Practice Problems: Lesson 2 - Pattern Recognition

Master these before the exam!

Part A: Quick Classification

For each equation, state: Order, Linear/Nonlinear, Autonomous/Non-autonomous, Special Type

1. $y' = \dfrac{y}{x} + \dfrac{x}{y}$
2. $y'' + y' \tan x = \sin 2x$
3. $(1 + e^x)yy' = e^x$
4. $y' = \sqrt{x^2 + y^2}$
5. $xy' - y = x^2 e^x$

Part B: Hidden Orders

Determine the actual order of these equations:

6. Let $p = y'$. The equation is: $p^2 + xp' = y$
7. Let $v = y/x$. After substitution, we get: $v + x\frac{dv}{dx} = v^2$
8. The equation $\frac{d}{dx}\left(x\frac{dy}{dx}\right) = y$ after expansion

Part C: Linearity Tricks

Determine if linear. If nonlinear, identify the nonlinear term(s):

9. $y'' + \dfrac{1}{x}y' + \left(1 - \dfrac{n^2}{x^2}\right) y = 0$
10. $y' = \dfrac{y \ln y}{x}$
11. $y'' + (\sin y)y' = 0$
12. $\dfrac{d}{dx}\left[\dfrac{1}{y'}\right] + y = 0$

Part D: Special Type Identification

Match each equation to its type and explain why:

13. $y' - \dfrac{2y}{x} = -x^2y^2$
14. $(2xy - 3)dx + (x^2 + 4y)dy = 0$
15. $y' = \dfrac{x^2 - y^2}{x^2 + y^2}$
16. $y' + y = xy^3$

Part E: Autonomy Analysis

17. Which of these are autonomous? Find all equilibrium solutions where applicable:
    (a) $y' = y^2 - 4y + 3$
    (b) $y' = \sin(y)\cos(y)$
    (c) $y' = y\sin(x)$
    (d) $y'' + \sin(y) = 0$
18. For the autonomous equation $y' = y(1 - y)(y - 2)$:
    (a) Find all equilibrium solutions
    (b) If $y(5) = 1.5$, what is $\lim_{x\to\infty} y(x)$?
    (c) Sketch the phase line

Part F: Exam-Style Recognition

19. Professor Ditkowski gives you: $y' = \dfrac{P(x)y^2 + Q(x)y + R(x)}{y}$
    (a) Rewrite in standard form
    (b) What type is this?
    (c) Under what condition on P, Q, R would this be linear?
20. Consider the substitution $u = y^{1-n}$ in the Bernoulli equation $y' + P(x)y = Q(x)y^n$:
    (a) What ODE does u satisfy?
    (b) Why does this fail for $n = 1$?
    (c) What happens when $n = 0$?
21. You're told an equation has the form $y' = f(ax + by + c)$ where a, b, c are constants:

(a) Is this autonomous?

(b) What substitution would you try?

(c) Give a specific example and solve it

Part G: Theoretical Questions

22. Prove that if y_1 and y_2 solve a linear homogeneous ODE, then $c_1y_1 + c_2y_2$ also solves it.

23. Show that $y' = |y|^{1/2}$ is not Lipschitz at $y = 0$. What does this imply about uniqueness?

24. Explain why every separable equation $y' = f(x)g(y)$ can be made autonomous by a change of variables.

Answer Key (Brief)

Part A: 1. 1st order, nonlinear, non-autonomous, neither standard type 2. 2nd order, linear, non-autonomous 3. 1st order, nonlinear (product yy'), non-autonomous, separable 4. 1st order, nonlinear, non-autonomous 5. 1st order, linear, non-autonomous

Part B: 6. Second order (p' = y") 7. Still first order (v is function of x) 8. Second order (expands to $xy'' + y' = y$)

Part D: 13. Riccati (quadratic in y) 14. Exact (check partials) 15. Homogeneous (degree 0) 16. Bernoulli with n = 3