5.6 Astrophysics and Cosmology

This topic covers gravitational fields and the physical interpretation of astronomical observations, the formation and evolution of stars and the history and future of the universe.

Candidates will be assessed on their ability to:

154	understand that a gravitational field (force field) is defined as a region where a mass experiences a force

Force due to gravity acts on all matter with mass and is always attractive

A **force field** is an area in which an object experiences a non-contact force

Force fields can be represented as vectors and be represented as diagrams containing field lines:

- Distance between field lines represents the strength of the force exerted by the field in that region
- Direction of arrow shows show the direction that a force acts on a mass a.k.a the direction of acceleration of a mass placed in the field

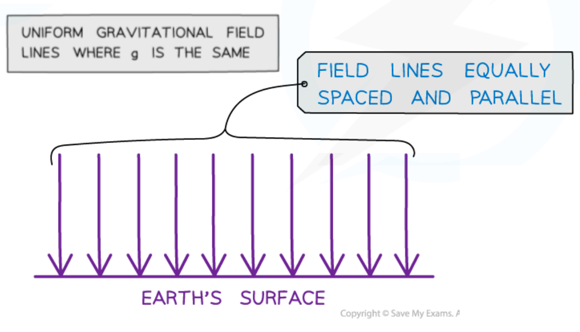
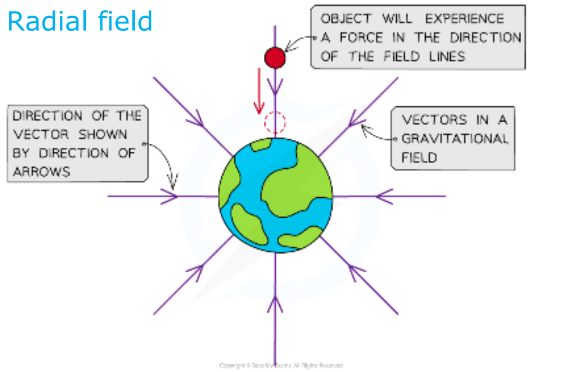
A **gravitational field** is a *force field in which objects with mass experience a force*

Since a gravitational field is a force field, they can also be represented as vectors

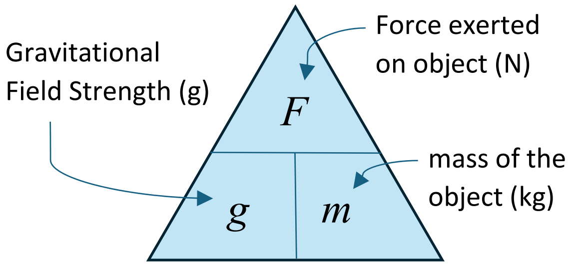
- Gravitational field lines are directed toward the centre of mass of a body because the gravitational force is always attractive

155	understand that gravitational field strength is defined as $g = \frac{F}{m}$ and be able to use this equation

There are two types of gravitational field; a **uniform** field or **radial** field

Uniform field	Radial field
	
A **uniform** field exerts the same gravitational force on a mass everywhere in the field, as shown by the parallel and equally spaced field lines	In a **radial** field, the force exerted depends on the position of the object in the field (as an object moves further away from the centre, the magnitude of force would decrease because the distance between field lines increases)

The Earth's gravitational field is *radial*, however very close to the surface it is almost completely *uniform*

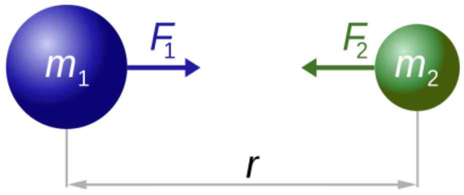


Gravitational field strength (g) is the force per unit charge experienced by an object in a gravitational field
This value is *constant in a uniform field*, but *varies in a radial field*

156 — be able to use the equation $F = \frac{Gm_1m_2}{r^2}$ (Newton's law of universal gravitation)

Newton's Law of Universal Gravitation

"The gravitational force F_G between two masses m_1 and m_2 is proportional to the product of their masses and inversely proportional to the square of the distance between their centres, r"



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

The $1/r^2$ relation is called the 'inverse square law'
This means that if the distance between two masses **doubles**, r becomes $2r$
Therefore, $1/r^2$ becomes $1/(2r)^2$, which is equal to $1/4r^2$
Hence, the gravitational force between the two masses **reduces** by a factor of **four**

157 — be able to derive and use the equation $g = \frac{Gm}{r^2}$ for the gravitational field due to a point mass

A *gravitational field formed due to a point mass is a radial gravitational field*

The **gravitational field strength** at a point describes how much gravitational force is experienced by a test mass (small m) at that point

$$g = \frac{GM}{r^2}$$

General Formula for Gravitational Field Strength $= g = \frac{F}{m}$

$$g = \frac{\frac{GM\cancel{m}}{r^2}}{\cancel{m}} = \frac{GM}{r^2} \qquad F = \frac{GMm}{r^2} \text{ (Gravitational Force)}$$

Derivation:

where m is the mass of test mass, M is mass of object causing gravitational field, r = distance from the centre of mass M to a chosen point in the field

