Real Analysis Midterm

Student Name:

ID:

Q1

2024
Prof. Lebesque

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Recall: Check assumptions: Finite measure? Non-negative?

Given: Let (X, M, μ) be a measure space. Let $f_n \to f$ pointwise a.e. and $|f_n| \leq g \in L^1$.

To Prove:

$$\lim_{n\to\infty} \int f_n d\mu = \int f d\mu$$

SCRATCHPAD

FORMAL PROOF

Q2

2023

Dr. Cauchy

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Recall:
Did you express δ in terms of ε?

Given: Prove the limit of the function $f(x) = 2x + 3$ as $x \to 1$.

To Prove:

$$\lim_{x \to 1}(2x + 3) = 5$$

SCRATCHPAD

FORMAL PROOF

Q3

2024
Prof. Noether

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Recall:
Is the map well-defined?

Given: $\varphi : G \to H$ **is a surjective homomorphism with kernel** K**.**

To Prove:

$$\frac{G}{K} \cong H$$

SCRATCHPAD

FORMAL PROOF

Hints & Summary

Q	Technique	Hint
1	Dominated Convergence	Apply Fatou's Lemma to $g - |f_n - f|$.
2	Epsilon-Delta	Start with $|f(x) - 5| < \varepsilon$ and solve for $|x - 1|$.
3	Isomorphism	Use the First Isomorphism Theorem.