A Proof of Proposition 4.1

PROOF.

- First we define the indicator function as follows:

$$\mathbf{1}_i^t(j) = \begin{cases} 1, & \text{if item } j \text{ appears in } h \text{ samples drawn without replacement from } \mathbf{p}_{s_i}^t(\alpha) \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

Based on Lemma E.1, we have $\mathbb{E}\left[\mathbf{1}_i^t(j)\right] \approx hp^t(\mathbf{s}_i^Y, \mathbf{v}_j; \alpha)$, then Equation 6 admits the following expectation representation:

$$\mathbf{u}_i(t+1) = \mathbf{u}_i(t) + \frac{\eta}{h}\mathbb{E}\left[\sum_{j=1}^{m} \mathbf{1}_i^t(j) w_i^t(j)\mathbf{v}_j\right] = \mathbf{u}_i(t) + \frac{\eta}{h}\sum_{j=1}^{m}\mathbb{E}\left[\mathbf{1}_i^t(j)\right]\mathbb{E}\left[w_i^t(j)\right]\mathbf{v}_j$$

$$= \mathbf{u}_i(t) + \frac{\eta}{h}\sum_{j=1}^{m} hp^t(\mathbf{s}_i^Y, \mathbf{v}_j; \alpha)(p_{pos}^t(\mathbf{u}_i, \mathbf{v}_{r_i^j}; \beta, \epsilon) - p_{neg}^t(\mathbf{u}_i, \mathbf{v}_{r_i^j}; \beta, \epsilon))\mathbf{v}_j = \mathbf{u}_i(t) + \eta\sum_{j=1}^{m} p^t(\mathbf{s}_i^Y, \mathbf{v}_j; \alpha) g^t(\mathbf{u}_i, \mathbf{v}_j; \beta, \epsilon)\mathbf{v}_j. \tag{18}$$

- First we have $\sum_{j=1}^m \mathbf{v}_j\mathbf{v}_j^T = \mathbf{V}\mathbf{V}^T, \sum_{j=1}^m \mathbf{v}_j \sum_{k=1}^m \mathbf{v}_k^T = \mathbf{V}\mathbf{1}_{m\times m}\mathbf{V}^T$,

$$p^t(\mathbf{s}_i^Y, \mathbf{v}_j; \alpha) = \frac{\exp\left(\alpha\mathbf{v}_j^T\mathbf{s}_i^Y(t)\right)}{\sum_{k=1}^m \exp\left(\alpha\mathbf{v}_k^T\mathbf{s}_i^Y(t)\right)} = \frac{1 + \alpha\mathbf{v}_j^T\mathbf{s}_i^Y(t) + O\left(\alpha^2(\mathbf{v}_j^T\mathbf{s}_i^Y(t))^2\right)}{\sum_{k=1}^m\left(1 + \alpha\mathbf{v}_k^T\mathbf{s}_i^Y(t) + O(\alpha^2(\mathbf{v}_k^T\mathbf{s}_i^Y(t))^2)\right)}$$

$$\approx \frac{1 + \alpha\mathbf{v}_j^T\mathbf{s}_i^Y(t)}{m\left(1 + \frac{\alpha}{m}\sum_{k=1}^m \mathbf{v}_k^T\mathbf{s}_i^Y(t)\right)} \approx \frac{1}{m}\left(1 + \alpha\mathbf{v}_j^T\mathbf{s}_i^Y(t) - \frac{\alpha}{m}\sum_{k=1}^m \mathbf{v}_k^T\mathbf{s}_i^Y(t)\right). \tag{19}$$

$$g^t(\mathbf{u}_i, \mathbf{v}_j; \beta, \epsilon) = \frac{\left(1 + \mathbf{v}_j^T\mathbf{u}_i(t)\right)^\beta - \left(1 - \mathbf{v}_j^T\mathbf{u}_i(t)\right)^\beta}{\left(1 + \mathbf{v}_j^T\mathbf{u}_i(t)\right)^\beta + \left(1 - \mathbf{v}_j^T\mathbf{u}_i(t)\right)^\beta} + \epsilon \approx \frac{\left(1 + \beta\mathbf{v}_j^T\mathbf{u}_i(t)\right) - \left(1 - \beta\mathbf{v}_j^T\mathbf{u}_i(t)\right)}{\left(1 + \beta\mathbf{v}_j^T\mathbf{u}_i(t)\right) + \left(1 - \beta\mathbf{v}_j^T\mathbf{u}_i(t)\right)} + \epsilon = \beta\mathbf{v}_j^T\mathbf{u}_i(t) + \epsilon. \tag{20}$$

Then Equation 18 equals:

$$\mathbf{u}_i(t+1) \approx \mathbf{u}_i(t) + \frac{\eta}{m}\sum_{j=1}^m(1 + \alpha\mathbf{v}_j^T\mathbf{s}_i^Y(t) - \frac{\alpha}{m}\sum_{k=1}^m \mathbf{v}_k^T\mathbf{s}_i^Y(t))(\beta\mathbf{v}_j^T\mathbf{u}_i(t) + \epsilon)\mathbf{v}_j \approx \mathbf{u}_i(t) + \frac{\eta}{m}\sum_{j=1}^m(\alpha\epsilon\mathbf{v}_j^T\mathbf{s}_i^Y(t) - \frac{\alpha\epsilon}{m}\sum_{k=1}^m \mathbf{v}_k^T\mathbf{s}_i^Y(t) + \beta\mathbf{v}_j^T\mathbf{u}_i(t) + \epsilon)\mathbf{v}_j$$

$$= \mathbf{u}_i(t) + \frac{\eta}{m}\sum_{j=1}^m((\alpha\epsilon\mathbf{v}_j^T - \frac{\alpha\epsilon}{m}\sum_{k=1}^m \mathbf{v}_k^T)(\gamma\mathbf{u}_i(t) + (1-\gamma)\frac{\sum_{j\in\mathcal{N}_i}\mathbf{u}_j(t)}{|\mathcal{N}_i|}) + \beta\mathbf{v}_j^T\mathbf{u}_i(t) + \epsilon)\mathbf{v}_j$$

$$= \frac{\eta\epsilon}{m}\sum_{j=1}^m \mathbf{v}_j + (\mathbf{I}_c + \frac{\eta(\alpha\epsilon\gamma + \beta)}{m}\sum_{j=1}^m \mathbf{v}_j\mathbf{v}_j^T - \frac{\eta\alpha\epsilon\gamma}{m^2}\sum_{j=1}^m \mathbf{v}_j\sum_{k=1}^m \mathbf{v}_k^T)\mathbf{u}_i(t) + (\frac{\eta\alpha\epsilon(1-\gamma)}{m}\sum_{j=1}^m \mathbf{v}_j\mathbf{v}_j^T - \frac{\eta\alpha\epsilon(1-\gamma)}{m^2}\sum_{j=1}^m \mathbf{v}_j\sum_{k=1}^m \mathbf{v}_k^T)\frac{\sum_{j\in\mathcal{N}_i}\mathbf{u}_j(t)}{|\mathcal{N}_i|}$$

