$\boxed{2015-2}$

$$f(x,y) = x^2 \cdot y^2 - 2xy^3$$

$$\frac{\partial f}{\partial x} = 2xy^2 - 2y^3$$

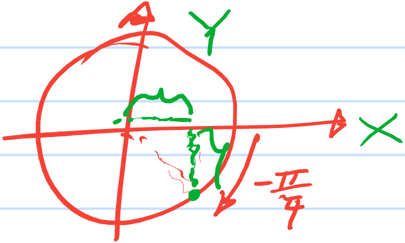
$$\frac{\partial f}{\partial y} = 2x^2y - 6xy^2$$

$$\frac{\partial f}{\partial x}(1,1) = 0$$

$$\frac{\partial f}{\partial y}(1,1) = 2-6 = -4$$

$$\Rightarrow \boxed{\nabla f(1,1) = (0,-4)}$$

$$\theta = -\frac{\pi}{4} \qquad \hat{n} = (\cos\theta, \sin\theta) = \Big(\underbrace{\cos\left(-\tfrac{\pi}{4}\right)}_{\frac{\sqrt{2}}{2}}, \underbrace{\sin\left(\tfrac{\pi}{4}\right)}_{-\frac{\sqrt{2}}{2}}\Big)$$



$$= \frac{\sqrt{2}}{2} \cdot (1, -1)$$

$$\nabla f(1,1) \cdot \hat{n} = (0, -4) \cdot \frac{\sqrt{2}}{2} \cdot (1, -1) = \frac{\sqrt{2}}{2} \cdot \left(0 \cdot 1 - 4 \cdot (-1)\right)$$

$$= \frac{\sqrt{2}}{2} \cdot (4) = \boxed{2\sqrt{2}}$$

$\boxed{2016-1}$

$f(x,y) = x^y$

$x^2 \quad x^3$

$(1,2)$

$2^y \quad 3^y$

$$\boxed{\begin{aligned} \frac{\partial f}{\partial x} &= y \cdot x^{y-1} \\ \frac{\partial f}{\partial y} &= \ln(x) \cdot x^y \end{aligned}}$$

$$\frac{\partial f}{\partial x}(1,2) = 2 \cdot 1^{2-1} = 2 \cdot 1^1 = 2$$

$$\Rightarrow \boxed{\nabla f(1,2) = (2,0)}$$

$$\frac{\partial f}{\partial y}(1,2) = \ln(1) \cdot 1^2 = 0$$

$\hat{w} = (1,1)$

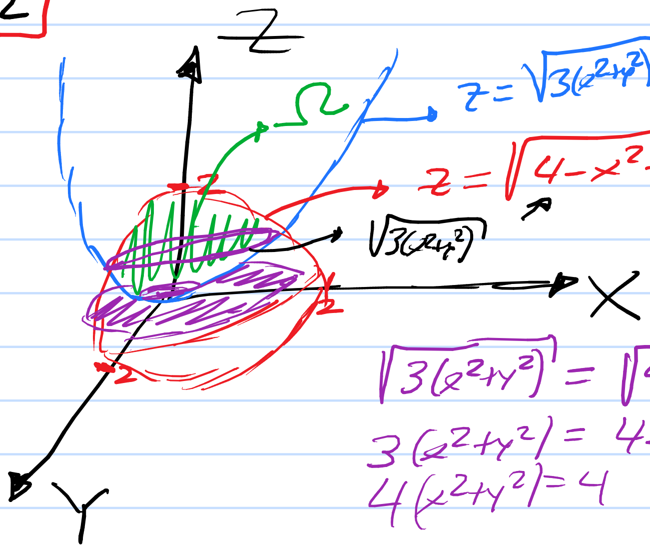
$\nabla f(1,2) \cdot \dfrac{\hat{w}}{\|\hat{w}\|}$

$$= (2,0) \cdot \frac{(1,1)}{\sqrt{2}} = \frac{(2 \cdot 1 + 0 \cdot 1)}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{2\sqrt{2}}{2}$$

$$= \sqrt{2} \Rightarrow \boxed{\sqrt{2}}$$

Alternativa c)

2016-2



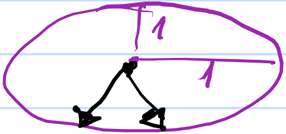
$z = \sqrt{3(x^2+y^2)}$

$z = \sqrt{4-x^2-y^2}$

$\sqrt{3(x^2+y^2)}$

$z = \sqrt{3(x^2+y^2)}$

$z^2+y^2+x^2=4$



$$\sqrt{3(x^2+y^2)} = \sqrt{4-x^2-y^2}$$

$$3(x^2+y^2) = 4-x^2-y^2$$

$$4(x^2+y^2)=4 \Rightarrow \boxed{x^2+y^2=1}$$

$$V = \iiint dz\,dx\,dy = \iiint dz \cdot r\,dr\,d\theta$$

$\theta \in [0, 2\pi]$

$r \in [0, 1]$

$z \in \left[\sqrt{3(x^2+y^2)}, \sqrt{4-x^2-y^2}\right]$

$$= \int_0^{2\pi} \int_0^1 \left[\int_{\sqrt{3(x^2+y^2)}}^{\sqrt{4-x^2-y^2}} dz \right] \cdot r\,dr\,d\theta$$

$$= \int_0^{2\pi} \int_0^1 \left[\underbrace{\sqrt{4-x^2-y^2} - \sqrt{3(x^2+y^2)}}_{x = r\cdot\cos\theta \quad y = r\cdot\sin\theta} \right] \cdot r\,dr\,d\theta$$

