Please read Remarks on last page first

Unit 1: Mechanics and Materials

1.3 Mechanics

This topic covers rectilinear motion, forces, energy and power. It may be studied using applications that relate to mechanics such as sports.

This unit includes many opportunities for developing experimental skills and techniques by carrying out more than just the core practical experiments.

Candidates will be assessed on their ability to:

<table>
<tr><td>1</td><td>be able to use the equations for uniformly accelerated motion in one dimension:
$s = \frac{(u+v)t}{2}$
$v = u + at$
$s = ut + ½ at^2$
$v^2 = u^2 + 2as$</td></tr>
<tr><td></td><td>Equations of Motion
The equations of motion, a.k.a. the SUVAT equations, are a set of formulas used to describe the motion of an object undergoing constant acceleration (but not zero) and motion is in a straight line
- These equations relate displacement (s), initial velocity (u), final velocity (v), acceleration (a), and time (t).
- Here are the four primary equations of motion:
1. $v = u + at$: (when displacement is not given)
2. $s = ut + (1/2)at^2$: (when final velocity is not given)
3. $v^2 = u^2 + 2as$: (when time taken is not given)
4. $s = \frac{(u+v)t}{2}$: (when acceleration is not given)
- It is important to note that the variables u, v, a and s are vector quantities so can be negative as well as positive depending on its direction
- Time is a scalar quantity so will not be negative

Useful tips!
○ Objects in motion often start or end at rest, making either u or v = 0
○ Objects in freefall have a constant acceleration of 9.81 m/s^2
○ When an object is slowing down it has a negative value for acceleration, this must be included in the calculation
○ Objects thrown or shot upwards also have negative acceleration (since they are slowing down as they ascend) and reach a final velocity of zero at the top of their path (at its maximum height)</td></tr>
</table>

2 be able to draw and interpret displacement-time, velocity-time and acceleration-time graphs

Displacement-Time Graph

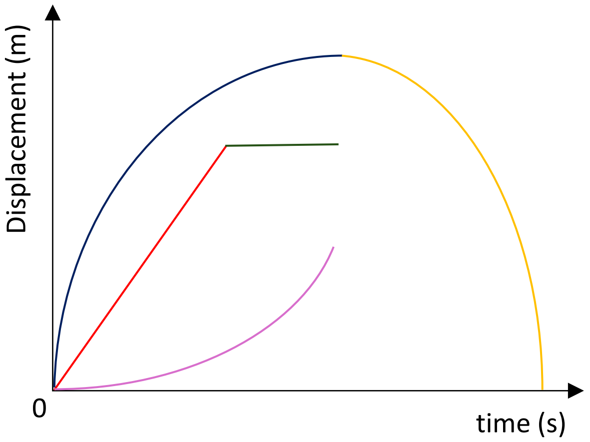
- **Gradient of graph = velocity**
- **Area under graph is meaningless**

A ***curve*** means that the object is *accelerating*
A ***straight diagonal*** **line** means that the object is moving with *constant velocity*
A ***straight horizontal line*** *(zero gradient)* means that the object is *stationary* (not moving)
A ***positive*** gradient represents motion in the positive direction
A ***negative*** gradient represents motion in the negative direction

A steep gradient means the object is moving at high constant velocity

A flat horizontal line means the object is stationary (not moving)

A curve with increasing gradient means the object is accelerating (can be at a constant or increasing rate depending on the shape of curve)



A curve with increasing negative gradient means the object is in negative acceleration (speeding up back to point it started)

A curve with decreasing positive gradient means the object is decelerating (slowing down)

Velocity-Time Graph

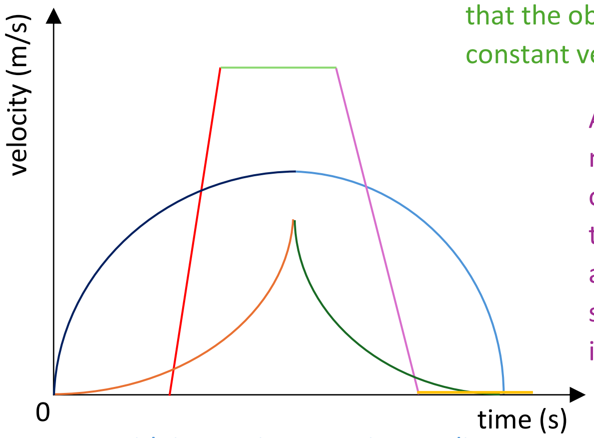
- **Gradient = acceleration**
- **Area under the curve = the displacement or distance travelled**

