

Exam: General

Question 1

Calculus

2024 | Prof. Gauss

Given:

Let $f(x) = x^2 e^x$.

To Prove:

Calculate $\int_0^1 f(x) \, dx$.

Hint: Use integration by parts twice.

Question 2

Combinatorics

2023 | Dr. Pascal

Given:

Expression $(x + y)^4$.

To Prove:

Expand using the Binomial Theorem.

Hint: Coefficients are 1, 4, 6, 4, 1.

Question 3

Complex Analysis

2024 | Prof. Euler

Given:

Consider $z^3 = 1$.

To Prove:

Find all roots of unity in the form $a + bi$.

Hint: Use De Moivre's Theorem: $e^{i\theta} = \cos(\theta) + i \sin(\theta)$.

Question 4

Limits

2023 | Dr. Smith

Given:

$$\text{Let } x_n = \frac{3n+1}{n-2}.$$

To Prove:

Prove that $\lim(x_n) = 3$.

Hint: Use the definition of convergence.

Question 5

Limits

2022 | Dr. Smith

Given:

$$\text{Let } x_n = \frac{\sin(n)}{n}.$$

To Prove:

Prove that $\lim(x_n) = 0$.

Hint: Use the Squeeze Theorem with $-\frac{1}{n} \leq x_n \leq \frac{1}{n}$.

Question 6

Linear Algebra

2025 | Prof. Noether

Given:

$$\text{Let } A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

To Prove:

Find the eigenvalues of A .

Hint: Solve $\det(A - \lambda I) = 0$.

Question 7

Logic

2025 | Dr. Boole

Given:

Propositions P and Q .

To Prove:

Show that $P \Rightarrow Q$ is equivalent to $\neg Q \Rightarrow \neg P$.

Hint: Construct a truth table.

Question 8

Series

2023 | Dr. Cauchy

Given:

Let $S = \sum_{n=1}^{\infty} \frac{n!}{n^n}$.

To Prove:

Determine if S converges.

Hint: Use the Ratio Test.

Question 9

Geometry

2024 | Prof. Euclid

Given:

Vectors $(u) = (1, 2)$ and $(v) = (3, -1)$.

To Prove:

Calculate the angle between (u) and (v) .

Hint: Use $(u) \cdot (v) = |(u)| |(v)| \cos(\theta)$.

Question 10**Set Theory**

2023 | Dr. Cantor

Given:

Let A and B be subsets of a universal set U .

To Prove:

Prove $(A \cup B)^c = A^c \cap B^c$.

Hint: Show mutual inclusion (LHS subset RHS and RHS subset LHS).

Question 11**Topology**

2025 | Prof. Haus

Given:

Let $X = \mathbb{R}$ with standard topology.

To Prove:

Prove that $(0, 1)$ is an open set.

Hint: Find a ball $B_{\varepsilon(x)}$ for every point.

End of Examination