

# Hiding in Multilayer Networks—Supplementary Materials

## Paper #6275

### 1 Proofs of Theoretical Results

**Observation 1.** *The problem of Multilayer Global Hiding is in P given the degree centrality measure. In fact, for a given problem instance either any  $A^*$  that connects  $\hat{v}$  with all contacts is a solution, or there are no solutions at all.*

*Proof.* Any valid solution to the problem  $A^*$  must connect the evader  $\hat{v}$  with all contacts. Therefore, after the addition of  $A^*$  the degree centrality of  $\hat{v}$  is  $|F|$ , while the degree centrality of every contact increases by 1. Hence, the degree centrality ranking in the network does not depend on the choice of layers in which  $\hat{v}$  gets connected with its contacts.  $\square$

**Theorem 1.** *The problem of Multilayer Global Hiding is NP-complete given the closeness centrality measure.*

*Proof.* The problem is trivially in NP, since after the addition of a given  $A^*$  the closeness centrality ranking can be computed in polynomial time.

Next, we prove that the problem is NP-hard. To this end, we show a reduction from the NP-complete problem of *Exact 3-Set Cover*. The decision version of this problem is defined by a set of subsets  $S = \{S_1, \dots, S_m\}$  of universe  $U = \{u_1, \dots, u_{3k}\}$ , such that  $\forall_i |S_i| = 3$ . The goal is to determine whether there exist  $k$  pairwise disjoint elements of  $S$  the sum of which equals  $U$ .

Given an instance of the problem of *Exact 3-Set Cover*, let us construct a multilayer network,  $M = (V_L, E_L, V', L)$ , as follows (Figure 1 depicts an instance of this network):

- **The set of nodes  $V'$ :** For every  $u_i \in U$  we create a node  $u_i$ , as well as 3 nodes  $w_{i,1}, w_{i,2}, w_{i,3}$ . We will denote the set of all nodes  $u_i$  by  $U$ , and the set of all nodes  $w_{i,j}$  by  $W$ . We also create the evader node  $\hat{v}$ , the node  $v'$ , and the following four sets of nodes:
  1.  $A = \{a_1, \dots, a_m\}$ ;
  2.  $B = \{b_1, \dots, b_{2k+2m}\}$ ;
  3.  $B' = \{b'_1, \dots, b'_{k+2m+1}\}$ ;
  4.  $B'' = \{b''_1, \dots, b''_{2k+m-1}\}$ .
- **The set of layers  $L$ :** For every  $S_i \in S$  we create a layer  $\alpha_i$ . We also create an additional layer  $\beta$ .
- **The set of occurrences of nodes in layers  $V_L$ :** Node  $u_j \in U$  appears in layer  $\alpha_i$  if and only if  $u_j \in S_i$ . Node  $w_{i,j} \in U$  appears only in layer  $\alpha_i$ . The evader  $\hat{v}$ , as well as all

nodes in  $A$  appear in every layer  $\alpha_i$ . Node  $v'$ , as well as all nodes in  $B$ ,  $B'$ , and  $B''$  appear only in layer  $\beta$ .

- **The set of edges  $E_L$ :** For every node that appears in multiple layers, we connect all occurrences of this node in a clique. For node  $u_j$  in layer  $\alpha_i$  we connect it with node  $a_i$ . In every layer  $\alpha_i$  we connect all nodes in  $A$  into a clique. Moreover, we connect every node  $b_i$  with node  $v'$ , and connect every node  $b'_i$  with node  $b_i$ . Finally, we connect every node  $b''_i$  with node  $b'_i$ .

Now, consider the following instance of the problem of Multilayer Global Hiding,  $(M, \hat{v}, F, c, d)$ , where:

- $M$  is the multilayer network we just constructed;
- $\hat{v}$  is the evader;
- $F = U \cup W$  is the set of contacts;
- $c$  is the closeness centrality measure;
- $d = 1$ .

Next, let us analyze the closeness centrality values of nodes in the network. Notice that every node  $w_{i,j}$  appears only in a single layer  $\alpha_i$ , hence  $\hat{v}$  has to connect with  $w_{i,j}$  in layer  $\alpha_i$ . Assume that the evader  $\hat{v}$  has connections with nodes in  $U$  in exactly  $x$  layers, i.e.,  $x = |\{\alpha_i \in L : \exists u_j (\hat{v}^{\alpha_i}, u_j^{\alpha_i}) \in A^*\}|$ . We then have:

