Fair Resource Allocation

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Overview

Motivating Example: Food Bank of the Southern Tier

- Serves six counties and nearly 4,000 square miles in New York
- Mobile Food Pantry provides food to clients a various distribution locations
- in 2019, serviced 70 regular sites, with 722 visits across all of them
- Often have to make decisions on what to stock for each location on the fly

Fair Resource Allocation

The Mobile Food Pantry also needs to make *fair* allocations. As measured by these three main desiderate

- *Pareto Efficiency*: for any location to benefit, another must be hurt
- *Envy-freeness*: no location prefers an allocation received by another
- *Proportionality*: each location prefers the allocation received versus equal allocation

The Model

- A principal wants to divide K commodities among n locations, with initial budget B,set of possible types (demands) Θ. Each customer has utility function u
- At each location, the principal makes an allocation $X \in \mathbb{R}^{|\Theta| \times K}$ where each row is how much of the budget he gives to each type
- Needs to make sure all customers are satisfied while maintaining a fair allocation

Offline Allocation

- In the *offline* allocation problem, the endowment (size) of each type is known at every location apriori
- In this scenario maximizing **Nash Social Welfare** yields a fair allocation

$$\text{maximize} \sum_{i=1}^{n} \sum_{\theta \in \Theta} N_{i,\theta} \log u(X_{i,\theta}, \theta) \tag{1}$$

$$\text{s.t} \sum_{i=1}^{n} \sum_{\theta \in \Theta} X_{i,\theta} \leq B \tag{2}$$

Online Allocation

- In the *online* setting, the endowments are not known apriori, but are generated from known distributions $\{\mathcal{F}_i | i \in [n]\}$
- Can be shown that in the online case achieving an allocation that fully satisfies all three fairness conditions is *impossible*
- Approximate maximizers of the Nash Social Welfare can be wildly unfair allocations

Online Resource Allocation as an MDP

Our MDP is defined as a five tuple $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathcal{T}, \mathcal{H})$

- State space $\mathcal{S} := \{(b, N) | b \in \mathbb{R}_+^k, N \in \mathbb{R}_+^{|\Theta|}\}$ where b is a vector of the remaining budget, and N is a vector of the endowments. Our initial state $S_0 = (B, N_0)$, where B is the full pre-planned budget and $N_0 \sim \mathcal{F}_0$
- The action-space in state (b, N) is defined as $\mathcal{A}_i := \{X \in \mathbb{R}_+^{|\Theta| \times K} | \sum_{\theta \in \Theta} N_\theta X_\theta \leq b\}$
- Our reward-space $\mathcal{R}$ is the Nash Social Welfare: while in state s and taking action a, we have $R(s, a) = \sum_{\theta \in \Theta} N_\theta \log u(X_\theta, \theta)$ where $u : \mathbb{R}^{|\Theta| \times K} \times \mathbb{R}^k \to \mathbb{R}_+$ is a utility function for the agents.
- Transitions. Given state $(b, N_i) \in \mathcal{S}$ and action $X \in \mathcal{A}$. we have our new state $s_{i+1} = (b - \sum_\theta N_\theta X_\theta, N_{i+1})$ where $N_{i+1} \sim \mathcal{F}_{i+1}$
- Each episode will have the same number of steps as there are locations. Thus $\mathcal{H} = n$

Heuristic Agent: Equal Allocation

The equal allocation agent will make allocations proportional to the expected endowment size of each location.

$$X_{i,\theta} = B\left(\frac{\mathbb{E}\left[N_{i,\theta}\right]}{\sum_{i,\theta}\mathbb{E}\left[N_{i,\theta}\right]}\right)$$

Additional Fairness Metrics

- While our reward is still the Nash Social Welfare, we still want to know how fair our algorithm's allocation is
- We do this by comparing online allocation X^{alg} to offline (hindsight) allocation X^{opt} for our fairness criteria

$$\Delta_{envy} := \max_{i,\theta} ||X^{alg}_{i,\theta} - X^{opt}_{i,\theta}||_{\infty} \tag{3}$$

$$\Delta_{efficiency} := \sum_{k=1}^{K} \left(B_k - \sum_{i,\theta} N_{i,\theta} X^{alg}_{i,\theta,k} \right) \tag{4}$$

- (should i include proportionality as well?)

Conclusion

- Some stuff
- Reinforcement Learning could produce more fair allocations than our heuristics, etc...