The inference process for the strong-weak influence model

Given the model description and hyperparameter J, the likelihood function can be determined as:

$$\begin{aligned}
&\zeta\left(o_{1:T}^{1:C}, q_{1:T}^{1:C}\middle| E^{1:C}, F^{1:C}, R(1:J), r_{1:T}\right) \\
&= \prod_{e}^{C} P(o_1^e|q_1^e)P(q_1^e) \\
&\times \prod_{t=2}^{T} \{P\left(r_t\right) P_{e=1}\left(o_t^e|q_t^e\right) P\left(q_t^e|q_{t-1}^1, q_{t-1}^2, ..., q_{t-1}^C\right) \\
&\times \prod_{e=2}^{C} P\left(o_t^e|q_t^e\right) P\left(q_t^e|q_t^1, q_t^2, ..., q_t^{e-1}, q_{t-1}^e, q_{t-1}^{e+1}, ..., q_{t-1}^C\right)\}
\end{aligned} \tag{.1}$$

Depending on whether the training set contains the state sequence or not, the learning algorithm of HMM or its variants is divided into two categories of approaches: supervised learning and unsupervised learning. Since well-labelled training data is usually very expensive and time consuming, unsupervised learning is the most popular learning algorithm, such as the forward-backward algorithm and variational Expectation-Maximization (EM). The dynamical influence model adopts the EM approach to learn parameters. However, in this paper, we have used two well labelled conversational datasets, which contains both the observation and the state sequence. Hence, we choose supervised approach to learn system parameters. Supervised learning estimates transition/emission probabilities from known samples via counting frequencies. Assume that there are two speakers A, B in the conversation, i.e., entity $C = 2$, and the first speaker of each turn tu is A, the second speaker is B. The inference process is as follows.

$$E_{s_i,s_j}^e|_{e\in\{A,B\}} = \frac{\sum_{tu} Count\left(q_{tu}^e = s_i, q_{tu+1}^e = s_j\right)}{\sum_{tu}\sum_s Count\left(q_{tu}^e = s_i, q_{tu+1}^e = s\right)} \tag{.2}$$

$$F_{s_i,s_j}^B = \frac{\sum_{tu} Count\left(q_{tu}^B = s_i, q_{tu+1}^A = s_j\right)}{\sum_{tu}\sum_s Count\left(q_{tu}^B = s_i, q_{tu+1}^A = s\right)} \tag{.3}$$

$$F_{s_i,s_j}^A = \frac{\sum_{tu} Count\left(q_{tu}^A = s_i, q_{tu}^B = s_j\right)}{\sum_{tu}\sum_s Count\left(q_{tu}^A = s_i, q_{tu}^B = s\right)} \tag{.4}$$

$$R_{e_1,e_2}^j = \begin{cases} \frac{F_{si,s_j}^{e_2}}{E_{s_i,sj}^{e_1}+F_{si,s_j}^{e_2}} & e_1 \neq e_2, r_t = j \\ \frac{E_{s_i,sj}^{e_1}}{E_{s_i,sj}^{e_1}+F_{s_i,s_j}^{e'}} & e_1 = e_2, e' = C - e_1, r_t = j \end{cases} \tag{.5}$$

and the emission probability,

$$b_{s_j}\left(o_k\right) = \frac{\sum_{tu} Count\left(q_{tu}^e = s_j, o_{tu}^e = o_k\right)}{\sum_{tu}\sum_o Count(q_{tu}^e = s_j, o_{tu}^e = o)} \tag{.6}$$