Lesson 22: Practice Problems - Finding Potential Functions

ODE 1 - Prof. Adi Ditkowski

Part A: Method 1 - Integration with respect to x (Problems 1-5)

1. Find the potential function and solve: $(2x + 3y)dx + (3x + 4y)dy = 0$
2. Solve using Method 1: $(y^2 + 2xy + 1)dx + (x^2 + 2xy + 2y)dy = 0$
3. Find H(x,y) for: $(3x^2y + 2x)dx + (x^3 + y^2)dy = 0$
4. Solve: $(e^x \sin y + 2x)dx + (e^x \cos y + 2y)dy = 0$
5. Find the solution passing through (1,0): $(2xy + y^2)dx + (x^2 + 2xy)dy = 0$

Part B: Method 2 - Integration with respect to y (Problems 6-10)

6. Solve using Method 2: $(2x + 3y)dx + (3x + 4y)dy = 0$
7. Find H(x,y): $(x^2 + y^2)dx + (2xy + 3y^2)dy = 0$
8. Solve: $(ye^{xy} + 2x)dx + (xe^{xy} + y)dy = 0$
9. Find the potential function: $(\ln y + x)dx + \left(\frac{x}{y} + 2y\right) dy = 0$
10. Solve with initial condition y(0) = 1: $(2x + y)dx + (x + 2y)dy = 0$

Part C: Method 3 - Line Integral Approach (Problems 11-15)

11. Use line integral method: $(2xy)dx + (x^2 + 1)dy = 0$
12. Solve using path integration: $(y + 1)dx + (x + y)dy = 0$

13. Find H(x,y) by line integral: $\left(\frac{x}{\sqrt{x^2+y^2}}\right)dx + \left(\frac{y}{\sqrt{x^2+y^2}}\right)dy = 0$
14. Use Method 3: $(2x + y^2)dx + (2xy + 3y^2)dy = 0$
15. Compare all three methods for: $(3x^2 + 2y)dx + (2x + 4y)dy = 0$

Part D: Mixed Practice (Problems 16-20)

16. Choose the best method and solve: $(y^2e^{xy} + 2x)dx + (2ye^{xy} + xy^2e^{xy})dy = 0$
17. Find the solution through (1,1): $(2xy + x^2)dx + (x^2 + 2y)dy = 0$
18. Solve: $\left(\frac{y}{x^2} + \frac{1}{y}\right)dx + \left(-\frac{1}{x} + \frac{x}{y^2}\right)dy = 0$
19. Find H(x,y): $(\cos(x + y) + y)dx + (\cos(x + y) + x)dy = 0$
20. Verify and solve: $(2x\sin y + y^2)dx + (x^2\cos y + 2xy)dy = 0$

Part E: Initial Value Problems (Problems 21-25)

21. Solve the IVP: $(2xy + 1)dx + (x^2 + 2y)dy = 0$, $y(1) = 2$
22. Find the particular solution: $(e^x + y)dx + (x + e^y)dy = 0$, $y(0) = 0$
23. Solve: $(3x^2 + 2y)dx + (2x + 4y)dy = 0$, $y(0) = -1$
24. Find the curve: $(2x + 3y^2)dx + (6xy + 4y^3)dy = 0$ through $(1, -1)$
25. Solve the IVP: $\left(\frac{1}{x} + y\right)dx + (x + \ln x)dy = 0$, $y(1) = 0$

Solutions and Hints

Problem 1: Method 1: $H = \int(2x + 3y)dx = x^2 + 3xy + g(y)$ $\frac{\partial H}{\partial y} = 3x + g'(y) = 3x + 4y$, so $g'(y) = 4y$, $g(y) = 2y^2$ Solution: $x^2 + 3xy + 2y^2 = C$

Problem 6: Method 2: $H = \int(3x+4y)dy = 3xy+2y^2+f(x)$ $\frac{\partial H}{\partial x} = 3y+f'(x) = 2x+3y$, so $f'(x) = 2x$, $f(x) = x^2$ Same result: $x^2 + 3xy + 2y^2 = C$

Problem 11: Using path $(0,0) \to (x,0) \to (x,y)$: $H = \int_0^x 0\,dt+\int_0^y(x^2+1)ds = (x^2+1)y$ Solution: $(x^2 + 1)y = C$

Problem 21: First find $H = x^2y+x+y^2$ At $(1,2)$: $C = (1)^2(2)+1+(2)^2 = 7$ Particular solution: $x^2y + x + y^2 = 7$