REPASO EDO

- Una ecuación diferencial ordinaria (EDO) que incluye a los términos $y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^m y}{dx^m}$ se dice que es una EDO de grado m.

$$F\left(y, \frac{dy}{dx}, \ldots, \frac{d^m y}{dx^m}\right) = g(x)$$

→ Si $\boxed{g(x)=0}$ se dice $\boxed{\text{homogenea}}$ y si $\boxed{g(x)\neq 0}$ se dice $\boxed{\text{NO homogenea}}$

- Una EDO se dice lineal si tomamos $y = y_1 + c\cdot y_2$ y se obtiene que:

$$F\left(y, \frac{dy}{dx}, \ldots, \frac{d^m y}{dx^m}\right) = F\left(y_1, \frac{dy_1}{dx}, \ldots, \frac{d^m y_1}{dx^m}\right)$$

$$+ c\cdot F\left(y_2, \frac{dy_2}{dx}, \ldots, \frac{d^m y_2}{dx^m}\right)$$

- Se dice NO lineal en caso contrario.

$$y'' - 2y' + y = x^2 \rightarrow \neq 0$$

Solución:
- EDO de segundo orden
- NO HOMOGÉNEA.

$$F(y, y', y'') = y'' - 2y' + y$$

$$F(y_1, y_1', y_1'')$$

$$c \cdot F(y_2, y_2', y_2'')$$

$$y = y_1 + c \cdot y_2 \Rightarrow \boxed{y' = y_1' + c \cdot y_2'} \quad \boxed{y'' = y_1'' + c \cdot y_2''}$$

$$F(y, y', y'') = y'' - 2y' + y = (y_1'' + c \cdot y_2'') - 2 \cdot (y_1' + c \cdot y_2')$$
$$+ (y_1 + c \cdot y_2) = (y_1'' - 2y_1' + y_1) + c \cdot (y_2'' - 2y_2' + y_2)$$

$$\boxed{F(y, y', y'') = F(y_1, y_1', y_1'') + c \cdot F(y_2, y_2', y_2'')}$$

- EDO es lineal.

FORMAS DE RESOLUCIÓN

① Ecuaciones separables:

$$\boxed{\frac{dy}{dx} = h(y)\cdot g(x)}$$

Res:

$$\frac{dy}{h(y)} = g(x)\cdot dx \qquad / \int$$

② Ecuaciones lineales de primer orden:

$$\boxed{\frac{dy}{dx} + P(x)\cdot y = Q(x)}$$

$$\boxed{y = e^{-\int P(x)dx}\left[\int Q(x)\cdot e^{-\int P(x)dx}dx + C\right]}$$

Res:

$$\boxed{\frac{d}{dx}\left(e^{\int P(x)dx}\right) = e^{\int P(x)dx}\,P(x)}$$

$$\frac{dy}{dx} + P(x)\cdot y = Q(x) \qquad \Big/ \cdot e^{\int P(x)dx}$$

$$\frac{dy}{dx}\cdot e^{\int P(x)dx} + \underbrace{P(x)\cdot e^{\int P(x)dx}}\cdot y = Q(x)\cdot e^{\int P(x)dx}$$

$$\underbrace{\frac{dy}{dx}\cdot e^{\int P(x)dx} + \frac{d}{dx}\left(e^{\int P(x)dx}\right)\cdot y} = Q(x)\cdot e^{\int P(x)dx}$$

$$\frac{d}{dx}\left(y\cdot e^{\int P(x)dx}\right) = Q(x)\cdot e^{\int P(x)dx}$$

$$y \cdot e^{\int P(x)dx} = \int Q(x) \cdot e^{\int P(x)dx} dx + C$$

$$\boxed{y = e^{-\int P(x)dx} \left[\int Q(x) \cdot e^{\int P(x)dx} dx + C \right]}$$

③ Ecuaciones no homogéneas

$$\boxed{\frac{dy}{dx} = F\left(\frac{y}{x}\right)} \to F(v)$$

Res: $v(x) = \frac{y(x)}{x} \Rightarrow \boxed{y = f(v, x) = v \cdot x}$

$$\frac{dy}{dx} = \underbrace{\frac{\partial f}{\partial v}}_{x} \cdot \underbrace{\frac{dv}{dx}} + \underbrace{\frac{\partial f}{\partial x}}_{v} \underbrace{\frac{dx}{dx}}_{1} = x \cdot \frac{dv}{dx} + v$$

$$\boxed{\frac{dy}{dx} = x \cdot \frac{dv}{dx} + v}$$

$$x \cdot \frac{dv}{dx} + v = F(v)$$

$$\Rightarrow \quad x \cdot \frac{dv}{dx} = F(v) - v \quad \Rightarrow \quad \boxed{\frac{dv}{F(v) - v} = \frac{dx}{x}}$$

