Biology

Hardy-Weinberg Equilibrium:
$p + q = 1$
$p^2 + 2pq + q^2 = 1$

Chemistry

Specific Heat:
$q = sm\Delta t$
Internal Energy:
$\Delta E = q + w$
Definition of Density:
$d = \frac{m}{V}$
Definition of Pressure:
$P = \frac{\vec{F}}{A}$
Boyle's Law:
$V_1P_1 = V_2P_2$
Charles's Law:
$\frac{V_1}{T_1} = \frac{V_2}{V_2}$
Gay-Lussac's Law:
$\frac{P_1}{T_1} = \frac{P_2}{T_2}$
Combined Gas Law:
$\frac{V_1P_1}{T_1} = \frac{V_2P_2}{T_2}$
Ideal Gas Law:
$PV = nRT = \frac{mRT}{MM}$
Dalton's Law:
$P_{total} = \sum P_i$
Definition of Molarity:
$M = \frac{n_{solute}}{V_{solution}}$
Definition of Molality:
$\bar{m} = \frac{n_{solute}}{m_{solvent}}$
Dilution:
$M_1V_1 = M_2V_2$
Neutralization:
$N_AV_A = N_BV_B$
Definition of pH and pOH:
$pH = -log_{10}H^+$
$pOH = -log_{10}OH^-$
Enthalpy of Formation:
$\Delta H^\circ = \sum nH_f^\circ - \sum mH_f^\circ$
Effective Nuclear Charge:
$Z_{eff} = Z - S$
Graham's Law of Effusion:
$\frac{r_1}{r_2} = \frac{\sqrt{M_2}}{\sqrt{M_1}}$
Rydberg Formula:
$E = \frac{hc}{\lambda}$
Plank's Equation:
$E = h\nu$
Wavelength-Frequency Equation:
$c = \nu\lambda$
Raoult's Law:
$P_{solution} = \chi_{solvent} \cdot P_{solvent}$
Boiling Point Elevation:
$\Delta T_{bp} = k_b\bar{m}i$
Freezing Point Depression:
$\Delta T_{fp} = k_f\bar{m}i$
Osmotic Pressure:
$\pi = iMRT$
Average Reaction Rate:
$\text{Rate} = \frac{-\Delta[R]}{\Delta t} = \frac{\Delta[P]}{\Delta t}$
Reaction Rate Law:
$\text{Rate} = k[A]^m[B]^n$
Integrated Rate Law of a Zero or First Order Reaction:
$\ln[A]_t = -kt + \ln[A]_0$
Integrated Rate Law of a Second or Higher Order Reaction:
$\frac{1}{[A]_t} = kt + \frac{1}{[A]_0}$
Activation Energy of a Reverse Reaction:
$E_{a\ (\text{Reverse})} = \Delta E_{(\text{Reverse})} + E_{a\ (\text{Forward})}$
Arrhenius Equation:
$k = Ae^{\frac{-E_a}{RT}}$
Half Life of a First Order Reaction:
$t_{\frac{1}{2}} = \frac{\ln 2}{k}$
Half Life of a Second or Higher Order

Reaction:
$t_{\frac{1}{2}} = \frac{1}{k[A]_0}$
Reaction Catalysis:
$\ln\left(\frac{k_1}{k_2}\right) = \frac{E_a}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$
Equilibrium Constant in Terms of Concentration:
$K_c = \frac{[\text{Products}]^{\text{Coefficient}}}{[\text{Reactants}]^{\text{Coefficient}}}$
Equilibrium Constant in Terms of Pressure:
$K_P = \frac{(P_{\text{products}})^{\text{Coefficient}}}{(P_{\text{reactants}})^{\text{Coefficient}}}$
The Relationship between the two Equilibrium Constants:
$K_P = K_c(RT)^{\Delta n}$

