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CONTEXT C0

SETS

TRAIN

CONSTANTS

a

b

WAY

AXIOMS

axiom1: $\{a, b\} \subseteq \mathbb{N}$

axiom2: $a < b$

axiom3: $WAY = a \dots b$

axiom4: $b - a \geq 20$

END

CONTEXT C2**EXTENDS** C0**SETS**

STATES

CONSTANTS

TTD

VSS

OCCUPIED

FREE

UNKNOWN

AMBIGUOUS

AXIOMS**axiom1:** $TTD \subseteq \mathbb{P}_1(WAY)$ **axiom2:** $union(TTD) = WAY$ **axiom3:** $inter(TTD) = \emptyset$ **axiom4:** $\forall ttd. (ttd \in TTD \Rightarrow (\exists p, q. (p .. q \subseteq WAY \wedge p < q \wedge ttd = p .. q)))$ **axiom5:** $VSS \subseteq \mathbb{P}_1(WAY)$ **axiom6:** $union(VSS) = WAY$ **axiom7:** $inter(VSS) = \emptyset$ **axiom8:** $\forall vss. (vss \in VSS \Rightarrow (\exists p, q, ttd. (ttd \in TTD \wedge p .. q \subseteq ttd \wedge p < q \wedge vss = p .. q)))$ **axiom9:** $partition(STATES, \{OCCUPIED\}, \{FREE\}, \{UNKNOWN\}, \{AMBIGUOUS\})$ **END**

MACHINE M0**SEES** C0**VARIABLES**

connectedTrain

front

rear

INVARIANTSinv0_1: $connectedTrain \in TRAIN \leftrightarrow BOOL$ inv0_2: $front \in dom(connectedTrain) \rightarrow WAY$ inv0_3: $rear \in dom(connectedTrain) \rightarrow WAY$ inv0_4: $\forall tr. (tr \in dom(rear) \Rightarrow rear(tr) < front(tr))$ **EVENTS****Initialisation****begin**act1: $connectedTrain := \emptyset$ act2: $front := \emptyset$ act3: $rear := \emptyset$ **end****Event** MoveTrainOnTrack *<ordinary>* $\hat{=}$ **any**

tr

len

wheregrd1: $tr \in connectedTrain^{-1}[\{TRUE\}]$ grd2: $len \in \mathbb{N}_1$ grd3: $front(tr) + len \in WAY$ **then**act1: $front(tr) := front(tr) + len$ act2: $rear := (\{TRUE \mapsto rear \Leftarrow \{tr \mapsto rear(tr) + len\}, FALSE \mapsto rear\})(bool(tr \in dom(rear)))$ **end****Event** _connectTrain *<ordinary>* $\hat{=}$ **any**

tr

fr

re

integer

wheregrd0: $TRAIN \setminus dom(connectedTrain) \neq \emptyset$ grd1: $tr \in TRAIN \setminus dom(connectedTrain)$ grd2: $fr \in WAY$ grd3: $integer \in BOOL$ grd4: $integer = TRUE \Rightarrow re \in WAY$ grd5: $re < fr$ **then**act1: $connectedTrain(tr) := TRUE$ act2: $front(tr) := fr$ act3: $rear := (\{TRUE \mapsto rear \Leftarrow \{tr \mapsto re\}, FALSE \mapsto rear\})(integer)$ **end****Event** _toggleTrainConnexionStatus *<ordinary>* $\hat{=}$ **any**

tr

wheregrd0: $dom(connectedTrain) \neq \emptyset$ grd1: $tr \in dom(connectedTrain)$ **then**act1: $connectedTrain := (\{TRUE \mapsto connectedTrain \Leftarrow \{tr \mapsto FALSE\}, FALSE \mapsto connectedTrain \Leftarrow \{tr \mapsto TRUE\}\})(bool(connectedTrain(tr) = TRUE))$ **end****END**

