1.4 Materials

This topic covers density, flow of liquids, Hooke's law, the Young modulus and elastic strain energy.

This topic should be studied using a variety of applications, for example making and testing food, engineering materials, spare-part surgery for joint replacement.

This unit includes many opportunities for developing experimental skills and techniques by carrying out more than just the core practical experiments.

Candidates will be assessed on their ability to:

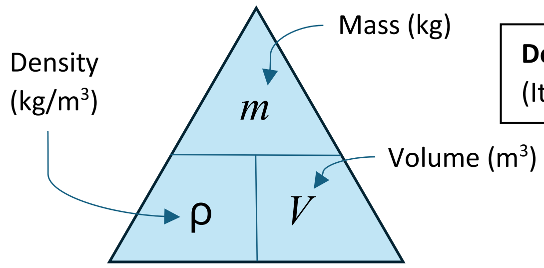
23 be able to use the equation: density $\rho = \frac{m}{V}$

Fluid Mechanics

- can be separated into two main branches: **Fluid Statics** and **Fluid Dynamics**
- **Fluid Statics** is the study of fluids at rest (which we'll dive into now)
- **Fluid Dynamics** is the study of fluids in motion (which we'll explore down below)

Fluid Statics

Fluid statics is the branch of fluid mechanics that studies fluids **at rest** and in a **state of equilibrium**, focusing on density, Archimede's principle, pressure and the forces acting in static fluids such as upthrust



Density is the mass per unit volume of an object
(It basically indicates how compact a substance is)

24 understand how to use the relationship upthrust = weight of fluid displaced

Archimede's principle states that:

"An object fully or partially submerged in a fluid at rest has an upward buoyancy force (upthrust) equal to the weight of the fluid displaced by object"

$$F_{upthrust} = \rho_{fluid} V_{displaced}\, g$$

In short: $F = \rho V g$

Before you ask! Yes, buoyant force & upthrust are the same thing! This force causes objects to float or sink and it depends on the density of the fluid and the volume of the displaced fluid

How to use this relationship to determine floating vs sinking:

- *Compare the calculated upthrust to the actual weight of the object*

Calculate upthrust using:	*Calculate weight using:*
Upthrust $= \rho_{fluid} V_{displaced}\, g$	*Weight* $= m_{object}\, g$

- *If Upthrust > Weight of object, the object will float*
- *If Upthrust < Weight of object, the object will sink*
- *If Upthrust = Weight of object, the object will float at a constant depth or be in equilibrium*

Some real-life applications

A. Submarines / Diving

- Adjust buoyancy by **changing internal volume of air** → changes **overall density**
- If average density < water → submarine rises
- If average density > water → submarine sinks

B. Hot Air Balloons

- Balloon rises because **heated air is less dense**
- Displaces heavier cool air → **upthrust > weight**

C. Ice Floating in Water

- Ice is less dense than water (~920 vs 1000 kg/m^3)
- Only part of it is submerged
- Upthrust = weight → equilibrium

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a be able to use the equation for viscous drag (Stokes' Law),

$$F = 6\pi\eta r v$$

b understand that this equation applies only to small spherical objects moving at low speeds with *laminar flow* (or in the absence of *turbulent flow*) and that viscosity is temperature dependent

Fluid Dynamics

Fluid dynamics is the branch of fluid mechanics that studies fluids **in motion,** focusing on viscosity, viscous drag and different types of flow such as laminar and turbulent

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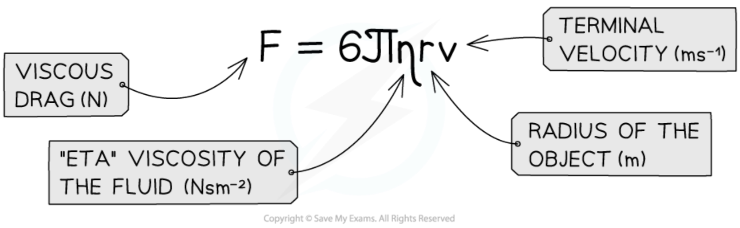
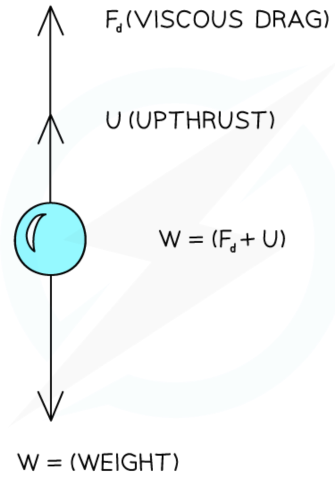
Stokes' Law describes that:

"The viscous drag force on a sphere moving through a fluid at a low, constant velocity is directly proportional to the fluid's viscosity, the sphere's radius and its velocity

- Stokes' Law is expressed as:





Viscous drag is:

"The frictional force which opposes the motion between an object through a fluid"

(i.e. how resistant a fluid is to flowing) (how thick the fluid is)

- It slows down objects moving through the fluid
- It also reduces flow rate of the fluid itself

Viscosity is:

"A measure of a fluid's internal resistance to flow, essentially its "thickness"

- Liquids like water have low viscosity, while substances like honey have high viscosity, as the latter resists deformation and requires more energy to flow
- This property is due to molecular interactions within the fluid, where stronger intermolecular forces in liquids increase resistance, whereas in gases, it's caused by intermolecular collisions
- More factors affecting viscosity are listed below!

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When object is moving at **terminal velocity**,
Resultant force = 0, Acceleration = 0, Upward forces = Downward forces which basically means: Weight = Drag + Upthrust (and object moves at constant speed)

At terminal velocity, $W_{(sphere)} = D + U$

$D = 6\pi\eta r v$

$W_s = m_s g = V_s \rho_s g$

$= \frac{4}{3}\pi r^3 \rho_s g$

U = W of fluid displaced

$W_f = m_f g = V_s \rho_f g$

$= \frac{4}{3}\pi r^3 \rho_f g$

Combining $\Rightarrow$ $\frac{4}{3}\pi r^3 \rho_s g = 6\pi\eta r v + \frac{4}{3}\pi r^3 \rho_f g$

v (terminal velocity) $= \dfrac{\frac{4}{3}\pi r^3 (\rho_s - \rho_f) g}{6\pi\eta r} = \dfrac{4\pi r^3 (\rho_s - \rho_f) g}{18\pi\eta r}$

$= \dfrac{2r^2(\rho_s - \rho_f) g}{9\eta}$

The final equation shows that **Terminal velocity** is:
- **Directly proportional to the square of the radius** of the sphere (meaning a larger sphere will fall faster) (doubling the radius quadruples the terminal velocity)
- **Inversely proportional to the viscosity** of the fluid

Note that viscous **drag increases with velocity**
BUT viscosity does not change (it is constant at a fixed temperature)

The coefficient of viscosity is a numerical value given to a fluid to indicate how much it resists flow

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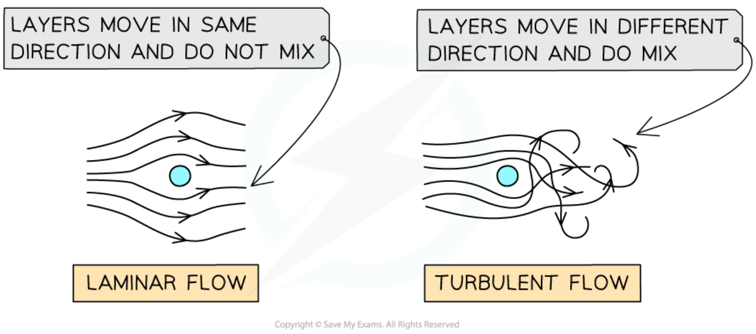
Conditions for Stokes' Law to apply:
- ***Small, smooth, spherical*** *objects moving at* ***low speeds***
- *The flow around that object must be* ***laminar flow*** *(not turbulent flow)*



Factors Affecting Viscosity

1. Temperature

For Liquids: Viscosity decreases as temperature increases
(As temperature increases, molecules gain kinetic energy, allowing them to overcome intermolecular forces and move more freely, thus decreasing viscosity)
(Example: Honey flows more easily when warm)
For Gases: Viscosity increases as temperature increases
(As temperature increases, molecules collide more frequently, increasing internal friction and thus increasing their viscosity)

2. Intermolecular Forces
3. Pressure
4. Shear rate
5. Particles and additives

Points 2 to 5 is not explained in detail further as it is not that relevant to the course but if you would like to find out more, you should definitely go search about it!

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<td>CORE PRACTICAL 2: Use a falling-ball method to determine the viscosity of a liquid</td>
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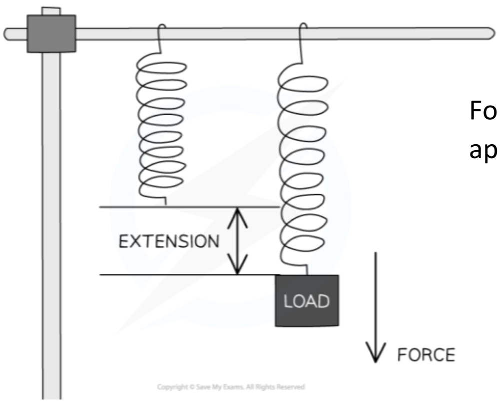
27 be able to use the Hooke's law equation, $\Delta F = k\Delta x$, where k is the stiffness of the object

Hooke's Law states that:
"The extension of an elastic object is directly proportional to the force applied, up to the limit of proportionality"

Spring constant has units of force per unit length (N/m)
Young modulus has units of force per unit area (Pa or N/m²)



Spring constant (k) indicates the stiffness of a material in N/m

Force (ΔF) applied in N

$$\boxed{\Delta F = k\Delta x}$$

Extension (Δx) in m

Note that the law only holds true for materials within their elastic limit
- If a spring is deformed beyond this limit (plastic deformation), it will not return to its original shape, and Hooke's Law no longer applies
- Only valid in the linear (elastic) region of the force-extension graph
- In this region, the spring returns to its original shape once the force is removed

What's the difference between **Spring constant, *k*** and **Young's modulus, *E***?

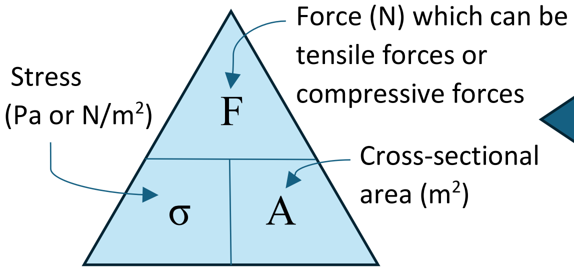
Spring constant, *k* from Hooke's law is a measure of the stiffness of a particular **object**, like springs (It indicates the force required to produce a unit of deformation)
And this depends on the object's material, size and shape
Young modulus, *E* is the stiffness constant for a **material** in general, regardless of size and shape
- It is a property that quantifies its stiffness and resistance to elastic deformation
- This means while Young's modulus is constant for a particular material, the spring constant can vary depending on the object's geometry (length and area)

For example, if you change the size, shape, length or cross-sectional area of a bar while keeping the material the same, the Young's modulus will remain constant, but the spring constant will change!

28	understand how to use the relationships • *(tensile or compressive) stress = force/cross-sectional area* • *(tensile or compressive) strain = change in length/original length* *Young modulus = stress/strain.*

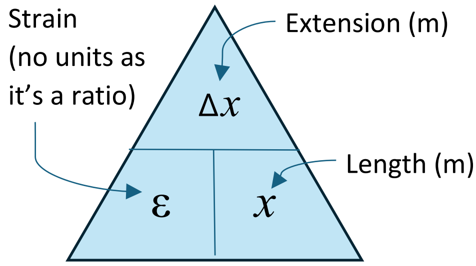
There are relationships that describe a material's elastic behavior under **tensile (pulling)** or **compressive (pushing)** forces (i.e. how much that material deforms) where **stress is force per area**, **strain is the relative change in length**, and **Young's modulus (E) is the ratio of stress to strain,** indicating stiffness

Stress & Strain



Stress is the force applied per unit cross-sectional area of a material
Tensile stress: Occurs when a force pulls the material, causing it to stretch
Compressive stress: Occurs when a force pushes on the material, causing it to compress

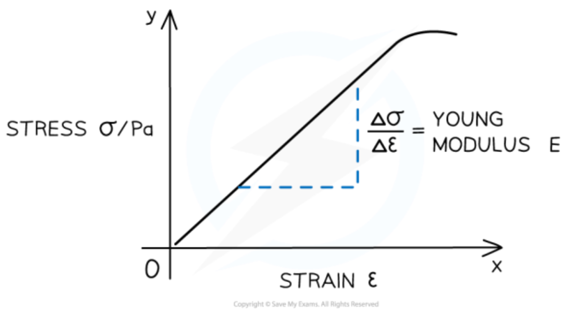
Strain is the change in length (extension or compression) per unit length of a material
Strain measures how much a material deforms in response to stress
It has **no units** since it's a ratio



Young Modulus (E)
- Young's modulus is a measure of a material's stiffness and is the ratio of stress to strain for a material in its linear (elastic) region
  (how much it deforms under a certain stress) (a material's ability to withstand changes in length with added load)
- It indicates how much stress is required to produce a certain amount of strain
  A higher Young's modulus means the material is stiffer and less deformable
  A lower Young's modulus means the material is quite flexible and stretchable
- The units are the same as stress, Pascals (Pa) because strain has no units

$$\boxed{E = \frac{\text{Stress } (\sigma)}{\text{Strain } (\varepsilon)} = \frac{F/A}{\Delta x/x} = \frac{Fx}{\Delta x A}}$$

For a material exhibiting elastic behaviour (i.e. up to the elastic limit or the limit of proportionality, stress and strain are directly proportional to each other just like with force-extension graphs)



In the linear region,
Gradient = Young modulus

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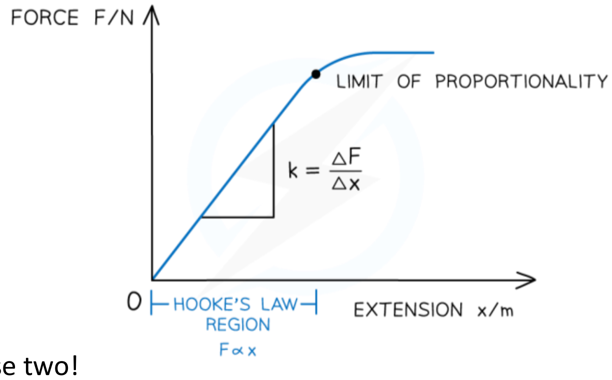
a be able to draw and interpret force-extension and force-compression graphs

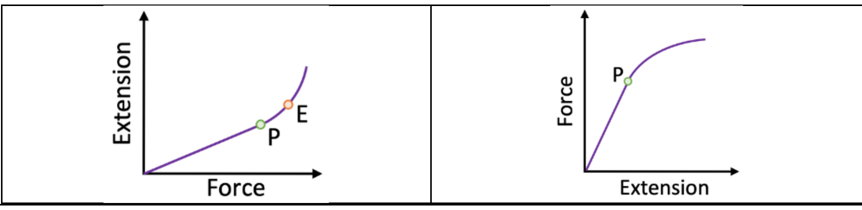
b understand the terms limit of proportionality, elastic limit, yield point, elastic deformation and plastic deformation and be able to apply them to these graphs

a

Force-extension and force-compression graphs



Be careful with these two!



The ***ultimate tensile stress*** is the maximum force per original cross-sectional area a wire can support until it breaks

b

Important terms related to force-extension graphs:

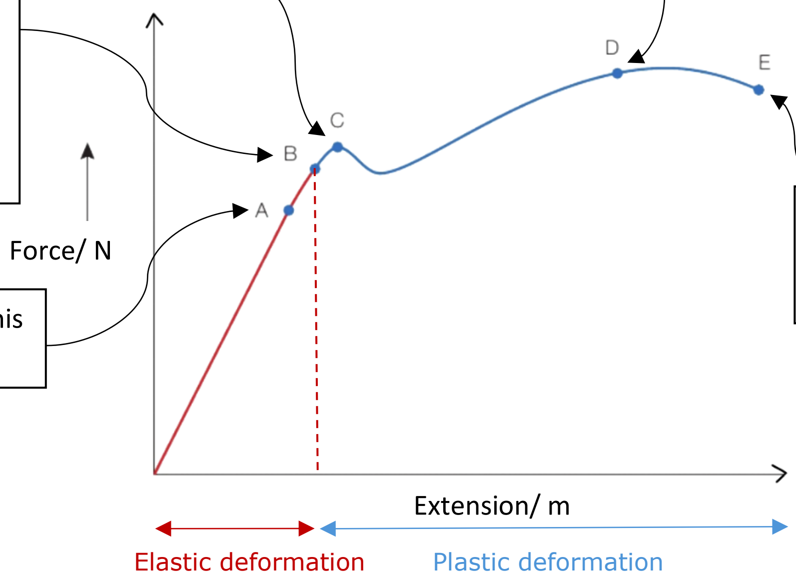
Yield point (beyond this point, material undergoes a large sudden plastic deformation, i.e. atoms start to 'slip' and take up new positions)

Ulimate tensile stress (highest possible stress within a material)

Elastic limit (up to this point, material may behave elastically, i.e. return to its original shape but may not obey Hooke's law) (beyond this point, material will not return to its original shape when stress/ force is removed)

Limit of proportionality (up to this point, object obeys Hooke's law)

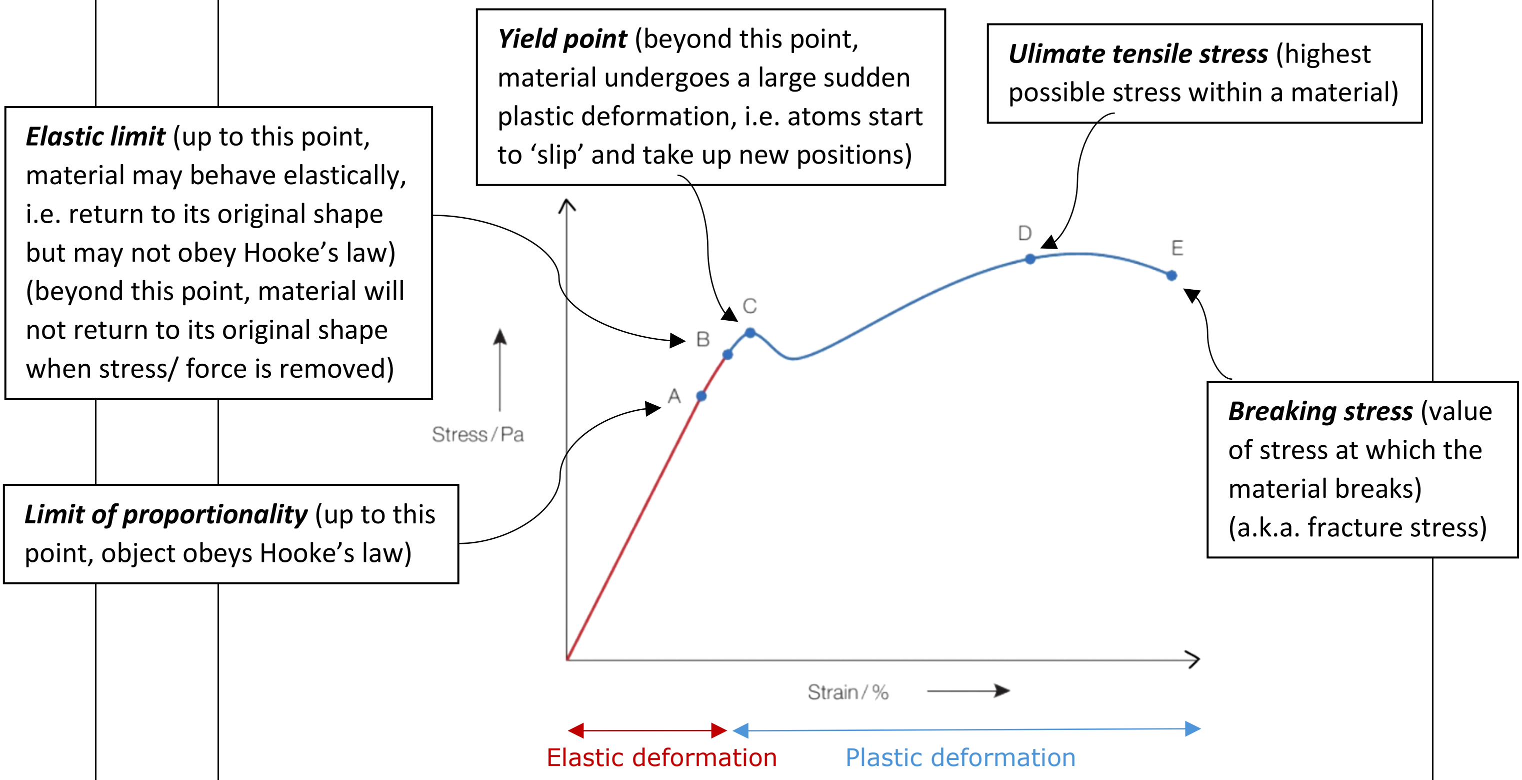
Breaking stress (value of stress at which the material breaks)



