Question 1

2024 | Prof. Lebesque

Given:
Let (X, M, μ) be a measure space. Let $f_n : X -> [0, \infty]$ be a sequence of measurable functions such that $f_n -> f$ pointwise almost everywhere. Assume there exists $g \in L^1(\mu)$ such that $|f_n| <= g$ for all n.

To Prove:
$\lim_{n->\infty} \int_X f_n d\mu = \int_X f d\mu$

THE ARSENAL: RELEVANT DEFINITIONS & THEOREMS

PROOF SKETCH / HEURISTIC

Rough ideas... Discretization? Contradiction? Split the domain?

Formal Proof

Citations

Question 2

2023 | Dr. Cauchy

Given:
Let $K \subset \mathbb{R}^n$ be a compact set and $U \supset K$ be an open set.

To Prove:
There exists a $\delta >; 0$ such that for all $x \in K$, the ball $B(x, \delta) \subset U$.

THE ARSENAL: RELEVANT DEFINITIONS & THEOREMS

PROOF SKETCH / HEURISTIC

Rough ideas... Discretization? Contradiction? Split the domain?

Formal Proof

Citations

Question 3

2024 | Prof. Noether

Given:
Let G be a group and N be a normal subgroup of G. Let $\varphi : G -> ; H$ be a surjective group homomorphism with kernel K.

To Prove:
$G/K \sim= H$.

THE ARSENAL: RELEVANT DEFINITIONS & THEOREMS

PROOF SKETCH / HEURISTIC

Rough ideas... Discretization? Contradiction? Split the domain?

Formal Proof

Citations

Hints & Techniques

Q	Technique	Hint
1	Dominated Convergence	Apply Fatou's Lemma to the non-negative sequences $g - \lvert f_n - f \rvert$.
2	Epsilon of Room	Consider the continuous function $f(x) = \operatorname{dist}(x, U^c)$. Use the Extreme Value Theorem.
3	Isomorphism Theorems	Define the map $\psi(gK) = \varphi(g)$ and show it is well-defined and bijective.