MACHINE M1**REFINES** M0**SEES** C0**VARIABLES**

connectedTrain

front

rear

MA

MAtemp

INVARIANTS**inv1.1:** $MA \in \text{dom}(\text{connectedTrain}) \leftrightarrow \mathbb{P}(\text{WAY})$ **inv1.2:** $\forall tr. (tr \in \text{dom}(MA) \Rightarrow (\exists p, q. (p \dots q \subseteq \text{WAY} \wedge p \leq q \wedge MA(tr) = p \dots q)))$ **inv1.3:** $\forall tr. (tr \in \text{dom}(MA) \Rightarrow \text{front}(tr) \in MA(tr))$ **inv1.4:** $\forall tr. (tr \in \text{dom}(\text{rear}) \cap \text{dom}(MA) \Rightarrow \text{rear}(tr) \in MA(tr))$ **inv1.5:** $\forall tr1, tr2. ((\{tr1, tr2\} \subseteq \text{dom}(MA) \wedge tr1 \neq tr2) \Rightarrow MA(tr1) \cap MA(tr2) = \emptyset)$ **inv1.6:** $MAtemp \in \text{dom}(\text{connectedTrain}) \leftrightarrow \mathbb{P}(\text{WAY})$ **inv1.7:** $\forall tr. (tr \in \text{dom}(MAtemp) \Rightarrow (\exists p, q. (p \dots q \subseteq \text{WAY} \wedge p \leq q \wedge MAtemp(tr) = p \dots q)))$ **SYSML/KAOS PROOF OBLIGATIONS****sysml_kaos_po.G1-Guard=>G-Guard:** (theorem)
$$\begin{aligned}
& \forall tr, p, q, len. ((\\
& (tr \in \text{connectedTrain}^{-1}[\{TRUE\}]) \\
& \wedge (p \dots q \subseteq \text{WAY} \wedge p \leq q) \\
& \wedge (\text{front}(tr) \in p \dots q) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q) \\
& \wedge (p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset) \\
& \wedge (len \in \mathbb{N}_1) \\
& \wedge (\text{front}(tr) + len \in \text{WAY}) \\
&) \Rightarrow \\
& (\\
& (tr \in \text{connectedTrain}^{-1}[\{TRUE\}]) \\
& \wedge (len \in \mathbb{N}_1) \\
& \wedge (\text{front}(tr) + len \in \text{WAY}) \\
&)
\end{aligned}$$
remplacement de toute reference a MAtemp par $((\{tr\} \triangleleft MAtemp) \cup \{tr \mapsto p \dots q\})$ **sysml_kaos_po.G1-Post=>G2-Guard:** (theorem)
$$\begin{aligned}
& \forall tr, p, q, len. ((\\
& (tr \in \text{connectedTrain}^{-1}[\{TRUE\}]) \\
& \wedge (p \dots q \subseteq \text{WAY} \wedge p \leq q) \\
& \wedge (\text{front}(tr) \in p \dots q) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q) \\
& \wedge (p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset) \\
& \wedge (len \in \mathbb{N}_1) \\
& \wedge (\text{front}(tr) + len \in \text{WAY}) \\
&) \Rightarrow \\
& (\\
& (tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(((\{tr\} \triangleleft MAtemp) \cup \{tr \mapsto p \dots q\}))) \\
& \wedge (\text{front}(tr) \in ((\{tr\} \triangleleft MAtemp) \cup \{tr \mapsto p \dots q\})(tr)) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in ((\{tr\} \triangleleft MAtemp) \cup \{tr \mapsto p \dots q\})(tr)) \\
& \wedge (((\{tr\} \triangleleft MAtemp) \cup \{tr \mapsto p \dots q\})(tr) \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset) \\
&)
\end{aligned}$$
remplacement de toute reference a MA par $((\{tr\} \triangleleft MA) \cup \{tr \mapsto MAtemp(tr)\})$ **sysml_kaos_po.G2-Post=>G3-Guard:** (theorem)
$$\begin{aligned}
& \forall tr, len. ((\\
& (tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MAtemp)) \\
& \wedge (\text{front}(tr) \in MAtemp(tr)) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in MAtemp(tr)) \\
& \wedge (MAtemp(tr) \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset) \\
&)
\end{aligned}$$

```

) ⇒
(
|  |
| --- |
| $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(((\{tr\} \triangleleft MA) \cup \{tr \mapsto MAtemp(tr)\}))$ |
| $\wedge (len \in \mathbb{N}_1)$ |
| $\wedge (\text{front}(tr) + len \in ((\{tr\} \triangleleft MA) \cup \{tr \mapsto MAtemp(tr)\})(tr))$ |


))
sysml_kaos_po.G3-Post=>G-Post: <theorem>
  ∀tr, len. (
    (
       $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MA)$ 
       $\wedge (len \in \mathbb{N}_1)$ 
       $\wedge (\text{front}(tr) + len \in MA(tr))$ 
    ) ⇒
    (
       $\text{front}(tr) + len = \text{front}(tr) + len$ 
       $\wedge ((\{TRUE \mapsto rear \triangleleft \{tr \mapsto rear(tr) + len\}, FALSE \mapsto rear\})(\text{bool}(tr \in \text{dom}(rear)))) = (\{TRUE \mapsto rear \triangleleft \{tr \mapsto rear(tr) + len\}, FALSE \mapsto rear\})(\text{bool}(tr \in \text{dom}(rear))))$ 
    )
  )

```

EVENTS

Initialisation

begin

```

act1: connectedTrain := ∅
act2: front := ∅
act3: rear := ∅
act4: MA := ∅
act5: MAtemp := ∅

```

end

Event ComputeTrainMA **<ordinary>** $\hat{=}$

any

```

tr
p
q
len

```

where

```

grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}]$ 
grd2:  $p \dots q \subseteq WAY \wedge p \leq q$ 
grd3:  $\text{front}(tr) \in p \dots q$ 
grd4:  $tr \in \text{dom}(rear) \Rightarrow rear(tr) \in p \dots q$ 
grd5:  $p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
grd6:  $len \in \mathbb{N}_1$ 
grd7:  $\text{front}(tr) + len \in WAY$ 

```

then

```

act1: MAtemp(tr) := p .. q

```

end

Event AssignMAtoTrain **<ordinary>** $\hat{=}$

any

```

tr

```

where

```

grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MAtemp)$ 
grd2:  $\text{front}(tr) \in MAtemp(tr)$ 
grd3:  $tr \in \text{dom}(rear) \Rightarrow rear(tr) \in MAtemp(tr)$ 
grd4:  $MAtemp(tr) \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 

```

then

```

act1: MA(tr) := MAtemp(tr)

```

end

Event MoveTrainFollowingItsMA **<ordinary>** $\hat{=}$

refines MoveTrainOnTrack

```

any
  tr
  len
where
  grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MA)$ 
  grd2:  $len \in \mathbb{N}_1$ 
  grd3:  $\text{front}(tr) + len \in MA(tr)$ 
then
  act1:  $\text{front}(tr) := \text{front}(tr) + len$ 
  act2:  $\text{rear} := (\{TRUE \mapsto \text{rear} \Leftarrow \{tr \mapsto \text{rear}(tr) + len\}, FALSE \mapsto \text{rear}\})(\text{bool}(tr \in \text{dom}(\text{rear})))$ 
end
Event _connectTrain  $\langle \text{ordinary} \rangle \hat{=}$ 
extends _connectTrain
any
  tr
  fr
  re
  integer
where
  grd0:  $TRAIN \setminus \text{dom}(\text{connectedTrain}) \neq \emptyset$ 
  grd1:  $tr \in TRAIN \setminus \text{dom}(\text{connectedTrain})$ 
  grd2:  $fr \in WAY$ 
  grd3:  $integer \in BOOL$ 
  grd4:  $integer = TRUE \Rightarrow re \in WAY$ 
  grd5:  $re < fr$ 
then
  act1:  $\text{connectedTrain}(tr) := TRUE$ 
  act2:  $\text{front}(tr) := fr$ 
  act3:  $\text{rear} := (\{TRUE \mapsto \text{rear} \Leftarrow \{tr \mapsto re\}, FALSE \mapsto \text{rear}\})(integer)$ 
end
Event _toggleTrainConnexionStatus  $\langle \text{ordinary} \rangle \hat{=}$ 
extends _toggleTrainConnexionStatus
any
  tr
where
  grd0:  $\text{dom}(\text{connectedTrain}) \neq \emptyset$ 
  grd1:  $tr \in \text{dom}(\text{connectedTrain})$ 
then
  act1:  $\text{connectedTrain} := (\{TRUE \mapsto \text{connectedTrain} \Leftarrow \{tr \mapsto FALSE\}, FALSE \mapsto \text{connectedTrain} \Leftarrow \{tr \mapsto TRUE\}\})(\text{bool}(\text{connectedTrain}(tr) = TRUE))$ 
end
END