- $c_{clos}(\hat{v}) = 3k + 3m + \frac{x}{2} + \frac{m-x}{3} \geq 3k + 3\frac{1}{3}m$  as  $\hat{v}$  is a neighbor of  $3k$  nodes in  $U$  and  $3m$  nodes in  $W$ , while for any  $a_i \in A$  the distance between  $a_i$  and  $\hat{v}$  is 2 if  $\hat{v}$  is connected with any  $u_j$  in layer  $\alpha_i$  and 3 otherwise;
- $c_{clos}(u_i) \leq 1 + m + \frac{3k-1}{2} + \frac{3m}{2} = 1\frac{1}{2}k + 2\frac{1}{2}m + \frac{1}{2} < c_{clos}(\hat{v})$  as  $u_i$  is a neighbor of  $\hat{v}$  and at most  $m$  nodes in  $A$ , while the distance to all other nodes is at least 2;
- $c_{clos}(a_i) \leq 3 + m - 1 + \frac{1}{2} + \frac{3k-3}{2} + \frac{3m}{3} = 1\frac{1}{2}k + 2m + 1 < c_{clos}(\hat{v})$  as  $a_i$  is a neighbor of 3 nodes from  $U$  and all other  $m - 1$  nodes in  $A$ , while the distance to  $\hat{v}$  and all other nodes in  $U$  is 2, and the distance to all nodes in  $W$  is at least 3;
- $c_{clos}(w_{i,j}) < c_{clos}(\hat{v})$  as for any other node  $v$  we have  $\lambda(w_{i,j}, v) = \lambda(\hat{v}, v) + 1$ , since the shortest paths between  $w_{i,j}$  and all other nodes go through  $\hat{v}$ ;
- $c_{clos}(v') = 2k + 2m + \frac{k+2m+1}{2} + \frac{2k+m-1}{3} = 3k + 3m + \frac{k}{2} + \frac{m-k}{3} + \frac{1}{6}$  as  $v'$  is a neighbor of all  $2k + 2m$  nodes in

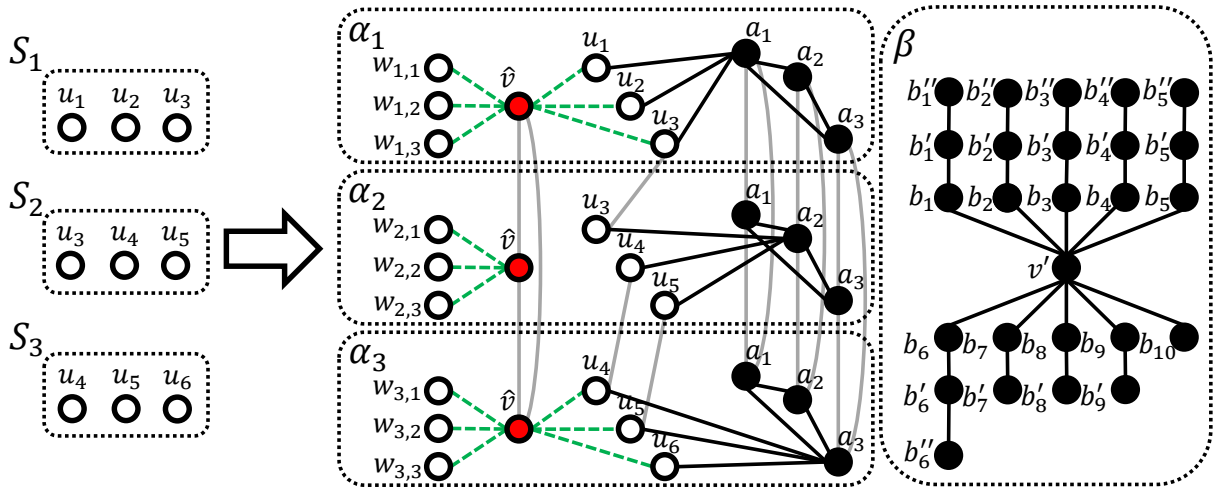


Figure 1: An illustration of the network used in the proof of Theorem 1. Edges connecting occurrences of the same node in different layers are highlighted in grey. The red node represents the evader, while the white nodes represent the contacts. Dashed (green) edges represent the solution to this problem instance.

$B$ , the distance to all  $k + 2m + 1$  nodes in  $B'$  is 2, while the distance to all  $m - k + 1$  nodes in  $B''$  is 3.