(Note that this means gravitational field strength at a certain point will be the same regardless of the mass of the test mass)

158 — be able to use the equation $V_{grav} = \frac{-Gm}{r}$ for a radial gravitational field

In a uniform field near Earth's surface, GPE is the energy an object has when lifted off the ground (GPE = mgh)

When using this equation, the GPE on the surface of the Earth is taken to be zero

- This means work is done to lift the object

In a radial field, GPE is defined as *the energy an object possesses due to its position in a gravitational field*

- The gravitational potential (V_{grav}) at a point is the gravitational potential energy per unit mass at that point

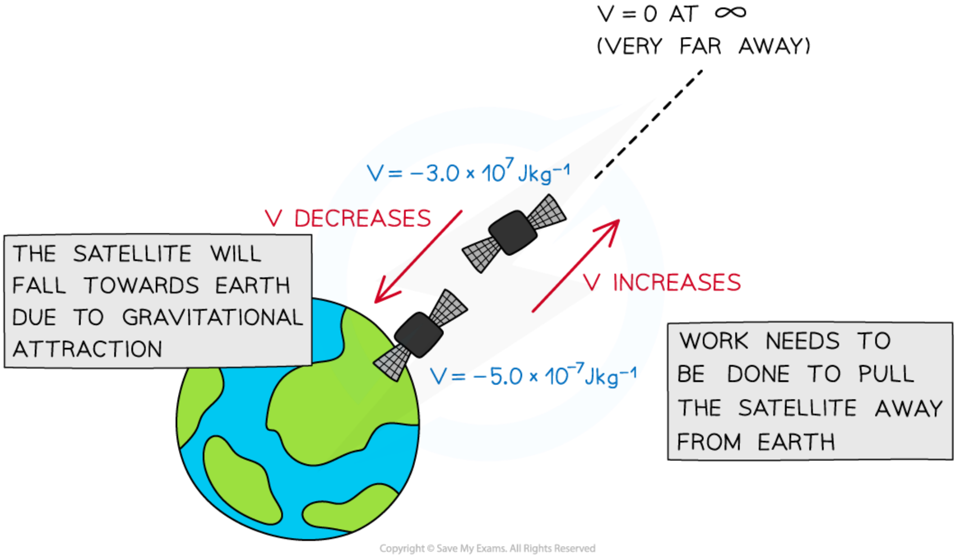
Gravitational potential (*V_{grav}*) at a point is defined as "*the work done per unit mass when moving an object (a test mass) from infinity to that point (in J/ kg)*"

- Gravitational potential at infinity is zero

- As an object moves from infinity to a point, energy is released as the gravitational potential energy is reduced, therefore gravitational potential is always negative

$$V = -\frac{GM}{r} \quad \text{(For a radial field)}$$

Where **M** is the mass of the object causing the field, **r** is the distance between the centres of the objects.

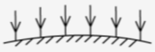
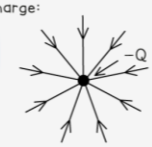
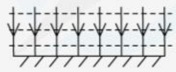
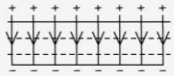


The **gravitational potential difference** (ΔV) is the *energy needed to move a unit mass between two points* and therefore can be used to find the work done when moving an object in a gravitational field

$$Work\ done = m\Delta V$$ Where **m** is the mass of the object moved.

159 be able to compare electric fields with gravitational fields

Similarities	**Differences**
Both fields have an infinite range	Gravitational field exerts a force on *particles with mass* whilst electric field exerts a force on *particles with charge*
Both fields follow *inverse square law* relationships	Gravitational field is always attractive whilst the electric field can be attractive or repulsive
Both the field lines around a *spherical mass and a spherical charge* are radial	The gravitational potential is always negative whilst the electric potential can be either negative or positive
Both the field lines in a uniform *gravitational and electric field are identical* (they are parallel and equally spaced)	The electric force between point charges is larger than the gravitational force between unit masses at a given separation
Both the *gravitational and electric field strength* have a *$1/r^2$ relationship* in a radial field	
Both the *gravitational and electric potential* have a *$1/r$ relationship*	

	Gravitational Fields	Electric Fields
Origin of the force	Mass	Charge
Force between two point masses/ charges	$F_G = \frac{GM_1M_2}{r^2}$	$F_E = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$
Type of Force	Attractive force	Attractive force (opposite charges) Repulsive force (like charges)
Field Strength	$g = \frac{F}{M}$	$E = \frac{F}{Q}$
Field strength due to a point mass/ charge	$g = \frac{GM}{r^2}$	$E = \frac{Q}{4\pi\varepsilon_0 r^2}$
Field Lines	Around a point mass:  In a uniform field (surface of a planet): 	Around a (negative) point charge:  In a uniform field (between charged) parallel plates: 
Potential	$V = -\frac{GM}{r}$	$V = \frac{Q}{4\pi\varepsilon_0 r}$
Equipotential Surfaces	Around a point mass:  In a uniform field (surface of a planet): 	Around a point charge:  In a uniform field (between charged) parallel plates 
Work Done on a Mass or Charge	$\Delta W = M\Delta V$	$\Delta W = Q\Delta V$

160 be able to apply Newton's laws of motion and universal gravitation to orbital motion