```

MACHINE M2**REFINES** M1**SEES** C2**VARIABLES**

connectedTrain
front
rear
MA
MAtemp
stateTTD
stateVSS

INVARIANTS

inv2.1: $stateTTD \in TTD \rightarrow \{OCCUPIED, FREE\}$

inv2.2: $stateVSS \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$

inv2.3: $\forall ttd, tr. ((tr \in dom(front) \setminus dom(rear) \wedge ttd \in TTD \wedge front(tr) \in ttd) \Rightarrow stateTTD(ttd) = OCCUPIED)$

inv2.4: $\forall ttd, tr. ((tr \in dom(rear) \wedge ttd \in TTD \wedge (rear(tr) .. front(tr)) \cap ttd \neq \emptyset) \Rightarrow stateTTD(ttd) = OCCUPIED)$

inv2.5: $\forall tr1, tr2. ((tr1 \in dom(rear) \wedge tr2 \in dom(rear) \wedge tr1 \neq tr2) \Rightarrow (rear(tr1) .. front(tr1)) \cap (rear(tr2) .. front(tr2)) = \emptyset)$

inv2.6: $\forall tr1, tr2. ((tr1 \in dom(rear) \wedge tr2 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr2) \Rightarrow front(tr2) < rear(tr1))$

inv2.7: $\forall tr1, tr2, ttd. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr2 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr2 \wedge ttd \in TTD \wedge front(tr1) \in ttd) \Rightarrow front(tr2) \notin ttd)$

SYSML/KAOS PROOF OBLIGATIONS

sysml_kaos_po.G1-Guard=>G-Guard: *(theorem)*

$\forall tr, p, q, len, ttds, ttds1, p0, p1, q1. (($
 $(tr \in connectedTrain^{-1}[\{TRUE\}])$
 $\wedge (ttds \subseteq stateTTD^{-1}[\{FREE\}])$
 $\wedge (union(ttds) = p1 .. q1)$
 $\wedge (p1 \geq front(tr))$
 $\wedge (ttds1 \subseteq TTD)$
 $\wedge (union(ttds1) = p0 .. (p1 - 1))$
 $\wedge (tr \in dom(rear) \Rightarrow rear(tr) \geq p0)$
 $\wedge (tr \notin dom(rear) \Rightarrow front(tr) \geq p0)$
 $\wedge (p .. q \subseteq union(ttds \cup ttds1))$
 $\wedge (p .. q \cap union(ran(\{tr\} \triangleleft MA)) = \emptyset)$
 $\wedge (front(tr) \in p .. q)$
 $\wedge (tr \in dom(rear) \Rightarrow rear(tr) \in p .. q)$
 $\wedge (len \in \mathbb{N}_1)$
 $\wedge (front(tr) + len \in WAY)$
 $\wedge (tr \notin dom(MAtemp) \vee MAtemp(tr) \neq p .. q)$
 $) \Rightarrow$
 $($
 $(tr \in connectedTrain^{-1}[\{TRUE\}])$
 $\wedge (p .. q \subseteq WAY \wedge p \leq q)$
 $\wedge (front(tr) \in p .. q)$
 $\wedge (tr \in dom(rear) \Rightarrow rear(tr) \in p .. q)$
 $\wedge (p .. q \cap union(ran(\{tr\} \triangleleft MA)) = \emptyset)$
 $\wedge (len \in \mathbb{N}_1)$
 $\wedge (front(tr) + len \in WAY)$
 $))$

sysml_kaos_po.G2-Guard=>G-Guard: *(theorem)*

$\forall tr, p, q, len, vsss, vsss1, p0, p1, q1, newstateVSS. (($
 $(newstateVSS \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\})$
 $\wedge (tr \in connectedTrain^{-1}[\{TRUE\}])$
 $\wedge (vsss \subseteq newstateVSS^{-1}[\{FREE\}])$
 $))$