We have shown that all nodes in  $A$ ,  $U$ , and  $W$  have smaller closeness centrality than  $\hat{v}$ . It is easy to check that  $v'$  has greater closeness centrality than all other nodes occurring in layer  $\beta$ . Hence,  $\hat{v}$  is hidden if and only if  $v'$  has greater closeness centrality than  $\hat{v}$ . This is true when:

$$3k + 3m + \frac{x}{2} + \frac{m-x}{3} < 3k + 3m + \frac{k}{2} + \frac{m-k}{3} + \frac{1}{6}$$

which can be simplified to  $x < k + 1$ . Since both  $x$  and  $k$  are in  $\mathbb{N}$  this is equivalent to  $x \leq k$ . Therefore,  $\hat{v}$  is hidden if and only if it has connections with nodes in  $U$  in at most  $k$  layers.

Now we will show that if there exists a solution to the given instance of the Exact 3-Set Cover problem, then there also exists a solution to the constructed instance of the Multilayer Global Hiding problem. Let  $S^*$  be an exact cover of  $U$ . In layer  $\alpha_i$  we connect  $\hat{v}$  with all nodes from  $W$  that occur in this layer. For every  $S_i \in S^*$  we connect  $\hat{v}$  with  $u_j \in S_i$  in layer  $\alpha_i$ . This way,  $\hat{v}$  becomes connected to all  $3k$  contacts from  $U$ , since all the sets in  $S^*$  are pairwise disjoint.

To complete the proof, we have to show that if there exists a solution  $A^*$  to the constructed instance of the Multilayer Global Hiding problem, then there also exists a solution to the given instance of the Exact 3-Set Cover problem. We have shown above that if  $\hat{v}$  is hidden, then it is connected to nodes in  $U$  in at most  $k$  layers from  $\{\alpha_1, \dots, \alpha_m\}$ . However, since  $\hat{v}$  must be connected with all  $3k$  nodes in  $U$  in order for  $A^*$  to be a correct solution, then  $\{S_i : \exists u_j (\hat{v}^{\alpha_i}, u_j^{\alpha_i}) \in A^*\}$  is a solution to the given instance of the Exact 3-Set Cover problem. This concludes the proof.  $\square$

**Theorem 2.** *The problem of Multilayer Global Hiding is NP-complete given the betweenness centrality measure.*

*Proof.* The problem is trivially in NP, since after the addition of a given  $A^*$  the betweenness centrality rankings can be computed in polynomial time.

Next, we prove that the problem is NP-hard. To this end, we show a reduction from the NP-complete problem of *Finding  $k$ -Clique*. The decision version of this problem is defined by a simple network,  $G = (V, E)$ , and a constant,  $k \in \mathbb{N}$ . The goal is to determine whether there exist  $k$  nodes in  $G$  that form a clique.

Given an instance of the problem of *Finding  $k$ -Clique*, defined by  $k$  and a simple network  $G = (V, E)$ , let us construct a multilayer network,  $M = (V_L, E_L, V', L)$ , as follows (Figure 2 depicts an instance of this network):

- **The set of nodes  $V'$ :** For every node,  $v_i \in V$ , we create a node  $v_i$ . Additionally, we create the evader node  $\hat{v}$ , node  $a$ , and the following three sets of nodes:
  1.  $B = \{b_1, b_2\}$ ;
  2.  $W = \{w_1, \dots, w_k\}$ ;
  3.  $C = \{c_1, \dots, c_{n+k}\}$ .
- **The set of layers  $L$ :** We create a layer  $\alpha$ , a layer  $\gamma$ , as well as  $n$  layers  $\beta_1, \dots, \beta_n$ .
- **The set of occurrences of nodes in layers  $V_L$ :** Node  $\hat{v}$  and node  $a$  appear in layer  $\alpha$  and all layers  $\{\beta_1, \dots, \beta_n\}$ . Each node  $v_i$  appears in layer  $\alpha$  and  $\beta_i$ . Nodes in  $W$  appear in all layers  $\{\beta_1, \dots, \beta_n\}$ . Nodes in  $B$  and  $C$  appear only in layer  $\gamma$ .
- **The set of edges  $E_L$ :** In layer  $\alpha$  we create an edge between two nodes  $v_i, v_j \in V$  if and only if this edge was present in  $G$ . In every layer where  $a$  appears we connect it with all occurring nodes from  $V$  and  $W$ . Finally, we connect every node  $c_i$  with both  $b_1$  and  $b_2$ .

