15 SYSTEMS MODELING

SYSTEMS–1

Which of the following matrices has an inverse?

$$\mathbf{A}_1 = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 2 & 1 \end{pmatrix} \qquad \mathbf{A}_2 = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 2 & 3 \\ 0 & 1 & 1 \end{pmatrix} \qquad \mathbf{A}_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ -1 & 2 & 3 \end{pmatrix}$$

(A) $\mathbf{A}_2$ only (B) $\mathbf{A}_1$ and $\mathbf{A}_2$ (C) $\mathbf{A}_1$ and $\mathbf{A}_3$ (D) $\mathbf{A}_2$ and $\mathbf{A}_3$

If, for matrix $\mathbf{A}$, the determinant is nonzero, the inverse matrix, $\mathbf{A}^{-1}$, exists.

$$\mathbf{D}_1 = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 9 + 2 + 2 - 6 - 6 - 1 = 0$$

$\mathbf{A}_1^{-1}$ does not exist.

$$\mathbf{D}_2 = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 2 & 3 \\ 0 & 1 & 1 \end{vmatrix} = 2 + 2 - 6 - 3 = -5$$

$\mathbf{A}_2^{-1}$ exists.

$$\mathbf{D}_3 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ -1 & 2 & 3 \end{vmatrix} = 12 - 2 - 12 - 2 = -4$$

$\mathbf{A}_3^{-1}$ exists.

Only $\mathbf{A}_2$ and $\mathbf{A}_3$ have inverses.

$\boxed{\text{The answer is (D).}}$

SYSTEMS–2

An investor is considering a stock portfolio that costs \$55. If he invests in the portfolio, there is a 0.5 probability that he will receive a total revenue of \$20. If that event does not occur, he will receive a total revenue of \$100. What will be the investor's expected profit if he decides to invest?

(A) \$5 (B) \$15 (C) \$55 (D) \$60

The expected profit is found by multiplying the expected revenues by their respective probabilities, adding them, and subtracting the initial cost.

$$\text{profit} = (0.5)(\$100) + (0.5)(\$20) - \$55 = \$5$$

The answer is (A).

SYSTEMS–3

For a function of a single variable, $f(x)$, to be convex, which of the following must be true?

(A) For each pair of values of x_1 and x_2, with $0 \leq \lambda \leq 1$,

$$f\big(\lambda x_2 + (1-\lambda)x_1\big) \leq \lambda f(x_2) + (1-\lambda) f(x_1)$$

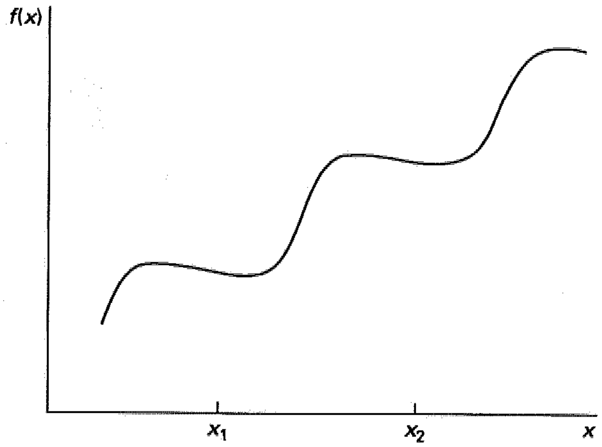
(B) For each pair of values of x_1 and x_2, with $0 \leq \lambda \leq 1$,

$$f\big(\lambda x_2 + (1-\lambda)x_1\big) \geq \lambda f(x_2) + (1-\lambda) f(x_1)$$

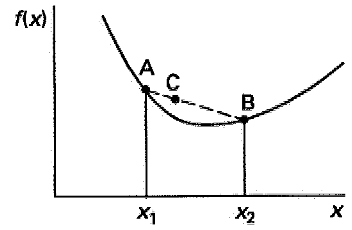
(C) For each pair of values of x_1 and x_2, with $0 \leq \lambda \leq 1$,

$$f\big(\lambda x_2 + (1-\lambda)x_1\big) = \lambda f(x_2) + (1-\lambda) f(x_1)$$

(D) Graphically, $f(x)$ is



A convex function always has a minimum value.



point A: $(x_1, f(x_1))$
point B: $(x_2, f(x_2))$
point C: $(x_1(1-\lambda)+x_2, f(x_1(1-\lambda)+x_2)$

The relationship in option (A) is the definition of a convex function, which implies that the function has a minimum value. For each pair of points A and B on the curve, the line segment joining these two points lies entirely above or on the graph of $f(x)$.

The answer is (A).

SYSTEMS–4

For a function of two variables, $f(x_1, x_2)$, and for all possible values of x_1 and x_2, which of the following conditions must exist in order for the function to be convex?

I. $\left(\frac{\partial^2 f(x_1,x_2)}{\partial x_1^2}\right)\left(\frac{\partial^2 f(x_1,x_2)}{\partial x_2^2}\right)-\left(\frac{\partial^2 f(x_1,x_2)}{\partial x_1 x_2}\right)^2 \geq 0$

II. $\frac{\partial^2 f(x_1,x_2)}{\partial x_1^2} \geq 0$

III. $\frac{\partial^2 f(x_1,x_2)}{\partial x_2^2} \geq 0$

(A) I only (B) I and II (C) I and III (D) I, II, and III

Second partial derivatives can be used to check functions of more than one variable to see if they are convex or concave. For two-variable functions, the partial derivatives in I, II, and III make up the determinant of the 2×2 Hessian matrix, which should be greater than or equal to zero. Thus, all three conditions must exist for the function to be convex.

The answer is (D).

SYSTEMS–5

If a function of n variables, $f(x_1, \ldots, x_n)$, is convex, which of the following is true about its $n \times n$ Hessian matrix?

(A) It is semidefinite.
(B) It is negative semidefinite.
(C) It is positive semidefinite.
(D) It is indefinite.

A positive semidefinite Hessian matrix implies two conditions.

1. For all values of x, the function $f(x_1, \ldots, x_n) \geq 0$.
2. There is at least one set of nonzero values of $x_1, \ldots, x_n$ such that $f(x_1, \ldots, x_n) = 0$.

These conditions are met when the determinant of the Hessian matrix is greater than or equal to zero, which occurs if and only if the function f is convex. Thus (C) is the correct answer.

The answer is (C).

SYSTEMS–6

Which of the following statements about linear programming is FALSE?

(A) In mathematical notation, linear programming problems are often written in the following form.

optimize:

$$Z = \sum_j C_j x_j$$

subject to the constraints:

$$\sum_i \sum_j a_{ij} x_j \leq b_i$$

($x_j \geq 0$, and a_{ij}, b_i, and C_j are constants.)

(B) Linear programming uses a mathematical model composed of a linear objective function and a set of linear constraints in the form of inequalities.

(C) The decision variables have physical significance only if they have integer values. The solution procedure yields integer values only.

(D) The simplex method is a technique used to solve linear programming problems.

By definition, $x_j \geq 0$ implies noninteger as well as integer values for the decision variable. Although it is sometimes the case that only integer values of the decision variables have physical significance, the solution procedure does not necessarily yield integer values.

The answer is (C).

SYSTEMS–7

Consider a nontrivial linear programming problem in one variable, x, with only lower- and upper-bound constraints on x. At optimum, where will x be in relation to these constraints?

(A) at its upper bound
(B) at its lower bound
(C) between its upper and lower bounds
(D) at its upper or lower bound

The constraints prevent the variable of a linear program from increasing or decreasing forever during maximization or minimization. The maximum or minimum will occur at either the upper or lower bound.

The answer is (D).

SYSTEMS–8

If all variables in a linear programming problem are restricted to be integers, which, if any, basic assumption of linear programming is violated?

(A) certainty (B) additivity (C) divisibility (D) proportionality

Divisibility implies that fractional levels of the decision variables must be permissible. By restricting all variables in the problem to be integers, divisibility is lost.

The answer is (C).

SYSTEMS–9

If a project that has diminishing returns with scale is modeled using a linear program, which basic assumption of linear programming will be violated?

(A) certainty (B) additivity (C) divisibility (D) proportionality

Proportionality assumes that a variable multiplied by a constant is equal to the contribution, regardless of scale.

The answer is (D).

SYSTEMS–10

Consider the following linear programming model.

