$\boxed{2015-2}$

$|x| = \sqrt{x^2} \Rightarrow |x|^2 = x^2 \qquad f(x) = x^2$

$\Rightarrow$ $\boxed{\text{Grafico iv)}}$

$\boxed{2016-1}$

$$f(x) = (-x)\cdot e^{-\frac{x^2}{2}}$$

$$f'(x) = (-x)\cdot\underbrace{\left(e^{-\frac{x^2}{2}}\right)'}_{e^{-\frac{x^2}{2}}\cdot\underbrace{\left(-\frac{2x}{2}\right)}_{-x}} + \underbrace{(-x)'}_{-1}\cdot e^{-\frac{x^2}{2}}$$

$$= (-x)\cdot(-x)\cdot e^{-\frac{x^2}{2}} - e^{-\frac{x^2}{2}} = x^2\cdot e^{-\frac{x^2}{2}} - e^{-\frac{x^2}{2}}$$

$$f'(x) = (x^2-1)\cdot e^{-\frac{x^2}{2}}$$

$$f'(x)=0 \Rightarrow (x^2-1)\cdot \underbrace{e^{-\frac{x^2}{2}}}_{\neq 0} = 0 \Rightarrow x^2-1=0$$

$$\Rightarrow \boxed{x=\pm 1} \quad \boxed{x_1=1} \quad \boxed{x_2=-1}$$

$$f(1) = -e^{-\frac{1}{2}} \qquad \boxed{f(-1)=e^{-\frac{1}{2}}} \qquad \left(-1, e^{-\frac{1}{2}}\right)$$

massimo

$\boxed{\text{Alternativa c)}}$

2016-2

$$f(x)=\frac{\sqrt{1-x^2+\frac{x^4}{2}}}{x^2+1}$$

$$f'(x)=\frac{\left(\sqrt{1-x^2+\frac{x^4}{2}}\right)'\cdot(x^2+1)-\overbrace{(x^2+1)'}^{2x}\cdot\sqrt{1-x^2+\frac{x^4}{2}}}{(x^2+1)^2}$$

$$\frac{1}{2\sqrt{1-x^2+\frac{x^4}{2}}}\cdot\underbrace{\left(1-x^2+\frac{x^4}{2}\right)'}_{2x^3-2x}=\frac{2x^3-2x}{2\sqrt{1-x^2+\frac{x^4}{4}}}=\frac{x^3-x}{\sqrt{1-x^2+\frac{x^4}{4}}}$$

$$f'(x) = \frac{\dfrac{x^3-x}{\sqrt{1-x^2+\frac{x^4}{2}}} \cdot (x^2+1) - 2x \cdot \sqrt{1-x^2+\frac{x^4}{2}}}{(x^2+1)^2}$$

$$\Rightarrow f'(x) = 0 \Rightarrow \frac{(x^3-x)(x^2+1)}{\sqrt{1-x^2+\frac{x^4}{2}}} = 2x\sqrt{1-x^2+\frac{x^4}{2}}$$

$$\Rightarrow \underbrace{x(x^2-1)(x^2+1)}_{(x^4-1)} = 2x\cdot\left(1-x^2+\frac{x^4}{2}\right)$$

$$\Rightarrow x\cdot(x^4-1) = 2x\cdot\left(1-x^2+\frac{x^4}{2}\right)$$

$$\Rightarrow x\cdot\left[(x^4-1)-2\cdot\left(1-x^2+\frac{x^4}{2}\right)\right]=0$$

$$\Rightarrow x\cdot\left[\cancel{x^4}\overbrace{-1-2}^{-3}+\underline{2x^2}-\cancel{x^4}\right]=0$$

$$\Rightarrow \underline{x} \cdot \underbrace{(2x^2-3)}_{=0} = 0 \Rightarrow \boxed{x_1 = 0}$$

$$\boxed{x_2 = \sqrt{\frac{3}{2}}} \quad x_2^2 = x_3^2$$

$$\boxed{x_3 = -\sqrt{\frac{3}{2}}}$$

$$f(x_1) = f(0) = \underbrace{\sqrt{1}}_{1} = 1 \Rightarrow \underline{\boxed{f(x_1) = 1}}$$