$$\boxed{V(x)} \qquad \boxed{Y(x) = V(x) \cdot X}$$

4. Ecuaciones de Bernoulli

$$\boxed{\frac{dy}{dx} + P(x)\cdot y = Q(x)\cdot y^{n}}$$

Res: $V(x) = y^{(1-n)} \Rightarrow y = V^{\frac{1}{1-n}}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(V^{\frac{1}{1-n}}\right) = \left(\frac{1}{1-n}\right)\cdot V^{\left(\frac{1}{1-n}-1\right)}\cdot\frac{dV}{dx}$$

$$\frac{1}{1-n} - \frac{(1-n)}{(1-n)} = \frac{n}{1-n}$$

$$= \frac{1}{(1-n)}\cdot V^{\frac{n}{1-n}}\cdot\frac{dV}{dx}$$

$$\Rightarrow \quad \boxed{\frac{dy}{dx} = \frac{1}{(1-n)} \cdot v^{\frac{n}{1-n}} \cdot \frac{dv}{dx}}$$

$$\frac{dy}{dx} + P(x) \cdot y = Q(x) \cdot y^n$$

$$\frac{1}{(1-n)} \cdot v^{\frac{n}{1-n}} \cdot \frac{dv}{dx} + P(x) \cdot v^{\frac{1}{(1-n)}} = Q(x) \cdot v^{\frac{n}{1-n}}$$

$$\cdot \frac{(1-n)}{v^{\frac{n}{1-n}}}$$

$$\frac{dv}{dx} + P(x)\cdot(1-n)\cdot\frac{v^{\frac{1}{1-n}}}{v^{\frac{n}{1-n}}} = \frac{Q(x)\,(1-n)}{1}.$$

$$\boxed{P'(x) = P(x)(1-n)} \qquad \boxed{Q'(x) = Q(x)(1-n)}$$

$$v^{\frac{1}{1-n} - \frac{n}{(1-n)}} = v^{\frac{1-n}{1-n}} = v$$

$$\boxed{\frac{dv}{dx} + P'(x)\cdot v = Q'(x)}$$

$$v(x) \qquad \underline{y(x) = v(x)^{\frac{1}{1-n}}}$$

5) Ecuaciones diferenciales exactas

$$\boxed{M_{(x,y)} \cdot dx + N_{(x,y)} \cdot dy = 0}$$

- $$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$F_{(x,y)} = C$$

$$\boxed{\frac{\partial F}{\partial x} = M_{(x,y)} \quad \frac{\partial F}{\partial y} = N_{(x,y)}}$$

$$F(x,y) = \underbrace{\int M(x,y)\,\partial x}_{g(x,y)} + f(y) = g(x,y) + f(y)$$

$$\frac{\partial F}{\partial y} = N(x,y) \Rightarrow \underbrace{\frac{\partial g(x,y)}{\partial y} + \boxed{\frac{d f(y)}{d y}}}_{} = N(x,y)$$

$$\Rightarrow f(y)$$

$$F(x,y) = g(x,y) + f(y)$$

Ejemplo 1:

$$\boxed{\frac{dy}{dx} - y = \frac{11}{8} \cdot e^{-\frac{x}{3}}} \qquad \boxed{y(0) = -1}$$

$$y(x) = e^{-\int P(x)dx} \cdot \left[\int Q(x) \cdot e^{\int P(x)dx} \, dx + C \right]$$

$$\boxed{P(x) = -1} \qquad \boxed{Q(x) = \frac{11}{8} \cdot e^{-\frac{x}{3}}} \qquad \int P(x)\,dx = \int -1\,dx = -x$$

$$y(x) = e^{-(-x)} \cdot \left[\int \frac{11}{8} \cdot \underbrace{e^{-\frac{x}{3}} \cdot e^{-x}}_{e^{-\frac{4}{3} \cdot x}} \, dx + C \right]$$

$$\Rightarrow y(x) = e^{x} \cdot \left[\frac{11}{8} \cdot \int e^{-\frac{4}{3}x} dx + C \right]$$

$$= e^{x} \cdot \left[\frac{11}{8} \cdot \frac{e^{-\frac{4}{3}x}}{\left(-\frac{4}{3}\right)} + C \right]$$

$$= e^{x} \cdot \left[-\frac{33}{32} \cdot e^{-\frac{4}{3}x} + C \right]$$

$$\boxed{y(x) = e^{x} \cdot \left[C - \frac{33}{32} \cdot e^{-\frac{4}{3}x} \right]}$$

$y(0) = -1$

$$y(0) = e^{0} \cdot \left[C - \frac{33}{32} \cdot e^{0} \right] = C - \frac{33}{32} = -1$$

$$\Rightarrow C = \frac{33}{32} - 1 = \frac{1}{32} \Rightarrow \boxed{C = \frac{1}{32}}$$

$$\boxed{y(x) = \frac{e^{x}}{32} \cdot \left[1 - 33 \cdot e^{-\frac{4}{3}x} \right]}$$