$$\begin{aligned}
& \wedge (\text{union}(vsss) = p1 \dots q1) \\
& \wedge (p1 \geq \text{front}(tr)) \\
& \wedge (vsss1 \subseteq VSS) \\
& \wedge (\text{union}(vsss1) = p0 \dots (p1 - 1)) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \geq p0) \\
& \wedge (tr \notin \text{dom}(\text{rear}) \Rightarrow \text{front}(tr) \geq p0) \\
& \wedge (p \dots q \subseteq \text{union}(vsss \cup vsss1)) \\
& \wedge (p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset) \\
& \wedge (\text{front}(tr) \in p \dots q) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q) \\
& \wedge (tr \notin \text{dom}(MAtemp) \vee MAtemp(tr) \neq p \dots q) \\
& \wedge (len \in \mathbb{N}_1) \\
& \wedge (\text{front}(tr) + len \in WAY) \\
& \wedge (tr \notin \text{dom}(MAtemp) \vee MAtemp(tr) \neq p \dots q) \\
&) \Rightarrow \\
& (\\
& (tr \in \text{connectedTrain}^{-1}[\{TRUE\}]) \\
& \wedge (p \dots q \subseteq WAY \wedge p \leq q) \\
& \wedge (\text{front}(tr) \in p \dots q) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q) \\
& \wedge (p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset) \\
& \wedge (len \in \mathbb{N}_1) \\
& \wedge (\text{front}(tr) + len \in WAY) \\
&) \\
& \text{sysml_kaos_po.G1-Post} \Rightarrow \text{G-Post: } \langle \text{theorem} \rangle \\
& \forall tr, p, q, len, ttds, ttds1, p0, p1, q1. ((\\
& (tr \in \text{connectedTrain}^{-1}[\{TRUE\}]) \\
& \wedge (ttds \subseteq \text{stateTTD}^{-1}[\{FREE\}]) \\
& \wedge (\text{union}(ttds) = p1 \dots q1) \\
& \wedge (p1 \geq \text{front}(tr)) \\
& \wedge (ttds1 \subseteq TTD) \\
& \wedge (\text{union}(ttds1) = p0 \dots (p1 - 1)) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \geq p0) \\
& \wedge (tr \notin \text{dom}(\text{rear}) \Rightarrow \text{front}(tr) \geq p0) \\
& \wedge (p \dots q \subseteq \text{union}(ttds \cup ttds1)) \\
& \wedge (p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset) \\
& \wedge (\text{front}(tr) \in p \dots q) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q) \\
& \wedge (len \in \mathbb{N}_1) \\
& \wedge (\text{front}(tr) + len \in WAY) \\
& \wedge (tr \notin \text{dom}(MAtemp) \vee MAtemp(tr) \neq p \dots q) \\
&) \Rightarrow \\
& (\\
& (p \dots q = p \dots q) \\
&) \\
&) \\
& \text{sysml_kaos_po.G2-Post} \Rightarrow \text{G-Post: } \langle \text{theorem} \rangle \\
& \forall tr, p, q, len, vsss, vsss1, p0, p1, q1, newstateVSS. ((\\
& (newstateVSS \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOW, AMBIGUOUS\}) \\
& \wedge (tr \in \text{connectedTrain}^{-1}[\{TRUE\}]) \\
& \wedge (vsss \subseteq \text{newstateVSS}^{-1}[\{FREE\}]) \\
& \wedge (\text{union}(vsss) = p1 \dots q1) \\
& \wedge (p1 \geq \text{front}(tr)) \\
& \wedge (vsss1 \subseteq VSS) \\
& \wedge (\text{union}(vsss1) = p0 \dots (p1 - 1)) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \geq p0) \\
& \wedge (tr \notin \text{dom}(\text{rear}) \Rightarrow \text{front}(tr) \geq p0) \\
& \wedge (p \dots q \subseteq \text{union}(vsss \cup vsss1)) \\
& \wedge (p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset)
\end{aligned}$$

```


$$\wedge (front(tr) \in p \dots q)$$


$$\wedge (tr \in dom(rear) \Rightarrow rear(tr) \in p \dots q)$$


$$\wedge (len \in \mathbb{N}_1)$$


$$\wedge (front(tr) + len \in WAY)$$


$$\wedge (tr \notin dom(MAtemp) \vee MAtemp(tr) \neq p \dots q)$$


$$\Rightarrow$$


$$(\wedge (p \dots q = p \dots q))$$


$$\wedge$$


$$(\text{remplacement de MAtemp par } ((\{tr\} \triangleleft MAtemp) \cup \{tr \mapsto p \dots q\}))$$

sysml_kaos_po.G1-Post=>not(G2-Guard): (theorem)

$$\forall tr, p, q, len, ttds, ttds1, p0, p1, q1. (($$


$$(tr \in connectedTrain^{-1}[\{TRUE\}])$$


$$\wedge (ttds \subseteq stateTTD^{-1}[\{FREE\}])$$


$$\wedge (union(ttds) = p1 \dots q1)$$


$$\wedge (p1 \geq front(tr))$$


$$\wedge (ttds1 \subseteq TTD)$$


$$\wedge (union(ttds1) = p0 \dots (p1 - 1))$$


$$\wedge (tr \in dom(rear) \Rightarrow rear(tr) \geq p0)$$


$$\wedge (tr \notin dom(rear) \Rightarrow front(tr) \geq p0)$$


$$\wedge (p \dots q \subseteq union(ttds \cup ttds1))$$


$$\wedge (p \dots q \cap union(ran(\{tr\} \triangleleft MA)) = \emptyset)$$


$$\wedge (front(tr) \in p \dots q)$$


$$\wedge (tr \in dom(rear) \Rightarrow rear(tr) \in p \dots q)$$


$$\wedge (len \in \mathbb{N}_1)$$


$$\wedge (front(tr) + len \in WAY)$$


$$\wedge (tr \notin dom(MAtemp) \vee MAtemp(tr) \neq p \dots q)$$


$$\Rightarrow$$


$$\neg(\exists vsss, vsss1, newstateVSS. ($$


$$(newstateVSS \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\})$$


$$\wedge (tr \in connectedTrain^{-1}[\{TRUE\}])$$


$$\wedge (vsss \subseteq newstateVSS^{-1}[\{FREE\}])$$


$$\wedge (union(vsss) = p1 \dots q1)$$


$$\wedge (p1 \geq front(tr))$$


$$\wedge (vsss1 \subseteq VSS)$$


$$\wedge (union(vsss1) = p0 \dots (p1 - 1))$$


$$\wedge (tr \in dom(rear) \Rightarrow rear(tr) \geq p0)$$


$$\wedge (tr \notin dom(rear) \Rightarrow front(tr) \geq p0)$$


$$\wedge (p \dots q \subseteq union(vsss \cup vsss1))$$


$$\wedge (p \dots q \cap union(ran(\{tr\} \triangleleft MA)) = \emptyset)$$


$$\wedge (front(tr) \in p \dots q)$$


$$\wedge (tr \in dom(rear) \Rightarrow rear(tr) \in p \dots q)$$


$$\wedge (len \in \mathbb{N}_1)$$


$$\wedge (front(tr) + len \in WAY)$$


$$\wedge (tr \notin dom(((\{tr\} \triangleleft MAtemp) \cup \{tr \mapsto p \dots q\})) \vee ((\{tr\} \triangleleft MAtemp) \cup \{tr \mapsto p \dots q\})(tr) \neq p \dots q)$$


$$\wedge$$


$$\wedge$$


$$\wedge$$


$$(\text{remplacement de MAtemp par } ((\{tr\} \triangleleft MAtemp) \cup \{tr \mapsto p \dots q\}))$$

sysml_kaos_po.G2-Post=>not(G1-Guard): (theorem)

$$\forall tr, p, q, len, vsss, vsss1, p0, p1, q1, newstateVSS. (($$


$$(newstateVSS \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\})$$


$$\wedge (tr \in connectedTrain^{-1}[\{TRUE\}])$$


$$\wedge (vsss \subseteq newstateVSS^{-1}[\{FREE\}])$$


$$\wedge (union(vsss) = p1 \dots q1)$$


$$\wedge (p1 \geq front(tr))$$


$$\wedge (vsss1 \subseteq VSS)$$


$$\wedge (union(vsss1) = p0 \dots (p1 - 1))$$