$$f(x_2) = \frac{\sqrt{1 - \frac{3}{2} + \left(\frac{3}{2}\right)^2 \cdot \frac{1}{2}}}{\frac{3}{2} + 1} = \frac{\sqrt{1 - \frac{3}{2} + \frac{9}{8}}}{\frac{5}{2}}$$

$$= \frac{\sqrt{\frac{8}{8} - \frac{12}{8} + \frac{9}{8}}}{\frac{5}{2}} = \frac{\sqrt{\frac{5}{8}}}{\frac{5}{2}} = \frac{\frac{\sqrt{5}}{2\sqrt{2}}}{\frac{5}{2}} = \frac{\cancel{\sqrt{5}} \cdot \cancel{2}}{\underbrace{\cancel{5}}_{\sqrt{5}} \cdot \cancel{2}\sqrt{2}} = \frac{1}{\sqrt{10}} < 1$$

$$\boxed{f(x_2) = \frac{1}{\sqrt{10}}}$$

$$\boxed{f(x_1) = 1} \qquad (0,1)$$

$$\boxed{f(x_3) = \frac{1}{\sqrt{10}}} \Rightarrow \boxed{\text{Alternativa a)}}$$

$\boxed{2017-1}$ x_1 y x_2 son soluciones de la fórmula anterior

$$ax^2+bx+c = k\cdot(x-x_1)\cdot(x-x_2)$$

$$= k\cdot\left[x^2-(x_1+x_2)\cdot x+x_1\cdot x_2\right]$$

$$= k\cdot x^2 \underbrace{-k(x_1+x_2)}\cdot x+k\cdot x_1\cdot x_2$$

1. $\boxed{k=a}$
2. $-k(x_1+x_2)=b$
3. $c=k\cdot x_1\cdot x_2$

$$ax^2+bx+c = a\cdot(x-x_1)(x-x_2)$$

$$(A+B)\cdot x+(-Ax_2-Bx_1)$$

$$\frac{1}{ax^2+bx+c} = \frac{1}{a}\cdot\left[\frac{1}{(x-x_1)(x-x_2)}\right]$$

$$\frac{1}{(x-x_1)(x-x_2)} = \frac{A}{(x-x_1)} + \frac{B}{(x-x_2)} \quad \Big|\cdot(x-x_1)(x-x_2)$$

$$1 = A\cdot(x-x_2)+B(x-x_1) = Ax-A\cdot x_2+Bx-Bx_1$$

$$1 = (A+B)x + (-A\cdot x_2 - Bx_1)$$

(i) $A+B=0 \Rightarrow \boxed{A=-B}$

(ii) $-A\cdot x_2 - Bx_1 = 1 \Rightarrow -A\cdot x_2 + Ax_1 = 1 \Rightarrow A(x_1-x_2)=1$

$$\Rightarrow \underbrace{\boxed{A=\frac{1}{x_1-x_2}}, \quad \boxed{B=-\frac{1}{(x_1-x_2)}}}$$

$$\frac{1}{ax^2+bx+c} = \frac{1}{a} \cdot \left[\frac{1}{(x-x_1)} \cdot \frac{1}{(x-x_2)} \right]$$

$$= \frac{1}{a} \cdot \left[\boxed{\frac{1}{(x_1-x_2)} \cdot \frac{1}{(x-x_1)} - \frac{1}{(x_1-x_2)} \cdot \frac{1}{(x-x_2)}} \right]$$

$$= \frac{1}{a} \cdot \frac{1}{(x_1-x_2)} \cdot \left[\frac{1}{(x-x_1)} - \frac{1}{(x-x_2)} \right]$$

$$\frac{1}{ax^2+bx+c} = \frac{1}{a(x_1-x_2)} \cdot \left[\frac{1}{(x-x_1)} - \frac{1}{(x-x_2)}\right]$$

$$\int \frac{dx}{ax^2+bx+c} = \int \frac{1}{a(x_1-x_2)} \cdot \left[\frac{1}{(x-x_1)} - \frac{1}{(x-x_2)}\right] dx$$

$$= \frac{1}{a(x_1-x_2)} \cdot \left[\int \frac{dx}{x-x_1} - \int \frac{dx}{x-x_2}\right]$$

$$\ln\left(\left|\frac{x-x_1}{x-x_2}\right|\right)$$

$$= \frac{1}{a(x_1-x_2)} \cdot \left[\ln(|x-x_1|) - \ln(|x-x_2|)\right]$$

$$\boxed{\frac{1}{a(x_1-x_2)} \cdot \ln\left(\frac{|x-x_1|}{|x-x_2|}\right)} \Rightarrow \text{Alternativa d)}$$

