx)$

$$\boxed{y_1(x)=e^{x}\cdot\cos(x)} \qquad \boxed{y_2(x)=e^{x}\cdot\sin(x)}$$

$c_1 = c_2 = 1$

$$y(x) = c_1 \cdot y_1(x) + c_2 \cdot y_2(x)$$

$$= c_1 \cdot e^x \cdot \cos(x) + c_2 \cdot e^x \cdot \sin(x)$$

$$\boxed{y(x) = c_1 \cdot e^{x} \cdot \cos(x) + c_2 \cdot e^{x} \cdot \sin(x)} \quad (1)$$

$$y'(x) = c_1 \cdot \left[\underbrace{(e^x)'}_{e^x} \cdot \cos(x) + e^x \cdot \underbrace{(\cos(x))'}_{-\sin(x)} \right]$$

$$+ c_2 \left[\underbrace{(e^x)'}_{e^x} \cdot \sin(x) + e^x \cdot \underbrace{(\sin(x))'}_{\cos(x)} \right]$$

$$\boxed{\begin{aligned} y'(x) = {} & C_1 \cdot \left[\overset{1}{e^x} \cdot \overset{1}{\cos(x)} - \overset{1}{e^x} \cdot \overset{0}{\sin(x)} \right] \\ & + C_2 \cdot \left[\overset{1}{e^x} \cdot \overset{0}{\sin(x)} + \overset{1}{e^x} \cdot \overset{1}{\cos(x)} \right] \end{aligned}} \qquad (2)$$

$y(0) = 1 \Rightarrow y(0) = C_1 = 1 \Rightarrow \boxed{C_1 = 1}$

$y'(0) = 2 \Rightarrow y'(0) = C_1 + C_2 = 2 \Rightarrow 1 + C_2 = 2$

$\Rightarrow C_2 = 2 - 1 = 1 \Rightarrow \boxed{C_2 = 1}$

$$\boxed{y(x) = e^{x} \cdot [\cos(x) + \sin(x)]} \rightarrow \boxed{\text{Alternativa c)}}$$

$\boxed{2017-1}$

$$\boxed{dp - r\cdot p\left(1-\frac{p}{K}\right)dt = 0}$$

$$dp = 1\cdot r\cdot p\left(1-\frac{p}{K}\right)dt$$

$$\frac{dp}{\underbrace{p\left(1-\frac{p}{K}\right)}} = r\cdot dt \Rightarrow \frac{K\cdot dp}{p(K-p)} = r\,dt$$

$$p\left(1-\frac{p}{K}\right) = \frac{p}{K}\cdot K\left(1-\frac{p}{K}\right) = \frac{p}{K}\cdot\left(K-p\right) = \frac{p(K-p)}{K}$$

$$\boxed{\frac{dp}{p(k-p)} = \frac{r}{k}\,dt}$$

$$\int$$

$$\boxed{\int \frac{dp}{p(k-p)} = \int \frac{r}{k}\,dt}$$

$$\frac{r}{k} \cdot t + C$$

$$\Rightarrow \boxed{\int \frac{dp}{p(k-p)} = \frac{r}{k} \cdot t + C} \quad (*)$$

$$\frac{1}{p(k-p)} = \frac{A}{p} + \frac{B}{(k-p)} \quad \Big/ \cdot p(k-p)$$

$$1 = A\cdot(k-p) + B\cdot p \; \Big\} = A\cdot k - A\cdot p + B p$$
$$= A\cdot k + (B-A)\cdot p$$

$$A\cdot k + \underbrace{(B-A)}\cdot p = 1 \Rightarrow B-A=0 \Rightarrow \boxed{B=A}$$
$$A\cdot k = 1 \Rightarrow \boxed{A=\frac{1}{k}} \quad \boxed{B=\frac{1}{k}}$$

$$\frac{1}{\rho(k-\rho)} = \frac{1}{k}\cdot\left[\frac{1}{\rho} + \frac{1}{k-\rho}\right] \quad \Big/ \int d\rho$$

$$\int\frac{d\rho}{\rho(k-\rho)} = \frac{1}{k}\cdot\left[\int\frac{d\rho}{\rho} + \int\frac{d\rho}{(k-\rho)}\right]$$