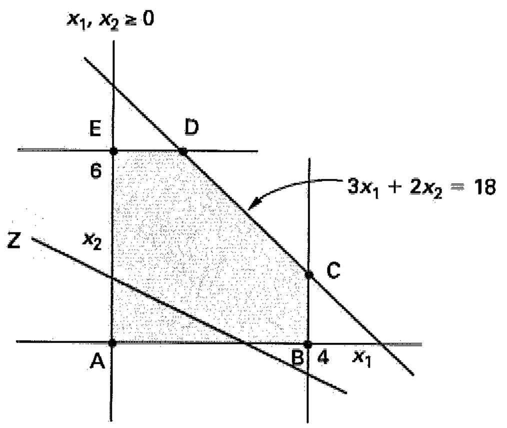
maximize:

$$Z = 3x_1 + 5x_2$$

subject to the constraints:

$$\begin{aligned} 3x_1 + 2x_2 &\leq 18 \\ x_1 &\leq 4 \\ x_2 &\leq 6 \end{aligned}$$

The graphical solution is



At what point does the optimum solution occur?

(A) A (B) B (C) C (D) D

If the objective function Z is plotted and moved along within the feasible region, the last point of contact will be point D. Therefore, Z will be maximized while satisfying all constraints at point D.

The answer is (D).

SYSTEMS–11

For which of the following linear programming problems can an optimum solution be found?

I. maximize:

$$Z = 20x + 10y$$

subject to the constraints:

$$\begin{aligned} x + y &\leq 4 \\ 3x + y &\leq 6 \\ x, y &\geq 0 \end{aligned}$$

II. maximize:

$$Z = 20x + 10y$$

subject to the constraints:

$$\begin{aligned} x + y &\geq 4 \\ 3x + 2y &\leq 6 \\ x, y &\geq 0 \end{aligned}$$

III. maximize:

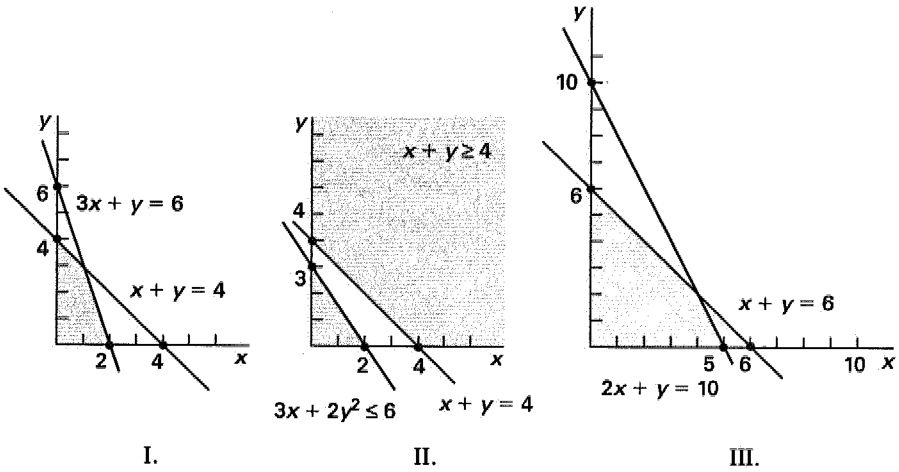
$$Z = 20x + 10y$$

subject to the constraints:

$$\begin{aligned} 2x + y &\leq 10 \\ x + y &\leq 6 \\ x, y &\geq 0 \end{aligned}$$

(A) I only (B) I and II (C) I and III (D) II and III

For an optimum solution to exist, there must be a feasibility region. The graphs of the feasibility regions I, II, and III are as shown.



For II, there is no region where all four conditions are met. I and III have feasibility regions and, therefore, have optimum solutions.

$$\boxed{\text{The answer is (C).}}$$

SYSTEMS–12

Which of the following linear programming problems have multiple optimum points that yield the same optimum solution?

I. maximize:

$$Z = 20x + 10y$$

subject to the constraints:

$$\begin{aligned} x + y &\leq 4 \\ 3x + y &\leq 6 \\ x, y &\geq 0 \end{aligned}$$

II. maximize:

$$Z = 20x + 10y$$

subject to the constraints:

$$\begin{aligned} x + y &\geq 4 \\ 3x + 2y &\leq 6 \\ x, y &\geq 0 \end{aligned}$$

III. maximize:

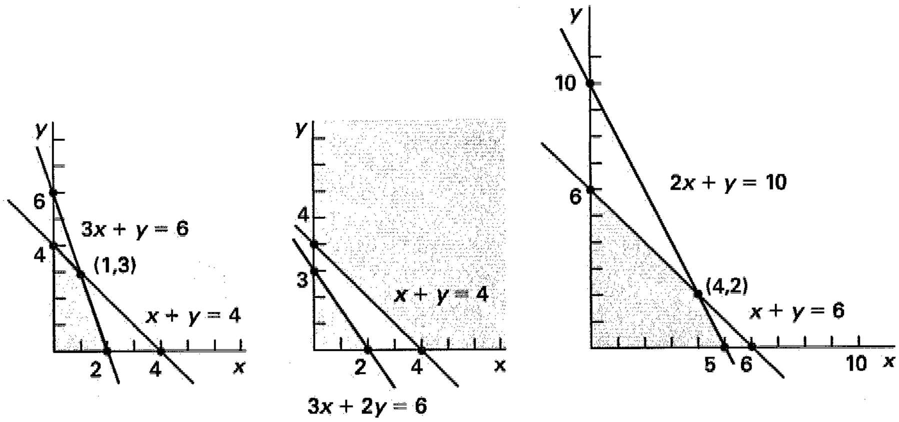
$$Z = 20x + 10y$$

subject to the constraints:

$$\begin{aligned} 2x + y &\leq 10 \\ x + y &\leq 6 \\ x, y &\geq 0 \end{aligned}$$

(A) I only (B) II only (C) III only (D) I and III

The graphs of I, II, and III are as shown.



I. II. III.

For I, the optimum solution is at the point (1,3) where $Z = 20+30 = 50$; this solution has a unique optimum point. II has no feasibility region and, thus, has no optimum solution. For III, the points (4,2) and (5,0) both yield the optimum solution $Z = 100$; III is the only choice with multiple optimum points. In fact, any point on the line segment adjoining (5,0) and (4,2) will yield the optimum solution $Z = 100$.

The answer is (C).

SYSTEMS–13

What is the maximum value of Z for the following integer linear programming problem? (x and y are integers.)

maximize:

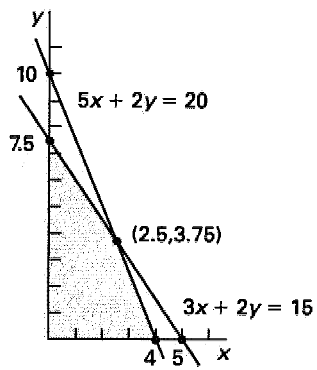
$$Z = 6x + 5y$$

subject to the constraints:

$$\begin{aligned} 5x + 2y &\leq 20 \\ 3x + 2y &\leq 15 \\ x, y &\geq 0 \\ x, y &\quad \text{integers} \end{aligned}$$

(A) 28 (B) 33 (C) 35 (D) 36

The maximum Z value is found from the extreme points in the illustration.



extreme point	Z
(0,0)	0
(4,0)	24
(0,7.5)	37.5
(2.5,3.75)	33.75

The largest value of Z over all real numbers is given at (0,7.5). Since x and y are integers, the largest Z value will be given at either $x = 0$ and $y = 7$, or $x = 1$ and $y = 6$. The first combination gives $Z = 35$, but the second gives $Z = 36$.

The answer is (D).

SYSTEMS–14

The simplex method is extremely efficient in solving which of the following classic problems?

(A) the transportation problem
(B) the assignment problem
(C) the transhipment problem
(D) the allocation problem

Theoretically, all problem categories can be solved using the simplex method. However, the assignment, transportation, and transhipment problems are numerically inefficient when solved by the general simplex method. For allocation problems, the variables are continuous and, therefore, manageable enough in size to be solved using the simplex method.

The answer is (D).

SYSTEMS–15

Upon which of the following properties of linear programming is the simplex method based?

I. The collection of feasible solutions constitutes a convex set.
II. If a feasible solution exists, a basic solution exists where the feasible solutions correspond to the extreme points of the set of feasible solutions.
III. Only a finite number of basic feasible solutions exist.
IV. If the objective function possesses a finite maximum, at least one optimum solution is a basic feasible solution.