$$= \frac{\eta\epsilon}{m}\sum_{j=1}^m \mathbf{v}_j + (\mathbf{I}_c + \frac{\eta(\alpha\epsilon\gamma + \beta)}{m}\mathbf{V}\mathbf{V}^T - \frac{\eta\alpha\epsilon\gamma}{m^2}\mathbf{V}\mathbf{1}_{m\times m}\mathbf{V}^T)\mathbf{u}_i(t) + (\frac{\eta\alpha\epsilon(1-\gamma)}{m}\mathbf{V}\mathbf{V}^T - \frac{\eta\alpha\epsilon(1-\gamma)}{m^2}\mathbf{V}\mathbf{1}_{m\times m}\mathbf{V}^T)\frac{\sum_{j\in\mathcal{N}_i}\mathbf{u}_j(t)}{|\mathcal{N}_i|} \tag{21}$$

Under the definitions of X, Y, and Z given in Proposition 4.1, we have:

$$[\mathbf{u}_1(t+1), \mathbf{u}_2(t+1), \dots, \mathbf{u}_n(t+1)] = \frac{\eta\epsilon}{m}[\sum_{j=1}^m \mathbf{v}_j, \sum_{j=1}^m \mathbf{v}_j, \dots, \sum_{j=1}^m \mathbf{v}_j] + \mathbf{Y}[\mathbf{u}_1(t), \mathbf{u}_2(t), \dots, \mathbf{u}_n(t)] + \mathbf{Z}[\frac{\sum_{j\in\mathcal{N}_1}\mathbf{u}_j(t)}{|\mathcal{N}_1|}, \frac{\sum_{j\in\mathcal{N}_2}\mathbf{u}_j(t)}{|\mathcal{N}_2|}, \dots, \frac{\sum_{j\in\mathcal{N}_n}\mathbf{u}_j(t)}{|\mathcal{N}_n|}]$$

$$\mathbf{U}(t+1) = \frac{\eta\epsilon}{m}\mathbf{V}\mathbf{1}_{m\times n} + \mathbf{Y}\mathbf{U}(t) + \mathbf{Z}\mathbf{U}(t)(\mathrm{diag}(\mathbf{S}\mathbf{1}_{n\times 1})^{-1}\mathbf{S})^T = \mathbf{X} + \mathbf{Y}\mathbf{U}(t) + \mathbf{Z}\mathbf{U}(t)\tilde{\mathbf{S}}^T \tag{22}$$

□

B Proof of Proposition 4.2

PROOF.

Under the column-wise vectorization operator $\mathrm{vec}(\cdot): \mathbb{R}^{c\times n} \to \mathbb{R}^{cn}$, Equation 8 transforms to:

$$\mathrm{vec}(\mathbf{U}(t+1)) = \mathrm{vec}(\mathbf{X}) + \mathrm{vec}(\mathbf{Y}\mathbf{U}(t)) + \mathrm{vec}(\mathbf{Z}\mathbf{U}(t)\tilde{\mathbf{S}}^T). \tag{23}$$

Based on Lemma E.2, we have

$$\mathrm{vec}(\mathbf{Y}\mathbf{U}(t)) = \mathrm{vec}(\mathbf{Y}\mathbf{U}(t)\mathbf{I}_n^T) = (\mathbf{I}_n \otimes \mathbf{Y})\,\mathrm{vec}(\mathbf{U}(t)) \;\;;\;\; \mathrm{vec}(\mathbf{Z}\mathbf{U}(t)\tilde{\mathbf{S}}^T) = (\tilde{\mathbf{S}} \otimes \mathbf{Z})\,\mathrm{vec}(\mathbf{U}(t)). \tag{24}$$

$$\mathrm{vec}(\mathbf{U}(t+1)) = \mathrm{vec}(\mathbf{X}) + (\mathbf{I}_n \otimes \mathbf{Y})\,\mathrm{vec}(\mathbf{U}(t)) + (\tilde{\mathbf{S}} \otimes \mathbf{Z})\,\mathrm{vec}(\mathbf{U}(t)) = \mathrm{vec}(\mathbf{X}) + (\mathbf{I}_n \otimes \mathbf{Y} + \tilde{\mathbf{S}} \otimes \mathbf{Z})\,\mathrm{vec}(\mathbf{U}(t)). \tag{25}$$

Assume that $\mathbf{U}(t)$ in Equation 25 converges to $\mathbf{U}^*$, we have

$$(\mathbf{I}_{nc} - (\mathbf{I}_n \otimes \mathbf{Y} + \tilde{\mathbf{S}} \otimes \mathbf{Z}))\,\mathrm{vec}(\mathbf{U}^*) = \mathrm{vec}(\mathbf{X}) \tag{26}$$

Now, the convergence of Equation 8 is equivalent to the invertibility of $(\mathbf{I}_{nc} - (\mathbf{I}_n \otimes \mathbf{Y} + \tilde{\mathbf{S}} \otimes \mathbf{Z}))$. To establish that $\eta(\frac{\beta}{2} + \frac{\alpha\epsilon\gamma}{2} + \frac{\beta^2}{8\alpha\epsilon\gamma}) < 1$ is sufficient for $(\mathbf{I}_{nc} - (\mathbf{I}_n \otimes \mathbf{Y} + \tilde{\mathbf{S}} \otimes \mathbf{Z}))^{-1} exists$, we proceed through a sequence of logical implications:

(1) We first demonstrate that $\eta(\frac{\beta}{2} + \frac{\alpha\epsilon\gamma}{2} + \frac{\beta^2}{8\alpha\epsilon\gamma}) < 1 \Rightarrow \|\mathbf{Y}\|_\infty + \|\mathbf{Z}\|_\infty < 1$;

Proof. We previously defined the representation of item j as $\mathbf{v}_j = \left[v_j^{(1)}, v_j^{(2)}, \dots, v_j^{(c)}\right]^T, j \in \{1, 2, \dots, m\}$, if item j belongs to category k, then $v_j^{(k)}v_j^{(k)} = 1, v_j^{(p)}v_j^{(q)} = 0 \;for\; p \neq k \;or\; q \neq k$, and $\sum_{j=1}^m v_j^{(i)} = n_i, \sum_{i=1}^c n_i = m$.