Now, consider the following instance of the problem of Multilayer Local Hiding,  $(M, \hat{v}, F, c, (d^\alpha)_{\alpha \in L})$ , where:

- $M$  is the multilayer network we just constructed;
- $\hat{v}$  is the evader;
- $F = V \cup W$  is the set of contacts;

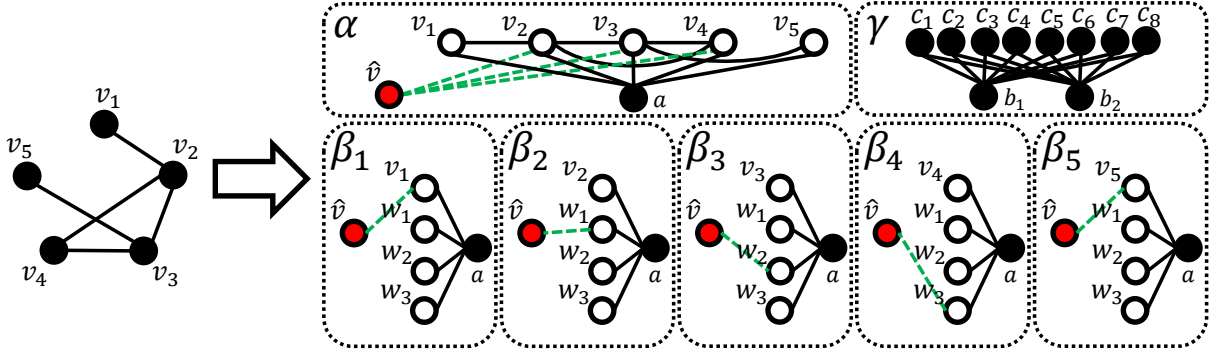


Figure 2: An illustration of the network used in the proof of Theorem 2. Edges connecting occurrences of the same node in different layers are highlighted in grey. The red node represents the evader, while the white nodes represent the contacts. Dashed (green) edges represent the solution to this problem instance.

- $c$  is the betweenness centrality measure;
- $d = 2n + 2k + 3$  is the safety margin.

Notice that, since  $d = 2n + 2k + 3$ , all other nodes must have greater betweenness centrality than the evader in order for  $\hat{v}$  to be hidden. Notice also that the betweenness centrality of every node  $c_i$  is  $\frac{1}{n+k}$ . Moreover, after adding  $A^*$  all nodes other than  $\hat{v}$  have non-zero betweenness centrality. If  $\hat{v}$  gets connected to at least two nodes from  $F$  that are not connected to each other, then  $\hat{v}$  controls one of at most  $n + k - 1$  shortest path between them (other paths can only go through nodes in  $V \cup W \cup \{a\}$ ) and thus the betweenness centrality of  $\hat{v}$  is at least  $\frac{1}{n+k-1}$ . Therefore, in order to get hidden,  $\hat{v}$  cannot control any shortest paths in the network. This implies that, if  $\hat{v}$  is hidden then all nodes that are connected to  $\hat{v}$  in layer  $\alpha$  must form a clique, and also implies that in every layer  $\beta$ , the evader  $\hat{v}$  can be connected to at most one node (otherwise  $\hat{v}$  controls one of the shortest paths between its two neighbors without an edges between them).

Now we will show that if there exists a solution to the given instance of the Finding  $k$ -Clique problem, then there also exists a solution to the constructed instance of the Multilayer Global Hiding problem. Let  $V^*$  be a group of  $k$  nodes forming a clique in  $G$ . Let us create  $A^*$  by connecting  $\hat{v}$  to nodes from  $V^*$  in layer  $\alpha$ . Now, we connect every  $v_i \in V \setminus V^*$  to  $\hat{v}$  in layer  $\beta_i$ . In the remaining layers from  $\{\beta_1, \dots, \beta_n\}$  (corresponding to elements  $v_i \in V^*$ ) we connect  $\hat{v}$  to all nodes in  $W$ . As argued above, for such  $A^*$ , the evader  $\hat{v}$  is hidden, hence  $A^*$  is a solution to the constructed instance of the Multilayer Global Hiding problem.

To complete the proof we have to show that if there exists a solution  $A^*$  to the constructed instance of the Multilayer Global Hiding problem, then there also exists a solution to the given instance of the Finding  $k$ -Clique problem. As argued above, in each layer  $\beta_i$  the evader  $\hat{v}$  can be connected to at most one node. Since all  $k$  nodes from  $W$  appear only in layers from  $\{\beta_1, \dots, \beta_n\}$ , the evader  $\hat{v}$  can be connected to at most  $n - k$  nodes from  $V$  in layers from  $\{\beta_1, \dots, \beta_n\}$ . Therefore,  $\hat{v}$  has to have at least  $k$  neighbors from  $V$  in layer  $\alpha$ . As shown above, in order for  $\hat{v}$  to be hidden in  $\alpha$ , all of its neighbors must form a clique. Hence, the neighbors of  $\hat{v}$  in

layer  $\alpha$  form a clique in  $G$ . This concludes the proof.  $\square$

**Theorem 3.** *The problem of Multilayer Local Hiding is NP-complete given the degree centrality measure.*

*Proof.* The problem is trivially in NP, since after the addition of a given  $A^*$  the degree centrality rankings for all layers can be computed in polynomial time.

