

Exam: General (KEY)

Question 1

Calculus

2024 | Prof. Gauss

Given:

Let $f(x) = x^2 e^x$.

To Prove:

Calculate $\int_0^1 f(x) \, dx$.

Hint: Use integration by parts twice.

Official Solution:

Step 1 (Calculation)

Let $u = x^2$ and $dv = e^x \, dx$.

Final Answer (Result)

$e - 2$.

Question 2

Combinatorics

2023 | Dr. Pascal

Given:

Expression $(x + y)^4$.

To Prove:

Expand using the Binomial Theorem.

Hint: Coefficients are 1, 4, 6, 4, 1.

Official Solution:

Expansion (Result)

$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$.

Question 3

Complex Analysis

2024 | Prof. Euler

Given:

Consider $z^3 = 1$.

To Prove:

Find all roots of unity in the form $a + bi$.

Hint: Use De Moivre's Theorem: $e^{i\theta} = \cos(\theta) + i \sin(\theta)$.

Official Solution:

Roots (Result)

$1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$.

Question 4

Limits

2023 | Dr. Smith

Given:

Let $x_n = \frac{3n+1}{n-2}$.

To Prove:

Prove that $\lim(x_n) = 3$.

Hint: Use the definition of convergence.

Official Solution:

Scratch Work (Proof)

Find N such that $|x_n - 3| < \varepsilon$.

Question 5

Limits

2022 | Dr. Smith

Given:

$$\text{Let } x_n = \frac{\sin(n)}{n}.$$

To Prove:

Prove that $\lim(x_n) = 0$.

Hint: Use the Squeeze Theorem with $-\frac{1}{n} \leq x_n \leq \frac{1}{n}$.

Official Solution:

Bound (Proof)

Since $|\sin(n)| \leq 1$, we have $-1 \leq \sin(n) \leq 1$.

Question 6

Linear Algebra

2025 | Prof. Noether

Given:

$$\text{Let } A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

To Prove:

Find the eigenvalues of A .

Hint: Solve $\det(A - \lambda I) = 0$.

Official Solution:

Characteristic Eq (Calculation)

$$(2 - \lambda)^2 - 1 = 0.$$

Question 7

Logic

2025 | Dr. Boole

Given:

Propositions P and Q .

To Prove:

Show that $P \Rightarrow Q$ is equivalent to $\neg Q \Rightarrow \neg P$.

Hint: Construct a truth table.

Official Solution:

Contrapositive (Proof)

This is the Law of Contrapositive.

Question 8

Series

2023 | Dr. Cauchy

Given:

Let $S = \sum_{n=1}^{\infty} \frac{n!}{n^n}$.

To Prove:

Determine if S converges.

Hint: Use the Ratio Test.

Official Solution:

Ratio (Calculation)

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{e} < 1.$$

Question 9

Geometry

2024 | Prof. Euclid

Given:

Vectors $(u) = (1, 2)$ and $(v) = (3, -1)$.

To Prove:

Calculate the angle between (u) and (v) .

Hint: Use $(u) \cdot (v) = |(u)| |(v)| \cos(\theta)$.

Official Solution:

Dot Product (Calculation)

$$1(3) + 2(-1) = 1.$$

Question 10

Set Theory

2023 | Dr. Cantor

Given:

Let A and B be subsets of a universal set U .

To Prove:

Prove $(A \cup B)^c = A^c \cap B^c$.

Hint: Show mutual inclusion (LHS subset RHS and RHS subset LHS).

Official Solution:

Forward Direction (Proof)

Let $x \in (A \cup B)^c$. Then $x \notin A \cup B$.

Question 11**Topology**2025 | Prof. Haus

Given:

Let $X = \mathbb{R}$ with standard topology.

To Prove:

Prove that $(0, 1)$ is an open set.

Hint: Find a ball $B_{\varepsilon(x)}$ for every point.

Official Solution:**Radius (Proof)**

For any x , choose $\varepsilon = \min(x, 1 - x)$.

End of Examination