

$ax^2+bx+c=0$, let us consider the Solution Matrix of x of this solution to be $x = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$

$$\Rightarrow x^2 = x \cdot x = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} \cdot \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} = \begin{bmatrix} \alpha^2 - \beta^2 & 2\alpha\beta \\ -2\alpha\beta & \alpha^2 - \beta^2 \end{bmatrix}$$

$$\Rightarrow ax^2+bx+c=0$$

$$\Rightarrow a \begin{bmatrix} \alpha^2 - \beta^2 & 2\alpha\beta \\ -2\alpha\beta & \alpha^2 - \beta^2 \end{bmatrix} + b \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a\alpha^2 - a\beta^2 + b\alpha + c & 2\alpha\beta + b\beta \\ -2(a)\alpha\beta - b\beta & a\alpha^2 - a\beta^2 + b\alpha + c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow a(\alpha^2 - \beta^2) + b\alpha + c = 0 \text{ \&}$$

$$\Rightarrow 2\alpha\beta + b\beta = 0$$

$$\Rightarrow 2\beta(\alpha + b) = 0$$

$$\Rightarrow \beta = 0$$

$$\Rightarrow a\alpha^2 + b\alpha + c = 0$$

$$\Rightarrow \alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$ Solution Matrix of x of this Equation $ax^2+bx+c=0$

$$\Rightarrow \alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \beta = 0$$

$$\Rightarrow x = \begin{bmatrix} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & 0 \\ 0 & \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{bmatrix}$$

Let's consider $D = b^2 - 4ac$

$$\Rightarrow \begin{bmatrix} \frac{-b \pm \sqrt{D}}{2a} & 0 \\ 0 & \frac{-b \pm \sqrt{D}}{2a} \end{bmatrix}$$

$$\Rightarrow \frac{b}{2a} \begin{bmatrix} -1 \pm \sqrt{D}/b & 0 \\ 0 & -1 \pm \sqrt{D}/b \end{bmatrix}$$

$$\Rightarrow \frac{b}{2a} \left\{ \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \pm \begin{bmatrix} \sqrt{D}/b & 0 \\ 0 & \sqrt{D}/b \end{bmatrix} \right\}$$

$$\Rightarrow \frac{b}{2a} \{ (-1) \cdot I \pm \begin{bmatrix} \sqrt{D}/b & 0 \\ 0 & \sqrt{D}/b \end{bmatrix} \}$$

$$\Rightarrow -\frac{b}{2a} \{ I \mp \begin{bmatrix} \sqrt{D}/b & 0 \\ 0 & \sqrt{D}/b \end{bmatrix} \}$$

for $D < 0$, $\sqrt{D} = i\sqrt{-D}$

So,

Most important Findings/observation -#1

it applies a phase of $\Pi/2$ from $|+\rangle$ state Along z axis & lands at $|i+\rangle$ state if $D < 0$

Say for example for higher order of solution for $X^n = 1$ I mean to say for finding the nth root of 1 This Solution could be useful, right, cause $(n - 1)$ Roots are imaginary. say for example for $X^5 = 1$ please do kindly note that Most of the roots, I mean to say the 4 roots of 1 are imaginary.