$$\sqrt{4-x^2-y^2} = \sqrt{4-r^2\cdot\cos^2\theta - r^2\cdot\sin^2\theta}$$

$$= \sqrt{4-r^2\cdot\underbrace{(\cos^2\theta+\sin^2\theta)}_{1}} = \sqrt{4-r^2}$$

$$\sqrt{3(x^2+y^2)} = \sqrt{3(r^2\cdot\cos^2\theta + r^2\cdot\sin^2\theta)} = \sqrt{3(r^2)} = \sqrt{3}\cdot|r|$$

$$= \sqrt{3}\,r$$

$$= \int_0^{2\pi} \int_0^1 \left(\sqrt{4-r^2} - \sqrt{3}\, r\right) \cdot r \, dr \, d\theta$$

$$\cdot 1$$

$$= \int_0^{2\pi} \int_0^1 \underbrace{\left(\sqrt{4-r^2} - \sqrt{3}\, r\right) \cdot r}_{\text{Solo depende de } r} \, dr \, d\theta$$

$$= \left(\int_0^{2\pi} d\theta\right)\left(\int_0^1 \left(\sqrt{4-r^2} - \sqrt{3} \cdot r\right) \cdot r \, dr\right)$$

$$= 2\pi \cdot \Big[\underbrace{\int_0^1 \sqrt{4-r^2} \cdot r \, dr}_{I_1} - \underbrace{\sqrt{3} \int_0^1 r^2 \, dr}_{I_2} \Big]$$

$$= 2\pi \cdot [I_1 - I_2]$$

$$(*) \quad \boxed{V = 2\pi \cdot [I_1 - I_2]}$$

$$I_2 = \sqrt{3} \cdot \int_0^1 r^2 dr = \sqrt{3} \cdot \frac{r^3}{3}\Big|_0^1 = \sqrt{3} \cdot \left[\frac{1}{3} - \frac{0}{3}\right]$$

$$= \frac{\sqrt{3}}{3} \Rightarrow \boxed{I_2 = \frac{\sqrt{3}}{3}}$$

$$w = 4 - r^2$$

$$dw = -2r\,dr$$

$$\boxed{-\frac{1}{2} \cdot dw = r\,dr}$$

$$I_1 = \int_0^1 \sqrt{4 - r^2} \cdot r\,dr$$

$$= \int_4^3 w^{\frac{1}{2}} \left(-\frac{1}{2} dw\right)$$

$$= -\frac{1}{2} \cdot \int_4^3 w^{\frac{1}{2}} dw = \frac{1}{2} \cdot \int_3^4 w^{\frac{1}{2}} dw = \frac{1}{2} \frac{w^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} \Big|_3^4$$

$$= \frac{1}{2} \cdot \frac{2}{3} \cdot \left(w\sqrt{w}\right)\Big|_{w=3}^{w=4} = \frac{1}{3} \cdot \left(4 \cdot 2 - 3\sqrt{3}\right)$$

$$= \frac{1}{3} \cdot \left(8 - 3\sqrt{3}\right) = \frac{8}{3} - \sqrt{3} \Rightarrow \boxed{I_1 = \frac{8}{3} - \sqrt{3}}$$

$$\boxed{I_1 = \frac{8}{3} - \sqrt{3}} \qquad \boxed{I_2 = \frac{\sqrt{3}}{3}} \qquad V = 2\pi \cdot [I_1 - I_2]$$

$$V = 2\pi \cdot \left[\frac{8}{3} - \sqrt{3} - \frac{\sqrt{3}}{3}\right] = 2\pi \cdot \left[\frac{8}{3} - \frac{3\sqrt{3}}{3} - \frac{\sqrt{3}}{3}\right]$$

$$= 2\pi \cdot \left[\frac{8}{3} - \frac{4\sqrt{3}}{3}\right] = \frac{2\pi}{3} \cdot [8 - 4\sqrt{3}] = \frac{2\pi}{3} \cdot 8 \cdot \left[1 - \frac{\sqrt{3}}{2}\right]$$

$$= \frac{16\pi}{3} \cdot \left[1 - \frac{\sqrt{3}}{2}\right] \Rightarrow \boxed{\text{Alternativa d)}}$$

$\boxed{2017-1}$

1

$z = \sqrt{1-x^2-y^2}$

$z \in \left[\sqrt{1-x^2-y^2}, 1\right]$

$r \in [0,1]$

$\theta \in \left[0, \frac{\pi}{2}\right]$

$$V = \int dV = \iiint dz \cdot r\,dr\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 \left[\int_{\sqrt{1-x^2-y^2}}^{1} 1 \cdot dz \right] r\,dr\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 \underbrace{\left(1 - \sqrt{1-x^2-y^2} \right)}_{x,y \quad r,\theta} r\,dr\,d\theta$$

$$\sqrt{1-x^2-y^2}$$

$$= \sqrt{1 - r^2 \cdot \cos^2\theta - r^2 \cdot \sin^2\theta}$$

$$= \sqrt{1 - r^2 \cdot [\cos^2\theta + \sin^2\theta]}$$

$$= \sqrt{1-r^2}$$

$$= \int_0^{\frac{\pi}{2}} \left[\int_0^1 1 \cdot \left(1 - \sqrt{1-r^2}\right) \cdot r \, dr \right] d\theta$$

$$\boxed{V = \frac{\pi}{2} \cdot I}$$

$$= \left(\int_0^{\frac{\pi}{2}} d\theta \right) \left(\int_0^1 \left(1 - \sqrt{1-r^2}\right) r \, dr \right)$$

$$= \frac{\pi}{2} \cdot \Big(\underbrace{\int_0^1 \left(r - r\sqrt{1-r^2}\right) dr}_{I} \Big)$$

