VMLS - Study Guide Chapter 1 **Name:** Alex Cullen

Please always include this title page with your PDF. Include your name above.

- Submit your work in Gradescope as a PDF - you will identify where your “questions are.”
- Identify the question number as you submit. Since we grade "blind" if the questions are NOT identified, the work WILL NOT BE GRADED and a 0 will be recorded. Always leave enough time to identify the questions when submitting.
- One section per page (if a page or less) - We prefer to grade the main solution in a single page, extra work can be included on the following page.
- Long instructions may be removed to fit on a single page.
- **Do not start a new question in the middle of a page.**
- Solutions to book questions are provided for reference.
- You may NOT submit given solutions - this includes minor modifications - as your own.
- Solutions that do not show individual engagement with the solutions will be marked as no credit and can be considered a violation of honor code.
- If you use the given solutions you must reference or explain how you used them, in particular...

For full credit, EACH book exercise in the Study Guides must use one or more of the following methods and FOR EACH QUESTION. Identify the number the method by number to ensure full credit.

Method 1 - Provide original examples which demonstrate the ideas of the exercise in addition to your solution.

Method 2 - Include and discuss the specific topics needed from the chapter and how they relate to the question.

Method 3 - Include original Python code, of reasonable length (as screenshot r text) to show how the topic or concept was explored.

Method 4 - Expand the given solution in a significant way, with additional steps and comments. All steps are justified. This is a good method for a proof for which you are only given a basic outline.

Method 5 - Attempt the exercise without looking at the solution and then the solution is used to check work. Words are used to describe the results.

Method 6 - Provide an analysis of the strategies used to understand the exercise, describing in detail what was challenging, who helped you or what resources were used. The process of understanding is described.

1. (20 pts) Reading the book carefully is essential in this class and in all advanced mathematics. This is an exercise in annotation.

For this first exercise, pick a section or page from Chapter one of VMLS.

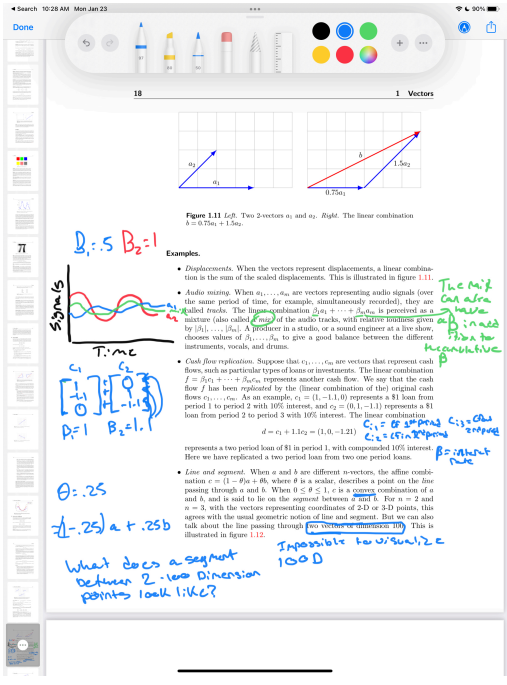
Read straight through the section once for an overview. Include the following 3 items for #1.

a) Write down your initial thoughts, questions, and first impressions (‘whaaat???’)
b) Now, return to the section and slowly work through each line using a pencil or pen.
   - Expand equations.
   - Identify key concepts and explain in your own words.
   - Make note - is that a vector or a scalar?
   - Fill in missing ideas or steps.
   - Include a screenshot of your annotation.

c) How has your understanding of the section changed?

Include a screenshot of your annotation.

Initial thoughts on page 18 of chapter 1 from VMLS:
This is definitely stretching my mind a bit. The description of Audio mixing as the linear combination of two vectors I found to be so fascinating. It made me realize that when we hear audio, that most of what we're hearing is an illusion. It sounds like multiple instruments playing simultaneously, however there's only a single audio wave that is the combination of the waves underneath it, and there's only one audio wave.
Similarly, my background is finance, so thinking about cash flows on a loan as a linear combination of vectors is fascinating to me, as I would never have characterized it that way.
Finally, in the last part, it talks about affine combinations between two n-vectors, and implines that there can be a segment between two 2D points, 3D points, but also, 100D points. Which is hard to even visualize what a line segment between two 100D points would even look like



How has my understanding changed post annotation?

