

# Biology

**Hardy-Weinberg Equilibrium:**

$$p + q = 1$$

$$p^2 + 2pq + q^2 = 1$$

# Chemistry

**Specific Heat:**

$$q = sm\Delta t$$

**Internal Energy:**

$$\Delta E = q + w$$

**Definition of Density:**

$$d = \frac{m}{V}$$

**Definition of Pressure:**

$$P = \frac{\vec{F}}{A}$$

**Boyle's Law:**

$$V_1 P_1 = V_2 P_2$$

**Charles's Law:**

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

**Gay-Lussac's Law:**

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

**Combined Gas Law:**

$$\frac{V_1 P_1}{T_1} = \frac{V_2 P_2}{T_2}$$

**Ideal Gas Law:**

$$PV = nRT = \frac{mRT}{MM}$$

**Dalton's Law:**

$$P_{total} = \sum P_i$$

**Definition of Molarity:**

$$M = \frac{n_{solute}}{V_{solution}}$$

**Definition of Molality:**

$$\bar{m} = \frac{n_{solute}}{m_{solvent}}$$

**Dilution:**

$$M_1 V_1 = M_2 V_2$$

**Neutralization:**

$$N_A V_A = N_B V_B$$

**Definition of pH and pOH:**

$$pH = -\log_{10} H^+$$

$$pOH = -\log_{10} OH^-$$

**Enthalpy of Formation:**

$$\Delta H^\circ = \sum n H_f^\circ - \sum m H_f^\circ$$

**Effective Nuclear Charge:**

$$Z_{eff} = Z - S$$

**Graham's Law of Effusion:**

$$\frac{r_1}{r_2} = \frac{\sqrt{M_2}}{\sqrt{M_1}}$$

**Rydberg Formula:**

$$E = \frac{hc}{\lambda}$$

**Plank's Equation:**

$$E = h\nu$$

**Wavelength-Frequency Equation:**

$$c = \nu\lambda$$

**Raoult's Law:**

$$P_{solution} = \chi_{solvent} \cdot P_{solvent}$$

**Boiling Point Elevation:**

$$\Delta T_{bp} = k_b \bar{m} i$$

**Freezing Point Depression:**

$$\Delta T_{fp} = k_f \bar{m} i$$

**Osmotic Pressure:**

$$\pi = iMRT$$

**Average Reaction Rate:**

$$\text{Rate} = \frac{-\Delta[R]}{\Delta t} = \frac{\Delta[P]}{\Delta t}$$

**Reaction Rate Law:**

$$\text{Rate} = k[A]^m[B]^n$$

**Integrated Rate Law of a Zero or First Order Reaction:**

$$\ln[A]_t = -kt + \ln[A]_0$$

**Integrated Rate Law of a Second or Higher Order Reaction:**

$$\frac{1}{[A]_t} = kt + \frac{1}{[A]_0}$$

**Activation Energy of a Reverse Reaction:**

$$E_a \text{ (Reverse)} = \Delta E \text{ (Reverse)} + E_a \text{ (Forward)}$$

**Arrhenius Equation:**

$$k = A e^{-\frac{E_a}{RT}}$$

**Half Life of a First Order Reaction:**

$$t_{\frac{1}{2}} = \frac{\ln 2}{k}$$

**Half Life of a Second or Higher Order Reaction:**

$$t_{\frac{1}{2}} = \frac{1}{k[A]_0}$$

**Reaction Catalysis:**

$$\ln \left( \frac{k_1}{k_2} \right) = \frac{E_a}{R} \left[ \frac{1}{T_2} - \frac{1}{T_1} \right]$$

**Equilibrium Constant in Terms of Concentration:**

$$K_c = \frac{[\text{Products}]^{\text{Coefficient}}}{[\text{Reactants}]^{\text{Coefficient}}}$$

**Equilibrium Constant in Terms of Pressure:**

$$K_P = \frac{(P_{\text{products}})^{\text{Coefficient}}}{(P_{\text{reactants}})^{\text{Coefficient}}}$$

**The Relationship between the two Equilibrium Constants:**

$$K_P = K_c(RT)^{\Delta n}$$

## Physics

**Mass-Energy Equivalence:**

$$E = mc^2$$

**Newtonian Mechanics**

**Newton's Second Law:**

$$\vec{F}_{\text{net}} = m_{\text{sys}} \vec{a} = \frac{d\vec{p}}{dt}$$

**Newton's Third Law:**

$$\vec{F}_{a \rightarrow b} = -\vec{F}_{b \rightarrow a}$$

**Definitions of Displacement, Velocity, and Acceleration:**

$$\Delta x = x_f - x_i$$

$$\vec{v} = \frac{\Delta x}{\Delta t}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

