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CONTEXT C0

SETS

TRAIN

CONSTANTS

a

b

WAY

AXIOMS

axiom1: $\{a, b\} \subseteq \mathbb{N}$

axiom2: $a < b$

axiom3: $WAY = a \dots b$

axiom4: $b - a \geq 20$

END

CONTEXT C2**EXTENDS** C0**SETS**

STATES

CONSTANTS

TTD

VSS

OCCUPIED

FREE

UNKNOWN

AMBIGUOUS

AXIOMS**axiom1:** $TTD \subseteq \mathbb{P}_1(WAY)$ **axiom2:** $union(TTD) = WAY$ **axiom3:** $inter(TTD) = \emptyset$ **axiom4:** $\forall ttd. (ttd \in TTD \Rightarrow (\exists p, q. (p .. q \subseteq WAY \wedge p < q \wedge ttd = p .. q)))$ **axiom5:** $VSS \subseteq \mathbb{P}_1(WAY)$ **axiom6:** $union(VSS) = WAY$ **axiom7:** $inter(VSS) = \emptyset$ **axiom8:** $\forall vss. (vss \in VSS \Rightarrow (\exists p, q, ttd. (ttd \in TTD \wedge p .. q \subseteq ttd \wedge p < q \wedge vss = p .. q)))$ **axiom9:** $partition(STATES, \{OCCUPIED\}, \{FREE\}, \{UNKNOWN\}, \{AMBIGUOUS\})$ **END**

MACHINE M0**SEES** C0**VARIABLES**

connectedTrain

front

rear

INVARIANTSinv0_1: $connectedTrain \in TRAIN \leftrightarrow BOOL$ inv0_2: $front \in dom(connectedTrain) \rightarrow WAY$ inv0_3: $rear \in dom(connectedTrain) \rightarrow WAY$ inv0_4: $\forall tr. (tr \in dom(rear) \Rightarrow rear(tr) < front(tr))$ **EVENTS****Initialisation****begin**act1: $connectedTrain := \emptyset$ act2: $front := \emptyset$ act3: $rear := \emptyset$ **end****Event** MoveTrainOnTrack *<ordinary>* $\hat{=}$ **any**

tr

len

wheregrd1: $tr \in connectedTrain^{-1}[\{TRUE\}]$ grd2: $len \in \mathbb{N}_1$ grd3: $front(tr) + len \in WAY$ **then**act1: $front(tr) := front(tr) + len$ act2: $rear := (\{TRUE \mapsto rear \Leftarrow \{tr \mapsto rear(tr) + len\}, FALSE \mapsto rear\})(bool(tr \in dom(rear)))$ **end****Event** _connectTrain *<ordinary>* $\hat{=}$ **any**

tr

fr

re

integer

wheregrd0: $TRAIN \setminus dom(connectedTrain) \neq \emptyset$ grd1: $tr \in TRAIN \setminus dom(connectedTrain)$ grd2: $fr \in WAY$ grd3: $integer \in BOOL$ grd4: $integer = TRUE \Rightarrow re \in WAY$ grd5: $re < fr$ **then**act1: $connectedTrain(tr) := TRUE$ act2: $front(tr) := fr$ act3: $rear := (\{TRUE \mapsto rear \Leftarrow \{tr \mapsto re\}, FALSE \mapsto rear\})(integer)$ **end****Event** _toggleTrainConnexionStatus *<ordinary>* $\hat{=}$ **any**

tr

wheregrd0: $dom(connectedTrain) \neq \emptyset$ grd1: $tr \in dom(connectedTrain)$ **then**act1: $connectedTrain := (\{TRUE \mapsto connectedTrain \Leftarrow \{tr \mapsto FALSE\}, FALSE \mapsto connectedTrain \Leftarrow \{tr \mapsto TRUE\}\})(bool(connectedTrain(tr) = TRUE))$ **end****END**

MACHINE M1**REFINES** M0**SEES** C0**VARIABLES**

connectedTrain

front

rear

MA

MAtemp

INVARIANTS**inv1.1:** $MA \in \text{dom}(\text{connectedTrain}) \leftrightarrow \mathbb{P}(\text{WAY})$ **inv1.2:** $\forall tr. (tr \in \text{dom}(MA) \Rightarrow (\exists p, q. (p \dots q \subseteq \text{WAY} \wedge p \leq q \wedge MA(tr) = p \dots q)))$ **inv1.3:** $\forall tr. (tr \in \text{dom}(MA) \Rightarrow \text{front}(tr) \in MA(tr))$ **inv1.4:** $\forall tr. (tr \in \text{dom}(\text{rear}) \cap \text{dom}(MA) \Rightarrow \text{rear}(tr) \in MA(tr))$ **inv1.5:** $\forall tr1, tr2. ((\{tr1, tr2\} \subseteq \text{dom}(MA) \wedge tr1 \neq tr2) \Rightarrow MA(tr1) \cap MA(tr2) = \emptyset)$ **inv1.6:** $MAtemp \in \text{dom}(\text{connectedTrain}) \leftrightarrow \mathbb{P}(\text{WAY})$ **inv1.7:** $\forall tr. (tr \in \text{dom}(MAtemp) \Rightarrow (\exists p, q. (p \dots q \subseteq \text{WAY} \wedge p \leq q \wedge MAtemp(tr) = p \dots q)))$ **EVENTS****Initialisation****begin****act1:** $\text{connectedTrain} := \emptyset$ **act2:** $\text{front} := \emptyset$ **act3:** $\text{rear} := \emptyset$ **act4:** $MA := \emptyset$ **act5:** $MAtemp := \emptyset$ **end****Event** ComputeTrainMA **<ordinary>** $\hat{=}$ **any**

tr

p

q

where**grd1:** $tr \in \text{connectedTrain}^{-1}[\{TRUE\}]$ **grd2:** $p \dots q \subseteq \text{WAY} \wedge p \leq q$ **grd3:** $\text{front}(tr) \in p \dots q$ **grd4:** $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q$ **grd5:** $p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ **then****act1:** $MAtemp(tr) := p \dots q$ **end****Event** AssignMAtoTrain **<ordinary>** $\hat{=}$ **any**

tr

where**grd1:** $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MAtemp)$ **grd2:** $\text{front}(tr) \in MAtemp(tr)$ **grd3:** $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in MAtemp(tr)$ **grd4:** $MAtemp(tr) \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ **then****act1:** $MA(tr) := MAtemp(tr)$ **end****Event** MoveTrainFollowingItsMA **<ordinary>** $\hat{=}$ **refines** MoveTrainOnTrack**any**

tr

len

```

where
  grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MA)$ 
  grd2:  $len \in \mathbb{N}_1$ 
  grd3:  $\text{front}(tr) + len \in MA(tr)$ 
then
  act1:  $\text{front}(tr) := \text{front}(tr) + len$ 
  act2:  $\text{rear} := (\{TRUE \mapsto \text{rear} \Leftarrow \{tr \mapsto \text{rear}(tr) + len\}, FALSE \mapsto \text{rear}\})(\text{bool}(tr \in \text{dom}(\text{rear})))$ 
end
Event _connectTrain <ordinary>  $\hat{=}$ 
extends _connectTrain
  any
    tr
    fr
    re
    integer
  where
    grd0:  $TRAIN \setminus \text{dom}(\text{connectedTrain}) \neq \emptyset$ 
    grd1:  $tr \in TRAIN \setminus \text{dom}(\text{connectedTrain})$ 
    grd2:  $fr \in WAY$ 
    grd3:  $integer \in BOOL$ 
    grd4:  $integer = TRUE \Rightarrow re \in WAY$ 
    grd5:  $re < fr$ 
  then
    act1:  $\text{connectedTrain}(tr) := TRUE$ 
    act2:  $\text{front}(tr) := fr$ 
    act3:  $\text{rear} := (\{TRUE \mapsto \text{rear} \Leftarrow \{tr \mapsto re\}, FALSE \mapsto \text{rear}\})(integer)$ 
  end
Event _toggleTrainConnexionStatus <ordinary>  $\hat{=}$ 
extends _toggleTrainConnexionStatus
  any
    tr
  where
    grd0:  $\text{dom}(\text{connectedTrain}) \neq \emptyset$ 
    grd1:  $tr \in \text{dom}(\text{connectedTrain})$ 
  then
    act1:  $\text{connectedTrain} := (\{TRUE \mapsto \text{connectedTrain} \Leftarrow \{tr \mapsto FALSE\}, FALSE \mapsto \text{connectedTrain} \Leftarrow \{tr \mapsto TRUE\}\})(\text{bool}(\text{connectedTrain}(tr) = TRUE))$ 
  end
END