I think that going back, I had to really spend time focusing on the cash flow replication to understand what each value in the vectors represented. I think there's actual a few different ways to interpret the meaning of each element, and each interpretation is equally valid. There's also some information missing that would clarify the interpretation, like was the loan fully repaid, or is this a calculation of outstanding balance? Who's perspective is this from, the loaner or the loanee?

Additionally, the line and segment section was really interesting, because it made me realize that while a line can be between two points on a 2D space (which is how I had only ever conceived of it) you could have a segment between 3D points, and also between 3D+ points, but that's something I don't fully understand since I can't visualize it.

2. (10pts) Solve the Random exercise from the video and Piazza in your own words here.

Random Exercise - symptoms vector

Approach

My approach to this problem will be to attempt this independently and then watch the video and see how my approach/solution is different from the lecturers, annotating my code etc.

Problem Statement: Symptoms vector. A 20-vector "s" records whether each of 20 different symptoms is present in a medical patient, with "s_i" = 1 meaning the patient has the symptom and "s_i" = 0 meaning she does not. Expressing the following using vector notation.

```
a) The total number of symptoms the patient has.
b) The patient exhibits five out of the first ten symptoms
```

My Attempt

Part A

In [56]:
```
'''
The total number of symptoms the patient has is equivalent to the sum of each element of the vector since
each element of the vector is a boolean value where 0 = false (false meaning the patient does not have the symptom)
and each true = 1

total number of symptoms in patient symptom vector s = s_1 + s_2 + ... + s_20
written

sum of s_i where i = 1 to i = 20 and s is a vector of size 20

in python sum(s) could be a useful shorthand or maybe its s.sum(), I'm not sure what methods are available
on lists to support this and will need to investigate
the list data structure is a good proxy for a 1D vector, and I will use that here. To simulate the patient
'''
#I thought the library was rand, it's actually random when I looked at the documentation
#import rand
from random import random

def total_number_symptoms(s):
    print(f'total number of patient symptoms (sum of s_i for i=1...20): {sum(s)}')
    return sum(s)

def generate_patient_symptom_vector():
    #I'll need some form of randomness, so will probably use the python native rand library imported at the top of the
    #I'll also need to populate each cell, range generates a range of integer values from 0 to n-1. n here is the size
    #of the vector
    s = []
    for i in range(20):
        #append is the function needed to add elements to the end of a list. random generates pseudorandom
        #number between 0 and 1. round will round to the nearest number so x > 0.5 -> 1 and x < .5 -> 0
        s.append(round(random()))
    print(f'symptom vector "s": {s}')
    return s
#run and print output where generated vector is input to total_number_symptoms function
total_number_symptoms(generate_patient_symptom_vector())
```

```
symptom vector "s": [0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1]
total number of patient symptoms (sum of s_i for i=1...20): 10
```

Out[56]: 10

In [16]:
```
#testing that sum(list) is short hand for sum each element in the list
sum([1,2,3])
```

Out[16]: 6

In [17]:
```
#testing that random.random() will generate a numer between 0 and 1 as described in python documentation
random()
```

Out[17]: 0.33797746000041395

Part B

In [31]:
```
'''
The wording on Part B is somewhat ambiguous, it's not clear to me if it's asking that we evaluate whether the patient
exhibits 5 of 10 first symptoms, or to create a vector where the patient exhibits five out of first ten symptoms.

If I'm going to express in vector notation. Then a subvector of s, we will call v, that is size 10 where v_i = s_i
and 0 < i <= 10, and a patient that exhibits 5 of 10 first symptoms is one where:

(sum of v_i where i = 1 and n = 10) >= 5

'''
def five_symptom_patient(patient):
    print(f"subvector v: {patient[:10]}")
    if sum(patient[:10]) >= 5:
        print("Patient has 5 of first 10 symptoms")
        return True
    else:
        print("Patient does not have 5 of first 10 symptoms")
        return False
patient = generate_patient_symptom_vector()
total_symptoms = total_number_symptoms(patient)
five_symptom_patient(patient)
```

```
symptom vector "s": [0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1]
total number of patient symptoms (sum of s_i for i=1...20): 10
vector v: [0, 0, 0, 1, 0, 0, 0, 1, 1, 1]
Patient does not have 5 of first 10 symptoms
```