Newton's Law of Gravitation & Orbits

Centripetal Force = Gravitational Force

$$\frac{\cancel{m}v^2}{r} = \frac{GM\cancel{m}}{r^2}$$

$$v^2 = \frac{GM}{r}$$

Time Period & Orbital Radius Relation

Kepler's third law is that the square of the orbital period (T) is directly proportional to the cube of the radius (r): $T^2 \propto r^3$

Derivation:

Centripetal Force = Gravitational Force

$$\frac{\cancel{m}v^2}{r} = \frac{GM\cancel{m}}{r^2}$$

$$v = \frac{2\pi r}{T}$$

$$v^2 = \frac{GM}{r}$$

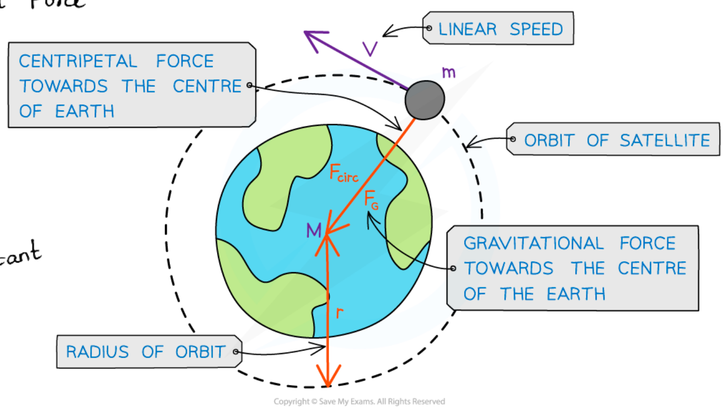
$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

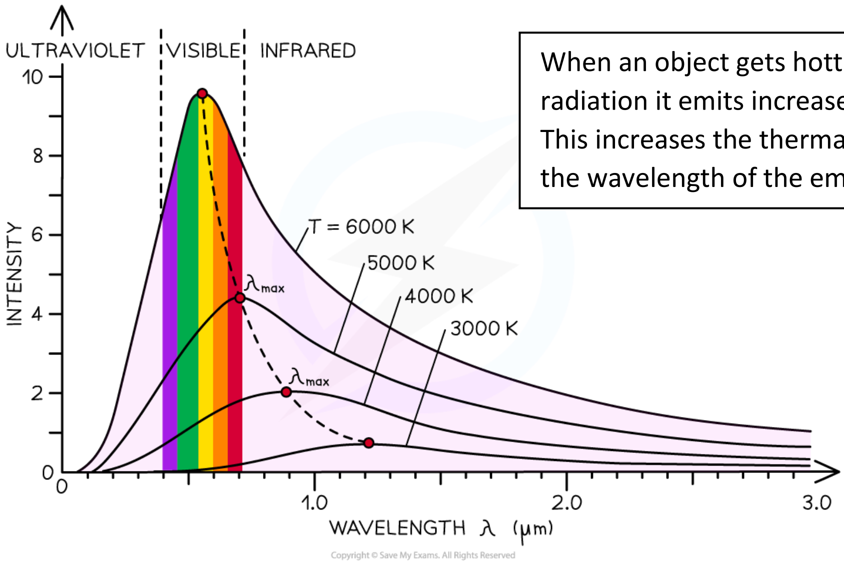
$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

constant

$$T^2 \propto r^3$$



where M is the mass of object being orbited

161	understand what is meant by a *black body radiator* and be able to interpret radiation curves for such a radiator
	A **black body radiator** is defined as **"the *perfect emitter and absorber of all possible wavelengths of radiation"*** (does not reflect or transmit any radiation) Black body radiation is the name given to the thermal radiation emitted by all bodies *All objects emit black body radiation* (thermal radiation) regardless of temperature, in the form of electromagnetic waves All bodies emit a spectrum of thermal radiation in the form of electromagnetic waves The intensity and wavelength distribution of any emitted waves depends on the temperature of the body (Black body) **Radiation curves** are graphs of intensity against wavelength of radiation emitted by objects at different temperatures - As the temperature increases, the peak of the curve moves  When an object gets hotter, the amount of thermal radiation it emits increases This increases the thermal energy emitted and so the wavelength of the emitted radiation decreases An ideal black body radiator is one that absorbs and emits all wavelengths, but it is a theoretical object (though, stars are the best approximation there is)
162	be able to use the Stefan-Boltzmann law equation $L = \sigma A T^4$ for black body radiators
	An object's luminosity depends on two factors: 1. Its surface temperature 2. Its surface area **Stefan-Boltzmann law** states that *the **power output** (a.k.a luminosity, L, a.k.a the total energy emitted per unit area) of a black body radiator **is directly proportional to its surface area (A) and its (surface temperature)^4*** which means $L \propto AT^4$ $$L = \sigma A T^4$$ *Where T is the surface temperature of star and σ is the Stefan-Boltzmann constant* This law can be used to compare the power output, temperature and size of stars

Surface area of a spherical object (star) = $4\pi r^2$

163 be able to use Wien's law equation $\lambda_{max}T$ = 2.898 x 10^{-3} m K for black body radiators