(A) I only
(B) IV only
(C) I and II
(D) I, II, III, and IV

The simplex method is based upon all of the given linear programming properties.

The answer is (D).

SYSTEMS–16

As the simplex algorithm progresses from one solution to the next in a linear programming maximization problem, what will happen to the value of the objective function?

(A) It will increase and then decrease.
(B) It will decrease and then increase.
(C) It will increase or stay the same.
(D) It will decrease or stay the same.

A characteristic of the simplex algorithm is that the value of the objective function improves (does not worsen) between iterations. For a maximization problem, only option (C) can be true.

The answer is (C).

SYSTEMS–17

In the following simplex tableau, the first row of numbers represents the objective function of a maximization problem, and subsequent rows represent constraints. What are the basic variables?

x_1	x_2	x_3	x_4	x_5	right side
−10	−6	0	0	0	0
1	0	1	0	0	3
0	1	0	1	0	15
2	3	0	0	1	17

(A) x_1 and x_2
(B) x_1, x_4, and x_5
(C) x_2, x_4, and x_5
(D) x_3, x_4, and x_5

The system of equations represented by the tableau implies that $Z = 10x_1 + 6x_2$, and

$$\mathbf{A}x = \begin{pmatrix} 3 \\ 15 \\ 17 \end{pmatrix}$$

Given the negative coefficients of the objective function, Z, the values of the variables must be $x_1 = 0$, $x_2 = 0$, $x_3 = 3$, $x_4 = 15$, and $x_5 = 17$. x_3, x_4, and x_5 are "used" by the columns of the basis and are, therefore, the basic variables.

The answer is (D).

SYSTEMS–18

After several iterations, the following simplex tableau is developed.

	Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	right side
Z	1	−4	6	0	0	−3	1	0	14
x_4	0	7	2	0	1	−2	−6	0	3
x_7	0	6	2.5	0	0	4	0.8	1	12
x_3	0	3	3.5	1	0	2.5	5	0	2

Which of the following describes the solution found?

(A) It is feasible but not optimum.
(B) It is optimum but not feasible.
(C) It is optimum and feasible.
(D) It is neither optimum nor feasible.

The solution is feasible because there are no negative numbers on the right side. It is not, however, an optimum solution since there are negative numbers in the objective function.

The answer is (A).

SYSTEMS–19

In the following simplex tableau, the first row of numbers represents the objective function of a maximization problem. What are the current values of x_1 and x_2?

x_1	x_2	x_3	x_4	x_5	right side
−7.5	0	−4.5	0	0	0
1	1	0	0	0	3
0	0	1	1	0	15
2	0	3	0	1	17

(A) $x_1 = 0$, $x_2 = 0$
(B) $x_1 = 0$, $x_2 = 3$
(C) $x_1 = 2$, $x_2 = 15$
(D) $x_1 = 3$, $x_2 = 3$

Since the coefficients of x_1 and x_3 in the objective function are negative, x_1 and x_3 must be zero. The tableau represents a system of equations with the following solution: $x_1 = 0$, $x_2 = 3$, $x_3 = 0$, $x_4 = 15$, and $x_5 = 17$.

The answer is (B).

SYSTEMS–20

In the following simplex tableau, identify the current entering and exiting basic variables.

	Z	x_1	x_2	x_3	x_4	x_5	x_6	right side
Z	1	4	−6	−1	0	0	0	0
x_4	0	−3	2	−3	1	0	0	10
x_5	0	4	−8	3	0	1	0	24
x_6	0	7	3	6	0	0	1	30

(A) x_2 entering, x_4 exiting
(B) x_2 entering, x_6 exiting
(C) x_3 entering, x_5 exiting
(D) x_3 entering, x_4 exiting

The first row of the tableau gives

$$Z = -4x_1 + 6x_2 + x_3$$

x_2 will cause the greatest increase in Z when it goes from 0 to 1. The entering variable is, therefore, x_2. With $x_1 = x_3 = 0$, the x_4 row gives

$$2x_2 + x_4 = 10$$
$$x_2 = 5$$
$$x_4 = 0$$

The x_6 row gives

$$3x_2 + x_6 = 30$$
$$x_2 = 10$$
$$x_6 = 0$$

The exiting variable is the one that goes to zero first as x_2 is increased from zero. The exiting variable is x_4.

The answer is (A).

SYSTEMS–21

In the following simplex tableau, the first row of numbers represents the objective function of a maximization problem. What will be the next variable to enter the basis?

x_1	x_2	x_3	x_4	x_5	right side
−5	−3	0	0	0	0
1	0	1	0	0	3
0	1	0	1	0	15
2	3	0	0	1	17

(A) x_1 (B) x_2 (C) x_3 (D) none of the above

If the first row represents the objective function of a maximization problem, then the tableau is already optimum, since for a maximization problem a tableau is optimum when all $C_i \leq 0$. If this had been the tableau for a minimization problem, x_1 would have been the next variable to enter the basis.

The answer is (D).

SYSTEMS–22

Find the "pivot" value in the following simplex tableau.

	x_1	x_2	u	v	
u	3	2	1	0	7
v	7	5	0	1	12
	−50	−40	0	0	

(A) 2 (B) 3 (C) 5 (D) 7

The first step in the method of pivot searching is to select the pivotal column by determining the column with the most negative entry in the objective row. In this problem it is the x_1 column.

Next, find the pivotal row by dividing each row's rightmost value by that row's pivotal column value. The row that has the lower quotient is the pivotal row. The quotient for row 1 is $^7/_3$, and for row 2 it is $^{12}/_7$. Thus, the second row is the pivotal row.

Finally, the pivot is the value that is at the intersection of the pivotal row and column. For this problem, it is 7.

The answer is (D).

SYSTEMS–23

Find the optimum value of the slack variable x_3.

	x_1	x_2	x_3	x_4	x_5	
x_3	1	−1	1	0	0	2
x_4	2	1	0	1	0	4
x_5	−3	2	0	0	1	6
	−5	−3	0	0	0	0

(A) 7/38 (B) 36/7 (C) 38/7 (D) 39/7

The simplex tableau becomes

	x_1	x_2	x_3	x_4	x_5	
x_3	1	−1	1	0	0	2
x_4	2	1	0	1	0	4
x_5	−3	2	0	0	1	6
	−5	−3	0	0	0	0

	x_1	x_2	x_3	x_4	x_5	
x_1	1	−1	1	0	0	2
x_4	0	3	−2	1	0	0
x_5	0	−1	3	0	1	12
	0	−8	5	0	0	10

	x_1	x_2	x_3	x_4	x_5	
x_1	1	0	$\frac{1}{3}$	$\frac{1}{3}$	0	2
x_2	0	1	$-\frac{2}{3}$	$\frac{1}{3}$	0	0
x_5	0	0	$\frac{7}{3}$	$\frac{1}{3}$	1	12
	0	0	$-\frac{1}{3}$	$\frac{8}{3}$	0	10

	x_1	x_2	x_3	x_4	x_5	
x_1	1	0	0	$\frac{2}{7}$	$-\frac{1}{7}$	$\frac{2}{7}$
x_2	0	1	0	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{24}{7}$
x_3	0	0	1	$\frac{1}{7}$	$\frac{3}{7}$	$\frac{36}{7}$
	0	0	0	$\frac{59}{21}$	$\frac{1}{7}$	$\frac{82}{7}$

The slack variables are, therefore, $x_3 = {}^{36}/_{7}$, $x_4 = 0$, and $x_5 = 0$. The optimum value is ${}^{36}/_{7}$.

The answer is (B).

SYSTEMS–24

The heights of several thousand fifth grade boys in Santa Clara County were measured. It was found that the mean of the height was 1.20 m and the variance was 25×10^{-4} m^2. Approximately what percentage of these boys has a height greater than 1.23 m?

(A) 27% (B) 31% (C) 69% (D) 73%

To convert the normal distribution to unit normal distribution, the new variable, z, is constructed from the height h, mean μ, and variance σ^2.

$$z = \frac{h - \mu}{\sigma}$$

For a height greater than 1.23 m,

$$z = \frac{1.23 \text{ m} - 1.20 \text{ m}}{\sqrt{25 \times 10^{-4}} \text{ m}} = 0.6$$

From a unit normal distribution table, the cumulative distribution function at $z = 0.6$ is 0.726. Therefore, the percentage of boys having height greater than 1.23 m is

$$\text{percentage taller than 1.23 m} = 100\% - 72.6\% = 27.4\%$$

The answer is (A).