Out[31]:
```
False
```

Reflection after Reviewing instructor solution

The response to the question should have been expressed in vector notation, and while I did express it in series summation notation, and was able to implement useful functions around that, it was not aligned with the initial expectations of the question.

A series summation can also be expressed as the inner product of an n-vector and a 1-vector of the same size, which is how the lecturer described it in the solution video. The correct answer was $s^T 1$ where the inner product of a vector and a 1's vector of the same size will yield a single value equal to the sume of each element of the vector multiplied by 1.

Similarly, for the second part of the question, after reviewing the lecture, I think that I did not explicitly express the solution in vector notation. The correct answer is $s_{1:10} = 5$. Also, I created a new vector v to represent the subvector of $s_{1:10}$. I also misunderstood the statement and used the >= operation indicating that any value over 5 for the total number of symptoms in $s_{1:10}$ was an acceptable answer. In actuality, only a strict equality of '= 5' is acceptable.

1.8 Problem

3. (10 pts) Explain the solution to 1.8 here in your own words. (Since you are given a solution, you will be graded on your ability to explain).

1.8 Problem

My Approach

I am going to attempt this independently and then refer to the solution afterwards to see where I was wrong

Problem Statement

- Company sells n different products or items.
- vector p gives the profit in dollars per unit for each of the n items. (negative entries indicate loss leaders)
- vector s gives total sales of each item sold over some period

Express the total profit in terms of p and s using vector notation

My Attempt

The contribution to total profit of a single item can be represented as: total profit of item = (profit per unit) * (# of units sold within the period)

or $x_i = p_i * s_i$ where x represents total profit, i is the index of the element in the vector, and vectors p and s are the same size.

The inner product is defined as the sum of the products of the corresponding entries.

Thus, by the definition of inner product, the total profit x can be described as the inner product of the vector of profits per unit and s is the vector of sales $x = p^T s$.

$\sum_{i=1}^{n} p_i * s_i == p^T s$

4. (10 pts) Explain the solution to 1.16 here in your own words. (Since you are given a solution, you will be graded on your ability to explain).

1.16 Problem

Inner product of nonnegative vectors. All entries must be nonnegative. a) Explain why the inner product of two nonnegative vectors is nonnegative? b) Suppose the inner product of two nn vectors is zero. What can you say about them? Your answer should be in terms of their respective sparsity patterns. i.e., which entries are zero and nonzero.

My attempt

a) Proof by contradiction:

Hypothesis: the inner product of two nonnegative vectors is negative.

$n_1^T n_2 < 0$ where $\forall_i (n_{1i} \in n_1 | n_{1i} >= 0)$ and $ \forall_{j}(n_{2j} \in n_{2} | n_{2j} >= 0)

Proof:

given: $\forall_i (n_{1i} \in n_1 | n_{1i} >= 0)$ and $\forall_j (n_{2j} \in n_2 | n_{2j} >= 0)$

by definition of inner product:

$n_1^T n_2 = n_{11} n_{21} + n_{12} n_{22} + \ldots + n_{1q} n_{2r}$

because n_{1i} and n_{2i} are both positive numbers, then by the property of real numbers, the sum and product of two positive numbers is also a positive number.

Contradiction:

The sum of the series $\sum_{i=1}^{n} n_{1i} n_{2j}$ will always be a positive number given every element in the each vector is a positive number, therefore the inner product of two nonnegative vectors will always be nonnegative.