```

MACHINE M2**REFINES** M1**SEES** C2**VARIABLES**

connectedTrain
front
rear
MA
MAtemp
stateTTD
stateVSS

INVARIANTS

inv2.1: $stateTTD \in TTD \rightarrow \{OCCUPIED, FREE\}$
inv2.2: $stateVSS \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$
inv2.3: $\forall ttd, tr. ((tr \in dom(front) \setminus dom(rear) \wedge ttd \in TTD \wedge front(tr) \in ttd) \Rightarrow stateTTD(ttd) = OCCUPIED)$
inv2.4: $\forall ttd, tr. ((tr \in dom(rear) \wedge ttd \in TTD \wedge (rear(tr) .. front(tr)) \cap ttd \neq \emptyset) \Rightarrow stateTTD(ttd) = OCCUPIED)$
inv2.5: $\forall tr1, tr2. ((tr1 \in dom(rear) \wedge tr2 \in dom(rear) \wedge tr1 \neq tr2) \Rightarrow (rear(tr1) .. front(tr1)) \cap (rear(tr2) .. front(tr2)) = \emptyset)$
inv2.6: $\forall tr1, tr2. ((tr1 \in dom(rear) \wedge tr2 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr2) \Rightarrow front(tr2) < rear(tr1))$
inv2.7: $\forall tr1, tr2, ttd. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr2 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr2 \wedge ttd \in TTD \wedge front(tr1) \in ttd) \Rightarrow front(tr2) \notin ttd)$

EVENTS**Initialisation****begin**

act1: $connectedTrain := \emptyset$
act2: $front := \emptyset$
act3: $rear := \emptyset$
act4: $MA := \emptyset$
act5: $MAtemp := \emptyset$
act6: $stateTTD := TTD \times \{OCCUPIED\}$
act7: $stateVSS := VSS \times \{UNKNOWN\}$

end**Event** ComputeTrainMAFollowingTTDStates *<ordinary>* $\hat{=}$ **refines** ComputeTrainMA**any**

tr
ttds
p
q
ttds1
p0
p1
q1 ttds1 designe l'ensemble des ttd sur lesquels le train est susceptible de se trouver

where

grd1: $tr \in connectedTrain^{-1}[\{TRUE\}]$
grd2: $ttds \subseteq stateTTD^{-1}[\{FREE\}]$
grd3: $union(ttds) = p1 .. q1$
grd4: $p1 \geq front(tr)$
grd5: $ttds1 \subseteq TTD$
grd6: $union(ttds1) = p0 .. (p1 - 1)$
grd7: $tr \in dom(rear) \Rightarrow rear(tr) \geq p0$
grd8: $tr \notin dom(rear) \Rightarrow front(tr) \geq p0$
grd9: $p .. q \subseteq union(ttds \cup ttds1)$

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    grd10:  $p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
    grd11:  $\text{front}(tr) \in p \dots q$ 
    grd12:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q$ 
  then
    act1:  $MAtemp(tr) := p \dots q$ 
  end
Event ComputeTrainMAFollowingVSSStates  $\langle \text{ordinary} \rangle \triangleq$ 
refines ComputeTrainMA
  any
    tr
    vsss
    p
    q
    vsss1
    p0
    p1
    q1
    newstateVSS vsss1 designe l'ensemble des vss sur lesquels le train est susceptible de se trouver
  where
    grd0:  $\text{newstateVSS} \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$ 
    grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}]$ 
    grd2:  $vsss \subseteq \text{newstateVSS}^{-1}[\{FREE\}]$ 
    grd3:  $\text{union}(vsss) = p1 \dots q1$ 
    grd4:  $p1 \geq \text{front}(tr)$ 
    grd5:  $vsss1 \subseteq VSS$ 
    grd6:  $\text{union}(vsss1) = p0 \dots (p1 - 1)$ 
    grd7:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \geq p0$ 
    grd8:  $tr \notin \text{dom}(\text{rear}) \Rightarrow \text{front}(tr) \geq p0$ 
    grd9:  $p \dots q \subseteq \text{union}(vsss \cup vsss1)$ 
    grd10:  $p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
    grd11:  $\text{front}(tr) \in p \dots q$ 
    grd12:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q$ 
  then
    act1:  $MAtemp(tr) := p \dots q$ 
    act2:  $\text{stateVSS} := \text{newstateVSS}$ 
  end
Event AssignMAtoTrain  $\langle \text{ordinary} \rangle \triangleq$ 
extends AssignMAtoTrain
  any
    tr
  where
    grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MAtemp)$ 
    grd2:  $\text{front}(tr) \in MAtemp(tr)$ 
    grd3:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in MAtemp(tr)$ 
    grd4:  $MAtemp(tr) \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
  then
    act1:  $MA(tr) := MAtemp(tr)$ 
  end
Event MoveTrainFollowingItsMA  $\langle \text{ordinary} \rangle \triangleq$ 
extends MoveTrainFollowingItsMA
  any
    tr
    len
    ttds
  where
    grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MA)$ 
    grd2:  $len \in \mathbb{N}_1$ 
    grd3:  $\text{front}(tr) + len \in MA(tr)$ 