SYSTEMS–25

Using the simplex method, in what form would one write the given objective function in order to maximize it?

$$Z = |x_1| - 3x_2$$

(A) $Z = x_1^+ + x_1^- - 3x_2$; subject to $x_1^+, x_1^- \geq 0$
(B) $Z - x_1^+ + x_1^- + 3x_2 = 0$; subject to $x_1^+, x_1^- \geq 0$
(C) $Z + x_1^+ - x_1^- - 3x_2 = 0$; subject to $x_1^+, x_1^- \geq 0$
(D) $Z - |x_1| + 3x_2 = 0$; subject to $x_1 \geq 0$

The absolute value term is written as $x_1^+ - x_1^-$, with x_1^+ and x_1^- greater than or equal to zero. The proper form for the simplex method is to have all terms on the same side of the equal sign equal to zero.

The answer is (B).

SYSTEMS–26

One constraint for a linear program is as follows: $3x_1 - 2x_2 + 4x_3 \geq 6$. What is the proper form of this constraint for use in the simplex method?

(A) $3x_1 - 2x_2 + 4x_3 - 6 = 0$
(B) $3x_1 - 2x_2 + 4x_3 + 6 \leq 0$
(C) $3x_1 - 2x_2 + 4x_3 + x_4 \geq -6$
(D) $3x_1 - 2x_2 + 4x_3 + x_4 = 6$

The proper form for a constraint uses a slack variable to account for the inequality. For this problem, the slack variable x_4 is added.

The answer is (D).

SYSTEMS–27

How would the following problem be written if the simplex solution method is to be used?

maximize:

$$Z = 12x_1 - 4x_2$$

subject to the constraints:

$$x_1 + 3x_2 = 42$$
$$x_1, x_2 \geq 0$$

(A) $Z = 12x_1 - 4x_2$; $x_1 + 3x_2 - 42 = 0$
(B) $Z - 12x_1 + 4x_2 = 0$; $x_1 + 3x_2 + x_3 = 42$
(C) $Z - 12x_1 + 4x_2 + Mx_3 = 0$; $x_1 + 3x_2 + x_3 = 42$; M is some large number.
(D) $Z - 12x_1 + 4x_2 = -Mx_3$; $x_1 + 3x_2 - 42 = \bar{x}_3$

The slack variable x_3 is added to the objective function, multiplied by a constant, M. The restriction becomes $x_1 + 3x_2 + x_3 = 42$. Although option (D) has all the correct terms, they are not in the proper position.

$\boxed{\text{The answer is (C).}}$

SYSTEMS–28

Consider the following linear programming problem.

maximize:

$$Z = 6x_1 + 5x_2$$

subject to the constraints:

$$\begin{aligned} x_1 + x_2 &\leq 4 \\ 5x_1 + 3x_2 &\leq 15 \\ x_1, x_2 &\geq 0 \end{aligned}$$

If $x_1 + x_2 + x_3 = 4 + \Delta b$, find the range of Δb over which the basis remains unchanged.

(A) $-1 \leq \Delta b \leq 1$ (B) $0 \leq \Delta b \leq 1$ (C) $-2 \leq \Delta b \leq 1$ (D) $-2 \leq \Delta b \leq 2$

Using the slack variables x_3 and x_4, the constraints become

$$\begin{aligned} x_1 + x_2 + x_3 &= 4 \\ 5x_1 + 3x_2 + x_4 &= 15 \end{aligned}$$

The simplex tableaus are shown.

	x_1	x_2	x_3	x_4	
x_3	1	1	1	0	4
x_4	5	3	0	1	15
	−6	−5	0	0	0

	x_1	x_2	x_3	x_4	
x_3	0	$\frac{2}{5}$	1	$-\frac{1}{5}$	1
x_1	1	$\frac{3}{5}$	0	$\frac{1}{5}$	3
	0	$-\frac{7}{5}$	0	$\frac{6}{5}$	18

	x_1	x_2	x_3	x_4	
x_2	0	1	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$
x_1	1	0	$-\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$
	0	0	$\frac{7}{2}$	$\frac{1}{2}$	$\frac{43}{2}$

For the basis to remain unchanged, Z must also remain unchanged. Thus, for Z to remain at $^{43}/_{2}$,

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}$$

$$x' = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \Delta b \begin{pmatrix} \frac{5}{2} \\ -\frac{3}{2} \end{pmatrix} \geq 0$$

This gives

$$\frac{5}{2} + \Delta b \left(\frac{5}{2}\right) \geq 0$$

$$\Delta b \geq -1$$

$$\frac{3}{2} - \Delta b \left(\frac{3}{2}\right) \geq 0$$

$$\Delta b \leq 1$$

Therefore, $-1 \leq \Delta b \leq 1$.

$\boxed{\text{The answer is (A).}}$

SYSTEMS–29

Which of the following statements is INCORRECT for the primal linear programming problem in the form given?

maximize:

$$Z_x = \sum_j C_j x_j$$

subject to the constraints:

$$\sum_j \sum_i a_{ij} x_j \le b_i$$

$$x_j \ge 0$$

(A) The dual problem is

minimize:

$$Z_y = \sum_i b_i y_i$$

subject to the constraints:

$$\sum_j \sum_i a_{ij} y_i \ge C_j$$

$$y_i \ge 0$$

(B) The dual problem is the same as in option (A), but with the inequality signs reversed.

(C) y_i is unrestrictive in sign if the inequality signs in the primal problem are replaced by equality signs.

(D) x_j is unrestrictive in sign if the inequality signs in the dual problem are replaced by equality signs.

By definition, the dual of a primal linear programming problem is exactly the reverse of the primal, including the reversal of the inequality signs.

$\boxed{\text{The answer is (B).}}$

SYSTEMS–30

For the following problem, what are the constraints of the dual problem?

maximize:

$$Z = 6x_1 + 3x_2 + 4x_3$$

subject to the constraints:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &\leq 12 \\ x_1 + 4x_2 + 3x_3 &= 15 \\ x_1, x_3 &\geq 0 \end{aligned}$$

The dual problem statement is

minimize:

$$Z' = 12w_1 + 15w_2$$

(A)
$$\begin{aligned} w_1 + w_2 &= 6 \\ 2w_1 + 4w_2 &\geq 3 \\ 3w_1 + 3w_2 &\geq 4 \\ w_1, w_2 &\geq 0 \end{aligned}$$

(B)
$$\begin{aligned} w_1 + w_2 &\geq 6 \\ 2w_1 + 4w_2 &\geq 3 \\ 3w_1 + 3w_2 &= 4 \\ w_2 &\geq 0 \end{aligned}$$

(C)
$$\begin{aligned} w_1 + w_2 &\geq 6 \\ 2w_1 + 4w_2 &= 3 \\ 3w_1 + 3w_2 &\geq 4 \\ w_1 &\geq 0 \end{aligned}$$

(D)
$$\begin{aligned} w_1 + w_2 &\geq 6 \\ 2w_1 + 4w_2 &\geq 3 \\ 3w_1 + 3w_2 &= 4 \\ w_1, w_2 &\geq 0 \end{aligned}$$

Each of the constraints, C_i, in the primal problem corresponds to a respective variable, w_i, in the dual problem. The coefficients of the objective function in the primal problem are the constants on the right-hand side of the constraints in the dual problem. The coefficients of the ith constraint in the primal problem are the coefficients of the ith variable in the dual problem. If the jth variable in the primal problem is restricted, the jth constraint in the dual problem is an inequality. If the primal constraint is an equality, then the corresponding variable will be unrestricted in the dual problem.

Thus, w_2 is unrestricted in the dual problem, eliminating options (A), (B), and (D). In the dual problem, if a variable w_i is unrestricted, the ith constraint is an equality. If the variable is restricted, the ith constraint is an inequality. Therefore, $2w_1 + 4w_2 = 3$.

The answer is (C).

SYSTEMS–31

The mathematical model for the classic transportation problem is as follows.

Find $x_{ij} (i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$ in order to maximize

$$\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$$

subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i \quad [i = 1, \ldots, m]$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad [j = 1, \ldots, n]$$

$$x_{ij} \geq 0$$

Which of the following is FALSE?

(A) m can be regarded as the number of factories supplying n warehouses with a certain product.