QED

b) If the inner product of two nn vectors is zero then the product of each corresponding entry must equal 0. This means that either:

1.) both vectors are 0 vectors

2.) one vector is a 0 vector

3.) one or both elements in each of the corresponding entries is equal to 0

$\forall_{(x,y)} (x = 0 \vee y = 0)$

where x and y are corresponding entries in equal size vectors.

5. (50 points) Create a Jupyter Notebook of the following:

The Python **Companion** (linked at the top of the course) includes examples of code that can help you throughout the course and provides the basis for the fundamentals of vector operations we will use throughout the course.

Every week you can use these code elements to illustrate concepts and ideas from the weeks.

This week we guide you through an example.

1. Find the Python Companion and sections 1.1 - 1.4,

2. Select at least 10 code blocks (you may want to do all of them) that look useful.

3. Rewrite (best) or copy and paste each code cell **into your own Jupyter Notebook**.

4. Add notes in your own words in a text block above about the code using the notes given AND incorporate ideas from the book (cite the book page).

5. For each section of code include at least one additional specific example of using the code.

6. **Attach a PDF of your Jupyter Notebook here** to include in the single PDF study guide.

Random Exercise - symptoms vector

Approach

My approach to this problem will be to attempt this independently and then watch the video and see how my approach/solution is different from the lecturers, annotating my code etc.

Problem Statement: Symptoms vector. A 20-vector "s" records whether each of 20 different symptoms is present in a medical patient, with "s_i" = 1 meaning the patient has the symptom and "s_i" = 0 meaning she does not. Expressing the following using vector notation.

```
a) The total number of symptoms the patient has.
b) The patient exhibits five out of the first ten symptoms
```

My Attempt

Part A

In [56]:
```
'''
The total number of symptoms the patient has is equivalent to the sum of each element of
each element of the vector is a boolean value where 0 = false (false meaning the patient
and each true = 1

total number of symptoms in patient symptom vector s = s_1 + s_2 + ... + s_20
written

sum of s_i where i = 1 to i = 20 and s is a vector of size 20

in python sum(s) could be a useful shorthand or maybe its s.sum(), I'm not sure what met
on lists to support this and will need to investigate

the list data structure is a good proxy for a 1D vector, and I will use that here. To si
'''
#I thought the library was rand, it's actually random when I looked at the documentation
#import rand
from random import random

def total_number_symptoms(s):
    print(f'total number of patient symptoms (sum of s_i for i=1...20): {sum(s)}')
    return sum(s)

def generate_patient_symptom_vector():
    #I'll need some form of randomness, so will probably use the python native rand libr
    #I'll also need to populate each cell, range generates a range of integer values fro
    #of the vector
    s = []
    for i in range(20):
        #append is the function needed to add elements to the end of a list. random gene
        #number between 0 and 1. round will round to the nearest number so x > 0.5 -> 1
        s.append(round(random()))
    print(f'symptom vector "s": {s}')
    return s
#run and print output where generated vector is input to total_number_symptoms function
total_number_symptoms(generate_patient_symptom_vector())
```

```
symptom vector "s": [0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1]
```

```
total number of patient symptoms (sum of s_i for i=1...20): 10
```

Out[56]:
```
10
```

In [16]:
```
#testing that sum(list) is short hand for sum each element in the list
sum([1,2,3])
```

Out[16]:
```
6
```

In [17]:
```
#testing that random.random() will generate a numer between 0 and 1 as described in pyth
random()
```

Out[17]:
```
0.33797746000041395
```

Part B

In [31]:
```
'''
The wording on Part B is somewhat ambiguous, it's not clear to me if it's asking that we
exhibits 5 of 10 first symptoms, or to create a vector where the patient exhibits five o

If I'm going to express in vector notation. Then a subvector of s, we will call v, that
and 0 < i <= 10, and a patient that exhibits 5 of 10 first symptoms is one where:

(sum of v_i where i = 1 and n = 10) >= 5

'''
def five_symptom_patient(patient):
    print(f"subvector v: {patient[:10]}")
    if sum(patient[:10]) >= 5:
        print("Patient has 5 of first 10 symptoms")
        return True
    else:
        print("Patient does not have 5 of first 10 symptoms")
        return False
patient = generate_patient_symptom_vector()
total_symptoms = total_number_symptoms(patient)
five_symptom_patient(patient)
```