```

$$\begin{aligned}
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \geq p0) \\
& \wedge (tr \notin \text{dom}(\text{rear}) \Rightarrow \text{front}(tr) \geq p0) \\
& \wedge (p \dots q \subseteq \text{union}(\text{vsss} \cup \text{vsss1})) \\
& \wedge (p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset) \\
& \wedge (\text{front}(tr) \in p \dots q) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q) \\
& \wedge (\text{len} \in \mathbb{N}_1) \\
& \wedge (\text{front}(tr) + \text{len} \in \text{WAY}) \\
& \wedge (tr \notin \text{dom}(MAtemp) \vee MAtemp(tr) \neq p \dots q) \\
&) \Rightarrow \\
& \neg(\exists ttds, ttds1. (\\
& (tr \in \text{connectedTrain}^{-1}[\{TRUE\}]) \\
& \wedge (ttds \subseteq \text{stateTTD}^{-1}[\{FREE\}]) \\
& \wedge (\text{union}(ttds) = p1 \dots q1) \\
& \wedge (p1 \geq \text{front}(tr)) \\
& \wedge (ttds1 \subseteq TTD) \\
& \wedge (\text{union}(ttds1) = p0 \dots (p1 - 1)) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \geq p0) \\
& \wedge (tr \notin \text{dom}(\text{rear}) \Rightarrow \text{front}(tr) \geq p0) \\
& \wedge (p \dots q \subseteq \text{union}(ttds \cup ttds1)) \\
& \wedge (p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset) \\
& \wedge (\text{front}(tr) \in p \dots q) \\
& \wedge (tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q) \\
& \wedge (\text{len} \in \mathbb{N}_1) \\
& \wedge (\text{front}(tr) + \text{len} \in \text{WAY}) \\
& \wedge (tr \notin \text{dom}(((\{tr\} \triangleleft MAtemp) \cup \{tr \mapsto p \dots q\})) \vee ((\{tr\} \triangleleft MAtemp) \cup \{tr \mapsto p \dots q\})(tr) \neq p \dots q) \\
&) \\
&) \\
&)
\end{aligned}$$

EVENTS

Initialisation

begin

act1: $\text{connectedTrain} := \emptyset$
act2: $\text{front} := \emptyset$
act3: $\text{rear} := \emptyset$
act4: $MA := \emptyset$
act5: $MAtemp := \emptyset$
act6: $\text{stateTTD} := TTD \times \{OCCUPIED\}$
act7: $\text{stateVSS} := VSS \times \{UNKNOWN\}$

end

Event ComputeTrainMAFollowingTTDStates *(ordinary)* $\hat{=}$

any

tr
 $ttds$
 p
 q
 $ttds1$
 $p0$
 $p1$
 $q1$

len $ttds1$ designe l'ensemble des ttd sur lesquels le train est susceptible de se trouver

where

grd1: $tr \in \text{connectedTrain}^{-1}[\{TRUE\}]$
grd2: $ttds \subseteq \text{stateTTD}^{-1}[\{FREE\}]$
grd3: $\text{union}(ttds) = p1 \dots q1$
grd4: $p1 \geq \text{front}(tr)$
grd5: $ttds1 \subseteq TTD$
grd6: $\text{union}(ttds1) = p0 \dots (p1 - 1)$
grd7: $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \geq p0$

```

    grd8:  $tr \notin \text{dom}(\text{rear}) \Rightarrow \text{front}(tr) \geq p0$ 
    grd9:  $p \dots q \subseteq \text{union}(\text{ttDs} \cup \text{ttDs1})$ 
    grd10:  $p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
    grd11:  $\text{front}(tr) \in p \dots q$ 
    grd12:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q$ 
    grd13:  $\text{len} \in \mathbb{N}_1$ 
    grd14:  $\text{front}(tr) + \text{len} \in \text{WAY}$ 
    grd15:  $tr \notin \text{dom}(MAtemp) \vee MAtemp(tr) \neq p \dots q$ 
  then
    act1:  $MAtemp(tr) := p \dots q$ 
  end
Event ComputeTrainMAFollowingVSSStates <ordinary>  $\hat{=}$ 
  any
    tr
    vsss
    p
    q
    vsss1
    p0
    p1
    q1
    newstateVSS
    len vsss1 designe l'ensemble des vss sur lesquels le train est susceptible de se trouver
  where
    grd0:  $\text{newstateVSS} \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$ 
    grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}]$ 
    grd2:  $vsss \subseteq \text{newstateVSS}^{-1}[\{FREE\}]$ 
    grd3:  $\text{union}(vsss) = p1 \dots q1$ 
    grd4:  $p1 \geq \text{front}(tr)$ 
    grd5:  $vsss1 \subseteq VSS$ 
    grd6:  $\text{union}(vsss1) = p0 \dots (p1 - 1)$ 
    grd7:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \geq p0$ 
    grd8:  $tr \notin \text{dom}(\text{rear}) \Rightarrow \text{front}(tr) \geq p0$ 
    grd9:  $p \dots q \subseteq \text{union}(vsss \cup vsss1)$ 
    grd10:  $p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
    grd11:  $\text{front}(tr) \in p \dots q$ 
    grd12:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q$ 
    grd13:  $\text{len} \in \mathbb{N}_1$ 
    grd14:  $\text{front}(tr) + \text{len} \in \text{WAY}$ 
    grd15:  $tr \notin \text{dom}(MAtemp) \vee MAtemp(tr) \neq p \dots q$ 
  then
    act1:  $MAtemp(tr) := p \dots q$ 
    act2:  $\text{stateVSS} := \text{newstateVSS}$ 
  end
Event MoveTrainFollowingItsMA <ordinary>  $\hat{=}$ 
extends MoveTrainFollowingItsMA
  any
    tr
    len
    ttDs
  where
    grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MA)$ 
    grd2:  $\text{len} \in \mathbb{N}_1$ 
    grd3:  $\text{front}(tr) + \text{len} \in MA(tr)$ 
    grd4:  $\text{ttDs} \subseteq \text{stateTTD}^{-1}[\{FREE\}]$ 
    grd5:  $\forall \text{ttd}. (\text{ttd} \in \text{stateTTD}^{-1}[\{FREE\}] \wedge ((\text{front}(tr) + \text{len} \in \text{ttd}) \vee (tr \in \text{dom}(\text{rear}) \wedge ((\text{rear}(tr) + \text{len} \dots \text{front}(tr) + \text{len}) \cap \text{ttd} \neq \emptyset))) \Rightarrow \text{ttd} \in \text{ttDs})$ 
    grd6:  $tr \in \text{dom}(\text{rear}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow (\text{rear}(tr1) \dots \text{front}(tr1)) \cap (\text{rear}(tr) + \text{len} \dots \text{front}(tr) + \text{len}) = \emptyset))$ 