Next, we prove that the problem is NP-hard. To this end, we show a reduction from the NP-complete problem of *Exact 3-Set Cover*. The decision version of this problem is defined by a set of subsets  $S = \{S_1, \dots, S_m\}$  of universe  $U = \{u_1, \dots, u_{3k}\}$ , such that  $\forall_i |S_i| = 3$ . The goal is to determine whether there exist  $k$  pairwise disjoint elements of  $S$  the sum of which equals  $U$ .

Given an instance of the problem of *Exact 3-Set Cover*, let us construct a multilayer network,  $M = (V_L, E_L, V', L)$ , as follows (Figure 3 depicts an instance of this network):

- **The set of nodes  $V'$ :** For every element,  $u_i \in U$ , we create a node  $u_i$ . We also create  $2(m - k)$  nodes  $w_1, \dots, w_{2(m-k)}$ . Additionally, we create the evader node  $\hat{v}$  and three nodes  $a_1, a_2, a_3$ . We will denote the set of all nodes  $a_i$  as  $A$ , the set of all nodes  $u_i$  as  $U$ , and the set of all nodes  $w_i$  as  $W$ .
- **The set of layers  $L$ :** For every  $S_i \in S$  we add a layer  $\alpha_i$ .
- **The set of occurrences of nodes in layers  $V_L$ :** Node  $u_j \in U$  appears in layer  $\alpha_i$  if and only if  $u_j \in S_i$ . The evader  $\hat{v}$ , as well as all nodes in  $A$  and  $W$ , appear in all layers.
- **The set of edges  $E_L$ :** In every layer we connect every node  $u_j \in U$  occurring in this layer to every node in  $A$ .

Now, consider the following instance of the problem of Multilayer Local Hiding,  $(M, \hat{v}, F, c, (d^\alpha)_{\alpha \in L})$ , where:

- $M$  is the multilayer network we just constructed;
- $\hat{v}$  is the evader;
- $F = U \cup W$  is the set of contacts;
- $c$  is the degree centrality measure;
- $d^{\alpha_i} = 3$  for every  $\alpha_i \in L$ .

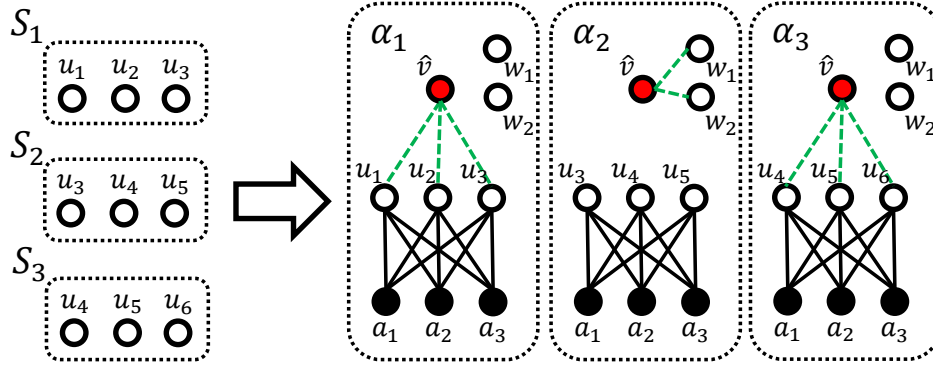


Figure 3: An illustration of the network used in the proof of Theorem 3. The red node represents the evader, while the white nodes represent the contacts. Dashed (green) edges represent the solution to this problem instance.

Next, let us consider what are the sets of edges that can be added between the evader  $\hat{v}$  and the contacts  $F$  in each layer, so that the evader is hidden. In every layer  $\alpha_i$  the nodes in  $A$  as well as the nodes  $u_j \in S_i$  have degree 3, while all other nodes have degree 0. We can connect  $\hat{v}$  to any two or less contacts and  $\hat{v}$  will still be hidden. If we connect the evader to three contacts, they have to be nodes in  $S_i$  (as these are the only nodes that potentially can have degree greater than 3). We cannot connect  $\hat{v}$  to more than three contacts and still have  $\hat{v}$  hidden.

