

$ax^2+bx+c=0$ , let us consider the Solution Matrix of  $x$  of this solution to be  $x = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$

$$\Rightarrow x^2 = x \cdot x = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} \cdot \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} = \begin{bmatrix} \alpha^2 - \beta^2 & 2\alpha\beta \\ -2\alpha\beta & \alpha^2 - \beta^2 \end{bmatrix}$$

$$\Rightarrow ax^2+bx+c=0$$

$$\Rightarrow a \begin{bmatrix} \alpha^2 - \beta^2 & 2\alpha\beta \\ -2\alpha\beta & \alpha^2 - \beta^2 \end{bmatrix} + b \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a\alpha^2 - a\beta^2 + b\alpha + c & 2\alpha\beta + b\beta \\ -2(a)\alpha\beta - b\beta & a\alpha^2 - a\beta^2 + b\alpha + c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow a(\alpha^2 - \beta^2) + b\alpha + c = 0 \text{ \& }$$

$$\Rightarrow 2\alpha\beta + b\beta = 0$$

$$\Rightarrow 2\beta(\alpha + b) = 0$$

$$\Rightarrow \beta = 0$$

$$\Rightarrow a\alpha^2 + b\alpha + c = 0$$

$$\Rightarrow \alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$  Solution Matrix of  $x$  of this Equation  $ax^2+bx+c=0$

$$\Rightarrow \alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \beta = 0$$

$$\Rightarrow x = \begin{bmatrix} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & 0 \\ 0 & \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{bmatrix}$$

Let's consider  $D = b^2 - 4ac$

$$\Rightarrow \begin{bmatrix} \frac{-b \pm \sqrt{D}}{2a} & 0 \\ 0 & \frac{-b \pm \sqrt{D}}{2a} \end{bmatrix}$$

$$\Rightarrow \frac{b}{2a} \begin{bmatrix} -1 \pm \sqrt{D}/b & 0 \\ 0 & -1 \pm \sqrt{D}/b \end{bmatrix}$$

$$\Rightarrow \frac{b}{2a} \left\{ \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \pm \begin{bmatrix} \sqrt{D}/b & 0 \\ 0 & \sqrt{D}/b \end{bmatrix} \right\}$$

$$\Rightarrow \frac{b}{2a} \{ (-1) \cdot I \pm \begin{bmatrix} \sqrt{D}/b & 0 \\ 0 & \sqrt{D}/b \end{bmatrix} \}$$

$$\Rightarrow -\frac{b}{2a} \{ I \mp \begin{bmatrix} \sqrt{D}/b & 0 \\ 0 & \sqrt{D}/b \end{bmatrix} \}$$

for  $D < 0$ ,  $\sqrt{D} = i\sqrt{-D}$

So,

**Most important Findings/observation -#1**

it applies a phase of  $\Pi/2$  from  $|+\rangle$  state Along z axis & lands at  $|i+\rangle$  state if  $D < 0$

Say for example for higher order of solution for  $X^n = 1$  I mean to say for finding the nth root of 1 This Solution could be useful, right, cause  $(n - 1)$  Roots are imaginary. say for example for  $X^5 = 1$  please do kindly note that Most of the roots, I mean to say the 4 roots of 1 are imaginary.

**Example Solving:**

$$X^5 = 1$$

$$\Rightarrow X^5 - 1^5 = 0$$

$$\Rightarrow (X-1)(X^4 + X^3 + X^2 + X + 1) = 0$$

$$\Rightarrow \text{Either } X = 1$$

$$\Rightarrow \text{Or } (X^4 + X^3 + X^2 + X + 1) = 0$$

$$\Rightarrow X \neq 0 \text{ so Dividing by } X^2$$

$$\Rightarrow (X^2 + X + 1 + 1/X + 1/X^2) = 0$$

$$\Rightarrow (X^2 + 1/X^2 + X + 1/X + 1) = 0$$

$$\Rightarrow \{(X + 1/X)^2 + 2 \cdot X \cdot 1/X + (X + 1/X) + 1\} = 0$$

$$\Rightarrow \{(X + 1/X)^2 + 2 + (X + 1/X) + 1\} = 0$$

$$\Rightarrow \text{Let us consider } (X + 1/X) = Y$$

$$\Rightarrow Y^2 + Y + 3 = 0$$

$$\Rightarrow Y = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$

$$\Rightarrow Y = \frac{-1 \pm \sqrt{-11}}{2 \cdot 1}$$

$$\Rightarrow Y = \frac{-1 \pm i\sqrt{11}}{2 \cdot 1}$$

$\Rightarrow$  So, solving further this Equation for  $(X + 1/X) = Y$  we can find the 4 Imaginary Roots of X that is the 5<sup>th</sup> Roots of 1.