$$\epsilon \asymp \text{polylog}(n) \div \sqrt{n}\rho_n$$