```

```

grd7:   $tr \in dom(rear) \Rightarrow (\forall tr1. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr) \Rightarrow front(tr1) <$ 
       $rear(tr) + len))$ 
grd8:   $tr \in dom(front) \setminus dom(rear) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow front(tr) + len <$ 
       $rear(tr1)))$ 
grd9:   $tr \in dom(front) \setminus dom(rear) \Rightarrow (\forall tr1, ttd. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr \wedge ttd \in$ 
       $TTD \wedge front(tr1) \in ttd) \Rightarrow front(tr) + len \notin ttd))$ 
then
  act1:  $front(tr) := front(tr) + len$ 
  act2:  $rear := (\{TRUE \mapsto rear \Leftarrow \{tr \mapsto rear(tr) + len\}, FALSE \mapsto rear\})(bool(tr \in dom(rear)))$ 
  act3:  $stateTTD := stateTTD \Leftarrow (ttds \times \{OCCUPIED\})$ 
end
END

```

MACHINE M3**REFINES** M2**SEES** C0,C2**VARIABLES**

connectedTrain
front
rear
MA
MAtemp
stateTTD
stateVSS
newstateVSScomputed

INVARIANTS

inv3.1: $\text{newstateVSScomputed} \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$

EVENTS**Initialisation****begin**

act1: $\text{connectedTrain} := \emptyset$
act2: $\text{front} := \emptyset$
act3: $\text{rear} := \emptyset$
act4: $\text{MA} := \emptyset$
act5: $\text{MAtemp} := \emptyset$
act6: $\text{stateTTD} := TTD \times \{OCCUPIED\}$
act7: $\text{stateVSS} := VSS \times \{UNKNOWN\}$
act8: $\text{newstateVSScomputed} := VSS \times \{UNKNOWN\}$

end**Event** ComputeVSSStates $\langle \text{ordinary} \rangle \triangleq$ **any**

$\text{newstateVSScomputed1}$

where

grd0: $\text{newstateVSScomputed1} \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$

then

act1: $\text{newstateVSScomputed} := \text{newstateVSScomputed1}$

end**Event** ComputeTrainMAUsingVSSStates $\langle \text{ordinary} \rangle \triangleq$ **any**

tr
vsss
p
q
vsss1
p0
p1
q1

newstateVSS vsss1 designe l'ensemble des vss sur lesquels le train est susceptible de se trouver

where

grd0: $\text{newstateVSS} = \text{newstateVSScomputed}$
grd1: $\text{tr} \in \text{connectedTrain}^{-1}[\{TRUE\}]$
grd2: $\text{vsss} \subseteq \text{newstateVSS}^{-1}[\{FREE\}]$
grd3: $\text{union}(\text{vsss}) = p1 \dots q1$
grd4: $p1 \geq \text{front}(\text{tr})$
grd5: $\text{vsss1} \subseteq VSS$
grd6: $\text{union}(\text{vsss1}) = p0 \dots (p1 - 1)$
grd7: $\text{tr} \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(\text{tr}) \geq p0$
grd8: $\text{tr} \notin \text{dom}(\text{rear}) \Rightarrow \text{front}(\text{tr}) \geq p0$
grd9: $p \dots q \subseteq \text{union}(\text{vsss} \cup \text{vsss1})$
grd10: $p \dots q \cap \text{union}(\text{ran}(\{\text{tr}\} \triangleleft \text{MA})) = \emptyset$

```
    grd11:  $front(tr) \in p .. q$   
    grd12:  $tr \in dom(rear) \Rightarrow rear(tr) \in p .. q$   
  then  
    act1:  $MAtemp(tr) := p .. q$   
    act2:  $stateVSS := newstateVSS$   
  end  
END
```

MACHINE M4**REFINES** M3**SEES** C0,C2**VARIABLES**

connectedTrain
 front
 rear
 MA
 MAtemp
 stateTTD
 stateVSS
 newstateVSScomputed

EVENTS**Initialisation****begin**

act1: $connectedTrain := \emptyset$
act2: $front := \emptyset$
act3: $rear := \emptyset$
act4: $MA := \emptyset$
act5: $MAtemp := \emptyset$
act6: $stateTTD := TTD \times \{OCCUPIED\}$
act7: $stateVSS := VSS \times \{UNKNOWN\}$
act8: $newstateVSScomputed := VSS \times \{UNKNOWN\}$

end**Event** ComputeVSSStatesFollowingTTDStates *ordinary* $\hat{=}$ **any**

newstateVSScomputed1

where*grd0*: $newstateVSScomputed1 \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$ **then***act1*: $newstateVSScomputed := newstateVSScomputed1$ **end****Event** ComputeVSSStateswoTTDStates *ordinary* $\hat{=}$ **any**

newstateVSScomputed1

where*grd0*: $newstateVSScomputed1 \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$ **then***act1*: $newstateVSScomputed := newstateVSScomputed1$ **end****END**

MACHINE M5**REFINES** M4**SEES** C0,C2**VARIABLES**

connectedTrain

front

rear

MA

MAtemp

stateTTD

stateVSS

newstateVSScomputed **SYSML/KAOS PROOF OBLIGATIONS****INVARIANTS****sysml_kaos_po.G1-Guard=>G-Guard:** *(theorem)*

$$\begin{aligned} & \forall vss, vss1, vss2, vss3, vss4, newstateVSScomputed1. ((\\ & (vss = stateVSS^{-1}[\{UNKNOWN\}]) \\ & \wedge (partition(vss, vss1, vss2, vss3, vss4)) \\ & \wedge (newstateVSScomputed1 = stateVSS \Leftarrow ((vss1 \times \{OCCUPIED\}) \cup (vss2 \times \{FREE\}) \cup (vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOWN\}))) \\ &) \Rightarrow \\ & (\\ & (newstateVSScomputed1 \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}) \\ &)) \end{aligned}$$
sysml_kaos_po.G2-Guard=>G-Guard: *(theorem)*

