E Supporting Lemmas

Lemma E.1.

$$\mathbb{E}\left[\mathbf{1}_i^t(j)\right] \approx hp^t(\mathbf{s}_i^{\gamma}, \mathbf{v}_j; \alpha) \tag{47}$$

Proof.
First we define the selection indicator:

$$\mathbf{1}_{i,k}^t(j) = \begin{cases} 1, & \text{if item } j \text{ is selected at the } k\text{-th draw without replacement from } \mathbf{p}_i^t(\alpha) \\ 0, & \text{otherwise} \end{cases} \tag{48}$$

For simplicity, we denote $p^t(\mathbf{s}_i^{\gamma}, \mathbf{v}_j; \alpha)$ as p_j, because of $h \ll m, p_j \to 0$, we have:

$$\mathbb{E}\left[\mathbf{1}_i^t(j)\right] = \sum_{k=1}^{h} \mathbb{E}\left[\mathbf{1}_{i,k}^t(j)\right] \approx \sum_{k=1}^{h} p_j = hp_j \tag{49}$$

Then we compute the error:

$$\delta = \sum_{k=1}^{h} p_j - \sum_{k=1}^{h} \mathbb{E}\left[\mathbf{1}_{i,k}^t(j)\right] \approx hp_j - (p_j + (h-1)\sum_{l \neq j} p_l p_j) = hp_j - (p_j + (h-1)(1-p_j)p_j) = (h-1)p_j^2 = (h-1)(p^t(\mathbf{s}_i^{\gamma}, \mathbf{v}_j; \alpha))^2 \tag{50}$$

□

Lemma E.2.

$$\text{vec}(\mathbf{AUB}^T) = (\mathbf{B} \otimes \mathbf{A})\,\text{vec}(\mathbf{U}) \tag{51}$$

where $\mathbf{A} \in \mathbb{R}^{c \times c}, \mathbf{U} \in \mathbb{R}^{c \times n}, \mathbf{B} \in \mathbb{R}^{n \times n}$.

Proof.
$\forall i, j, (\mathbf{AUB}^T)_{ij} = \sum_{k=1}^{c} \sum_{l=1}^{n} A_{ik} U_{kl} B_{lj}^T$ is equal to the $(j-1)c + i$-th row element of matrix $\text{vec}(\mathbf{AUB}^T)$.
Furthermore,

$$\mathbf{B} \otimes \mathbf{A} = \begin{bmatrix} B_{11}\mathbf{A} & \cdots & B_{1n}\mathbf{A} \\ \vdots & \ddots & \vdots \\ B_{n1}\mathbf{A} & \cdots & B_{nn}\mathbf{A} \end{bmatrix} \in \mathbb{R}^{cn \times cn}, \tag{52}$$

the $(j-1)c + i$-th row element of matrix $(\mathbf{B} \otimes \mathbf{A})\,\text{vec}(\mathbf{U})$ is equal to $\sum_{k=1}^{c} \sum_{l=1}^{n} A_{ik} U_{kl} B_{lj}^T$. Therefore, all corresponding elements of $\text{vec}(\mathbf{AUB}^T)$ and $(\mathbf{B} \otimes \mathbf{A})\,\text{vec}(\mathbf{U})$ are equal, i.e., $\text{vec}(\mathbf{AUB}^T) = (\mathbf{B} \otimes \mathbf{A})\,\text{vec}(\mathbf{U})$. □

Lemma E.3.

$$\|\mathbf{AB}\|_\infty \le \|\mathbf{A}\|_\infty \|\mathbf{B}\|_\infty \tag{53}$$

Proof.
For a matrix $\mathbf{A} \in \mathbb{R}_{m \times n}$, the infinity norm is defined as $\|\mathbf{A}\|_\infty = \max_{1 \le i \le m} \sum_{j=1}^{n} |A_{ij}|$. Since $\sum_i |B_{ji}| \le \max_j \sum_i |B_{ji}| = \|\mathbf{B}\|_\infty$, $\sum_j |A_{kj}| \le \max_k \sum_j |A_{kj}| = \|\mathbf{A}\|_\infty$.