$$= \frac{1}{k}\cdot\left[\ln(|\rho|) - \ln(|k-\rho|)\right] + C_2$$

$u = k-\rho$

$du = -d\rho$

$$\int -\frac{du}{u} = -\int\frac{du}{u} = -\ln|u| = -\ln|k-\rho|$$

$$\frac{1}{k}\cdot\left[\underbrace{\ln|P| - \ln|k-P|}\right] + C_2 = \frac{r}{k}\cdot t + C \;/\cdot k$$

$$\ln\left|\frac{P}{k-P}\right| + C_2\cdot k = r\cdot t + C\cdot k$$

$$\ln\left|\frac{P}{k-P}\right| = r\cdot t + \underbrace{C\cdot k - C_2\cdot k}_{C_3}$$

$$\ln\left|\frac{P}{K-P}\right| = r\cdot t + C_3$$

$$A>0$$

$$\Rightarrow e^{r\cdot t + C_3} = \left|\frac{P}{K-P}\right| \Rightarrow e^{r\cdot t}\cdot \overbrace{e^{C_3}}^{A>0} = \left|\frac{P}{K-P}\right|$$

$$e^{rt}\cdot A = \left|\frac{P}{K-P}\right| \Rightarrow \frac{P}{K-P} = \underbrace{\pm A}_{B=}\, e^{r\cdot t}$$

$$\frac{P}{K-P} = B\cdot e^{rt} \Rightarrow P = B\cdot e^{r\cdot t}\cdot K - P\cdot B\cdot e^{rt}$$

$$P + P\cdot B\cdot e^{rt} = B\cdot e^{rt}\cdot K$$

$$P(1+B\cdot e^{rt}) = B\cdot e^{rt}\cdot K$$

$$P(t) = \frac{B\cdot \cancel{e^{rt}}\cdot K \rightarrow \frac{1}{\cancel{e^{rt}}}}{1+B\cancel{e^{rt}} \rightarrow \frac{1}{\cancel{e^{rt}}}} = \frac{B\cdot K}{e^{-rt}+B}$$

$$\boxed{P(t) = \frac{B \cdot K}{e^{-r \cdot t} + B}} \qquad P(0) = P_0$$

$$P(0) = \frac{B \cdot K}{1 + B} = P_0 \Rightarrow BK = P_0 + B \cdot P_0$$

$$\Rightarrow B(K - P_0) = P_0 \Rightarrow \boxed{B = \frac{P_0}{K - P_0}}$$

$$P(t) = \frac{\frac{P_0}{(K-P_0)} \cdot K \cdot (K-P_0)}{\left(e^{-rt} + \left(\frac{P_0}{K-P_0}\right)\right) \cdot (K-P_0)}$$

$\boxed{\text{Alternative b)}}$

$$\boxed{= \frac{P_0 \cdot K}{(K-P_0) \cdot e^{-rt} + P_0} = P(t)}$$

$\boxed{2017\text{-}2}$

$$\boxed{\begin{aligned} \frac{dx}{dt} &= 3x(t) - 5y(t) \\ \frac{dy}{dt} &= x(t) - y(t) \end{aligned}}$$

$$\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \underbrace{\begin{pmatrix} 3 & -5 \\ 1 & -1 \end{pmatrix}}_{A = \begin{bmatrix} 3 & -5 \\ 1 & -1 \end{bmatrix}} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \qquad \vec{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$$\Rightarrow \boxed{\vec{x}\,'(t) = A \cdot \vec{x}(t)}$$

$$\boxed{\vec{x}(t) = e^{\lambda t}\cdot\vec{v}} \Rightarrow \boxed{\vec{x}\,'(t) = \lambda\cdot e^{\lambda t}\cdot\vec{v}}$$

$$\lambda\cdot\cancel{e^{\lambda t}}\cdot\vec{v} = A\cdot\cancel{e^{\lambda t}}\cdot\vec{v} \Rightarrow \boxed{A\vec{v} = \lambda\cdot\vec{v}}$$