$$\mathbf{V}\mathbf{V}^T = \sum_{j=1}^m \mathbf{v}_j\mathbf{v}_j^T = \begin{bmatrix} \sum_{j=1}^m v_j^{(1)}v_j^{(1)} & \cdots & \sum_{j=1}^m v_j^{(1)}v_j^{(c)} \\ \vdots & \ddots & \vdots \\ \sum_{j=1}^m v_j^{(c)}v_j^{(1)} & \cdots & \sum_{j=1}^m v_j^{(c)}v_j^{(c)} \end{bmatrix} = \begin{bmatrix} n_1 & 0 & \cdots & 0 \\ 0 & n_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & n_c \end{bmatrix} \tag{27}$$

$$\mathbf{V}\mathbf{1}_{m\times m}\mathbf{V}^T = \sum_{j=1}^m \mathbf{v}_j\sum_{k=1}^m \mathbf{v}_k^T = \begin{bmatrix} \sum_{j=1}^m v_j^{(1)}\sum_{k=1}^m v_k^{(1)} & \cdots & \sum_{j=1}^m v_j^{(1)}\sum_{k=1}^m v_k^{(c)} \\ \vdots & \ddots & \vdots \\ \sum_{j=1}^m v_j^{(c)}\sum_{k=1}^m v_k^{(1)} & \cdots & \sum_{j=1}^m v_j^{(c)}\sum_{k=1}^m v_k^{(c)} \end{bmatrix} = \begin{bmatrix} n_1\cdot n_1 & \cdots & n_1\cdot n_c \\ \vdots & \ddots & \vdots \\ n_c\cdot n_1 & \cdots & n_c\cdot n_c \end{bmatrix} \tag{28}$$

$$\mathbf{Y} \approx \frac{\eta(\alpha\epsilon\gamma + \beta)}{m}\begin{bmatrix} n_1 & 0 & \cdots & 0 \\ 0 & n_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & n_c \end{bmatrix} - \frac{\eta\alpha\epsilon\gamma}{m^2}\begin{bmatrix} n_1\cdot n_1 & \cdots & n_1\cdot n_c \\ \vdots & \ddots & \vdots \\ n_c\cdot n_1 & \cdots & n_c\cdot n_c \end{bmatrix} \tag{29}$$

$$\mathbf{Z} = \frac{\eta\alpha\epsilon(1-\gamma)}{m}(\begin{bmatrix} n_1 & 0 & \cdots & 0 \\ 0 & n_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & n_c \end{bmatrix} - \frac{1}{m}\begin{bmatrix} n_1\cdot n_1 & \cdots & n_1\cdot n_c \\ \vdots & \ddots & \vdots \\ n_c\cdot n_1 & \cdots & n_c\cdot n_c \end{bmatrix}) \tag{30}$$

$$\|\mathbf{Y}\|_\infty = \max_{1\le l\le c}\left(\frac{\eta\alpha\epsilon\gamma}{m^2}n_l(\sum_{i=1}^c n_i) - \frac{\eta\alpha\epsilon\gamma}{m^2}n_l^2 + \frac{\eta(\alpha\epsilon\gamma + \beta)}{m}n_l - \frac{\eta\alpha\epsilon\gamma}{m^2}n_l^2\right) = \max_{1\le l\le c}\left(\frac{\eta(2\alpha\epsilon\gamma + \beta)}{m}n_l - \frac{2\eta\alpha\epsilon\gamma}{m^2}n_l^2\right) \le \eta\frac{(2\alpha\epsilon\gamma + \beta)^2}{8\alpha\epsilon\gamma} \tag{31}$$

$$\|\mathbf{Z}\|_\infty = \frac{\eta\alpha\epsilon(1-\gamma)}{m}\max_{1\le l\le c}\left(\frac{1}{m}n_l(\sum_{i=1}^c n_i) - \frac{1}{m}n_l^2 + n_l - \frac{1}{m}n_l^2\right) = \frac{2\eta\alpha\epsilon(1-\gamma)}{m}\max_{1\le l\le c}\left(n_l - \frac{1}{m}n_l^2\right) \le \frac{2\eta\alpha\epsilon(1-\gamma)}{m}\cdot\frac{m}{4} = \eta\frac{\alpha\epsilon(1-\gamma)}{2} \tag{32}$$

Then $\|\mathbf{Y}\|_\infty + \|\mathbf{Z}\|_\infty \le \eta\frac{(2\alpha\epsilon\gamma+\beta)^2}{8\alpha\epsilon\gamma} + \eta\frac{\alpha\epsilon(1-\gamma)}{2} = \eta(\frac{\beta}{2} + \frac{\alpha\epsilon\gamma}{2} + \frac{\beta^2}{8\alpha\epsilon\gamma}) < 1$. □

(2) Next, we demonstrate that $\|\mathbf{Y}\|_\infty + \|\mathbf{Z}\|_\infty < 1 \Rightarrow \|\mathbf{I}_n \otimes \mathbf{Y} + \tilde{\mathbf{S}} \otimes \mathbf{Z}\|_\infty < 1$;

Proof. Given the triangle inequality property of norms and Lemma E.6, we derive:

$$\|\mathbf{I}_n \otimes \mathbf{Y} + \tilde{\mathbf{S}} \otimes \mathbf{Z}\|_\infty \le \|\mathbf{I}_n \otimes \mathbf{Y}\|_\infty + \|\tilde{\mathbf{S}} \otimes \mathbf{Z}\|_\infty = \|\mathbf{I}_n\|_\infty\|\mathbf{Y}\|_\infty + \|\tilde{\mathbf{S}}\|_\infty\|\mathbf{Z}\|_\infty$$

Furthermore, $\|\mathbf{I}_n\|_\infty = 1$, and $\tilde{\mathbf{S}} = \mathrm{diag}(\mathbf{S}\mathbf{1}_{n\times 1})^{-1}\mathbf{S}$, then $\forall k, \sum_i |\tilde{S}_{ki}| = 1, \|\tilde{\mathbf{S}}\|_\infty = 1$. Consequently, we obtain $\|\mathbf{I}_n \otimes \mathbf{Y} + \tilde{\mathbf{S}} \otimes \mathbf{Z}\|_\infty \le \|\mathbf{Y}\|_\infty + \|\mathbf{Z}\|_\infty < 1$. □

(3) Finally, based on Lemma E.5, we have $\|\mathbf{I}_n \otimes \mathbf{Y} + \tilde{\mathbf{S}} \otimes \mathbf{Z}\|_\infty < 1 \Rightarrow (\mathbf{I}_{nc} - (\mathbf{I}_n \otimes \mathbf{Y} + \tilde{\mathbf{S}} \otimes \mathbf{Z}))^{-1} exists$.

Therefore, we prove that $\eta(\frac{\beta}{2} + \frac{\alpha\epsilon\gamma}{2} + \frac{\beta^2}{8\alpha\epsilon\gamma}) < 1$ is sufficient for $(\mathbf{I}_{nc} - (\mathbf{I}_n \otimes \mathbf{Y} + \tilde{\mathbf{S}} \otimes \mathbf{Z}))^{-1} exists$, i.e., the $\mathbf{U}(t)$ of Equation 8 converges to $\mathbf{U}^* = \mathrm{unvec}((\mathbf{I}_{nc} - (\mathbf{I}_n \otimes \mathbf{Y} + \tilde{\mathbf{S}} \otimes \mathbf{Z}))^{-1}\,\mathrm{vec}(\mathbf{X}))$. □

C Proof of COROLLARY 4.3

PROOF.

