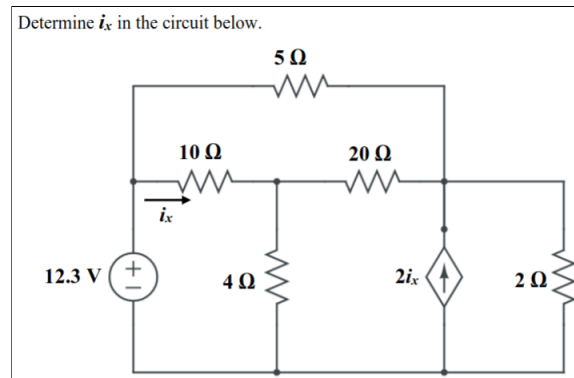


## EXAMPLES OF MATHEMATICAL CALCULATIONS IN CIRCUIT ANALYSIS

This document provides an example in which GPT-4o is asked to identify the arithmetic errors in a solution. Through this example, we demonstrate the limitations of GPT-4o in providing reliable homework assessment and feedback without the aid of the official solution.

*Note:* For the responses of GPT-4o, we mark the incorrect parts with a red background and the dubious contents with a yellow background. In addition, we mark with the green background the response segments that we would like to highlight as correct.

The considered problem is given as follows.



The correct solution to this problem is presented as follows.

Denote the voltage at the junction of the  $10\ \Omega$ ,  $4\ \Omega$ , and  $20\ \Omega$  resistors as  $v_1$ , and the voltage at the junction of the  $5\ \Omega$ ,  $20\ \Omega$ , and  $2\ \Omega$  resistors and the current-controlled current source as  $v_2$ , respectively.

Step 1: According to Ohm's law, we have

$$i_x = \frac{12.3 - v_1}{10}$$

which indicates

$$2i_x = \frac{12.3 - v_1}{5}$$

Step 2: By applying Kirchhoff's Current Law (KCL) to node 1, where the voltage is  $v_1$ , we have

$$\frac{v_1 - 12.3}{10} + \frac{v_1}{4} + \frac{v_1 - v_2}{20} = 0$$

which can be simplified to

$$8v_1 - v_2 = 24.6$$

Step 3: By applying KCL to node 2, where the voltage is  $v_2$ , we have

$$\frac{v_2 - v_1}{20} + \frac{v_2 - 12.3}{5} + \frac{v_2}{2} - \frac{12.3 - v_1}{5} = 0$$

which can be simplified to

$$v_1 + 5v_2 = 32.8$$

Solving the system of equations  $8v_1 - v_2 = 24.6$  and  $v_1 + 5v_2 = 32.8$ , we have

$$v_1 = 3.8 \text{ V}, v_2 = 5.8 \text{ V}$$

Thus,

$$i_x = \frac{12.3 - 3.8}{10} = 0.85 \text{ A}$$

The following box represents an incorrect solution, in which we intentionally introduce an arithmetic error.

Denote the voltage at the junction of the  $10 \Omega$ ,  $4 \Omega$ , and  $20 \Omega$  resistors as  $v_1$ , and the voltage at the junction of the  $5 \Omega$ ,  $20 \Omega$ , and  $2 \Omega$  resistors and the current-controlled current source as  $v_2$ , respectively.

Step 1: According to Ohm's law, we have

$$i_x = \frac{12.3 - v_1}{10}$$

which indicates

$$2i_x = \frac{12.3 - v_1}{5}$$

Step 2: By applying Kirchhoff's Current Law (KCL) to node 1, where the voltage is  $v_1$ , we have

$$\frac{v_1 - 12.3}{10} + \frac{v_1}{4} + \frac{v_1 - v_2}{20} = 0$$

which can be simplified to

$$8v_1 - v_2 = -24.6$$

Step 3: By applying KCL to node 2, where the voltage is  $v_2$ , we have

$$\frac{v_2 - v_1}{20} + \frac{v_2 - 12.3}{5} + \frac{v_2}{2} - \frac{12.3 - v_1}{5} = 0$$

which can be simplified to

$$v_1 + 5v_2 = -32.8$$

Solving the system of equations  $8v_1 - v_2 = -24.6$  and  $v_1 + 5v_2 = -32.8$ , we have

$$v_1 = -3.8 \text{ V}, v_2 = -5.8 \text{ V}$$

Thus,

$$i_x = \frac{12.3 - (-3.8)}{10} = 1.61 \text{ A}$$

We prompt GPT-4o to identify the arithmetic error in the incorrect solution provided above. Due to GPT-4o's unreliability of image recognition, we directly use the LaTeX source of the incorrect solution in the prompt. Although GPT-4o correctly derived the intermediate equations  $8v_1 - v_2 = 24.6$  and  $v_1 + 5v_2 = 32.8$ , it consistently used the wrong equation  $8v_1 - v_2 = -24.6$  in the following calculation processes, leading to another incorrect answer.

The following solution in LaTeX shows a student solution to a circuit analysis problem. Can you identify if there is any arithmetic error in the solution?

Denote the voltage at the junction of the  $10\ \Omega$ ,  $4\ \Omega$ , and  $20\ \Omega$  resistors as  $v_1$ , and the voltage at the junction of the  $5\ \Omega$ ,  $20\ \Omega$ , and  $2\ \Omega$  resistors and the current-controlled current source as  $v_2$ , respectively.

Step 1: According to Ohm's law, we have

$$i_x = \frac{12.3 - v_1}{10}$$

which indicates

$$2i_x = \frac{12.3 - v_1}{5}$$

Step 2: By applying Kirchhoff's Current Law (KCL) to node 1, where the voltage is  $v_1$ , we have

$$\frac{v_1 - 12.3}{10} + \frac{v_1}{4} + \frac{v_1 - v_2}{20} = 0$$

which can be simplified to

$$8v_1 - v_2 = -24.6$$

Step 3: By applying KCL to node 2, where the voltage is  $v_2$ , we have

$$\frac{v_2 - v_1}{20} + \frac{v_2 - 12.3}{5} + \frac{v_2}{2} - \frac{12.3 - v_1}{5} = 0$$

which can be simplified to

$$v_1 + 5v_2 = -32.8$$

Solving the system of equations  $(8v_1 - v_2 = -24.6)$  and  $(v_1 + 5v_2 = -32.8)$ , we have

$$v_1 = -3.8\ \text{V}, v_2 = -5.8\ \text{V}$$

Thus,

$$i_x = \frac{12.3 - (-3.8)}{10} = 1.61 \text{ A}$$

Let's go through the solution step by step and check for arithmetic errors.

### Step 1: Current $i_x$

Given:

$$i_x = \frac{12.3 - v_1}{10}$$

$$2i_x = \frac{12.3 - v_1}{5}$$

These are consistent with Ohm's law and the given problem statement.

### Step 2: KCL at Node 1

Applying KCL at node 1 ( $v_1$ ):

$$\frac{v_1 - 12.3}{10} + \frac{v_1}{4} + \frac{v_1 - v_2}{20} = 0$$

Combining the fractions over a common denominator:

$$\frac{2(v_1 - 12.3) + 5v_1 + (v_1 - v_2)}{20} = 0$$

$$2v_1 - 24.6 + 5v_1 + v_1 - v_2 = 0$$

$$8v_1 - v_2 = 24.6$$

It should be:

$$8v_1 - v_2 = -24.6$$

This step seems to be correct.

### Step 3: KCL at Node 2

Applying KCL at node 2 ( $v_2$ ):

$$\frac{v_2 - v_1}{20} + \frac{v_2 - 12.3}{5} + \frac{v_2}{2} - \frac{12.3 - v_1}{5} = 0$$

Combining the fractions over a common denominator:

$$\frac{v_2 - v_1}{20} + \frac{v_2 - 12.3}{5} + \frac{v_2}{2} - \frac{12.3 - v_1}{5}$$

Simplifying each term:

$$\begin{aligned} & \frac{v_2 - v_1}{20} + \frac{v_2 - 12.3}{5} + \frac{v_2}{2} - \frac{12.3 - v_1}{5} \\ &= \frac{v_2 - v_1}{20} + \frac{v_2 - 12.3}{5} + \frac{v_2}{2} - \frac{12.3 - v_1}{5} \\ &= \frac{v_2 - v_1}{20} + \frac{v_2 - 12.3 + v_1 - 12.3}{5} + \frac{v_2}{2} \\ &= \frac{v_2 - v_1}{20} + \frac{v_2 + v_1 - 24.6}{5} + \frac{v_2}{2} \end{aligned}$$

Combining terms over a common denominator:

$$\begin{aligned} & \frac{v_2 - v_1}{20} + 4 \left( \frac{v_2 + v_1 - 24.6}{20} \right) + 10 \left( \frac{v_2}{20} \right) = 0 \\ & \frac{v_2 - v_1 + 4v_2 + 4v_1 - 98.4 + 10v_2}{20} = 0 \\ & \frac{15v_2 + 3v_1 - 98.4}{20} = 0 \\ & 15v_2 + 3v_1 - 98.4 = 0 \\ & 3v_1 + 15v_2 = 98.4 \\ & v_1 + 5v_2 = 32.8 \end{aligned}$$

This equation should be:

$$v_1 + 5v_2 = 32.8$$

**Solving the system of equations:**

The system of equations is:

1.  $8v_1 - v_2 = -24.6$
2.  $v_1 + 5v_2 = 32.8$

Solving these simultaneously:

Multiply the second equation by 8 to eliminate  $v_1$ :

$$8v_1 + 40v_2 = 262.4$$

Subtract the first equation from this result:

$$(8v_1 + 40v_2) - (8v_1 - v_2) = 262.4 + 24.6$$

$$41v_2 = 287$$

$$v_2 = \frac{287}{41}$$

$$v_2 = 7$$

Substituting  $v_2 = 7$  into  $v_1 + 5v_2 = 32.8$ :

$$v_1 + 35 = 32.8$$

$$v_1 = 32.8 - 35$$

$$v_1 = -2.2$$

Thus:

$$v_1 = -2.2 \text{ V},$$

$$v_2 = 7 \text{ V}$$

Thus,

$$i_x = \frac{12.3 - (-2.2)}{10}$$

$$i_x = \frac{12.3 + 2.2}{10}$$

$$i_x = \frac{14.5}{10}$$

$$i_x = 1.45 \text{ A}$$

So, there was an arithmetic error in solving the system of equations. The corrected values should be  $v_1 = -2.2 \text{ V}$  and  $v_2 = 7 \text{ V}$ , leading to  $i_x = 1.45 \text{ A}$ , not  $1.61 \text{ A}$ .