**Displacement with Constant Acceleration:**

$$x_f = x_i + \vec{v}_x \Delta t + \frac{\vec{a}_x (\Delta t)^2}{2}$$

**Velocity with Constant Acceleration:**

$$\vec{v}_{xf} = \vec{v}_{xi} + \vec{a}_x \Delta t$$

**Velocity-Displacement Relation with Constant Acceleration:**

$$\vec{v}_{xf}^2 = \vec{v}_{xi}^2 + 2\vec{a}_x \Delta x$$

**Vector Equations:**

$$\vec{A}_x = \vec{A} \cos \theta$$

$$\vec{A}_y = \vec{A} \sin \theta$$

$$\vec{A} = \sqrt{\vec{A}_x^2 + \vec{A}_y^2}$$

$$\theta = \arctan \frac{\vec{A}_y}{\vec{A}_x}$$

**Center of Mass:**

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

**Definition of Weight:**

$$\vec{F}_w = m(g + \vec{a}_y) = -\vec{F}_n$$

**Maximum Static Friction:**

$$\vec{F}_{sf \max} = \mu \vec{F}_n$$

**Kinetic Friction:**

$$\vec{F}_{kf} = \mu_k \vec{F}_n$$

**Hooke's Law:**

$$\vec{F}_{sp \ x} = -k \Delta x$$

**Newton's Law of Gravitation:**

$$\vec{F}_g = \frac{G m_1 m_2}{r^2}$$

**Kepler's Third Law:**

$$t^2 = \frac{4\pi^2 R^3}{MG}$$

**Time to Orbit:**

$$t = \frac{2\pi r}{\vec{v}}$$

**Minimum Velocity to Orbit:**

$$\vec{v}_{\min} = \sqrt{gr}$$

**Circular Acceleration:**

$$\vec{a}_c = \frac{\vec{v}^2}{r}$$

**Work:**

$$w = \vec{F} d \cos \theta$$

**Translational Kinetic Energy:**

$$k = \frac{mv^2}{2}$$

**Gravitational Potential Energy:**

$$U_g = mgy$$

**Elastic Potential Energy:**

$$U_s = \frac{k \Delta x^2}{2}$$

**Work-Energy Theorem:**

$$w = \Delta k$$

**Definition of Power:**

$$P = \frac{\Delta E}{\Delta t} = \frac{w}{\Delta t} = \vec{F} \vec{v} \cos \theta$$

**Definition of Impulse:**

$$\vec{J} = \vec{F}_{\text{avg}} \Delta t$$

**Definition of Momentum:**

$$\vec{p} = m\vec{v}$$

**Conservation of Momentum:**

$$\vec{p}_f - \vec{p}_i = 0$$

**Impulse-Momentum Theorem:**

$$\vec{J} = \Delta \vec{p} = m\Delta \vec{v} = \vec{F} \Delta t$$

**Orbital Velocity:**

$$\vec{v} = \sqrt{\frac{Gm}{r}}$$

**Orbital Gravitational Potential Energy:**

$$U_g = \frac{-Gm_1m_2}{r}$$

**Escape Velocity:**

$$\vec{v}_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

**Period of a Pendulum:**

$$t_p = 2\pi \sqrt{\frac{l}{g}}$$

**Period of a Spring:**

$$t_s = 2\pi \sqrt{\frac{m}{k}}$$

## Rotational Mechanics

**Definitions of Angular Displacement, Velocity, and Acceleration:**

$$\theta = \frac{s}{r}$$

$$\Delta\theta = \theta_f - \theta_i$$

$$\vec{\omega}_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$$

$$\vec{\alpha}_{\text{avg}} = \frac{\Delta\vec{\omega}}{\Delta t}$$

**Angular Velocity with Constant Acceleration:**

$$\vec{\omega}_f = \vec{\omega}_i + \vec{\alpha}\Delta t$$

**Angular Displacement with Constant Acceleration:**

$$\theta_f = \theta_i + \vec{\omega}_i \Delta t + \frac{\vec{\alpha}\Delta t^2}{2}$$

**Angular Velocity-Displacement Relation with Constant Acceleration:**

$$\vec{\omega}_f^2 = \vec{\omega}_i^2 + 2\vec{\alpha}\Delta\theta$$

**Angular to Linear Motion:**

$$\Delta x = r\Delta\theta$$

$$\vec{v} = r\vec{\omega}$$

$$\vec{a}_T = r\vec{\alpha}$$

**Torque:**

$$\vec{\tau} = r\vec{F} \sin\theta$$

**Archimedes's Law of Levers:**

$$\frac{\vec{F}_2}{\vec{F}_1} = \frac{D_1}{D_2}$$

**Moment of Inertia:**

$$I = Cmr^2$$

$$I_{\text{sys}} = \sum C_i m_i r_i^2$$

**Parallel Axis Theorem:**

$$I' = I_{\text{cm}} + mx^2$$

**Newton's Second Law for Rotational Motion:**

$$\vec{\alpha} = \frac{\vec{\tau}_{\text{net}}}{I_{\text{sys}}}$$

**Newton's Third Law for Rotational Motion:**

$$\Delta \vec{L}_{a \rightarrow b} = -\Delta \vec{L}_{b \rightarrow a}$$

**Rotational Kinetic Energy:**

$$k_{\text{rot}} = \frac{I\vec{\omega}^2}{2}$$

**Rotational Work:**

$$w = \vec{\tau} \Delta\theta$$

**Rotational Work-Energy Theorem:**

$$w = \Delta k_{\text{rot}}$$

**Angular Momentum:**

$$\vec{L} = I\vec{\omega}$$

**Orbital Angular Momentum:**

$$\vec{L} = rm\vec{v} \sin\theta$$

**Angular Impulse-Momentum Theorem:**

$$\Delta \vec{L} = I\Delta\vec{\omega} = \vec{\tau}\Delta t$$

**Conservation of Angular Momentum:**

$$\vec{L}_f - \vec{L}_i = 0$$

## Fluid Mechanics

**Fluid Pressure:**

$$P = P_{\text{atm}} + dgy$$

**Buoyant Force:**

$$\vec{F}_b = d_{\text{fluid}} V_{\text{disp}} g$$

**Fluid Flow Rate:**

$$Q = \frac{V}{t}$$

**Bernoulli's Equation:**

$$P_1 + dgy_1 + \frac{d\vec{v}_1^2}{2} = P_2 + dgy_2 + \frac{d\vec{v}_2^2}{2}$$

**Torricelli's Theorem:**

$$\vec{v}_2 = \sqrt{2g\Delta y}$$

## Geometry

**Definition of Pi:**

$$\pi = \frac{C}{d} = \frac{\tau}{2}$$

**Definition of Tau:**

$$\tau = \frac{C}{r} = 2\pi$$

### Two Dimensional

**Pythagorean Theorem:**

$$a^2 + b^2 = c^2$$

**Angles of a Regular N-Gon:**

$$\theta_I = \frac{180^\circ(n-2)}{n} = \frac{\pi(n-2)}{n} = \frac{\tau(n-2)}{2n}$$

$$\theta_E = \frac{360^\circ}{n} = \frac{2\pi}{n} = \frac{\tau}{n}$$

**Euler's Formula:**

$$F + V = E + 2$$

**Area of a Triangle:**

$$A = \frac{bh}{2}$$

**Alternative Area of a Triangle:**

$$A = \frac{bc \sin A}{2}$$

**Heron's Formula:**

$$s = \frac{a+b+c}{2}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

**Area of a Parallelogram:**

$$A = bh$$

**Area of a Square:**

$$A = s^2$$

**Area of a Trapezoid:**

$$A = \frac{h(b_1 b_2)}{2}$$

**Area of a Rhombus:**

$$A = bh$$

**Alternative Area of a Rhombus:**

$$A = \frac{d_1 d_2}{2}$$

**Area of a Kite:**

$$A = \frac{d_1 d_2}{2}$$

**Area of a Regular N-Gon:**

$$A = \frac{ap}{2}$$

**Area of a Circle:**

$$A = \pi r^2 = \frac{\tau r^2}{2}$$

### Three Dimensional

**Lateral and Surface Area of a Prism:**

$$A_L = ph$$

$$A_S = A_L + 2b$$

**Lateral and Surface Area of a Cylinder:**

$$A_L = 2\pi rh = \tau rh$$

$$A_S = A_L + 2\pi r^2 = 2\pi r(r+h) = \tau r(r+h)$$

**Lateral and Surface Area of a Pyramid:**

$$A_L = \frac{pl}{2}$$

$$A_S = A_L + A_b$$

**Lateral and Surface Area of a Cone:**

$$A_L = \pi rl = \frac{\tau rl}{2}$$

$$A_S = A_L + \pi r^2 = \pi r(l+r)$$

**Surface Area of a Sphere:**

$$A_S = 4\pi r^2 = 2\tau r^2$$

**Volume of a Prism:**

$$V = bh$$

**Volume of a Cylinder:**

$$V = \pi r^2 h = \frac{\tau r^2 h}{2}$$

**Volume of a Pyramid:**

$$V = \frac{bh}{3}$$

**Volume of a Cone:**

$$V = \frac{\pi r^2 h}{3} = \frac{\tau r^2 h}{6}$$

**Volume of a Sphere:**

$$V = \frac{4\pi r^3}{3} = \frac{2\tau r^3}{3}$$

## Trigonometry

**Definition of Sine:**

$$\sin A = \frac{a}{c}$$

**Definition of Cosine:**

$$\cos A = \frac{b}{c}$$

**Definition of Tangent:**

$$\tan A = \frac{a}{b}$$

**Definition of Cosecant:**

$$\csc A = \frac{c}{a}$$

**Definition of Secant:**

$$\sec A = \frac{c}{b}$$

**Definition of Cotangent:**

$$\cot A = \frac{b}{a}$$

**Law of Sines:**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

**Law of Cosines:**

$$c^2 = a^2 + b^2 - 2ab \cos C$$

**Law of Tangents:**

$$\frac{\tan \frac{A-B}{2}}{\tan \frac{A+B}{2}} = \frac{a-b}{a+b}$$

**Law of Cotangents:**

$$s = \frac{a+b+c}{2}$$

$$\frac{\cot \frac{A}{2}}{s-a} = \frac{\cot \frac{B}{2}}{s-b} = \frac{\cot \frac{C}{2}}{s-c}$$

**Pythagorean Identities:**

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

**Sum/Difference Identities:**

$$\sin(x \pm y) = \sin(x) \cdot \cos(y) \pm \cos(x) \cdot \sin(y)$$

$$\cos(x \pm y) = \cos(x) \cdot \cos(y) \mp \sin(x) \cdot \sin(y)$$

$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x) \cdot \tan(y)}$$

$$\csc(x \pm y) = \frac{\sec(x) \sec(y) \csc(x) \csc(y)}{\sec(x) \csc(y) \pm \csc(x) \sec(y)}$$

$$\sec(x \pm y) = \frac{\sec(x) \sec(y) \csc(x) \csc(y)}{\csc(x) \csc(y) \mp \sec(x) \sec(y)}$$

$$\cot(x \pm y) = \frac{\cot(x) \cot(y) \mp 1}{\cot(y) \pm \cot(x)}$$

**Double Angle Identities:**

$$\sin(2x) = 2 \sin(x) \cdot \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

$$\csc(2x) = \frac{\sec(x) \cdot \csc(x)}{2}$$

$$\sec(2x) = \frac{\sec^2(x)}{2 - \sec^2(x)}$$

$$\cot(2x) = \frac{\cot^2(x) - 1}{2 \cot(x)}$$

**Half Angle Identities:**

$$\sin\left(\frac{x}{2}\right) = \operatorname{sgn}\left(\sin\left(\frac{x}{2}\right)\right) \sqrt{\frac{1 - \cos(x)}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \operatorname{sgn}\left(\cos\left(\frac{x}{2}\right)\right) \sqrt{\frac{1 + \cos(x)}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin(x)}{1 + \cos(x)}$$

$$\csc\left(\frac{x}{2}\right) = \operatorname{sgn}\left(\sin\left(\frac{x}{2}\right)\right) \sqrt{\frac{2}{1 - \cos(x)}}$$

$$\sec\left(\frac{x}{2}\right) = \operatorname{sgn}\left(\cos\left(\frac{x}{2}\right)\right) \sqrt{\frac{2}{1 + \cos(x)}}$$

$$\cot\left(\frac{x}{2}\right) = \frac{1 + \cos(x)}{\sin(x)}$$

**Product to Sum Identities:**

$$\sin(x) \cdot \sin(y) = \frac{\cos(x-y) - \cos(x+y)}{2}$$

$$\cos(x) \cdot \cos(y) = \frac{\cos(x-y) + \cos(x+y)}{2}$$

$$\sin(x) \cdot \cos(y) = \frac{\sin(x-y) - \sin(x+y)}{2}$$

$$\tan(x) \cdot \tan(y) = \frac{\cos(x-y) - \cos(x+y)}{\cos(x-y) + \cos(x+y)}$$

$$\tan(x) \cdot \cot(y) = \frac{\cos(x-y) + \cos(x+y)}{\cos(x-y) - \cos(x+y)}$$

**Sum to Product Identities:**

$$\sin(x) \pm \sin(y) = 2 \sin\left(\frac{x \mp y}{2}\right) \cos\left(\frac{x \mp y}{2}\right)$$

$$\cos(x) + \cos(y) = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos(x) - \cos(y) = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

$$\tan(x) \pm \tan(y) = \frac{\sin(x \pm y)}{\cos(x) \cdot \cos(y)}$$

**Polar Coordinate Equations:**

$$x^2 + y^2 = r^2$$

$$\tan \theta = \frac{y}{x}$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

**Polar Form of Conic Sections:**

$$r = \frac{de}{1 + e \sin \theta}$$

$$r = \frac{de}{1 + e \cos \theta}$$

## Hyperbolic Trig

**Definition of Hyperbolic Sine:**

$$\sinh x = \frac{e^x - e^{-x}}{2} = -i \sin ix$$

**Definition of Hyperbolic Cosine:**

$$\cosh x = \frac{e^x + e^{-x}}{2} = \cos ix$$

**Definition of Hyperbolic Tangent:**

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = -i \tan ix$$

**Definition of Hyperbolic Cosecant:**

$$\operatorname{csch} x = \frac{2}{e^x - e^{-x}} = i \csc ix$$

**Definition of Hyperbolic Secant:**

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}} = \sec ix$$

**Definition of Hyperbolic Cotangent:**

$$\operatorname{coth} x = \frac{e^x + e^{-x}}{e^x - e^{-x}} = i \cot ix$$

**Hyperbolic Pythagorean Identities:**

$$\sinh^2 x - \cosh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\operatorname{coth}^2 x - 1 = \operatorname{csch}^2 x$$

**Hyperbolic Sum/Difference Identities:**

$$\sinh(x \pm y) = \sinh(x) \cdot \cosh(y) \pm \cosh(x) \cdot \sinh(y)$$

$$\cosh(x \pm y) = \cosh(x) \cdot \cosh(y) \pm \sinh(x) \cdot \sinh(y)$$

$$\tanh(x \pm y) = \frac{\tanh(x) \pm \tanh(y)}{1 \pm \tanh(x) \cdot \tanh(y)}$$

**Hyperbolic Double Angle Identities:**

$$\sinh(2x) = 2 \sinh(x) \cdot \cosh(x)$$

$$\cosh(2x) = \sinh^2(x) + \cosh^2(x)$$

$$\tanh(2x) = \frac{2 \tanh(x)}{1 + \tanh^2(x)}$$

**Hyperbolic Half Angle Identities:**

$$\sinh\left(\frac{x}{2}\right) = \operatorname{sgn}(x) \sqrt{\frac{\cosh(x) - 1}{2}}$$

$$\cosh\left(\frac{x}{2}\right) = \sqrt{\frac{\cosh(x) + 1}{2}}$$

$$\tanh\left(\frac{x}{2}\right) = \frac{\sinh(x)}{\cosh(x) + 1}$$

**Hyperbolic Sum to Product Identities:**

$$\sinh(x) + \sinh(y) = 2 \sinh\left(\frac{x+y}{2}\right) \cosh\left(\frac{x-y}{2}\right)$$

$$\sinh(x) - \sinh(y) = 2 \cosh\left(\frac{x+y}{2}\right) \sinh\left(\frac{x-y}{2}\right)$$

$$\cosh(x) + \cosh(y) = 2 \cosh\left(\frac{x+y}{2}\right) \cosh\left(\frac{x-y}{2}\right)$$

$$\cosh(x) - \cosh(y) = -2 \sinh\left(\frac{x+y}{2}\right) \sinh\left(\frac{x-y}{2}\right)$$

## Algebra

**Definition of the Imaginary Unit:**

$$i^2 = -1$$

**Formulae****Distance Formula:**

$$D = \sqrt{x^2 + y^2}$$

**Slope Formula:**

$$m = \frac{\Delta x}{\Delta y} = \frac{x_2 - x_1}{y_2 - y_1}$$

**Discriminant of a Quadratic Equation:**

$$\Delta = b^2 - 4ac$$

**Quadratic Formula:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Cubic Formula:**

$$\frac{-1 + \sqrt{-3}}{2}$$

$$\Delta_0 = b^2 - 3ac$$

$$\Delta_1 = 2b^2 - 9abc + 27a^2d$$

$$C = \sqrt[3]{\frac{\Delta_1 \pm \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}$$

$$x_k = \frac{-1}{3a} \left( b + \zeta^k C + \frac{\Delta_0}{\zeta^k C} \right); k \in \{0, 1, 2\}$$

**Binomial Formula:**

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

## Functions

**Slope-Intercept Form of a Line:**

$$y = mx + b$$

**Point-Slope Form of a Line:**

$$y - y_1 = m(x - x_1)$$

**Standard Form of a Line:**

$$Ax + By = C$$

**Exponential Function:**

$$y = a \cdot b^x$$

**Vertex Form of a Quadratic:**

$$y = a(x - h)^2 + k$$

**Factored Form of a Quadratic:**

$$y = (x - r_1)(x - r_2)$$

**Standard Form of a Quadratic:**

$$y = ax^2 + bx + c$$

**Standard Form of a Polynomial:**

$$ax^n + bx^{n-1} + cx^{n-2} \dots + vx^2 + wx + z = 0$$

**Standard Form of an Absolute Value Function:**

$$y = a|x - h| + k$$

**Standard Form of a Square Root Function:**

$$y = a\sqrt{x - h} + k$$

## Conic Sections

**Equation of a Circle:**

$$x^2 + y^2 = r^2$$

**Equation of a Horizontal Ellipse:**

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

**Equation of a Vertical Ellipse:**

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$$

**Equation of a Horizontal Hyperbola:**

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

**Equation of a Vertical Hyperbola:**

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

**Equation of a Horizontal Parabola:**

$$x = \frac{1}{4p}(y - k)^2 + h$$

**Equation of a Vertical Parabola:**

$$y = \frac{1}{4p}(x - h)^2 + k$$

**Standard Form of a Conic Section:**

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

## Sequences and Series

**Recursive Arithmetic Sequence:**

$$a_1 = x$$

$$a_n = a_{n-1} + d$$

**Explicit Arithmetic Sequence:**

$$a_n = d(n - 1) + a_1$$

**Recursive Geometric Sequence:**

$$a_1 = x$$

$$a_n = a_{n-1} \cdot r$$

**Explicit Geometric Sequence:**

$$a_n = a_1(r)^{n-1}$$

**Sum of an Arithmetic Series when the Last Term is Known:**

$$S_n = \frac{n(a_1 + a_n)}{2}$$

**Sum of an Arithmetic Series when the Last Term is Unknown:**

$$S_n = \frac{n(2a_1 + d(n - 1))}{2}$$

**Sum of a Geometric Series:**

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

**Sum of an Infinite Geometric Series:**

$$S = \frac{a_1}{1 - r}; 0 < |r| < 1$$

## Methods

**Difference of Squares:**

$$a^2 - b^2 = (a + b)(a - b)$$

**Sum of Squares:**

$$a^2 + b^2 = (a + bi)(a - bi)$$

**Difference of Cubes:**

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

**Sum of Cubes:**

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

## Various Summation of Single Terms:

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=1}^n k^4 = \frac{n(2n+1)(n+1)(3n^2+3n-1)}{30}$$

$$\sum_{k=1}^n k^5 = \frac{n^2(2n^2+2n-1)(n+1)^2}{12}$$

$$\sum_{k=1}^n k^6 = \frac{n(2n+1)(n+1)(3n^4+6n^3-3n+1)}{42}$$

$$\sum_{k=1}^n k^7 = \frac{n^2(3n^4+6n^3-n^2-4n+2)(n+1)^2}{24}$$

## Combinatorics

### Number of Permutations of a Given System:

$${}_nP_r = \frac{n!}{(n-r)!}$$

### Number of Combinations of a Given System:

$${}_nC_r = {}^n_r = \frac{n!}{r!(n-1)!}$$

### Permutations of Repeated Objects:

$${}_nP_n = \frac{n!}{a! \cdot b! \cdot c! \dots}$$

### Circular Permutations:

$${}_nP_n = (n-1)!$$

### Circular Permutations with Repeated Objects:

$${}_nP_n = \frac{(n-1)!}{a! \cdot b! \cdot c! \dots}$$

### Circular Permutations of Flipable Circles:

$${}_nP_n = \frac{(n-1)!}{2}$$

## Statistics

### Calculation of the Mean Average:

$$\bar{x} = \frac{\sum x_i}{n}$$

### Calculation of Sample Standard Distribution

$$S_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

## Probability

### Probability Laws:

$$P(\neg A) = 1 - P(A)$$

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

$$P(A \wedge B) = P(A) \cdot P(B|A)$$

$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

### Bayes's Theorem:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

## Calculus

### Fundamental Theorem of Calculus, Part One:

$$f(x) = \int_0^{h(x)} g(t)dt$$

$$\Rightarrow \frac{d}{dx} f(x) = (g \circ h)(x) \cdot \frac{d}{dx} h(x)$$

### Fundamental Theorem of Calculus, Part Two:

$$\int_a^b \frac{d}{dx} f(x) dx = f(b) - f(a)$$

## Limit

### L'ôpital's Rule:

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{\frac{d}{dx} f(x)}{\frac{d}{dx} g(x)}$$

$$[|f(c)| = |g(c)| = \infty] \vee [|f(c)| = |g(c)| = 0]$$

### Common Limits:

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x$$

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0$$

### Common Trigonometric Limits:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{-1 + \cos x}{x} = 0$$

$$\lim_{x \rightarrow \infty} \frac{-1 + \cos x}{x} = 0$$

$$\lim_{x \rightarrow \infty} \frac{1 + \cos x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

### Differential

#### Definition of the Derivative:

$$\frac{d}{dx} f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(h)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

#### Definition of the Taylor Series:

$$f_a(x) = \sum_{n=0}^{\infty} \frac{\frac{d}{dx} f(a)}{n!} (x-a)^n$$

#### Newton's Method:

$$x_1 = a$$

$$x_n = \frac{-f(x_{n-1})}{\frac{d}{dx} f(x_{n-1})} + x_{n-1}$$

#### Trigonometric Derivatives:

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \csc x = -\csc x \cdot \cot x$$

$$\frac{d}{dx} \sec x = \sec x \cdot \tan x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

### Hyperbolic Trigonometric Derivatives:

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$$

$$\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \cdot \coth x$$

$$\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \cdot \tanh x$$

$$\frac{d}{dx} \coth x = -\operatorname{csch}^2 x$$

### Inverse Trigonometric Derivatives:

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arccos x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arctan x = \frac{1}{x^2+1}$$

$$\frac{d}{dx} \operatorname{arccsc} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx} \operatorname{arcsec} x = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx} \operatorname{arccot} x = \frac{-1}{x^2+1}$$

### Inverse Hyperbolic Trigonometric Derivatives:

$$\frac{d}{dx} \operatorname{arsinh} x = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx} \operatorname{arcosh} x = \frac{1}{\sqrt{x^2+1}}; x > 1$$

$$\frac{d}{dx} \operatorname{arcoth} x = \frac{1}{1-x^2}; |x| < 1$$

$$\frac{d}{dx} \operatorname{aresch} x = \frac{-1}{|x|\sqrt{1+x^2}}; x \neq 0$$

$$\frac{d}{dx} \operatorname{arsech} x = \frac{-1}{x\sqrt{1+x^2}}; 0 < x < 1$$

$$\frac{d}{dx} \operatorname{arcoth} x = \frac{1}{1-x^2}; |x| > 1$$

### Integral

#### Definition of the Riemann Integral:

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x; \Delta x = \frac{b-a}{n}$$

### Trigonometric Integrals:

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \tan x dx = \ln |\sec x| + C$$

$$\int \csc x dx = \ln |\csc x - \cot x| + C$$

$$\int \sec x dx = \ln |\sec x + \tan x| + C$$

$$\int \cot x dx = -\ln |\csc x| + C$$

### Hyperbolic Trigonometric Integrals:

$$\int \sinh x dx = \cosh x + C$$

$$\int \cosh x dx = \sinh x + C$$

$$\int \tanh x dx = \ln \cosh x + C$$

$$\int \operatorname{csch} x dx = \ln \left| \tanh \frac{x}{2} \right| + C$$

$$\int \operatorname{sech} x dx = \arctan \sinh x + C$$

$$\int \coth x dx = \ln |\sinh x| + C$$

### Inverse Trigonometric Integrals:

$$\int \arcsin x dx = x \arcsin x + \sqrt{1-x^2} + C; |x| \leq 1$$

$$\int \arccos x dx = x \arccos x - \sqrt{1-x^2} + C; |x| \leq 1$$

$$\int \arctan x dx = x \arctan x - \frac{\ln |1+x^2|}{2} + C$$

$$\int \operatorname{arccsc} x dx = x \operatorname{arccsc} x + \ln \left| x \left( 1 + \sqrt{1-x^{-2}} \right) \right| + C$$

$$\int \operatorname{arcsec} x dx = x \operatorname{arcsec} x - \ln \left| x \left( 1 + \sqrt{1-x^{-2}} \right) \right| + C$$

$$\int \operatorname{arccot} x dx = x \operatorname{arccot} x + \frac{\ln |1+x^2|}{2} + C$$

### Implementations

#### Arc Length:

$$L_{f(x)}|_a^b = \int_a^b \sqrt{1 + \left( \frac{d}{dx} f(x) \right)^2} dx$$

#### Surface Area of a Rotational Solid:

$$S_{f(x)}|_a^b = 2\pi \int_a^b r \cdot \sqrt{1 + \left( \frac{d}{dx} f(x) \right)^2} dx$$

#### Volume of a Rotational Solid:

$$V_{f(x)}|_a^b = 2\pi \int_a^b f(x)^2 dx$$

#### Definition of Euler's Number:

$$e = \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n$$

#### Exponential McLaurin Series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

#### Euler's Formula:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

#### Euler's Identity:

$$e^{i\pi} + 1 = 0$$

$$e^{i\tau} = 1$$

#### Power Series of Trigonometric Functions:

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot x^{2n+1}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot x^{2n}$$

$$\tan x = \sum_{n=0}^{\infty} \frac{U_{2n+1}}{(2n+1)!} \cdot x^{2n+1}; |x| < \frac{\pi}{4}$$

$$\csc x = \sum_{n=0}^{\infty} \frac{(-1)^n 2(2^{2n-1} - 1) B_{2n}}{(2n+1)!} \cdot x^{2n}; 0 < x < \frac{\pi}{2}$$

$$\sec x = \sum_{n=0}^{\infty} \frac{U_{2n}}{(2n)!} \cdot x^{2n}; |x| < \frac{\pi}{4}$$

$$\cot x = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n-1} B_{2n}}{(2n+1)!} \cdot x^{2n+1}; 0 < x < \frac{\pi}{2}$$

## Analysis

**Definition of the Gamma Function:**

$$\Gamma(z) = (z-1)! = \int_0^{\infty} x^{z-1} e^{-x} dx$$

**Definition of the Riemann Zeta Function:**

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

**Definition of the Fourier Transform:**

$$\hat{g}(f) = \int_{-\infty}^{\infty} g(t) e^{-\tau ift} dt$$

**Definition of the Laplace Transform:**

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} f(t) e^{-st} dt$$

## Number Theory

**Fermat's Last Theorem:**

$$\neg [\forall n \in \mathbb{Z} : [n \geq 3] \exists [a, b, c \in \mathbb{Z}] : [a^n + b^n = c^n]]$$

**Fermat's Little Theorem:**

$$\forall [a \in \mathbb{Z}] \wedge [n \in \mathbb{P}] [(a^n - a) \bmod n = 0]$$

**Wilson's Theorem:**

$$[((n-1)! - 1) \bmod n = 0] \iff [n \in \mathbb{P}]$$

**The Fibonacci Sequence:**

$$F_n = F_{n-1} + F_{n-2}; F_0 = 0, F_1 = 1$$

**Definition of the Golden Ratio:**

$$\phi = \frac{1 + \sqrt{5}}{2} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \dots}}}}$$

**Pythagorean Triples:**

$$\left. \begin{array}{l} a=m^2-n^2 \\ b=2mn \\ c=m^2+n^2 \end{array} \right\} (m > n) \wedge (m, n \in \mathbb{Z})$$

## Economics

**Simple Interest:**

$$A = P(1 + rt)$$

**Compound Interest:**

$$A = P\left(1 + \frac{1}{n}\right)^{nt}$$

**Continual Compound Interest:**

$$A = Pe^{rt}$$

**Future Value of an Ordinary Annuity:**

$$A = \frac{Pn \left[ \left(1 + \frac{r}{n}\right)^{nt} - 1 \right]}{r}$$

## Electrical Engineering

**Ohm's Law:**

$$V = IR$$

## Cryptology

**Caesar Cipher:**

$$y = (x + n) \bmod 26$$

**Affine Cipher:**

$$y = (ax + b) \bmod 26$$

**Password Entropy:**

$$E = L \cdot \log_2 R$$

## Medical Science

**Glaister Equation:**

$$\Delta t_{death} \approx \frac{T_{baseline} - T_{rectal}}{R_{cooling}}$$