$$A-\lambda I = \begin{pmatrix} 3-\lambda & -5 \\ 1 & -1-\lambda \end{pmatrix}$$

$$\det(A-\lambda I) = (3-\lambda)(-1-\lambda) - (-5)\cdot 1 = (\lambda-3)(\lambda+1)+5$$

$$= \lambda^2 - 2\lambda - 3 + 5 = \lambda^2 - 2\lambda + 2$$

$$\det(A - \lambda I) = \boxed{\lambda^2 - 2\lambda + 2 = 0}$$

$$\boxed{\begin{aligned} a &= 1 \\ b &= -2 \\ c &= 2 \end{aligned}}$$

$$\lambda_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 2}}{2} = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2 \cdot \sqrt{-1}}{2} = 1 \pm \sqrt{-1} = 1 \pm i$$

$$\boxed{\begin{aligned} \lambda_1 &= 1 + i \\ \lambda_2 &= 1 - i \end{aligned}}$$

$$\boxed{\lambda_1 = 1+i}$$

$$A - \lambda I = A - (1+i) = \begin{pmatrix} 3-(1+i) & -5 \\ 1 & -1-(1+i) \end{pmatrix}$$

$$= \begin{pmatrix} 2-i & -5 \\ 1 & -2-i \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$(2-i)v_1 - 5v_2 = 0 \Rightarrow (2-i)v_1 = 5v_2 \Rightarrow \boxed{v_2 = \frac{(2-i)}{5} \cdot v_1}$$

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ \frac{(2-i)}{5} \cdot v_1 \end{pmatrix} = \underline{\begin{pmatrix} 1 \\ \frac{(2-i)}{5} \end{pmatrix}} \cdot v_1 =$$

$$\boxed{\vec{v}_1 = \begin{pmatrix} 5 \\ 2-i \end{pmatrix}}$$

$$\boxed{\lambda_2 = 1 - i}$$

$$A - (1-i)\cdot I = \begin{pmatrix} 3-(1-i) & -5 \\ 1 & -1-(1-i) \end{pmatrix}$$

$$= \begin{pmatrix} 2+i & -5 \\ 1 & -2+i \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$(2+i)\cdot v_1 - 5 v_2 = 0 \Rightarrow (2+i) v_1 = 5 v_2 \Rightarrow \boxed{v_2 = \frac{(2+i)}{5} v_1}$$

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ \frac{(2+i)}{5} v_1 \end{pmatrix} = v_1 \cdot \begin{pmatrix} 1 \\ \frac{(2+i)}{5} \end{pmatrix} =$$

$$\boxed{\vec{v_2} = \begin{pmatrix} 5 \\ 2+i \end{pmatrix}}$$

$$\boxed{\lambda_1 = 1+i} \qquad \boxed{\vec{v}_1 = \begin{pmatrix} 5 \\ 2-i \end{pmatrix}} \qquad \boxed{\lambda_2 = 1-i} \qquad \boxed{\vec{v}_2 = \begin{pmatrix} 5 \\ 2+i \end{pmatrix}}$$

$$\boxed{\vec{x}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}}$$

$$e^t \cdot e^{it} = e^t \cdot [\cos(t) + i \cdot \sin(t)]$$

$$\vec{x}(t) = C_1 \cdot \overbrace{e^{(1+i)\cdot t}} \begin{pmatrix} 5 \\ 2-i \end{pmatrix} + C_2 \cdot \underbrace{e^{(1-i)\cdot t}} \begin{pmatrix} 5 \\ 2+i \end{pmatrix}$$

$$\hookrightarrow e^{(1-i)\cdot t}$$

$$= e^t \cdot e^{-it}$$

$$= e^t [\cos(-t) + i \sin(-t)]$$

$$= e^t [\cos(t) - i \sin(t)]$$

$$= C_1 \cdot \left[e^t \cdot (\cos(t) + i \cdot \sin(t)) \right] \begin{pmatrix} 5 \\ 2-i \end{pmatrix}$$

$$+ C_2 \cdot \left[e^t \cdot (\cos(t) - i \sin(t)) \right] \cdot \begin{pmatrix} 5 \\ 2+i \end{pmatrix}$$

