5.5 Oscillations

This topic covers simple harmonic motion and damping.

This topic may be studied using applications that relate to, for example, the construction of buildings in earthquake zones.

This unit includes many opportunities for developing experimental skills and techniques by carrying out more than just the core practical experiments.

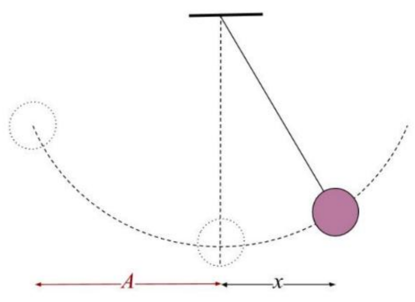
Candidates will be assessed on their ability to:

143 understand that the condition for simple harmonic motion is $F = -kx$, and hence understand how to identify situations in which simple harmonic motion will occur

Simple Harmonic Motion

- ***An object experiencing simple harmonic motion is one which experiences a restoring force, which always acts towards the centre of equilibrium***
- ***This restoring force is directly proportional to the object's distance from the equilibrium position***
- This can be described using the equation: $\boldsymbol{F = -kx}$

Where F is the restoring force, k is a constant which depends on the oscillating system and x is the distance from the equilibrium position



An example of a simple harmonic oscillator is the simple pendulum (The pendulum oscillates around a central midpoint known as the equilibrium position)

X is the measure of displacement and *A* is the amplitude of the oscillations (*A* is the maximum displacement)

The *time period* (*T*) of the oscillations can be found out by measuring the time taken by the pendulum to move from the equilibrium position to the maximum displacement to the left, then to the maximum displacement to the right and back to the equilibrium position

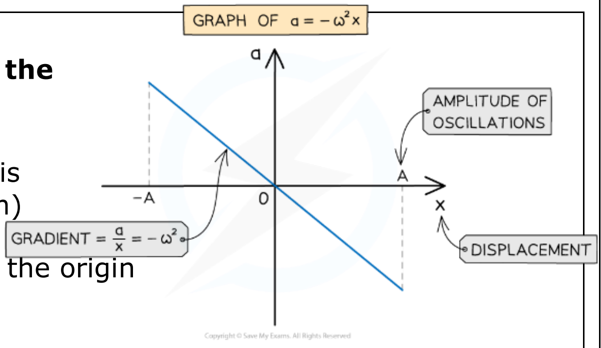
In a simple pendulum, the restoring force is provided by the horizontal component of gravity acting on the pendulum bob

144 be able to use the equations $a = -\omega^2 x$, $x = A\cos\omega t$, $v = -A\omega\sin\omega t$, $a = -A\omega^2\cos\omega t$, and $T = \frac{1}{f} = \frac{2\pi}{\omega}$ and $\omega = 2\pi f$ as applied to a simple harmonic oscillator

Equations for Simple Harmonic Motion

Acceleration

- **The acceleration is proportional to the displacement** ($a \propto -x$)
- **The acceleration is in the opposite direction to the displacement** (this is indicated by the minus sign of equation)
- Graph of acceleration against displacement is a straight line through the origin sloping downwards



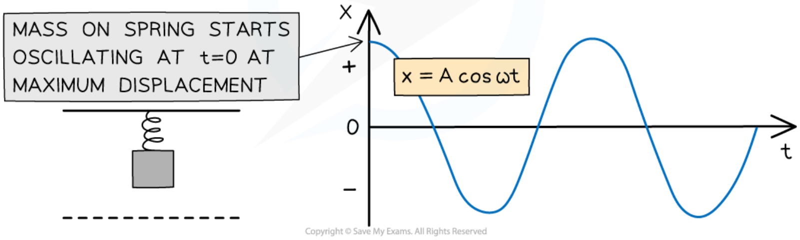
$$a = -\omega^2 x$$

Where a is acceleration, ω is angular velocity and x is displacement from equilibrium position

Displacement

- When an object is oscillating from its amplitude position (x = A or x = $-A$, at t= 0), the displacement equation is:

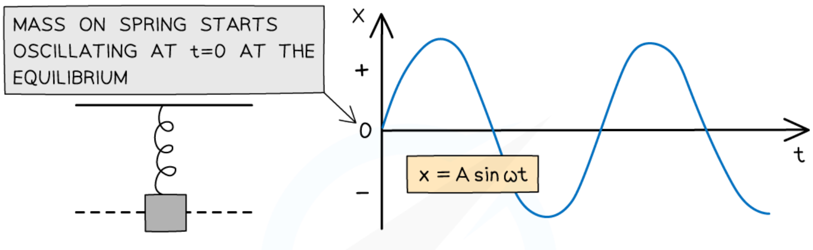
$$x = A\cos(\omega t)$$



- The displacement will be at its maximum when cos(ωt) equals 1 or −1, when x = A

- When an object is oscillating from its equilibrium position (x = 0 at t = 0) then the displacement equation will be:

$$x = A\sin(\omega t)$$



- The displacement will be at its maximum when sin(ωt) equals 1 or −1, when x = A
- This is because the sine graph starts at 0, whereas the cosine graph starts at a maximum

Speed

- The v_{max} of an oscillator is at the equilibrium position i.e. when its displacement is x = 0
- $v_{max} = 2\pi f A$

Angular speed (ω) is the angle an object moves through per unit time

$$\omega = 2\pi f$$

By rearranging the above formula so that its subject is frequency, you can derive the following formula for the time period of oscillations (T)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Using these measurements: x (displacement) A (amplitude) and t (time period), you can use the following formulas with simple harmonic oscillators:

$$x = A\cos\omega t$$

$$v = -A\omega\sin\omega t$$

$$a = -A\omega^2\cos\omega t$$

145 be able to use equations

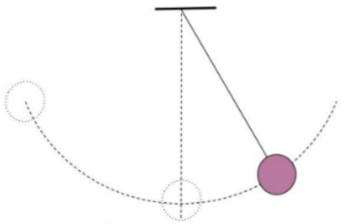
for a simple harmonic oscillator $T = 2\pi\sqrt{\frac{m}{k}}$, and a simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

Examples of simple harmonic systems (systems oscillating with simple harmonic motion)

Notice that in mass-spring system,
The restoring force equation is the same as Hooke's law
F = −kx

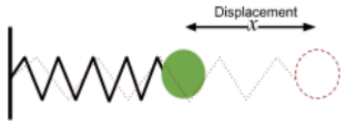
Simple Pendulum
- An object moving from side to side
- Hanging from a string of length l
- Attached to a fixed point above

$$T = 2\pi\sqrt{\frac{l}{g}}$$



Where l is the length of string

Mass-spring system (two types)
- Spring is vertical
- Spring is horizontal



$$T = 2\pi\sqrt{\frac{m}{k}}$$

Where k is the spring constant

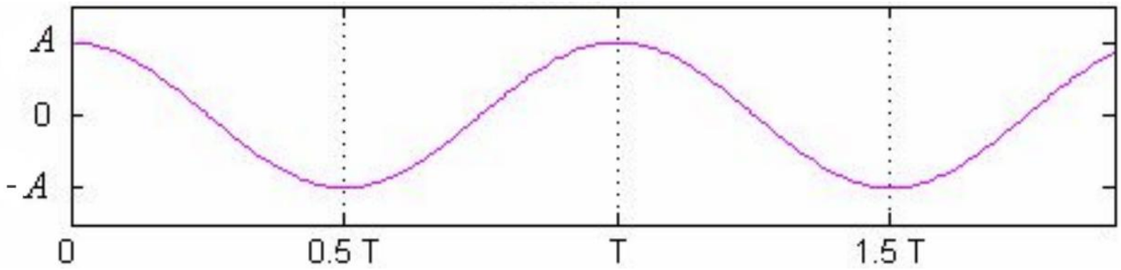
146 be able to draw and interpret a displacement-time graph for an object oscillating and know that the gradient at a point gives the velocity at that point

Displacement-time graph

All undamped simple harmonic motion graphs are represented by **periodic functions**
This means it can be either a sine or cosine graph depending on where the object starts oscillating at t = 0:

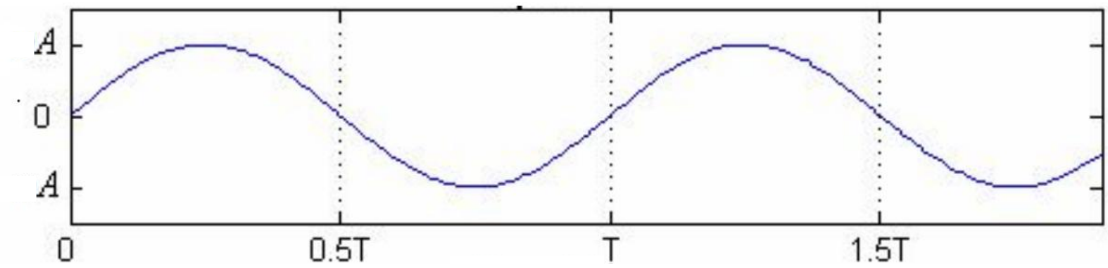
1. **If object starts oscillating on either side of the equilibrium** (cosine graph)

   (i.e.) If at t = 0, x = max, then $x = A\cos \omega t$



2. **If object starts oscillating at the equilibrium** (sine graph)

   (i.e.) If at t = 0, x = 0, then $x = A\sin \omega t$

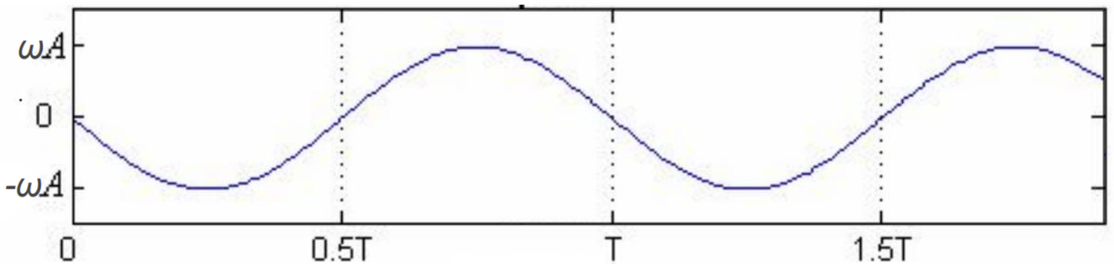


147	be able to draw and interpret a velocity-time graph for an oscillating object and know that the gradient at a point gives the acceleration at that point

Velocity-time graph

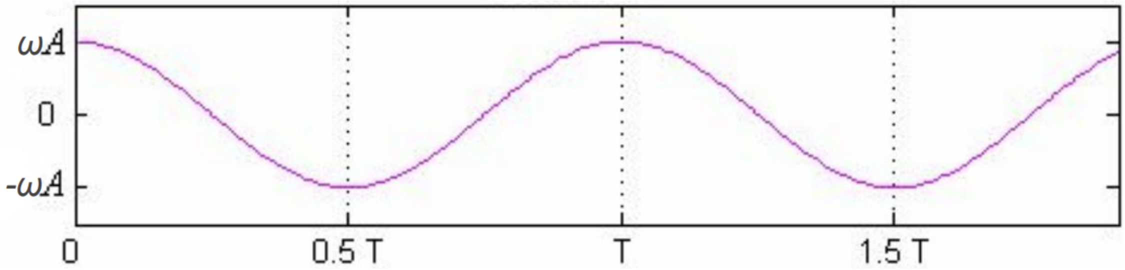
1. **If object starts oscillating on either side of the equilibrium** ($-$sine graph)

   (i.e.) If at t = 0, x = max, then $v = -A\omega \sin \omega t$



2. **If object starts oscillating at the equilibrium** (cosine graph)

   (i.e.) If at t = 0, x = 0, then $v = A\omega \cos \omega t$



Key features of the velocity-time graph:

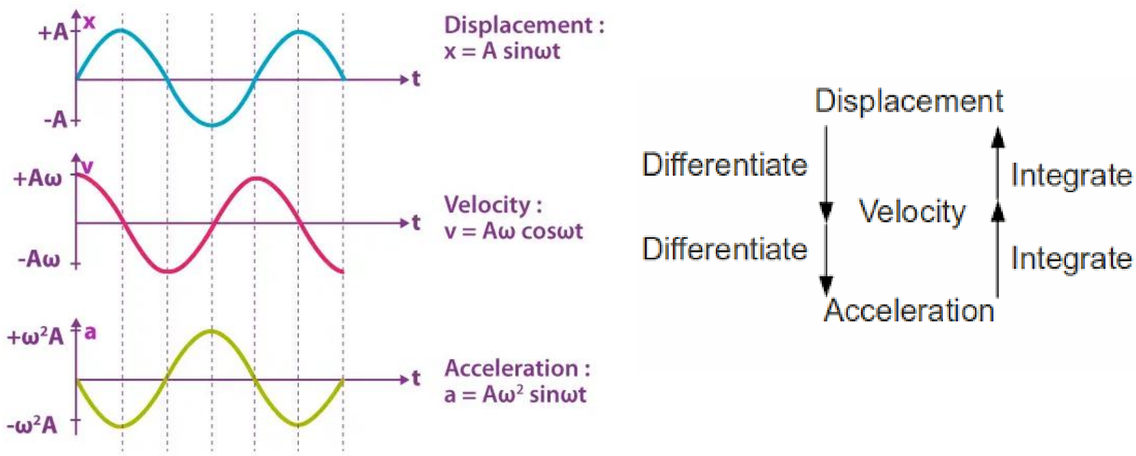
- It is 90° out of phase with the displacement-time graph
- Velocity = the rate of change of displacement = gradient of x-t graph
- Gradient of v-t graph = acceleration at that point

An oscillator moves the fastest at its equilibrium position

Therefore, the "maximum velocity is when the displacement is zero"

Integration and Differentiation

It's useful to know this!



148 understand what is meant by *resonance*

Resonance is defined as:

"When the frequency of the applied force to an oscillating system is equal to its natural frequency, the amplitude of the resulting oscillations increases significantly"

- The frequency of forced oscillations is referred to as the **driving frequency**, *f*, or the frequency of the applied force
- All oscillating systems have a **natural frequency**, f_0, this is defined as the frequency of oscillation when the oscillating system is allowed to oscillate freely

At resonance, there is **maximum energy transfer** from driver to oscillating system

Resonance has many applications for example:

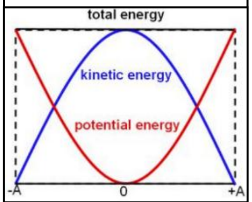
- Instruments - An instrument such as a flute has a long tube in which air resonates, causing a stationary sound wave to be formed

149 **CORE PRACTICAL 16: Determine the value of an unknown mass using the resonant frequencies of the oscillation of known masses**

150 understand how to apply conservation of energy to damped and undamped oscillating systems

Damping occurs at all frequencies, not just at resonance

In an undamped system, no energy is lost so the total energy of the system remains constant



Damping is defined as:

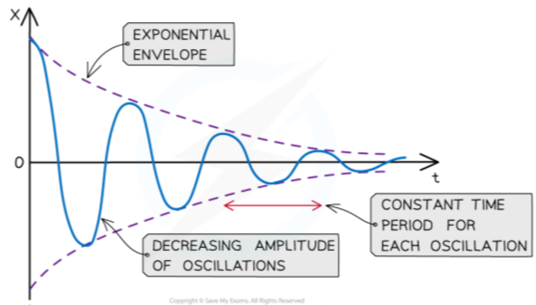
"The reduction in energy and amplitude of oscillations due to resistive forces on the oscillating system"

- Note that the *frequency of damped oscillations does not change* as the amplitude decreases (eg: swinging pendulum)
- Total energy of the system decreases over time

Types of Damping

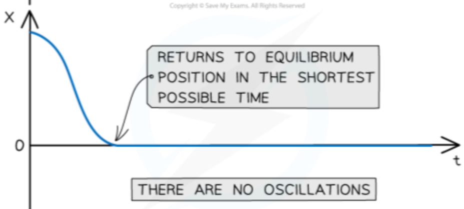
1. **Light Damping**

When a lightly damped oscillator is displaced from the equilibrium, it will oscillate with **exponentially decreasing amplitude**