(B) a_i is the number of units produced at factory i, while b_j is the number of units required for delivery to warehouse j.

(C) C_{ij} is the shipping cost from factory i to warehouse j. x_{ij} is the decision variable, the amount shipped from factory i to warehouse j.

(D) x_{ij} has physical significance only for noninteger values.

A transportation problem has physical significance only when decision variables are integers.

$\boxed{\text{The answer is (D).}}$

SYSTEMS–32

Which of the following describes the optimum solution to a transportation problem?

(A) It can be determined using the simplex algorithm.

(B) It cannot be found if there are no upper-bound constraints on supplies from several sources.

(C) It can only be found if there are no upper bounds on supplies from several sources.

(D) It is trivial if the demand is unstable.

Transportation problems are special linear programming problems that allow for the presence or lack of upper bounds on variables and assume constant demand. Only option (A) is true.

The answer is (A).

SYSTEMS–33

How must the following cost and requirements table for a transportation problem be altered so that linear programming methods can be used to find an optimum solution?

	destination				
source	A	B	C	D	supply
1	C_{1A}	C_{1B}	$\cdots$		4
2	C_{2A}	C_{2B}			6
3	$\vdots$				2
	6	2	7	3	
	demand				

(A) A dummy source must be added to supply six units.
(B) A dummy destination must be added to increase the demand by six units.
(C) The sum of the costs in each row must be made equal by inclusion of a dummy cost.
(D) The sum of the costs in each column must be made equal by inclusion of a dummy cost.

For the model to have a feasible solution, the supply and demand must be equal. A dummy source must be created which will supply an additional six units.

The answer is (A).

SYSTEMS–34

How many basic variables are there for the following transportation problem cost and requirements table?

	destination				
source	A	B	C	D	supply
1	C_{1A}	C_{1B}	$\cdots$		s_1
2	C_{2A}	C_{2B}			s_2
3	$\vdots$				s_3
	d_1	d_2	d_3	d_4	
	demand				

(A) three (B) four (C) six (D) seven

The number of basic variables is equal to the number of sources plus the number of destinations minus one. Thus, there are $3 + 4 - 1 = 6$ basic variables.

The answer is (C).

SYSTEMS–35

The *northwest corner rule* is to be used to find an initial solution to the following transportation simplex problem. What is the value of the fourth basic variable?

	destination					
source	A	B	C	D	E	supply
1						20
2						30
3						40
4						30
	40	20	10	30	20	
	demand					

(A) 10 (B) 20 (C) 30 (D) 40

The procedure under the *northwest corner rule* for obtaining an initial basic feasible solution is as follows.

1. Start with the cell in the upper left-hand corner.
2. Allocate the maximum feasible amount.
3. If there is supply remaining, move one cell to the right. If there is no remaining supply, move one cell down. Stop when it is impossible to do either of these. Repeat the process beginning at step 2 for the new cell. Each new cell represents a new basic variable.

Carrying out this procedure for the given problem results in the following table.

	destination					
source	*A*	*B*	*C*	*D*	*E*	supply
1	20					20
2	20	10				30
3		10	10	20		40
4				10	20	30
	40	20	10	30	20	
	demand					

The fourth basic variable is equal to 10.

> The answer is (A).

SYSTEMS–36

Four technicians—Tom, Scott, Ed, and Jeri—are each assigned a project on which to work. The costs for each technician to complete each project are estimated as follows.

	project			
	1	2	3	4
Tom	10	14	15	12
Scott	9	13	17	10
Ed	8	12	14	11
Jeri	12	15	12	12

What is the optimum project assignment scheme such that all projects are completed at the minimum cost? (The order of technicians listed in the answer choices corresponds to project 1, project 2, project 3, and project 4.)

(A) Tom, Scott, Ed, Jeri
(B) Scott, Tom, Jeri, Ed
(C) Ed, Jeri, Scott, Tom
(D) Ed, Tom, Jeri, Scott

The cost matrix can be reduced by subtracting any constant from a row, as long as the row entries remain greater than or equal to zero. Subtract eight from each row. The matrix is then

	job			
	1	2	3	4
Tom	2	6	7	4
Scott	1	5	9	2
Ed	0	4	6	3
Jeri	4	7	4	4

Thus, for minimum cost, Ed should do job 1, and Scott should do job 4. Jeri should do job 3, and Tom should do job 2. The correct order is Ed, Tom, Jeri, and Scott.

There is another assignment scheme that will result in the same minimum cost: Tom, Ed, Jeri, Scott. However, this is not one of the options.

The answer is (D).

SYSTEMS–37

If an integer programming problem is solved as a linear programming problem and the resulting values of the decision variable are rounded off, which of the following will result?

(A) an optimum integer solution
(B) a noninteger solution that is optimum
(C) an integer solution that may be optimum, or close to it
(D) an integer solution that is not optimum

After rounding off, the solution will be an integer. However, it will no longer be an exact solution. Thus, it may provide an optimum solution or only one that is close to optimum.

The answer is (C).

SYSTEMS–38

Which of the following is NOT a good application of network analysis?

(A) electrical engineering
(B) information theory
(C) the study of transportation systems
(D) inventory theory

Inventory problems are generally not solved using network analysis. Network analysis involves maximizing the flow through a network connecting a source and a destination. Inventory theory involves the optimization of the problem of stocking goods; it is not concerned with the flow through a network.

The answer is (D).

SYSTEMS–39

For which of the following is the Program Evaluation and Review Technique (PERT) NOT used?

(A) construction projects
(B) computer programming assignments
(C) preparation of bids and proposals
(D) queueing problems

PERT is used to predict the completion time for large projects. All of the choices given except option (D) are projects that have a finite completion time.

The answer is (D).

SYSTEMS–40

Identify the FALSE statement.

(A) The primary objective of PERT is to determine the probability of meeting specified deadlines.

(B) PERT identifies the activities that are most likely to "bottleneck" and the activities that are most likely to stay on schedule.

(C) PERT evaluates the sensitivity to changes in the program.

(D) To apply PERT, one should develop a network representation of the project plan.

Linear programming automatically performs a sensitivity analysis of the variables as a by-product of the solution process. PERT, however, cannot provide a similar sensitivity analysis to evaluate the effect of changes in the program parameters.

The answer is (C).

SYSTEMS–41

What are the basic features of dynamic programming problems?

I. The problem can be divided into stages with a policy decision required at each stage.

II. Each stage has a number of states associated with it.

III. The effect of the policy decisions at each stage is to transform the current state into a state associated with the next stage.

IV. The problem formulation is dependent on the probability distribution associated with it.

(A) I only (B) IV only (C) I and II (D) I, II, and III

Statement IV is irrelevant. There may not be a probability distribution when deterministic problems are solved using dynamic programming. The formulation of the problem depends only on the first three statements.

The answer is (D).

SYSTEMS–42

Which of the following statements about dynamic programming is FALSE?

(A) Dynamic programming is a mathematical technique that is often useful for making a sequence of interrelated decisions.

(B) Dynamic programming provides a systematic procedure for determining the combination of decisions that maximize overall effectiveness.

(C) Dynamic programming can be represented in standard mathematical formulation.

(D) Dynamic programming is a conceptual approach to problem solving.

Dynamic programming is a conceptual approach to problem solving, not a mathematical one. Its formulation depends on the specifics of the problem.

The answer is (C).

SYSTEMS–43

Queueing theory provides a large number of alternative mathematical models for describing which of the following?

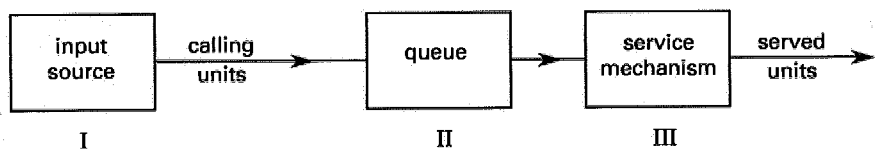
(A) network problems
(B) probabilistic arrivals
(C) probabilistic service facilities
(D) waiting line problems

Queueing theory involves the mathematical study of waiting lines or "queues."

The answer is (D).

SYSTEMS–44

The various elements of the queueing process are depicted as follows.



Which elements of the figure make up the queueing system?

(A) I only (B) II only (C) III only (D) II and III

The queue (II) and the service mechanism (III) make up the actual queueing system.

The answer is (D).

SYSTEMS–45

In a queueing process of customers in a store, what statistical pattern will most likely describe the arrival of customers over time?

