

## Equality

**Reflexive Property:**

$$(a = b) \iff (b = a)$$

**Transitive Property:**

$$(a = b) \wedge (b = c) \implies (a = c)$$

## Addition

**Commutative Property:**

$$a + b = b + a$$

**Associative Property:**

$$(a + b) + c = a + (b + c)$$

**Additive Identity Property:**

$$a + 0 = a$$

**Inverse Property:**

$$a + (-a) = 0$$

## Multiplication

**Commutative Property:**

$$a \cdot b = b \cdot a$$

**Associative Property:**

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

**Multiplicative Identity Property:**

$$a \cdot 1 = a$$

**Inverse Property:**

$$a \cdot \frac{1}{a} = 1; a \neq 0$$

**Distributive Property:**

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

**Zero Product Property:**

$$a \cdot 0 = 0$$

## Exponents

**Exponential Identity Property:**

$$a^1 = a$$

**Zero Power Property:**

$$a^0 = 1; a \neq 0$$

**Power of One Property:**

$$1^a = 1$$

**Power of Zero Property:**

$$0^a = 0; a \neq 0$$

**Product of Powers Property:**

$$x^a \cdot x^b = x^{a+b}$$

**Quotient of Powers Property:**

$$\frac{x^a}{x^b} = x^{a-b}; x \neq 0$$

**Power of a Power Property:**

$$(x^a)^b = x^{a \cdot b}$$

**Power of a Product Property:**

$$(x \cdot y)^a = x^a \cdot y^a$$

**Power of a Quotient Property:**

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}; y \neq 0$$

**Negative Power Property:**

$$x^{-a} = \frac{1}{x^a}; x \neq 0$$

**Fractional Power Property:**

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}; b \neq 0$$

## Roots

**Product of Roots Property:**

$$\sqrt[a]{x} \cdot \sqrt[a]{y} = \sqrt[a]{x \cdot y}; a \neq 0 \wedge x, y \in \mathbb{R}^+$$

**Quotient of Roots:**

$$\frac{\sqrt[a]{x}}{\sqrt[a]{y}} = \sqrt[a]{\frac{x}{y}}; a, y \neq 0$$

**Radical Identity Property:**

$$\sqrt[a]{x^a} = x; a \neq 0$$

## Logarithms

**Logarithmic Identity Properties:**

$$\log_x 1 = 0$$

$$\log_x x = 1$$

**Product Property:**

$$\log_a(x \cdot y) = \log_a x + \log_a y$$

**Quotient Property:**

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

**Power Property:**

$$\log_a x^y = y \cdot \log_a x$$

**Base Change Property:**

$$\log_a x = \frac{\log_n x}{\log_n a}$$

## Summation

**Constant Factorization:**

$$\sum_{k=1}^n c \cdot a_k = c \cdot \sum_{k=1}^n a_k$$

**Sum or Difference of Sequences:**

$$\sum_{k=1}^n (a_k \pm b_k) = \sum_{k=1}^n a_k \pm \sum_{k=1}^n b_k$$

**Summation of a Constant:**

$$\sum_{k=1}^n c = n \cdot c$$

**Index Shift:**

$$\sum_{k=1}^n a_k = \sum_{k=1+p}^{n+p} a_{k-p}$$

## Products

**Associative Property:**

$$\prod_{k=1}^n (a_k \cdot b_k) = \left( \prod_{k=1}^n a_k \right) \cdot \left( \prod_{k=1}^n b_k \right)$$

**Commutative Property:**

$$\left( \prod_{k=1}^n a_k \right)^x = \prod_{k=1}^n a_k^x$$

## Limits

**Sum of Limits**

$$\lim_{x \rightarrow c} (f + g)(x) = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x)$$

**Difference of Limits:**

$$\lim_{x \rightarrow c} (f - g)(x) = \lim_{x \rightarrow c} f(x) - \lim_{x \rightarrow c} g(x)$$

**Product of Limits:**

$$\lim_{x \rightarrow c} (f \cdot g)(x) = \lim_{x \rightarrow c} f(x) \cdot \lim_{x \rightarrow c} g(x)$$

**Quotient of Limits:**

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)}; \lim_{x \rightarrow c} g(x) \neq 0$$

**Power of Limits:**

$$\lim_{x \rightarrow c} f(x)^{\frac{n}{d}} = \left( \lim_{x \rightarrow c} f(x) \right)^{\frac{n}{d}}; \frac{n}{d} \in \mathbb{R}$$

**Composition of Limits:**

$$\lim_{x \rightarrow c} (f \circ g)(x) = f(\lim_{x \rightarrow c} g(x))$$

**Constant Multiple Rule:**

$$\lim_{x \rightarrow c} (k \cdot f(x)) = k \cdot \lim_{x \rightarrow c} f(x)$$

**Constant Rule:**

$$\lim_{x \rightarrow c} k = k$$

**Identity Rule:**

$$\lim_{x \rightarrow c} x = c$$

## Derivatives

**Power Rule:**

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

**Functional Power Rule:**

$$\frac{d}{dx} (f(x)^{g(x)}) = f(x)^{g(x)} \left( \frac{g(x) \cdot \frac{df}{dx}}{f(x)} + \frac{dg}{dx} \cdot \ln f(x) \right)$$

**Sum Rule:**

$$\frac{d}{dx} (f + g)(x) = \frac{df}{dx} + \frac{dg}{dx}$$

**Product Rule:**

$$\frac{d}{dx} (f \cdot g)(x) = f(x) \cdot \frac{dg}{dx} + g(x) \cdot \frac{df}{dx}$$

**General Leibniz Rule:**

$$\frac{d^n}{dx^n} (f \cdot g)(x) = \sum_{k=0}^n \binom{n}{k} \frac{d^{n-k} f}{dx^{n-k}} \cdot \frac{d^k g}{dx^k}$$

**Quotient Rule:**

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{g(x) \cdot \frac{df}{dx} - f(x) \frac{dg}{dx}}{g(x)^2}$$

**Chain Rule:**

$$\frac{d}{dx} (f \circ g)(x) = \frac{df}{dg} \cdot \frac{dg}{dx}$$

**Exponential Derivatives:**

$$\frac{d}{dx} n^x = n^x \ln(n); n > 0$$

**Logarithmic Derivatives:**

$$\frac{d}{dx} \log_b x = \frac{1}{x \ln b}$$

## Definite Integrals

**Sum of Integrals:**

$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$$

**Identity Rule:**

$$\int_a^a f(x) dx = 0$$

**Inverse Rule:**

$$\int_a^b f(x)dx = - \int_b^a f(x)dx$$

## Indefinite Integrals

**Inverse Power Rule:**

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$$

**Sum Rule:**

$$\int (f + g)(x)dx = \int f(x)dx + \int g(x)dx$$

**Integration by Parts:**

$$\int (f \cdot g)(x)dx = f(x) \cdot \int g(x)dx - \int g(x) \cdot \frac{df}{dx}dx$$

$$\int u dv = u \cdot v - \int v du$$

**Integration by Substitution:**

$$\int \frac{dg}{dx} \cdot (f \circ g)(x)dx = \int f(u)du; u = g(x)$$

**Exponential Integrals:**

$$\int n^x dx = \frac{n^x}{\ln(n)} + C$$

**Logarithmic Integrals:**

$$\int \log_b x dx = \frac{x}{\ln b}(\ln x - 1) + C$$