Wien's displacement law

states that the peak wavelength (λ_{max}) of emitted radiation is inversely proportional to the surface temperature (T) of the object

The maximum wavelength (λ_{max}) is the wavelength of light released at peak intensity (can be seen on black-body curve)

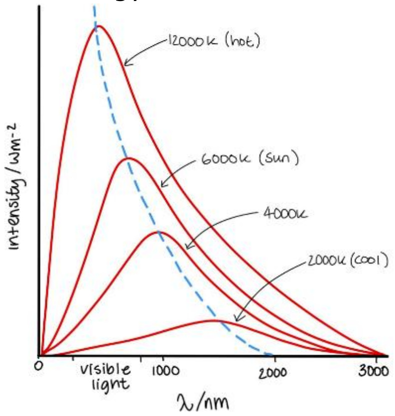
Note that the unit shows metres-Kelvin, and not milliKelvin

$$\lambda_{max}T = \text{constant} = 2.898 \times 10^{-3}\ \text{m K}$$

Wein's law shows that the maximum wavelength of emitted radiation by a black body radiator decreases as its surface temperature increases (as wavelength decreases, the frequency of emitted radiation increases which results in the energy of the wave increasing)

This law tells us the higher the temperature of a body:

- The shorter the wavelength at the peak intensity, so hotter objects tend to be white or blue, and cooler objects tend to be red or yellow
- The greater the intensity of the radiation at each wavelength



164 be able to use the equation, intensity $I = \frac{L}{4\pi d^2}$ where L is luminosity and d is distance from the source

Inverse Square Law of Intensity

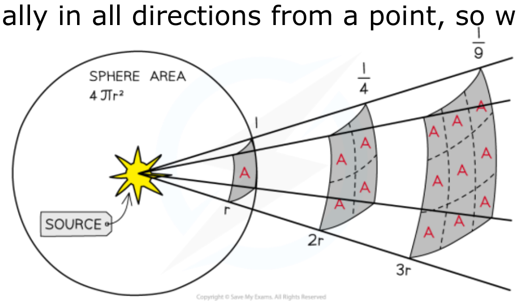
Luminosity (L) is the *rate of light energy released (or) the power output of a star* (W)

Intensity (I) is the *power received from a star (its luminosity) per unit area* (W m^{-2})

The inverse square law of intensity states that the intensity of a star is inversely proportional to the square of the distance between the star and the observer

It is assumed that light is emitted uniformally in all directions from a point, so will spread out (in the shape of a sphere)

$$I = \frac{L}{4\pi d^2}$$



Where I is observed intensity on Earth,
L is luminosity of source (star), d is the distance from from star to Earth

This equation assumes:

- The power from the star radiates uniformly through space
- No radiation is absorbed between the star and the Earth

165	understand how astronomical distances can be determined using trigonometric parallax

Determining distances using trigonometric parallax

Stellar Parallax is defined as:

"The apparent change in position of a nearby star in comparison to a backdrop of distant stars, *as a result of the orbit of the Earth around the Sun"*

It involves observing how the position of a nearby stars changes over a period of time (usually at least a 6-month period) against a fixed background of distant stars

- From observer's position, the background of distant stars do not appear to move
- However, the nearby star does appear to move to a different position
- This apparent movement of the nearby star is called the *stellar parallax*

The property is measured by the angle of parallax (θ)

- The angle of parallax can be found by measuring the angle to a star and seeing how this angle changes as the Earth changes position
- The greater the angle of parallax, the closer the star is to the Earth.

➔ **Astronomical Unit (AU)** - The average distance between the centre of the Earth and the centre of the Sun.

1 AU = 1.5×10^{11} m

➔ **Parsec (pc)** - The distance at which the angle of parallax is 1 arcsecond (1/3600th of a degree).

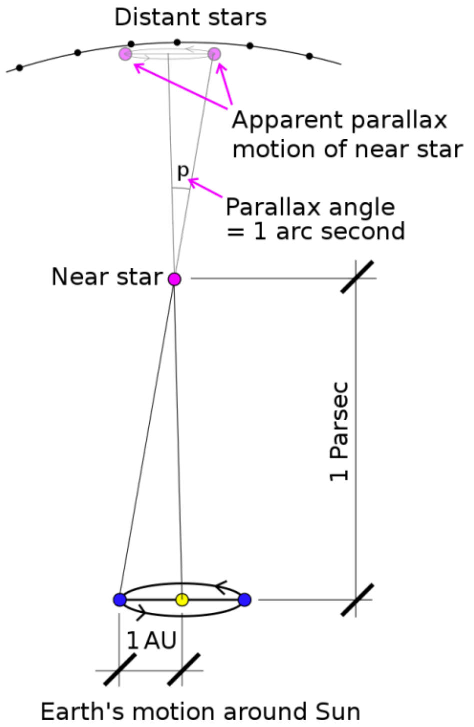
1 pc = 2.06×10^{5} AU = 3.08×10^{16} m = 3.26 ly