Physics

Mass-Energy Equivalence:
$E = mc^2$

Newtonian Mechanics

Newton's Second Law:
$\vec{F}_{\text{net}} = m_{\text{sys}}\vec{a} = \frac{d\vec{p}}{dt}$
Newton's Third Law:
$\vec{F}_{a\to b} = -\vec{F}_{b\to a}$
Definitions of Displacement, Velocity, and Acceleration:
$\Delta x = x_f - x_i$
$\vec{v} = \frac{\Delta x}{\Delta t}$
$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$
Displacement with Constant Acceleration:
$x_f = x_i + \vec{v}_x\Delta t + \frac{\vec{a}_x(\Delta t)^2}{2}$
Velocity with Constant Acceleration:
$\vec{v}_{xf} = \vec{v}_{xi} + \vec{a}_x\Delta t$
Velocity-Displacement Relation with Constant Acceleration:
$\vec{v}_{xf}^2 = \vec{v}_{xi}^2 + 2\vec{a}_x\Delta x$
Vector Equations:
$\vec{A}_x = \vec{A}\cos\theta$
$\vec{A}_y = \vec{A}\sin\theta$
$\vec{A} = \sqrt{\vec{A}_x^2 + \vec{A}_y^2}$
$\theta = \arctan\frac{\vec{A}_y}{\vec{A}_x}$
Center of Mass:
$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$
Definition of Weight:
$\vec{F}_w = m(g + \vec{a}_y) = -\vec{F}_n$
Maximum Static Friction:
$\vec{F}_{\text{sf max}} = \mu\vec{F}_n$
Kinetic Friction:
$\vec{F}_{\text{kf}} = \mu_k\vec{F}_n$
Hooke's Law:
$\vec{F}_{\text{sp x}} = -k\Delta x$
Newton's Law of Gravitation:
$\vec{F}_g = \frac{Gm_1m_2}{r^2}$
Kepler's Third Law:
$t^2 = \frac{4\pi^2R^3}{MG}$
Time to Orbit:
$t = \frac{2\pi r}{\vec{v}}$
Minimum Velocity to Orbit:
$\vec{v}_{\text{min}} = \sqrt{gr}$
Circular Acceleration:
$\vec{a}_c = \frac{\vec{v}^2}{r}$
Work:
$w = \vec{F}d\cos\theta$
Translational Kinetic Energy:
$k = \frac{m\vec{v}^2}{2}$
Gravitational Potential Energy:
$U_g = mgy$
Elastic Potential Energy:
$U_s = \frac{k\Delta x^2}{2}$
Work-Energy Theorem:
$w = \Delta k$

Definition of Power:
$P = \frac{\Delta E}{\Delta t} = \frac{w}{\Delta t} = \vec{F}\vec{v}\cos\theta$
Definition of Impulse:
$\vec{J} = \vec{F}_{\text{avg}}\Delta t$
Definition of Momentum:
$\vec{p} = m\vec{v}$
Conservation of Momentum:
$\vec{p}_f - \vec{p}_i = 0$
Impulse-Momentum Theorem:
$\vec{J} = \Delta\vec{p} = m\Delta\vec{v} = \vec{F}\Delta t$
Orbital Velocity:
$\vec{v} = \sqrt{\frac{Gm}{r}}$
Orbital Gravitational Potential Energy:
$U_g = \frac{-Gm_1 m_2}{r}$
Escape Velocity:
$\vec{v}_{\text{esc}} = \sqrt{\frac{2GM}{r}}$
Period of a Pendulum:
$t_p = 2\pi\sqrt{\frac{l}{g}}$
Period of a Spring:
$t_s = 2\pi\sqrt{\frac{m}{k}}$

Rotational Mechanics

Definitions of Angular Displacement, Velocity, and Acceleration:
$\theta = \frac{s}{r}$
$\Delta\theta = \theta_f - \theta_i$
$\vec{\omega}_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$
$\vec{\alpha}_{\text{avg}} = \frac{\Delta\vec{\omega}}{\Delta t}$
Angular Velocity with Constant Acceleration:
$\vec{\omega}_f = \vec{\omega}_i + \vec{\alpha}\Delta t$
Angular Displacement with Constant Acceleration:
$\theta_f = \theta_i + \vec{\omega}_i\Delta t + \frac{\vec{\alpha}\Delta t^2}{2}$
Angular Velocity-Displacement Relation with Constant Acceleration:
$\vec{\omega}_f^2 = \vec{\omega}_i^2 + 2\vec{\alpha}\Delta\theta$
Angular to Linear Motion:
$\Delta x = r\Delta\theta$
$\vec{v} = r\vec{\omega}$
$\vec{a}_T = r\vec{\alpha}$
Torque:
$\vec{\tau} = r\vec{F}\sin\theta$
Archimedes's Law of Levers:
$\frac{\vec{F}_2}{\vec{F}_1} = \frac{D_1}{D_2}$
Moment of Inertia:
$I = Cmr^2$
$I_{\text{sys}} = \sum C_i m_i r_i^2$
Parallel Axis Theorem:
$I' = I_{\text{cm}} + mx^2$
Newton's Second Law for Rotational Motion:
$\vec{\alpha} = \frac{\vec{\tau}_{\text{net}}}{I_{\text{sys}}}$
Newton's Third Law for Rotational Motion:
$\Delta\vec{L}_{a\to b} = -\Delta\vec{L}_{b\to a}$
Rotational Kinetic Energy:
$k_{\text{rot}} = \frac{I\vec{\omega}^2}{2}$
Rotational Work:
$w = \vec{\tau}\Delta\theta$
Rotational Work-Energy Theorem:
$w = \Delta k_{\text{rot}}$
Angular Momentum:
$\vec{L} = I\vec{\omega}$
Orbital Angular Momentum:
$\vec{L} = rm\vec{v}\sin\theta$
Angular Impulse-Momentum Theorem:
$\Delta\vec{L} = I\Delta\vec{\omega} = \vec{\tau}\Delta t$
Conservation of Angular Momentum:
$\vec{L}_f - \vec{L}_i = 0$