$$I = \int_0^1 \left(1 - \sqrt{1-r^2}\right) \cdot r\,dr = \int_0^1 r\,dr - \int_0^1 \sqrt{1-r^2}\,r\,dr$$

$$= \frac{r^2}{2}\Big|_0^1 - \int_0^1 \sqrt{1-r^2} \cdot r\,dr = \frac{1}{2} - \int_0^1 \underbrace{\sqrt{1-r^2}}_{u=1-r^2}\, r\,dr$$

$$du = -2r\,dr$$

$$\left(\frac{du}{-2}\right) = r\,dr$$

$$= \frac{1}{2} - \int_1^0 u^{\frac{1}{2}} \frac{du}{(-2)}$$

$$= \frac{1}{2} + \frac{1}{2} \cdot \int_1^0 u^{\frac{1}{2}} du = \frac{1}{2} - \frac{1}{2} \int_0^1 u^{\frac{1}{2}} du$$

$$= \frac{1}{2} - \frac{1}{2} \cdot \frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\Bigg|_{u=0}^{u=1} = \frac{1}{2} - \frac{1}{2} \cdot \frac{2}{3} \cdot \left[u\sqrt{u}\right]_{u=0}^{u=1}$$

$$= \frac{1}{2} - \frac{1}{3} \cdot \left[1\sqrt{1} - 0 \cdot \sqrt{0}\right] = \frac{1}{2} - \frac{1}{3} \cdot [1] = \frac{1}{2} - \frac{1}{3}$$

$$= \frac{3}{6} - \frac{2}{6} = \frac{1}{6} \Rightarrow I = \frac{1}{6}$$

$$V = \frac{\pi}{2} \cdot I = \frac{\pi}{2} \cdot \frac{1}{6} = \frac{\pi}{12} \Rightarrow \boxed{\text{Alternativa b)}}$$

$\boxed{2017-2}$

$$f(x,y) = \sin\left(\sqrt{1+[\ln(xy)]^2}\right) \qquad \ln(xy) = \ln(x) + \ln(y)$$

$$\frac{\partial f}{\partial x} = \cos\left(\sqrt{1+[\ln(xy)]^2}\right) \cdot \frac{\partial}{\partial x}\left(\sqrt{1+[\ln(xy)]^2}\right)$$

$$= \cos\left(\sqrt{1+[\ln(xy)]^2}\right) \cdot \frac{1}{2\sqrt{1+[\ln(xy)]^2}} \cdot \frac{\partial}{\partial x}\left(1+[\ln(xy)]^2\right)$$

$$= \cos\left(\sqrt{1+[\ln(xy)]^2}\right) \cdot \frac{1}{\cancel{2}\sqrt{1+[\ln(xy)]^2}} \cdot \cancel{2} \cdot \ln(xy) \cdot \underbrace{\left[\frac{\partial}{\partial x}(\ln(xy))\right]}_{\frac{1}{x}}$$

$\frac{1}{x}$

$$= \cos\left(\sqrt{1+[\ln(xy)]^2}\right) \cdot \frac{\ln(xy)}{x} \cdot \frac{1}{\sqrt{1+[\ln(xy)]^2}}$$

$$\boxed{\frac{\partial f}{\partial x} = \cos\left(\sqrt{1+[\ln(xy)]^2}\right) \cdot \frac{\ln(xy)}{x \cdot \sqrt{1+[\ln(xy)]^2}}}$$

$x = 2$

$y = \frac{1}{2}$

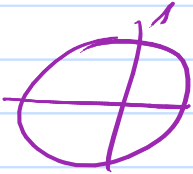
$xy = 1$

$\ln(1)$

$$\boxed{\frac{\partial f}{\partial y} = \cos\left(\sqrt{1+[\ln(xy)]^2}\right) \cdot \frac{\ln(xy)}{y \cdot \sqrt{1+[\ln(xy)]^2}}}$$

$$\frac{\partial f}{\partial x} = \cos\left(\sqrt{1+[\underbrace{\ln(1)}_{0}]^2}\right) \cdot \frac{\overbrace{\ln(1)}^{0}}{2\cdot\sqrt{1+[\ln(1)]^2}} = 0$$

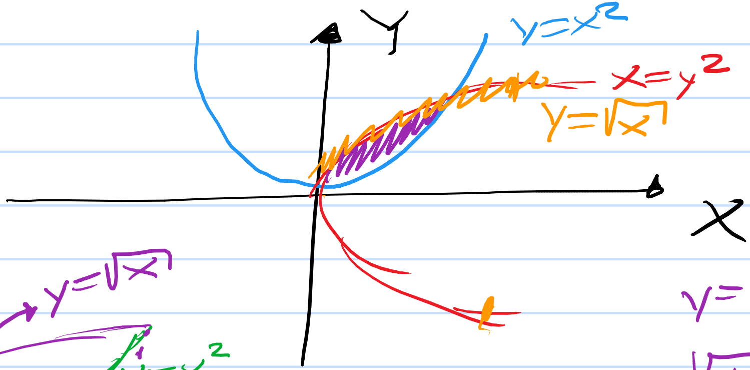
$$\frac{\partial f}{\partial y} = 0 \quad \Rightarrow \quad \boxed{\nabla f\left(2,\tfrac{1}{2}\right) = (0,0)}$$



$$\theta = \frac{\pi}{2} \qquad \hat{n} = \left(\cos\left(\tfrac{\pi}{2}\right), \sin\left(\tfrac{\pi}{2}\right)\right) = (0,1)$$