A curve indicates changing (non-uniform) acceleration
A straight diagonal line represents uniform acceleration
A **straight horizontal line** (zero gradient) represents motion with constant velocity
A **positive** gradient represents an increase in velocity in the positive direction
A **negative** gradient represents an increase in velocity in the negative direction

A straight line with a positive gradient means object is travelling with constant positive acceleration

A straight line at the bottom (v = 0) means that the object is stationary

A flat straight horizontal line means that the object is moving with constant velocity (zero acceleration)



A straight line with a negative gradient means object is accelerating in the negative direction at a constant rate (in this specific case, the object is said to be decelerating)

A curve with increasing negative gradient means it is accelerating in the negative direction at an increasing rate

A curve with decreasing negative gradient means it is accelerating in the negative direction at a decreasing rate

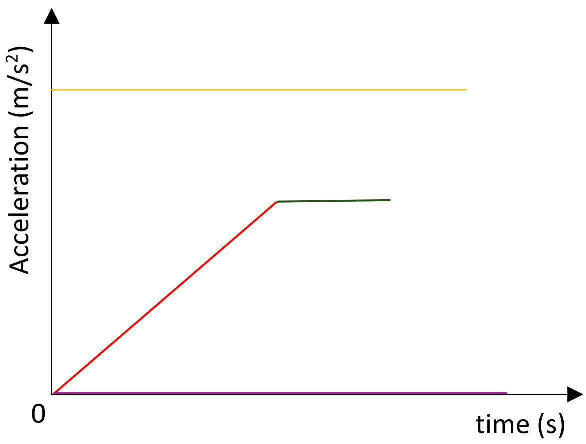
Acceleration-Time Graph

- **Area under graph = change in velocity**

A ***straight diagonal*** **line** means that the object is accelerating at a constant changing rate
A straight horizontal line at the bottom (a=0) *(zero gradient)* means that the object is *moving at constant velocity*
A straight horizontal line (not at a = 0) means that the object is *moving at constant acceleration*

A ***straight diagonal*** **line** means that the object is accelerating at a constant increasing rate

A straight horizontal line at the bottom (a=0) *(zero gradient)* means that the object is *moving at constant velocity*



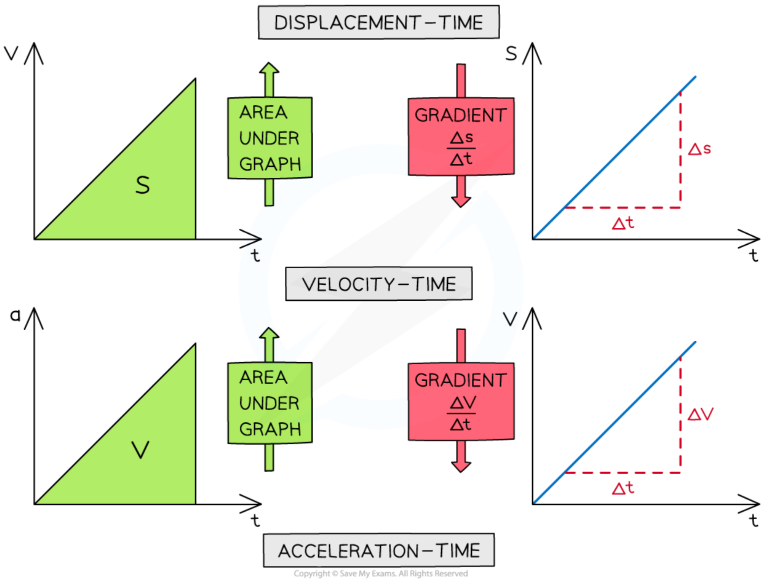
A flat horizontal line means the object is at constant accelerating

A straight horizontal line that is not at a = 0 means the object is moving at constant acceleration

!Just to clear things up:
Negative acceleration and Deceleration are related but NOT exactly the same thing
Negative acceleration means acceleration in the opposite direction of chosen positive direction (i.e. when gradient of velocity-time graph is negative)
It can be both speeding up in the opposite direction or slowing down in the same direction
Meanwhile, Deceleration only means slowing down regardless of direction so deceleration can be said to be a specific case of negative acceleration

3	know the physical quantities derived from the slopes and areas of displacement-time, velocity-time and acceleration-time graphs, including cases of non-uniform acceleration and understand how to use the quantities

Cases of non-uniform acceleration
Non-uniform acceleration such as the free fall of a skydiver will produce a **curved** velocity-time graph.
To find out its gradient at a certain point or time, you'll have to draw a tangent at that point and calculate its gradient.
When drawing tangents, make it as long as possible!