Fluid Mechanics

Fluid Pressure:
$P = P_{\text{atm}} + dgy$
Buoyant Force:
$\vec{F}_b = d_{\text{fluid}} V_{\text{disp}} g$
Fluid Flow Rate:
$Q = \frac{V}{t}$
Bernoulli's Equation:
$P_1 + dgy_1 + \frac{d\vec{v}_1^2}{2} = P_2 + dgy_2 + \frac{d\vec{v}_2^2}{2}$
Torricelli's Theorem:
$\vec{v}_2 = \sqrt{2g\Delta y}$

Geometry

Definition of Pi:
$\pi = \frac{C}{d} = \frac{\tau}{2}$
Definition of Tau:
$\tau = \frac{C}{r} = 2\pi$

Two Dimensional

Pythagorean Theorem:
$a^2 + b^2 = c^2$
Angles of a Regular N-Gon:
$\theta_I = \frac{180°(n-2)}{n} = \frac{\pi(n-2)}{n} = \frac{\tau(n-2)}{2n}$
$\theta_E = \frac{360°}{n} = \frac{2\pi}{n} = \frac{\tau}{n}$
Euler's Formula:
$F + V = E + 2$
Area of a Triangle:
$A = \frac{bh}{2}$
Alternative Area of a Triangle:
$A = \frac{bc \sin A}{2}$
Heron's Formula:
$s = \frac{a+b+c}{2}$
$A = \sqrt{s(s-a)(s-b)(s-c)}$
Area of a Parallelogram:
$A = bh$
Area of a Square:
$A = s^2$
Area of a Trapezoid:
$A = \frac{h(b_1 b_2)}{2}$
Area of a Rhombus:
$A = bh$
Alternative Area of a Rhombus:
$A = \frac{d_1 d_2}{2}$
Area of a Kite:
$A = \frac{d_1 d_2}{2}$
Area of a Regular N-Gon:
$A = \frac{ap}{2}$
Area of a Circle:
$A = \pi r^2 = \frac{\tau r^2}{2}$

Three Dimensional

Lateral and Surface Area of a Prism:
$A_L = ph$
$A_S = A_L + 2b$
Lateral and Surface Area of a Cylinder:
$A_L = 2\pi rh = \tau rh$
$A_S = A_L + 2\pi r^2 = 2\pi r(r+h) = \tau r(r+h)$
Lateral and Surface Area of a Pyramid:
$A_L = \frac{pl}{2}$
$A_S = A_L + A_b$
Lateral and Surface Area of a Cone:
$A_L = \pi rl = \frac{\tau rl}{2}$
$A_S = A_L + \pi r^2 = \pi r(l + r)$
Surface Area of a Sphere:
$A_S = 4\pi r^2 = 2\tau r^2$
Volume of a Prism:
$V = bh$
Volume of a Cylinder:
$V = \pi r^2 h = \frac{\tau r^2 h}{2}$
Volume of a Pyramid:
$V = \frac{bh}{3}$

Volume of a Cone:
$V = \frac{\pi r^2 h}{3} = \frac{\tau r^2 h}{6}$
Volume of a Sphere:
$V = \frac{4\pi r^3}{3} = \frac{2\tau r^3}{3}$

