

Equality

Reflexive Property:

$$(a = b) \iff (b = a)$$

Transitive Property:

$$(a = b) \wedge (b = c) \implies (a = c)$$

Addition

Commutative Property:

$$a + b = b + a$$

Associative Property:

$$(a + b) + c = a + (b + c)$$

Additive Identity Property:

$$a + 0 = a$$

Inverse Property:

$$a + (-a) = 0$$

Multiplication

Commutative Property:

$$a \cdot b = b \cdot a$$

Associative Property:

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Multiplicative Identity Property:

$$a \cdot 1 = a$$

Inverse Property:

$$a \cdot \frac{1}{a} = 1; a \neq 0$$

Distributive Property:

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

Zero Product Property:

$$a \cdot 0 = 0$$

Exponents

Exponential Identity Property:

$$a^1 = a$$

Zero Power Property:

$$a^0 = 1; a \neq 0$$

Power of One Property:

$$1^a = a$$

Power of Zero Property:

$$0^a = 0; a \neq 0$$

Product of Powers Property:

$$x^a \cdot x^b = x^{a+b}$$

Quotient of Powers Property:

$$\frac{x^a}{x^b} = x^{a-b}; x \neq 0$$

Power of a Power Property:

$$(x^a)^b = x^{a \cdot b}$$

Power of a Product Property:

$$(x \cdot y)^a = x^a \cdot y^a$$

Power of a Quotient Property:

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}; y \neq 0$$

Negative Power Property:

$$x^{-a} = \frac{1}{x^a}; x \neq 0$$

Fractional Power Property:

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}; b \neq 0$$

Roots

Product of Roots Property:

$$\sqrt[a]{x} \cdot \sqrt[a]{y} = \sqrt[a]{x \cdot y}; a \neq 0 \wedge x, y \in \mathbb{R}^+$$

Quotient of Roots:

$$\frac{\sqrt[a]{x}}{\sqrt[a]{y}} = \sqrt[a]{\frac{x}{y}}; a, y \neq 0$$

Radical Identity Property:

$$\sqrt[a]{x^a} = x; a \neq 0$$

Logarithms

Logarithmic Identity Properties:

$$\log_x 1 = 0$$

$$\log_x x = 1$$

Product Property:

$$\log_a(x \cdot y) = \log_a x + \log_a y$$

Quotient Property:

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

Power Property:

$$\log_a x^y = y \cdot \log_a x$$

Base Change Property:

$$\log_a x = \frac{\log_n x}{\log_n a}$$

Summation

Constant Factorization:

$$\sum_{k=1}^n c \cdot a_k = c \cdot \sum_{k=1}^n a_k$$

Sum or Difference of Sequences:

$$\sum_{k=1}^n (a_k \pm b_k) = \sum_{k=1}^n a_k \pm \sum_{k=1}^n b_k$$

Summation of a Constant:

$$\sum_{k=1}^n c = n \cdot c$$

Index Shift:

$$\sum_{k=1}^n a_k = \sum_{k=1+p}^{n+p} a_{k-p}$$

Products

Associative Property:

$$\prod_{k=1}^n (a_k \cdot b_k) = \left(\prod_{k=1}^n a_k \right) \cdot \left(\prod_{k=1}^n b_k \right)$$

Commutative Property:

$$\left(\prod_{k=1}^n a_k \right)^x = \prod_{k=1}^n a_k^x$$

Limits

Sum of Limits

$$\lim_{x \rightarrow c} (f + g)(x) = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x)$$

Difference of Limits:

$$\lim_{x \rightarrow c} (f - g)(x) = \lim_{x \rightarrow c} f(x) - \lim_{x \rightarrow c} g(x)$$

Product of Limits:

$$\lim_{x \rightarrow c} (f \cdot g)(x) = \lim_{x \rightarrow c} f(x) \cdot \lim_{x \rightarrow c} g(x)$$

Quotient of Limits:

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)}; \lim_{x \rightarrow c} g(x) \neq 0$$

Power of Limits:

$$\lim_{x \rightarrow c} f(x)^{\frac{n}{d}} = \left(\lim_{x \rightarrow c} f(x) \right)^{\frac{n}{d}}; \frac{n}{d} \in \mathbb{R}$$

Composition of Limits:

$$\lim_{x \rightarrow c} (f \circ g)(x) = f(\lim_{x \rightarrow c} g(x))$$

Constant Multiple Rule:

$$\lim_{x \rightarrow c} (k \cdot f(x)) = k \cdot \lim_{x \rightarrow c} f(x)$$

Constant Rule:

$$\lim_{x \rightarrow c} k = k$$

Identity Rule:

$$\lim_{x \rightarrow c} x = c$$

Derivatives

Power Rule:

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

Functional Power Rule:

$$\frac{d}{dx} (f(x)^{g(x)}) = f(x)^{g(x)} \left(\frac{g(x) \cdot \frac{df}{dx}}{f(x)} + \frac{dg}{dx} \cdot \ln f(x) \right)$$

Sum Rule:

$$\frac{d}{dx} (f + g)(x) = \frac{df}{dx} + \frac{dg}{dx}$$

Product Rule:

$$\frac{d}{dx} (f \cdot g)(x) = f(x) \cdot \frac{dg}{dx} + g(x) \cdot \frac{df}{dx}$$

General Leibniz Rule:

$$\frac{d^n}{dx^n} (f \cdot g)(x) = \sum_{k=0}^n \binom{n}{k} \frac{d^{n-k} f}{dx^{n-k}} \cdot \frac{d^k g}{dx^k}$$

Quotient Rule:

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \cdot \frac{df}{dx} - f(x) \frac{dg}{dx}}{g(x)^2}$$

Chain Rule:

$$\frac{d}{dx} (f \circ g)(x) = \frac{df}{dg} \cdot \frac{dg}{dx}$$

Exponential Derivatives:

$$\frac{d}{dx} n^x = n^x \ln(n); n > 0$$

Logarithmic Derivatives:

$$\frac{d}{dx} \log_b x = \frac{1}{x \ln b}$$

Definite Integrals

Sum of Integrals:

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

Identity Rule:

$$\int_a^a f(x)dx = 0$$

Inverse Rule:

$$\int_a^b f(x)dx = - \int_b^a f(x)dx$$

Indefinite Integrals

Inverse Power Rule:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$$

Sum Rule:

$$\int (f+g)(x)dx = \int f(x)dx + \int g(x)dx$$

Integration by Parts:

$$\int (f \cdot g)(x)dx = f(x) \cdot \int g(x)dx - \int g(x) \cdot \frac{df}{dx} dx$$

$$\int u dv = u \cdot v - \int v du$$

Integration by Substitution:

$$\int \frac{dg}{dx} \cdot (f \circ g)(x)dx = \int f(u)du; u = g(x)$$

Exponential Integrals:

$$\int n^x dx = \frac{n^x}{\ln(n)} + C$$

Logarithmic Integrals:

$$\int \log_b x dx = \frac{x}{\ln b} (\ln x - 1) + C$$