4 understand scalar and vector quantities and know examples of each type of quantity and recognise vector notation

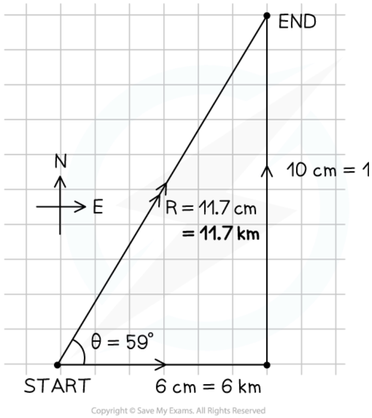
Scalar quantities only have **magnitude**
Vector quantities have both **magnitude and direction**

Scalar	Vector
Distance	Displacement
Speed	Velocity
Mass	Weight
Energy/ Work done	Force
Volume	Acceleration
Density	Momentum
Temperature	Moment
Power	Electric/ Gravitational field strength
Charge	
Efficiency	
Electric/ Gravitational potential	

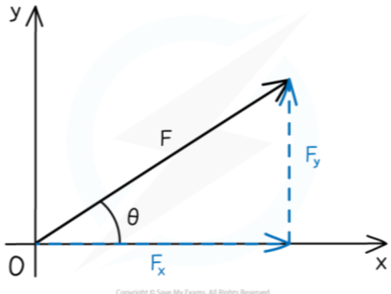
5 be able to resolve a vector into two components at right angles to each other by drawing and by calculation

Resolving Vectors by Scale Drawing

- Choose a suitable scale which fits to the page
- Draw the resultant vector with the correct length corresponding to its magnitude using a ruler and correct angle according to its direction using protractor
- Draw a horizontal line from the tail of the resultant vector and then connect the head of this horizontal vector with the head of the resultant vector
- The vector components must be perpendicular to each other



Resolving Vectors by Calculation

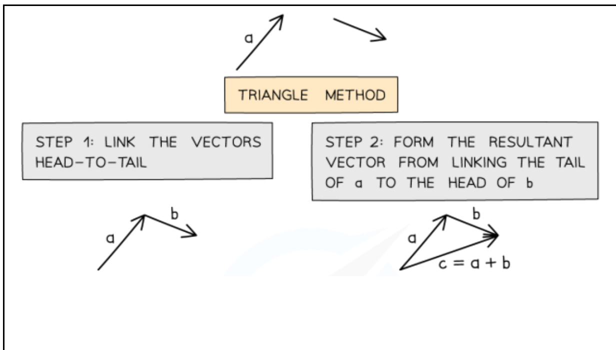


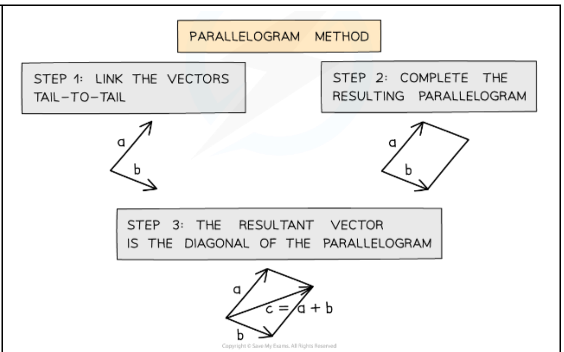
Using trigonometry, the resultant vector F can be resolved into its components:
For the **horizontal** component, $\boldsymbol{F_x = F\cos\vartheta}$
For the **vertical** component, $\boldsymbol{F_y = F\sin\vartheta}$

6 be able to find the resultant of two coplanar vectors at any angle to each other by drawing, and at right angles to each other by calculation

