Econ 521: Econometric Methods I
Assignment 1

Name(s): Your name(s) go(es) here.

1. Consider the following regression model

$$\begin{aligned} y &= g(\mathbf{x}, \mathbf{z}_1) + \varepsilon; \qquad && \mathbf{z} = [\mathbf{z}_1', \mathbf{z}_2']', \\ \mathbf{x} &= \mathbf{h}(\mathbf{z}) + \mathbf{u}; && \mathbb{E}[\mathbf{u}|\mathbf{z}] = 0,\ \mathbb{E}[\varepsilon|\mathbf{z}, \mathbf{u}] = \mathbb{E}[\varepsilon|\mathbf{u}], \end{aligned}$$

   where y is an observable scalar random variable, g denotes a known scalar function, $\mathbf{x}$ is a $d_{\mathbf{x}} \times 1$ vector of explanatory variables, $\mathbf{z}_1$ and $\mathbf{z}_2$ are $d_1 \times 1$ and $d_2 \times 1$ vectors of instrumental variables, $\mathbf{h} := [h_1, \ldots, h_{d_{\mathbf{x}}}]'$ is a $d_{\mathbf{x}} \times 1$ vector of functions of instruments $\mathbf{z}$, and $\mathbf{u}$ and ε are disturbances. What does $\mathbb{E}[y|\mathbf{x}, \mathbf{z}, \mathbf{u}]$ equal to?

   Answer: Your answer goes here.

2. Let x be an absolutely continuous random variable with strictly increasing cdf F_x. Let $\widehat{q}$ be the value that minimizes $\mathbb{E}[\rho_\tau(x-q)]$ with respect to q, where $\rho_\tau(u) = u[\tau - \mathbb{I}(u < 0)]$ and $\mathbb{I}(A)$ is called the indicator function that equals one if A is true and 0 otherwise. Show that $\widehat{q} \equiv F_x^{-1}(\tau)$. Hint: Use the Leibniz integral rule.

   Answer: Your answer goes here.

3. Let y be the response variable variable, $\mathbf{x}$ a set of $d_{\mathbf{x}} \times 1$ conditioning variables, and s a scalar binary group indicator (such as gender, college graduate versus non-college graduate, and so on). Define $\mu_0(\mathbf{x}) = \mathbb{E}[y|\mathbf{x}, s = 0]$ and $\mu_1(\mathbf{x}) = \mathbb{E}[y|\mathbf{x}, s = 1]$ to be the regression functions for the two groups.

   (a) Show that

$$\begin{aligned} \mathbb{E}[y|s=1] - \mathbb{E}[y|s=0] =& \{\mathbb{E}[\mu_1(\mathbf{x})|s=1] - \mathbb{E}[\mu_0(\mathbf{x})|s=1]\} \\ &+ \{\mathbb{E}[\mu_0(\mathbf{x})|s=1] - \mathbb{E}[\mu_0(\mathbf{x})|s=0]\}, \end{aligned}$$

   Hint: Use a suitable representation of $\mathbb{E}[y|\mathbf{x}, s]$ as a function of $\mu_0(\mathbf{x})$ <u>and</u> $\mu_1(\mathbf{x})$, and then apply the *Law of Iterated Expectations*.
   Answer: Your answer goes here.

   (b) Suppose both expectations are linear: $\mu_s(\mathbf{x}) = \mathbf{x}'\beta_s$, $s \in \{0, 1\}$. Show that

$$\mathbb{E}[y|s=1] - \mathbb{E}[y|s=0] = \mathbb{E}[\mathbf{x}'|s=1] \times \{\beta_1 - \beta_0\} + \{\mathbb{E}[\mathbf{x}'|s=1] - \mathbb{E}[\mathbf{x}'|s=0]\} \times \beta_0.$$

   Can you interpret this decomposition?

   Answer: Your answer goes here.