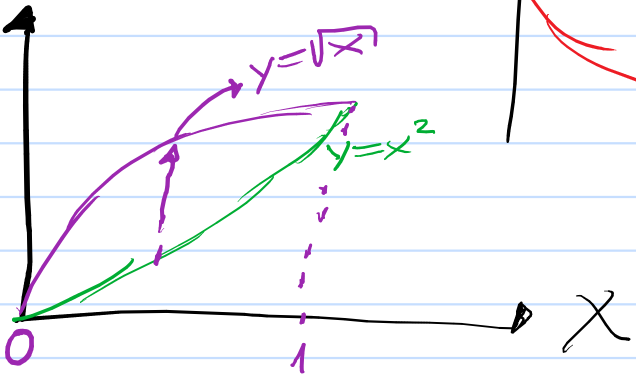
$$\boxed{0} \Rightarrow \boxed{\text{Alternativa c)}}$$

$\boxed{2018-1}$



$x = y^2$

$y = \pm\sqrt{x}$



$y = \sqrt{x} \qquad y = x^2$

$\sqrt{x} = x^2 \quad /()^2$

$x^2 = x^4 \qquad 1 - x^2 = 0 \to x = \pm 1$

$x^2 - x^4 = 0$

$x^2(1 - x^2) = 0 \qquad x = 0$

$$\boxed{R = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 1,\ \underline{x^2 \leq y \leq \sqrt{x}}\}}$$

$$\rho(x,y) = \frac{dm}{dA} \Rightarrow \boxed{dm = \rho(x,y) \cdot dA}$$

$$\boxed{\bar{x} = \frac{M_x}{M}} \qquad M_x = \int x \cdot dm \qquad \boxed{\bar{y} = \frac{M_y}{M}}$$

$$M_x = \iint_R x \cdot dm = \iint_R x \cdot \rho(x,y)\, dA = \iint_R x \cdot \rho(x,y) \cdot dx\,dy$$

$$= \iint_R x \cdot \sqrt{x} \cdot dx\,dy = \int_0^1 \left[\int_{x^2}^{\sqrt{x}} \underline{x\sqrt{x}}\, dy \right] dx$$

$$= \int_0^1 x\sqrt{x} \left(\int_{x^2}^{\sqrt{x}} dy \right) dx = \int_0^1 x\sqrt{x} \cdot \left[\sqrt{x} - x^2\right] dx$$

$$= \int_0^1 x^{\frac{3}{2}} \cdot \left[x^{\frac{1}{2}} - x^2 \right] dx = \int_0^1 \left(x^2 - x^{\frac{7}{2}} \right) dx$$

$$\frac{3}{2} + 2 = \frac{3}{2} + \frac{4}{2} = \frac{7}{2}$$

$$= \left(\frac{x^3}{3} - \frac{x^{\frac{9}{2}}}{\left(\frac{9}{2}\right)} \right) \Bigg|_{x=0}^{x=1} = \left(\frac{x^3}{3} - \frac{2}{9} \cdot x^{\frac{9}{2}} \right) \Bigg|_{x=0}^{x=1}$$

$$= \left(\frac{1}{3} - \frac{2}{9} \right) = \frac{3}{9} - \frac{2}{9} = \frac{1}{9} \Rightarrow \boxed{M_x = \frac{1}{9}}$$

$$M_y = \iint_R y\,dm = \int_0^1 \int_{x^2}^{\sqrt{x}} y \cdot \rho(x,y)\,dy\,dx$$

$$= \int_0^1 \left[\int_{x^2}^{\sqrt{x}} y \cdot \boxed{\sqrt{x}}\,dy \right] dx = \int_0^1 \sqrt{x} \cdot \left[\int_{x^2}^{\sqrt{x}} y\,dy \right] dx$$

$$= \int_0^1 x^{\frac{1}{2}} \cdot \left(\frac{y^2}{2} \Big|_{x^2}^{\sqrt{x}} \right) dx$$

$$\frac{x}{2} - \frac{x^4}{2} = \frac{(x - x^4)}{2}$$

$$= \int_0^1 x^{\frac{1}{2}} \cdot \frac{\left(x^1 - x^4\right)}{2} \, dx$$

$$= \frac{1}{2} \cdot \int_0^1 \left(x^{\frac{3}{2}} - x^{\frac{9}{2}}\right) dx$$

$$= \frac{1}{2} \cdot \left[\frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} - \frac{x^{\frac{11}{2}}}{\left(\frac{11}{2}\right)} \right]_{x=0}^{x=1} = \frac{1}{2} \cdot \left[\frac{1}{\left(\frac{5}{2}\right)} - \frac{1}{\left(\frac{11}{2}\right)} \right]$$

$$= \frac{1}{2} \cdot \left[\frac{2}{5} - \frac{2}{11} \right] = \frac{1}{2} \cdot \left[\frac{2 \cdot 11}{5 \cdot 11} - \frac{2 \cdot 5}{5 \cdot 11} \right] = \frac{1}{2} \cdot \left(\frac{22 - 10}{55} \right)$$

$$= \frac{1}{2} \cdot \left[\frac{12}{55} \right] = \frac{6}{55} \Rightarrow \boxed{M_y = \frac{6}{55}}$$

$$M = \int dm = \int \sqrt{x}\, dA = \iint \sqrt{x}\, dx\, dy$$

$$= \int_0^1 \left[\int_{x^2}^{\sqrt{x}} \sqrt{x}\, dy \right] dx = \int_0^1 \sqrt{x} \cdot \left[\int_{x^2}^{\sqrt{x}} dy \right] dx$$