$$\text{Then } \forall k, \sum_i |(\mathbf{AB})_{ki}| = \sum_i |\sum_j A_{kj} B_{ji}| \le \sum_i \sum_j |A_{kj}||B_{ji}| = \sum_j |A_{kj}| \sum_i |B_{ji}| \le \sum_j |A_{kj}| \|\mathbf{B}\|_\infty \le \|\mathbf{A}\|_\infty \|\mathbf{B}\|_\infty \tag{54}$$

Therefore $\|\mathbf{AB}\|_\infty = \max_k \sum_i |(\mathbf{AB})_{ki}| \le \|\mathbf{A}\|_\infty \|\mathbf{B}\|_\infty$.

□

Lemma E.4.

$$\rho(\mathbf{A}) \le \|\mathbf{A}\|_\infty \tag{55}$$

where $\rho(\mathbf{A})$ *denotes the spectral radius of matrix* $\mathbf{A}$.

Proof.
Let λ be an eigenvalue of matrix $\mathbf{A}$, and let $\mathbf{x}$ be a corresponding eigenvector such that $\mathbf{Ax} = \lambda\mathbf{x}$ and $\mathbf{x} \neq 0$. Then $\|\mathbf{Ax}\|_\infty = \|\lambda\mathbf{x}\|_\infty = |\lambda|\|\mathbf{x}\|_\infty$.
Since $\|\mathbf{Ax}\|_\infty \le \|\mathbf{A}\|_\infty \|\mathbf{x}\|_\infty$ based on Lemma E.3, we get $|\lambda|\|\mathbf{x}\|_\infty \le \|\mathbf{A}\|_\infty \|\mathbf{x}\|_\infty$.
And $\|\mathbf{x}\|_\infty > 0$ with $\mathbf{x} \neq 0$, we have $|\lambda| \le \|\mathbf{A}\|_\infty$, this inequality holds for all eigenvalues λ of matrix $\mathbf{A}$. Therefore,

$$\rho(\mathbf{A}) = \max\{|\lambda| | \lambda \in \text{eigenvalues of } \mathbf{A}\} \le \|\mathbf{A}\|_\infty. \tag{56}$$

□

Lemma E.5.
A sufficient condition for the invertibility of $\mathbf{I} - \mathbf{A}$ *is that the infinity norm of* $\mathbf{A}$ *is strictly less than* 1, *i.e.,*

$$\|\mathbf{A}\|_\infty < 1 \Rightarrow (\mathbf{I} - \mathbf{A})^{-1} \text{ exists.} \tag{57}$$

Proof.
Based on Lemma E.4, we get $\rho(\mathbf{A}) \le \|\mathbf{A}\|_\infty$. Given that $\|\mathbf{A}\|_\infty < 1$, it follows that $\rho(\mathbf{A}) < 1$.
This implies that all eigenvalues λ of $\mathbf{A}$ satisfy $|\lambda| < 1$, i.e., all eigenvalues of $\mathbf{I} - \mathbf{A}$ satisfy $\mu = 1 - \lambda > 0$.
Hence, $\mathbf{I} - \mathbf{A}$ has no zero eigenvalues, which implies that $\det(\mathbf{I} - \mathbf{A}) \neq 0$.
Therefore, $\mathbf{I} - \mathbf{A}$ is invertible. □

Lemma E.6.

$$\|\mathbf{A} \otimes \mathbf{B}\|_\infty = \|\mathbf{A}\|_\infty \|\mathbf{B}\|_\infty \tag{58}$$

Proof.
Since

$$\|\mathbf{A}\|_\infty = \max_l \sum_i |A_{li}|, \|\mathbf{B}\|_\infty = \max_k \sum_r |A_{kr}|. \tag{59}$$

Then

$$\|\mathbf{A} \otimes \mathbf{B}\|_\infty = \max_{l,k} \sum_i \sum_r |A_{li} B_{kr}| = \max_l \sum_i |A_{li}| \cdot \max_k \sum_r |A_{kr}| = \|\mathbf{A}\|_\infty \|\mathbf{B}\|_\infty \tag{60}$$

□

Lemma E.7.