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    grd4:  $ttds \subseteq stateTTD^{-1}[\{FREE\}]$ 
    grd5:  $\forall ttd. (ttd \in stateTTD^{-1}[\{FREE\}] \wedge ((front(tr) + len \in ttd) \vee (tr \in dom(rear) \wedge ((rear(tr) + len \dots front(tr) + len) \cap ttd \neq \emptyset))) \Rightarrow ttd \in ttds)$ 
    grd6:  $tr \in dom(rear) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow (rear(tr1) \dots front(tr1)) \cap (rear(tr) + len \dots front(tr) + len) = \emptyset))$ 
    grd7:  $tr \in dom(rear) \Rightarrow (\forall tr1. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr) \Rightarrow front(tr1) < rear(tr) + len))$ 
    grd8:  $tr \in dom(front) \setminus dom(rear) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow front(tr) + len < rear(tr1)))$ 
    grd9:  $tr \in dom(front) \setminus dom(rear) \Rightarrow (\forall tr1, ttd. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr \wedge ttd \in TTD \wedge front(tr1) \in ttd) \Rightarrow front(tr) + len \notin ttd))$ 
  then
    act1:  $front(tr) := front(tr) + len$ 
    act2:  $rear := (\{TRUE \mapsto rear \Leftarrow \{tr \mapsto rear(tr) + len\}, FALSE \mapsto rear\})(bool(tr \in dom(rear)))$ 
    act3:  $stateTTD := stateTTD \Leftarrow (ttds \times \{OCCUPIED\})$ 
  end
Event _connectTrain <ordinary>  $\hat{=}$ 
extends _connectTrain
any
  tr
  fr
  re
  integer
  ttds
where
  grd0:  $TRAIN \setminus dom(connectedTrain) \neq \emptyset$ 
  grd1:  $tr \in TRAIN \setminus dom(connectedTrain)$ 
  grd2:  $fr \in WAY$ 
  grd3:  $integer \in BOOL$ 
  grd4:  $integer = TRUE \Rightarrow re \in WAY$ 
  grd5:  $re < fr$ 
  grd6:  $ttds \subseteq stateTTD^{-1}[\{FREE\}]$ 
  grd7:  $\forall ttd. (ttd \in stateTTD^{-1}[\{FREE\}] \wedge ((fr \in ttd) \vee ((integer = TRUE) \wedge ((re \dots fr) \cap ttd \neq \emptyset))) \Rightarrow ttd \in ttds)$ 
  grd8:  $(integer = TRUE) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow (rear(tr1) \dots front(tr1)) \cap (re \dots fr) = \emptyset))$ 
  grd9:  $(integer = TRUE) \Rightarrow (\forall tr1. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr) \Rightarrow front(tr1) < re))$ 
  grd10:  $(integer = FALSE) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow fr < rear(tr1)))$ 
  grd11:  $(integer = FALSE) \Rightarrow (\forall tr1, ttd. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr \wedge ttd \in TTD \wedge front(tr1) \in ttd) \Rightarrow fr \notin ttd))$ 
  then
    act1:  $connectedTrain(tr) := TRUE$ 
    act2:  $front(tr) := fr$ 
    act3:  $rear := (\{TRUE \mapsto rear \Leftarrow \{tr \mapsto re\}, FALSE \mapsto rear\})(integer)$ 
    act4:  $stateTTD := stateTTD \Leftarrow (ttds \times \{OCCUPIED\})$ 
  end
Event _toggleTrainConnexionStatus <ordinary>  $\hat{=}$ 
extends _toggleTrainConnexionStatus
any
  tr
where
  grd0:  $dom(connectedTrain) \neq \emptyset$ 
  grd1:  $tr \in dom(connectedTrain)$ 
  then
    act1:  $connectedTrain := (\{TRUE \mapsto connectedTrain \Leftarrow \{tr \mapsto FALSE\}, FALSE \mapsto connectedTrain \Leftarrow \{tr \mapsto TRUE\}\})(bool(connectedTrain(tr) = TRUE))$ 
  end
Event _freeTtd <ordinary>  $\hat{=}$ 
any

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    ttd
  where
    grd0:  $ttd \in stateTTD^{-1}[\{OCCUPIED\}]$ 
    grd1:  $\forall tr. (tr \in (dom(front) \setminus dom(rear)) \Rightarrow front(tr) \notin ttd)$ 
    grd2:  $\forall tr. (tr \in dom(rear) \Rightarrow (rear(tr) .. front(tr)) \cap ttd = \emptyset)$ 
  then
    act1:  $stateTTD(ttd) := FREE$ 
  end
END
```

MACHINE M3**REFINES** M2**SEES** C0,C2**VARIABLES**

connectedTrain
front
rear
MA
MAtemp
stateTTD
stateVSS
newstateVSScomputed

INVARIANTS

inv3_1: $\text{newstateVSScomputed} \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$

EVENTS**Initialisation****begin**

act1: $\text{connectedTrain} := \emptyset$
act2: $\text{front} := \emptyset$
act3: $\text{rear} := \emptyset$
act4: $\text{MA} := \emptyset$
act5: $\text{MAtemp} := \emptyset$
act6: $\text{stateTTD} := TTD \times \{OCCUPIED\}$
act7: $\text{stateVSS} := VSS \times \{UNKNOWN\}$
act8: $\text{newstateVSScomputed} := VSS \times \{UNKNOWN\}$

end**Event** ComputeVSSStates $\langle \text{ordinary} \rangle \hat{=}$ **any**

$\text{newstateVSScomputed1}$

where

grd0: $\text{newstateVSScomputed1} \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$

then

act1: $\text{newstateVSScomputed} := \text{newstateVSScomputed1}$

end**Event** ComputeTrainMA $\langle \text{ordinary} \rangle \hat{=}$ **refines** ComputeTrainMAFollowingVSSStates**any**

tr

vsss

p

q

vsss1

p0

p1

q1

newstateVSS vsss1 designe l'ensemble des vss sur lesquels le train est susceptible de se trouver

where

grd0: $\text{newstateVSS} = \text{newstateVSScomputed}$

grd1: $\text{tr} \in \text{connectedTrain}^{-1}[\{TRUE\}]$

grd2: $\text{vsss} \subseteq \text{newstateVSS}^{-1}[\{FREE\}]$

grd3: $\text{union}(\text{vsss}) = p1 \dots q1$

grd4: $p1 \geq \text{front}(\text{tr})$

grd5: $\text{vsss1} \subseteq VSS$

grd6: $\text{union}(\text{vsss1}) = p0 \dots (p1 - 1)$

grd7: $\text{tr} \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(\text{tr}) \geq p0$

grd8: $\text{tr} \notin \text{dom}(\text{rear}) \Rightarrow \text{front}(\text{tr}) \geq p0$

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    grd9:  $p \dots q \subseteq \text{union}(vsss \cup vsss1)$ 
    grd10:  $p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
    grd11:  $\text{front}(tr) \in p \dots q$ 
    grd12:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q$ 
  then
    act1:  $MAtemp(tr) := p \dots q$ 
    act2:  $\text{stateVSS} := \text{newstateVSS}$ 
  end
Event AssignMAtoTrain <ordinary>  $\hat{=}$ 
extends AssignMAtoTrain
  any
    tr
  where
    grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MAtemp)$ 
    grd2:  $\text{front}(tr) \in MAtemp(tr)$ 
    grd3:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in MAtemp(tr)$ 
    grd4:  $MAtemp(tr) \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
  then
    act1:  $MA(tr) := MAtemp(tr)$ 
  end
Event MoveTrainFollowingItsMA <ordinary>  $\hat{=}$ 
extends MoveTrainFollowingItsMA
  any
    tr
    len
    ttds
  where
    grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MA)$ 
    grd2:  $len \in \mathbb{N}_1$ 
    grd3:  $\text{front}(tr) + len \in MA(tr)$ 
    grd4:  $ttds \subseteq \text{stateTTD}^{-1}[\{FREE\}]$ 
    grd5:  $\forall ttd. (ttd \in \text{stateTTD}^{-1}[\{FREE\}] \wedge ((\text{front}(tr) + len \in ttd) \vee (tr \in \text{dom}(\text{rear}) \wedge ((\text{rear}(tr) + len \dots \text{front}(tr) + len) \cap ttd \neq \emptyset))) \Rightarrow ttd \in ttds)$ 
    grd6:  $tr \in \text{dom}(\text{rear}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow (\text{rear}(tr1) \dots \text{front}(tr1)) \cap (\text{rear}(tr) + len \dots \text{front}(tr) + len) = \emptyset))$ 
    grd7:  $tr \in \text{dom}(\text{rear}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow \text{front}(tr1) < \text{rear}(tr) + len))$ 
    grd8:  $tr \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow \text{front}(tr) + len < \text{rear}(tr1)))$ 
    grd9:  $tr \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \Rightarrow (\forall tr1, ttd. ((tr1 \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \wedge tr1 \neq tr \wedge ttd \in TTD \wedge \text{front}(tr1) \in ttd) \Rightarrow \text{front}(tr) + len \notin ttd))$ 
  then
    act1:  $\text{front}(tr) := \text{front}(tr) + len$ 
    act2:  $\text{rear} := (\{TRUE \mapsto \text{rear} \triangleleft \{tr \mapsto \text{rear}(tr) + len\}, FALSE \mapsto \text{rear}\})(\text{bool}(tr \in \text{dom}(\text{rear})))$ 
    act3:  $\text{stateTTD} := \text{stateTTD} \triangleleft (ttds \times \{OCCUPIED\})$ 
  end
Event _connectTrain <ordinary>  $\hat{=}$ 
extends _connectTrain
  any
    tr
    fr
    re
    integer
    ttds
  where
    grd0:  $TRAIN \setminus \text{dom}(\text{connectedTrain}) \neq \emptyset$ 
    grd1:  $tr \in TRAIN \setminus \text{dom}(\text{connectedTrain})$ 
    grd2:  $fr \in WAY$ 

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```

    grd3:  $integer \in \text{BOOL}$ 
    grd4:  $integer = \text{TRUE} \Rightarrow re \in \text{WAY}$ 
    grd5:  $re < fr$ 
    grd6:  $ttds \subseteq \text{stateTTD}^{-1}[\{\text{FREE}\}]$ 
    grd7:  $\forall ttd. (ttd \in \text{stateTTD}^{-1}[\{\text{FREE}\}] \wedge ((fr \in ttd) \vee ((integer = \text{TRUE}) \wedge ((re .. fr) \cap ttd \neq \emptyset))) \Rightarrow ttd \in ttds)$ 
    grd8:  $(integer = \text{TRUE}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow (\text{rear}(tr1) .. \text{front}(tr1)) \cap (re .. fr) = \emptyset))$ 
    grd9:  $(integer = \text{TRUE}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow \text{front}(tr1) < re))$ 
    grd10:  $(integer = \text{FALSE}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow fr < \text{rear}(tr1)))$ 
    grd11:  $(integer = \text{FALSE}) \Rightarrow (\forall tr1, ttd. ((tr1 \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \wedge tr1 \neq tr \wedge ttd \in \text{TTD} \wedge \text{front}(tr1) \in ttd) \Rightarrow fr \notin ttd))$ 
  then
    act1:  $\text{connectedTrain}(tr) := \text{TRUE}$ 
    act2:  $\text{front}(tr) := fr$ 
    act3:  $\text{rear} := (\{\text{TRUE} \mapsto \text{rear} \Leftarrow \{tr \mapsto re\}, \text{FALSE} \mapsto \text{rear}\})(integer)$ 
    act4:  $\text{stateTTD} := \text{stateTTD} \Leftarrow (ttds \times \{\text{OCCUPIED}\})$ 
  end
Event _toggleTrainConnexionStatus <ordinary>  $\hat{=}$ 
extends _toggleTrainConnexionStatus
  any
     $tr$ 
  where
    grd0:  $\text{dom}(\text{connectedTrain}) \neq \emptyset$ 
    grd1:  $tr \in \text{dom}(\text{connectedTrain})$ 
  then
    act1:  $\text{connectedTrain} := (\{\text{TRUE} \mapsto \text{connectedTrain} \Leftarrow \{tr \mapsto \text{FALSE}\}, \text{FALSE} \mapsto \text{connectedTrain} \Leftarrow \{tr \mapsto \text{TRUE}\}\})(\text{bool}(\text{connectedTrain}(tr) = \text{TRUE}))$ 
  end
Event _freeTtd <ordinary>  $\hat{=}$ 
extends _freeTtd
  any
     $ttd$ 
  where
    grd0:  $ttd \in \text{stateTTD}^{-1}[\{\text{OCCUPIED}\}]$ 
    grd1:  $\forall tr. (tr \in (\text{dom}(\text{front}) \setminus \text{dom}(\text{rear})) \Rightarrow \text{front}(tr) \notin ttd)$ 
    grd2:  $\forall tr. (tr \in \text{dom}(\text{rear}) \Rightarrow (\text{rear}(tr) .. \text{front}(tr)) \cap ttd = \emptyset)$ 
  then
    act1:  $\text{stateTTD}(ttd) := \text{FREE}$ 
  end
END