- Transient Homogenization

When $\epsilon = 0, \gamma = 1$, Equation 21 reduces to

$$\mathbf{u}_i(t+1) = (\mathbf{I}_c + \frac{\eta\beta}{m}\mathbf{V}\mathbf{V}^T)\mathbf{u}_i(t), \tag{33}$$

Let $\lambda = \frac{\eta\beta}{m}$, and applying Equation 27, we have

$$\begin{bmatrix} u_i^{(1)}(t+1) \\ u_i^{(2)}(t+1) \\ \vdots \\ u_i^{(c)}(t+1) \end{bmatrix} = \begin{bmatrix} 1+\lambda n_1 & 0 & \cdots & 0 \\ 0 & 1+\lambda n_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1+\lambda n_c \end{bmatrix}\begin{bmatrix} u_i^{(1)}(t) \\ u_i^{(2)}(t) \\ \vdots \\ u_i^{(c)}(t) \end{bmatrix} = \begin{bmatrix} (1+\lambda n_1)u_i^{(1)}(t) \\ (1+\lambda n_2)u_i^{(2)}(t) \\ \vdots \\ (1+\lambda n_c)u_i^{(c)}(t) \end{bmatrix} \tag{34}$$

If $\exists k$, s.t. $u_i^{(k)}(t)u_j^{(k)}(t) \ge \frac{\|\mathbf{u}_i(t)\|_2\|\mathbf{u}_j(t)\|_2}{\sqrt{1+(\frac{2n_k+\lambda n_k^2}{\sum_{l\neq k}2n_l+\lambda n_l^2})^2}}$, based on Lemma E.7, we have

$$\min_{l\neq k} u_i^{(l)}(t)u_j^{(l)}(t) \ge -\sqrt{\|\mathbf{u}_i(t)\|_2^2\|\mathbf{u}_j(t)\|_2^2 - (\frac{\|\mathbf{u}_i(t)\|_2\|\mathbf{u}_j(t)\|_2}{\sqrt{1+(\frac{2n_k+\lambda n_k^2}{\sum_{l\neq k}2n_l+\lambda n_l^2})^2}})^2} = -\frac{\|\mathbf{u}_i(t)\|_2\|\mathbf{u}_j(t)\|_2(2n_k+\lambda n_k^2)}{\sum_{l\neq k}(2n_l+\lambda n_l^2)\sqrt{1+(\frac{2n_k+\lambda n_k^2}{\sum_{l\neq k}2n_l+\lambda n_l^2})^2}} \tag{35}$$

Then

$$\mathbf{u}_i^T(t+1)\mathbf{u}_j(t+1) - \mathbf{u}_i^T(t)\mathbf{u}_j(t) = \sum_l(2\lambda n_l + \lambda^2 n_l^2)u_i^{(l)}u_j^{(l)}$$

$$= \lambda\left((2n_k+\lambda n_k^2)u_i^{(k)}u_j^{(k)} + \sum_{l\neq k}(2n_l+\lambda n_l^2)u_i^{(l)}u_j^{(l)}\right) \ge \lambda\left((2n_k+\lambda n_k^2)u_i^{(k)}u_j^{(k)} + \sum_{l\neq k}(2n_l+\lambda n_l^2)\min_{p\neq k}u_i^{(p)}u_j^{(p)}\right)$$

$$\ge \lambda\left((2n_k+\lambda n_k^2)\frac{\|\mathbf{u}_i(t)\|_2\|\mathbf{u}_j(t)\|_2}{\sqrt{1+(\frac{2n_k+\lambda n_k^2}{\sum_{l\neq k}2n_l+\lambda n_l^2})^2}} - \sum_{l\neq k}(2n_l+\lambda n_l^2)\frac{\|\mathbf{u}_i(t)\|_2\|\mathbf{u}_j(t)\|_2(2n_k+\lambda n_k^2)}{\sum_{l\neq k}(2n_l+\lambda n_l^2)\sqrt{1+(\frac{2n_k+\lambda n_k^2}{\sum_{l\neq k}2n_l+\lambda n_l^2})^2}}\right) = 0 \tag{36}$$

- Steady-state Homogenization

$$\text{Let } \mathbf{D} = \begin{bmatrix} 1+\lambda n_1 & 0 & \cdots & 0 \\ 0 & 1+\lambda n_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1+\lambda n_c \end{bmatrix}, d_i = D_{ii} = 1 + \lambda n_i$$

Proof by Mathematical Induction:

If $k = \underset{o\in\{1,2,\dots,c\}}{\mathrm{argmax}}\; n_o$, then $\forall\tau \in \mathbb{Z}^+$, we have

$$u_i^{(k)}(t+\tau-1)u_j^{(k)}(t+\tau-1) \ge \frac{\|\mathbf{u}_i(t+\tau-1)\|_2\|\mathbf{u}_j(t+\tau-1)\|_2}{\sqrt{1+\left(\frac{2n_k+\lambda n_k^2}{\sum_{l\neq k}2n_l+\lambda n_l^2}\right)^2}}. \tag{37}$$

Base Case: We have verified that the inequality holds when we let $\tau = 1$, i.e.,

$$u_i^{(k)}(t)u_j^{(k)}(t) \ge \frac{\|\mathbf{u}_i(t)\|_2\|\mathbf{u}_j(t)\|_2}{\sqrt{1+(\frac{2n_k+\lambda n_k^2}{\sum_{l\neq k}2n_l+\lambda n_l^2})^2}}, \tag{38}$$

and we also have $d_k = 1 + \lambda n_k = 1 + \lambda\max_o n_o = \max_o(1 + \lambda n_o) = \max_o d_o$.

Inductive Hypothesis: Assume that the inequality holds for $\tau = m \in \mathbb{Z}^+$, i.e.,

$$u_i^{(k)}(t+m-1)u_j^{(k)}(t+m-1) \ge \frac{\|\mathbf{u}_i(t+m-1)\|_2\|\mathbf{u}_j(t+m-1)\|_2}{\sqrt{1+(\frac{2n_k+\lambda n_k^2}{\sum_{l\neq k}2n_l+\lambda n_l^2})^2}}. \tag{39}$$