$$= C_1 \cdot \left[e^t \cdot \cos(t) + i \cdot e^t \cdot \sin(t) \right] \cdot \begin{pmatrix} 5 \\ 2-i \end{pmatrix} \Big\}$$

$$+ C_2 \cdot \left[e^t \cdot \cos(t) - i \cdot e^t \cdot \sin(t) \right] \cdot \begin{pmatrix} 5 \\ 2+i \end{pmatrix}$$

$$= \begin{pmatrix} C_1 \cdot e^t \cdot \cos(t) \cdot 5 + C_1 \cdot i \cdot e^t \cdot \sin(t) \cdot 5 \\ (2-i) \cdot C_1 \cdot e^t \cdot \cos(t) + (2-i) \cdot C_1 \cdot e^t \cdot \sin(t) \cdot i \end{pmatrix}$$

$$+\begin{pmatrix} C_2 \cdot e^t \cdot \cos(t) \cdot 5 - i \cdot e^t \cdot \sin(t) \cdot C_2 \cdot 5 \\ C_1 \cdot e^t \cdot \cos(t)\,(2+i) - C_2 \cdot e^t \cdot i \cdot \sin(t)(2+i) \end{pmatrix}$$

$$= \begin{pmatrix} 5C_1 \cdot e^t \cdot \overbrace{\cos(t)}^{1} + 5C_1 \cdot i \cdot e^t \cdot \overbrace{\sin(t)}^{0} + 5C_2 \cdot e^t \cdot \overbrace{\cos(t)}^{1} \\ - 5C_2 \cdot i \cdot e^t \cdot \sin(t) \\ (2-i)C_1 \overbrace{e^t \cdot \cos(t)}^{1} + (2-i) \cdot C_1 \cdot e^t \overbrace{\sin(t) \cdot i}^{0} \\ + C_2 \cdot \underbrace{e^t \cdot \cos(t)}_{1} \cdot (2+i) - C_2 \cdot \underbrace{e^t \cdot i \cdot \sin(t)}_{0} \cdot (2+i) \end{pmatrix}$$

$$\vec{x}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\vec{x}(0) = \begin{pmatrix} 5c_1 + 5c_2 \\ (2-i)\cdot c_1 + c_2(2+i) \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

i $\boxed{c_1 + c_2 = \frac{3}{5}}$ ii $\boxed{(2-i)\cdot c_1 + (2+i)\cdot c_2 = 1}$

$$\boxed{c_1 = \frac{3}{5} - c_2}$$

$$(2-i)\cdot\underline{\left(\frac{3}{5}-c_2\right)}+(2+i)\cdot(c_2)=1$$

$$\Rightarrow \frac{6}{5}-\cancel{2c_2}-\frac{i\cdot 3}{5}+\cancel{i\cdot c_2}+\cancel{2c_2}+\cancel{i\cdot c_2}=1$$

$$2i\cdot c_2+\frac{3}{5}\cdot(2-i)=1$$

$$\underline{2i\cdot c_2}=1-\frac{3}{5}(2-i)=\frac{5}{5}-\frac{(6-3i)}{5}=\underline{\underline{\frac{3i-1}{5}}}$$

$$2i\cdot c_2 = \left(\frac{3i-1}{5}\right) \Rightarrow c_2 = \left(\frac{3i-1}{5}\right)\cdot\frac{1}{2i}$$

$$= \frac{1}{10}\cdot\left(\frac{3i-1}{i}\right) = \frac{1}{10}\left(\frac{3i^2-i}{i^2}\right) = \frac{1}{10}\left(\frac{-3-i}{-1}\right)$$

$$= \frac{1}{10}\cdot(3+i) \qquad \boxed{c_2 = \left(\frac{3+i}{10}\right)} \quad \boxed{c_1 = \left(\frac{3-i}{10}\right)}$$

$$c_1 = \frac{3}{5} - c_2 = \frac{3}{5} - \frac{(3+i)}{10} = \frac{6}{10} - \frac{(3+i)}{10} = \frac{6-3-i}{10} = \boxed{\frac{3-i}{10}}$$