```

MACHINE M4**REFINES** M3**SEES** C0,C2**VARIABLES**

connectedTrain
 front
 rear
 MA
 MAtemp
 stateTTD
 stateVSS
 newstateVSScomputed

EVENTS**Initialisation****begin**

act1: $connectedTrain := \emptyset$
act2: $front := \emptyset$
act3: $rear := \emptyset$
act4: $MA := \emptyset$
act5: $MAtemp := \emptyset$
act6: $stateTTD := TTD \times \{OCCUPIED\}$
act7: $stateVSS := VSS \times \{UNKNOWN\}$
act8: $newstateVSScomputed := VSS \times \{UNKNOWN\}$

end**Event** ComputeVSSStatesFollowingTTDStates *<ordinary>* $\hat{=}$ **refines** ComputeVSSStates**any**

newstateVSScomputed1

where*grd0*: $newstateVSScomputed1 \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$ **then***act1*: $newstateVSScomputed := newstateVSScomputed1$ **end****Event** ComputeVSSStateswoTTDDStates *<ordinary>* $\hat{=}$ **refines** ComputeVSSStates**any**

newstateVSScomputed1

where*grd0*: $newstateVSScomputed1 \in VSS \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$ **then***act1*: $newstateVSScomputed := newstateVSScomputed1$ **end****Event** ComputeTrainMA *<ordinary>* $\hat{=}$ **extends** ComputeTrainMA**any***tr**vsss**p**q**vsss1**p0**p1**q1**newstateVSS* *vsss1* designe l'ensemble des vss sur lesquels le train est susceptible de se trouver**where***grd0*: $newstateVSS = newstateVSScomputed$

```

    grd1:  $tr \in connectedTrain^{-1}[\{TRUE\}]$ 
    grd2:  $vsss \subseteq newstateVSS^{-1}[\{FREE\}]$ 
    grd3:  $union(vsss) = p1 .. q1$ 
    grd4:  $p1 \geq front(tr)$ 
    grd5:  $vsss1 \subseteq VSS$ 
    grd6:  $union(vsss1) = p0 .. (p1 - 1)$ 
    grd7:  $tr \in dom(rear) \Rightarrow rear(tr) \geq p0$ 
    grd8:  $tr \notin dom(rear) \Rightarrow front(tr) \geq p0$ 
    grd9:  $p .. q \subseteq union(vsss \cup vsss1)$ 
    grd10:  $p .. q \cap union(ran(\{tr\} \triangleleft MA)) = \emptyset$ 
    grd11:  $front(tr) \in p .. q$ 
    grd12:  $tr \in dom(rear) \Rightarrow rear(tr) \in p .. q$ 
  then
    act1:  $MAtemp(tr) := p .. q$ 
    act2:  $stateVSS := newstateVSS$ 
  end
Event AssignMAtoTrain <ordinary>  $\hat{=}$ 
extends AssignMAtoTrain
  any
    tr
  where
    grd1:  $tr \in connectedTrain^{-1}[\{TRUE\}] \cap dom(MAtemp)$ 
    grd2:  $front(tr) \in MAtemp(tr)$ 
    grd3:  $tr \in dom(rear) \Rightarrow rear(tr) \in MAtemp(tr)$ 
    grd4:  $MAtemp(tr) \cap union(ran(\{tr\} \triangleleft MA)) = \emptyset$ 
  then
    act1:  $MA(tr) := MAtemp(tr)$ 
  end
Event MoveTrainFollowingItsMA <ordinary>  $\hat{=}$ 
extends MoveTrainFollowingItsMA
  any
    tr
    len
    ttds
  where
    grd1:  $tr \in connectedTrain^{-1}[\{TRUE\}] \cap dom(MA)$ 
    grd2:  $len \in \mathbb{N}_1$ 
    grd3:  $front(tr) + len \in MA(tr)$ 
    grd4:  $ttds \subseteq stateTTD^{-1}[\{FREE\}]$ 
    grd5:  $\forall ttd. (ttd \in stateTTD^{-1}[\{FREE\}] \wedge ((front(tr) + len \in ttd) \vee (tr \in dom(rear) \wedge ((rear(tr) + len .. front(tr) + len) \cap ttd \neq \emptyset))) \Rightarrow ttd \in ttds)$ 
    grd6:  $tr \in dom(rear) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow (rear(tr1) .. front(tr1)) \cap (rear(tr) + len .. front(tr) + len) = \emptyset))$ 
    grd7:  $tr \in dom(rear) \Rightarrow (\forall tr1. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr) \Rightarrow front(tr1) < rear(tr) + len))$ 
    grd8:  $tr \in dom(front) \setminus dom(rear) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow front(tr) + len < rear(tr1)))$ 
    grd9:  $tr \in dom(front) \setminus dom(rear) \Rightarrow (\forall tr1, ttd. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr \wedge ttd \in TTD \wedge front(tr1) \in ttd) \Rightarrow front(tr) + len \notin ttd))$ 
  then
    act1:  $front(tr) := front(tr) + len$ 
    act2:  $rear := (\{TRUE \mapsto rear \triangleleft \{tr \mapsto rear(tr) + len\}, FALSE \mapsto rear\})(bool(tr \in dom(rear)))$ 
    act3:  $stateTTD := stateTTD \triangleleft (ttds \times \{OCCUPIED\})$ 
  end
Event _connectTrain <ordinary>  $\hat{=}$ 
extends _connectTrain
  any
    tr