$$= \int_0^1 x^{\frac{1}{2}} \cdot \left[x^{\frac{1}{2}} - x^2 \right] dx$$

$$= \int_0^1 \left(x^1 - x^{\frac{5}{2}} \right) dx = \left(\frac{x^2}{2} - \frac{x^{\frac{7}{2}}}{\left(\frac{7}{2}\right)} \right) \Bigg|_{x=0}^{x=1}$$

$$= \left(\frac{1}{2} - \frac{1}{\left(\frac{7}{2}\right)} \right) = \left(\frac{1}{2} - \frac{2}{7} \right) = \frac{7}{14} - \frac{4}{14} = \frac{3}{4}$$

$$\Rightarrow \boxed{\Pi = \frac{3}{14}} \qquad \boxed{\Pi_x = \frac{1}{9}} \qquad \boxed{\Pi_y = \frac{6}{55}}$$

$$\bar{x} = \frac{\Pi_x}{\Pi} = \frac{\frac{1}{9}}{\frac{3}{14}} = \frac{14}{9 \cdot 3} = \frac{14}{27} \Rightarrow \boxed{\bar{x} = \frac{14}{27}}$$

$$\bar{y} = \frac{\Pi_y}{\Pi} = \frac{\frac{6}{55}}{\frac{3}{14}} = \frac{14 \cdot \cancel{6}^{\,2}}{55 \cdot \cancel{3}} = \frac{28}{55} \Rightarrow \boxed{\bar{y} = \frac{28}{55}}$$

$\Rightarrow$ $\boxed{\text{Alternativa c)}}$

$\boxed{2019-1}$ > 0

b) $z = 1x^2 + 2y^2 - 3x + 6y + 3$

d) $z = 1x^2 + 2y^2 + 3x - 6y + 3$

a) $z = x^2 - 2y^2 - 3x + 6y + 7$

$= (x^2 - 3x) + (-2y^2 + 6y) + 7$

$$= (x^2-3x) - 2(y^2-3y) + 7$$

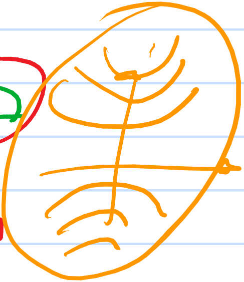
$$x^2-3x = x^2 - 2\cdot\left(\frac{3}{2}\right)\cdot x = x^2 - 2\cdot\left(\frac{3}{2}\right)\cdot x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2$$

$$= \Big[\underbrace{x^2 - 2\cdot\left(\frac{3}{2}\right)\cdot x + \left(\frac{3}{2}\right)^2}_{\left(x-\frac{3}{2}\right)^2}\Big] - \frac{9}{4} = \left(x-\frac{3}{2}\right)^2 - \frac{9}{4}$$

$$y^2-3y = \left(y-\frac{3}{2}\right)^2 - \frac{9}{4}$$

$$= \left[\left(x-\frac{3}{2}\right)^2 - \frac{9}{4}\right] - 2\cdot\left[\left(y-\frac{3}{2}\right)^2 - \frac{9}{4}\right] + 7$$

$$= \left(x-\frac{3}{2}\right)^2 - \frac{9}{4} - 2\cdot\left(y-\frac{3}{2}\right)^2 + \frac{9}{2} + 7$$



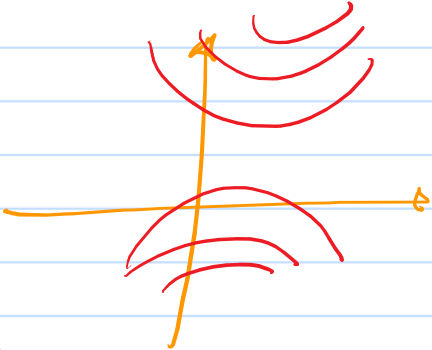
$$-\frac{9}{4} + \frac{9}{2} + 7$$

$$= -\frac{9}{4} + \frac{18}{4} + \frac{28}{4}$$

$$= \frac{9}{4} + \frac{28}{4} = \frac{37}{4}$$

$$= \left(x-\frac{3}{2}\right)^2 - 2\left(y-\frac{3}{2}\right)^2 + \frac{37}{4}$$

$x^2 - 2y^2 + C$ $\quad \left(\frac{3}{2}, \frac{3}{2}\right)$



c) $z = x^2 - 2y^2 + 3x - 6y + 7$

$= x^2 + 3x + (-2y^2 - 6y) + 7 = (x^2 + 3x)$

$-2 \cdot (y^2 + 3y) + 7$

$$\boxed{z = (x^2+3x) - 2(y^2+3y) + 7}$$

$$x^2+3x = x^2 + 2\cdot\left(\frac{3}{2}\right)x = x^2 + 2\left(\frac{3}{2}\right)x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2$$

$$= \left[x^2 + 2\cdot\left(\frac{3}{2}\right)x + \left(\frac{3}{2}\right)^2\right] - \frac{9}{4}$$

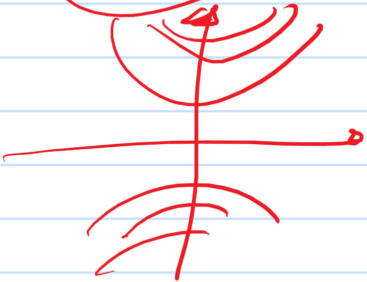
$$= \left(x+\frac{3}{2}\right)^2 - \frac{9}{4}$$