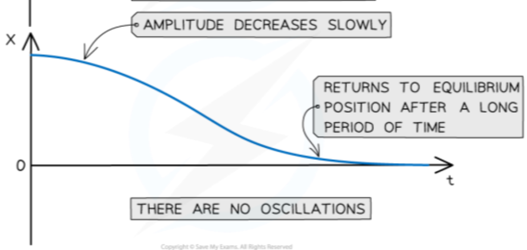
2. **Critical Damping**

When a critically damped oscillator is displaced from the equilibrium, it will return to rest at its equilibrium position in the shortest possible time **without** oscillating (eg: car suspension system)



3. **Heavy Damping**

When a heavily damped oscillator is displaced from the equilibrium, it will take a long time to return to its equilibrium position **without** oscillating



151 understand the distinction between *free* and *forced oscillations*

Free oscillations are defined as:

"Oscillations where there are only internal forces (and no external forces) acting and there is no energy input"

- A free vibration always oscillates at its **resonant (natural) frequency**
- Amplitude stays constant over time

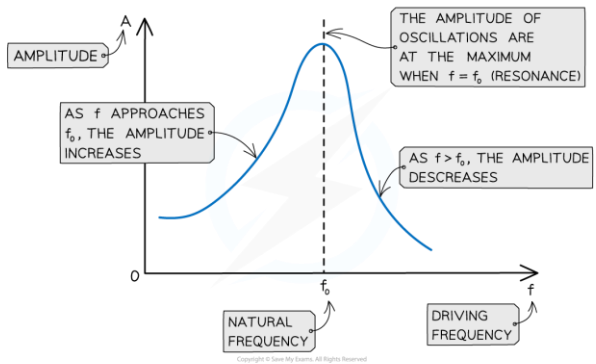
Forced oscillations are defined as:

"Oscillations acted on by a periodic external force where energy is given in order to sustain oscillations"

- To sustain oscillations in a simple harmonic system, a periodic force (external driving force) must be applied to replace the energy lost in **damping**
- Forced oscillations are made to oscillate at the same frequency as the oscillator creating the external, periodic driving force

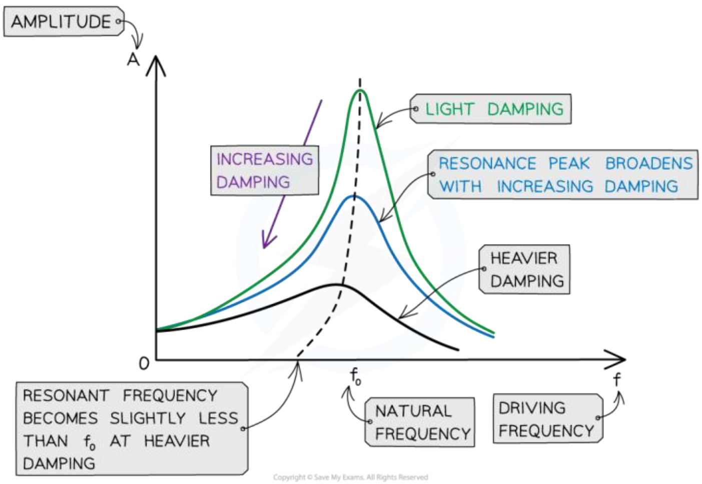
152	understand how the amplitude of a forced oscillation changes at and around the natural frequency of a system and know, qualitatively, how damping affects resonance

Resonance Graphs



How Damping affects Resonance

- Damping **reduces** the amplitude at resonance frequency
- Damping reduces the sharpness of resonance



153	understand how damping and the plastic deformation of ductile materials reduce the amplitude of oscillation

Damping an oscillator affects its **amplitude** of oscillation:

- When **damping** is **increased** the **amplitude decreased**
- **damping** and **amplitude** are **inversely proportional** to each other

A **ductile** material is one which can **undergo a large amount of plastic deformation** before fracturing, meaning it will be *permanently deformed*

The **plastic deformation** of a ductile material can be used to reduce the amplitude of oscillations, this happens because energy is used to deform the material, decreasing the kinetic energy of the system and so the amplitude of oscillations decreases