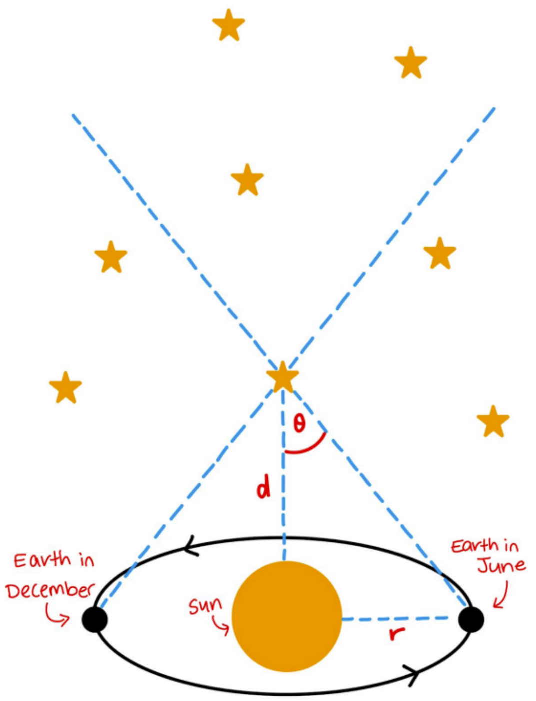
➔ **Light year (ly)** - The distance that an EM waves travels in a year in a vacuum.

1 ly = 9.46×10^{15} m

$$\tan\theta = \frac{opp}{adj} \rightarrow \tan\theta = \frac{r}{d} \rightarrow \quad d = \frac{r}{\theta}$$ As tan θ ≈ θ for small θ

Where **d** and **r** are in metres and **θ** is in radians. These are labelled on the diagram below on the right.





If the distance to the nearby star is to be measured in parsec, equation becomes:

$$(\text{distance to star}) = \frac{1}{(\text{parallax angle})}$$

This equation is accurate for distances of up to 100 pc

- For distances larger than 100 pc the angles involved are so small they are hard to measure accurately

166	understand how astronomical distances can be determined using measurements of intensity received from standard candles (objects of known luminosity)

Determining distances using standard candles

A **standard candle** is defined as:

"An astronomical object which has a known luminosity due to a characteristic quality possessed by that class of object"

Examples of standard candles are:

- Cepheid variable stars (A type of pulsating star which increases and decreases in brightness over a set time period)
- Type 1a supernovae (A supernova explosion involving a white dwarf where the luminosity at the time of the explosion is always the same)

First, the standard candle is located and the radiant flux intensity (a.k.a. the observed intensity on Earth) of the electromagnetic radiation arriving at the Earth is measured

Since luminosity is known (as the object is a standard candle), the distance can then be calculated using the *inverse square law of intensity*

$$I = \frac{L}{4\pi d^2}$$

Where I is observed intensity on Earth,
L is luminosity of source (star), d is the distance from from star to Earth

***13** The average distance between galaxies is many times greater than the size of our galaxy. Astronomers use standard candles to determine the distances to other galaxies.

Describe how astronomers determine the luminosity of a standard candle and can use this to determine the distance to a nearby galaxy.

Astronomers use stellar parallax to determine distance to a nearby standard candle and measure the intensity of radiation from the standard candle

Then, the inverse square law is used to calculate the luminosity of the standard candle

Locate standard candle (in nearby galaxy)

Standard candle has a known luminosity

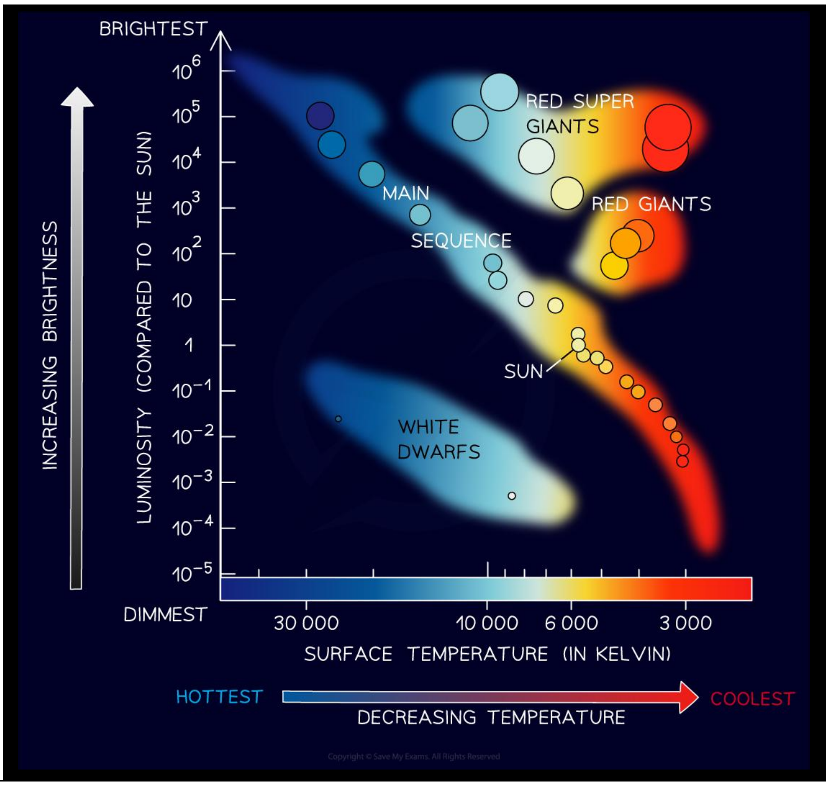
Measure intensity of radiation from the standard candle and use inverse square law to calculate distance to nearby galaxy

