

## Exam: General

### Question 1

Calculus

2024 | Prof. Gauss

**Given:**

Let  $f(x) = x^2 e^x$ .

**To Prove:**

Calculate  $\int_0^1 f(x) dx$ .

*Hint: Use integration by parts twice.*

### Question 2

Combinatorics

2023 | Dr. Pascal

**Given:**

Expression  $(x + y)^4$ .

**To Prove:**

Expand using the Binomial Theorem.

*Hint: Coefficients are 1, 4, 6, 4, 1.*

### Question 3

Complex Analysis

2024 | Prof. Euler

**Given:**

Consider  $z^3 = 1$ .

**To Prove:**

Find all roots of unity in the form  $a + bi$ .

*Hint: Use De Moivre's Theorem:  $e^{i\theta} = \cos(\theta) + i \sin(\theta)$ .*

## Question 4

Limits

2023 | Dr. Smith

**Given:**

Let  $x_n = \frac{3n+1}{n-2}$ .

**To Prove:**

Prove that  $\lim(x_n) = 3$ .

**Hint:** Use the definition of convergence.

## Question 5

Limits

2022 | Dr. Smith

**Given:**

Let  $x_n = \frac{\sin(n)}{n}$ .

**To Prove:**

Prove that  $\lim(x_n) = 0$ .

**Hint:** Use the Squeeze Theorem with  $-\frac{1}{n} \leq x_n \leq \frac{1}{n}$ .

## Question 6

Linear Algebra

2025 | Prof. Noether

**Given:**

Let  $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ .

**To Prove:**

Find the eigenvalues of  $A$ .

**Hint:** Solve  $\det(A - \lambda I) = 0$ .

## Question 7

Logic

2025 | Dr. Boole

**Given:**

Propositions  $P$  and  $Q$ .

**To Prove:**

Show that  $P \Rightarrow Q$  is equivalent to  $\neg Q \Rightarrow \neg P$ .

**Hint:** Construct a truth table.

## Question 8

Series

2023 | Dr. Cauchy

**Given:**

Let  $S = \sum_{n=1}^{\infty} \frac{n!}{n^n}$ .

**To Prove:**

Determine if  $S$  converges.

**Hint:** Use the Ratio Test.

## Question 9

Geometry

2024 | Prof. Euclid

**Given:**

Vectors  $(u) = (1, 2)$  and  $(v) = (3, -1)$ .

**To Prove:**

Calculate the angle between  $(u)$  and  $(v)$ .

**Hint:** Use  $(u) \cdot (v) = |(u)| |(v)| \cos(\theta)$ .

## Question 10

**Set Theory**

2023 | Dr. Cantor

**Given:**

Let  $A$  and  $B$  be subsets of a universal set  $U$ .

**To Prove:**

Prove  $(A \cup B)^c = A^c \cap B^c$ .

**Hint:** Show mutual inclusion (LHS subset RHS and RHS subset LHS).

## Question 11

**Topology**

2025 | Prof. Haus

**Given:**

Let  $X = \mathbb{R}$  with standard topology.

**To Prove:**

Prove that  $(0, 1)$  is an open set.

**Hint:** Find a ball  $B_{\varepsilon(x)}$  for every point.

**End of Examination**