

$ax^2+bx+c = 0$ , let us consider the Solution Matrix of  $x$  of this solution to be  $x = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$

$$\Rightarrow x^2 = x \cdot x = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} \cdot \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} = \begin{bmatrix} \alpha^2 - \beta^2 & 2\alpha\beta \\ -2\alpha\beta & \alpha^2 - \beta^2 \end{bmatrix}$$

$$\Rightarrow ax^2+bx+c = 0$$

$$\Rightarrow a \begin{bmatrix} \alpha^2 - \beta^2 & 2\alpha\beta \\ -2\alpha\beta & \alpha^2 - \beta^2 \end{bmatrix} + b \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a\alpha^2 - a\beta^2 + b\alpha + c & 2\alpha\beta + b\beta \\ -2(a)\alpha\beta - b\beta & a\alpha^2 - a\beta^2 + b\alpha + c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow a(\alpha^2 - \beta^2) + b\alpha + c = 0 \text{ \& }$$

$$\Rightarrow 2\alpha\beta + b\beta = 0$$

$$\Rightarrow 2\beta(\alpha + b) = 0$$

$$\Rightarrow \beta = 0$$

$$\Rightarrow a\alpha^2 + b\alpha + c = 0$$

$$\Rightarrow \alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$  Solution Matrix of  $x$  of this Equation  $ax^2+bx+c = 0$

$$\Rightarrow \alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \beta = 0$$

$$\Rightarrow x = \begin{bmatrix} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & 0 \\ 0 & \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{bmatrix}$$

Let's consider  $D = b^2 - 4ac$

$$\Rightarrow \begin{bmatrix} \frac{-b \pm \sqrt{D}}{2a} & 0 \\ 0 & \frac{-b \pm \sqrt{D}}{2a} \end{bmatrix}$$

for  $D < 0$ ,  $\sqrt{D} = i\sqrt{D}$

So,

### **Most important Findings/observation -#1**

it applies a phase of  $\Pi/2$  from  $|+\rangle$  state Along  $z$  axis & lands at  $|i+\rangle$  state if  $D < 0$