$$\vec{x}(t) = \begin{pmatrix} 5c_1 \cdot e^t \cdot \cos(t) + 5c_1 \cdot i \cdot e^t \cdot \sin(t) \\ + 5c_2 \cdot e^t \cdot \cos(t) - 5c_2 \cdot i \cdot e^t \cdot \sin(t) \\ \hline (2-i) \cdot c_1 \cdot e^t \cdot \cos(t) + (2-i) \cdot c_1 \cdot e^t \cdot \sin(t) \cdot i \\ + c_2 \cdot e^t \cdot \cos(t)(2+i) - c_2 e^t \cdot i \sin(t)(2+i) \end{pmatrix}$$

$$c_1 = \frac{3-i}{10}$$

$$c_2 = \frac{3+i}{10}$$

$$5 \cdot \frac{(3-i)}{10} \cdot e^t \cdot \cos(t) + 5 \cdot \left(\frac{3-i}{10}\right) \cdot i \cdot e^t \cdot \sin(t)$$

$$+ 5 \cdot \left(\frac{3+i}{10}\right) \cdot e^t \cos(t) - 5 \cdot \left(\frac{3+i}{10}\right) \cdot i \cdot e^t \cdot \sin(t)$$

$$= \frac{(3-i)}{2} \cdot e^t \cdot \cos(t) + \frac{(3-i)}{2} \cdot i \cdot e^t \cdot \sin(t)$$

$$\frac{3i-i^2}{2} = \frac{3i-(-1)}{2} = \left(\frac{3i+1}{2}\right)$$

$$+\left(\frac{3+i}{2}\right) e^t \cdot \cos(t) - \frac{(3+i)}{2} \cdot i \cdot e^t \cdot \sin(t)$$

$$\frac{3i+i^2}{2} = \frac{3i-1}{2}$$

$$\left(\frac{3-i}{2}\right)+\left(\frac{3+i}{2}\right) = \frac{6}{2} = 3$$

$$\frac{(3i+1)}{2} - \frac{(3i-1)}{2} = \frac{2}{2} = 1$$

$$= \frac{(3-i)}{2} \cdot e^t \cdot \cos(t) + \left(\frac{3i+1}{2}\right) \cdot e^t \cdot \sin(t)$$

$$+ \frac{(3+i)}{2} \cdot e^t \cdot \cos(t) - \frac{(3i-1)}{2} e^t \cdot \sin(t)$$

$$= 3 \cdot e^t \cdot \cos(t) + e^t \cdot \sin(t)$$

$$\vec{x}(t) = \begin{pmatrix} 3e^t \cdot \cos(t) + e^t \cdot \sin(t) \\ e^t \cdot \sin(t) + e^t \cdot \cos(t) \end{pmatrix}$$

$\Rightarrow$ Alternativa b)

$$c_1 = \frac{3-i}{10}$$

$$= (2-i)\, c_1 e^t \cdot \cos(t) + (2-i) \cdot c_1 \cdot e^t \cdot \sin(t) \cdot i$$

$$+ c_2 \cdot e^t \cdot \cos(t)\,(2+i) - c_2 \cdot e^t \cdot i \cdot \sin(t)\,(2+i)$$

$$c_2 = \frac{3+i}{10}$$

$$= \left(\frac{(2-i)(3-i)}{10}\right) e^t \cdot \cos(t) + \left(\frac{(2-i)\cdot(3-i)}{10} \cdot i\right) \cdot e^t \cdot \sin(t)$$

$$+ \left(\frac{(3+i)}{10} \cdot (2+i)\right) \cdot e^t \cos(t) - \frac{(3+i)}{10} \cdot i \cdot (2+i) \cdot e^t \cdot \sin(t)$$

$$-1 \qquad 12 \qquad \frac{10}{10} - 1 = \frac{10}{10} \cdot 1$$

$$\frac{(2-i)(3-i)}{10} + \frac{(3+i)(2+i)}{10} = \frac{6 - 2i - 3i + i^2}{10} + \frac{6 + 3i + 2i + i^2}{10}$$