```

```

    fr
    re
    integer
    ttds
  where
    grd0:  $TRAIN \setminus dom(connectedTrain) \neq \emptyset$ 
    grd1:  $tr \in TRAIN \setminus dom(connectedTrain)$ 
    grd2:  $fr \in WAY$ 
    grd3:  $integer \in BOOL$ 
    grd4:  $integer = TRUE \Rightarrow re \in WAY$ 
    grd5:  $re < fr$ 
    grd6:  $ttds \subseteq stateTTD^{-1}[\{FREE\}]$ 
    grd7:  $\forall ttd. (ttd \in stateTTD^{-1}[\{FREE\}] \wedge ((fr \in ttd) \vee ((integer = TRUE) \wedge ((re .. fr) \cap ttd \neq \emptyset))) \Rightarrow ttd \in ttds)$ 
    grd8:  $(integer = TRUE) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow (rear(tr1) .. front(tr1)) \cap (re .. fr) = \emptyset))$ 
    grd9:  $(integer = TRUE) \Rightarrow (\forall tr1. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr) \Rightarrow front(tr1) < re))$ 
    grd10:  $(integer = FALSE) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow fr < rear(tr1)))$ 
    grd11:  $(integer = FALSE) \Rightarrow (\forall tr1, ttd. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr \wedge ttd \in TTD \wedge front(tr1) \in ttd) \Rightarrow fr \notin ttd))$ 
  then
    act1:  $connectedTrain(tr) := TRUE$ 
    act2:  $front(tr) := fr$ 
    act3:  $rear := (\{TRUE \mapsto rear \Leftarrow \{tr \mapsto re\}, FALSE \mapsto rear\})(integer)$ 
    act4:  $stateTTD := stateTTD \Leftarrow (ttds \times \{OCCUPIED\})$ 
  end
Event _toggleTrainConnexionStatus <ordinary>  $\hat{=}$ 
extends _toggleTrainConnexionStatus
  any
    tr
  where
    grd0:  $dom(connectedTrain) \neq \emptyset$ 
    grd1:  $tr \in dom(connectedTrain)$ 
  then
    act1:  $connectedTrain := (\{TRUE \mapsto connectedTrain \Leftarrow \{tr \mapsto FALSE\}, FALSE \mapsto connectedTrain \Leftarrow \{tr \mapsto TRUE\}\})(bool(connectedTrain(tr) = TRUE))$ 
  end
Event _freeTtd <ordinary>  $\hat{=}$ 
extends _freeTtd
  any
    ttd
  where
    grd0:  $ttd \in stateTTD^{-1}[\{OCCUPIED\}]$ 
    grd1:  $\forall tr. (tr \in (dom(front) \setminus dom(rear)) \Rightarrow front(tr) \notin ttd)$ 
    grd2:  $\forall tr. (tr \in dom(rear) \Rightarrow (rear(tr) .. front(tr)) \cap ttd = \emptyset)$ 
  then
    act1:  $stateTTD(ttd) := FREE$ 
  end
END

```


MACHINE M5**REFINES** M4**SEES** C0,C2**VARIABLES**

connectedTrain
 front
 rear
 MA
 MAtemp
 stateTTD
 stateVSS
 newstateVSScomputed
 newstateVSScomputedTmp

INVARIANTS

inv5.1: $\text{newstateVSScomputedTmp} \in \text{VSS} \rightarrow \{OCCUPIED, FREE, UNKNOWN, AMBIGUOUS\}$

EVENTS**Initialisation****begin**

act1: $\text{connectedTrain} := \emptyset$
act2: $\text{front} := \emptyset$
act3: $\text{rear} := \emptyset$
act4: $\text{MA} := \emptyset$
act5: $\text{MAtemp} := \emptyset$
act6: $\text{stateTTD} := \text{TTD} \times \{OCCUPIED\}$
act7: $\text{stateVSS} := \text{VSS} \times \{UNKNOWN\}$
act8: $\text{newstateVSScomputed} := \text{VSS} \times \{UNKNOWN\}$
act9: $\text{newstateVSScomputedTmp} := \text{VSS} \times \{UNKNOWN\}$

end**Event** ComputeStatesOfVSSinUnknowState *ordinary* $\hat{=}$ **any**

vss
 vss1
 vss2
 vss3
 vss4

where

grd1: $\text{vss} = \text{stateVSS}^{-1}[\{UNKNOWN\}]$
grd2: $\text{partition}(\text{vss}, \text{vss1}, \text{vss2}, \text{vss3}, \text{vss4})$

then

act1: $\text{newstateVSScomputedTmp} := \text{newstateVSScomputedTmp} \Leftarrow ((\text{vss1} \times \{OCCUPIED\}) \cup (\text{vss2} \times \{FREE\}) \cup (\text{vss3} \times \{AMBIGUOUS\}) \cup (\text{vss4} \times \{UNKNOWN\}))$

end**Event** ComputeStatesOfVSSinOccupiedState *ordinary* $\hat{=}$ **any**

vss
 vss1
 vss2
 vss3
 vss4

where

grd1: $\text{vss} = \text{stateVSS}^{-1}[\{OCCUPIED\}]$
grd2: $\text{partition}(\text{vss}, \text{vss1}, \text{vss2}, \text{vss3}, \text{vss4})$

then

act1: $\text{newstateVSScomputedTmp} := \text{newstateVSScomputedTmp} \Leftarrow ((\text{vss1} \times \{OCCUPIED\}) \cup (\text{vss2} \times \{FREE\}) \cup (\text{vss3} \times \{AMBIGUOUS\}) \cup (\text{vss4} \times \{UNKNOWN\}))$

end

Event ComputeStatesOfVSSinAmbiguousState $\langle \text{ordinary} \rangle \hat{=}$

any

vss
vss1
vss2
vss3
vss4

where

grd1: $vss = \text{stateVSS}^{-1}[\{ \text{AMBIGUOUS} \}]$
grd2: $\text{partition}(vss, vss1, vss2, vss3, vss4)$

then

act1: $\text{newstateVSScomputedTmp} := \text{newstateVSScomputedTmp} \Leftarrow ((vss1 \times \{ \text{OCCUPIED} \}) \cup (vss2 \times \{ \text{FREE} \}) \cup (vss3 \times \{ \text{AMBIGUOUS} \}) \cup (vss4 \times \{ \text{UNKNOWN} \}))$

end

Event ComputeStatesOfVSSinFreeState $\langle \text{ordinary} \rangle \hat{=}$

any

vss
vss1
vss2
vss3
vss4

where

grd1: $vss = \text{stateVSS}^{-1}[\{ \text{FREE} \}]$
grd2: $\text{partition}(vss, vss1, vss2, vss3, vss4)$