Inductive Step: When $\tau = m + 1$, based on Lemma E.8, we have

$$\frac{u_i^{(k)}(t+m)u_j^{(k)}(t+m)}{\|\mathbf{u}_i(t+m)\|_2\|\mathbf{u}_j(t+m)\|_2} = \frac{d_k^2u_i^{(k)}(t+m-1)u_j^{(k)}(t+m-1)}{\|\mathbf{D}\mathbf{u}_i(t+m-1)\|_2\|\mathbf{D}\mathbf{u}_j(t+m)\|_2} \ge \frac{d_k^2u_i^{(k)}(t+m-1)u_j^{(k)}(t+m-1)}{(\max_i d_i)^2\|\mathbf{u}_i(t+m-1)\|_2\|\mathbf{u}_j(t+m-1)\|_2}$$

$$\ge \frac{d_k^2u_i^{(k)}(t+m-1)u_j^{(k)}(t+m-1)}{d_k^2\|\mathbf{u}_i(t+m-1)\|_2\|\mathbf{u}_j(t+m-1)\|_2} = \frac{u_i^{(k)}(t+m-1)u_j^{(k)}(t+m-1)}{\|\mathbf{u}_i(t+m-1)\|_2\|\mathbf{u}_j(t+m-1)\|_2} \ge \frac{1}{\sqrt{1+(\frac{2n_k+\lambda n_k^2}{\sum_{l\neq k}2n_l+\lambda n_l^2})^2}} \tag{40}$$

$$\text{i.e. } u_i^{(k)}(t+m)u_j^{(k)}(t+m) \ge \frac{\|\mathbf{u}_i(t+m)\|_2\|\mathbf{u}_j(t+m)\|_2}{\sqrt{1+(\frac{2n_k+\lambda n_k^2}{\sum_{l\neq k}2n_l+\lambda n_l^2})^2}}.$$

After leveraging the inductive hypothesis, we show the inequality continues to hold when $\tau = m + 1$.

Conclusion:

By the principle of mathematical induction, $\forall\tau \in \mathbb{Z}^+$, we have $u_i^{(k)}(t+\tau-1)u_j^{(k)}(t+\tau-1) \ge \frac{\|\mathbf{u}_i(t+\tau-1)\|_2\|\mathbf{u}_j(t+\tau-1)\|_2}{\sqrt{1+(\frac{2n_k+\lambda n_k^2}{\sum_{l\neq k}2n_l+\lambda n_l^2})^2}}$, so we also have $\forall\tau \in \mathbb{Z}^+, \mathbf{u}_i^T(t+\tau)\mathbf{u}_j(t+\tau) \ge \mathbf{u}_i^T(t+\tau-1)\mathbf{u}_j(t+\tau-1)$.

□

D Proof of COROLLARY 4.4

PROOF.

From Equation 34: $\forall o, i, u_i^{(o)}(t+1) = (1 + \lambda n_o)u_i^{(o)}(t)$, and this is equivalent to $\Delta t = 1$, $\frac{u_i^{(o)}(t+\Delta t) - u_i^{(o)}(t)}{\Delta t} = \lambda n_o u_i^{(o)}(t)$, when $\Delta t \to 0$, we similarly obtain

$$\frac{\mathrm{d}u_i^{(o)}(t)}{\mathrm{d}t} = \frac{u_i^{(o)}(t+\Delta t) - u_i^{(o)}(t)}{\Delta t} = \lambda n_o u_i^{(o)}(t) \tag{41}$$

Then

$$p_{i,o}^t := \frac{s_i^o(t)}{\sum_{k=1}^c s_i^k(t)} = P(user\ i\ receives\ an\ item\ from\ Category\ o) = \frac{n_o e^{\alpha u_i^{(o)}(t)}}{\sum_k n_k e^{\alpha u_i^{(k)}(t)}} \tag{42}$$

The time derivative of recommended categories entropy for user i at time t is given by

$$\frac{\mathrm{d}H_i(t)}{\mathrm{d}t} = -\sum_{o=1}^c(\frac{\mathrm{d}p_{i,o}^t}{\mathrm{d}t}\ln p_{i,o}^t + p_{i,o}^t\frac{\mathrm{d}\ln p_{i,o}^t}{\mathrm{d}t}) = -\sum_{o=1}^c\ln p_{i,o}^t\frac{\mathrm{d}p_{i,o}^t}{\mathrm{d}t} - \sum_{o=1}^c\frac{\mathrm{d}p_{i,o}^t}{\mathrm{d}t} = -\sum_{o=1}^c\ln p_{i,o}^t\frac{\mathrm{d}p_{i,o}^t}{\mathrm{d}t} - \frac{\mathrm{d}\sum_{o=1}^c p_{i,o}^t}{\mathrm{d}t} = -\sum_{o=1}^c\ln p_{i,o}^t\frac{\mathrm{d}p_{i,o}^t}{\mathrm{d}t} \tag{43}$$

And

$$\frac{\mathrm{d}p_{i,o}^t}{\mathrm{d}t} = \frac{\frac{\mathrm{d}(n_o e^{\alpha u_i^{(o)}(t)})}{\mathrm{d}t}(\sum_k n_k e^{\alpha u_i^{(k)}(t)}) - \frac{\mathrm{d}(\sum_k n_k e^{\alpha u_i^{(k)}(t)})}{\mathrm{d}t}(n_o e^{\alpha u_i^{(o)}(t)})}{(\sum_k n_k e^{\alpha u_i^{(k)}(t)})^2} = \frac{(n_o e^{\alpha u_i^{(o)}(t)}\alpha\lambda n_o u_i^{(o)}(t))(\sum_k n_k e^{\alpha u_i^{(k)}(t)})}{(\sum_k n_k e^{\alpha u_i^{(k)}(t)})^2}$$

$$-\frac{(\sum_k n_k e^{\alpha u_i^{(k)}(t)}\alpha\lambda n_k u_i^{(k)}(t))(n_o e^{\alpha u_i^{(o)}(t)})}{(\sum_k n_k e^{\alpha u_i^{(k)}(t)})^2} = \lambda\alpha\frac{n_o e^{\alpha u_i^{(o)}(t)}}{\sum_k n_k e^{\alpha u_i^{(k)}(t)}}\left(n_o u^{(o)}(t) - \sum_k n_k u_i^{(k)}(t)\frac{n_k e^{\alpha u_i^{(k)}(t)}}{\sum_k n_k e^{\alpha u_i^{(k)}(t)}}\right)$$

$$= \lambda\alpha p_{i,o}^t(n_o u_i^{(o)}(t) - \sum_k n_k u_i^{(k)}(t)p_{i,k}^t) \tag{44}$$

Then Equation 43 equals:

$$\frac{\mathrm{d}H_i(t)}{\mathrm{d}t} = -\sum_{o=1}^c\ln p_{i,o}^t\lambda\alpha p_{i,o}^t(n_o u_i^{(o)}(t) - \sum_{k=1}^c n_k u_i^{(k)}(t)p_{i,k}^t) = -\lambda\alpha\left(\sum_{o=1}^c(n_o u_i^{(o)}(t)\ln p_{i,o}^t)p_{i,o}^t - \sum_{k=1}^c(n_k u_i^{(k)}(t))p_{i,k}^t)(\sum_{o=1}^c(\ln p_{i,o}^t)p_{i,o}^t\right) \tag{45}$$