$$\begin{aligned} & \forall vss, vss1, vss2, vss3, vss4, newstateVSScomputed1. ((\\ & (vss = stateVSS^{-1}[\{OCCUPIED\}]) \\ & \wedge (partition(vss, vss1, vss2, vss3, vss4)) \\ & \wedge (newstateVSScomputed1 = stateVSS \Leftarrow ((vss1 \times \{OCCUPIED\}) \cup (vss2 \times \{FREE\}) \cup (vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOWN\}))) \\ &) \Rightarrow \\ & (\\ & (newstateVSScomputed1 \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}) \\ &)) \end{aligned}$$
sysml_kaos_po.G3-Guard=>G-Guard: *(theorem)*

$$\begin{aligned} & \forall vss, vss1, vss2, vss3, vss4, newstateVSScomputed1. ((\\ & (vss = stateVSS^{-1}[\{AMBIGUOUS\}]) \\ & \wedge (partition(vss, vss1, vss2, vss3, vss4)) \\ & \wedge (newstateVSScomputed1 = stateVSS \Leftarrow ((vss1 \times \{OCCUPIED\}) \cup (vss2 \times \{FREE\}) \cup (vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOWN\}))) \\ &) \Rightarrow \\ & (\\ & (newstateVSScomputed1 \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}) \\ &)) \end{aligned}$$
sysml_kaos_po.G4-Guard=>G-Guard: *(theorem)*

$$\begin{aligned} & \forall vss, vss1, vss2, vss3, vss4, newstateVSScomputed1. ((\\ & (vss = stateVSS^{-1}[\{FREE\}]) \\ & \wedge (partition(vss, vss1, vss2, vss3, vss4)) \\ & \wedge (newstateVSScomputed1 = stateVSS \Leftarrow ((vss1 \times \{OCCUPIED\}) \cup (vss2 \times \{FREE\}) \cup (vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOWN\}))) \\ &) \Rightarrow \\ & (\\ & (newstateVSScomputed1 \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}) \\ &)) \end{aligned}$$
sysml_kaos_po.G1-G2-G3-G4-G5-Post=>G-Post : *(theorem)*

$$\begin{aligned} & \forall vss1, vss11, vss12, vss13, vss14, vss2, vss21, vss22, vss23, vss24, vss3, vss31, vss32, vss33, vss34, vss4, vss41, vss42, vss43, \\ & (\\ & (vss1 = stateVSS^{-1}[\{UNKNOWN\}]) \end{aligned}$$

$$\begin{aligned}
& \wedge (\text{partition}(vss1, vss11, vss12, vss13, vss14)) \\
& \wedge (vss2 = \text{stateVSS}^{-1}[\{OCCUPIED\}]) \\
& \wedge (\text{partition}(vss2, vss21, vss22, vss23, vss24)) \\
& \wedge (vss3 = \text{stateVSS}^{-1}[\{AMBIGUOUS\}]) \\
& \wedge (\text{partition}(vss3, vss31, vss32, vss33, vss34)) \\
& \wedge (vss4 = \text{stateVSS}^{-1}[\{FREE\}]) \\
& \wedge (\text{partition}(vss4, vss41, vss42, vss43, vss44)) \\
&) \Rightarrow \\
& (\\
& (\text{stateVSS} \Leftarrow ((vss11 \times \{OCCUPIED\}) \cup (vss12 \times \{FREE\}) \cup (vss13 \times \{AMBIGUOUS\}) \cup (vss14 \times \{UNKNOWN\}))) \\
& \cup (\text{stateVSS} \Leftarrow ((vss21 \times \{OCCUPIED\}) \cup (vss22 \times \{FREE\}) \cup (vss23 \times \{AMBIGUOUS\}) \cup (vss24 \times \{UNKNOWN\}))) \\
& \cup (\text{stateVSS} \Leftarrow ((vss31 \times \{OCCUPIED\}) \cup (vss32 \times \{FREE\}) \cup (vss33 \times \{AMBIGUOUS\}) \cup (vss34 \times \{UNKNOWN\}))) \\
& \cup (\text{stateVSS} \Leftarrow ((vss41 \times \{OCCUPIED\}) \cup (vss42 \times \{FREE\}) \cup (vss43 \times \{AMBIGUOUS\}) \cup (vss44 \times \{UNKNOWN\}))) \\
& \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\} \\
&) \\
&)
\end{aligned}$$

EVENTS

Initialisation

begin

act1: *connectedTrain* := \emptyset
act2: *front* := \emptyset
act3: *rear* := \emptyset
act4: *MA* := \emptyset
act5: *MAtemp* := \emptyset
act6: *stateTTD* := $TTD \times \{OCCUPIED\}$
act7: *stateVSS* := $VSS \times \{UNKNOWN\}$
act8: *newstateVSScomputed* := $VSS \times \{UNKNOWN\}$

end

Event ComputeStatesOfVSSinUnknowState $\langle \text{ordinary} \rangle \hat{=}$

any

vss
vss1
vss2
vss3
vss4
newstateVSScomputed1

where

grd1: *vss* = $\text{stateVSS}^{-1}[\{UNKNOWN\}]$
grd2: *partition*(*vss*, *vss1*, *vss2*, *vss3*, *vss4*)
grd3: *newstateVSScomputed1* = $\text{stateVSS} \Leftarrow ((vss1 \times \{OCCUPIED\}) \cup (vss2 \times \{FREE\}) \cup (vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOWN\}))$

then

act1: *newstateVSScomputed* := *newstateVSScomputed1*

end

Event ComputeStatesOfVSSinOccupiedState $\langle \text{ordinary} \rangle \hat{=}$