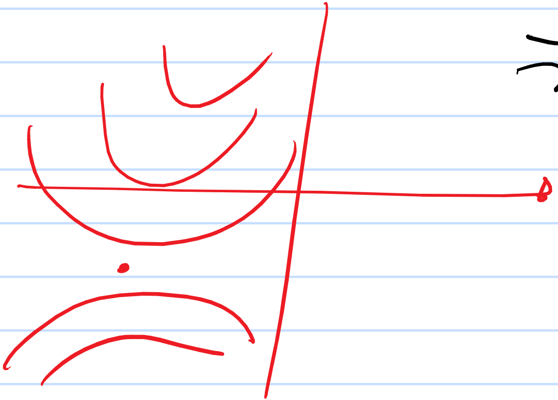
$$= \left(x+\frac{3}{2}\right)^2 - \frac{9}{4} - 2\cdot\left[\left(y+\frac{3}{2}\right)^2 - \frac{9}{4}\right] + 7$$

$$= \left(x+\frac{3}{2}\right)^2 - \frac{9}{4} - 2\cdot\left(y+\frac{3}{2}\right)^2 + \frac{9}{2} + 7$$

$$= \left(x+\frac{3}{2}\right)^2 - 2\left(y+\frac{3}{2}\right)^2 + C$$

$x^2 - 2y^2 + C_1$ $\quad \left(-\frac{3}{2}, -\frac{3}{2}\right)$





$\Rightarrow$ $\boxed{\text{Alternative a)}}$

$\boxed{2019\text{-}2}$

$$\boxed{f(x,y) = \sqrt{x^2+y^2}}$$

$$\frac{x^2+y^2}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2+y^2}} \cdot \frac{\partial}{\partial x}(x^2+y^2) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$$

$$\boxed{\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}} \qquad \boxed{\frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}}$$

$$\nabla f(1,1) = \left(\frac{1}{\sqrt{1^2+1^2}}, \frac{1}{\sqrt{1^2+1^2}}\right) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}(1,1)$$

$$\boxed{\nabla f(1,1) = \frac{1}{\sqrt{2}}(1,1)}$$

$$\theta = \frac{\pi}{4}$$

$$\vec{n} = \left(\cos\left(\frac{\pi}{4}\right), \sin\left(\frac{\pi}{4}\right)\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(1,1)$$

$$\boxed{\vec{n} = \frac{\sqrt{2}}{2}(1,1)}$$

$$\nabla f(1,1)\cdot \hat{n} = \frac{1}{\sqrt{2}}\cdot(1,1)\cdot\frac{\sqrt{2}}{2}\cdot(1,1)$$

$$= \left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{2}\right)\cdot\left(1\cdot 1+1\cdot 1\right) = \frac{1}{2}\cdot(1+1) = \frac{2}{2} = 1$$

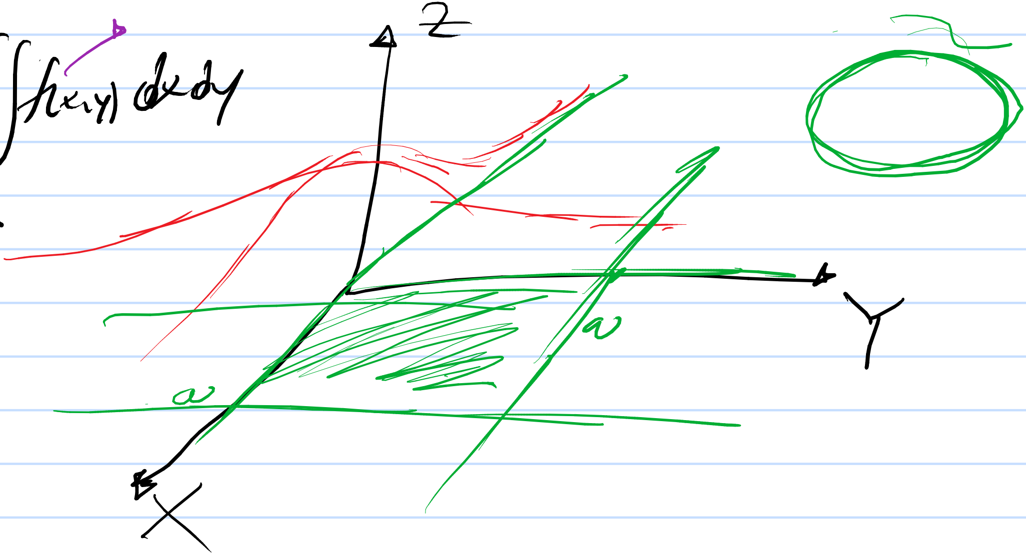
$\boxed{\text{Alternativa 2)}}$

$\boxed{2018-2}$

$$z \cdot (a^2+x^2)^{\frac{3}{2}} = a^4$$

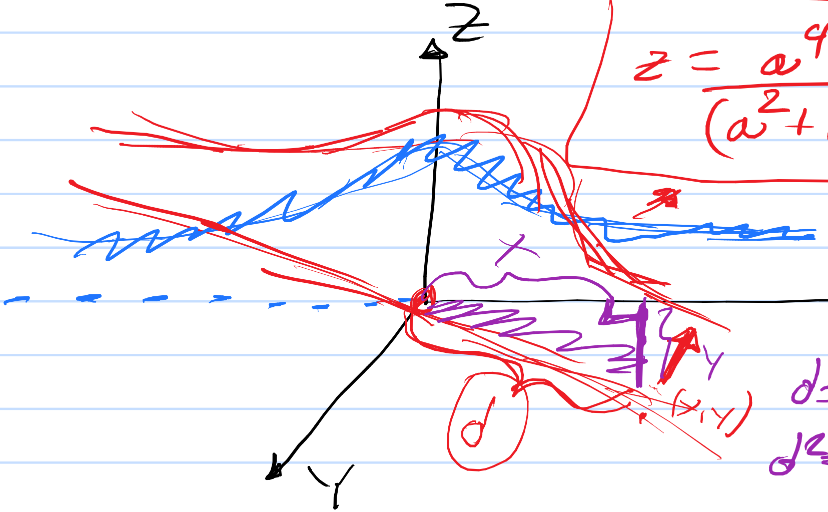
$$\Rightarrow \boxed{z = \frac{a^4}{(a^2+x^2)^{\frac{3}{2}}}}$$