2017-2

$$f(x) = e^{\sin(|x|)+\ln(x)} = e^{\ln(x)} \cdot e^{\sin(|x|)}$$

$$= \frac{1}{x} \cdot e^{\sin(|x|)}$$

si es periodica

$e^{(\cdot)}$ valores positivos

No es periodica

$$f(x) = e^{[Algo]} = e^{\sin(|x|)+\ln|x|}$$

Se descartan los graficos ii) y iv)

i) es periodico $\Rightarrow$ No es periodico $\Rightarrow$ iii)

$\boxed{2018-1}$

$$\int \frac{dx}{x^{\frac{1}{5}}+2}$$

$$u = x^{\frac{1}{5}}+2 \qquad u-2 = x^{\frac{1}{5}}$$

$$\Rightarrow du = \frac{1}{5}\cdot x^{\left(\frac{1}{5}-1\right)}dx$$

$$= \frac{1}{5x^{\frac{4}{5}}}dx$$

$$\Rightarrow 5x^{\frac{4}{5}}\cdot du = dx$$

$$\Rightarrow 5\cdot(u-2)^4 du = dx$$

$$= \int \frac{5\,du\,(u-2)^4}{u}$$

$$= 5 \cdot \left[\int \frac{(w-2)^4}{w} \, dw \right]$$

$$= 5 \cdot \left[\int \frac{(w^2-4w+4)^2}{w} \, dw \right]$$

$$(w^2-4w+4)^2 = \left[(w^2-4w)+4 \right]^2 = (w^2-4w)^2$$

$$+2 \cdot 4 (w^2-4w)+16 = \left[w(w-4) \right]^2 + 8(w^2-4w)+16$$

$$= w^2 \cdot (w^2-8w+16) + 8(w^2-4w) + 16$$

$$= w^4 - 8w^3 + \boxed{16w^2}\boxed{+8w^2} - 32w + 16$$

$$= w^4 - 8w^3 + 24w^2 - 32w + 16$$

$$5 \cdot \int \left(\frac{w^4 - 8w^3 + 24w^2 - 32w + 16}{w} \right) dw$$

$$= 5 \cdot \int \left(w^3 - 8w^2 + 24w - 32 + \frac{16}{w} \right) dw$$

$$= 5 \cdot \left[\frac{u^4}{4} - \frac{8u^3}{3} + \frac{24u^2}{2} - 32u + 16 \cdot \ln(|u|) \right] \qquad u = x^{\frac{1}{5}}+2$$

$$= 5 \cdot \left[\frac{(x^{\frac{1}{5}}+2)^4}{4} - \frac{8(x^{\frac{1}{5}}+2)^3}{3} + 12 \cdot (x^{\frac{1}{5}}+2)^2 - 32(x^{\frac{1}{5}}+2) + 16 \cdot \ln\left(|x^{\frac{1}{5}}+2|\right) \right]$$

8

$$\boxed{\left(x^{\frac{1}{5}}+2\right)^2 = x^{\frac{2}{5}}+4x^{\frac{1}{5}}+4} \quad (1)$$

$$\left(x^{\frac{1}{5}}+2\right)^3 = \left(x^{\frac{1}{5}}+2\right)^2\cdot\left(x^{\frac{1}{5}}+2\right) = \left[x^{\frac{2}{5}}+4x^{\frac{1}{5}}+4\right]\left(x^{\frac{1}{5}}+2\right)$$

$$= \left[\cancel{x^{\frac{3}{5}}}+\cancel{4x^{\frac{2}{5}}}+\cancel{4x^{\frac{1}{5}}}+\cancel{2x^{\frac{2}{5}}}+\cancel{8x^{\frac{1}{5}}}+8\right]$$