Let X be a random variable with possible values $\{n_1u_i^{(1)}(t), n_2u_i^{(2)}(t), \dots, n_cu_i^{(c)}(t)\}$, let Y be a random variable with possible values $\{\ln p_{i,1}^t, \ln p_{i,2}^t, \dots, \ln p_{i,c}^t\}$, and $P(X = n_ou_i^{(o)}(t)) = P(Y = \ln p_{i,o}^t) = p_{i,o}^t$. Then Equation 45 equals:

$$\frac{\mathrm{d}H_i(t)}{\mathrm{d}t} = -\lambda\alpha(\mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]) = -\lambda\alpha\mathrm{Cov}(X, Y) \tag{46}$$

Because $p_{i,o}^t = \frac{n_o e^{\alpha u_i^{(o)}(t)}}{\sum_k n_k e^{\alpha u_i^{(k)}(t)}}$, an increase in $u_i^{(o)}(t)$ results in increases in both $n_ou_i^{(o)}(t)$ and $\ln p_{i,o}^t$, indicating a positive correlation between X and Y, which implies $\mathrm{Cov}(X, Y) > 0$ and $\frac{\mathrm{d}H_i(t)}{\mathrm{d}t} < 0$. □

E Supporting Lemmas

Lemma E.1.

$$\mathbb{E}\left[\mathbf{1}_i^t(j)\right] \approx hp^t(\mathbf{s}_i^\gamma, \mathbf{v}_j; \alpha) \tag{47}$$

Proof.
First we define the selection indicator:

$$\mathbf{1}_{i,k}^t(j) = \begin{cases} 1, & \text{if item } j \text{ is selected at the } k\text{-th draw without replacement from } \mathbf{p}_i^t(\alpha) \\ 0, & \text{otherwise} \end{cases} \tag{48}$$

For simplicity, we denote $p^t(\mathbf{s}_i^\gamma, \mathbf{v}_j; \alpha)$ as p_j, because of $h \ll m, p_j \to 0$, we have:

$$\mathbb{E}\left[\mathbf{1}_i^t(j)\right] = \sum_{k=1}^{h} \mathbb{E}\left[\mathbf{1}_{i,k}^t(j)\right] \approx \sum_{k=1}^{h} p_j = hp_j \tag{49}$$

Then we compute the error:

$$\delta = \sum_{k=1}^{h} p_j - \sum_{k=1}^{h} \mathbb{E}\left[\mathbf{1}_{i,k}^t(j)\right] \approx hp_j - (p_j + (h-1)\sum_{l \neq j} p_l p_j) = hp_j - (p_j + (h-1)(1-p_j)p_j) = (h-1)p_j^2 = (h-1)(p^t(\mathbf{s}_i^\gamma, \mathbf{v}_j; \alpha))^2 \tag{50}$$

□

Lemma E.2.

$$\mathrm{vec}(\mathbf{AUB}^T) = (\mathbf{B} \otimes \mathbf{A})\,\mathrm{vec}(\mathbf{U}) \tag{51}$$

where $\mathbf{A} \in \mathbb{R}^{c \times c}, \mathbf{U} \in \mathbb{R}^{c \times n}, \mathbf{B} \in \mathbb{R}^{n \times n}$.

Proof.
$\forall i, j, (\mathbf{AUB}^T)_{ij} = \sum_{k=1}^{c} \sum_{l=1}^{n} A_{ik} U_{kl} B_{lj}^T$ is equal to the $(j-1)c + i$-th row element of matrix $\mathrm{vec}(\mathbf{AUB}^T)$.

Furthermore,

$$\mathbf{B} \otimes \mathbf{A} = \begin{bmatrix} B_{11}\mathbf{A} & \cdots & B_{1n}\mathbf{A} \\ \vdots & \ddots & \vdots \\ B_{n1}\mathbf{A} & \cdots & B_{nn}\mathbf{A} \end{bmatrix} \in \mathbb{R}^{cn \times cn}, \tag{52}$$

the $(j-1)c + i$-th row element of matrix $(\mathbf{B} \otimes \mathbf{A})\,\mathrm{vec}(\mathbf{U})$ is equal to $\sum_{k=1}^{c} \sum_{l=1}^{n} A_{ik} U_{kl} B_{lj}^T$. Therefore, all corresponding elements of $\mathrm{vec}(\mathbf{AUB}^T)$ and $(\mathbf{B} \otimes \mathbf{A})\,\mathrm{vec}(\mathbf{U})$ are equal, i.e., $\mathrm{vec}(\mathbf{AUB}^T) = (\mathbf{B} \otimes \mathbf{A})\,\mathrm{vec}(\mathbf{U})$. □

Lemma E.3.

$$||\mathbf{AB}||_\infty \leq ||\mathbf{A}||_\infty ||\mathbf{B}||_\infty \tag{53}$$

Proof.
For a matrix $\mathbf{A} \in \mathbb{R}_{m \times n}$, the infinity norm is defined as $||\mathbf{A}||_\infty = \max_{1 \leq i \leq m} \sum_{j=1}^{n} |A_{ij}|$. Since $\sum_i |B_{ji}| \leq \max_j \sum_i |B_{ji}| = ||\mathbf{B}||_\infty$, $\sum_j |A_{kj}| \leq \max_k \sum_j |A_{kj}| = ||\mathbf{A}||_\infty$.

$$\text{Then } \forall k, \sum_i |(\mathbf{AB})_{ki}| = \sum_i |\sum_j A_{kj} B_{ji}| \leq \sum_i \sum_j |A_{kj}||B_{ji}| = \sum_j |A_{kj}| \sum_i |B_{ji}| \leq \sum_j |A_{kj}| ||\mathbf{B}||_\infty \leq ||\mathbf{A}||_\infty ||\mathbf{B}||_\infty \tag{54}$$

Therefore $||\mathbf{AB}||_\infty = \max_k \sum_i |(\mathbf{AB})_{ki}| \leq ||\mathbf{A}||_\infty ||\mathbf{B}||_\infty$.

□

Lemma E.4.

$$\rho(\mathbf{A}) \leq ||\mathbf{A}||_\infty \tag{55}$$

where $\rho(\mathbf{A})$ *denotes the spectral radius of matrix* $\mathbf{A}$.