(A) the normal law of probability
(B) the Poisson distribution
(C) the uniform law of probability
(D) the exponential distribution

The common assumption is that calling units in a queueing process arrive according to a Poisson distribution.

The answer is (B).

SYSTEMS–46

In a queueing process of customers in a store, what type of distribution most likely governs the time between consecutive arrivals of customers?

(A) a normal probability distribution
(B) an exponential distribution
(C) a uniform probability distribution
(D) a Poisson distribution

The "interarrival time" is commonly assumed to be exponentially distributed.

The answer is (B).

SYSTEMS–47

The jobs to be performed by a particular machine arrive according to a Poisson input process with a mean rate of 1 per hour. If the machine breaks down and requires 2 h to be repaired, what is the probability that the number of new jobs that arrive during the 2 h period is zero?

(A) e^{-2} (B) e^{-1} (C) 1 (D) e

For a Poisson distribution, the probability, p, of x jobs arriving in a time, t, is given by

$$p\{x\} = \lambda t^x \left(\frac{e^{-\lambda t}}{x!}\right)$$

λ is the mean arrival rate. Therefore,

$$\begin{aligned} p\{x\} &= \left(1\ \frac{\text{job}}{\text{h}}\right)(2\ \text{h})^0 \left(\frac{e^{-(1/\text{h})(2\ \text{h})}}{0!}\right) \\ &= e^{-2} \end{aligned}$$

The answer is (A).

SYSTEMS–48

In a queueing system that has an arrival rate of 5 customers/h, the expected waiting time for any customer in the system, including service time, is 40 min. What is the expected number of customers in the system under steady-state conditions?

(A) 5/40 customers (B) 2/15 customers (C) 10/3 customers (D) 8 customers

Little's formula states

$$L = \lambda W$$

L is the expected number of customers in the system, λ is the mean arrival rate of customers per hour, and W is the expected waiting time for each customer. Thus,

$$\begin{aligned} L &= \left(5\ \frac{\text{customers}}{\text{h}}\right)\left(\frac{2}{3}\ \text{h}\right) \\ &= 10/3 \text{ customers} \end{aligned}$$

The answer is (C).

SYSTEMS–49

Consider a queueing system with three servers, such that the mean service rate for each busy server is 2 customers/h. If the mean arrival rate of customers is 5/h, what is the expected fraction of total time that all servers will be busy? Assume steady-state conditions.

(A) $^2/_5$ (B) $^3/_5$ (C) $^2/_3$ (D) $^5/_6$

The expected fraction of time that all servers will be busy is

$$\begin{aligned}\text{fraction of } t &= (\text{arrival rate})\left(\frac{1}{(\text{no. of servers})(\text{service rate})}\right)\\ &= \left(5\ \frac{\text{customers}}{\text{h}}\right)\left(\frac{1}{(3)\left(2\ \frac{\text{customers}}{\text{h}}\right)}\right)\\ &= 5/6\end{aligned}$$

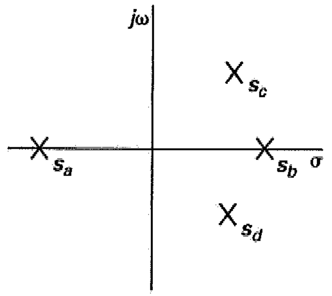
The answer is (D).

SYSTEMS–50

Consider the following equation.

$$F(s) = \frac{A_1}{s - a_1} + \frac{A_2}{s - a_2} + \frac{A_3}{s + a_3}$$

If s_1, s_2, and s_3 are the poles corresponding to the three terms in $F(s)$, respectively, which point on the graph may represent s_3 if a_3 is real and $a_3 > 0$?



(A) s_a (B) s_b (C) s_c (D) s_d

$s_3 = -a_3$. Since $a_3 > 0$ and real, s_3 is located at the negative real axis. The only point that may represent s_3 is s_a.

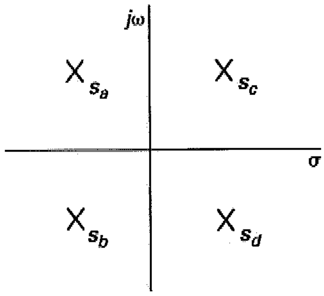
$\boxed{\text{The answer is (A).}}$

SYSTEMS–51

The pole diagram is shown for the following equation.

$$F(s) = \frac{P(s)}{Q(s)} = \frac{P(s)}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(s - s_3)}$$

If s_1, s_2, and s_3 are the poles corresponding to the solution of $(s^2 + 2\zeta\omega_n s + \omega_n^2)$ and $(s - s_3)$, respectively, what points on the diagram correspond to s_1 and s_2? ($\zeta > 0$, $\omega_n > 0$, and $\zeta < 1$. s_3 is not plotted.)



(A) s_a and s_b (B) s_a and s_c (C) s_a and s_d (D) s_b and s_c

$$s_1,\ s_2 = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

The roots of $s^2 + 2\zeta\omega_n s + \omega_n^2$ must be either s_a and s_b or s_c and s_d, since the $j\omega$ components are of the same magnitude but different in sign. The algebraic expression gives negative roots. Therefore, the solution is on the left side of the plot.

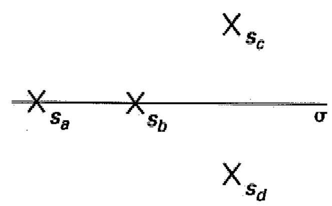
$\boxed{\text{The answer is (A).}}$

SYSTEMS–52

The following function is plotted on a pole-zero diagram.

$$F(s) = \frac{K(s - z_1)}{s(s - p_1)(s - p_2)}$$

The z_1 value and the $j\omega$ axis are not shown. The magnitude of p_1 is larger than the magnitude of p_2, and both are positive numbers. Determine which of the following statements is true.



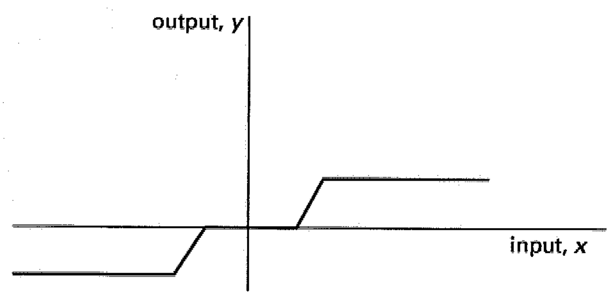
(A) $p_1 = s_a$, $p_2 = s_b$
(B) $p_1 = s_b$, $p_2 = s_a$
(C) $p_1 = s_c$, $p_2 = s_d$
(D) p_1 and p_2 are real and described by s_c and s_d.

Since p_1 and p_2 are positive numbers, they are real numbers. Therefore, p_1 and p_2 fall on the σ-axis. p_1 is of greater magnitude than p_2 and is, therefore, to the right of p_2.

The answer is (B).

SYSTEMS–53

Which of the following best describes the function shown?



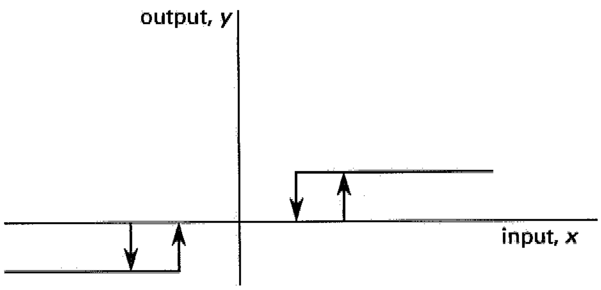
(A) It has a dead zone.
(B) It is a saturated zone system.
(C) There is both a dead zone and a saturated system zone.
(D) It is an impulse zone system.

The dead zone occurs in the region where there is no amplitude near the origin. The positive and negative saturation occurs after a short linear increase or decrease, respectively. A ramp would only have a single, continuously increasing function. An impulse is a narrow major increase, such as a single square-wave pulse.

The answer is (C).

SYSTEMS–54

Which of the following is true about the function shown?



(A) It has a dead zone with linear output outside the dead zone.
(B) It has a dead zone with saturation.
(C) There is no dead zone.
(D) It has a dead zone with hysteresis.

This is a classic hysteresis input/output curve with a dead zone. There is no information given about the stability of the system. There is no ramp present.

The answer is (D).