$$= \left(e^{t} \cdot \cos(t) + e^{t} \cdot \sin(t) \right)$$

$$\frac{(2-i)(3-i) \cdot i}{10} - \frac{(3+i) \cdot (2+i) \cdot i}{10}$$

$$= \frac{[6-2i-3i+i^2] \cdot i}{10} - \frac{[6+3i+2i+i^2] \cdot i}{10}$$

$$= \frac{1}{10} \cdot i \cdot \left[\frac{\cancel{6} \overbrace{-2i-3i}^{-5i} + \cancel{i^2} - \cancel{6} \underbrace{-3i-2i}_{-5i} - \cancel{i^2}}{\cancel{10}} \right] - \frac{1}{\cancel{10}} \cdot i \, (-\cancel{10}i) = 1$$

$\boxed{2018\text{-}1}$

a) Si $m=2,\ n=1,\ p=1,\ q=1,\ t=0$

$$\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right) + y = 0$$

No lineal

No homogenea

De segundo orden

b) S: $m = 1$, $n = 1$, $p = 0$, $q = 1$, $t = 0$

$$\left(\frac{dy}{dx}\right) + 1 = \underline{x}$$

Lineal
No homogenea
Primer Orden

c) Si $m=1,\ n=2,\ p=1,\ q=1,\ t=1$

$$\left(\frac{dy}{dx}\right)\cdot\left(\frac{dy}{dx}\right) + x\cdot y = 2x$$

No homogenea
No lineal
De primer orden

d) $m=2,\ n=1,\ p=1,\ q=1,\ t=0$

$$\left(\frac{d^2y}{dx^2}\right)\cdot\left(\frac{dy}{dx}\right) + 0\,y = x$$

No lineal
No homogenea
Segundo orden
Coeficientes constantes

Alternativa d)

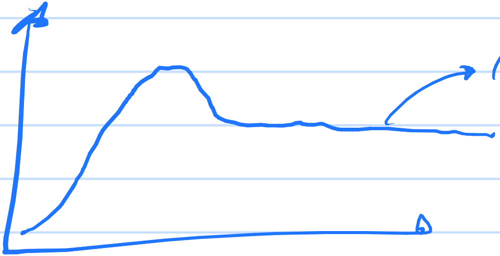
$\boxed{2018-2}$

$$\frac{d\rho}{dt} = c \cdot r \cdot \left(1 - \left(\frac{\rho(t)}{K}\right) - \left(\frac{\rho(t)}{K}\right)^2\right)$$

$$\overbrace{\frac{d\rho}{dt}}^{\rho'(t)} = c \cdot r \cdot \left(1 - \left[\frac{\rho(t)}{K}\right] - \left[\frac{\rho(t)}{K}\right]^2\right) \Big/ \lim_{t\to\infty}$$

$$\underbrace{\lim_{t\to\infty} \rho'(t)}_{0} = c \cdot r \left(1 - \left[\frac{\lim_{t\to\infty} \rho(t)}{K}\right] - \left[\frac{\lim_{t\to\infty} \rho(t)}{K}\right]^2\right)$$

$$\lim_{t\to\infty} \rho(t) = \rho_{TER}$$



$p'(t) = 0$

$$u = \frac{p_{tem}}{K}$$

$$0 = 1 - \left(\frac{p_{tem}}{K}\right) - \left(\frac{p_{tem}}{K}\right)^2$$

$a = 1$
$b = 1$
$c = -1$

$$0 = 1 - u - u^2 \rightarrow u^2 + u - 1 = 0$$

$$u_{1,2} = \frac{-1 \pm \sqrt{1 - 4(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$u_{1,2} = \frac{-1 \pm \sqrt{5}}{2} \qquad u_1 = \frac{-1-\sqrt{5}}{2} < 0$$

$$u_2 = \frac{\sqrt{5}-1}{2} \checkmark \quad u_2 = \frac{\rho_{term}}{k} \Rightarrow \frac{\rho_{term}}{k} = \left(\frac{\sqrt{5}-1}{2}\right)$$

$$\boxed{\rho_{term} = \left(\frac{\sqrt{5}-1}{2}\right) \cdot K} \qquad \boxed{\text{Alternativa b)}}$$