Now we will show that if there exists a solution to the given instance of the Exact 3-Set Cover problem, then there also exists a solution to the constructed instance of the Multilayer Local Hiding problem. Let  $S^*$  be an exact cover of  $U$ . For every  $S_i \in S^*$  we connect  $\hat{v}$  to every  $u_j \in S_i$  in layer  $\alpha_i$ . This way,  $\hat{v}$  becomes connected to all  $3k$  contacts from  $U$ , since all the sets in  $S^*$  are pairwise disjoint. For every  $S_i \notin S^*$  we connect  $\hat{v}$  to two nodes from  $W$  in layer  $\alpha_i$  (since there are  $m - k$  such layers, we can connect  $\hat{v}$  to all  $2(m - k)$  contacts from  $W$  this way).

To complete the proof we have to show that if there exists a solution to the constructed instance of the Multilayer Local Hiding problem, then there also exists a solution to the given instance of the Exact 3-Set Cover problem. Let  $x$  be the number of layers from  $\{\alpha_1, \dots, \alpha_m\}$  in which  $\hat{v}$  has at most two neighbors, and let  $m - x$  be the number of layers from  $\{\alpha_1, \dots, \alpha_m\}$  where  $\hat{v}$  has exactly three neighbors. Since  $\hat{v}$  has to be connected to all  $3k + 2(m - k)$  contacts, we have  $2x + 3(m - x) \geq 3k + 2(m - k)$ , which gives us  $x \leq m - k$ . However, since  $\hat{v}$  can connect to nodes from  $W$  in layer  $\alpha_i$  if and only if it connects to at most two nodes in  $\alpha_i$ , we also have  $2x \geq 2(m - k)$ . Hence, we have  $x = m - k$ , i.e.,  $\hat{v}$  is connected with all nodes from  $W$  in  $m - k$  layers from  $\{\alpha_1, \dots, \alpha_m\}$ . Therefore, in the remaining  $k$  layers from  $\{\alpha_1, \dots, \alpha_m\}$ , the evader  $\hat{v}$  has to connect to all  $3k$  nodes from  $U$ . Since the evader cannot connect to more than three nodes in any layer  $\alpha_i$ , all these sets of neighbors from  $U$  have to be disjoint, thus forming the solution to the given instance of the Exact 3-Set Cover problem. This concludes the proof.  $\square$

**Theorem 4.** *The problem of Multilayer Local Hiding is NP-*

*complete given the closeness centrality measure.*

*Proof.* The problem is trivially in NP, since after the addition of a given  $A^*$  the closeness centrality rankings for all layers can be computed in polynomial time.

Next, we prove that the problem is NP-hard. To this end, we show a reduction from the NP-complete problem of *Exact 3-Set Cover*. The decision version of this problem is defined by a set of subsets  $S = \{S_1, \dots, S_m\}$  of universe  $U = \{u_1, \dots, u_{3k}\}$ , such that  $\forall_i |S_i| = 3$ . The goal is to determine whether there exist  $k$  pairwise disjoint elements of  $S$  the sum of which equals  $U$ .

Given an instance of the problem of *Exact 3-Set Cover*, let us construct a multilayer network,  $M = (V_L, E_L, V', L)$ , as follows (Figure 4 depicts an instance of this network):

- **The set of nodes  $V'$ :** For every  $u_i \in U$  we create a node  $u_i$ . In addition, we create the nodes  $w_1, \dots, w_{2(m-k)}$  and  $a_1, \dots, a_{2(m-k)}$ . Finally, we create the evader node  $\hat{v}$  and 5 nodes  $c_1, \dots, c_5$ . We will denote the set of all nodes  $a_i$  by  $A$ , the set of all nodes  $c_i$  by  $C$ , the set of all nodes  $u_i$  by  $U$ , and the set of all nodes  $w_i$  by  $W$ .
- **The set of layers  $L$ :** For every  $S_i \in S$  we add a layer  $\alpha_i$ .
- **The set of occurrences of nodes in layers  $V_L$ :** Node  $u_j \in U$  appears in layer  $\alpha_i$  if and only if  $u_j \in S_i$ . The evader  $\hat{v}$ , as well as all nodes in  $A$ ,  $C$ , and  $W$ , appear in all layers.
- **The set of edges  $E_L$ :** In all layers we connect every node  $w_i$  with the node  $a_i$ , and we create edges  $(c_1, c_2), (c_1, c_3), (c_1, c_4), (c_4, c_5)$ .

Now, consider the following instance of the problem of Multilayer Local Hiding,  $(M, \hat{v}, F, c, (d^\alpha)_{\alpha \in L})$ , where:

- $M$  is the multilayer network we just constructed;
- $\hat{v}$  is the evader;
- $F = U \cup W$  is the set of contacts;
- $c$  is the closeness centrality measure;
- $d^{\alpha_i} = 1$  for every  $\alpha_i \in L$ .