any

vss
vss1
vss2
vss3
vss4
newstateVSScomputed1

where

grd1: *vss* = $\text{stateVSS}^{-1}[\{OCCUPIED\}]$
grd2: *partition*(*vss*, *vss1*, *vss2*, *vss3*, *vss4*)

```

    grd3:  $newstateVSScomputed1 = stateVSS \Join ((vss1 \times \{OCCUPIED\}) \cup (vss2 \times \{FREE\}) \cup$ 
       $(vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOWN\}))$ 
  then
    act1:  $newstateVSScomputed := newstateVSScomputed1$ 
  end
Event ComputeStatesOfVSSinAmbiguousState  $\langle ordinary \rangle \hat{=}$ 
  any
    vss
    vss1
    vss2
    vss3
    vss4
    newstateVSScomputed1
  where
    grd1:  $vss = stateVSS^{-1}[\{AMBIGUOUS\}]$ 
    grd2:  $partition(vss, vss1, vss2, vss3, vss4)$ 
    grd3:  $newstateVSScomputed1 = stateVSS \Join ((vss1 \times \{OCCUPIED\}) \cup (vss2 \times \{FREE\}) \cup$ 
       $(vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOWN\}))$ 
  then
    act1:  $newstateVSScomputed := newstateVSScomputed1$ 
  end
Event ComputeStatesOfVSSinFreeState  $\langle ordinary \rangle \hat{=}$ 
  any
    vss
    vss1
    vss2
    vss3
    vss4
    newstateVSScomputed1
  where
    grd1:  $vss = stateVSS^{-1}[\{FREE\}]$ 
    grd2:  $partition(vss, vss1, vss2, vss3, vss4)$ 
    grd3:  $newstateVSScomputed1 = stateVSS \Join ((vss1 \times \{OCCUPIED\}) \cup (vss2 \times \{FREE\}) \cup$ 
       $(vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOWN\}))$ 
  then
    act1:  $newstateVSScomputed := newstateVSScomputed1$ 
  end
END

```

MACHINE M6**REFINES** M5**SEES** C0,C2**VARIABLES**

connectedTrain
 front
 rear
 MA
 MAtemp
 stateTTD
 stateVSS
 newstateVSScomputed
 freeVssChangingtoFree
 freeVssChangingtoUnknow
 freeVssChangingtoOccupied
 freeVssChangingtoAmbiguous

INVARIANTS

inv6_1: $freeVssChangingtoFree \subseteq VSS$
 inv6_2: $freeVssChangingtoUnknow \subseteq VSS$
 inv6_3: $freeVssChangingtoOccupied \subseteq VSS$
 inv6_4: $freeVssChangingtoAmbiguous \subseteq VSS$

EVENTS**Initialisation****begin**

act1: $connectedTrain := \emptyset$
 act2: $front := \emptyset$
 act3: $rear := \emptyset$
 act4: $MA := \emptyset$
 act5: $MAtemp := \emptyset$
 act6: $stateTTD := TTD \times \{OCCUPIED\}$
 act7: $stateVSS := VSS \times \{UNKNOWN\}$
 act8: $newstateVSScomputed := VSS \times \{UNKNOWN\}$
 act10: $freeVssChangingtoFree := \emptyset$
 act11: $freeVssChangingtoUnknow := \emptyset$
 act12: $freeVssChangingtoOccupied := \emptyset$
 act13: $freeVssChangingtoAmbiguous := \emptyset$

end**Event** ComputeStatesOfVSSinFreeStateWhenTTDisFree *(ordinary)* $\hat{=}$ **any**

vssTtdFree

where

grd1: $vssTtdFree \subseteq stateVSS^{-1}[\{FREE\}]$
 grd2: $\forall vss. (vss \in vssTtdFree \Rightarrow vss \subseteq union(stateTTD^{-1}[\{FREE\}]))$

thenact1: $freeVssChangingtoFree := freeVssChangingtoFree \cup vssTtdFree$ **end****Event** ComputeStatesOfVSSinFreeStateWhenTTDisOccupiedandNoTrainisLocatedonTTD *(ordinary)* $\hat{=}$ **any**

vssTtdOccupiedwithNoTrain

where

grd1: $vssTtdOccupiedwithNoTrain \subseteq stateVSS^{-1}[\{FREE\}]$
 grd2: $\forall vss. (vss \in vssTtdOccupiedwithNoTrain \Rightarrow vss \subseteq union(stateTTD^{-1}[\{OCCUPIED\}]))$
 grd3: $\forall vss, p, q. ((vss \in vssTtdOccupiedwithNoTrain \wedge p \dots q \in TTD \wedge vss \subseteq p \dots q) \Rightarrow (\forall tr. tr \in connectedTrain^{-1}[\{TRUE\}] \wedge tr \in dom(rear) \Rightarrow (front(tr) < p \vee rear(tr) > q)))$

```

    grd4:  $\forall vss, p, q. ((vss \in vssTtdOccupiedwithNoTrain \wedge p \dots q \in TTD \wedge vss \subseteq p \dots q) \Rightarrow (\forall tr. tr \in$ 
       $connectedTrain^{-1}[\{TRUE\}] \wedge tr \notin dom(rear) \Rightarrow (front(tr) < p \vee front(tr) > q)))$ 
  then
    act1:  $freeVssChangingtoUnknow := freeVssChangingtoUnknow \cup vssTtdOccupiedwithNoTrain$ 
  end
Event ComputeStatesOfVSSinFreeStateWhenTTDisOccupiedandNoMAisIssued <ordinary>  $\hat{=}$ 
  any
    vssTtdOccupiedwithNoMA
  where
    grd1:  $vssTtdOccupiedwithNoMA \subseteq stateVSS^{-1}[\{FREE\}]$ 
    grd2:  $\forall vss. (vss \in vssTtdOccupiedwithNoMA \Rightarrow vss \subseteq union(stateTTD^{-1}[\{OCCUPIED\}]))$ 
    grd3:  $\forall vss, ttd. ((vss \in vssTtdOccupiedwithNoMA \wedge ttd \in TTD \wedge vss \subseteq ttd) \Rightarrow (union(ran(MA)) \cap$ 
       $ttd = \emptyset))$ 
  then
    act1:  $freeVssChangingtoUnknow := freeVssChangingtoUnknow \cup vssTtdOccupiedwithNoMA$ 
  end
Event FullComputeStatesOfVSSinFreeState <ordinary>  $\hat{=}$ 
  any
    vss
    vss1
    vss2
    vss3
    vss4
    newstateVSScomputed1
  where
    grd1:  $vss = stateVSS^{-1}[\{FREE\}]$ 
    grd2:  $partition(vss, vss1, vss2, vss3, vss4)$ 
    grd3:  $freeVssChangingtoFree \subseteq vss2$ 
    lorsque toutes les transitions seront implementees, ceci deviendra une egalite
    grd4:  $freeVssChangingtoUnknow \subseteq vss4$ 
    lorsque toutes les transitions seront implementees, ceci deviendra une egalite
    grd5:  $newstateVSScomputed1 = stateVSS \Leftarrow ((vss1 \times \{OCCUPIED\}) \cup (vss2 \times \{FREE\}) \cup$ 
       $(vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOWN\}))$ 
  then
    act1:  $newstateVSScomputed := newstateVSScomputed1$ 
  end
END

```