Ejemplo 2:

$$(6xy-y^3)dx + (4y+3x^2-3xy^2)dy=0$$

$$\boxed{M(x,y) = 6xy - y^3} \qquad \boxed{N(x,y) = 4y+3x^2-3xy^2}$$

$$\boxed{\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}}$$

$$\frac{\partial N}{\partial x} = 6x - 3y^2$$

$$\frac{\partial M}{\partial y} = 6x - 3y^2$$

✓ EXACTA

$6y \cdot \frac{x^2}{2} = 3x^2y$

$$\boxed{F(x,y)} = \int M(x,y)\,\partial x + f(y) = \int (6xy - y^3)\,\partial x + f(y)$$

$-x \cdot y^3$

$$= \boxed{3x^2y - xy^3 + f(y)}$$

$$\frac{\partial F}{\partial y} = N(x,y)$$

$$\boxed{F(x,y) = 3x^2y - xy^3 + f(y)}$$

$$\frac{\partial F}{\partial y} = 3x^2 - 3y^2 x + f'(y) = N(x,y)$$

$$= 4y + 3x^2 - 3xy^2$$

$$\Rightarrow 3x^2 - 3y^2x + f'(y) = 4y + 3x^2 - 3xy^2$$

$$\Rightarrow \boxed{f'(y) = 4y} \Rightarrow \frac{\partial f}{\partial y} = 4y$$

$$\Rightarrow \partial f = 4y\,\partial y \quad \int \quad \boxed{f(y) = 2y^2 + C_1}$$

$$\boxed{F(x,y) = 3x^2y - xy^3 + 2y^2 + C_1} \quad F(x,y) = c$$

$$3x^2y - y^3x + 2y^2 + c = C_1$$

$$\boxed{C_2 = C_1 - c}$$

$$\boxed{3x^2y - y^3x + 2y^2 = C_2}$$

$$y(x=0) = 1$$

$x=0 \quad y=1$

x con y

$$2 = C_2 \Rightarrow \boxed{2 = C_2}$$

$$y(x) = f(x)$$

$$\boxed{3x^2y - y^3x + 2y^2 = 2}$$

implícita y en función de x.

- **Crecimiento anual de poblaciones**

Sea r la tasa de crecimiento anual y P_0 la población actual, entonces tenemos que en t años la población será:

$$\boxed{P(t) = P_0(1+r)^t}$$

- **Exponencial imaginaria**

Sea t un numero real e "i" la unidad imaginaria

$(i^2=-1\ ,\ i=\sqrt{-1})$

$$\boxed{e^{i\cdot t} = \cos(t) + i\cdot \sin(t)}$$

Nota: $\lambda = 1+i$

$$e^{\lambda\cdot t} = e^{(1+i)\cdot t} = e^{t+i\cdot t} = e^{t}\cdot e^{it}$$

$$= \boxed{e^{t} \cdot \left[\cos(t) + i \cdot \sin(t)\right]}$$

• **Ecuaciones lineales de segundo orden**

$$\boxed{a \cdot y'' + b y' + c \cdot y = 0}$$

Se supone una solución del tipo $y(x) = e^{\lambda x}$

$\Rightarrow y'(x) = \lambda \cdot e^{\lambda x} \Rightarrow y''(x) = \lambda^2 \cdot e^{\lambda x}$

$$a \cdot \lambda^2 \cdot e^{\lambda x} + b \cdot \lambda \cdot e^{\lambda x} + c \cdot e^{\lambda x} = 0 \quad / e^{\lambda x}$$

$$\boxed{a \lambda^2 + b\lambda + c = 0}$$

NOTA: Sean $y_1(x)$, $y_2(x)$ soluciones de la ecuación lineal de segundo orden, entonces:

$$\boxed{y(x) = C_1 \cdot y_1(x) + C_2 \cdot y_2(x)}$$

} Es solución de la ecuación lineal de segundo orden también.

• SISTEMAS DE ECUACIONES

$$\boxed{\begin{aligned} a\cdot x(t) + b\cdot y(t) &= x'(t) \\ c\cdot x(t) + d\cdot y(t) &= y'(t) \end{aligned}}$$

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{A} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix}$$

$$\vec{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$$\vec{x}\,'(t) = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix}$$

$$\Rightarrow \boxed{A\cdot\vec{x}(t) = \vec{x}\,'(t)}$$

$$\vec{x}(t) = e^{\lambda t} \cdot \vec{v} \Rightarrow \vec{x}\,'(t) = \lambda \cdot e^{\lambda t} \cdot \vec{v}$$

$$A \cdot e^{\lambda t} \cdot \vec{v} = \underbrace{\lambda \cdot e^{\lambda t} \cdot \vec{v}}_{\lambda e^{\lambda t} \cdot I \cdot \vec{v}} \Rightarrow \boxed{(A - \lambda I) \cdot \vec{v} \cdot \cancel{e^{\lambda t}} = 0}$$

$$\boxed{(A - \lambda I) \cdot \vec{v} = 0}$$

1 $\det(A - \lambda I) = 0$

2 $\lambda \Rightarrow \boxed{(A - \lambda I) \cdot \vec{v} = 0}$