SYSTEMS–55

The frequency response of a system is given by

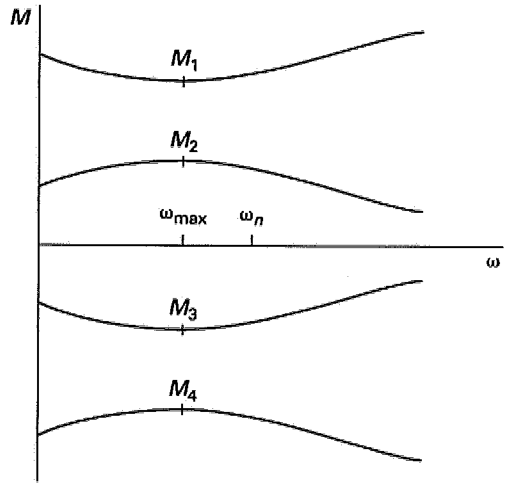
$$\frac{M}{\alpha} = \frac{X_2(j\omega)}{X_1(j\omega)}$$

By differentiation, the peak value of M, M_{max}, and the frequency at which it occurs, ω_{max}, are expressed in terms of the damping ratio, ζ, and natural frequency, ω_n.

$$M_{\text{max}} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\omega_{\text{max}} = \omega_n\sqrt{1-2\zeta^2}$$

Determine which curve in the figure is a correct representation for M to be the largest response.



(A) $M = M_1$ (B) $M = M_2$ (C) $M = M_3$ (D) $M = M_4$

As ω goes to ω_{max}, M should increase to its peak value. Therefore, the shape of M_2 and M_4 are both correct. However, since M_{max} is always positive, only M_2 is correct.

The answer is (B).

SYSTEMS–56

The frequency response of a system is given by

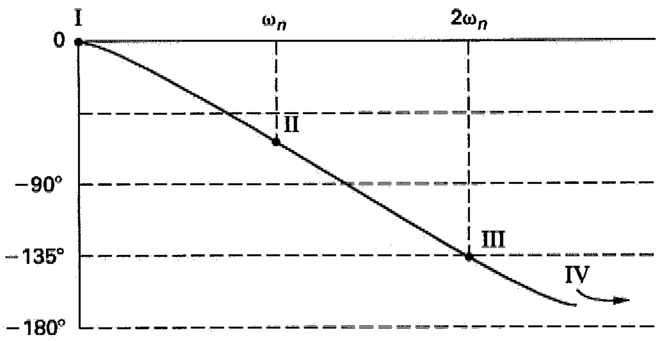
$$\frac{M}{\alpha} = \frac{X_2(j\omega)}{X_1(j\omega)}$$

By differentiation, the peak value of M, M_{max}, and the frequency at which it occurs, ω_{max}, are expressed in terms of the damping ratio, ζ, and natural frequency, ω_n.

$$M_{\text{max}} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\omega_{\text{max}} = \omega_n\sqrt{1-2\zeta^2}$$

Considering the polar plot shown, for what range of ω does the peak response in amplitude occur?



(A) ω is at point II.
(B) ω is at point IV.
(C) ω is between points I and II.
(D) ω is between points II and III.

A plot of M_{max} shows that the peak falls between 0 and ω_n. The polar plot shows two conjugate poles for a simple second-order system. The damping ratio, ζ, and the natural frequency are used to determine ω_{max}. The peak is at $\omega_{\text{max}} = \omega_n\sqrt{1-2\zeta^2}$, except when $\zeta = 0$ (completely undamped), $\omega_{\text{max}} < \omega_n$.

The answer is (C).

SYSTEMS–57

Which of the following $Q(s)$ equations can be stable?

I. $4s^4 + 8s^2 + 3s + 2 = 0$

II. $4s^4 + 2s^3 + 8s^2 + 3s + 2 = 0$

III. $4s^4 + 2s^3 + 8js^2 + 5s + 2 = 0$

IV. $4s^4 + 2s^3 + 8s^2 - 3s + 2 = 0$

(A) I only (B) II only (C) I and IV (D) II and III

The Routh test indicates that the necessary conditions for a polynomial to have all its roots in the left-hand plane (i.e., the system is stable) are (a) all of the terms must have the same sign; and (b) all of the powers between the highest and the lowest value must have nonzero coefficients, unless all even-power or all odd-power terms are missing. Condition (a) also implies that the coefficient cannot be imaginary. Equation II is the only equation that meets the criteria.

The answer is (B).

SYSTEMS–58

The Routhian array for the following equation is given.

$$Q(s) = s^4 + 6s^3 + 13s^2 + (20 + K)s + K = 0$$

s^4	1	13	K
s^3	6	$20 + K$	0
s^2	$\frac{58 - K}{6}$	K	0
s^1	$\frac{(58 - K)(20 + K) - 36K}{58 - K}$	0	0
s^0	K	0	0

For what range of K will the system be stable?

(A) $0 < K < 33$ (B) $33 < K < 58$ (C) $58 < K < 116$ (D) $0 < K < 116$

For the system to be stable, there can be no sign changes in the first column of the Routhian array. Since the first two entries of that column are positive, the last three entries must also be positive. The entry in the s^2 row gives $K < 58$, while the entry in the s^0 row gives $K > 0$. The numerator in the s^1 row is equal to $-K^2 + 2K + 116$, with roots of -35.1 and 33.1, or $-35.1 < K < 33.1$. For K to satisfy all these restrictions, $0 < K < 33$.

The answer is (A).

SYSTEMS–59

The characteristic equation for a system is

$$Q(s) = s^4 + 5s^3 + 10s^2 + Ks - 1 = 0$$
$$K > 0$$

The Routhian array is

s^4	1	10	-1
s^3	5	K	0
s^2	$\dfrac{50-K}{5}$	-1	0
s^1	$\dfrac{K(50-K)+25}{(5)(50-K)}$	0	0
s^0	5	0	0

Which of the following statements is true?

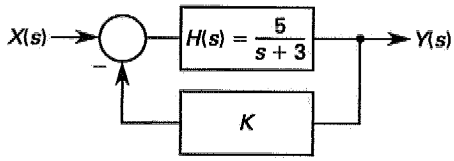
(A) The system is unstable at all points.
(B) The system is unstable for $K < 50$.
(C) The system is stable for $0 < K < 50$.
(D) The system is stable for some point above $K = 50$.

For the system to be stable, there can be no sign changes in the first column of the Routhian array. The $(50-K)$ term in the s^2 row means that the system should be stable for $0 < K < 50$. The term in the s^1 row requires that $-0.49 < K < 50.5$. Thus, the range of values for K for the system to be stable is $0 < K < 50$.

The answer is (C).

SYSTEMS–60

A control system is constructed from linear time-invariant elements as shown. What is the requirement of the constant K so that the closed-loop system is stable (i.e., so that bounded input yields bounded output)?



(A) $K \leq -5/3$ (B) $K \leq -3/5$ (C) $K \geq -3/5$ (D) $K \geq 0$

System transfer function $G(s)$ for the closed-loop system is

$$\begin{aligned} G(s) &= \frac{H(s)}{1+KH(s)} \\ &= \frac{\dfrac{5}{s+3}}{\dfrac{1+5K}{s+3}} \\ &= \frac{\dfrac{5}{s+3}}{1+K\left(\dfrac{5}{s+3}\right)} \\ &= \frac{5}{s+3+5K} \end{aligned}$$

In order for the system to be stable, the pole of the closed-loop system transfer function needs to be at the left hand side of the plane. Therefore,

$$\begin{aligned} s_1 &= -3 - 5K \leq 0 \\ K &\geq -3/5 \end{aligned}$$

The answer is (C).

SYSTEMS–61

Given the following transfer functions, which of these statements is true? K is a constant.

$$G_1(s) = \frac{K}{s\left(s+\dfrac{1}{B_1}\right)}$$

$$G_2(s) = \frac{K\left(s+\dfrac{1}{B_2}\right)}{s\left(s+\dfrac{1}{B_1}\right)}$$

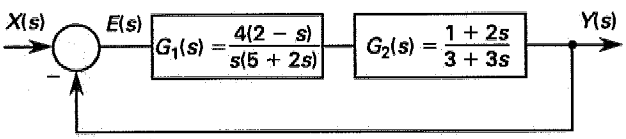
(A) $G_2(s)$ is as stable as $G_1(s)$.
(B) $G_2(s)$ is less stable than $G_1(s)$.
(C) $G_2(s)$ has slower transients than $G_1(s)$.
(D) none of the above

A system's stability is determined by its poles. $G_1(s)$ and $G_2(s)$ have the same poles, 0 and $-1/B_1$. The addition of another zero at $1/B_2$ in system $G_2(s)$ does not change the stability. The addition of the zero also does not change the decay rate of the function. Therefore, the correct answer is option (A).