Transitory phases happen quickly in relation to the lifetime of a star

Black holes cannot be seen since they emit no light

167	be able to sketch and interpret a simple Hertzsprung-Russell diagram that relates stellar luminosity to surface temperature

Hertzsprung-Russell diagram



For main sequence stars, *luminosity increases with surface temperature*

Red giants, and red super giants (on the top right corner) show an increase in luminosity at cooler temperatures (because they are much larger than main sequence stars)

To the bottom left, the white dwarf stars are hot, but not very luminous (so they must be much **smaller** than main sequence stars)

The Hertzsprung-Russell Diagram only shows stars that are in **stable phases**

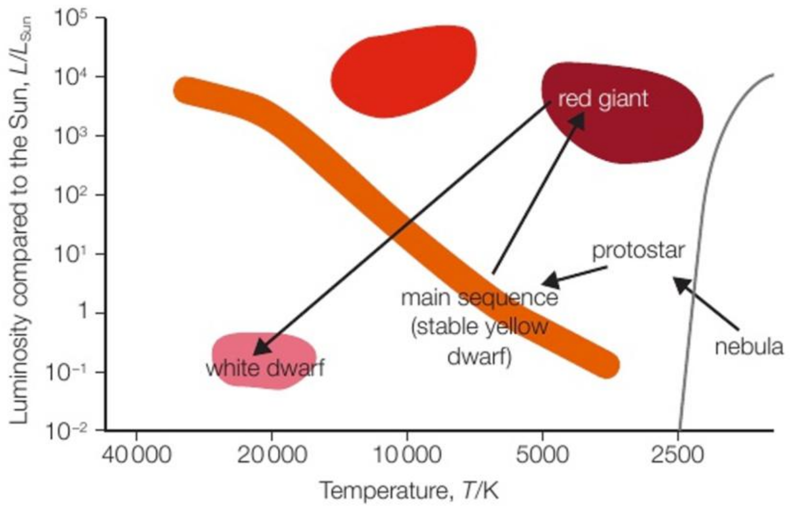
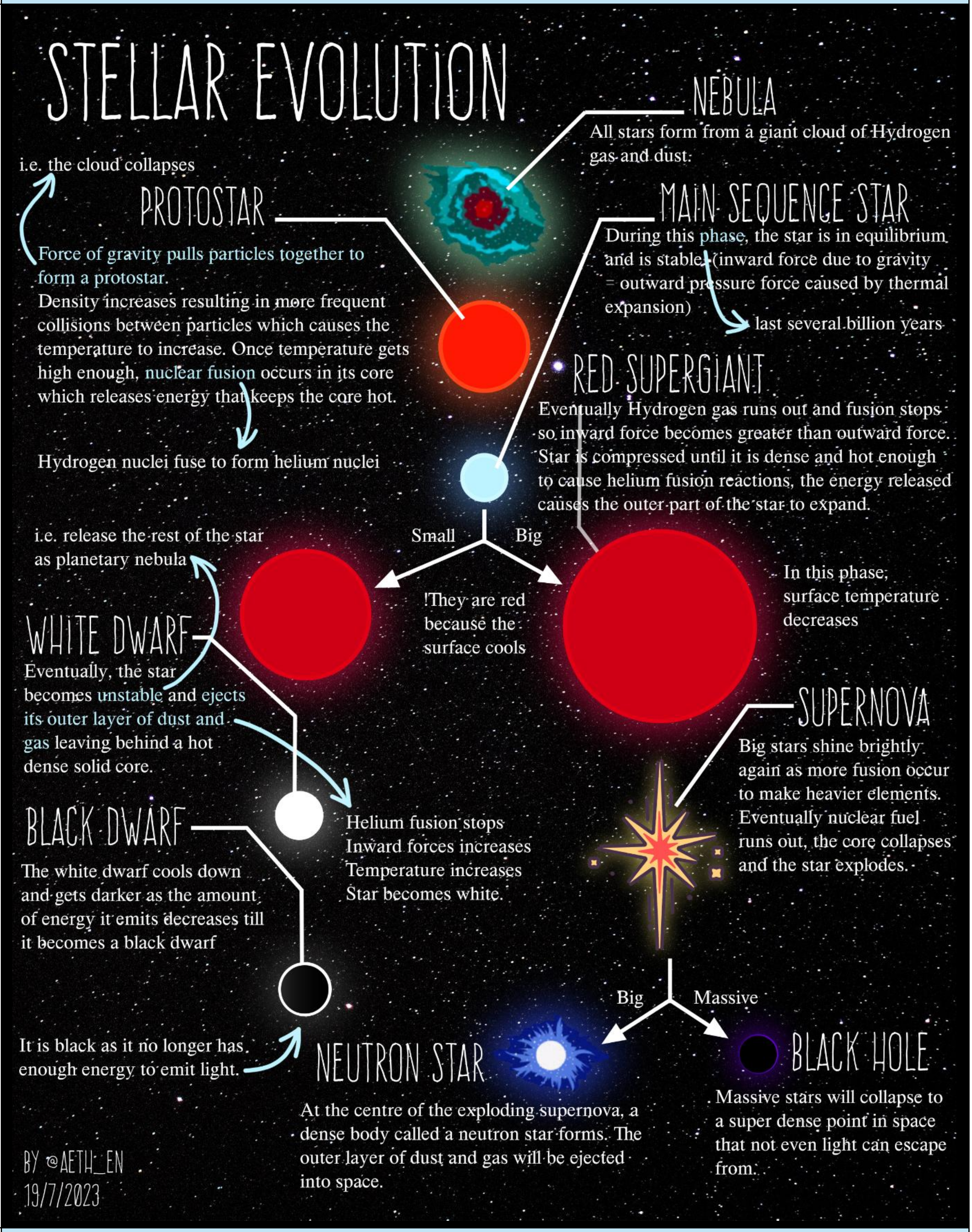