Next, let us consider what are the sets of edges that can be added between the evader  $\hat{v}$  and the contacts  $F$  in each layer, so that the evader is hidden. Notice that closeness centrality

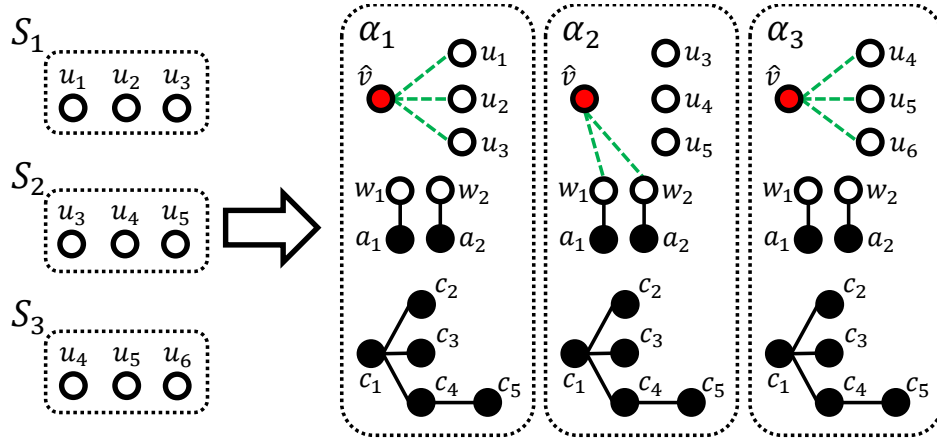


Figure 4: An illustration of the network used in the proof of Theorem 4. The red node represents the evader, while white the nodes represent the contacts. Dashed (green) edges represent the solution to this problem instance.

of the node  $c_1$  is  $3\frac{1}{2}$  and it is not affected by the edges added to  $\hat{v}$ . Assume that we connect node  $\hat{v}$  with  $x$  nodes from  $U$  and  $y$  nodes from  $W$ . We then have the following (for easier comparison we express the centrality values as fractions with the common denominator 6):

- $c_{clos}(\hat{v}) = x + \frac{3y}{2} = \frac{6x+9y}{6}$ ;
- $c_{clos}(w_i) = \frac{x}{2} + \frac{5y}{6} + \frac{7}{6} = \frac{3x+5y+7}{6}$  if  $w_i \in N(\hat{v})$ ;
- $c_{clos}(c_1) = \frac{7}{2} = \frac{21}{6}$ ;

No other node can have greater closeness centrality than  $\hat{v}$ . We can connect  $\hat{v}$  with at most two of any of the contacts, as node  $c_1$  will still have greater closeness centrality. If we want to connect  $\hat{v}$  with three contacts, these contacts have to be nodes from  $U$ . If  $x + y = 3$  and  $y > 0$ , or if  $x + y > 3$ , then the closeness centrality of  $\hat{v}$  is the highest in the network, meaning that  $\hat{v}$  is not hidden.

Now we will show that if there exists a solution to the given instance of the Exact 3-Set Cover problem, then there also exists a solution to the constructed instance of the Multilayer Local Hiding problem. Let  $S^*$  be an exact cover of  $U$ . For every  $S_i \in S^*$  we connect  $\hat{v}$  to every  $u_j \in S_i$  in layer  $\alpha_i$ . This way,  $\hat{v}$  becomes connected to all  $3k$  contacts from  $U$ , since all the sets in  $S^*$  are pairwise disjoint. For every  $S_i \notin S^*$  we connect  $\hat{v}$  to two nodes from  $W$  in layer  $\alpha_i$  (since there are  $m - k$  such layers, we can connect  $\hat{v}$  to all  $2(m - k)$  contacts from  $W$  this way).

To complete the proof, we have to show that if there exists a solution  $A^*$  to the constructed instance of the Multilayer Local Hiding problem, then there also exists a solution to the given instance of the Exact 3-Set Cover problem. Let  $z$  be the number of layers from  $\{\alpha_1, \dots, \alpha_m\}$  where  $\hat{v}$  has at most two neighbors, and let  $z - x$  be the number of layers from  $\{\alpha_1, \dots, \alpha_m\}$  where  $\hat{v}$  has exactly three neighbors. Since we have to connect  $\hat{v}$  to all  $3k + 2(m - k)$  contacts, we have  $2z + 3(m - z) \geq 3k + 2(m - k)$ , which gives us  $z \leq m - k$ . However, since  $\hat{v}$  can connect to nodes from  $W$  in layer  $\alpha_i$  if and only if it connects to at most two nodes in  $\alpha_i$ , we also have  $2z \geq 2(m - k)$ . Hence, we have  $z = m - k$ , i.e.,  $\hat{v}$  connects to all nodes from  $W$  in  $m - k$  layers from  $\{\alpha_1, \dots, \alpha_m\}$ .