Trigonometry

Definition of Sine:
$\sin A = \frac{a}{c}$
Definition of Cosine:
$\cos A = \frac{b}{c}$
Definition of Tangent:
$\tan A = \frac{a}{b}$
Definition of Cosecant:
$\csc A = \frac{c}{a}$
Definition of Secant:
$\sec A = \frac{c}{b}$
Definition of Cotangent:
$\cot A = \frac{b}{a}$
Law of Sines:
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Law of Cosines:
$c^2 = a^2 + b^2 - 2ab\cos C$
Law of Tangents:
$\frac{\tan\frac{A-B}{2}}{\tan\frac{A+B}{2}} = \frac{a-b}{a+b}$
Law of Cotangents:
$s = \frac{a+b+c}{2}$
$\frac{\cot\frac{A}{2}}{s-a} = \frac{\cot\frac{B}{2}}{s-b} = \frac{\cot\frac{C}{2}}{s-c}$
Pythagorean Identities:
$\sin^2 x + \cos^2 x = 1$
$\tan^2 x + 1 = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$
Sum/Difference Identities:
$\sin(x \pm y) = \sin(x)\cdot\cos(y) \pm \cos(x)\cdot\sin(y)$
$\cos(x \pm y) = \cos(x)\cdot\cos(y) \mp \sin(x)\cdot\sin(y)$
$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\cdot\tan(y)}$
$\csc(x \pm y) = \frac{\sec(x)\sec(y)\csc(x)\csc(y)}{\sec(x)\csc(y) \pm \csc(x)\sec(y)}$
$\sec(x \pm y) = \frac{\sec(x)\sec(y)\csc(x)\csc(y)}{\csc(x)\csc(y) \mp \sec(x)\sec(y)}$
$\cot(x \pm y) = \frac{\cot(x)\cot(y) \mp 1}{\cot(y) \pm \cot(x)}$
Double Angle Identities:
$\sin(2x) = 2\sin(x)\cdot\cos(x)$
$\cos(2x) = \cos^2(x) - \sin^2(x)$
$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$
$\csc(2x) = \frac{\sec(x)\cdot\csc(x)}{2}$
$\sec(2x) = \frac{\sec^2(x)}{2 - \sec^2(x)}$
$\cot(2x) = \frac{\cot^2(x) - 1}{2\cot(x)}$
Half Angle Identities:
$\sin\left(\frac{x}{2}\right) = \operatorname{sgn}\left(\sin\left(\frac{x}{2}\right)\right)\sqrt{\frac{1-\cos(x)}{2}}$
$\cos\left(\frac{x}{2}\right) = \operatorname{sgn}\left(\cos\left(\frac{x}{2}\right)\right)\sqrt{\frac{1+\cos(x)}{2}}$
$\tan\left(\frac{x}{2}\right) = \frac{\sin(x)}{1+\cos(x)}$
$\csc\left(\frac{x}{2}\right) = \operatorname{sgn}\left(\sin\left(\frac{x}{2}\right)\right)\sqrt{\frac{2}{1-\cos(x)}}$
$\sec\left(\frac{x}{2}\right) = \operatorname{sgn}\left(\cos\left(\frac{x}{2}\right)\right)\sqrt{\frac{2}{1+\cos(x)}}$
$\cot\left(\frac{x}{2}\right) = \frac{1+\cos(x)}{\sin(x)}$
Product to Sum Identities:
$\sin(x)\cdot\sin(y) = \frac{\cos(x-y) - \cos(x+y)}{2}$
$\cos(x)\cdot\cos(y) = \frac{\cos(x-y) + \cos(x+y)}{2}$
$\sin(x)\cdot\cos(y) = \frac{\sin(x-y) - \sin(x+y)}{2}$
$\tan(x)\cdot\tan(y) = \frac{\cos(x-y) - \cos(x+y)}{\cos(x-y) + \cos(x+y)}$
$\tan(x)\cdot\cot(y) = \frac{\cos(x-y)+\cos(x+y)}{\cos(x-y)-\cos(x+y)}$
Sum to Product Identities:

$\sin(x) \pm \sin(y) = 2\sin\left(\frac{x \pm y}{2}\right)\cos\left(\frac{x \mp y}{2}\right)$

$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

$\tan(x) \pm \tan(y) = \frac{\sin(x \pm y)}{\cos(x)\cdot\cos(y)}$

Polar Coordinate Equations:

$x^2 + y^2 = r^2$

$\tan\theta = \frac{y}{x}$

$x = r\cos\theta$

$y = r\sin\theta$

Polar Form of Conic Sections:

$r = \frac{de}{1 + e\sin\theta}$

$r = \frac{de}{1 + e\cos\theta}$

Hyperbolic Trig

Definition of Hyperbolic Sine:

$\sinh x = \frac{e^x - e^{-x}}{2} = -i\sin ix$

Definition of Hyperbolic Cosine:

$\cosh x = \frac{e^x + e^{-x}}{2} = \cos ix$

Definition of Hyperbolic Tangent:

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = -i\tan ix$

Definition of Hyperbolic Cosecant:

$\operatorname{csch} x = \frac{2}{e^x - e^{-x}} = i\csc ix$

Definition of Hyperbolic Secant:

$\operatorname{sech} x = \frac{2}{e^x + e^{-x}} = \sec ix$

Definition of Hyperbolic Cotangent:

$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}} = i\cot ix$

Hyperbolic Pythagorean Identities:

$\sinh^2 x - \cosh^2 x = 1$

$1 - \tanh^2 x = \operatorname{sech}^2 x$

$\coth^2 x - 1 = \operatorname{csch}^2 x$

Hyperbolic Sum/Difference Identities:

$\sinh(x \pm y) = \sinh(x)\cdot\cosh(y) \pm \cosh(x)\cdot\sinh(y)$

$\cosh(x \pm y) = \cosh(x)\cdot\cosh(y) \pm \sinh(x)\sinh(y)$

$\tanh(x \pm y) = \frac{\tanh(x) \pm \tanh(y)}{1 \pm \tanh(x)\cdot\tanh(y)}$

Hyperbolic Double Angle Identities:

$\sinh(2x) = 2\sinh(x)\cdot\cosh(x)$

$\cosh(2x) = \sinh^2(x) + \cosh^2(x)$

$\tanh(2x) = \frac{2\tanh(x)}{1 + \tanh^2(x)}$

Hyperbolic Half Angle Identities:

$\sinh\left(\frac{x}{2}\right) = \operatorname{sgn}(x)\sqrt{\frac{\cosh(x) - 1}{2}}$

$\cosh\left(\frac{x}{2}\right) = \sqrt{\frac{\cosh(x) + 1}{2}}$

$\tanh\left(\frac{x}{2}\right) = \frac{\sinh(x)}{\cosh(x) + 1}$

Hyperbolic Sum to Product Identities:

$\sinh(x) + \sinh(y) = 2\sinh\left(\frac{x+y}{2}\right)\cosh\left(\frac{x-y}{2}\right)$

$\sinh(x) - \sinh(y) = 2\cosh\left(\frac{x+y}{2}\right)\sinh\left(\frac{x-y}{2}\right)$

$\cosh(x) + \cosh(y) = 2\cosh\left(\frac{x+y}{2}\right)\cosh\left(\frac{x-y}{2}\right)$

$\cosh(x) - \cosh(y) = -2\sinh\left(\frac{x+y}{2}\right)\sinh\left(\frac{x-y}{2}\right)$

Algebra

Fundamental Theorem of Algebra:

$\forall f : [f(x) = ax^n + bx^{n-1} + ... + zx + c]$

$\exists[\chi_i \in \mathbb{C} : f(\chi_i) = 0]; i \in \mathbb{Z}_{1 \le i \le n}$

Definition of the Imaginary Unit:

$i^2 = -1$

Formulae

Distance Formula:

$D = \sqrt{x^2 + y^2}$

Slope Formula:

$m = \frac{\Delta x}{\Delta y} = \frac{x_2 - x_1}{y_2 - y_1}$

Discriminant of a Quadratic Equation:

$\Delta = b^2 - 4ac$

Quadratic Formula:
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Cubic Formula:
$\xi = \frac{-1 + \sqrt{-3}}{2}$
$\Delta_0 = b^2 - 3ac$
$\Delta_1 = 2b^2 - 9abc + 27a^2d$
$C = \sqrt[3]{\frac{\Delta_1 \pm \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}$
$x_k = \frac{-1}{3a}\left(b + \zeta^k C + \frac{\Delta_0}{\zeta^k C}\right); k \in \{0, 1, 2\}$
Binomial Formula:
$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

Functions

Slope-Intercept Form of a Line:
$y = mx + b$
Point-Slope Form of a Line:
$y - y_1 = m(x - x_1)$
Standard Form of a Line:
$Ax + By = C$
Exponential Function:
$y = a \cdot b^x$
Vertex Form of a Quadratic:
$y = a(x-h)^2 + k$
Factored Form of a Quadratic:
$y = (x - r_1)(x - r_2)$
Standard Form of a Quadratic:
$y = ax^2 + bx + c$
Standard Form of a Polynomial:
$ax^n + bx^{n-1} + cx^{n-2} ... + vx^2 + wx + z = 0$
Standard Form of an Absolute Value Function:
$y = a|x-h| + k$
Standard Form of a Square Root Function:
$y = a\sqrt{x-h} + k$