Let $\mathbf{a} = [a^{(1)}, a^{(2)}, \ldots, a^{(n)}]^T \in \mathbb{R}^n$, $\mathbf{b} = [b^{(1)}, b^{(2)}, \ldots, b^{(n)}]^T \in \mathbb{R}^n$.
If there exists an index $k \in \{1, \ldots, n\}$ *such that* $a^{(k)} b^{(k)} \ge x > 0$, *then a lower bound for the remaining products is given by*

$$\min_{i \neq k} a^{(i)} b^{(i)} \ge -\sqrt{\|\mathbf{a}\|_2^2 \|\mathbf{b}\|_2^2 - x^2}. \tag{61}$$

Proof.
First

$$\sum_i (a^{(i)} b^{(i)})^2 = \sum_i (a^{(i)})^2 (b^{(i)})^2 \le \sum_i (a^{(i)})^2 \sum_i (b^{(i)})^2 = \|\mathbf{a}\|_2^2 \|\mathbf{b}\|_2^2 \tag{62}$$

Given $a^{(k)} b^{(k)} \ge x > 0$,

$$\sum_{i \neq k} (a^{(i)} b^{(i)})^2 \le \|\mathbf{a}\|_2^2 \|\mathbf{b}\|_2^2 - x^2. \tag{63}$$

This implies that for all $i \neq k$,

$$|a^{(i)} b^{(i)}| \le \sqrt{\|\mathbf{a}\|_2^2 \|\mathbf{b}\|_2^2 - x^2}, \tag{64}$$

which leads to:

$$\min_{i \neq k} a^{(i)} b^{(i)} \ge -\sqrt{\|\mathbf{a}\|_2^2 \|\mathbf{b}\|_2^2 - x^2}. \tag{65}$$

□

Lemma E.8.
Let $\mathbf{D} = \text{diag}(d_1, d_2, \ldots, d_n)$ *be a diagonal matrix with strictly positive entries, i.e.,* $d_i > 0$ *for all* $i \in \{1, 2, \ldots, n\}$. *Then, for any vector* $\mathbf{b} \in \mathbb{R}^n$, *the following inequality holds:*

$$\|\mathbf{Db}\|_2 \le \left(\max_i d_i\right) \cdot \|\mathbf{b}\|_2. \tag{66}$$

Proof.
Let $\mathbf{b} = [b_1, b_2, \ldots, b_n]^T$. Then,

$$\mathbf{Db} = [d_1 b_1, d_2 b_2, \ldots, d_n b_n]^T, \tag{67}$$

and

$$\|\mathbf{Db}\|_2 = \sqrt{\sum_{i=j}^{n} (d_j b_j)^2} \le \sqrt{\sum_{j=1}^{n} (\max_i d_i)^2 (b_j)^2} = (\max_i d_i) \sqrt{\sum_{j=1}^{n} (b_j)^2} = (\max_i d_i) \|\mathbf{b}\|_2 \tag{68}$$

□

F Additional Experiments on Parameter Analysis

We perform the same analysis of the four factors on the Epinions and synthetic datasets, and the results show a high degree of consistency with those on the Ciao dataset.

F.1 Analysis of α, β, γ and ϵ on Epinions Dataset

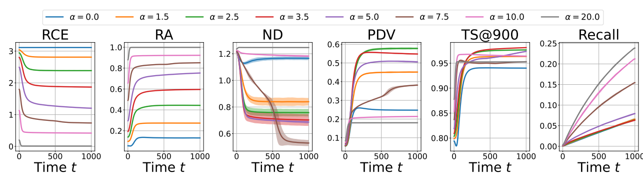

Figure 13: Metric trends over time under varying α on Epinions dataset.