$\boxed{2019\text{-}1}$

$$\frac{dA}{dt} = k \cdot A^2 \Rightarrow dA \cdot A^{-2} = k\,dt$$

$$\int A^{-2}\,dA = \int k\,dt + C \Rightarrow \left(\frac{A^{-1}}{-1}\right) = k \cdot t + C$$

$$\Rightarrow \cdot -\frac{1}{A} = kt + C \Rightarrow \boxed{-\frac{1}{(kt+C)} = A(t)}$$

$$A(0) = 100 \Rightarrow -\frac{1}{C} = 100 \Rightarrow \boxed{C = -\frac{1}{100}}$$

$$A(1) = -\frac{1}{(k+c)} = 50$$

$$\Rightarrow \left(\frac{1}{k+c}\right) = -50 \Rightarrow k+c = -\frac{1}{50}$$

$$k - \frac{1}{100} = -\frac{1}{50} \Rightarrow k = -\frac{1}{50} + \frac{1}{100} = -\frac{2}{100} + \frac{1}{100}$$

$$= -\frac{1}{100}$$

$$\boxed{k = -\frac{1}{100}}$$

$$A(t) = -\frac{1}{(k\cdot t + c)} = -\frac{1}{\left(-\frac{1}{100}\cdot t - \frac{1}{100}\right)}$$

$$= \frac{1}{\left(\frac{t+1}{100}\right)} = \frac{100}{t+1}$$

$$\boxed{A(t) = \frac{100}{t+1}}$$

$$A(4) = \frac{100}{4+1} = \frac{100}{5} = 20\ [\mathrm{gr}]$$

Alternative c)

$\boxed{2019\text{-}2}$

$$\frac{dT}{dt} = k \cdot (A - T)$$

$u = A - T \quad du = -dT$

$-du = dT$

$\int \frac{-du}{u} = -\ln|u|$

$= -\ln|A-T|$

$$\Rightarrow \frac{dT}{A-T} = k \cdot dt \Rightarrow \int \frac{dT}{A-T} = k \cdot t + C_1$$

$$\Rightarrow -\ln|A-T| = kt + C_1 \Rightarrow \ln|A-T| = -kt - C_1$$

$$e^{-k \cdot t} \cdot \underbrace{e^{-C_1}}_{B} = |A-T| \Rightarrow B e^{-k \cdot t} = |A-T| \, ?$$

$B > 0$

$$\Rightarrow \quad A - T = \underbrace{\pm B}\cdot e^{-k\cdot t} \qquad T(0) = A - C = 2A$$

$$C = \pm B \qquad \Rightarrow \quad A - 2A = C$$

$$\boxed{C = -A}$$

$$\Rightarrow \quad A - T = C\cdot e^{-k\cdot t}$$

$$\boxed{A - C\cdot e^{-kt} = T}$$

$$\boxed{T(t) = A - C\cdot e^{-k\cdot t}} \Rightarrow T(t) = A - (-A)e^{-k\cdot t}$$

$$= A + A e^{-kt}$$

$$= A\cdot(1+e^{-kt}) \qquad \boxed{f(t) = A\cdot(1+e^{-kt})}$$

$$f(t_0) = 1{,}5\cdot A \Rightarrow f(t_0) = \frac{3}{2}\cdot A$$

$$\Rightarrow \cancel{A}\cdot(1+e^{-k\cdot t_0}) = \frac{3\cancel{A}}{2}$$

$$\frac{3}{2}-1 = \frac{3}{2}-\frac{2}{2} = \frac{1}{2}$$

$$1+e^{-k\cdot t_0} = \frac{3}{2} \Rightarrow e^{-kt} = \frac{3}{2}-1 = \frac{1}{2}$$

$$e^{-k\cdot t^*} = \left(+\frac{1}{2}\right) \qquad / \ln()$$

$$\underbrace{\ln\left(e^{-k\cdot t^*}\right)} = \underbrace{\ln\left(\frac{1}{2}\right)}$$

$$-k\cdot t^* = -\ln(2) \Rightarrow k\cdot t^* = \ln(2)$$

$$\boxed{t^* = \frac{\ln(2)}{k}} \Rightarrow$$

Alternative 2)