Conic Sections

Equation of a Circle:
$x^2 + y^2 = r^2$
Equation of a Horizontal Ellipse:
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
Equation of a Vertical Ellipse:
$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
Equation of a Horizontal Hyperbola:
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
Equation of a Vertical Hyperbola:
$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Equation of a Horizontal Parabola:
$x = \frac{1}{4p}(y-k)^2 + h$
Equation of a Vertical Parabola:
$y = \frac{1}{4p}(x-h)^2 + k$
Standard Form of a Conic Section:
$ax^2 + bxy + cy^2 + dx + ey + f = 0$

Sequences and Series

Recursive Arithmetic Sequence:
$a_1 = x$
$a_n = a_{n-1} + d$
Explicit Arithmetic Sequence:
$a_n = d(n-1) + a_1$
Recursive Geometric Sequence:
$a_1 = x$
$a_n = a_{n-1} \cdot r$
Explicit Geometric Sequence:
$a_n = a_1(r)^{n-1}$
Sum of an Arithmetic Series when the Last Term is Given:
$S_n = \frac{n(a_1 + a_n)}{2}$
Sum of an Arithmetic Series when the Last Term is Not Given:
$S_n = \frac{n(2a_1 + d(n-1))}{2}$
Sum of a Geometric Series:

$S_n = \dfrac{a_1(1-r^n)}{1-r}$

Sum of an Infinite Geometric Series:

$S = \dfrac{a_1}{1-r}; 0 < |x| < 1$

Methods

Difference of Squares:

$a^2 - b^2 = (a+b)(a-b)$

Sum of Squares:

$a^2 + b^2 = (a+bi)(a-bi)$

Difference of Cubes:

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

Sum of Cubes:

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

Various Summation of Single Terms:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=1}^{n} k^4 = \frac{n(2n+1)(n+1)(3n^2+3n-1)}{30}$$

$$\sum_{k=1}^{n} k^5 = \frac{n^2(2n^2+2n-1)(n+1)^2}{12}$$

$$\sum_{k=1}^{n} k^6 = \frac{n(2n+1)(n+1)(3n^4+6n^3-3n+1)}{42}$$

$$\sum_{k=1}^{n} k^7 = \frac{n^2(3n^4+6n^3-n^2-4n+2)(n+1)^2}{24}$$

Statistics

Calculation of the Mean Average:

$\bar{x} = \dfrac{\sum x_i}{n}$

Calculation of Sample Standard Distribution:

$S_x = \sqrt{\frac{\sum (x_i - \bar{x})}{n-1}}$

Probability

Probability Laws:

$P(\neg A) = 1 - P(A)$

$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$

$P(A \wedge B) = P(A) \cdot P(B|A)$

$P(A|B) = \dfrac{P(A \wedge B)}{P(B)}$

Bayes's Theorem:

$P(A|B) = \dfrac{P(B|A) \cdot P(A)}{P(B)}$

Combinatorics

Number of Permutations of a Given System:

${}_nP_r = \dfrac{n!}{(n-r)!}$

Number of Combinations of a Given System:

${}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-1)!}$

Permutations of Repeated Objects:

${}_nP_n = \dfrac{n!}{a! \cdot b! \cdot c! \ldots}$

Circular Permutations:

${}_nP_n = (n-1)!$

Circular Permutations with Repeated Objects:

${}_nP_n = \dfrac{(n-1)!}{a! \cdot b! \cdot c! \ldots}$

Circular Permutations of Flipable Circles:

${}_nP_n = \dfrac{(n-1)!}{2}$

Calculus

Fundamental Theorem of Calculus, Part One:

$$f(x) = \int_0^{h(x)} g(t)dt$$
$$\implies \frac{d}{dx}f(x) = (g \circ h)(x) \cdot \frac{d}{dx}h(x)$$

Fundamental Theorem of Calculus, Part Two:
$$\int_a^b \frac{d}{dx}f(x)dx = f(b) - f(a)$$

Limit

L'ôpital's Rule:
$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}$$
$$[|f(c)| = |g(c)| = \infty] \vee [|f(c)| = |g(c)| = 0]$$

Squeeze Theorem:
$$\forall f(x) : [g(x) \leq f(x) \leq h(x) \forall x \in [b, c]]$$
$$\exists [\lim_{x \to a} f(x)] = \lim_{x \to a} g(x) = \lim_{x \to a} h(x)$$
$$\iff \exists [\lim_{x \to a} g(x) = \lim_{x \to a} h(x)] \wedge [a \in [b, c]]$$

Common Limits:
$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$
$$\lim_{x \to \infty} \left(1 + \frac{x}{n}\right)^n = e^x$$
$$\lim_{x \to \infty} \frac{\ln x}{x} = 0$$