then

act1: $\text{newstateVSScomputedTmp} := \text{newstateVSScomputedTmp} \Leftarrow ((vss1 \times \{ \text{OCCUPIED} \}) \cup (vss2 \times \{ \text{FREE} \}) \cup (vss3 \times \{ \text{AMBIGUOUS} \}) \cup (vss4 \times \{ \text{UNKNOWN} \}))$

end

Event updateVSSStates $\langle \text{ordinary} \rangle \hat{=}$

refines ComputeVSSStatesFollowingTTDDStates

any

newstateVSScomputed1

where

grd0: $\text{newstateVSScomputed1} = \text{newstateVSScomputedTmp}$

then

act1: $\text{newstateVSScomputed} := \text{newstateVSScomputed1}$

end

Event ComputeTrainMA $\langle \text{ordinary} \rangle \hat{=}$

extends ComputeTrainMA

any

tr
vsss
p
q
vsss1
p0
p1
q1

newstateVSS vsss1 designe l'ensemble des vss sur lesquels le train est susceptible de se trouver

where

grd0: $\text{newstateVSS} = \text{newstateVSScomputed}$
grd1: $tr \in \text{connectedTrain}^{-1}[\{ \text{TRUE} \}]$
grd2: $vsss \subseteq \text{newstateVSS}^{-1}[\{ \text{FREE} \}]$
grd3: $\text{union}(vsss) = p1 \dots q1$
grd4: $p1 \geq \text{front}(tr)$
grd5: $vsss1 \subseteq \text{VSS}$
grd6: $\text{union}(vsss1) = p0 \dots (p1 - 1)$
grd7: $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \geq p0$

```

    grd8:  $tr \notin \text{dom}(\text{rear}) \Rightarrow \text{front}(tr) \geq p0$ 
    grd9:  $p \dots q \subseteq \text{union}(vsss \cup vsss1)$ 
    grd10:  $p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
    grd11:  $\text{front}(tr) \in p \dots q$ 
    grd12:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q$ 
  then
    act1:  $MAtemp(tr) := p \dots q$ 
    act2:  $\text{stateVSS} := \text{newstateVSS}$ 
  end
Event AssignMAtoTrain  $\langle \text{ordinary} \rangle \hat{=}$ 
extends AssignMAtoTrain
  any
     $tr$ 
  where
    grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MAtemp)$ 
    grd2:  $\text{front}(tr) \in MAtemp(tr)$ 
    grd3:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in MAtemp(tr)$ 
    grd4:  $MAtemp(tr) \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
  then
    act1:  $MA(tr) := MAtemp(tr)$ 
  end
Event MoveTrainFollowingItsMA  $\langle \text{ordinary} \rangle \hat{=}$ 
extends MoveTrainFollowingItsMA
  any
     $tr$ 
     $len$ 
     $ttds$ 
  where
    grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MA)$ 
    grd2:  $len \in \mathbb{N}_1$ 
    grd3:  $\text{front}(tr) + len \in MA(tr)$ 
    grd4:  $ttds \subseteq \text{stateTTD}^{-1}[\{FREE\}]$ 
    grd5:  $\forall ttd. (ttd \in \text{stateTTD}^{-1}[\{FREE\}] \wedge ((\text{front}(tr) + len \in ttd) \vee (tr \in \text{dom}(\text{rear}) \wedge ((\text{rear}(tr) + len \dots \text{front}(tr) + len) \cap ttd \neq \emptyset))) \Rightarrow ttd \in ttds)$ 
    grd6:  $tr \in \text{dom}(\text{rear}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow (\text{rear}(tr1) \dots \text{front}(tr1)) \cap (\text{rear}(tr) + len \dots \text{front}(tr) + len) = \emptyset))$ 
    grd7:  $tr \in \text{dom}(\text{rear}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow \text{front}(tr1) < \text{rear}(tr) + len))$ 
    grd8:  $tr \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow \text{front}(tr) + len < \text{rear}(tr1)))$ 
    grd9:  $tr \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \Rightarrow (\forall tr1, ttd. ((tr1 \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \wedge tr1 \neq tr \wedge ttd \in TTD \wedge \text{front}(tr1) \in ttd) \Rightarrow \text{front}(tr) + len \notin ttd))$ 
  then
    act1:  $\text{front}(tr) := \text{front}(tr) + len$ 
    act2:  $\text{rear} := (\{TRUE \mapsto \text{rear} \triangleleft \{tr \mapsto \text{rear}(tr) + len\}, FALSE \mapsto \text{rear}\})(\text{bool}(tr \in \text{dom}(\text{rear})))$ 
    act3:  $\text{stateTTD} := \text{stateTTD} \triangleleft (ttds \times \{OCCUPIED\})$ 
  end
Event _connectTrain  $\langle \text{ordinary} \rangle \hat{=}$ 
extends _connectTrain
  any
     $tr$ 
     $fr$ 
     $re$ 
     $integer$ 
     $ttds$ 
  where
    grd0:  $TRAIN \setminus \text{dom}(\text{connectedTrain}) \neq \emptyset$ 
    grd1:  $tr \in TRAIN \setminus \text{dom}(\text{connectedTrain})$ 

```

```

    grd2:  $fr \in WAY$ 
    grd3:  $integer \in BOOL$ 
    grd4:  $integer = TRUE \Rightarrow re \in WAY$ 
    grd5:  $re < fr$ 
    grd6:  $ttds \subseteq stateTTD^{-1}[\{FREE\}]$ 
    grd7:  $\forall ttd. (ttd \in stateTTD^{-1}[\{FREE\}] \wedge ((fr \in ttd) \vee ((integer = TRUE) \wedge ((re .. fr) \cap ttd \neq \emptyset))) \Rightarrow ttd \in ttds)$ 
    grd8:  $(integer = TRUE) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow (rear(tr1) .. front(tr1)) \cap (re .. fr) = \emptyset))$ 
    grd9:  $(integer = TRUE) \Rightarrow (\forall tr1. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr) \Rightarrow front(tr1) < re))$ 
    grd10:  $(integer = FALSE) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow fr < rear(tr1)))$ 
    grd11:  $(integer = FALSE) \Rightarrow (\forall tr1, ttd. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr \wedge ttd \in TTD \wedge front(tr1) \in ttd) \Rightarrow fr \notin ttd))$ 
  then
    act1:  $connectedTrain(tr) := TRUE$ 
    act2:  $front(tr) := fr$ 
    act3:  $rear := (\{TRUE \mapsto rear \Leftarrow \{tr \mapsto re\}, FALSE \mapsto rear\})(integer)$ 
    act4:  $stateTTD := stateTTD \Leftarrow (ttds \times \{OCCUPIED\})$ 
  end
Event _toggleTrainConnexionStatus <ordinary>  $\hat{=}$ 
extends _toggleTrainConnexionStatus
  any
     $tr$ 
  where
    grd0:  $dom(connectedTrain) \neq \emptyset$ 
    grd1:  $tr \in dom(connectedTrain)$ 
  then
    act1:  $connectedTrain := (\{TRUE \mapsto connectedTrain \Leftarrow \{tr \mapsto FALSE\}, FALSE \mapsto connectedTrain \Leftarrow \{tr \mapsto TRUE\}\})(bool(connectedTrain(tr) = TRUE))$ 
  end
Event _freeTtd <ordinary>  $\hat{=}$ 
extends _freeTtd
  any
     $ttd$ 
  where
    grd0:  $ttd \in stateTTD^{-1}[\{OCCUPIED\}]$ 
    grd1:  $\forall tr. (tr \in (dom(front) \setminus dom(rear)) \Rightarrow front(tr) \notin ttd)$ 
    grd2:  $\forall tr. (tr \in dom(rear) \Rightarrow (rear(tr) .. front(tr)) \cap ttd = \emptyset)$ 
  then
    act1:  $stateTTD(ttd) := FREE$ 
  end
END

