

Figure 1: Our ongoing work on learning  $w_{\text{max}}$  when a sharp bound is unknown. This is a preliminary result of the convergence of  $\hat{w}_{\text{max}}$  estimated by our new algorithm, which tries to resolve the requirement of knowing a sharp bound of  $w_{\text{max}}$ . Experiment setting: consider a linear autonomous systems  $x_{t+1} = Ax_t + w_t$ , where  $A \in \mathbb{R}^{10 \times 10}$ , its each entry is generated i.i.d. from Unif[0, 1], then rescale A to make it stable. The norm of A used for this figure is  $||A||_2 = 0.83$ . The disturbances  $w_t$  are generated i.i.d from  $N(0, I_{10})$  truncated to  $[-3, 3]^{10}$ . Algorithm: Given data  $\{x_0\}_{t=1}^T$ , compute the Chebyshev estimate (Cramer, 1964; Rider, 1957) of A by:  $\hat{A}_{\text{cheb}} = \arg\min_A \max_{0 \le t \le T-1} ||x_{t+1} - Ax_t||_{\infty}$ . Then, estimate  $w_{\text{max}}$  by the  $\hat{A}_{\text{cheb}} = \max_{0 \le t \le T-1} ||x_{t+1} - Ax_t||_{\infty}$ . Observation: The estimation error of  $w_{\text{max}}$  quickly decreases to close to 0. This is promising since we can apply the estimated  $\hat{w}_{\text{max}}$  to SM. With proper two-time-scale updates and analysis, we expect that SM is able to converge even without knowing a sharp bound  $w_{\text{max}}$ . The non-asymptotic analysis is heavily based on the novel proof techniques developed in Appendix D.3. However, the two-time-scale updates involves more complicated analysis, which is our ongoing work.