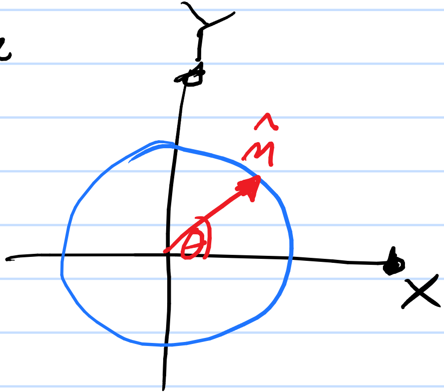
REPASO Cálculo III

- Vectores unitarios en el plano polar

$$\boxed{\hat{n} = (\cos\theta, \operatorname{sen}\theta)}$$

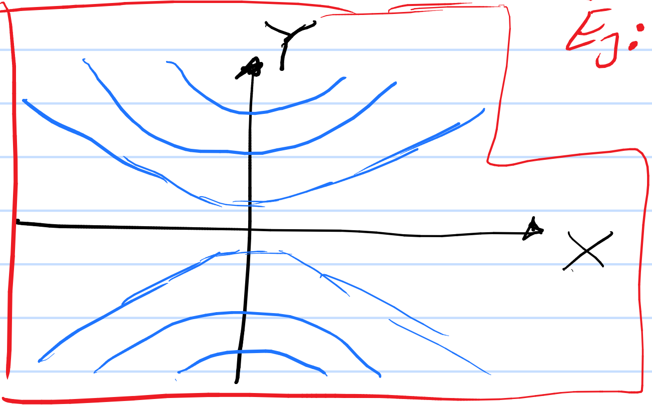


- **Curvas de nivel**

→ **Del tipo hiperbolas:** $z = k_1 \cdot x^2 - k_2 \cdot y^2 + k_3$

Con $k_1, k_2 > 0$ $k_3 \in \mathbb{R}$

Ej: $x^2 - y^2 + 5$



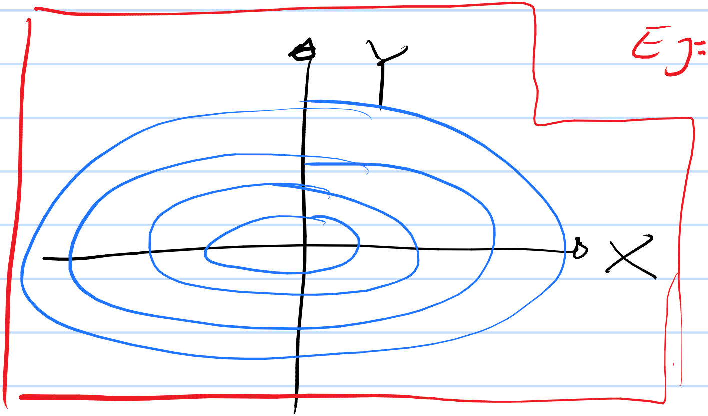
→ Del tipo elípticas:

$z = k_1 \cdot x^2 + k_2 \cdot y^2 + k_3$

con $k_1, k_2 > 0$

$k_3 \in \mathbb{R}$

Ej: $x^2 + y^2 + 9$



Derivadas parciales

$$\boxed{\frac{\partial f(x,y)}{\partial x} = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}}$$

$$\boxed{\frac{\partial f(x,y)}{\partial y} = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}}$$

- **Derivadas direccionales**: La derivada direccional en el punto (x,y) en la dirección $\vec{d} \neq 0$ es:

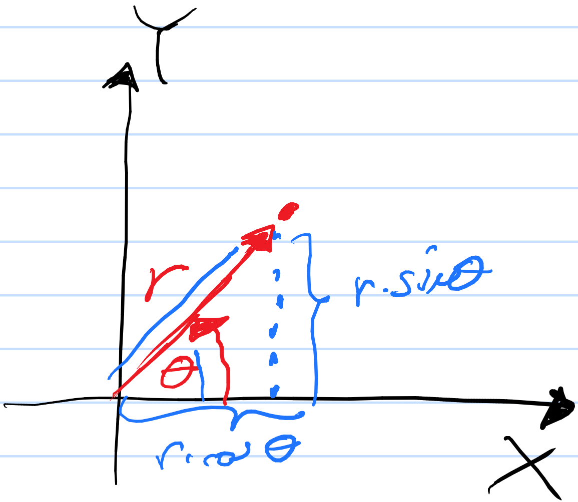
$$\boxed{\nabla f(x,y) \cdot \frac{\vec{d}}{\|\vec{d}\|}}$$

SISTEMAS COORDENADOS

- **Coordenadas polares:**

$$\boxed{\begin{aligned} x &= r \cdot \cos\theta \\ y &= r \cdot \sin\theta \end{aligned}}$$