$$= x^{\frac{3}{5}}+6x^{\frac{2}{5}}+12x^{\frac{1}{5}}+8.$$

$$\boxed{\left(x^{\frac{1}{5}}+2\right)^3 = x^{\frac{3}{5}}+6x^{\frac{2}{5}}+12x^{\frac{1}{5}}+8} \quad (2)$$

$$\left(x^{\frac{1}{5}}+2\right)^4 = \left[\left(x^{\frac{1}{5}}+2\right)^2\right]^2 = \left[x^{\frac{2}{5}}+4x^{\frac{1}{5}}+4\right]^2$$

$$x^{\frac{1}{5}}\cdot\left(x^{\frac{1}{5}}+4\right)$$

$$=\left[\left(x^{\frac{2}{5}}+4x^{\frac{1}{5}}\right)+4\right]^2 = \left(x^{\frac{2}{5}}+4x^{\frac{1}{5}}\right)^2 +$$

$$8\left(x^{\frac{2}{5}}+4x^{\frac{1}{5}}\right)+16 = \left[x^{\frac{1}{5}}\cdot\left(x^{\frac{1}{5}}+4\right)\right]^2$$

$$+8x^{\frac{2}{5}}+32x^{\frac{1}{5}}+16 = x^{\frac{2}{5}}\cdot\left(\underline{x^{\frac{1}{5}}+4}\right)^2+8x^{\frac{2}{5}}$$

$$+32x^{\frac{1}{5}}+16 = \underline{x^{\frac{2}{5}}\cdot\left[x^{\frac{2}{5}}+8x^{\frac{1}{5}}+16\right]}+8x^{\frac{2}{5}}$$

$$+32x^{\frac{1}{5}}+16 = \underline{x^{\frac{4}{5}}} + \underline{8x^{\frac{3}{5}}} + \underbrace{16x^{\frac{2}{5}} + 8x^{\frac{2}{5}}}_{24x^{\frac{2}{5}}}$$

$$+\underline{32x^{\frac{1}{5}}}\ \underline{+16}$$

$$\boxed{\left(x^{\frac{1}{5}}+2\right)^4 = x^{\frac{4}{5}} + 8x^{\frac{3}{5}} + 24x^{\frac{2}{5}} + 32x^{\frac{1}{5}} + 16} \quad (3)$$

$$5\cdot\left[\frac{(x^{\frac{1}{5}}+2)^4}{4}-\frac{8}{3}\cdot(x^{\frac{1}{5}}+2)^3+12\cdot(x^{\frac{1}{5}}+2)^2\right.$$

$$\left.-32\cdot(x^{\frac{1}{5}}+2)+16\cdot\ln(|x^{\frac{1}{5}}+2|)\right]$$

2

$-\frac{40{,}6}{3}$

$4\cdot 40$

$\frac{\cancel{12}\cdot 5\cdot 8}{\cancel{3}}$

$8\cdot\frac{8}{3}\cdot 5=\frac{16\cdot 10\cdot 2}{3}$

$$=5\cdot\left[\frac{1}{4}\cdot\left(x^{\frac{4}{5}}+\cancel{8}x^{\frac{3}{5}}+2\cancel{4}x^{\frac{2}{5}}+\cancel{3}2x^{\frac{1}{5}}+\cancel{1}6\right)\right. \quad \frac{320}{3}$$

$$-\frac{8}{3}\cdot\left(x^{\frac{3}{5}}+6x^{\frac{2}{5}}+12x^{\frac{1}{5}}+8\right)+12\left(x^{\frac{2}{5}}+4x^{\frac{1}{5}}+4\right)$$

$$\left.-32\left(x^{\frac{1}{5}}+2\right)+16\ln|x^{\frac{1}{5}}+2|\right]$$

$$= \cancel{80 \cdot \ln|x^{\frac{1}{5}}+2|} + \left(-32x^{\frac{1}{5}} \cancel{-64} + \cancel{12x^{\frac{2}{5}}} \right.$$

$60x^{\frac{2}{5}}$

$80+40$

-160

$$\left. + 48x^{\frac{1}{5}} + \cancel{48} \right) \cdot 5 + \cancel{-\frac{40}{3}x^{\frac{3}{5}}} \cancel{-80x^{\frac{2}{5}}} - 160x^{\frac{1}{5}}$$