Out[31]:
```
symptom vector "s": [0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1]
total number of patient symptoms (sum of s_i for i=1...20): 10
vector v: [0, 0, 0, 1, 0, 0, 0, 1, 1, 1]
Patient does not have 5 of first 10 symptoms
False
```

Reflection after Reviewing instructor solution

The response to the question should have been expressed in vector notation, and while I did express it in series summation notation, and was able to implement useful functions around that, it was not aligned with the initial expectations of the question.

A series summation can also be expressed as the inner product of an n-vector and a 1-vector of the same size, which is how the lecturer described it in the solution video. The correct answer was $s^T 1$ where the inner product of a vector and a 1's vector of the same size will yield a single value equal to the sume of each element of the vector multiplied by 1.

Similarly, for the second part of the question, after reviewing the lecture, I think that I did not explicitly express the solution in vector notation. The correct answer is $s_{1:10} = 5$. Also, I created a new vector v to represent the subvector of $s_{1:10}$. I also misunderstood the statement and used the >= operation

indicating that any value over 5 for the total number of symptoms in $s_{1:10}$ was an acceptable answer. In actuality, only a strict equality of '= 5' is acceptable.

1.8 Problem

My Approach

I am going to attempt this independently and then refer to the solution afterwards to see where I was wrong

Problem Statement

- Company sells n different products or items.
- vector p gives the profit in dollars per unit for each of the n items. (negative entries indicate loss leaders)
- vector s gives total sales of each item sold over some period

Express the total profit in terms of p and s using vector notation

My Attempt

The contribution to total profit of a single item can be represented as: total profit of item = (profit per unit) * (# of units sold within the period)

or $x_i = p_i * s_i$ where x represents total profit, i is the index of the element in the vector, and vectors p and s are the same size.

The inner product is defined as the sum of the products of the corresponding entries.

Thus, by the definition of inner product, the total profit x can be described as the inner product of the vector of profits per unit and s is the vector of sales $x = p^T s$.

$\sum_{i=1}^{n} p_i * s_i == p^T s$

In [ ]:

1.16 Problem

Inner product of nonnegative vectors. All entries must be nonnegative. a) Explain why the inner product of two nonnegative vectors is nonnegative? b) Suppose the inner product of two nn vectors is zero. What can you say about them? Your answer should be in terms of their respective sparsity patterns. i.e., which entries are zero and nonzero.

My attempt

a) Proof by contradiction:

Hypothesis: the inner product of two nonnegative vectors is negative.

$n_1^T n_2 < 0$ where $\forall_i(n_{1i} \in n_1 | n_{1i} >= 0)$ and $ \forall{j}(n{2j} \in n{2} / n{2j} >= 0)

Proof:

given: $\forall_i(n_{1i} \in n_1 | n_{1i} >= 0)$ and $\forall_j(n_{2j} \in n_2 | n_{2j} >= 0)$

by definition of inner product:

$n_1^T n_2 = n_{11}n_{21} + n_{12}n_{22} + \ldots + n_{1q}n_{2r}$

because n_{1i} and n_{2i} are both positive numbers, then by the property of real numbers, the sum and product of two positive numbers is also a positive number.

Contradiction:

The sum of the series $\sum_{i=1}^{n} n_{1i}n_{2j}$ will always be a positive number given every element in the each vector is a positive number, therefore the inner product of two nonnegative vectors will always be nonnegative.

QED

b) If the inner product of two nn vectors is zero then the product of each corresponding entry must equal 0. This means that either:

1.) both vectors are 0 vectors

2.) one vector is a 0 vector

3.) one or both elements in each of the corresponding entries is equal to 0

$\forall_{(x,y)}(x = 0 \vee y = 0)$

where x and y are corresponding entries in equal size vectors.