Proof.
Let λ be an eigenvalue of matrix $\mathbf{A}$, and let $\mathbf{x}$ be a corresponding eigenvector such that $\mathbf{Ax} = \lambda\mathbf{x}$ and $\mathbf{x} \neq 0$. Then $||\mathbf{Ax}||_\infty = ||\lambda\mathbf{x}||_\infty = |\lambda|||\mathbf{x}||_\infty$.
Since $||\mathbf{Ax}||_\infty \leq ||\mathbf{A}||_\infty ||\mathbf{x}||_\infty$ based on Lemma E.3, we get $|\lambda|||\mathbf{x}||_\infty \leq ||\mathbf{A}||_\infty ||\mathbf{x}||_\infty$.
And $||\mathbf{x}||_\infty > 0$ with $\mathbf{x} \neq 0$, we have $|\lambda| \leq ||\mathbf{A}||_\infty$, this inequality holds for all eigenvalues λ of matrix $\mathbf{A}$. Therefore,

$$\rho(\mathbf{A}) = \max\{|\lambda| | \lambda \in \text{eigenvalues of } \mathbf{A}\} \leq ||\mathbf{A}||_\infty. \tag{56}$$

□

Lemma E.5.
A sufficient condition for the invertibility of $\mathbf{I} - \mathbf{A}$ *is that the infinity norm of* $\mathbf{A}$ *is strictly less than* 1, *i.e.,*

$$||\mathbf{A}||_\infty < 1 \Rightarrow (\mathbf{I} - \mathbf{A})^{-1} \textit{ exists.} \tag{57}$$

Proof.
Based on Lemma E.4, we get $\rho(\mathbf{A}) \leq ||\mathbf{A}||_\infty$. Given that $||\mathbf{A}||_\infty < 1$, it follows that $\rho(\mathbf{A}) < 1$.
This implies that all eigenvalues λ of $\mathbf{A}$ satisfy $|\lambda| < 1$, i.e., all eigenvalues of $\mathbf{I} - \mathbf{A}$ satisfy $\mu = 1 - \lambda > 0$.
Hence, $\mathbf{I} - \mathbf{A}$ has no zero eigenvalues, which implies that $\det(\mathbf{I} - \mathbf{A}) \neq 0$.
Therefore, $\mathbf{I} - \mathbf{A}$ is invertible. □

Lemma E.6.

$$||\mathbf{A} \otimes \mathbf{B}||_\infty = ||\mathbf{A}||_\infty ||\mathbf{B}||_\infty \tag{58}$$

Proof.
Since

$$||\mathbf{A}||_\infty = \max_l \sum_i |A_{li}|, ||\mathbf{B}||_\infty = \max_k \sum_r |A_{kr}|. \tag{59}$$

Then

$$||\mathbf{A} \otimes \mathbf{B}||_\infty = \max_{l,k} \sum_i \sum_r |A_{li} B_{kr}| = \max_l \sum_i |A_{li}| \cdot \max_k \sum_r |A_{kr}| = ||\mathbf{A}||_\infty ||\mathbf{B}||_\infty \tag{60}$$

□

Lemma E.7.

$$\textit{Let } \mathbf{a} = [a^{(1)}, a^{(2)}, \ldots, a^{(n)}]^T \in \mathbb{R}^n,\ \mathbf{b} = [b^{(1)}, b^{(2)}, \ldots, b^{(n)}]^T \in \mathbb{R}^n.$$

If there exists an index $k \in \{1, \ldots, n\}$ *such that* $a^{(k)} b^{(k)} \geq x > 0$, *then a lower bound for the remaining products is given by*

$$\min_{i \neq k} a^{(i)} b^{(i)} \geq -\sqrt{||\mathbf{a}||_2^2 ||\mathbf{b}||_2^2 - x^2}. \tag{61}$$

Proof.
First

$$\sum_i (a^{(i)} b^{(i)})^2 = \sum_i (a^{(i)})^2 (b^{(i)})^2 \leq \sum_i (a^{(i)})^2 \sum_i (b^{(i)})^2 = ||\mathbf{a}||_2^2 ||\mathbf{b}||_2^2 \tag{62}$$

Given $a^{(k)} b^{(k)} \geq x > 0$,

$$\sum_{i \neq k} (a^{(i)} b^{(i)})^2 \leq ||\mathbf{a}||_2^2 ||\mathbf{b}||_2^2 - x^2. \tag{63}$$

This implies that for all $i \neq k$,

$$|a^{(i)} b^{(i)}| \leq \sqrt{||\mathbf{a}||_2^2 ||\mathbf{b}||_2^2 - x^2}, \tag{64}$$

which leads to:

$$\min_{i \neq k} a^{(i)} b^{(i)} \geq -\sqrt{||\mathbf{a}||_2^2 ||\mathbf{b}||_2^2 - x^2}. \tag{65}$$

□

Lemma E.8.
Let $\mathbf{D} = \mathrm{diag}(d_1, d_2, \ldots, d_n)$ *be a diagonal matrix with strictly positive entries, i.e.,* $d_i > 0$ *for all* $i \in \{1, 2, \ldots, n\}$. *Then, for any vector* $\mathbf{b} \in \mathbb{R}^n$, *the following inequality holds:*

$$||\mathbf{Db}||_2 \leq \left(\max_i d_i\right) \cdot ||\mathbf{b}||_2. \tag{66}$$

Proof.
Let $\mathbf{b} = [b_1, b_2, \ldots, b_n]^T$. Then,

$$\mathbf{Db} = [d_1 b_1, d_2 b_2, \ldots, d_n b_n]^T, \tag{67}$$

and

$$||\mathbf{Db}||_2 = \sqrt{\sum_{i=j}^{n} (d_j b_j)^2} \leq \sqrt{\sum_{j=1}^{n} (\max_i d_i)^2 (b_j)^2} = (\max_i d_i) \sqrt{\sum_{j=1}^{n} (b_j)^2} = (\max_i d_i) ||\mathbf{b}||_2 \tag{68}$$

□

F Additional Experiments on Parameter Analysis

We perform the same analysis of the four factors on the Epinions and synthetic datasets, and the results show a high degree of consistency with those on the Ciao dataset.

F.1 Analysis of α, β, γ and ϵ on Epinions Dataset

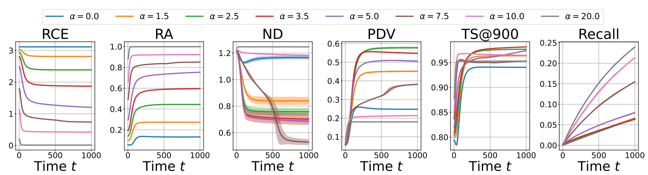

Figure 13: Metric trends over time under varying α on Epinions dataset.

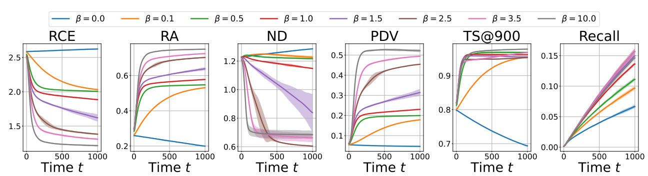

Figure 14: Metric trends over time under varying β on Epinions dataset.

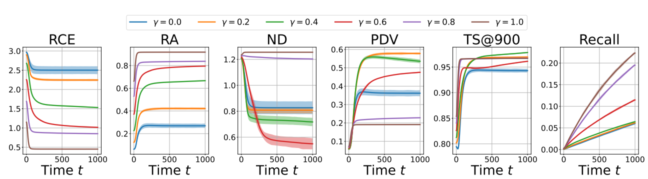

Figure 15: Metric trends over time under varying γ on Epinions dataset.

