

Real Analysis Exam

Student Name: _____

ID: _____

Question 1

Measure Theory

2024 / Prof. Lebesgue

Given: Let (X, M, μ) be a measure space. Let $f_n \rightarrow f$ pointwise a.e. and $|f_n| \leq g \in L^1$.

To Prove:

$$\lim_{n \rightarrow \infty} \int f_n d\mu = \int f d\mu$$

SCRATCHPAD

FORMAL PROOF

Given: Prove the limit of the function $f(x) = 2x + 3$ as $x \rightarrow 1$.

To Prove:

$$\lim_{x \rightarrow 1} (2x + 3) = 5$$

SCRATCHPAD

FORMAL PROOF

Given: Let $\varphi : G \rightarrow H$ be a surjective homomorphism with kernel K .

To Prove:

$$\frac{G}{K} \cong H$$

SCRATCHPAD

FORMAL PROOF