Common Trigonometric Limits:
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$
$$\lim_{x \to \infty} \frac{\sin x}{x} = 0$$
$$\lim_{x \to 0} \frac{-1 + \cos x}{x} = 0$$
$$\lim_{x \to \infty} \frac{-1 + \cos x}{x} = 0$$
$$\lim_{x \to \infty} \frac{1 + \cos x}{x} = 0$$
$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

Differential

Definition of the Derivative:
$$\frac{d}{dx}f(x) = \lim_{h \to 0} \frac{f(x + h) - f(h)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Mean Value Theorem for Derivatives:
$$\forall f \exists c \in [a, b] : \frac{f(b) - f(a)}{b - a} = \frac{d}{dx}f(c)$$

Definition of the Taylor Series:
$$f_a(x) = \sum_{n=0}^{\infty} \frac{\frac{d}{dx}f(a)}{n!}(x - a)^n$$

Newton's Method:
$$x_1 = a$$
$$x_n = \frac{-f(x_{n-1})}{\frac{d}{dx}f(x_{n-1})} + x_{n-1}$$

Trigonometric Derivatives:
$$\frac{d}{dx}\sin x = \cos x$$
$$\frac{d}{dx}\cos x = -\sin x$$
$$\frac{d}{dx}\tan x = \sec^2 x$$
$$\frac{d}{dx}\csc x = -\csc x \cdot \cot x$$
$$\frac{d}{dx}\sec x = \sec x \cdot \tan x$$
$$\frac{d}{dx}\cot x = -\csc^2 x$$

Hyperbolic Trigonometric Derivatives:
$$\frac{d}{dx}\sinh x = \cosh x$$
$$\frac{d}{dx}\cosh x = \sinh x$$
$$\frac{d}{dx}\tanh x = \text{sech}^2 x$$
$$\frac{d}{dx}\text{csch } x = -\text{csch } x \cdot \coth x$$
$$\frac{d}{dx}\text{sech } x = -\text{sech } x \cdot \tanh x$$
$$\frac{d}{dx}\coth x = -\text{csch}^2 x$$

Inverse Trigonometric Derivatives:
$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx}\arctan x = \frac{1}{x^2 + 1}$$
$$\frac{d}{dx}\text{arccsc } x = \frac{1}{|x|\sqrt{x^2 - 1}}$$
$$\frac{d}{dx}\text{arcsec } x = \frac{-1}{|x|\sqrt{x^2 - 1}}$$

$\frac{d}{dx}\text{arccot }x = \frac{-1}{x^2+1}$

Inverse Hyperbolic Trigonometric Derivatives:

$\frac{d}{dx}\text{arsinh }x = \frac{1}{\sqrt{x^2+1}}$

$\frac{d}{dx}\text{arcosh }x = \frac{1}{\sqrt{x^2+1}}; x > 1$

$\frac{d}{dx}\text{arcoth }x = \frac{1}{1-x^2}; |x| < 1$

$\frac{d}{dx}\text{arcsch }x = \frac{-1}{|x|\sqrt{1+x^2}}; x \neq 0$

$\frac{d}{dx}\text{arsech }x = \frac{-1}{x\sqrt{1+x^2}}; 0 < x < 1$

$\frac{d}{dx}\text{arcoth }x = \frac{1}{1-x^2}; |x| > 1$

Integral

Definition of the Riemann Integral:

$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x; \Delta x = \frac{b-a}{n}$

Mean Value Theorem for Integrals:

$\forall f \exists c \in [a,b] : \frac{\int_a^b f(x)dx}{b-a} = f(c)$

Trigonometric Integrals:

$\int \sin x dx = -\cos x + C$

$\int \cos x dx = \sin x + C$

$\int \tan x dx = \ln|\sec x| + C$

$\int \csc x dx = \ln|\csc x - \cot x| + C$

$\int \sec x dx = \ln|\sec x + \tan x| + C$

$\int \cot x dx = -\ln|\csc x| + C$

Hyperbolic Trigonometric Integrals:

$\int \sinh x dx = \cosh x + C$

$\int \cosh x dx = \sinh x + C$

$\int \tanh x dx = \ln\cosh x + C$

$\int \text{csch }x dx = \ln\left|\tanh\frac{x}{2}\right| + C$

$\int \text{sech }x dx = \arctan\sinh x + C$

$\int \coth x dx = \ln|\sinh x| + C$

Inverse Trigonometric Integrals:

$\int \arcsin x dx = x\arcsin x + \sqrt{1-x^2} + C; |x| \leq 1$

$\int \arccos x dx = x\arccos x - \sqrt{1-x^2} + C; |x| \leq 1$

$\int \arctan x dx = x\arctan x - \frac{\ln|1+x^2|}{2} + C$

$\int \text{arccsc }x dx = x \text{ arccsc }x + \ln\left|x\left(1+\sqrt{1-x^{-2}}\right)\right| + C; |x| \geq 1$

$\int \text{arcsec }x dx = x \text{ arcsec }x - \ln\left|x\left(1+\sqrt{1-x^{-2}}\right)\right| + C; |x| \geq 1$

$\int \text{arccot }x dx = x \text{ arccot }x + \frac{\ln|1+x^2|}{2} + C$

Implementations

Arc Length:

$L_{f(x)}\big|_a^b = \int_a^b \sqrt{1+\left(\frac{d}{dx}f(x)\right)^2}dx$

Surface Area of a Rotational Solid:

$S_{f(x)}\big|_a^b = 2\pi\int_a^b r \cdot \sqrt{1+\left(\frac{d}{dx}f(x)\right)^2}dx$

Volume of a Rotational Solid:

$V_{f(x)}\big|_a^b = 2\pi\int_a^b f(x)^2 dx$

Definition of Euler's Number:

$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

Exponential McLauren Series:

$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$

Euler's Formula:

$e^{i\theta} = \cos\theta + i\sin\theta$

Euler's Identity:
$e^{i\pi} + 1 = 0$
$e^{i\tau} = 1$
Power Series of Trigonometric Functions:

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot x^{2n+1}$$
$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot x^{2n}$$
$$\tan x = \sum_{n=0}^{\infty} \frac{U_{2n+1}}{(2n+1)!} \cdot x^{2n+1}; |x| < \frac{\tau}{4}$$
$$\csc x = \sum_{n=0}^{\infty} \frac{(-1)^n 2(2^{2n-1}-1)B_{2n}}{(2n+1)!} \cdot x^{2n}; 0 < x < \frac{\tau}{2}$$
$$\sec x = \sum_{n=0}^{\infty} \frac{U_{2n}}{(2n)!} \cdot x^{2n}; |x| < \frac{\tau}{4}$$
$$\cot x = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n-1} B_{2n}}{(2n+1)!} \cdot x^{2n+1}; 0 < x < \frac{\tau}{2}$$

Analysis

Definition of the Gamma Function:
$$\Gamma(z) = (z-1)! = \int_0^{\infty} x^{z-1} e^{-x} dx$$
Definition of the Riemann Zeta Function:
$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$
Definition of the Fourier Transform:
$$\hat{g}(f) = \int_{-\infty}^{\infty} g(t) e^{-\tau i f t} dt$$
Definition of the Laplace Transform:
$$\mathcal{L}\{f\}(s) = \int_0^{\infty} f(t) e^{-st} dt$$

Number Theory

Fermat's Last Theorem:
$\neg[\forall n \in \mathbb{Z} : [n \geq 3] \exists [a, b, c \in \mathbb{Z}] : [a^n + b^n = c^n]]$
Fermat's Little Theorem:
$\forall [a \in \mathbb{Z}] \wedge [n \in \mathbb{P}] [(a^n - a) \bmod n = 0]$
Wilson's Theorem:
$[((n-1)! - 1) \bmod n = 0] \iff [n \in \mathbb{P}]$
The Fibonacci Sequence:
$F_n = F_{n-1} + F_{n-2}; F_0 = 0, F_1 = 1$
Definition of the Golden Ratio:
$$\phi = \frac{1+\sqrt{5}}{2} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}$$
Pythagorean Triples:
$$\left.\begin{smallmatrix} a=m^2-n^2 \\ b=2mn \\ c=m^2+n^2 \end{smallmatrix}\right\} (m > n) \wedge (m, n \in \mathbb{Z})$$

Economics

Simple Interest:
$A = P(1 + rt)$
Compound Interest:
$A = P(1 + \frac{1}{n})^{nt}$
Continual Compound Interest:
$A = Pe^{rt}$
Future Value of an Ordinary Annuity:
$$A = \frac{Pn\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{r}$$

Electrical Engineering

Ohm's Law:
$V = IR$

Cryptology

Caesar Cipher:
$y = (x + n) \bmod 26$
Affine Cipher:
$y = (ax + b) \bmod 26$
Password Entropy:
$E = L \cdot \log_2 R$

Medical Science

Glaister Equation:
$\Delta t_{death} \approx \frac{T_{baseline} - T_{rectal}}{R_{cooling}}$