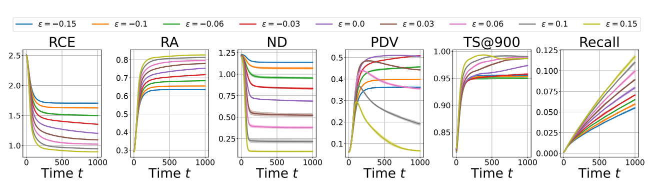

Figure 16: Metric trends over time under varying ϵ on Epinions dataset.

F.2 Analysis of α, β, γ and ϵ on Synthetic Dataset

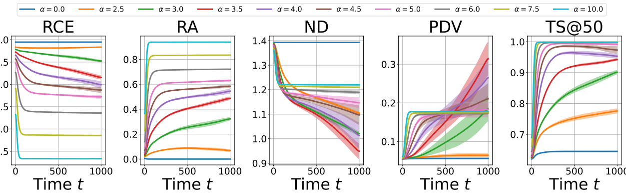

Figure 17: Metric trends over time under varying α on Synthetic dataset.

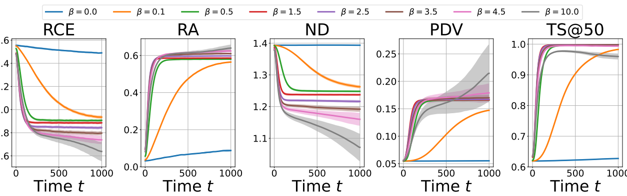

Figure 18: Metric trends over time under varying β on Synthetic dataset.

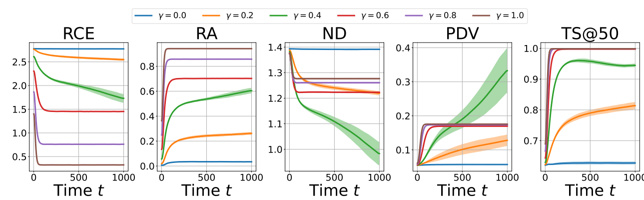

Figure 19: Metric trends over time under varying γ on Synthetic dataset.

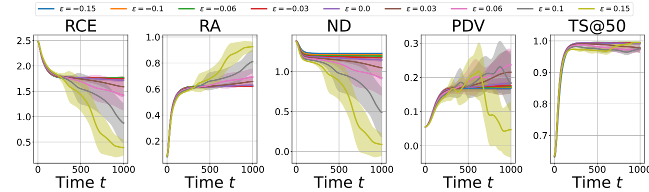

Figure 20: Metric trends over time under varying ϵ on Synthetic dataset.

Table 4: Parameter sensitivity analysis of four strategies on Ciao dataset.

Ciao		RCE↑	RA↑	ND↑	PDV↓	TS@300↓
UAα	$\alpha = 1$	$1.21_{\pm 0.004}$	$0.74_{\pm 0.001}$	$0.90_{\pm 0.005}$	$0.30_{\pm 0.002}$	$0.96_{\pm 0.001}$
	$\alpha = 5$	$1.24_{\pm 0.003}$	$0.73_{\pm 0.001}$	$0.94_{\pm 0.001}$	$0.29_{\pm 0.002}$	$0.96_{\pm 0.001}$
	$\alpha = 10$	$1.28_{\pm 0.000}$	$0.72_{\pm 0.000}$	$0.95_{\pm 0.002}$	$0.28_{\pm 0.002}$	$0.96_{\pm 0.001}$
	$\alpha = 20$	$1.32_{\pm 0.004}$	$0.71_{\pm 0.001}$	$0.98_{\pm 0.001}$	$0.27_{\pm 0.000}$	$0.95_{\pm 0.000}$
FUA	$\rho = -0.01$	$1.16_{\pm 0.004}$	$0.75_{\pm 0.001}$	$0.86_{\pm 0.004}$	$0.32_{\pm 0.003}$	$0.96_{\pm 0.001}$
	$\rho = 0.02$	$1.27_{\pm 0.009}$	$0.73_{\pm 0.002}$	$0.92_{\pm 0.007}$	$0.29_{\pm 0.005}$	$0.96_{\pm 0.000}$
	$\rho = 0.05$	$1.36_{\pm 0.005}$	$0.70_{\pm 0.001}$	$0.96_{\pm 0.003}$	$0.27_{\pm 0.002}$	$0.95_{\pm 0.000}$
	$\rho = 0.08$	$1.44_{\pm 0.004}$	$0.68_{\pm 0.001}$	$1.00_{\pm 0.001}$	$0.26_{\pm 0.002}$	$0.95_{\pm 0.001}$
DPP	$\theta = 0.500$	$1.13_{\pm 0.001}$	$0.78_{\pm 0.000}$	$0.89_{\pm 0.006}$	$0.30_{\pm 0.003}$	$0.96_{\pm 0.000}$
	$\theta = 0.501$	$1.38_{\pm 0.001}$	$0.74_{\pm 0.000}$	$0.97_{\pm 0.004}$	$0.27_{\pm 0.002}$	$0.95_{\pm 0.000}$
	$\theta = 0.502$	$1.61_{\pm 0.001}$	$0.70_{\pm 0.000}$	$1.04_{\pm 0.003}$	$0.24_{\pm 0.003}$	$0.94_{\pm 0.001}$
	$\theta = 0.503$	$1.80_{\pm 0.001}$	$0.66_{\pm 0.000}$	$1.09_{\pm 0.002}$	$0.22_{\pm 0.001}$	$0.94_{\pm 0.001}$
SAR	$\omega = 100$	$1.22_{\pm 0.005}$	$0.72_{\pm 0.001}$	$0.85_{\pm 0.016}$	$0.31_{\pm 0.004}$	$0.96_{\pm 0.002}$
	$\omega = 1000$	$1.26_{\pm 0.009}$	$0.68_{\pm 0.003}$	$0.94_{\pm 0.008}$	$0.28_{\pm 0.004}$	$0.95_{\pm 0.001}$
	$\omega = 2000$	$1.24_{\pm 0.003}$	$0.69_{\pm 0.001}$	$0.90_{\pm 0.019}$	$0.29_{\pm 0.005}$	$0.95_{\pm 0.000}$
	$\omega = 5000$	$1.25_{\pm 0.004}$	$0.68_{\pm 0.001}$	$0.92_{\pm 0.013}$	$0.28_{\pm 0.004}$	$0.95_{\pm 0.001}$

G Additional Experiments with Mitigation Strategies on Ciao Dataset

Through further parameter tuning of the four mitigation strategies on the Ciao dataset in Table 4, we observe that sacrificing a certain degree of accuracy can lead to more effective alleviation of echo chambers and user homogenization.