The answer is (A).

SYSTEMS–62

For the following control system, what is the steady-state error $e_{ss}(t)$ for a ramp input function?



(A) 0 (B) 1/4 (C) 15/8 (D) ∞

Rearrange the open-loop transfer function to Canonic form.

$$G(s) = G_1(s)G_2(s) = \left(\frac{4(2-s)}{s(5+2s)}\right)\left(\frac{1+2s}{3+3s}\right)$$

$$= \frac{\left(\frac{8}{15}\right)\left(\frac{1-s}{2}\right)\left(\frac{1+s}{0.5}\right)}{s\left(\frac{1+s}{2.5}\right)(1+s)}$$

Therefore, $K_b = 8/15$ and $T = 1$.

This is a type 1 system. Using the steady-state error analysis table, the steady-state error $e_{ss}(t)$ for a ramp input function is

$$e_{ss}(t) = \frac{1}{K_B} = 15/8$$

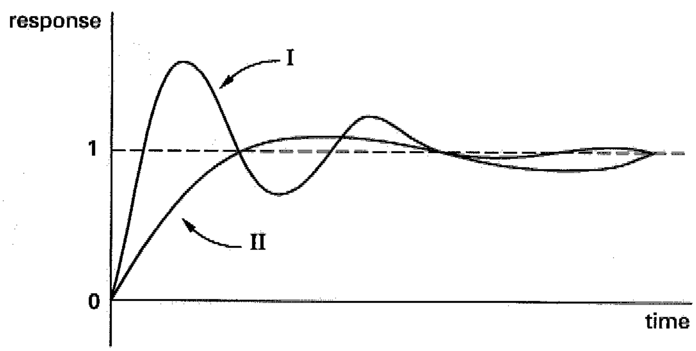
The answer is (C).

SYSTEMS–63

Both of the curves shown represent a system response of the form

$$G(s) = \frac{K}{s(1+s)(1+0.5s)}$$

K_1 is the gain for curve I, and K_2 is the gain for curve II. Which of these statements is true?



(A) K_1 is greater than K_2.
(B) K_1 is less than K_2.
(C) The size of K has no effect on the response.
(D) K is the same for both functions.

Generally, the gain, K, has a direct effect on the type of response obtained. Larger values of gain give larger overshoots or longer settling times. Thus, the gain of curve I is larger than the gain of curve II.

The answer is (A).

SYSTEMS–64

A control system has a control response ratio of

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + 0.3s + 1}$$

Given that the damping ratio is $\zeta < 0.707$, how many peaks occur in the transient prior to reaching steady-state conditions?

(A) zero
(B) one
(C) two
(D) The function gives a minimum.

The control ratio has two complex poles, which are dominant, but it has no zeros. The poles are $(-0.15 + j0.88)$ and $(-0.15 - j0.88)$. For a damping ratio of $\zeta < 0.707$, a peak occurs. The control response is in the form

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Then,

$$\omega_n^2 = 1$$
$$2\zeta = 0.3$$
$$\zeta = 0.15$$

The system is underdamped. Therefore, there is only one peak value given by the formula.

$$M_{\text{max}} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{1}{(2)(0.15)\sqrt{1-(0.15)^2}}$$
$$= 3.371$$

The answer is (B).

SYSTEMS–65

Which of the following is the correct transform for the following function?

$$F(s) = \frac{1}{s(s+a)(s+b)}$$

(A) $\left(\frac{1}{ab}\right)\left(1+\frac{be^{-at}}{a-b}-\frac{ae^{-bt}}{a-b}\right)$

(B) $\left(\frac{1}{ab}\right)\left(1+\frac{be^{-at}}{b-a}+\frac{ae^{-bt}}{b-a}\right)$

(C) $\left(\frac{1}{ab}\right)\left(1+\frac{be^{-at}}{b-a}-\frac{ae^{-bt}}{b-a}\right)$

(D) $\left(\frac{1}{ab}\right)\left(1-\frac{be^{-at}}{b-a}-\frac{ae^{-bt}}{b-a}\right)$

$$F(s) = \frac{A}{s} + \frac{B}{s+a} + \frac{C}{s+b}$$
$$A = \left(\frac{1}{s(s+a)(s+b)}\right) s \,\bigg|_{s=0} = \frac{1}{ab}$$
$$B = \left(\frac{1}{s(s+a)(s+b)}\right) (s+a) \,\bigg|_{s=-a} = \frac{1}{a(a-b)}$$
$$C = \left(\frac{1}{s(s+a)((s+b)}\right) (s+b) \,\bigg|_{s=-b} = -\frac{1}{b(a-b)}$$

$$s_0 f(t) = L^{-1}(F(s)) = A + Be^{-at} + ce^{-bt} \quad [\text{for } t \geq 0]$$

$$= \frac{1}{ab} + \left(\frac{1}{a(a-b)}\right)e^{-at} - \left(\frac{1}{b(a-b)}\right)e^{-bt}$$

$$= \left(\frac{1}{ab}\right)\left(1 + \left(\frac{b}{a-b}\right)e^{-at} - \left(\frac{a}{a-b}\right)e^{-bt}\right)$$

The answer is (A).

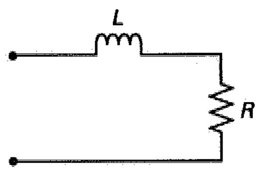
SYSTEMS–66

A circuit with inductance L and resistance R has the transfer function

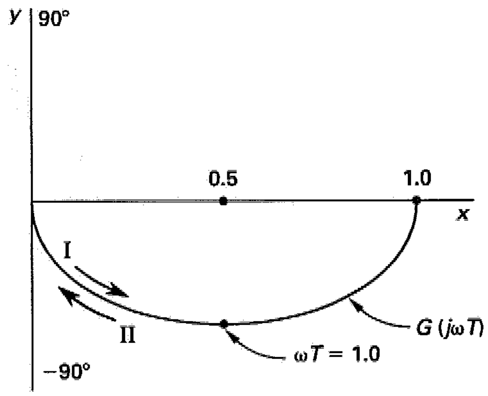
$$G(j\omega) = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

Alternatively,

$$G(j\omega T) = \frac{1}{1 + j\omega T}$$



Which of these statements is true about the polar plot of the function?



(A) I shows ω going to zero.
(B) II shows ω going to zero.
(C) If the inductor is replaced by a capacitor, the plot will be in the upper half of the phase plane.
(D) Both options (A) and (C) are true.

As ω approaches ∞, $|G(j\omega)|$ approaches 0. As ω approaches 0, $|G(j\omega)|$ approaches 1. I shows ω going to zero. If the inductor is a capacitor, the plot would be "reflected" on the horizontal axis into the top half of the plane.

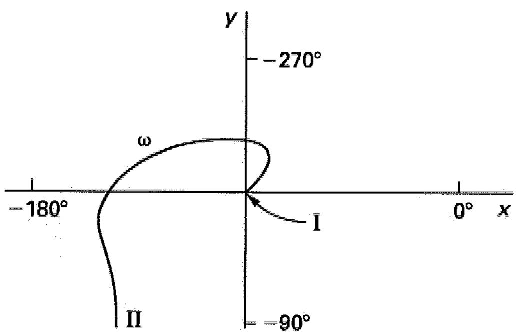
The answer is (D).

SYSTEMS–67

A typical transfer function is as follows.

$$G(j\omega) = \frac{C(j\omega)}{E(j\omega)}$$
$$= \frac{K}{j\omega(1 + j\omega T_1)(1 + j\omega T_2)(1 + j\omega T_3)}$$

The plot is as shown.



Which of the following is true?

(A) I is where $G(j\omega) = 0$.
(B) I is where $G(j\omega)$ approaches ∞.
(C) II is where $G(j\omega)$ approaches ∞.
(D) II is where $G(j\omega)$ approaches $-\infty$.

In the equation, as ω approaches 0, $G(j\omega)$ approaches ∞, and as ω approaches ∞, $G(j\omega)$ approaches 0. Therefore, point I is where $G(j\omega)$ approaches ∞.

The answer is (B).