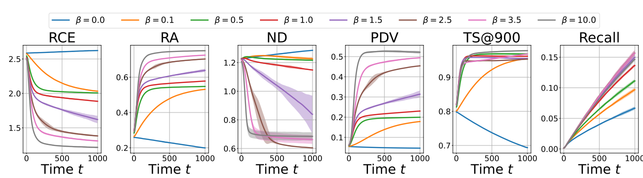

Figure 14: Metric trends over time under varying β on Epinions dataset.

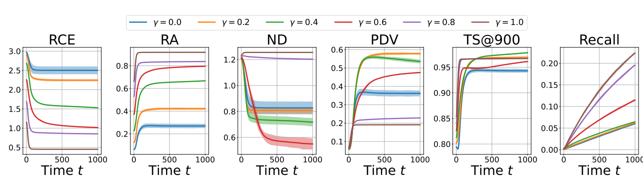

Figure 15: Metric trends over time under varying γ on Epinions dataset.

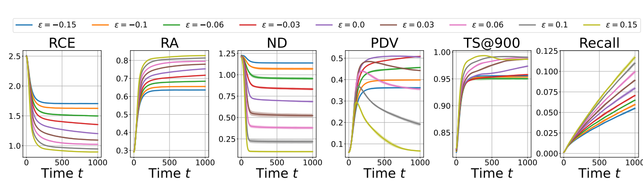

Figure 16: Metric trends over time under varying ϵ on Epinions dataset.

F.2 Analysis of α, β, γ and ϵ on Synthetic Dataset

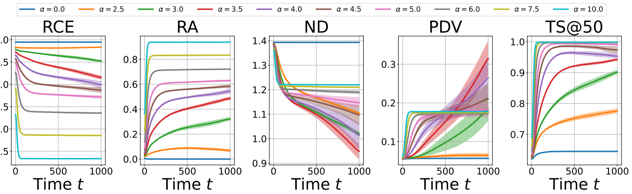

Figure 17: Metric trends over time under varying α on Synthetic dataset.

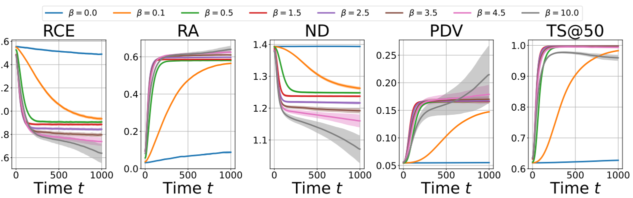

Figure 18: Metric trends over time under varying β on Synthetic dataset.

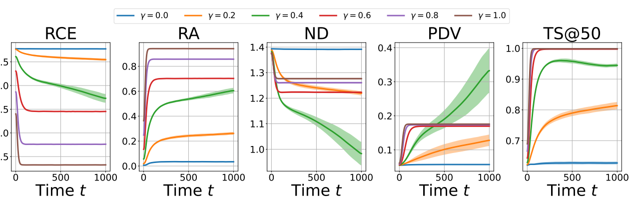

Figure 19: Metric trends over time under varying γ on Synthetic dataset.

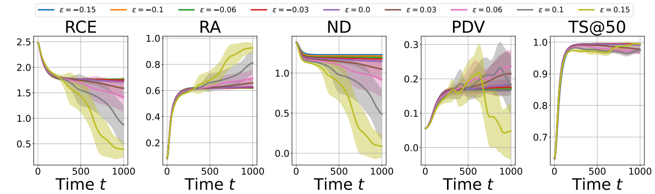

Figure 20: Metric trends over time under varying ϵ on Synthetic dataset.

Table 4: Parameter sensitivity analysis of four strategies on Ciao dataset.