```

MACHINE M6**REFINES** M5**SEES** C0,C2**VARIABLES**

connectedTrain
 front
 rear
 MA
 MAtemp
 stateTTD
 stateVSS
 newstateVSScomputed
 newstateVSScomputedTmp
 freeVssChangingtoFree
 freeVssChangingtoUnknow
 freeVssChangingtoOccupied
 freeVssChangingtoAmbiguous

INVARIANTS

inv6_1: $freeVssChangingtoFree \subseteq VSS$
inv6_2: $freeVssChangingtoUnknow \subseteq VSS$
inv6_3: $freeVssChangingtoOccupied \subseteq VSS$
inv6_4: $freeVssChangingtoAmbiguous \subseteq VSS$

EVENTS**Initialisation****begin**

act1: $connectedTrain := \emptyset$
act2: $front := \emptyset$
act3: $rear := \emptyset$
act4: $MA := \emptyset$
act5: $MAtemp := \emptyset$
act6: $stateTTD := TTD \times \{OCCUPIED\}$
act7: $stateVSS := VSS \times \{UNKNOWN\}$
act8: $newstateVSScomputed := VSS \times \{UNKNOWN\}$
act9: $newstateVSScomputedTmp := VSS \times \{UNKNOWN\}$
act10: $freeVssChangingtoFree := \emptyset$
act11: $freeVssChangingtoUnknow := \emptyset$
act12: $freeVssChangingtoOccupied := \emptyset$
act13: $freeVssChangingtoAmbiguous := \emptyset$

end**Event** ComputeStatesOfVSSinFreeStateWhenTTDisFree **<ordinary>** $\hat{=}$ **any** $vssTtdFree$ **where**

grd1: $vssTtdFree \subseteq stateVSS^{-1}[\{FREE\}]$
grd2: $\forall vss. (vss \in vssTtdFree \Rightarrow vss \subseteq union(stateTTD^{-1}[\{FREE\}]))$

then**act1:** $freeVssChangingtoFree := freeVssChangingtoFree \cup vssTtdFree$ **end****Event** ComputeStatesOfVSSinFreeStateWhenTTDisOccupiedandNoTrainisLocatedonTTD **<ordinary>** $\hat{=}$ **any** $vssTtdOccupiedwithNoTrain$ **where**

