Review:
Expertise and knowledge: a modal logic perspective

This paper, included in the field of modal logic, furthers the study of a framework, introduced by Singleton (2021), which allows to reason about expertise and 'soundness of information'. This is modelled on a modal logic framework by the intuitive idea, later formalized, that an agent is an expert on ϕ whenever she is able to refute ϕ in any possible world wherein ϕ does not hold. Formally, this is achieved by the inclusion of 'expertise sets' in the models.

The present paper considers less constrictive conditions on expertise sets than those present in Singleton (2021), and provides the sound and complete logic for different classes of 'expertise frames'. It further extends this framework by considering a multi-agent version, wherein the expertise of the different agents can be combined in similar ways as to how knowledge is usually combined into common and distributed knowledge in the setting of epistemic logic.

The prose is impeccable and the presentation is very careful. Although many of the proofs employ rather standard techniques, with the exception of the (very well presented) proof of Lemma 9, the amount of results and the inherent usefulness of the framework studied here inform my recommendation that the paper should be accepted for publication, with minimal (almost negligible) changes.

Below I detail the (very few) typos I encountered, some comments of the presentation, and a few open questions and suggestions for the author(s) which I do not expect they will incorporate into the paper.

Specific comments

Introduction. The introduction could perhaps use some slight rewriting: while reading, I had a few questions which, although I was confident they would be addressed in the main text, made me slightly confused.

- In order to fully understand the intuitive idea of 'soundness', I had to go to Singleton (2021). The way it's phrased here, one would think that everything which entails a proposition about which the source is an expert would be sound. Of course, this would mean that reporting a contradiction like $p \wedge \neg p$ would be vacuously sound, which we see is not the case when the semantics is introduced.

- 'If A is an expert on policy area X and B is an expert on how X affects Y, then together they will have expertise on Y.'

  I know what this sentence is driving at, but I don't quite agree with it on an intuitive level. Say A has a PhD on how the weather affects mental health (but she's not an expert on mental health broadly speaking), and B is a weatherman; between the two of them they do not make an expert on mental health, unless A was an expert already. I guess there is a subtle but important distinction between being an expert on the assertion 'ϕ implies ψ' and being an expert on the way in which ϕ affects ψ.

In general, I think this introduction could do a better job at motivating the paper, and could perhaps benefit from being a few lines longer (in case of lack of space, perhaps the proof of Lemma 3 can be put in the appendix or made into a reference).

Section 2.

- Page 3: please introduce the set Prop before using it for the first time. mistake on the reviewer

- Mention of neighbourhood semantics: while Pacuit's book is an excellent reference text, neighbourhood models were invented by Dana Scott and Richard Montague in 1970.

  While the models in this text can indeed be seen as neighbourhood models (in the same way that one could think of any subset space as a neighbourhood model) I am not sure what the semantic correspondence is here. The 'protagonist' modal operator in the language neighbourhood semantics would be read as $M, x \models \Box\phi$ iff $\|\phi\|_M \in P$, this is exactly the semanti which does not correspond to any of the modal operators discussed in this text.

Section 4.

- Page 9: in definition 3, I would encircle ($y \in A \implies x \in A$) in parentheses.

- Page 13: In 1-agent S5 models one can assume without loss of generality that the 'knowledge' modality is universal, for the equivalence class partitions the model into the generated submodels given by the possible resolution: we work with the language with modalities K and A. if we consider bi-relational models w

equivalence classes, each of them with a universal relation. Of course, if we attempt to do this in this framework we obtain that the A and the K modality have the same semantics; I would have loved to see some discussion on why this does not work in the present framework.

Section 5.

- Small typos on table 1: $S\phi$ and $E\phi$ are used instead of $\mathsf{S}\phi$ and $\mathsf{E}\phi$ (lines 3 and 6).
- Page 15: if I'm not mistaken, Corollary 1 follows not from the immediately previous Lemma 8 but from Lemma 7 plus the (EA) axiom: if $E\phi \in \Sigma$ then (by (EA)) $AE\phi \in \Sigma$ which implies (by the definition of R) $E\phi \in \Gamma$.

the reviewer is correct on th

- Page 16: 'Now, suppose $\hat{M}_\Sigma, (\Gamma, t) \models \mathsf{E}\phi$. Then by induction hypothesis $|\phi|_\Sigma \times \mathbb{R} \in \hat{P}_\Sigma$'

  I feel that it could be made more clear how the induction hypothesis is applied in this step.
- Right after the definition of the set $\mathcal{D}$:

  'since N is at most countable, so too is $\mathcal{D}$' $\to$ 'since N is at most countable, so is $\mathcal{D}$'
- I find footnote 8 unnecessary.
- Page 18: perhaps remind the reader that $\hat{\mathsf{S}}$ is the dual of S, since this notation is introduced in passing many pages ago.

i'll ignore this. KT4 is a muc

- Why talk about **KT4** instead of the more common nomenclature **S4**?
- Page 19: the abbreviation $\mathbb{M}_{int-compl} = \mathbb{M}_{int} \cap \mathbb{M}_{compl}$ is introduced and the only used once three lines later; it is not even that much shorter in terms of the number of characters, so I would consider not using it.
- Please mention something about the proof of Theorem 7, even if it is the fact that it is in the appendix and that it is very similar to previous proofs.

  (In general the last paragraph of Section 5 feels like it was cut short; I do not like that the section ends with the blunt statement of a theorem).

Section 6.

- Page 21: since the P_j's in this section are Alevandrov topologies, it would be nice to bring up the notion of topological join when defining

P_J^{dist}. If I am not mistaken P_J^{dist} is exactly the topology generated by the basis

$$\left\{ \bigcap_{j \in J} A_j : A_j \in P_j \right\};$$

this corresponds to the join $\bigvee_{j \in J} P_j$ in the lattice of topologies on X.

- Page 22: please mention that the proof of Prop. 5 is in the appendix.

In the bibliography, 'van Benthem' should be alphabetized by the letter B, not by the letter v.