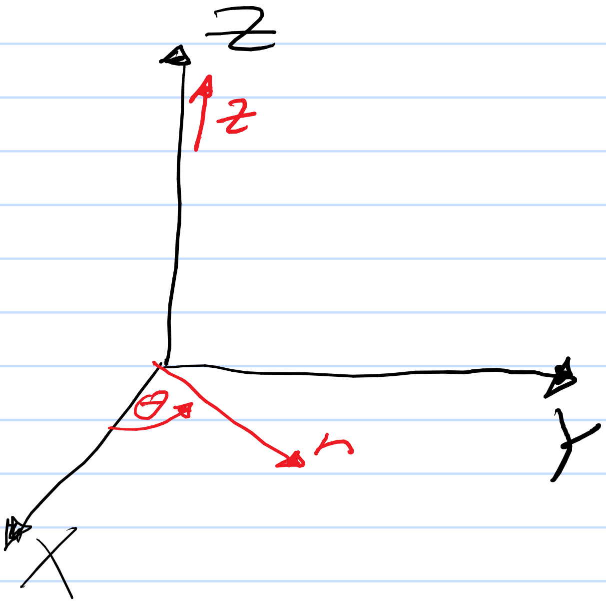
$$\boxed{dA = dx\,dy = r\,dr\,d\theta}$$



- Coordenadas cilíndricas

$$\boxed{\begin{aligned} z &= z \\ x &= r\cdot\cos\theta \\ y &= r\cdot\sin\theta \end{aligned}}$$

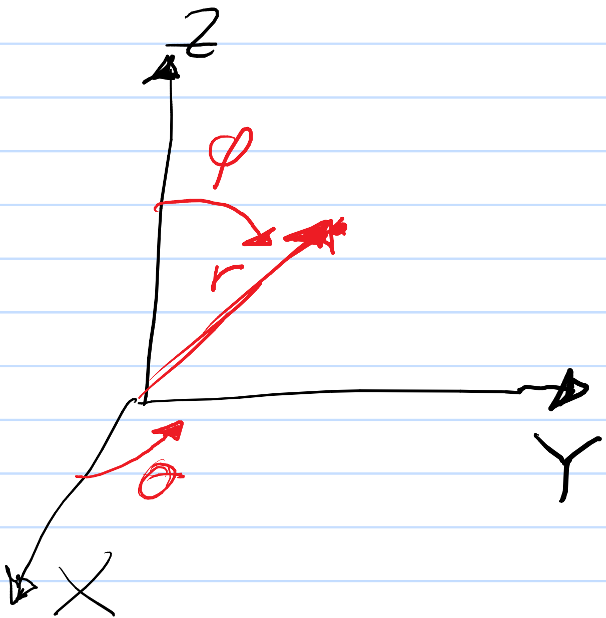
$$\boxed{dV = dx\,dy\,dz = r\,dr\,d\theta\,dz}$$



- **Coordenadas esferas**

$$\boxed{\begin{aligned} x &= r\cdot \operatorname{sen}(\varphi)\cdot \cos\theta \\ y &= r\cdot \operatorname{sen}(\varphi)\cdot \operatorname{sen}\theta \\ z &= r\cdot \cos(\varphi) \end{aligned}}$$

$$\boxed{dV = r^2 \cdot \operatorname{sen}(\varphi)\, dr\, d\varphi\, d\theta}$$



• Calculo de masa:

$$\boxed{m = \int dm}$$

→ Densidad 1D:

$$\boxed{\rho = \frac{dm}{dl}} \Rightarrow \boxed{dm = \rho \cdot dl}$$

→ Densidad 2D:

$$\boxed{\rho = \frac{dm}{dA}} \Rightarrow \boxed{dm = \rho \cdot dA}$$

→ Densidad 3D:

$$\boxed{\rho = \frac{dm}{dV}} \Rightarrow \boxed{dm = \rho \, dV}$$

- Centros de massa

$dV = dx\,dy\,dz$

$$\boxed{\bar{x} = \frac{\int x\,dm}{\int dm} = \frac{\int x \cdot \rho\,dV}{\int \rho\,dV}}$$

$$\boxed{\bar{y} = \frac{\int y\,dm}{\int dm} = \frac{\int y\rho\,dV}{\int \rho\,dV}}$$

$$\boxed{\bar{z} = \frac{\int z\,dm}{\int dm} = \frac{\int z \cdot \rho \cdot dV}{\int \rho\, dV}} \rightarrow \frac{\cancel{\rho} \cdot \int z\,dV}{\cancel{\rho} \int dV} = \frac{\int z\,dV}{\int dV}$$

Nota: Cuando no se especifica densidad, se asume densidad constante

$$\boxed{\bar{x} = \frac{\int x\,dV}{\int dV}} \qquad \boxed{\bar{y} = \frac{\int y\,dV}{\int dV}} \qquad \boxed{\bar{z} = \frac{\int z\,dV}{\int dV}}$$

Caso em que a densidade é constante.