

# Practice Problems: Lesson 1 - ODE Classification

Work these out before checking solutions!

## Part A: Basic Classification

Classify each ODE by order, linearity, and autonomy:

1.  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = \cos(2x)$

2.  $\left(\frac{dy}{dx}\right)^3 + y = x^2$

3.  $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

4.  $\frac{dy}{dx} = \frac{y^2 - 1}{y^2 + 1}$

5.  $x^3\frac{d^3y}{dx^3} + x\frac{dy}{dx} = y$

## Part B: Degree Determination

Find the degree of each ODE (or state if undefined):

6.  $\left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx} + y^2 = 0$

7.  $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = xy$

8.  $e^{y'} + y = x$

9.  $\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} + y = 0$

## Part C: Special Types

Identify which special type each equation belongs to (if any):

10.  $\frac{dy}{dx} + \frac{2y}{x} = x^2y^3$
11.  $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$
12.  $\frac{dy}{dx} = y^2 - 2y + 1 - x^2$
13.  $(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

## Part D: Conceptual Questions

14. Give an example of a second-order, nonlinear, autonomous ODE.
15. Can a first-order ODE have degree 3? If yes, give an example.
16. Why is  $y' = \sqrt{y}$  problematic at  $y = 0$ ? What classification issue does this raise?
17. The equation  $y' = y^2$  is autonomous. If  $y(x)$  is a solution, explain why  $y(x - c)$  is also a solution for any constant  $c$ .

## Part E: Exam-Style Problems

18. Consider the parametric family:  $y' + p(x)y = q(x)y^n$ 
  - (a) For which values of  $n$  is this equation linear?
  - (b) What special type is this for  $n \neq 0, 1$ ?
  - (c) Is this ever autonomous? Explain.
19. Given that an ODE has the form  $F(y, y', y'') = 0$  (autonomous):
  - (a) How many arbitrary constants will the general solution have?
  - (b) If  $y = 2$  is a solution, what can you say about  $y' = f(y)$ ?
  - (c) Can this equation have a solution of the form  $y = mx + b$ ?
20. Professor Ditkowski writes:  $(1 + y'^2)^{3/2} = y''$ 
  - (a) What is the order?
  - (b) What is the degree after rationalization?
  - (c) Is it linear? Autonomous?
  - (d) This appears in geometry - what does it represent?

## Solutions Guide

**Part A:** 1. 2nd order, linear, non-autonomous 2. 1st order, nonlinear (cubed derivative), non-autonomous 3. 2nd order, nonlinear (product  $yy''$ ), autonomous 4. 1st order, nonlinear (rational in  $y$ ), autonomous 5. 3rd order, linear, non-autonomous

**Part B:** 6. Degree 2 (after expanding: highest derivative power is 2) 7. Degree 2 (after squaring both sides to clear radical) 8. Degree undefined (exponential is not polynomial) 9. Degree 3 (highest derivative to third power)

**Part C:** 10. Bernoulli equation with  $n = 3$  11. Homogeneous equation (can use  $v = y/x$  substitution) 12. Riccati equation (quadratic in  $y$ ) 13. Legendre equation with  $n = 1$

**Check your work:** Did you identify all the key features? Practice until classification becomes automatic!