Ciao		RCE↑	RA↑	ND↑	PDV↓	TS@300↓
UAα	$\alpha = 1$	$1.21_{\pm 0.004}$	$0.74_{\pm 0.001}$	$0.90_{\pm 0.005}$	$0.30_{\pm 0.002}$	$0.96_{\pm 0.001}$
	$\alpha = 5$	$1.24_{\pm 0.003}$	$0.73_{\pm 0.001}$	$0.94_{\pm 0.001}$	$0.29_{\pm 0.002}$	$0.96_{\pm 0.001}$
	$\alpha = 10$	$1.28_{\pm 0.000}$	$0.72_{\pm 0.000}$	$0.95_{\pm 0.002}$	$0.28_{\pm 0.002}$	$0.96_{\pm 0.001}$
	$\alpha = 20$	$1.32_{\pm 0.004}$	$0.71_{\pm 0.001}$	$0.98_{\pm 0.001}$	$0.27_{\pm 0.000}$	$0.95_{\pm 0.000}$
FUA	$\rho = -0.01$	$1.16_{\pm 0.004}$	$0.75_{\pm 0.001}$	$0.86_{\pm 0.004}$	$0.32_{\pm 0.003}$	$0.96_{\pm 0.001}$
	$\rho = 0.02$	$1.27_{\pm 0.009}$	$0.73_{\pm 0.002}$	$0.92_{\pm 0.007}$	$0.29_{\pm 0.005}$	$0.96_{\pm 0.000}$
	$\rho = 0.05$	$1.36_{\pm 0.005}$	$0.70_{\pm 0.001}$	$0.96_{\pm 0.003}$	$0.27_{\pm 0.002}$	$0.95_{\pm 0.000}$
	$\rho = 0.08$	$1.44_{\pm 0.004}$	$0.68_{\pm 0.001}$	$1.00_{\pm 0.001}$	$0.26_{\pm 0.002}$	$0.95_{\pm 0.001}$
DPP	$\theta = 0.500$	$1.13_{\pm 0.001}$	$0.78_{\pm 0.000}$	$0.89_{\pm 0.006}$	$0.30_{\pm 0.003}$	$0.96_{\pm 0.000}$
	$\theta = 0.501$	$1.38_{\pm 0.001}$	$0.74_{\pm 0.000}$	$0.97_{\pm 0.004}$	$0.27_{\pm 0.002}$	$0.95_{\pm 0.000}$
	$\theta = 0.502$	$1.61_{\pm 0.001}$	$0.70_{\pm 0.000}$	$1.04_{\pm 0.003}$	$0.24_{\pm 0.003}$	$0.94_{\pm 0.001}$
	$\theta = 0.503$	$1.80_{\pm 0.001}$	$0.66_{\pm 0.000}$	$1.09_{\pm 0.002}$	$0.22_{\pm 0.001}$	$0.94_{\pm 0.001}$
SAR	$\omega = 100$	$1.22_{\pm 0.005}$	$0.72_{\pm 0.001}$	$0.85_{\pm 0.016}$	$0.31_{\pm 0.004}$	$0.96_{\pm 0.002}$
	$\omega = 1000$	$1.26_{\pm 0.009}$	$0.68_{\pm 0.003}$	$0.94_{\pm 0.008}$	$0.28_{\pm 0.004}$	$0.95_{\pm 0.001}$
	$\omega = 2000$	$1.24_{\pm 0.003}$	$0.69_{\pm 0.001}$	$0.90_{\pm 0.019}$	$0.29_{\pm 0.005}$	$0.95_{\pm 0.000}$
	$\omega = 5000$	$1.25_{\pm 0.004}$	$0.68_{\pm 0.001}$	$0.92_{\pm 0.013}$	$0.28_{\pm 0.004}$	$0.95_{\pm 0.001}$

G Additional Experiments with Mitigation Strategies on Ciao Dataset

Through further parameter tuning of the four mitigation strategies on the Ciao dataset in Table 4, we observe that sacrificing a certain degree of accuracy can lead to more effective alleviation of echo chambers and user homogenization.