Therefore, in the remaining  $k$  layers from  $\{\alpha_1, \dots, \alpha_m\}$ , the evader  $\hat{v}$  has to connect with all  $3k$  nodes from  $U$ . Since the evader cannot connect to more than three nodes in any layer  $\alpha_i$ , all these sets of neighbors from  $U$  have to be disjoint, thus forming a solution to the given instance of the Exact 3-Set Cover problem. This concludes the proof.  $\square$

## 2 Simulation Results

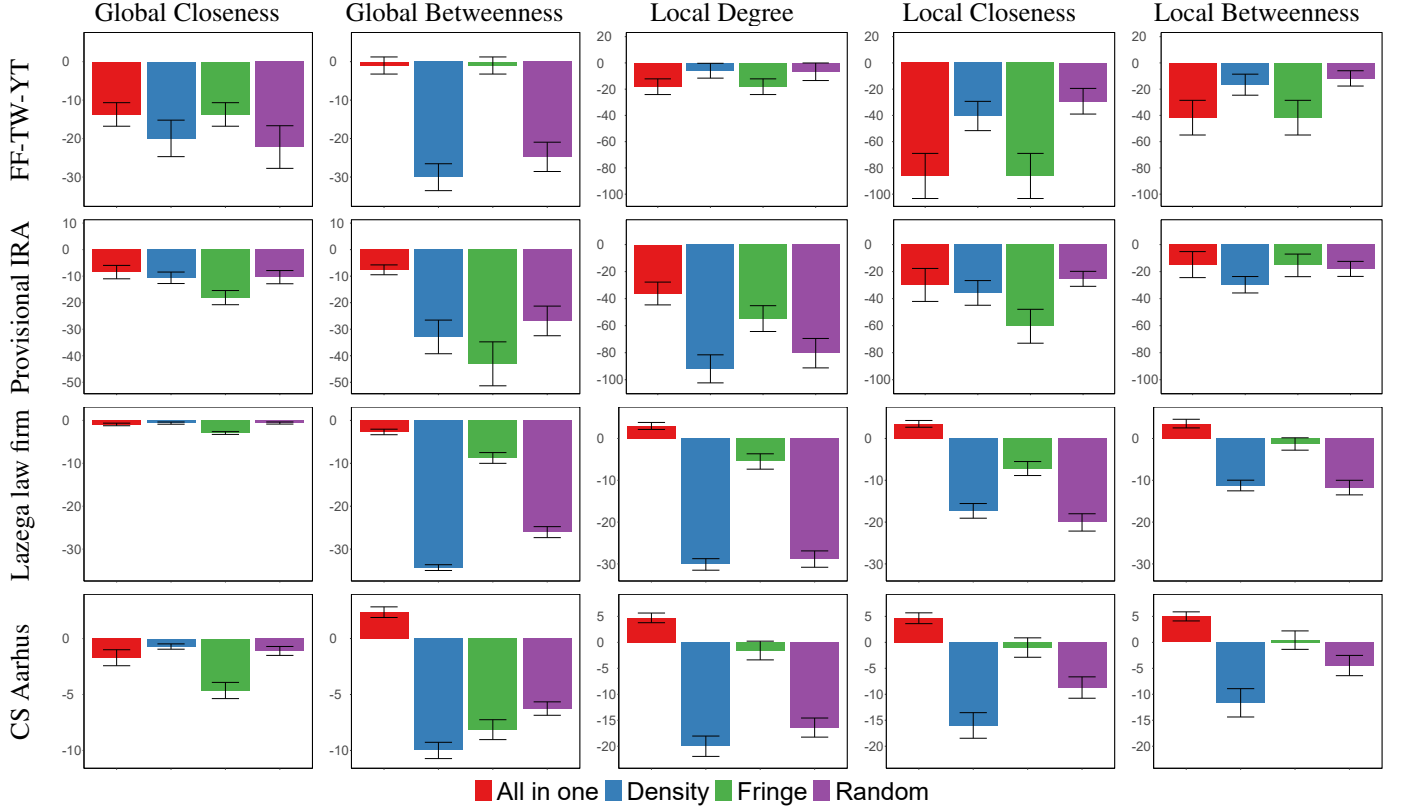


Figure 5: Given different centrality measures and different networks, the figure depicts the average change in centrality ranking of 10 different evaders as a result of execution of different hiding heuristics. For the randomly generate networks the experiment is repeated 100 times, with a new network generated each time. Error bars represent 95% confidence intervals.