CODE BLOCKS

Block 1

Notes: Anything in square brackets is automatically interpreted by python as a `list` which is a data structure similar to an array. This is appropriate for representing a vector which is an ordered finite list of numbers [page 3]. We could also represent a vector as a tuple, of the same size using parentheses, however, lists and tuples have different methods and properties.

In [1]:
```
x = [-1.1, 0.0, 3.6, -7.2]
len(x)
```

Out[1]:
```
4
```

Additional example: v is a vector of boolean values represented as 0's for False and 1's for True.

$$v = (0, 1, 0, 1)$$

In [16]:
```
v = [0, 1, 0 , 1]
# or
v = (0, 1, 0, 1)
```

Block 2

Notes: Numpy is not a native library and must be imported. We can pass a list as a parameter to the array method on the numpy class imported from the numpy module.

Numpy array's support linear algebra operations like vector addition and scalar multiplication [page 15]

In [22]:
```
import numpy as np
x = np.array([-1.1, 0.0, 3.6, -7.2])
len(x)
```

Out[22]:
```
4
```

Additional Example: If I multiply a python list by a scalar, 4, it just concatenates the list to itself 4 times. However, if I multiply a numpy array by a scalar, it correctly performs the linear algebra operation of scaling each element by 4.

In [28]:
```
y = [-1.1, 0.0, 3.6, -7.2]
print (f'wrong: {y * 4}')
print (f'right: {4 * x}')
```

```
wrong: [-1.1, 0.0, 3.6, -7.2, -1.1, 0.0, 3.6, -7.2, -1.1, 0.0, 3.6, -7.2, -1.1, 0.0, 3.
6, -7.2]
right: [ -4.4   0.   14.4 -28.8]
```

BLOCK 3

Notes: You can index into an np array with the x[i] shorthand, where i is equal to the index of an element in an n-vector array, and the expression returns the element in the list at i. However, unlike mathematical notation in the book, which is 1-indexed, the indexing here will be 0-indexed [page 5]

In [30]:
```
x = np.array([-1.1, 0.0, 3.6, -7.2])
x[2]
```

Out[30]:
```
3.6
```

Additional Example:

the n-vector above is of size n=4. Therefore in mathematical notation $x_4 = -7.2$ however, as we can see below, the np array is 0 indexed and x[4] returns an index error

In [32]:
```
x[4]
```

```
---------------------------------------------------------------------------
IndexError                                Traceback (most recent call last)
Cell In[32], line 1
----> 1 x[4]

IndexError: index 4 is out of bounds for axis 0 with size 4
```

BLOCK 4

Notes:

Array indexing on the left-hand side of an assignment statement to change the value of the corresponding element is useful python shorthand, however it is not a valid mathematical expression. After the below code is run, there is effectively a new vector that has been generated x_2, although the python program just assigns the vector to the original variable name x

In [4]:
```
x[2] = 20.0
print(x)
```

```
[-1.1  0.  20.  -7.2]
```

Additional example: We can assign any valid vector index a new value

In [35]:
```
x[3] = 1
print(x)
```

```
[-1.1  0.   3.6  1. ]
```

Block 5

Notes: Negative indexing is not a valid operation in mathematics, but is useful shorthand for python. The book says the following about integer indexes: "If we denote an n-vector using the symbol a, the ith element of the vector a is denoted ai, where the subscript i is an integer index that runs $from\ 1\ to\ n$, the size of the vector." [page 3]

In [37]:
```
x[-1]
```

Out[37]:
```
1.0
```

Additional Example: Negative indexing can also return second from last element in the vector, third from last, etc.

In [39]:
```
x[-2]
```

Out[39]:
```
3.6
```

Block 6

Notes: If we are going to change a vector, it's better convention to copy the vector into a new variable. Whenever we update an array directly the original data is mutated or destroyed. Additionally if we don't copy and assign a new variable to the value of an existing variable, a change to either variable would mutate both, potentially introducing unintended bugs.