Coplanar vectors are vectors that lie within the same plane

Adding vectors at any angle by scale drawing

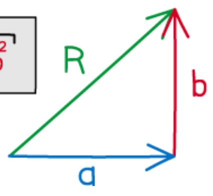


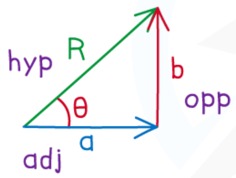


Adding vectors at right angles by calculation

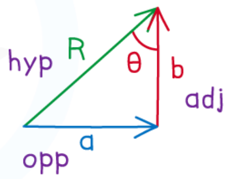
- Finding the **direction** of the resultant using **trigonometry**
- Finding the **magnitude** of the resultant using **Pythagoras**

$$R = \sqrt{a^2 + b^2}$$





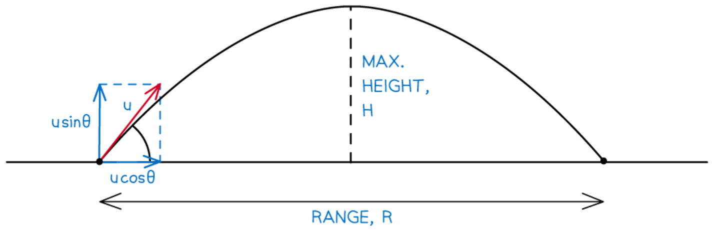
$$\tan\theta = \frac{opp}{adj} = \frac{b}{a}$$



$$\tan\theta = \frac{opp}{adj} = \frac{a}{b}$$

7 understand how to make use of the independence of vertical and horizontal motion of a projectile moving freely under gravity

Projectile motion



VERTICAL MOTION (↑)

INITIAL SPEED, $u = u\sin\theta$

ACCELERATION, $a = 9.81\ ms^{-2}$

DISPLACEMENT = 0

TIME OF FLIGHT

$u = u\sin\theta \quad v = 0 \quad a = -g \quad t = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v = u + at$

$0 = u\sin\theta - gt$

$t = \frac{u\sin\theta}{g}$

$2t = \frac{2u\sin\theta}{g}$

IF THE TIME TO MAXIMUM HEIGHT IS t, THEN THE TIME OF FLIGHT IS 2t

MAXIMUM HEIGHT ATTAINED

$u = u\sin\theta \quad v = 0 \quad a = -g \quad H = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v^2 = u^2 + 2as$

$0 = (u\sin\theta)^2 - 2gH$

$2gH = (u\sin\theta)^2$

$H = \frac{(u\sin\theta)^2}{2g}$

HORIZONTAL MOTION (→)

INITIAL SPEED, $u = u\cos\theta$

ACCELERATION, $a = 0$

DISPLACEMENT = R

RANGE (R)

$u = u\cos\theta \quad t = \frac{2u\sin\theta}{g} \quad a = 0 \quad R = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

DISTANCE = SPEED × TIME

$R = (u\cos\theta)t$

$R = \frac{2u^2\sin\theta\cos\theta}{g}$

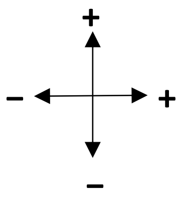
$R = \frac{u^2\sin 2\theta}{g}$

USING THE TRIG IDENTITY: $2\sin\theta\cos\theta = \sin 2\theta$

Sign convention

You may choose any direction to be positive, so long as you are consistent with it in your calculations

So it is advised to add a label so that the examiner can clearly tell which direction you chose. Generally:



The trajectory of an object undergoing projectile motion can be resolved into a **vertical component** and a **horizontal component**

The time of flight is the same for both vertical and horizontal components (common *t*)

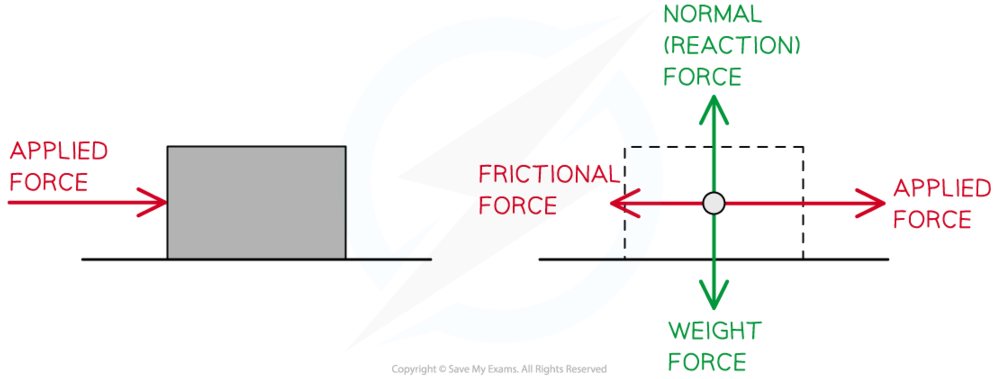
At maximum height, the projectile is momentarily at rest (v = 0)

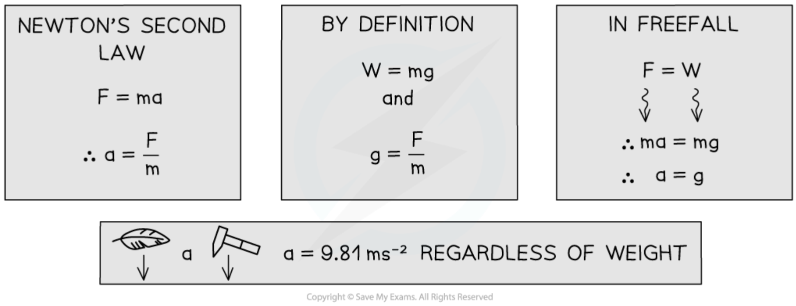
Range is the horizontal distance traveled by the projectile

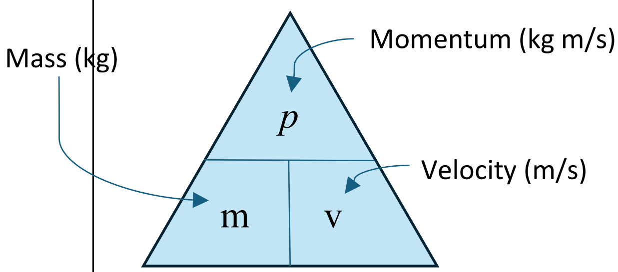
Sign Convention

Generally, we consider the direction the object is initially travelling in as positive

So all vectors in the direction of motion will be positive and opposing vectors, such as drag forces, will be negative

8	be able to draw and interpret free-body force diagrams to represent forces on a particle or on an extended but rigid body using the concept of *centre of gravity* of an extended body
	We can use a free-body diagram to represent forces acting on a particle or on an **extended but rigid body** (which simply means that all parts stay in the same position relative to each other when the object moves) You can draw a free-body diagram by drawing the centre of gravity (centre of mass) and drawing the forces acting on that body using vectors with correct direction and proportional length • Free body diagrams simplify problems by reducing complex shapes into simple one • Any object can be thought of as a particle (or a point mass) 
9	be able to use the equation $\Sigma F = ma$, and understand how to use this equation in situations where *m* is constant (Newton's second law of motion), including Newton's first law of motion where $a = 0$, objects at rest or travelling at constant velocity *Use of the term 'terminal velocity' is expected.*
ΣF means sum of all forces acting on object	**Newton's First Law (Law of Inertia)** states that: ***"An object at rest will stay at rest and an object in motion will stay in motion with the same speed and direction unless acted upon by an unbalanced external force"*** - This means if the forces in any direction are balanced, then the resultant force ΣF is 0 - Hence, the acceleration is also 0 - Hence, the velocity is constant (or uniform) - The velocity (i.e. speed and direction) **can only change** if a **resultant force** acts on the object **Newton's Second Law (Law of Force and Acceleration)** states that: ***"The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass"*** This is expressed as the formula $\Sigma F = ma$ - If the resultant force is **along the direction of motion**, the body will speed up (**accelerate**) or slow down (**decelerate**) - If the resultant force is at an **angle**, the body will change **direction** **Terminal Velocity** On Earth, there is always some force that acts on objects in motion like friction Terminal velocity is reached when the forces in the direction of motion are balanced by the forces opposing motion .

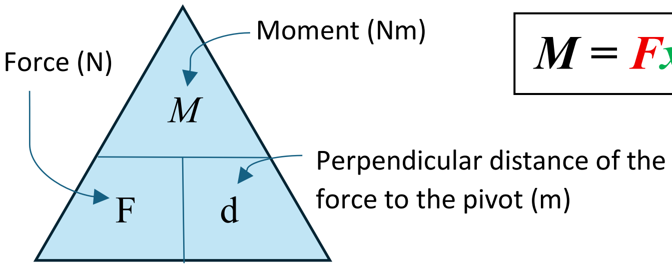
<table>
<tr><td>10</td><td>be able to use the equations for gravitational field strength $g = \frac{F}{m}$ and weight $W = mg$</td></tr>
<tr><td></td><td>


</td></tr>
<tr><td>11</td><td>CORE PRACTICAL 1: Determine the acceleration of a freely-falling object</td></tr>
<tr><td>12</td><td>know and understand Newton's third law of motion and know the properties of pairs of forces in an interaction between two bodies</td></tr>
<tr><td></td><td>Newton's third law of motion states that:
"Whenever two bodies interact, the force they exert on each other are equal in magnitude and opposite in direction"
- All forces arise in <u>pairs</u> – If A exerts a force on B, then B exerts an equal and opposite force on A
- Force pairs are of the <u>same type</u> – if A exerts a gravitational force, then B also exerts an equal and opposite gravitational force
- <u>Two bodies</u> are exerting the same type of force on one another, not one body exerting two forces in opposite directions</td></tr>
<tr><td>13</td><td>understand that momentum is defined as $p = mv$</td></tr>
<tr><td></td><td>Momentum


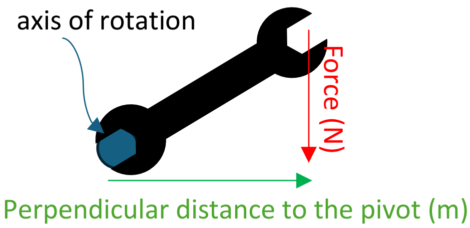
$$\boxed{p = mv}$$
!Note that an object at rest has no momentum
Momentum of an object will change if:
- The object accelerates or decelerates
- Changes direction
- Mass changes</td></tr>
<tr><td>14</td><td>know the principle of conservation of linear momentum, understand how to relate this to Newton's laws of motion and understand how to apply this to problems in one dimension</td></tr>
<tr><td></td><td>Law of conservation of linear momentum states that:
"The total momentum before a collision = the total momentum after a collision provided no external force acts (i.e. in an isolated/closed system)"</td></tr>
</table>

15 be able to use the equation for the moment of a force, moment of force = Fx where x is the perpendicular distance between the line of action of the force and the axis of rotation

Moment



$$M = Fx$$



> Don't forget to take components if the given length is not perpendicular to the force applied

Moment is defined as: **The turning effect of a force about a pivot** (in Nm)

16 be able to use the concept of centre of gravity of an extended body and apply the principle of moments to an extended body in equilibrium

Centre of gravity (centre of mass) is defined as:
"The point through which the weight of an object acts"

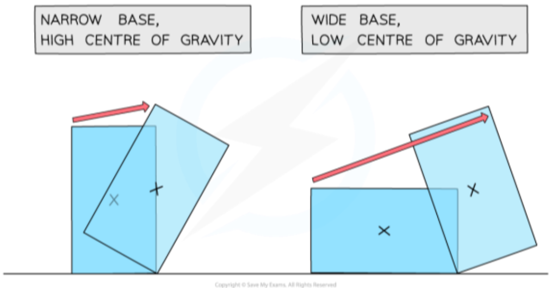
- For a symmetrical object of uniform density, the centre of gravity is located at the point of symmetry
- For an irregular object, its centre of gravity is found by locating its balance point

!Note that centre of gravity can lie inside or outside of a body

Stability

An object is stable when its centre of gravity lies above its base

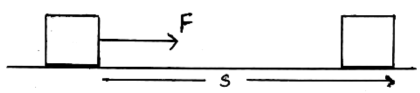
The **wider** base an object has, the **lower** its centre of gravity and it is more **stable**
The **narrower** base an object has, the **higher** its centre of gravity and the object is more likely to topple over if pushed



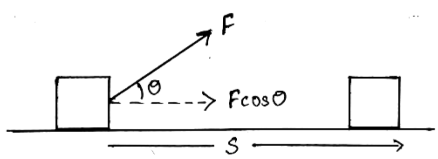
Principle of moments states that:
"For an object in equilibrium, sum of clockwise moment about a point = sum of anti-clockwise moment about the same point"

17 be able to use the equation for work $\Delta W = F\Delta s$, including calculations when the force is not along the line of motion

Work is defined as:
"The amount of energy transferred when an external force causes an object to move over a certain distance"



If the force applied is parallel to the direction of motion:
$W = F\cos 0\ s = Fs$



If the force applied is at an angle θ to the direction of motion:
$W = F\cos\theta\ s$

18 be able to use the equation $E_k = ½\,mv^2$ for the kinetic energy of a body

Kinetic energy (E_k or KE) is the energy an object has due to its **motion** (or velocity)

This means the faster an object moves, the greater its kinetic energy

Formula derivation of KE = ½ mv^2

$$W = F \times s$$

$$= ma \times \frac{v^2 - u^2}{2a}$$

$$W = \frac{1}{2} m (v^2 - u^2)$$

If u = 0, (object starts at rest)

$$W = \frac{1}{2} mv^2$$

Work done = Change in kinetic energy

$$\therefore \quad E_k = \frac{1}{2} mv^2.$$

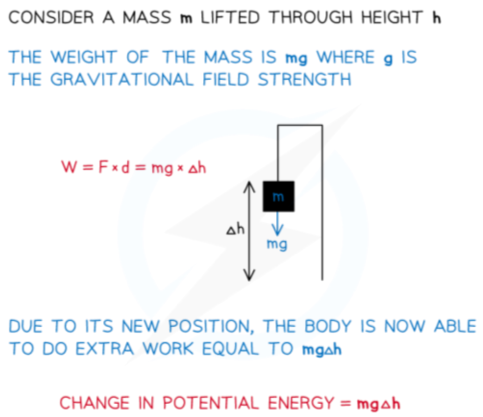
19 be able to use the equation $\Delta E_{grav} = mg\Delta h$ for the difference in gravitational potential energy near the Earth's surface

Gravitational potential energy (E_{grav} or GPE) is the energy stored in a mass due to its position in a gravitational field

- If a mass is **lifted** up, it will **gain** GPE (converted **from** other forms of energy)
- If a mass **falls**, it will **lose** GPE (and be converted **to** other forms of energy)

GPE near the Earth's surface where it is uniform has the equation $\Delta E_{grav} = mg\Delta h$

Formula derivation of GPE = mgΔh



20 know, and understand how to apply, the principle of conservation of energy including use of work done, gravitational potential energy and kinetic energy

The Principle of Conservation of Energy states that:

"Energy can neither be created nor destroyed, it can only be transferred from one store to another"

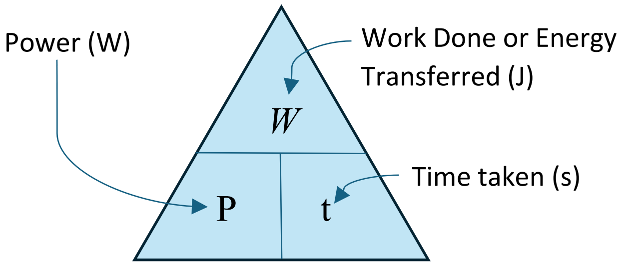
This means the total energy input should always be equal to the total energy output

- Conservation of energy is often applied in questions about exchanges between **kinetic energy** and **gravitational potential energy**

Example:

21	be able to use the equations relating power, time and energy transferred or work done $P = E/t$ or $P = W/t$
	Power is defined as: "The rate of doing work" (or) "The rate of transfer of energy" "The work done per unit time" (or) "The energy transfer per unit time"
22	be able to use the equations $\text{efficiency} = \dfrac{\text{useful energy output}}{\text{total energy input}}$ and $\text{efficiency} = \dfrac{\text{useful power output}}{\text{total power input}}$
	Efficiency is defined as: **"The ratio of the useful power OR energy transfer output from a system to its total power or energy transfer input"** Efficiency has no units as it is a ratio and the units on the numerator and denominator cancel each other out

1 Watt = 1 J/s



$$\boxed{P = W/t}$$

$$\boxed{P = E/t}$$

$$\text{EFFICIENCY} = \frac{\text{USEFUL POWER OUTPUT}}{\text{TOTAL POWER INPUT}} \times 100\%$$

$$\text{EFFICIENCY} = \frac{\text{USEFUL ENERGY OUTPUT}}{\text{TOTAL ENERGY INPUT}} \times 100\%$$