fig F The Sun's life cycle will move it around the H–R diagram.

168	understand how to relate the Hertzsprung-Russell diagram to the life cycle of stars



169	understand how the movement of a source of waves relative to an observer/detector gives rise to a shift in frequency (Doppler effect)

The **Doppler effect** is defined as:

"The apparent change in wavelength (and frequency) of the radiation from a source when the source of the waves is moving away or towards an observer"

It can be observed by comparing the light spectrum produced from a close object, such as our Sun, with that of a distant galaxy

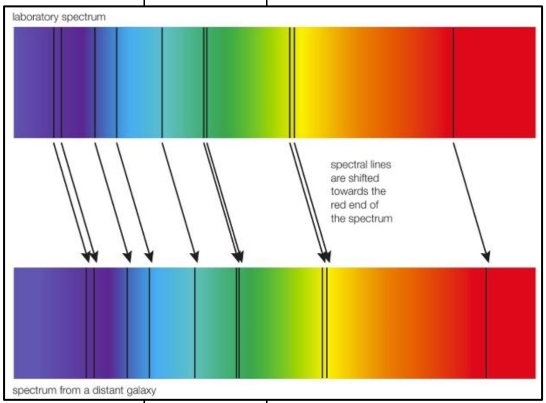
- The light from the distant galaxy is shifted towards the red end of the spectrum
- This provides evidence that the universe is expanding

170 be able to use the equations for redshift

$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$$

for a source of electromagnetic radiation moving relative to an observer and $v = H_0 d$ for objects at cosmological distances

Redshift is defined as:



"The fractional increase in wavelength (or decrease in frequency) due to the source and observer receding (moving away) from each other"

On the other hand, **blueshift** is "*the fractional decrease in wavelength (or increase in frequency) due to the source and observer moving towards each other"*

For **non-relativistic** galaxies, Doppler redshift can be calculated using:

$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$$

Where $\Delta\lambda$ = shift in wavelength, λ = wavelength emitted from the source, v = speed of recession and c = speed of light in a vacuum

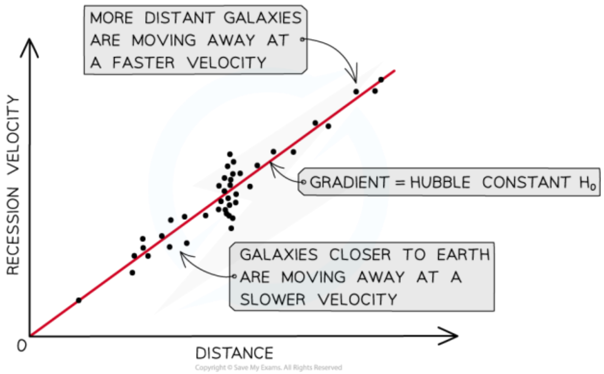
Hubble's law states:

"The recessional velocity, v, of a galaxy is proportional to its distance from Earth"

Age of universe = $1 / H_0$

$$v = H_0 d$$

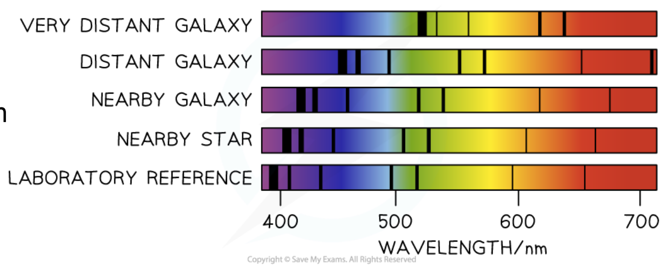
Where v = velocity of an object moving away from an observer (km s^{-1}),
H_0 = Hubble constant (km s^{-1} Mpc^{-1})
d = distance between the object and the Earth (Mpc)



Hubble's law shows that the further away a star is from the Earth, the faster it moves away from us

He observed that light from more distant galaxies was shifted further towards the red end of the spectrum compared to closer galaxies

This provides evidence that the universe is expanding



171 understand the controversy over the age and ultimate fate of the universe associated with the value of the Hubble constant and the possible existence of dark matter