- Elastic deformation is when the material will return to its original shape when the stress/load/force is removed
- Plastic deformation is when the material will not return to its original shape when the stress/load/force is removed
- Gradient of force-extension graph represents spring constant, k (Note that these gradients must be from the linear part of the graph)
- Area under a force-extension graph represents the work done in stretching the material (i.e. the energy stored in the material when it is stretched) (which is stored as elastic potential energy if the material returns to its original shape, i.e. if material is within the elastic limit)

30	be able to draw and interpret tensile or compressive stress-strain graphs, and understand the term *breaking stress*

Stress-strain graphs



- Elastic deformation is when the material will return to its original shape when the stress/load/force is removed
- Plastic deformation is when the material will not return to its original shape when the stress/load/force is removed
- Gradient of stress-strain graph represents the Young Modulus, E
- Area under a stress-strain graph represents the energy absorbed per unit volume of a material as it deforms

So you may be wondering- what's the difference between Force-Extension Graph and Stress-Strain graph?
Though they may have the same shape and describe the same material properties, the key difference is that a **force-extension graph is specific to a particular object**, while a **stress-strain graph is a general property of the material** itself, independent of the object's dimensions

31	**CORE PRACTICAL 3: Determine the Young modulus of a material**
32	be able to calculate the elastic strain energy E_{el} in a deformed material sample, using the equation $\Delta E_{el} = ½ F\Delta x$, and from the area under the force-extension graph *the estimation of area and hence energy change for both linear and non-linear force-extension graphs is expected.*

When a material is stretched, work is done on it

Elastic strain energy

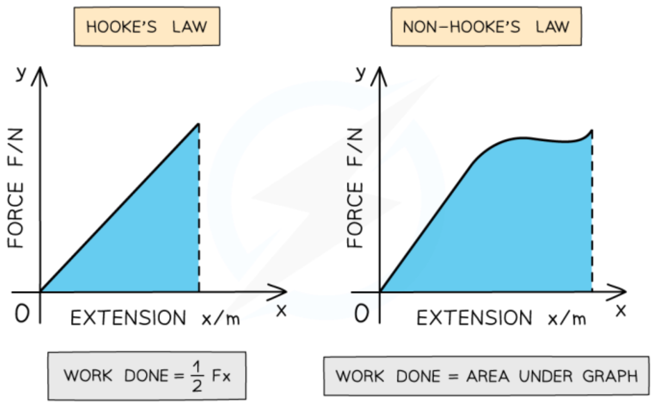
- To calculate the elastic strain energy (E_{el}) in a material, you can use the area under a force-extension graph like we mentioned above
- The method depends on whether the material's behavior is linear (obeys Hooke's Law) or non-linear (curved)
- What is elastic strain energy anyways?
- **Elastic strain energy** is the potential energy stored in a material as a result of a reversible deformation
- When the deforming force is removed, this stored energy can be released as the material returns to its original shape
- The **work done** by the force to deform the material is converted into this energy

Up to the elastic limit (i.e. whilst it obeys Hooke's law):

Work Done = Elastic Strain Energy = Area under force-extension graph

$\Delta E_{el} = ½ F\Delta x$ (or since F=kΔx) $\Delta E_{el} = ½ k(\Delta x)^2$

Since Work Done = F x d the average force applied during an extension of Δx is ½ F



! Note that "Work Done = Area under F-x graph" whether it obeys Hooke's law or not

For **linear graphs**, area under graph is the area of a right-angle triangle (A = ½bh)

For **non-linear graphs**, area under graph can be estimated using small squares, counting them and adding up the areas

Remarks

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- This booklet is primarily exam-based and has been produced for last-minute revision in your exams by making the information in the syllabus into a simpler and more compact form factor
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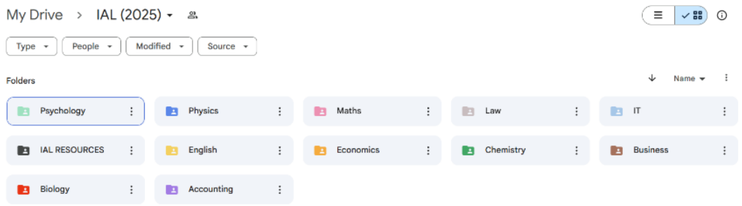

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