$$V = \iint_{R} f(x,y)\,dx\,dy$$



$R = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le a,\ 0 \le y \le a\}$

$$\boxed{z = \frac{a^4}{(a^2+d^2)^{\frac{3}{2}}}}$$



$d = \sqrt{x^2+y^2}$

$d^2 = x^2+y^2.$

$$\boxed{f(x,y) = \frac{a^4}{(a^2+x^2+y^2)^{\frac{3}{2}}}}$$

$$V = \int_0^a \left[\underbrace{\int_0^a \frac{a^4}{(a^2+x^2+y^2)^{\frac{3}{2}}}}\, dx \right] dy$$

$$\int_0^a \frac{a^4}{(a^2+x^2+y^2)^{\frac{3}{2}}}\,dx$$

$$\boxed{b=\sqrt{a^2+y^2}}$$

$$= \int_0^a \frac{a^4}{(b^2+x^2)^{\frac{3}{2}}}\,dx$$

$$u = \frac{1}{\sqrt{b^2+x^2}} = (b^2+x^2)^{-\frac{1}{2}}$$

$$\rightarrow du = \left(-\frac{1}{2}\right)\cdot\frac{1}{(b^2+x^2)^{\frac{3}{2}}}\cdot 2x\,dx = -\frac{x}{(b^2+x^2)^{\frac{3}{2}}}\,dx$$

$$\Rightarrow \boxed{-\frac{dw}{x} = \frac{dx}{(x^2+b^2)^{\frac{3}{2}}}} \qquad w = \frac{1}{\sqrt{b^2+x^2}}$$

$$\Rightarrow w^2 = \frac{1}{b^2+x^2} \Rightarrow w^2 \cdot b^2 + w^2 \cdot x^2 = 1 \qquad w^2 \cdot x^2 = 1 - w^2 b^2$$

$$x^2 = \frac{1-w^2b^2}{w^2} \Rightarrow x = \frac{\sqrt{1-w^2b^2}}{w} \qquad -\frac{1}{x} = -\frac{w}{\sqrt{1-w^2b^2}}$$

$$-\frac{dw \cdot w}{\sqrt{1-w^2b^2}}$$

$$\boxed{-dw \cdot \frac{w}{\sqrt{1-w^2b^2}} = \frac{dx}{(x^2+b^2)^{\frac{3}{2}}}}$$

$$\boxed{t = 1-w^2b^2}$$

$$dt = -2wb^2 dw$$

$$\int \frac{dx}{(b^2+x^2)^{\frac{3}{2}}} = \int -\frac{w}{\sqrt{1-w^2b^2}} dw = -\int \frac{w\,dw}{\sqrt{1-w^2b^2}}$$

$$\Rightarrow \left(-\frac{1}{2}\right) \cdot \frac{1}{b^2} dt = w\,dw \qquad = -\int \left(-\frac{1}{2}\right) \cdot \frac{1}{b^2} dt \cdot \frac{1}{\sqrt{t}}$$

$$= \frac{1}{2}\cdot\frac{1}{b^2}\cdot\int t^{-\frac{1}{2}}\,dt = \frac{1}{2b^2}\cdot\frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$$

$$= \frac{2}{2b^2}\cdot\sqrt{t} = \frac{1}{b^2}\cdot\sqrt{t} = \frac{1}{b^2}\sqrt{1-u^2b^2}$$

$$= \frac{1}{b^2}\cdot\sqrt{1-\left(\frac{1}{\sqrt{b^2+x^2}}\right)^2\cdot b^2} = \frac{1}{b^2}\cdot\sqrt{1-\frac{b^2}{b^2+x^2}}$$

$$= \frac{1}{b^2} \cdot \sqrt{\frac{b^2+x^2-b^2}{b^2+x^2}} = \frac{1}{b^2} \cdot \sqrt{\frac{x^2}{b^2+x^2}} = \frac{x}{b^2 \cdot \sqrt{b^2+x^2}}$$

$$= \frac{x}{(a^2+y^2) \cdot \sqrt{a^2+y^2+x^2}}$$

$$\int_0^a \frac{a^4}{(a^2+x^2+y^2)^{\frac{3}{2}}} dx = a^4 \cdot \left[\frac{x}{(a^2+y^2)\sqrt{a^2+x^2+y^2}} \right] \Bigg|_{x=0}^{x=a}$$

$$= \frac{a}{(a^2+y^2)\cdot\sqrt{a^2+a^2+y^2}} \cdot a^4 = \frac{a^5}{(a^2+y^2)\cdot\sqrt{2a^2+y^2}}$$

$$\boxed{V = \int_0^a \frac{a^5}{(a^2+y^2)\cdot\sqrt{2a^2+y^2}}\,dy}$$

$$\boxed{V = a^5 \cdot \left[\int_0^a \frac{dy}{(a^2+y^2)\sqrt{2a^2+y^2}}\right]}$$