The critical density is the specific average density of matter and energy in the universe that determines whether the universe will continue expanding forever or eventually stop and collapse

The Fate of the Universe

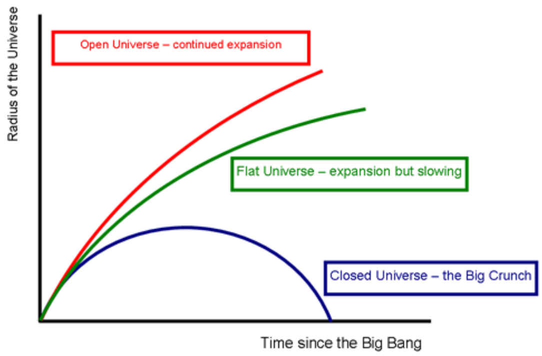
Right now, evidence currently points to the universe expanding. However, will this change in the future and what is the ultimate fate of the universe?

The truth is we don't know!

The answer depends on the average density of the universe and a critical density.

Different conditions would result in a different ultimate fate for the universe.

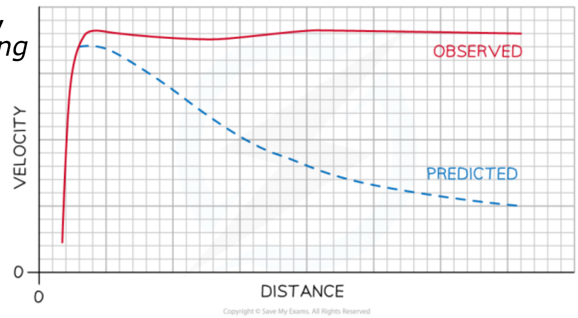
1. If the average density of the universe is greater than the critical density, the universe's expansion will slow down and will eventually contract (closed universe)
2. If the average density of the universe is less than the critical density, the universe will keep expanding forever (open universe)
3. If the average density of the universe equals the critical density, the universe will continue to expand forever, but the expansion rate will gradually decrease to 0 after an infinite amount of time (flat universe)



However, the reason why we are uncertain about the fate of the universe is because we are uncertain about the average density of the universe

This is because:

- We would expect *the velocity of an object within a galaxy to decrease as it moves away from the galaxy's centre because of weakening gravitational field strength*
- This is observed in smaller mass systems, like the solar system where *planets orbiting furthest from the Sun have the slowest orbital velocity*
- This is not the case in bigger mass systems like entire galaxies
- Hence, mass must not actually be concentrated in the centre of galaxies; it must be spread out
- However, all the "observable" mass of a galaxy is concentrated in its centre, so there must be another type of matter we can't see, called **Dark Matter**
- Astronomers were only able to detect its presence based on its gravitational effects (Galaxies are spinning so the stars must be experiencing a centripetal force towards the centre but the mass of the stars alone cannot make up for the large centripetal force generated to keep the galaxy spinning; in fact, it only accounts for 10% of the total mass needed. This means galaxies must contain a lot of mass that does not emit light)



Dark matter is defined as: ***"Matter which cannot be seen that does not emit or absorb electromagnetic radiation"***

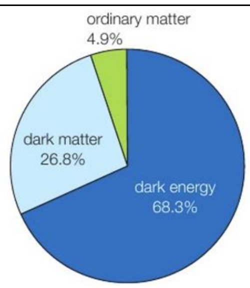
Hence, the reason why we cannot be certain about the average density of the universe is because we cannot be certain about the amount of dark matter in the universe

Dark Energy

New observations of distant supernovae have shown that the expansion of the Universe is not slowing but instead accelerating!

This suggests a new source of force that is causing this expansion of the universe to accelerate called **Dark Energy**

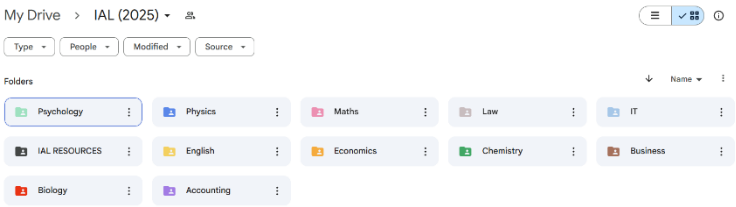
We have no idea what it is nor can we see it but what we do know is that dark energy is approximately two-thirds of everything that exists



Remarks

- Disclaimer: This Self-Study Booklet series are not intended as a textbook replacement but instead are meant to be used alongside it
- This booklet is primarily exam-based and has been produced to remove unnecessary information from the book to make it into a simpler and more compact form factor
- Thank you all for using this Self-Study Booklet and if you would like to find more resources like this one, do check out this google drive: https://drive.google.com/drive/folders/1qpY-sGR9iinJbnquB53hx8pLk49pyxd1?usp=drive_link
- You will be redirected here:

  (Note that there are only Self-Study Booklets/ CAQs for Physics and Chemistry)



- Do keep in mind that this is still a work-in-progress and you are welcome to add more resources to it- just drop a text to @aeth_en on discord!

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Help us improve

- This booklet is in its first edition so may contain mistakes here and there
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- Contact: mth.2021education@gmail.com or @aeth_en on discord
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