$16 \cdot 5 = 8 \cdot 10$

$$\cancel{-\frac{320}{3}} + \cancel{\frac{5}{4}x^{\frac{4}{5}}} + \cancel{10x^{\frac{2}{5}}} + \cancel{30x^{\frac{2}{5}}} + 40x^{\frac{1}{5}} + \cancel{240}$$

$10 - \frac{40}{3} - \frac{30}{3} - \frac{40}{3}$

$$= 80 \cdot \ln|x^{\frac{1}{5}}+2| + \frac{5}{4}x^{\frac{4}{5}} - \frac{10}{3} \cdot x^{\frac{3}{5}} + 10x^{\frac{2}{5}} - 40x^{\frac{1}{5}}$$

$\rightarrow$ $\boxed{\text{Alternativa b)}}$

$\boxed{2018-2}$

$$f(x) = \frac{ax^2+bx+c}{\underbrace{x+d}_{=0}}$$

i) $\boxed{x=-d} \Rightarrow$ Asintoto verticale

ii) $$m = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{ax^2+bx+c}{x(x+d)} =$$

$$\lim_{x\to\infty} \frac{ax^2+bx+c}{x^2+xd} = \lim_{x\to\infty} \frac{a+\frac{b}{x}+\frac{c}{x^2}}{1+\frac{d}{x}} = a \quad \boxed{m=a}$$

$$n = \lim_{x \to \infty} \left[f(x) - mx \right] = \lim_{x \to \infty} \left[\frac{ax^2+bx+c}{x+d} - ax \right]$$

$$= \lim_{x \to \infty} \left[\frac{ax^2+bx+c}{x+d} - \frac{ax(x+d)}{(x+d)} \right]$$

$$= \lim_{x \to \infty} \left[\frac{ax^2+bx+c-ax^2-axd}{(x+d)} \right]$$

$$= \lim_{x \to \infty} \left[\frac{\cancel{(b-a)x}\,(b-ad)x+c}{x+d} \right]$$

$$= \lim_{x\to\infty} \frac{(b-ad) + \frac{c}{x}}{1 + \frac{d}{x}} = (b-ad)$$

$\boxed{m = a}$ $\boxed{n = b-ad}$ $\boxed{y = ax + (b-ad)}$

$\Rightarrow$ $\boxed{\text{Alternativa } c)}$

$\boxed{2019-1}$

$\boxed{\ln(\ln(\ln(x)))}$

$= f(h(x))$

$[f(h(x))]' = f'(h(x)) \cdot h'(x)$

$f(w) = \ln(w) \Rightarrow \boxed{f'(w) = \frac{1}{w}}$

$\underline{h(w) = \ln(\ln(w))} \Rightarrow h'(w) = (\ln(\ln(w)))'$

$$\ln(\ln(\ln(x))) = f(h(x))$$

$$\left[\ln(\ln(\ln(x)))\right]' = f'(h(x)) \cdot h'(x)$$

$$= \frac{1}{h(x)} \cdot h'(x) = \frac{1}{\ln(\ln(x))} \cdot \boxed{(\ln(\ln(x)))'}$$

$$(\ln(\ln(x)))'$$

$$f(w) = \ln(w) \Rightarrow f'(w) = \frac{1}{w}$$

$$h(w) = \ln(w) \Rightarrow h'(w) = \frac{1}{w}$$

$$\ln(\ln(x)) = f(h(x)) \quad /()'$$

$$(\ln(\ln(x)))' = f'(h(x)) \cdot h'(x)$$

$$= \frac{1}{h(x)} \cdot h'(x) = \frac{1}{\ln(x)} \cdot \frac{1}{x}$$

$$= \frac{1}{\ln(\ln(x))} \cdot \frac{1}{\ln(x)} \cdot \frac{1}{x} \quad //$$

$\boxed{\text{Alternative 6)}}$

$\boxed{2019-2}$ $\quad f(x) = \sin(|x|)$

$$f(-x) = \sin(|-x|) = \sin(|x|) = f(x) \Rightarrow f(-x) = f(x)$$

$\underline{f \text{ es par}}$ Descartar i) ii)

Sinusoidal $\Rightarrow$ Descartar iii)

$f(x) = \sin(|x|)$ } $\Rightarrow$ Descartar iv) $\Rightarrow$ $\boxed{\text{Alternativa a)}}$