$$\int \frac{dy}{(a^2+y^2)\sqrt{2a^2+y^2}}$$

$$\boxed{u = \frac{y}{\sqrt{2a^2+y^2}}}$$

$$du = \left(\frac{\sqrt{2a^2+y^2} - y \cdot \frac{1}{2\sqrt{2a^2+y^2}} \cdot 2y}{\left[\sqrt{2a^2+y^2}\right]^2}\right) dy = \left(\frac{\sqrt{2a^2+y^2} - \frac{y^2}{\sqrt{2a^2+y^2}}}{\left(\sqrt{2a^2+y^2}\right)^2}\right) dy$$

$$= \frac{\left(\frac{2a^2+y^2}{\sqrt{2a^2+y^2}} - \frac{y^2}{\sqrt{2a^2+y^2}}\right)}{2a^2+y^2}\, dy = \frac{2a^2}{\sqrt{2a^2+y^2}\,(2a^2+y^2)}\, dy$$

$$\boxed{du = \frac{2a^2}{\sqrt{2a^2+y^2}\,(2a^2+y^2)}\, dy}$$

$$\boxed{du \cdot \frac{(2a^2+y^2)}{2a^2} = \frac{dy}{\sqrt{2a^2+y^2}}}$$

$$\left(\frac{1}{a^2+y^2}\right)$$

$$\boxed{du\,\frac{(2a^2+y^2)}{2a^2}\cdot\frac{1}{(a^2+y^2)} = \frac{dy}{\sqrt{2a^2+y^2}\,(a^2+y^2)}}$$

$$\frac{2a^2+y^2}{a^2+y^2} = \frac{a^2+(a^2+y^2)}{a^2+y^2} = \frac{a^2}{a^2+y^2}+1$$

u

$$\boxed{\left(1+\frac{a^2}{a^2+y^2}\right)\frac{1}{2a^2}\,du = \frac{dy}{\sqrt{2a^2+y^2}\,(a^2+y^2)}} \quad (*)$$

$$u = \frac{y}{\sqrt{2a^2+y^2}} \Rightarrow u^2 = \frac{y^2}{2a^2+y^2} \Rightarrow$$

$$u^2 \cdot 2a^2 + u^2 \cdot y^2 = y^2 \Rightarrow u^2 \cdot 2a^2 = y^2(1-u^2)$$

$$y^2 = \frac{u^2 \cdot 2a^2}{(1-u^2)} \quad / + a^2$$

$$a^2 \cdot u^2$$

$$y^2 + a^2 = \frac{u^2 \cdot 2a^2}{(1-u^2)} + \frac{a^2(1-u^2)}{(1-u^2)} = \frac{u^2 \cdot 2a^2 + a^2 - a^2 \cdot u^2}{(1-u^2)}$$

$$= \frac{a^2 \cdot u^2 + a^2}{(1-u^2)} \Rightarrow \boxed{y^2 + a^2 = \frac{a^2 u^2 + a^2}{(1-u^2)}}$$

$$y^2 + a^2 = \frac{a^2 \cdot (1+u^2)}{(1-u^2)} \quad \Big/ \frac{1}{a^2}$$

$$\frac{y^2+a^2}{a^2} = \frac{1+u^2}{1-u^2} \Rightarrow \frac{a^2}{y^2+a^2} = \frac{1-u^2}{1+u^2}$$

$$1+\frac{a^2}{a^2+y^2}=\frac{1-w^2}{1+w^2}+1=\overbrace{\frac{1-w^2}{1+w^2}}+\overbrace{\frac{1+w^2}{1+w^2}}=\frac{2}{1+w^2}$$

$$\frac{a^2}{a^2+y^2}+1=\frac{2}{1+w^2}$$

$$\frac{2}{(1+w^2)}$$

$$\left(1+\frac{a^2}{a^2+y^2}\right)\frac{1}{2w^2}\,dw=\frac{dy}{\sqrt{2w^2+y^2}\,(a^2+y^2)}$$

$$\frac{\cancel{2}}{(1+u^2)} \cdot \frac{1}{\cancel{2}a^2}\, du = \frac{dy}{(a^2+y^2)\sqrt{2a^2+y^2}} \quad / \int$$

$$\underbrace{\int \frac{du}{(1+u^2)\,a^2}} = \int \frac{dy}{(a^2+y^2)\sqrt{2a^2+y^2}}$$

$$\frac{1}{a^2} \cdot \int \frac{du}{(1+u^2)} = \frac{1}{a^2} \arctan(u) = \frac{1}{a^2} \cdot \arctan\left(\frac{y}{\sqrt{2a^2+y^2}}\right)$$

$$\int \frac{dy}{(a^2+y^2)\cdot\sqrt{2a^2+y^2}} = \frac{1}{a^2}\cdot \operatorname{arctg}\left(\frac{y}{\sqrt{2a^2+y^2}}\right)$$

$$V = a^5\cdot\left[\frac{1}{a^2}\cdot \operatorname{arctg}\left(\frac{y}{\sqrt{2a^2+y^2}}\right)\right]\Bigg|_{y=0}^{y=a}$$

$$a^5\cdot\left[\frac{1}{a^2}\cdot \operatorname{arctg}\underbrace{\left(\frac{a}{\sqrt{3a^2}}\right)}_{\frac{1}{\sqrt{3}}} - \cancel{\frac{1}{a^2}\cdot \operatorname{arctg}(0)}\right] \xrightarrow{\frac{\pi}{6}}$$

$$= a^5\cdot\frac{1}{a^2}\cdot \operatorname{tg}^{-1}\left(\frac{1}{\sqrt{3}}\right) = a^3\cdot\frac{\pi}{6}$$

$$\boxed{V = \frac{a^3 \cdot \pi}{6}} \Rightarrow \boxed{\text{Alternativa b)}}$$