grd1: $vssTtdOccupiedwithNoTrain \subseteq stateVSS^{-1}[\{FREE\}]$
grd2: $\forall vss. (vss \in vssTtdOccupiedwithNoTrain \Rightarrow vss \subseteq union(stateTTD^{-1}[\{OCCUPIED\}]))$

```

    grd3:  $\forall vss, p, q. ((vss \in vssTtdOccupiedwithNoTrain \wedge p \dots q \in TTD \wedge vss \subseteq p \dots q) \Rightarrow (\forall tr \cdot tr \in$ 
       $connectedTrain^{-1}[\{TRUE\}] \wedge tr \in dom(rear) \Rightarrow (front(tr) < p \vee rear(tr) > q)))$ 
    grd4:  $\forall vss, p, q. ((vss \in vssTtdOccupiedwithNoTrain \wedge p \dots q \in TTD \wedge vss \subseteq p \dots q) \Rightarrow (\forall tr \cdot tr \in$ 
       $connectedTrain^{-1}[\{TRUE\}] \wedge tr \notin dom(rear) \Rightarrow (front(tr) < p \vee front(tr) > q)))$ 
  then
    act1:  $freeVssChangingtoUnknow := freeVssChangingtoUnknow \cup vssTtdOccupiedwithNoTrain$ 
  end
Event ComputeStatesOfVSSinFreeStateWhenTTDisOccupiedandNoMAisIssued <ordinary>  $\hat{=}$ 
  any
    vssTtdOccupiedwithNoMA
  where
    grd1:  $vssTtdOccupiedwithNoMA \subseteq stateVSS^{-1}[\{FREE\}]$ 
    grd2:  $\forall vss. (vss \in vssTtdOccupiedwithNoMA \Rightarrow vss \subseteq union(stateTTD^{-1}[\{OCCUPIED\}]))$ 
    grd3:  $\forall vss, ttd. ((vss \in vssTtdOccupiedwithNoMA \wedge ttd \in TTD \wedge vss \subseteq ttd) \Rightarrow (union(ran(MA)) \cap$ 
       $ttd = \emptyset))$ 
  then
    act1:  $freeVssChangingtoUnknow := freeVssChangingtoUnknow \cup vssTtdOccupiedwithNoMA$ 
  end
Event FullComputeStatesOfVSSinFreeState <ordinary>  $\hat{=}$ 
refines ComputeStatesOfVSSinFreeState
  any
    vss
    vss1
    vss2
    vss3
    vss4
  where
    grd1:  $vss = stateVSS^{-1}[\{FREE\}]$ 
    grd2:  $partition(vss, vss1, vss2, vss3, vss4)$ 
    grd3:  $freeVssChangingtoFree \subseteq vss2$ 
    lorsque toutes les transitions seront implementees, ceci deviendra une egalite
    grd4:  $freeVssChangingtoUnknow \subseteq vss4$ 
    lorsque toutes les transitions seront implementees, ceci deviendra une egalite
  then
    act1:  $newstateVSScomputedTmp := newstateVSScomputedTmp \Leftarrow ((vss1 \times \{OCCUPIED\}) \cup$ 
       $(vss2 \times \{FREE\}) \cup (vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOW\}))$ 
  end
Event ComputeStatesOfVSSinUnknowState <ordinary>  $\hat{=}$ 
extends ComputeStatesOfVSSinUnknowState
  any
    vss
    vss1
    vss2
    vss3
    vss4
  where
    grd1:  $vss = stateVSS^{-1}[\{UNKNOW\}]$ 
    grd2:  $partition(vss, vss1, vss2, vss3, vss4)$ 
  then
    act1:  $newstateVSScomputedTmp := newstateVSScomputedTmp \Leftarrow ((vss1 \times \{OCCUPIED\}) \cup$ 
       $(vss2 \times \{FREE\}) \cup (vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOW\}))$ 
  end
Event ComputeStatesOfVSSinOccupiedState <ordinary>  $\hat{=}$ 
extends ComputeStatesOfVSSinOccupiedState
  any
    vss

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    vss1
    vss2
    vss3
    vss4
  where
    grd1:  $vss = stateVSS^{-1}[\{OCCUPIED\}]$ 
    grd2:  $partition(vss, vss1, vss2, vss3, vss4)$ 
  then
    act1:  $newstateVSScomputedTmp := newstateVSScomputedTmp \Leftarrow ((vss1 \times \{OCCUPIED\}) \cup$ 
       $(vss2 \times \{FREE\}) \cup (vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOWN\}))$ 
  end
Event ComputeStatesOfVSSinAmbiguousState  $\langle ordinary \rangle \hat{=}$ 
extends ComputeStatesOfVSSinAmbiguousState
  any
    vss
    vss1
    vss2
    vss3
    vss4
  where
    grd1:  $vss = stateVSS^{-1}[\{AMBIGUOUS\}]$ 
    grd2:  $partition(vss, vss1, vss2, vss3, vss4)$ 
  then
    act1:  $newstateVSScomputedTmp := newstateVSScomputedTmp \Leftarrow ((vss1 \times \{OCCUPIED\}) \cup$ 
       $(vss2 \times \{FREE\}) \cup (vss3 \times \{AMBIGUOUS\}) \cup (vss4 \times \{UNKNOWN\}))$ 
  end
Event updateVSSStates  $\langle ordinary \rangle \hat{=}$ 
extends updateVSSStates
  any
    newstateVSScomputed1
  where
    grd0:  $newstateVSScomputed1 = newstateVSScomputedTmp$ 
  then
    act1:  $newstateVSScomputed := newstateVSScomputed1$ 
  end
Event ComputeTrainMA  $\langle ordinary \rangle \hat{=}$ 
extends ComputeTrainMA
  any
    tr
    vsss
    p
    q
    vsss1
    p0
    p1
    q1
    newstateVSS vsss1 designe l'ensemble des vss sur lesquels le train est susceptible de se trouver
  where
    grd0:  $newstateVSS = newstateVSScomputed$ 
    grd1:  $tr \in connectedTrain^{-1}[\{TRUE\}]$ 
    grd2:  $vsss \subseteq newstateVSS^{-1}[\{FREE\}]$ 
    grd3:  $union(vsss) = p1 \dots q1$ 
    grd4:  $p1 \geq front(tr)$ 
    grd5:  $vsss1 \subseteq VSS$ 
    grd6:  $union(vsss1) = p0 \dots (p1 - 1)$ 
    grd7:  $tr \in dom(rear) \Rightarrow rear(tr) \geq p0$ 
    grd8:  $tr \notin dom(rear) \Rightarrow front(tr) \geq p0$ 
    grd9:  $p \dots q \subseteq union(vsss \cup vsss1)$ 

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    grd10:  $p \dots q \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
    grd11:  $\text{front}(tr) \in p \dots q$ 
    grd12:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in p \dots q$ 
  then
    act1:  $MAtemp(tr) := p \dots q$ 
    act2:  $\text{stateVSS} := \text{newstateVSS}$ 
  end
Event AssignMAtoTrain  $\langle \text{ordinary} \rangle \triangleq$ 
extends AssignMAtoTrain
  any
     $tr$ 
  where
    grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MAtemp)$ 
    grd2:  $\text{front}(tr) \in MAtemp(tr)$ 
    grd3:  $tr \in \text{dom}(\text{rear}) \Rightarrow \text{rear}(tr) \in MAtemp(tr)$ 
    grd4:  $MAtemp(tr) \cap \text{union}(\text{ran}(\{tr\} \triangleleft MA)) = \emptyset$ 
  then
    act1:  $MA(tr) := MAtemp(tr)$ 
  end
Event MoveTrainFollowingItsMA  $\langle \text{ordinary} \rangle \triangleq$ 
extends MoveTrainFollowingItsMA
  any
     $tr$ 
     $len$ 
     $ttds$ 
  where
    grd1:  $tr \in \text{connectedTrain}^{-1}[\{TRUE\}] \cap \text{dom}(MA)$ 
    grd2:  $len \in \mathbb{N}_1$ 
    grd3:  $\text{front}(tr) + len \in MA(tr)$ 
    grd4:  $ttds \subseteq \text{stateTTD}^{-1}[\{FREE\}]$ 
    grd5:  $\forall ttd. (ttd \in \text{stateTTD}^{-1}[\{FREE\}] \wedge ((\text{front}(tr) + len \in ttd) \vee (tr \in \text{dom}(\text{rear}) \wedge ((\text{rear}(tr) + len \dots \text{front}(tr) + len) \cap ttd \neq \emptyset))) \Rightarrow ttd \in ttds)$ 
    grd6:  $tr \in \text{dom}(\text{rear}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow (\text{rear}(tr1) \dots \text{front}(tr1)) \cap (\text{rear}(tr) + len \dots \text{front}(tr) + len) = \emptyset))$ 
    grd7:  $tr \in \text{dom}(\text{rear}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow \text{front}(tr1) < \text{rear}(tr) + len))$ 
    grd8:  $tr \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \Rightarrow (\forall tr1. ((tr1 \in \text{dom}(\text{rear}) \wedge tr1 \neq tr) \Rightarrow \text{front}(tr) + len < \text{rear}(tr1)))$ 
    grd9:  $tr \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \Rightarrow (\forall tr1, ttd. ((tr1 \in \text{dom}(\text{front}) \setminus \text{dom}(\text{rear}) \wedge tr1 \neq tr \wedge ttd \in TTD \wedge \text{front}(tr1) \in ttd) \Rightarrow \text{front}(tr) + len \notin ttd))$ 
  then
    act1:  $\text{front}(tr) := \text{front}(tr) + len$ 
    act2:  $\text{rear} := (\{TRUE \mapsto \text{rear} \triangleleft \{tr \mapsto \text{rear}(tr) + len\}, FALSE \mapsto \text{rear}\})(\text{bool}(tr \in \text{dom}(\text{rear})))$ 
    act3:  $\text{stateTTD} := \text{stateTTD} \triangleleft (ttds \times \{OCCUPIED\})$ 
  end
Event _connectTrain  $\langle \text{ordinary} \rangle \triangleq$ 
extends _connectTrain
  any
     $tr$ 
     $fr$ 
     $re$ 
     $integer$ 
     $ttds$ 
  where
    grd0:  $TRAIN \setminus \text{dom}(\text{connectedTrain}) \neq \emptyset$ 
    grd1:  $tr \in TRAIN \setminus \text{dom}(\text{connectedTrain})$ 
    grd2:  $fr \in WAY$ 
    grd3:  $integer \in BOOL$ 

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    grd4:  $integer = TRUE \Rightarrow re \in WAY$ 
    grd5:  $re < fr$ 
    grd6:  $ttds \subseteq stateTTD^{-1}[\{FREE\}]$ 
    grd7:  $\forall ttd. (ttd \in stateTTD^{-1}[\{FREE\}] \wedge ((fr \in ttd) \vee ((integer = TRUE) \wedge ((re .. fr) \cap ttd \neq \emptyset))) \Rightarrow ttd \in ttds)$ 
    grd8:  $(integer = TRUE) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow (rear(tr1) .. front(tr1)) \cap (re .. fr) = \emptyset))$ 
    grd9:  $(integer = TRUE) \Rightarrow (\forall tr1. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr) \Rightarrow front(tr1) < re))$ 
    grd10:  $(integer = FALSE) \Rightarrow (\forall tr1. ((tr1 \in dom(rear) \wedge tr1 \neq tr) \Rightarrow fr < rear(tr1)))$ 
    grd11:  $(integer = FALSE) \Rightarrow (\forall tr1, ttd. ((tr1 \in dom(front) \setminus dom(rear) \wedge tr1 \neq tr \wedge ttd \in TTD \wedge front(tr1) \in ttd) \Rightarrow fr \notin ttd))$ 
  then
    act1:  $connectedTrain(tr) := TRUE$ 
    act2:  $front(tr) := fr$ 
    act3:  $rear := (\{TRUE \mapsto rear \Leftarrow \{tr \mapsto re\}, FALSE \mapsto rear\})(integer)$ 
    act4:  $stateTTD := stateTTD \Leftarrow (ttds \times \{OCCUPIED\})$ 
  end
Event _toggleTrainConnexionStatus  $\langle ordinary \rangle \hat{=}$ 
extends _toggleTrainConnexionStatus
  any
     $tr$ 
  where
    grd0:  $dom(connectedTrain) \neq \emptyset$ 
    grd1:  $tr \in dom(connectedTrain)$ 
  then
    act1:  $connectedTrain := (\{TRUE \mapsto connectedTrain \Leftarrow \{tr \mapsto FALSE\}, FALSE \mapsto connectedTrain \Leftarrow \{tr \mapsto TRUE\}\})(bool(connectedTrain(tr) = TRUE))$ 
  end
Event _freeTtd  $\langle ordinary \rangle \hat{=}$ 
extends _freeTtd
  any
     $ttd$ 
  where
    grd0:  $ttd \in stateTTD^{-1}[\{OCCUPIED\}]$ 
    grd1:  $\forall tr. (tr \in (dom(front) \setminus dom(rear)) \Rightarrow front(tr) \notin ttd)$ 
    grd2:  $\forall tr. (tr \in dom(rear) \Rightarrow (rear(tr) .. front(tr)) \cap ttd = \emptyset)$ 
  then
    act1:  $stateTTD(ttd) := FREE$ 
  end
END

```