In [42]:
```
x = np.array([-1.1, 0.0, 3.6, -7.2])
y = x.copy()
x[2] = 20.0
print(y)
```

```
[-1.1  0.   3.6 -7.2]
```

Additional Example: If we mutate y, it will not have any impact on the value of x, because y is an

independent array from x

In [45]:
```
print(f'x {x}')
y[3] = 1
print(f'y {y}')
```

```
x [-1.1  0.  20.  -7.2]
y [-1.1  0.   3.6  1. ]
```

BLOCK 7

Notes: "Two vectors a and b are equal, which we denote a = b, if they have the same size, and each of the corresponding entries is the same. If a and b are n-vectors, then a = b means a1 = b1, . . . , an = bn." [page 3]

We can use the python == equality operator that will return a boolean vector x of size n, where x_i is true if vector $a_i = b_i$, and false otherwise

In [48]:
```
x = np.array([-1.1, 0.0, 3.6, -7.2])
y = x.copy()
x == y
```

Out[48]:
```
array([ True,  True,  True,  True])
```

Additional example: If we recreate the example above, but create an entirely new vector instead of copying, with the same values for each corresponding entry, then we get the same output as expected

In [58]:
```
y = np.array([-1.1, 0.0, 3.6, -7.2])
y == x
```

Out[58]:
```
array([ True,  True,  True,  True])
```

Block 8

Notes: "If $a_1 \dots a_m$ are n-vectors, and $1 \dots m$ are scalars, the n-vector $\beta_1 a_1 + \dots + \beta_1 a_m$ is called a linear combination of the vectors a1 ... an. The scalars 1 ... m are called the coe cients of the linear combination." [page 17]

We can see that the numpy module allows for us to express linear combinations in syntax that is similar to the formal mathematical notation.

In [59]:
```
import numpy as np
a = np.array([1,2])
b = np.array([3,4])
alpha = -0.5
beta = 1.5
c = alpha*a + beta*b
print(c)
```

```
[4. 5.]
```

Additional Example: If we didn't use numpy, but just lists, we can see that we would not be able to use similar syntax to express the linear combination. We get a type error that a list cannot be multiplied by a scalar.

In [60]:
```
a = [1, 2]
b = [3, 4]
alpha = -0.5
beta = 1.5
print (alpha *a + beta * b)
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[60], line 5
      3 alpha = -0.5
      4 beta = 1.5
----> 5 print (alpha *a + beta * b)

TypeError: can't multiply sequence by non-int of type 'float'
```

Block 9

Notes:

Another difference between python and mathematical notation of vectors is that a scalar is just a vector of size 1. [page 3] So where x = 2.4 and x is a scalar and y [2.4] is a vector of size 1. x and y are equivalent. However, as we can see below, when implemented in python the two are not equivalent, as one is of type list and the other is of type float/int

In [62]:
```
x = 2.4
y = [2.4]
x == y
```

Out[62]:
```
False
```

Additional Example: We can extract the value of the list of size 1 and show that the two are equivalent. y[0] returns an individual element which we is of the same type as x was originally intialized as.

In [63]:
```
y[0] == x
```

Out[63]:
```
True
```

Block 10

Notes: To find the inner product of two vectors, we must use a method on the np module, and pass as arguments to the `inner` function two numpy arrays.

The inner product is defined as: $a^T b = a_1 b_1 + a_2 b_2 + \qquad + a n b_n$

In [64]:
```
x = np.array([-1,2,2])
y = np.array([1,0,-3])
print(np.inner(x,y))
```

```
-7
```

Additional example: To implement without the `inner` method of np module I could define as follows

In [69]:
```
def inner_product(x, y):
    # we assume that x and y are of the same size
    #for this example
    #result is the accumulator variable
    result = 0
```

```
    for i in range(len(x)):
        #we index into corresponding elements in both lists
        # we add their product to the result accumulator
        result += x[i] * y[i]

    return result
x = [-1, 2, 2]
y = [1, 0, -3]
inner_product(x, y)
```

Out[69]: -7

In [ ]: